MATH 487 Deterministic Operations Research

GoldenOrbWeaver

Spring 2026

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1 Linear Programming

Definition 1.1. **Linear programming**: The optimization of a linear function subject to linear constraints.

Example. Suppose a starving artist is trying to plan a food budget. He is health conscious and wants a healthy diet that includes the following: at least 70 g of protein per day, at least 100 g of carbohydrates per day, exactly 15 mg of vitamin D per day, but no more than 75 g of fat per day.

Five foods to choose from (fix formatting later):

Food	Protein	Carbohydrates	Vitamin D	Fat	Cost
Hamburger	10g/oz	2g/oz	.5mg/oz	8g/oz	\$0.20/oz
Milk	2g/oz	3 g/oz	4mg/oz	2g/oz	\$0.02/oz
Cereal	3g/oz	23g/oz	2mg/oz	1g/oz	\$0.10/oz
Ch. N S	2g/oz	2g/oz	0 mg/oz	0.5g/oz	\$0.03/oz
Eggs	6g/egg	4g/egg	1mg/egg	5g/egg	\$0.10/egg

Question: How can he meet dietary goals while minimizing cost?

Answer. Set up **decision variables**:
H, M, C, CNS, and E are oz (or number) per day

Constraints:
Protein: $p = 10H + 2M + 3C + 2CNS + 6E \geq 70$
Carbs: $c = 2H + 3M + 23C + 2CNS + 4E \geq 100$
Vitamin D: $0.5H + 4M + 2C + E = 15$
Fat: $f = 8H + 2M + 1C + 0.5CNS + 5E \leq 75$
Nonnegativity: $H, M, C, CNS, E \geq 0$

Subject to these requirements, we wish to minimize cost:

$$cost = 20H + 2M + 10C + 3CNS + 10E$$

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Definition 1.2. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function of n variables, then f is called linear $\iff$ f is of the form

$$f(x_1, x_2, \dots x_n) = a_1x_1 + a_2x_2 + \cdots + a_nx_n + b_0$$

for some constraints

$$a_1, a_2, \dots a_n \text{ and } b_0$$

Definition 1.3. A **linear equation** is an equation of the form $f(x_1, \dots, x_n) = a$ constant.

Definition 1.4. A **linear inequality** is an inequality of the form $f(x_1, \dots, x_n) \leq$ a constant, or $f(x_1, \dots, x_n) \geq$ a constant.

Definition 1.5. A **linear constraint** is either a linear equation or a linear inequality.

Definition 1.6. A **linear program** is the optimization of a linear function subject to linear constraints.

Example. The Furniture Problem

Suppose you are in charge of a furniture factory. Your plant makes tables and chairs out of iron, wood, and labor.

Product	Iron (lbs)	Wood (ft)	Labor (hrs)	Profit ($)
Table	1	20	16	80
Chair	2	15	5	40

Suppose that your plant has access to 100 lbs of iron/day, 1000 lbs of wood/day, and it has 80 employees and thus 640 labor hours/day. What should their production plan be?

Answer. First, we need to decide on the decision variables. These should have two properties:

1. The direction manager must have control over them
2. Designation of optimal values solves the problem

We select two, T and C, the number of tables and chairs produced per day respectively.

Next, we need to select our objective function. Since we wish to maximize profit, our objective function is:

$$profit = \Pi = 80T + 40C$$

We also need to figure out constraints:

$$Iron : T + 2C \leq 100 Wood : 20T + 15C \leq 1000 Labor : 16T + 5C \leq 640 Nonnegativity : T, C \geq 0$$

We have a linear program:

$$max_{T,C} 80T + 40C s.t.$$

⊛

Remark. When there are two decision variables, we can graphically solve a linear program.

ADD DRAWING later

Definition 1.7. The **feasible region** of a linear program is the set of all points that satisfy all constraints.

Geometrically, we wish to find the highest isoprofit that intersects the feasible region, grazing the side of it. This will occur at a vertex (unless a constraint line is parallel to the isoprofit line). We can check all the vertices or we can analyze the slopes of the constraints and find the vertex of constraints with slopes above and below the slope of the isoprofits.

Definition 1.8. An **integer program** is a linear program where all of the decision variables must have integer values.

Example. Blending Model A scrap metal operator reviews an order for 24 lbs of tin, 15 lbs of copper, and 20 lbs of aluminum. She can buy two types of scrap metal which she can melt down:

Type	Tin	Copper	Aluminum	Cost ($0.01/lb)
Metal 1	40%	50%	10%	20
Metal 2	40%	10%	50%	10

Only 50 lbs of Metal 1 are available.

How can she meet the order most effectively?

Answer. Decision variables:
M_1 = Amount of metal 1 to buy (lbs)
M_2 = Amount of metal 2 to buy (lbs)
Linear program:
min $20M_1 + 10M_2 = cost$

s.t. $0.4M_1 + 0.4M_2 \geq 24$ (tin)
$0.5M_1 + 0.1M_2 \geq 15$ (copper)
$0.1M_1 + 0.5M_2 \geq 20$ (aluminum)
$M_1 \leq 50$ (availability)
Since there are only 2 decision variables, we can solve this graphically: ⊛

Example. Transportation Problems

Goods are located at sources and needed to be shipped to destinations. There is a per unit cost to ship from any particular source to an particular destination. The objective is to minimize the cost...

Suppose that the Frank Perdue Chicken Co. has 2000 tons of chickens on inventory, 500 of which are on a farm near San Francisco, 500 on a farm near Houston, and 1000 on a farm near Detroit. They wish to ship the chicken to four superstores located in New York, Los Angeles, Kansas City, and Miami. Demand is NYC 300 tons, LA 900 tons, KC 600 tons, and Mia 200 tons. The shipping costs per ton are:

From/To	NY	LA	KC	Mia
SF	80	10	65	80
Hou	30	50	20	20
Det	30	100	50	50

Define $x_{i,j}$ = the tonnage of chicken shipped from i to j
Linear program:

$$min_x 80x_{11} + 10x_{12} + 65x_{13} + 80x_{14} + 30x_{21} + 50x_{12} + 20x_{23} + 20x_{24} + 30x_{31} + 100x_{32} + 50x_{33} + 50x_{34}$$

s.t. $x_{11} + x_{12} + x_{13} + x_{14} \leq 500$
$x_{21} + x_{22} + x_{23} + x_{24} \leq 500$
$x_{31} + x_{32} + x_{33} + x_{34} \leq 1000$
$x_{11} + x_{21} + x_{31} \geq 300$
$x_{12+x_{22}+x_{32}\geq 900}$
$x_{13} + x_{23} + x_{33} \geq 600$
$x_{14} + x_{24} + x_{34} \geq 200$
$x_{i,j} \geq 0$

Remark. Note the specific special structure of the constraint matrix. This allows for specialized algorithms to solve transportation problems.

Remark. In this particular problem, the sum of the supplies at sources equals the sum of the demands at destinations. This implies that for any feasible solution, all the constraints hold with equality. In general transportation problems, the total supply at sources is greater than or equal to total demand at sinks.

Remark. In general, the transportation problem has the form:
$max_x \sum_{i=1}^{I} \sum_{j=1}^{J} C_{ij} X_{ij}$
s.t. $\sum_{i=1}^{I} X_{ij} \leq S_i$ for $i = 1, \ldots, I$
$\sum_{j=1}^{J} X_{ij} \geq D_j$ for $j = 1, \ldots, J$
$X_{ij} \geq 0$ for all i, j
where C_{ij} = cost per unit shipped from source i to destination j
I = number of sources
J = number of destinations
S_i = on hand at source i
d_j = demand at destination j