Notes on PRB.67.092408

1 spin field effect transistor

A type of quantum field effect transistor that operates purely on the flow of spin current in the absence of charge current. The rotating field induces a time-independent dc spin current, and at the same time generates no charge cur- rent. The physical principle of our SFET is due to a spin flip mecha- nism provided by the field.

2 Hamiltonian

A rotating magnetic field is

$$B_x = B_0 \sin\theta \, \cos(\omega \mathrm{t}) \tag{1}$$

$$B_y = B_0 \sin\theta \, \sin(\omega \mathrm{t}) \tag{2}$$

$$B_z = B_0 \cos\theta. \tag{3}$$

The Hamiltonian of system is

$$\begin{aligned} H = & \sum_{k,\sigma,\alpha=L,R} \epsilon_k C^+_{k\alpha\sigma} C_{k\alpha\sigma} + \sum_{n\sigma} \left[\epsilon_n + \sigma B_0 \cos\theta\right] d^+_{n\sigma} d_{n\sigma} \\ & + H'(t) + \sum_{k,n,\sigma,\alpha=L,R} \left[T_{k\alpha n} C^+_{k\alpha\sigma} d_{n\sigma} + \text{c.c.}\right] \end{aligned} \tag{4}$$

We assume that there are multiple orbits in the scattering region, which is different from the original paper, in which only one orbit is considered. Energy level of lead $\epsilon_{Lk} = \epsilon_{Rk} = \epsilon_k$.

A counterclock-wise rotating field allows a spin-down electron to absorb a photon and flip to spin-up, and it does not allow a spin-up electron to absorb a photon and flip to spin-down.

$$H'(t) = \sum_n \gamma \left[\exp(-i\omega t) d^+_{n\uparrow} d_{n\downarrow} + \exp(i\omega t) d^+_{n\downarrow} d_{n\uparrow}\right] \tag{5}$$

$$\gamma = B_0 \sin\theta \tag{6}$$

The scattering region is characterized by an energy level $\epsilon_n = \epsilon^0_n - qV_g$, controlled by the gate voltage V_g.

We solve the transport properties (charge and spin currents) of the model in both adiabatic and nonadiabatic regimes using the standard Keldysh nonequilibrium Green's function technique.

3 Operator evolution

EoM of $d^\dagger_\sigma$ is

$$\partial_{t'} d^\dagger_{n\sigma}(t') = i[H, d^\dagger_{n\sigma}] \tag{7}$$

$$[\sum_{n'\sigma'}[\epsilon_{n'}+\sigma B_0\cos\theta]\,d^\dagger_{n'\sigma'}d_{n'\sigma'},d^\dagger_{n\sigma}]=(\epsilon_n+\sigma B_0\cos\theta)d^\dagger_{n\sigma} \tag{8}$$

$$\begin{aligned}[d^\dagger_{n'\uparrow}d_{n'\downarrow},d^\dagger_{n\sigma}]&=d^\dagger_{n'\uparrow}\{d_{n'\downarrow},d^\dagger_{n\sigma}\}-\{d^\dagger_{n'\uparrow},d^\dagger_{n\sigma}\}d_{n'\downarrow}\\&=d^\dagger_{\uparrow}\delta_{nn'}\delta_{\sigma\downarrow},\end{aligned} \tag{9}$$

$$\begin{aligned}[d^\dagger_{n'\downarrow}d_{n'\uparrow},d^\dagger_{n\sigma}]&=d^\dagger_{n'\downarrow}\{d_{n'\uparrow},d^\dagger_{n\sigma}\}-\{d^\dagger_{n'\downarrow},d^\dagger_{n\sigma}\}d_{n'\uparrow}\\&=d^\dagger_{n'\uparrow}\delta_{nn'}\delta_{\sigma\uparrow}.\end{aligned} \tag{10}$$

Then

$$[H'(t),d^\dagger_{n\sigma}]=\gamma(e^{-i\omega t}d^\dagger_{n\uparrow}\delta_{\sigma\downarrow}+e^{i\omega t}d^\dagger_{n\downarrow}\delta_{\sigma\uparrow}) \tag{11}$$

$$\sum_{k,n',\sigma',\alpha=L,R}\Big[T_{n'k\alpha}C^\dagger_{k\alpha\sigma'}d_{n'\sigma'}+\text{c.c.}\ ,d^\dagger_{n\sigma}]=\sum_{k,\alpha=L,R}T_{k\alpha n}C^\dagger_{k\alpha\sigma} \tag{12}$$

Substitute into Eq. (7), we get

$$\partial_{t'}d^\dagger_{n\sigma}(t')=i[(\epsilon^0_n+\sigma B_0\cos\theta)d^\dagger_{n\sigma}+\gamma(e^{-i\omega t}d^\dagger_{n\uparrow}\delta_{\sigma\downarrow}+e^{i\omega t}d^\dagger_{n\downarrow}\delta_{\sigma\uparrow})+\sum_{k,\alpha=L,R}T_{k\alpha n}C^\dagger_{k\alpha\sigma}] \tag{13}$$

Equation of motion of particle operator $\hat{N}_{\alpha k\sigma}$ in the lead α is

$$\begin{aligned}\frac{d}{dt}\hat{N}_{\alpha k\sigma}&=\frac{i}{\hbar}[H,C^\dagger_{\alpha k\sigma}C_{\alpha k\sigma}]=\left[\sum_{k',\sigma',\alpha'=L,R}\left[T_{k'\alpha'}C^+_{k'\alpha'\sigma'}d_{\sigma'}+\text{c.c.}\right],C^\dagger_{\alpha k\sigma}C_{\alpha k\sigma}\right]\\&=\frac{i}{\hbar}\sum_{k',\sigma',\alpha'=L,R}\left[-T_{k'\alpha'}C^+_{k'\alpha'\sigma'}d_{\sigma'}\delta_{\alpha\alpha'}\delta_{kk'}\delta_{\sigma\sigma'}+\text{c.c.}\right]\\&=\frac{i}{\hbar}[-T_{k\alpha}C^\dagger_{k\alpha\sigma}d_\sigma+T^*_{k\alpha}d^\dagger_\sigma C_{k\alpha\sigma}]\end{aligned} \tag{14}$$

4 Charge current

So, the charge current due to L(R) lead with spin σ is given by

$$\begin{aligned}I_{L\sigma}(t)&=e\langle\frac{d}{dt}\hat{N}_\sigma(t)\rangle\\&=\frac{ie}{\hbar}\sum_{kn\alpha\in L}(\langle -T_{k\alpha n}C^\dagger_{k\alpha\sigma}(t)d_{n\sigma}(t)\rangle+\langle T^*_{k\alpha n}d^\dagger_{n\sigma}(t)C_{k\alpha\sigma}(t)\rangle)\end{aligned} \tag{15}$$

Define the lesser Green's function

$$G^<_{n\sigma',k\alpha\sigma}(\tau,\tau')=i\langle C^\dagger_{k\alpha\sigma}(\tau')d_{n\sigma'}(\tau)\rangle, \tag{16}$$

the current becomes

$$I_{L\sigma}(t)=\frac{-e}{\hbar}\sum_{kn\alpha\in L}(T_{k\alpha n}G^<_{n,k\alpha\sigma}(t,t)-T^*_{k\alpha n}G^<_{k\alpha,n}(t,t)\rangle) \tag{17}$$

More generally, we define the contour Green's function

$$G_{n\sigma',k\alpha\sigma}(\tau,\tau')=-i\langle d_{n\sigma'}(\tau)C^\dagger_{k\alpha\sigma}(\tau')\rangle. \tag{18}$$

EoM of operator $C_{k\alpha\sigma}^{\dagger}$ is

$$\partial_{t'} C_{k\alpha\sigma}^{\dagger}(t') = i[H, C_{k\alpha\sigma}^{\dagger}] = i(\varepsilon_k C_{k\alpha\sigma}^{\dagger} + \sum_n T_{k\alpha n} d_{n\sigma}^{\dagger}) \tag{19}$$

The equation-of-motion for the time-ordered Green function

$$\begin{aligned} -i\frac{\partial}{\partial t'} G_{n\sigma',k\alpha\sigma}^{t}\left(t,t'\right) &= \delta(t-t')\langle\{d_{n\sigma'}, C_{k\alpha\sigma}^{\dagger}\}\rangle - \langle T_c d_{n\sigma'} \partial_{t'} C_{k\alpha\sigma}^{\dagger}\rangle \\ &= \varepsilon_k G_{n\sigma',k\alpha\sigma}^{t}\left(t,t'\right) + \sum_m T_{k\alpha m}^{*} G_{n\sigma',m\sigma}^{t}(t,t') \end{aligned} \tag{20}$$

So, we have

$$(-i\frac{\partial}{\partial t'} - \varepsilon_k) G_{n\sigma',k\alpha\sigma}^{t}\left(t,t'\right) = \sum_m T_{k\alpha m}^{*} G_{n\sigma',m\sigma}^{t}(t,t') \tag{21}$$

in which

$$G_{n\sigma',m\sigma}^{t}(t,t') = -i\langle T_c d_{n\sigma'}(t) d_{m\sigma}^{\dagger}(t')\rangle. \tag{22}$$

Similarly, we evaluate the EoM for free Green's function $g_{k\alpha\sigma}^{t}(t,t')$ in lead α (note that $H = \sum_{k\sigma\alpha} \epsilon_k C_{k\alpha\sigma}^{\dagger} C_{k\alpha\sigma}$).

$$\begin{aligned} -i\frac{\partial}{\partial t'} g_{k\alpha\sigma}^{t}\left(t,t'\right) &= \delta(t-t')\langle\{C_{k\alpha\sigma}, C_{k\alpha\sigma}^{\dagger}\}\rangle - \langle T_c C_{k\alpha\sigma} \partial_{t'} C_{k\alpha\sigma}^{\dagger}\rangle \\ &= \delta(t-t') + \varepsilon_k g_{k\alpha\sigma}^{t}\left(t,t'\right), \end{aligned} \tag{23}$$

we have

$$(-i\frac{\partial}{\partial t'} - \varepsilon_k) g_{k\alpha\sigma}^{t}\left(t,t'\right) = \delta(t-t'). \tag{24}$$

Substitute Eq. (24) into Eq. (21) and integrate on both sides, we get an equation analogous to Jauho's notation [2],

$$G_{n,k\alpha}(\tau,\tau') = \sum_m \int d\tau_1 G_{nm}\left(\tau,\tau_1\right) t_{k\alpha m}^{*} g_{k\alpha}\left(\tau_1,\tau'\right),$$

we have

$$G_{n\sigma',k\alpha\sigma}^{t}(t,t') = \sum_m \int dt_1 G_{n\sigma',m\sigma}\left(t,t_1\right) T_{k\alpha m}^{*} g_{k\alpha\sigma}^{t}\left(\tau_1,\tau'\right). \tag{25}$$

When there is only one orbit presents, this equation reduces to

$$G_{\sigma',k\alpha\sigma}^{t}(t,t') = \int dt_1 G_{\sigma',\sigma}\left(t,t_1\right) T_{k\alpha}^{*} g_{k\alpha\sigma}^{t}\left(\tau_1,\tau'\right). \tag{26}$$

Since the contour Green's function has the same structure as real-time Green's function, the we have relation

$$G_{n\sigma',k\alpha\sigma}(\tau,\tau') = \sum_m \int d\tau_1 G_{n\sigma',m\sigma}\left(\tau,\tau_1\right) T_{k\alpha m}^{*} g_{k\alpha\sigma}\left(\tau_1,\tau'\right) \tag{27}$$

where $G_{n\sigma',k\alpha\sigma}(\tau,\tau')$ is contour Green's function defined in Eq. (16), and similarly the contour Green's function for non-interacting lead is defined as

$$g_{k\alpha\sigma}(\tau,\tau') = -i\langle T_c C_{k\alpha\sigma}(\tau) C_{k\alpha\sigma}^{\dagger}(\tau')\rangle \tag{28}$$

After analytic continuation, the current is formulated as

$$\begin{aligned} I_{L\sigma}(t) = -\frac{e}{\hbar} \int dt_1 \operatorname{Tr} \big[& G^{r}\left(t,t_1\right) \Sigma_{\alpha}^{<}\left(t_1,t\right) \\ & + G^{<}\left(t,t_1\right) \Sigma_{\alpha}^{a}\left(t_1,t\right) \big] + h.c. \end{aligned} \tag{29}$$

5 Adiabatic regime(ω is small)

So, the charge current is given by (why? DC?)

$$dQ_{\alpha\sigma}(t)/dt = q \int \frac{dE}{2\pi} \left(-\partial_E f\right) \left[\Gamma_\alpha \mathbf{G}^r(t) \mathbf{\Delta} \mathbf{G}^a(t)\right]_{\sigma\sigma} \tag{30}$$

References

[1] Y, K, Kato. Observation of the Spin Hall Effect in Semiconductors[J]. Science, 2004.

[2] Antti-Pekka Jauho, Quantum Kinetics in Transport and Optics of Semiconductors, P188.