

formula in PRB. 88, 220406(R) (2013)

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1 formulas in PRB. 88, 220406(R) (2013)

1.1 formula 1

System Hamiltonion:

$$H = H_L + H_R + H_{sd}. \quad (1)$$

Left lead is metallic

$$H_L = \sum_{k\sigma} (\varepsilon_{k\sigma} - \mu_\sigma) c_{k\sigma}^\dagger c_{k\sigma}, \quad (2)$$

right lead is insulating magnetic

$$H_R \approx \sum_q \hbar w_q a_q^\dagger a_q + \text{constant} . \quad (3)$$

The interfacial electron-magnon interaction is described by

$$H_{sd} = - \sum_{k,q} J_q \left[S_q^- c_{k\uparrow}^\dagger c_{k+q\downarrow} + S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \right] \quad (4)$$

where $S_q^- \approx \sqrt{2S_0}a_q^\dagger$, $S_q^+ \approx \sqrt{2S_0}a_q$ are in the momentum space and J_q denotes the effective exchange coupling at the interface. The magnonic spin current operator can be obtained by

$$\hat{I}_S = \frac{d\hat{N}_R}{dt} = \frac{d}{dt} \sum_q a_q^\dagger a_q, \quad (5)$$

the magnonic spin current is obtained by taking average over the nonequilibrium ground state $|\psi_0\rangle$ of the interacting system H :

$$I_S = \frac{dN_R}{dt} = \frac{d}{dt} \langle \sum_q a_q^\dagger a_q \rangle. \quad (6)$$

Using the Heisenberg equation, we get

$$I_S = \frac{i}{\hbar} \langle [H_{sd}, \sum_q a_q^\dagger a_q] \rangle. \quad (7)$$

$$[H_{sd}, \sum_q a_q^\dagger a_q] = [-\sum_{k,q} J_q (S_q^- c_{k\uparrow}^\dagger c_{k+q\downarrow} + S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}), \sum_q a_q^\dagger a_q], \quad (8)$$

in which,

$$[a_q^\dagger, \sum_{q'} a_{q'}^\dagger a_{q'}] = \delta_{qq'} [a_q^\dagger, a_{q'}^\dagger a_{q'}] = [a_q^\dagger, a_q^\dagger a_q] = a_q^\dagger [a_q^\dagger, a_q] = -a_q^\dagger. \quad (9)$$

Similarly,

$$[a_q, \sum_{q'} a_{q'}^\dagger a_{q'}] = [a_q, a_q^\dagger a_q] = a_q. \quad (10)$$

So,

$$\begin{aligned} I_S &= \frac{i}{\hbar} \langle -\sum_{kq} J_q (S_q^- c_{k\uparrow}^\dagger c_{k+q\downarrow} + S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}) \rangle \\ &= \frac{i}{\hbar} \sum_{kq} J_q (\langle S_q^- c_{k\uparrow}^\dagger c_{k+q\downarrow} \rangle - \langle S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \rangle). \end{aligned} \quad (11)$$

1.2 formula 2

$$\frac{d}{dt} \langle S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \rangle = \frac{i}{\hbar} \langle [H_L + H_{sd} + H_R, S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}] \rangle. \quad (12)$$

The rhs. of eq. (12) is decomposed into 3 terms. The first term reads

$$\begin{aligned} \langle [H_L, S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}] \rangle &= [\sum_{k'\sigma} (\varepsilon_{k'\sigma} - \mu_\sigma) c_{k'\sigma}^\dagger c_{k'\sigma}, S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}] \\ &= S_q^+ [\sum_{k'\sigma} (\varepsilon_{k'\sigma} - \mu_\sigma) c_{k'\sigma}^\dagger c_{k'\sigma}, c_{k+q\downarrow}^\dagger] c_{k\uparrow} \\ &\quad + S_q^+ c_{k+q\downarrow}^\dagger [\sum_{k'\sigma} (\varepsilon_{k'\sigma} - \mu_\sigma) c_{k'\sigma}^\dagger c_{k'\sigma}, c_{k\uparrow}] \end{aligned} \quad (13)$$

Note that,

$$[\sum_{k'\sigma} c_{k'\sigma}^\dagger c_{k'\sigma}, c_{k+q\downarrow}^\dagger] = \sum_{k'\sigma} c_{k'\sigma}^\dagger \delta_{k',k+q} \delta_{\sigma\downarrow} = c_{k+q\downarrow}^\dagger, \quad (14)$$

$$[\sum_{k'\sigma} c_{k'\sigma}^\dagger c_{k'\sigma}, c_{k\uparrow}] = -\sum_{k'\sigma} \{c_{k'\sigma}^\dagger, c_{k\uparrow}\} c_{k'\sigma} = -c_{k\uparrow}. \quad (15)$$

Eq. (14) (15) are derived using equity

$$\begin{aligned} [AB, C] &= A[B, C] + [A, C]B \\ &= A\{B, C\} - \{A, C\}B. \end{aligned} \quad (16)$$

So,

$$\left[H_L, S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \right] = (\varepsilon_{k+q\downarrow} - \varepsilon_{k\uparrow} + \mu_\uparrow - \mu_\downarrow) S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}. \quad (17)$$

If $\mu_\uparrow = \mu_\downarrow$, then eq. (17) reduces to

$$\left[H_L, S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \right] = (\varepsilon_{k+q\downarrow} - \varepsilon_{k\uparrow}) S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}. \quad (18)$$

The second term $H_R = \sum_q \hbar \omega_q a_q^\dagger a_q$, then using eq. (10), we get

$$\left[H_R, S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \right] = -\hbar \omega_q S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}. \quad (19)$$

The third term in eq. (12) reads

$$\begin{aligned} \left[H_{sd}, S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \right] &= -J_q \left[S_q^- c_{k\uparrow}^\dagger c_{k+q\downarrow}, S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \right] \\ &= J_q \left[S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}, S_q^- c_{k\uparrow}^\dagger c_{k+q\downarrow} \right] \end{aligned} \quad (20)$$

Combine these three terms, we get

$$\begin{aligned} \frac{d}{dt} \left\langle S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \right\rangle &= \frac{i}{\hbar} (\varepsilon_{k+q\downarrow} - \varepsilon_{k\uparrow} - \hbar \omega_q) \left\langle S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow} \right\rangle \\ &\quad + \frac{i}{\hbar} J_q \left\langle \left[S_q^+ c_{k+q\downarrow}^\dagger c_{k\uparrow}, S_q^- c_{k\uparrow}^\dagger c_{k+q\downarrow} \right] \right\rangle, \end{aligned} \quad (21)$$

which is also eq. (2) in PRB. 88, 220406(R) (2013).

2 NM-QD-MIL system

For system consists of quantum dot(QD) sandwiched by a left normal metal lead and a right magnetic insulating lead(MIL), the Hamiltonian is

$$H = H_L + H_{\text{QD}} + H_R + H_T. \quad (22)$$

$$H_L = \sum_{k\sigma} \varepsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma}, \quad (23)$$

$$H_{\text{QD}} = \sum_{\sigma} \varepsilon_{\sigma} d_{\sigma}^\dagger d_{\sigma}, \quad (24)$$

$$H_R \approx \sum_q \hbar w_q a_q^\dagger a_q, \quad (25)$$

$$H_T = V_L + V_R \quad (26)$$

V_L is the coupling between left lead and QD, while V_R is the coupling between right lead and QD.

$$V_L = \sum_{k\sigma} (t_{k\sigma} c_{k\sigma}^\dagger d_{\sigma} + t_{k\sigma}^* d_{\sigma}^\dagger c_{k\sigma}) \quad (27)$$

$$V_R = - \sum_q J_q \left[S_q^- d_{\uparrow}^\dagger d_{\downarrow} + S_q^+ d_{\downarrow}^\dagger d_{\uparrow} \right]. \quad (28)$$

where $S_q^- \approx \sqrt{2S_0} a_q^\dagger$, $S_q^+ \approx \sqrt{2S_0} a_q$ are in the momentum space and J_q denotes the effective exchange coupling between the QD and MIL.

2.1 Spin-dependent current in left lead

The spin-dependent current flow out of left lead is $I_{L\sigma}$:

$$I_{L\sigma} = \frac{d}{dt} \langle N_{L\sigma} \rangle \quad (29)$$

in which, $N_{L\sigma} = \sum_k c_{k\sigma}^\dagger c_{k\sigma}$ Heisenberg equation:

$$\frac{d}{dt} \langle N_{L\sigma} \rangle = \frac{i}{\hbar} \langle [H, N_{L\sigma}] \rangle \quad (30)$$

$$[H, N_{L\sigma}] = [H_T, N_{L\sigma}] = \sum_k \left(t_{k\sigma}^* d_{\sigma}^\dagger c_{k\sigma} - t_{k\sigma} c_{k\sigma}^\dagger d_{\sigma} \right) \quad (31)$$

so, the spin-dependent current

$$I_{L\sigma} = \frac{i}{\hbar} \sum_k \left(t_{k\sigma}^* \langle d_{\sigma}^\dagger c_{k\sigma} \rangle - t_{k\sigma} \langle c_{k\sigma}^\dagger d_{\sigma} \rangle \right). \quad (32)$$

Namely, the spin-up current is

$$I_{L\uparrow} = \frac{i}{\hbar} \sum_k \left(t_{k\uparrow}^* \langle d_{\uparrow}^\dagger c_{k\uparrow} \rangle - t_{k\uparrow} \langle c_{k\uparrow}^\dagger d_{\uparrow} \rangle \right), \quad (33)$$

the spin-down current is

$$I_{L\downarrow} = \frac{i}{\hbar} \sum_k \left(t_{k\downarrow}^* \langle d_{\downarrow}^\dagger c_{k\downarrow} \rangle - t_{k\downarrow} \langle c_{k\downarrow}^\dagger d_{\downarrow} \rangle \right), \quad (34)$$

The charge current in left lead is defined as

$$I_e = e(I_{L\uparrow} + I_{L\downarrow}). \quad (35)$$

The spin current in left lead is defined as

$$I_{LS} = \frac{1}{2}(I_{L\uparrow} - I_{L\downarrow}) \quad (36)$$

Substitute the spin-dependent current in, we get the spin current in left lead

$$I_{LS} = \frac{i}{2\hbar} \sum_k \left(t_{k\uparrow}^* \langle d_{\uparrow}^\dagger c_{k\uparrow} \rangle - t_{k\uparrow} \langle c_{k\uparrow}^\dagger d_{\uparrow} \rangle - t_{k\downarrow}^* \langle d_{\downarrow}^\dagger c_{k\downarrow} \rangle + t_{k\downarrow} \langle c_{k\downarrow}^\dagger d_{\downarrow} \rangle \right) \quad (37)$$

2.2 magnonic current in right lead

The magnonic current in right lead is

$$I_{RS} = \frac{d\langle N_R \rangle}{dt} = \frac{d}{dt} \langle \sum_q a_q^\dagger a_q \rangle, \quad (38)$$

From the Heisenberg equation, we have

$$\frac{d}{dt} \langle \sum_q a_q^\dagger a_q \rangle = \frac{i}{\hbar} \langle [H, \sum_q a_q^\dagger a_q] \rangle. \quad (39)$$

We have

$$\begin{aligned} & [H, \sum_q a_q^\dagger a_q] \\ &= [V_R, \sum_q a_q^\dagger a_q] \\ &= - \sum_q J_q \left([S_q^- d_{\uparrow}^\dagger d_{\downarrow}, \sum_{q'} a_{q'}^\dagger a_{q'}] + [S_q^+ d_{\downarrow}^\dagger d_{\uparrow}, \sum_{q'} a_{q'}^\dagger a_{q'}] \right) \\ &= \sum_q J_q (S_q^- d_{\uparrow}^\dagger d_{\downarrow} - S_q^+ d_{\downarrow}^\dagger d_{\uparrow}) \end{aligned} \quad (40)$$

So, the magnon current reads

$$I_{RS} = \frac{i}{\hbar} \sum_q J_q (\langle S_q^- d_{\uparrow}^\dagger d_{\downarrow} \rangle - \langle S_q^+ d_{\downarrow}^\dagger d_{\uparrow} \rangle). \quad (41)$$

2.3 spin-dependent current in QD

Similarly, the spin-dependent current in the central QD is defined as

$$I_{C\sigma} = \langle \frac{dN_{C\sigma}}{dt} \rangle = \langle \frac{d}{dt} d_{\sigma}^{\dagger} d_{\sigma} \rangle, \quad (42)$$

Heisenberg equation:

$$\frac{d}{dt} d_{\sigma}^{\dagger} d_{\sigma} = \frac{i}{\hbar} [H, d_{\sigma}^{\dagger} d_{\sigma}]. \quad (43)$$

Specifically, we have

$$[H, d_{\sigma}^{\dagger} d_{\sigma}] = [V_L, d_{\sigma}^{\dagger} d_{\sigma}] + [V_R, d_{\sigma}^{\dagger} d_{\sigma}] \quad (44)$$

in which,

$$\begin{aligned} [V_L, d_{\sigma}^{\dagger} d_{\sigma}] &= \sum_{k'\sigma'} (t_{k'\sigma'} [c_{k'\sigma'}^{\dagger} d_{\sigma'}, d_{\sigma}^{\dagger} d_{\sigma}] + t_{k'\sigma'}^* [d_{\sigma'}^{\dagger} c_{k'\sigma'}, d_{\sigma}^{\dagger} d_{\sigma}]) \\ &= \sum_{k'\sigma'} (t_{k'\sigma'} c_{k'\sigma'}^{\dagger} d_{\sigma'} \delta_{\sigma\sigma'} - t_{k'\sigma'}^* d_{\sigma'}^{\dagger} c_{k'\sigma'} \delta_{\sigma\sigma'}) \\ &= \sum_k (t_{k\sigma} c_{k\sigma}^{\dagger} d_{\sigma} - t_{k\sigma}^* d_{\sigma}^{\dagger} c_{k\sigma}) \end{aligned} \quad (45)$$

We change the summation index from k' to k in the last line, which doesn't change the result.

$$[V_R, d_{\sigma}^{\dagger} d_{\sigma}] = - \sum_q J_q (S_q^- [d_{\uparrow}^{\dagger} d_{\downarrow}, d_{\sigma}^{\dagger} d_{\sigma}] + S_q^+ [d_{\downarrow}^{\dagger} d_{\uparrow}, d_{\sigma}^{\dagger} d_{\sigma}]) \quad (46)$$

in which,

$$\begin{aligned} [d_{\uparrow}^{\dagger} d_{\downarrow}, d_{\sigma}^{\dagger} d_{\sigma}] &= d_{\uparrow}^{\dagger} [d_{\downarrow}, d_{\sigma}^{\dagger} d_{\sigma}] + [d_{\uparrow}^{\dagger}, d_{\sigma}^{\dagger} d_{\sigma}] d_{\downarrow} \\ &= d_{\uparrow}^{\dagger} d_{\downarrow} \delta_{\sigma\downarrow} - d_{\uparrow}^{\dagger} d_{\downarrow} \delta_{\sigma\uparrow} \end{aligned} \quad (47)$$

So,

$$[V_R, d_{\sigma}^{\dagger} d_{\sigma}] = - \sum_q J_q [S_q^- (d_{\uparrow}^{\dagger} d_{\downarrow} \delta_{\sigma\downarrow} - d_{\uparrow}^{\dagger} d_{\downarrow} \delta_{\sigma\uparrow}) + S_q^+ (d_{\downarrow}^{\dagger} d_{\uparrow} \delta_{\sigma\uparrow} - d_{\downarrow}^{\dagger} d_{\uparrow} \delta_{\sigma\downarrow})] \quad (48)$$

The spin-dependent current is

$$I_{C\uparrow} = \frac{i}{\hbar} [H, d_{\uparrow}^{\dagger} d_{\uparrow}] \quad (49)$$

$$I_{C\downarrow} = \frac{i}{\hbar} [H, d_{\downarrow}^{\dagger} d_{\downarrow}] \quad (50)$$

which gives

$$I_{C\uparrow} = \frac{i}{\hbar} \left[\sum_k (t_{k\uparrow} c_{k\uparrow}^{\dagger} d_{\uparrow} - t_{k\uparrow}^* d_{\uparrow}^{\dagger} c_{k\uparrow}) + \sum_q J_q (S_q^- d_{\uparrow}^{\dagger} d_{\downarrow} - S_q^+ d_{\downarrow}^{\dagger} d_{\uparrow}) \right] \quad (51)$$

and

$$I_{C\downarrow} = \frac{i}{\hbar} \left[\sum_k (t_{k\downarrow} c_{k\downarrow}^\dagger d_\downarrow - t_{k\downarrow}^* d_\downarrow^\dagger c_{k\downarrow}) + \sum_q J_q (S_q^+ d_\downarrow^\dagger d_\uparrow - S_q^- d_\uparrow^\dagger d_\downarrow) \right]. \quad (52)$$

The spin current in central dot is

$$I_{CS} = \frac{1}{2} (I_{C\uparrow} - I_{C\downarrow}) \quad (53)$$

2.4 Verifying continuity condition

2.4.1 Charge current

Since the right lead is a insulating lead, there is no charge current flow through it, so the charge current is

$$I_{Re} = 0 \quad (54)$$

where the subscript R denotes the right lead, while the subscript e denotes the charge current.

Meanwhile, the charge current flows in left lead and QD is

$$I_e = e \left(\sum_\sigma I_{L\sigma} + \sum_\sigma I_{C\sigma} \right) = 0 \quad (55)$$

2.4.2 Spin current

Spin current in the left lead and QD:

$$I_{LS} + I_{CS} = \frac{i}{\hbar} \sum_q J_q (\langle S_q^- d_\uparrow^\dagger d_\downarrow \rangle - \langle S_q^+ d_\downarrow^\dagger d_\uparrow \rangle). \quad (56)$$

The magnon current is

$$I_{RS} = \frac{i}{\hbar} \sum_q J_q (\langle S_q^- d_\uparrow^\dagger d_\downarrow \rangle - \langle S_q^+ d_\downarrow^\dagger d_\uparrow \rangle). \quad (57)$$

Thus, we have

$$\begin{aligned} I_{LS} + I_{CS} &= I_{RS} \\ &= \frac{i}{\hbar} \sum_q J_q (\langle S_q^- d_\uparrow^\dagger d_\downarrow \rangle - \langle S_q^+ d_\downarrow^\dagger d_\uparrow \rangle). \end{aligned} \quad (58)$$

3 Nonequilibrium Green's function technique