MATH 420 Mathematical Modeling

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Contents

1 One Variable Optimization

Approach to mathematical modeling:

1. Ask the question
2. Select the modeling approach
3. Formulate the model
4. Solve the problem
5. Answer the question

Example. A pig weighing 200 pounds gains 5 pounds per day and costs 45 cents a day to keep. The market price for pigs is 65 cents per pound, but is falling 1 center per day. When should the pig be sold?

Answer. Start at $t = 0$, with t in days. Then we find each component:
Weight of pig:

$$w(t) = 200 + 5t$$

Cost of pig:

$$c(t) = 0.45t$$

Price per pound to sell pig:

$$p(t) = 0.65 - 0.01t$$

Revenue from selling pig:

$$r(t) = w(t)p(t)$$

Profit from selling at time t:

$$p(t) = r(t) - c(t)$$

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However, we aren't sure that our values are accurate. The amount the price falls per day could be different, so we can instead set it to a function $r(t)$ then:

$$p(t) = 0.65 - rt$$

To find the optimal time to sell, solve $p'(t) = 0$ to get t_{max}. We are interested in how sensitive t_{max} is on changes in r, which we call $s(t, r)$:

$$s(t, v) = \lim \frac{\frac{\Delta t}{t}}{\frac{\Delta r}{r}} = \frac{r}{t} * \frac{dt}{dr}$$

We begin to solve:

$$p'(t) = r'(t) - c'(t) = w'(t)p'(t) = w(t)p'(t) - 0.45 = 5(0.65 - rt) + (200 + 5t)(-r) - 0.45 = -10rt - 200r + 2.80 = 0$$

Then, we solve for t_{max}:

$$t_{max} = \frac{2.80 - 200r}{10r} = 0.28r^{-1} - 20$$

Next, we find $\frac{dt_{max}}{dr}$:

$$\frac{dt_{max}}{dr} = -0.28r^{-2}$$

Therefore:

$$S(t, r) = \frac{-r}{t_{max}} * \frac{0.28}{r^2} = \frac{-0.28}{t_{max}r}$$

Substituting t_{max}:

$$S(t,r) = \frac{-0.28}{(0.28r^{-1} - 20)r} = \frac{-0.28}{0.28 - 20r}$$

At our best guess of $r = 0.01$:

$$S(t, 0.01) = \frac{-0.28}{0.28 - 0.2} = \frac{-0.28}{0.08} = \frac{-7}{2}$$

Thus, if r is wrong by $\pm 10\%$, then t_{max} will be wrong by $S(t,r) * 10\% = \frac{7}{2} * 10\% = \pm 35\%$

What if the rate of growth of the pig is also wrong? We redefine $w(t)$:

$$w(t) = 200 + gt$$

Then, we find $S(t, g)$:

$$S(t,g) = \frac{g}{t_{max}} * \frac{dt_{max}}{dg}|_{g=5} \approx 3.0625$$

We can also examine the sensitivity of the outcome:

$$S(P,r) = \frac{r}{P} * \frac{dP}{dr} = \frac{r}{P(t_{max})} * \frac{dP(t)}{dt} * \frac{dt}{dr}|_{t=8} = 0$$

This means that the profit doesn't change that much (to the first order) when the rate of change in the price of the pig varies.

(FIX LAST PART WITH dp/dr, on website)

Example. A manufacturer of color TV sets is planning the introduction of two new products, a 19-inch LCD flat panel set with a manufacturer's suggested retail price (MSRP) of \$339 and a 21-inch LCD flat panel set with an MSRP of \$399. The cost to the company is \$195 per 19-inch set and \$225 per 21-inch set, plus an additional \$400,000 in fixed costs. In the competitive market in which these sets will be sold, the number of sales per year will affect the average selling price. It is estimated that for each type of set, the average selling price drops by one cent for each additional unit sold. Furthermore, sales of the 19-inch set will affect sales of the 21-inch set and vice-versa. It is estimated that the average selling price for the 19-inch set will be reduced by an additional 0.3 cents for each 21-inch set sold, and the price for the 21-inch set will decrease by 0.4 cents for each 19-inch set sold. How many units of each type of set should be manufactured?

Answer. Let x_1 be the number of 19" TVs sold and x_2 be the number of 21" TVs sold. Then, our price for the 19" TVs is:

$$p_1(x_1, x_2) = 339 - 0.01x_1 - 0.003x_2$$

And our price for the 21" TVs is:

$$p_2(x_1, x_2) = 399 - 0.004x_1 - 0.01x_2$$

The total cost to produce TVs is:

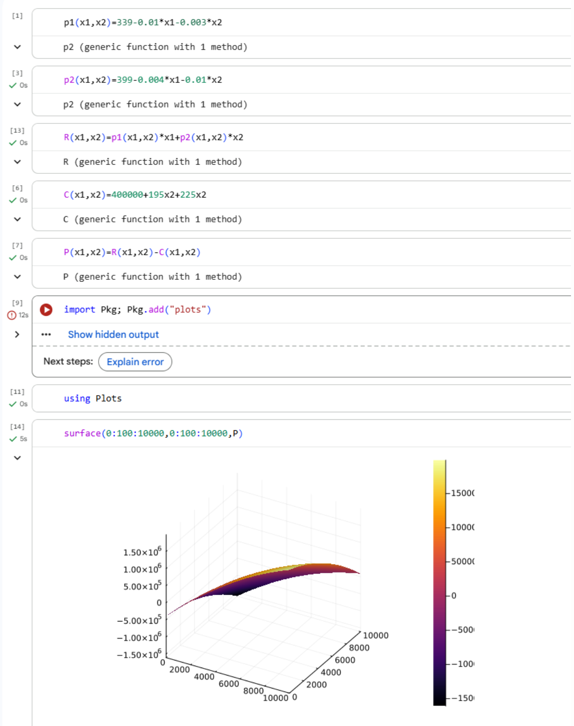
$$C(x_1, x_2) = 400000 + 195x_1 + 225x_2$$

Our revenue is:

$$R(x_1, x_2) = x_1p_1(x_1, x_2) + x_2p_2(x_1, x_2)$$

Our profit is:

$$P(x_1, x_2) = R(x_1, x_2) - C(x_1, x_2)$$



Note. When using computer algebra systems, use fractions instead of decimals because computers using floating point decimal systems, which can cause errors.

We are trying to maximize $P(x_1, x_2)$ on the set:

$$S = \{(x_1, x_2) : x_1 \geq 0, x_2 \leq 0\} \cap \mathbb{Z} \times \mathbb{Z}$$

$$\frac{\partial p}{\partial x_1} = \frac{\partial R}{\partial x_1} - \frac{\partial C}{}$$

(FINISH WRITING THIS SOLUTION, ADD JULIA PART) ⊛

Look into math modeling competitions (SIAM, consortium) download and learn julia. FIX FORMATTING

Maybe make notes for each lecture in separate documents and then combine? Figure out this weekend.

Definition 1.1. Sensitiv