NEGF Notes

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0.1 Hamiltonian

$$H = H_L + H_R + H_d + H_T + H_{sd} \tag{1}$$

$$H_L = \sum_{k\sigma} \epsilon_{k\sigma,L} c_{k\sigma}^\dagger c_{k\sigma} \tag{2}$$

$$H_R = \sum_q \omega_q a_q^\dagger a_q \tag{3}$$

$$H_d = \sum_{n\sigma} \epsilon_{n\sigma} d_{n\sigma}^\dagger d_{n\sigma} \tag{4}$$

$$H_T = \sum_{k\sigma n} \left(t_{k\sigma n} c_{k\sigma}^\dagger d_{n\sigma} + t_{k\sigma n}^* d_{n\sigma}^\dagger c_{k\sigma} \right) \tag{5}$$

$$H_{sd} = -\sum_{qnm} J_q \left(d_{n\uparrow}^\dagger d_{m\downarrow} a_q^\dagger + a_q d_{m\downarrow}^\dagger d_{n\uparrow} \right) \delta \left(\epsilon_{n\uparrow} - \epsilon_{m\downarrow} - \omega_q \right) \tag{6}$$

$$s_q^+ = \sum_{nm} d_{n\uparrow}^\dagger d_{m\downarrow} \delta_{\uparrow\downarrow} \tag{7}$$

$$s_q^- = \sum_{nm} d_{m\downarrow}^\dagger d_{n\uparrow} \delta_{\uparrow\downarrow} \tag{8}$$

0.1.1 check operators

$$i\dot{a}_q = \omega_q a_q - J_q s_q^+ \tag{9}$$

$$i\dot{c}_{k\sigma} = \epsilon_{k\sigma,L} c_{k\sigma} + \sum_{k'} t_{k\sigma n} d_{n\sigma} \tag{10}$$

$$i\dot{d}_{n\uparrow} = \epsilon_{n\uparrow} d_{n\uparrow} + \sum_k t_{k\uparrow n}^* c_{k\uparrow} - \sum_{q,m} J_q a_q^\dagger d_{m\downarrow} \delta(\epsilon_{n\uparrow} - \epsilon_{m\downarrow} - \omega_q) \tag{11}$$

$$i\dot{d}_{n\downarrow} = \epsilon_{n\downarrow} d_{n\downarrow} + \sum_k t_{k\downarrow n}^* c_{k\downarrow} - \sum_{q,m} J_q a_q d_{m\uparrow} \delta(\epsilon_{m\uparrow} - \epsilon_{n\downarrow} - \omega_q) \tag{12}$$

0.2 spin current ???

Define

$$G_{d,R}\left(\tau,\tau'\right)=-i\langle s_q^{+}(\tau)a_q^{\dagger}(\tau')\rangle. \tag{13}$$

The lesser Green's function is(s_q^{+} is fermionic but a_q is bosonic)

$$G_{d,R}^{<}\left(t,t'\right)=-i\langle a_q^{\dagger}(t')s_q^{+}(t)\rangle \tag{14}$$

We also define the Green's function that is related to the QD (not the Green's function of the QD),

$$G_d\left(\tau,\tau'\right)=-i\left\langle T_c S s_q^{+}(\tau)s_q^{-}\left(\tau'\right)\right\rangle. \tag{15}$$

We have

$$-i\partial_{\tau'}G_{d,R}\left(\tau,\tau'\right)=\omega_q G_{d,R}\left(\tau,\tau'\right)-J_q G_d \tag{16}$$

or

$$G_{d,R}g_{Rq}^{-1}=-J_q G_d \tag{17}$$

or

$$G_{d,R}\left(\tau,\tau'\right)=-J_q\int G_d\left(\tau,\tau_1\right)g_{Rq}\left(\tau_1,\tau'\right)d\tau_1 \tag{18}$$

the minus before J_q originates from the minus in H_{sd}. The rules of analytic continuation gives

$$G_{d,R}^{<}\left(t,t'\right)=-J_q\int_{-\infty}^{\infty}dt_1[G_d^r\left(t,t_1\right)g_{Rq}^{<}\left(t_1,t'\right)+G_d^{<}\left(t,t_1\right)g_{Rq}^{a}\left(t_1,t'\right)] \tag{19}$$

and

$$G_{R,d}^{<}\left(t,t'\right)=-J_q\int_{-\infty}^{\infty}dt_1[g_{Rq}^r\left(t,t_1\right)G_d^{<}\left(t_1,t'\right)+g_{Rq}^{<}\left(t,t_1\right)G_d^{a}\left(t_1,t'\right)] \tag{20}$$

The spin current flows out of right lead is

$$\begin{aligned} I_s &= i\sum_q J_q\left(\left\langle s_q^{+}a_q^{\dagger}\right\rangle-\left\langle a_q s_q^{-}\right\rangle\right) \\ &= -\sum_q J_q(G_{d,R}^{<}(t,t)-G_{R,d}^{<}(t,t)) \\ &= 2\mathrm{Re}\sum_{\mathrm{q}}\int \mathrm{dt_1}\,\mathrm{Tr}\left[\mathrm{G_d^r}\left(\mathrm{t},\mathrm{t_1}\right)\Sigma_{\mathrm{Rq}}^{<}\left(\mathrm{t_1},\mathrm{t'}\right)+\mathrm{G_d^{<}}\left(\mathrm{t},\mathrm{t_1}\right)\Sigma_{\mathrm{Rq}}^{\mathrm{a}}\left(\mathrm{t_1},\mathrm{t'}\right)\right] \end{aligned} \tag{21}$$

$$\Sigma_{Rq}^{\gamma}(\tau,\tau')=J_q^2 g_{Rq}^{\gamma}(\tau,\tau') \tag{22}$$

0.3 Calculation of G_d

Definition:

$$\begin{aligned} G_d\left(\tau,\tau'\right) &= -i\left\langle T_c S s_q^{+}(\tau)s_q^{-}\left(\tau'\right)\right\rangle \\ &= -i\sum_{mnm'n'}\langle T_c S d_{n\uparrow}^{\dagger}d_{m\downarrow}d_{m'\downarrow}^{\dagger}d_{n'\uparrow}\rangle\delta\left(\epsilon_{n\uparrow}-\epsilon_{m\downarrow}-\omega_q\right)\delta\left(\epsilon_{n'\uparrow}-\epsilon_{m'\downarrow}-\omega_q\right) \end{aligned} \tag{23}$$

When right lead is absent, the system Hamiltonian is

$$H=H_L+H_d+H_T. \tag{24}$$

$$G_d\left(\tau,\tau'\right)=-i\sum_{mnm'n'}G_{L,n'n\uparrow}(\tau',\tau)G_{L,mm'\downarrow}(\tau,\tau')\delta\left(\epsilon_{n\uparrow}-\epsilon_{m\downarrow}-\omega_q\right)\delta\left(\epsilon_{n'\uparrow}-\epsilon_{m'\downarrow}-\omega_q\right) \tag{25}$$

where

$$
\begin{aligned}
G_{L,mn\sigma}(\tau,\tau') &= -i\langle T_c d_{m\sigma}(\tau) d^\dagger_{n\sigma}(\tau')\rangle \\
&= g_{mn\sigma}(\tau,\tau')\delta_{mn} \\
&\quad + \iint d\tau_1 d\tau_2 g_{mm\sigma}(\tau,\tau_2)\sum_k t_{k\sigma n}t^*_{k\sigma m} g_{k\sigma}(\tau_2,\tau_1) g_{nn\sigma}(\tau_1,\tau') \\
&\quad + \cdots \\
&= g_{mn\sigma}(\tau,\tau')\delta_{mn} + \iint d\tau_1 d\tau_2 g_{mm\sigma}(\tau,\tau_2)\Sigma_{L,mn\sigma}(\tau_2,\tau_1) g_{nn\sigma}(\tau_1,\tau') \\
&\quad + \cdots \\
&= 1/\left[g^{-1}_{mn\sigma} - \Sigma_{L,mn\sigma}\right]
\end{aligned} \tag{26}
$$

$$
g_{mn\sigma}(\tau,\tau') = -i\langle T_c d_{m\sigma}(\tau) d^\dagger_{n\sigma}(\tau')\rangle_0 \tag{27}
$$

Self-energy of left lead

$$
\Sigma_{L,mn\sigma}(\tau_2,\tau_1) = \sum_k t_{k\sigma n} t^*_{k\sigma m} g_{k\sigma}(\tau_2,\tau_1) \tag{28}
$$

where

$$
g_{k\sigma}(\tau_2,\tau_1) = -i\langle T_c c_{k\sigma}(\tau_2) c^\dagger_{k\sigma}(\tau_1)\rangle_0. \tag{29}
$$

When left lead is absent, system Hamiltonian is

$$
H = H_d + H_R + H_{sd}. \tag{30}
$$

$$
\begin{aligned}
G_d(\tau,\tau') =& -i\sum_{mn} g_{n\uparrow}(\tau',\tau)\, g_{m\downarrow}(\tau,\tau')\,\delta(\varepsilon_{n\uparrow}-\varepsilon_{m\downarrow}-\omega_q) \\
& - \int d\tau_1 \int d\tau_2 \sum_{mnm'n'} g_{n\uparrow}(\tau_1,\tau)\, g_{m\downarrow}(\tau,\tau_1)\,\Sigma_{R,mnm'n'}(\tau_1,\tau_2)\, g_{n'\uparrow}(\tau',\tau_2)\, g_{m'\downarrow}(\tau_2,\tau') \\
& \quad \times \delta(\varepsilon_{n\uparrow}-\varepsilon_{m\downarrow}-\omega_{q_1})\delta(\varepsilon_{n'\uparrow}-\varepsilon_{m'\downarrow}-\omega_{q_1}) \\
& + \cdots \\
=& g_d(\tau,\tau') + \iint d\tau_1 d\tau_2 g_d(\tau,\tau_1)\Sigma_R(\tau_1,\tau_2) G_d(\tau_2,\tau')
\end{aligned} \tag{31}
$$

in which,

$$
g_d(\tau,\tau') = -i\sum_{mn} g_{n\uparrow}(\tau',\tau)\, g_{m\downarrow}(\tau,\tau')\,\delta(\varepsilon_{n\uparrow}-\varepsilon_{m\downarrow}-\omega_q), \tag{32}
$$

the self-energy of right lead is

$$
\Sigma_{R,mnm'n'}(\tau_1,\tau_2) = \sum_{q1} J^2_{q_1} g_{Rq_1}(\tau_1,\tau_2)\delta(\varepsilon_{n\uparrow}-\varepsilon_{m\downarrow}-\omega_{q_1})\delta(\varepsilon_{n'\uparrow}-\varepsilon_{m'\downarrow}-\omega_{q_1}) \tag{33}
$$

$$
g_{Rq_1}(\tau_1,\tau_2) = -i\langle T_c a_{q_1}(\tau_1) a^\dagger_{q_1}(\tau_2)\rangle_0 \tag{34}
$$

Hence, when both leads are present, we have

$$
\begin{aligned}
G_d(\tau,\tau') =& -i\sum_{mnm'n'} G_{L,nn'\uparrow}(\tau',\tau)\, G_{L,mm'\downarrow}(\tau,\tau')\,\delta(\epsilon_{n\uparrow}-\epsilon_{m\downarrow}-\omega_q)\,\delta(\epsilon_{n'\uparrow}-\epsilon_{m'\downarrow}-\omega_q) \\
& -i\sum_{mnm'n'} G_{L,nn'\uparrow}(\tau_1,\tau)\, G_{L,mm'\downarrow}(\tau,\tau_1)\,\Sigma_{R,mnm'n'}(\tau_1,\tau_2)\, G_d(\tau_2,\tau')\,\delta(\epsilon_{n\uparrow}-\epsilon_{m\downarrow}-\omega_q) \\
& \quad \times \delta(\epsilon_{n'\uparrow}-\epsilon_{m'\downarrow}-\omega_q)
\end{aligned} \tag{35}
$$

For the sack of convenience, we rewrite the above formula in matrix presentation as follows(the matrix indices are QD level indices m, n, not corrected yet!), and omit energy conservation constrain.

$$? \quad G_d\left(\tau,\tau'\right) = -iG_{L\uparrow}\left(\tau',\tau\right)G_{L\downarrow}\left(\tau,\tau'\right) - iG_{L\uparrow}\left(\tau_1,\tau\right)G_{L\downarrow}\left(\tau,\tau_1\right)\Sigma_R\left(\tau_1,\tau_2\right)G_d\left(\tau_2,\tau'\right) \tag{36}$$

0.4 continuation on Eq.(35)

$$A(\tau_1,\tau') \equiv \int d\tau_2 \Sigma_R\left(\tau_1,\tau_2\right)G_d\left(\tau_2,\tau'\right) \tag{37}$$

$$B(\tau,\tau_1) \equiv G_{L\uparrow}\left(\tau_1,\tau\right)G_{L\downarrow}\left(\tau,\tau_1\right) \tag{38}$$

$$C(\tau,\tau') \equiv -iG_{L\uparrow}\left(\tau_1,\tau\right)G_{L\downarrow}\left(\tau,\tau_1\right)A(\tau_1,\tau') \rightarrow \tag{39}$$

$$C(\tau,\tau') = -i\int d\tau_1 B(\tau,\tau_1)A(\tau_1,\tau') \tag{40}$$

$$D(\tau,\tau') \equiv -iG_{L\uparrow}\left(\tau',\tau\right)G_{L\downarrow}\left(\tau,\tau'\right) \tag{41}$$

So, we have

$$G_d(\tau,\tau') = D + C \tag{42}$$

Using the analytic continuation theorem, we have

$$D^< = -iG^>_{L\uparrow}G^<_{L\downarrow} \tag{43}$$

$$C^< = -i(B^r A^< + B^< A^a) \tag{44}$$

where

$$B^r = G^a_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^r_{L\downarrow} + G^a_{L\uparrow}G^r_{L\downarrow} \tag{45}$$

$$A^< = \Sigma^r_R G^<_d + \Sigma^<_R G^a_d \tag{46}$$

$$B^< = G^>_{L\uparrow}G^<_{L\downarrow} \tag{47}$$

$$A^a = \Sigma^a_R G^a_d \tag{48}$$

Then, the analytic continuation theorem on Eq.(35) yields

$$G^<_d = -iG^>_{L\uparrow}G^<_{L\downarrow} - i\left[(G^a_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^r_{L\downarrow} + G^a_{L\uparrow}G^r_{L\downarrow})(\Sigma^r_R G^<_d + \Sigma^<_R G^a_d) + (G^>_{L\uparrow}G^<_{L\downarrow})(\Sigma^a_R G^a_d)\right] \tag{49}$$

Similarly,

$$\begin{aligned} C^r &= -iB^r A^r \\ &= -i(G^a_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^r_{L\downarrow} + G^a_{L\uparrow}G^r_{L\downarrow})(\Sigma^r_R G^r_d) \end{aligned} \tag{50}$$

$$D^r = -i(G^a_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^r_{L\downarrow} + G^a_{L\uparrow}G^r_{L\downarrow}), \tag{51}$$

we have

$$\begin{aligned} G^r_d &= -i(G^a_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^r_{L\downarrow} + G^a_{L\uparrow}G^r_{L\downarrow}) - i(G^a_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^r_{L\downarrow} + G^a_{L\uparrow}G^r_{L\downarrow})(\Sigma^r_R G^r_d) \\ &= \frac{-i\left(G_{L\uparrow}G_{L\downarrow}\right)^r}{1 + i\left(G_{L\uparrow}G_{L\downarrow}\right)^r \Sigma^r_R} \end{aligned} \tag{52}$$

$$\left(G_{L\uparrow}G_{L\downarrow}\right)^r \equiv G^a_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^r_{L\downarrow} + G^a_{L\uparrow}G^r_{L\downarrow} \tag{53}$$

Now we calculate G^a_d.

$$C^a = -iB^a A^a \tag{54}$$

$$B^a = G^r_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^a_{L\downarrow} + G^r_{L\uparrow}G^a_{L\downarrow} \tag{55}$$

$$D^a = -i(G^r_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^a_{L\downarrow} + G^r_{L\uparrow}G^a_{L\downarrow}) \tag{56}$$

So we have

$$G^a_d = -i(G^r_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^a_{L\downarrow} + G^r_{L\uparrow}G^a_{L\downarrow}) - i(G^r_{L\uparrow}G^<_{L\downarrow} + G^>_{L\uparrow}G^a_{L\downarrow} + G^r_{L\uparrow}G^a_{L\downarrow})(\Sigma^a_R G^a_d) \tag{57}$$

From Eq.(49) we have

$$\begin{aligned} G^<_d &= -iG^>_{L\uparrow}G^<_{L\downarrow}\left(1+\Sigma^a_R G^a_d\right) - i\left(G_{L\uparrow}G_{L\downarrow}\right)^r\left(\Sigma^<_q G^a_d + \Sigma^r_R G^<_d\right) \\ &= \frac{-iG^>_{L\uparrow}G^<_{L\downarrow}\left(1+\Sigma^a_R G^a_d\right) - i\left(G_{L\uparrow}G_{L\downarrow}\right)^r \Sigma^<_R G^a_d}{1+i\left(G_{L\uparrow}G_{L\downarrow}\right)^r\Sigma^r_R} \\ &= \frac{-iG^>_{L\uparrow}G^<_{L\downarrow}\left(1+\Sigma^a_R G^a_d\right)}{1+i\left(G_{L\uparrow}G_{L\downarrow}\right)^r\Sigma^r_R} + G^r_d\Sigma^<_R G^a_d \\ &= -i(G^r_d\Sigma^r_R+1)G^>_{L\uparrow}G^<_{L\downarrow}\left(1+\Sigma^a_R G^a_d\right) + G^r_d\Sigma^<_R G^a_d \end{aligned} \tag{58}$$

Similarly,

$$G^>_d = -i\left(G^r_d\Sigma^r_R+1\right)G^<_{L\uparrow}G^>_{L\downarrow}\left(1+\Sigma^a_R G^a_d\right) + G^r_d\Sigma^>_R G^a_d \tag{59}$$

0.5 DC spin current

$$I_s = 2\mathrm{Re}\sum_{\mathrm{q}}\int\frac{\mathrm{dE}}{2\pi}\,\mathrm{Tr}\left[\left(\mathrm{G}^>_{\mathrm{d}} - \mathrm{G}^<_{\mathrm{d}}\right)\Sigma^<_{\mathrm{Rq}} + \mathrm{G}^<_{\mathrm{d}}\left(\Sigma^{\mathrm{a}}_{\mathrm{Rq}} - \Sigma^{\mathrm{r}}_{\mathrm{Rq}}\right)\right] \tag{60}$$

We have

$$G^>_d(E) - G^<_d(E) = -i\left(G^r_d\Sigma^r_R+1\right)\left(G^<_{L\uparrow}G^>_{L\downarrow} - G^>_{L\uparrow}G^<_{L\downarrow}\right)\left(1+\Sigma^a_R G^a_d\right) + G^r_d\left(\Sigma^>_R - \Sigma^<_R\right)G^a_d \tag{61}$$

Fourier transformation

$$G^<_d(E) = \int_{-\infty}^{+\infty} dt G^<_d(t-t')e^{iE(t-t')} \tag{62}$$

and inverse Fourier transformation

$$G^<_d(t-t') = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega G^<_d(E)e^{-iE(t-t')}, \tag{63}$$

are used, since the Green's functions only dependent on time difference. Then using Keldysh equation, we have

$$G^<_{L,mn\sigma}(E) = G^r_{L,mn\sigma}\Sigma^<_{L,mn\sigma}(E)G^a_{L,mn\sigma}(E), \tag{64}$$

where $G_{L,mn\sigma}$ is the Green's function when left free lead, QD and left coupling present. $\Sigma^<_{L,mn\sigma}$ is self-energy of left lead, defined in Eq. (29)

$$\Sigma^<_{L,mn\sigma} = if_{L\sigma}(E)\Gamma_{L,mn\sigma}(E). \tag{65}$$

so,

$$G^<_{L,mn\sigma}(E) = iG^r_{L,mn\sigma}f_{L\sigma}(E)\Gamma_{L,mn\sigma}(E)G^a_{L,mn\sigma}(E) \equiv iD_{L\sigma}f_{L\sigma}, \tag{66}$$

and

$$\begin{aligned} G^>_{L,mn\sigma}(E) &= -(G^<_{L,mn\sigma}(E))^\dagger \\ &= G^r_{L,mn\sigma}(E)\Sigma^>_{L,mn\sigma}(E)G^a_{L,mn\sigma}(E) \\ &= iD_{L\sigma}(f_{L\uparrow}(E)-1) \end{aligned} \tag{67}$$

in which, $D_{L\sigma}=G^r_{L\sigma}\Gamma_{L\sigma}G^a_{L\sigma}$, thus

$$\begin{aligned}G^<_{L\sigma}G^>_{L\sigma}-G^>_{L\sigma}G^<_{L\sigma}&=D_{L\uparrow}D_{L\downarrow}[(f_{L\uparrow}-1)f_{L\downarrow}-(f_{L\downarrow}-1)f_{L\uparrow}]\\&=D_{L\uparrow}D_{L\downarrow}(f_{L\uparrow}-f_{L\downarrow})\end{aligned}\tag{68}$$

$$\begin{aligned}\Sigma^<_R(E)&=\sum_{q_1}J^2_{q_1}g^<_{Rq_1}(E)\\&=if^B_R(E)\Gamma_R(E)\end{aligned}\tag{69}$$

$$\Sigma^a_R-\Sigma^r_R=\Sigma^<_R-\Sigma^>_R=i\Gamma_R(E).\tag{70}$$

$$G^>_d-G^<_d=-i\left[f_{L\uparrow}-f_{L\downarrow}\right]\left(G^r_d\Sigma^r_{Rq}+1\right)D_{L\uparrow}D_{L\downarrow}\left(1+\Sigma^a_{Rq}G^a_d\right)-iG^r_d\Gamma_{Rq}G^a_d\tag{71}$$

$$\begin{aligned}\left(G^>_d-G^<_d\right)\Sigma^<_{Rq}+G^<_d\left(\Sigma^a_{Rq}-\Sigma^r_{Rq}\right)&=\left[\left(f_{L\uparrow}-f_{L\downarrow}\right)f_R+\left(f_{L\uparrow}-1\right)f_{L\downarrow}\right]\\&\quad\times\left(G^r_d\Sigma^r_{Rq}+1\right)D_{L\uparrow}D_{L\downarrow}\left(1+\Sigma^a_{Rq}G^a_d\right)\Gamma_{Rq}\end{aligned}\tag{72}$$

The following formula exists

$$[f_{L\uparrow}(\varepsilon)-1]f_{L\downarrow}(\varepsilon)=-[f_{L\uparrow}(\varepsilon)-f_{L\downarrow}(\varepsilon)]f^B_L\tag{73}$$

where,

$$f_{L\sigma}(\epsilon)=\frac{1}{e^{\beta_L(\epsilon-\mu_\sigma)}+1}\tag{74}$$

$$f^B_L=\frac{1}{e^{\beta_L\Delta\mu_s}-1}\tag{75}$$

$\Delta\mu_s=\mu_\uparrow-\mu_\downarrow$ ($\omega=\varepsilon_\downarrow-\varepsilon_\uparrow$?). Note that this similar relation also exists,

$$\left(f_{L\uparrow}(\varepsilon)-1\right)f_{L\downarrow}(\varepsilon+\omega)=-[f_{L\uparrow}(\varepsilon)-f_{L\downarrow}(\varepsilon+\omega)]f^B_L(\omega)\tag{76}$$

$$f^B_L(\varepsilon)=\frac{1}{e^{\beta_L(\omega+\Delta\mu_s)}-1},\tag{77}$$

is the effective Boson-Einstein distribution of left electronic lead. Eq. (72) becomes

$$\begin{aligned}\left(G^>_d-G^<_d\right)\Sigma^<_{Rq}+G^<_d\left(\Sigma^a_{Rq}-\Sigma^r_{Rq}\right)&=\left[\left(f_{L\uparrow}-f_{L\downarrow}\right)\left(f_R-f^B_L\right)\right]\\&\quad\times\left(G^r_d\Sigma^r_{Rq}+1\right)D_{L\uparrow}D_{L\downarrow}\left(1+\Sigma^a_{Rq}G^a_d\right)\Gamma_{Rq}\end{aligned}\tag{78}$$

Substitute in Eq. (**??**), we get

$$I_{sR}=\int d\omega\rho_R(\omega)\left(f_R(\omega)-f^B_L(\omega)\right)\int dE\left(f_{L\uparrow}(E)-f_{L\downarrow}(E+\omega)\right)\mathrm{Tr}[A(E,\omega)],\tag{79}$$

$$A(E,\omega)=\left[G^r_d(E)\Sigma^r_{Rq}(\omega)+1\right]D_{L\uparrow}(E)D_{L\downarrow}(E+\omega)\left[1+\Sigma^a_{Rq}(\omega)G^a_d(E)\right].\tag{80}$$

Above $\rho_R(\omega)$ comes from the magnon q summation, is density of states of magnon lead, determined by magnon dispersion ω_q.

0.6 Spin current from the left lead

Define spin density operator

$$N_{sk}=d^\dagger_{k\uparrow}d_{k\uparrow}-d^\dagger_{k\downarrow}d_{k\downarrow}\tag{81}$$

$$I_{sL}=(1/2)\partial_tN_s=(1/2)\left(I_\uparrow-I_\downarrow\right)\tag{82}$$

$$I_\sigma=\mathrm{Tr}\left[\left(G^r_{d\sigma}-G^a_{d\sigma}\right)\Sigma^<_{L\sigma}+G^<_{d\sigma}\left(\Sigma^a_{L\sigma}-\Sigma^r_{L\sigma}\right)\right]\tag{83}$$

$$[G_{d\sigma}]_{nm}=-i\left\langle T_cSd_{n\sigma}d^\dagger_{m\sigma}\right\rangle\tag{84}$$

the factor of 1/2 comes from spin of electron while spin of magnon is 1.

References

[1] Y, K, Kato. Observation of the Spin Hall Effect in Semiconductors[J]. Science, 2004.

[2] Cao Zhan, Investigation on DC electronic transport in hybrid multiterminal quantum dot systems[D], 2017.