MATH 487 Deterministic Operations Research

GoldenOrbWeaver

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Contents

1 Linear Programming

Definition 1.1. Linear programming: The optimization of a linear function subject to linear constraints.

Example. Suppose a starving artist is trying to plan a food budget. He is health conscious and wants a healthy diet that includes the following: at least 70 g of protein per day, at least 100 g of carbohydrates per day, exactly 15 mg of vitamin D per day, but no more than 75 g of fat per day.

Five foods to choose from (fix formatting later):
Hamburger: 10g protein/oz, 2g carb/oz, .5mg vit D/oz, 8g fat/oz, \$0.20/oz
Milk: 2g protein/oz, 3 g carb/oz, 4mg vit D/oz, 2g fat/oz, \$0.02/oz
Cereal: 3g protein/oz, 23g carb/oz, 2mg vit D/oz, 1g fat/oz, \$0.10/oz
Ch. N S: 2g protein/oz, 2g carb/oz, 0 vit D/oz, 0.5g fat/oz, \$0.03/oz
Eggs: 6g protein/egg, 4g carb/egg, 1mg vit D/egg, 5g fat/egg, \$0.10/egg

Question: How can he meet dietary goals while minimizing cost?

Set up variables:
H, M, C, CNS, and E are oz (or number) per day, called **decision variables**

Constraints:
Protein: $p = 10H + 2M + 3C + 2CNS + 6E \geq 70$
Carbs: $c = 2H + 3M + 23C + 2CNS + 4E \geq 100$
Vitamin D: $0.5H + 4M + 2C + E = 15$
Fat: $f = 8H + 2M + 1C + 0.5CNS + 5E \leq 75$
Nonnegativity: $H, M, C, CNS, E \geq 0$

Subject to these requirements, we wish to minimize cost:

$$cost = 20H + 2M + 10C + 3CNS + 10E$$

Definition 1.2. Let $f : R^n \to R$ be a function of n variables, then f is called linear $\iff$ f is of the form

$$f(x_1, x_2, \ldots x_n) = a_1x_1 + a_2x_2 + \cdots + a_nx_n + b_0$$

for some constraints

$$a_1, a_2, \ldots a_n \text{and} b_0$$

Definition 1.3. A linear equation is an equation of the form $f(x_1, \ldots, x_n) = a$ constant.

Definition 1.4. A linear inequality is an inequality of the form $f(x_1, \ldots, x_n) \leq$ a constant, or $f(x_1, \ldots, x_n) \geq$ a constant.

Definition 1.5. A linear constraint is either a linear equation or a linear inequality.

Definition 1.6. A linear program is the optimization of a linear function subject to linear constraints.