

1 spin field effect transistor

A type of quantum field effect transistor that operates purely on the flow of spin current in the absence of charge current. The rotating field induces a time-independent dc spin current, and at the same time generates no charge current. The physical principle of our SFET is due to a spin flip mechanism provided by the field.

2 Hamiltonian

A rotating magnetic field is

$$B_x = B_0 \sin \theta \cos(\omega t) \quad (1)$$

$$B_y = B_0 \sin \theta \sin(\omega t) \quad (2)$$

$$B_z = B_0 \cos \theta. \quad (3)$$

The Hamiltonian of system is

$$\begin{aligned} H = & \sum_{k,\sigma,\alpha=L,R} \epsilon_k C_{k\alpha\sigma}^+ C_{k\alpha\sigma} + \sum_{n\sigma} [\epsilon_n + \sigma B_0 \cos \theta] d_{n\sigma}^+ d_{n\sigma} \\ & + H'(t) + \sum_{k,n,\sigma,\alpha=L,R} [T_{k\alpha n} C_{k\alpha\sigma}^+ d_{n\sigma} + \text{c.c.}] \end{aligned} \quad (4)$$

We assume that there are multiple orbits in the scattering region, which is different from the original paper, in which only one orbit is considered. Energy level of lead $\epsilon_{Lk} = \epsilon_{Rk} = \epsilon_k$.

A counterclock-wise rotating field allows a spin-down electron to absorb a photon and flip to spin-up, and it does not allow a spin-up electron to absorb a photon and flip to spin-down.

$$H'(t) = \sum_n \gamma \left[\exp(-i\omega t) d_{n\uparrow}^+ d_{n\downarrow} + \exp(i\omega t) d_{n\downarrow}^+ d_{n\uparrow} \right] \quad (5)$$

$$\gamma = B_0 \sin \theta \quad (6)$$

The scattering region is characterized by an energy level $\epsilon_n = \epsilon_n^0 - qV_g$, controlled by the gate voltage V_g .

We solve the transport properties (charge and spin currents) of the model in both adiabatic and nonadiabatic regimes using the standard Keldysh nonequilibrium Green's function technique.

3 Operator evolution

EoM of d_σ^\dagger is

$$\partial_t d_{n\sigma}^\dagger(t) = i[H, d_{n\sigma}^\dagger] \quad (7)$$

central dot part:

$$\left[\sum_{n'\sigma'} [\epsilon_{n'} + \sigma B_0 \cos \theta] d_{n'\sigma'}^\dagger d_{n'\sigma'}, d_{n\sigma}^\dagger \right] = (\epsilon_n + \sigma B_0 \cos \theta) d_{n\sigma}^\dagger \quad (8)$$

rotating magnetic field part:

$$\begin{aligned} [d_{n'\uparrow}^\dagger d_{n'\downarrow}, d_{n\sigma}^\dagger] &= d_{n'\uparrow}^\dagger \{d_{n'\downarrow}, d_{n\sigma}^\dagger\} - \{d_{n'\uparrow}^\dagger, d_{n\sigma}^\dagger\} d_{n'\downarrow} \\ &= d_{n'\uparrow}^\dagger \delta_{nn'} \delta_{\sigma\downarrow}, \end{aligned} \quad (9)$$

$$\begin{aligned} [d_{n'\downarrow}^\dagger d_{n'\uparrow}, d_{n\sigma}^\dagger] &= d_{n'\downarrow}^\dagger \{d_{n'\uparrow}, d_{n\sigma}^\dagger\} - \{d_{n'\downarrow}^\dagger, d_{n\sigma}^\dagger\} d_{n'\uparrow} \\ &= d_{n'\downarrow}^\dagger \delta_{nn'} \delta_{\sigma\uparrow}. \end{aligned} \quad (10)$$

Then

$$[H'(t), d_{n\sigma}^\dagger] = \gamma(e^{-i\omega t} d_{n\uparrow}^\dagger \delta_{\sigma\downarrow} + e^{i\omega t} d_{n\downarrow}^\dagger \delta_{\sigma\uparrow}) \quad (11)$$

Coupling part

$$\sum_{k,n',\sigma',\alpha=L,R} \left[T_{n'k\alpha} C_{k\alpha\sigma}^\dagger d_{n'\sigma'} + \text{c.c.}, d_{n\sigma}^\dagger \right] = \sum_{k,\alpha=L,R} T_{k\alpha n} C_{k\alpha\sigma}^\dagger \quad (12)$$

Substitute these three parts into Eq. (7), we get

$$\partial_t d_{n\sigma}^\dagger(t') = i[(\epsilon_n^0 + \sigma B_0 \cos \theta) d_{n\sigma}^\dagger + \gamma(e^{-i\omega t} d_{n\uparrow}^\dagger \delta_{\sigma\downarrow} + e^{i\omega t} d_{n\downarrow}^\dagger \delta_{\sigma\uparrow}) + \sum_{k,\alpha=L,R} T_{k\alpha n} C_{k\alpha\sigma}^\dagger] \quad (13)$$

Equation of motion of particle operator $\hat{N}_{\alpha k\sigma}$ in the lead α is

$$\begin{aligned} \frac{d}{dt} \hat{N}_{\alpha k\sigma} &= \frac{i}{\hbar} [H, C_{\alpha k\sigma}^\dagger C_{\alpha k\sigma}] = \left[\sum_{k',\sigma',\alpha'=L,R} [T_{k'\alpha'} C_{k'\alpha'\sigma'}^\dagger d_{\sigma'} + \text{c.c.}], C_{\alpha k\sigma}^\dagger C_{\alpha k\sigma} \right] \\ &= \frac{i}{\hbar} \sum_{k',\sigma',\alpha'=L,R} [-T_{k'\alpha'} C_{k'\alpha'\sigma'}^\dagger d_{\sigma'} \delta_{\alpha\alpha'} \delta_{kk'} \delta_{\sigma\sigma'} + \text{c.c.}] \\ &= \frac{i}{\hbar} [-T_{k\alpha} C_{k\alpha\sigma}^\dagger d_{\sigma} + T_{k\alpha}^* d_{\sigma}^\dagger C_{k\alpha\sigma}] \end{aligned} \quad (14)$$

4 Charge current

So, the charge current due to L(R) lead with spin σ is given by

$$\begin{aligned} I_{L\sigma}(t) &= e \left\langle \frac{d}{dt} \hat{N}_\sigma(t) \right\rangle \\ &= \frac{ie}{\hbar} \sum_{kn\alpha \in L} (\langle -T_{k\alpha n} C_{k\alpha\sigma}^\dagger(t) d_{n\sigma}(t) \rangle + \langle T_{k\alpha n}^* d_{n\sigma}^\dagger(t) C_{k\alpha\sigma}(t) \rangle) \end{aligned} \quad (15)$$

Define the lesser Green's function

$$G_{n\sigma',k\alpha\sigma}^<(\tau, \tau') = i \langle C_{k\alpha\sigma}^\dagger(\tau') d_{n\sigma'}(\tau) \rangle, \quad (16)$$

the current becomes

$$I_{L\sigma}(t) = \frac{-e}{\hbar} \sum_{kn\alpha \in L} (T_{k\alpha n} G_{n,k\alpha\sigma}^<(t, t) - T_{k\alpha n}^* G_{k\alpha,n}^<(t, t)) \quad (17)$$

More generally, we define the contour Green's function

$$G_{n\sigma',k\alpha\sigma}(\tau, \tau') = -i\langle d_{n\sigma'}(\tau)C_{k\alpha\sigma}^\dagger(\tau') \rangle. \quad (18)$$

EoM of operator $C_{k\alpha\sigma}^\dagger$ is

$$\partial_{t'} C_{k\alpha\sigma}^\dagger(t') = i[H, C_{k\alpha\sigma}^\dagger] = i(\varepsilon_k C_{k\alpha\sigma}^\dagger + \sum_n T_{k\alpha n} d_{n\sigma}^\dagger) \quad (19)$$

The equation-of-motion for the time-ordered Green function

$$\begin{aligned} -i\frac{\partial}{\partial t'} G_{n\sigma',k\alpha\sigma}^t(t, t') &= \delta(t-t')\langle \{d_{n\sigma'}, C_{k\alpha\sigma}^\dagger\} \rangle - \langle T_c d_{n\sigma'} \partial_{t'} C_{k\alpha\sigma}^\dagger \rangle \\ &= \varepsilon_k G_{n\sigma',k\alpha\sigma}^t(t, t') + \sum_m T_{k\alpha m}^* G_{n\sigma',m\sigma}^t(t, t') \end{aligned} \quad (20)$$

So, we have

$$(-i\frac{\partial}{\partial t'} - \varepsilon_k) G_{n\sigma',k\alpha\sigma}^t(t, t') = \sum_m T_{k\alpha m}^* G_{n\sigma',m\sigma}^t(t, t') \quad (21)$$

in which

$$G_{n\sigma',m\sigma}^t(t, t') = -i\langle T_c d_{n\sigma'}(t) d_{m\sigma}^\dagger(t') \rangle. \quad (22)$$

Similarly, we evaluate the EoM for free Green's function $g_{k\alpha\sigma}^t(t, t')$ in lead α (note that $H = \sum_{k\sigma\alpha} \varepsilon_k C_{k\alpha\sigma}^\dagger C_{k\alpha\sigma}$).

$$\begin{aligned} -i\frac{\partial}{\partial t'} g_{k\alpha\sigma}^t(t, t') &= \delta(t-t')\langle \{C_{k\alpha\sigma}, C_{k\alpha\sigma}^\dagger\} \rangle - \langle T_c C_{k\alpha\sigma} \partial_{t'} C_{k\alpha\sigma}^\dagger \rangle \\ &= \delta(t-t') + \varepsilon_k g_{k\alpha\sigma}^t(t, t'), \end{aligned} \quad (23)$$

we have

$$(-i\frac{\partial}{\partial t'} - \varepsilon_k) g_{k\alpha\sigma}^t(t, t') = \delta(t-t'). \quad (24)$$

Substitute Eq. (24) into Eq. (21) and integrate on both sides, we get an equation analogous to Jauho's notation [2],

$$G_{n,k\alpha}(\tau, \tau') = \sum_m \int d\tau_1 G_{nm}(\tau, \tau_1) t_{k\alpha m}^* g_{k\alpha}(\tau_1, \tau'),$$

we have

$$G_{n\sigma',k\alpha\sigma}^t(t, t') = \sum_m \int dt_1 G_{n\sigma',m\sigma}(t, t_1) T_{k\alpha m}^* g_{k\alpha\sigma}^t(\tau_1, \tau'). \quad (25)$$

When there is only one orbit presents, this equation reduces to

$$G_{\sigma',k\alpha\sigma}^t(t, t') = \int dt_1 G_{\sigma',\sigma}(t, t_1) T_{k\alpha}^* g_{k\alpha\sigma}^t(\tau_1, \tau'). \quad (26)$$

Since the contour Green's function has the same structure as real-time Green's function, the we have relation

$$G_{n\sigma',k\alpha\sigma}(\tau, \tau') = \sum_m \int d\tau_1 G_{n\sigma',m\sigma}(\tau, \tau_1) T_{k\alpha m}^* g_{k\alpha\sigma}(\tau_1, \tau') \quad (27)$$

where $G_{n\sigma',k\alpha\sigma}(\tau, \tau')$ is contour Green's function defined in Eq. (16), and similarly the contour Green's function for non-interacting lead is defined as

$$g_{k\alpha\sigma}(\tau, \tau') = -i\langle T_c C_{k\alpha\sigma}(\tau) C_{k\alpha\sigma}^\dagger(\tau') \rangle \quad (28)$$

After analytic continuation, the current is formulated as

$$I_{\alpha\sigma}(t) = -\frac{e}{\hbar} \int dt_1 \text{Tr} [G^r(t, t_1) \Sigma_{\alpha}^{<}(t_1, t) + G^{<}(t, t_1) \Sigma_{\alpha}^a(t_1, t)] + h.c. \quad (29)$$

Following Eq. (224) in WangJ's note, its Fourier transformation is

$$I_{\alpha\sigma}(\omega) = -\frac{e}{\hbar} \int \frac{dE}{2\pi} \frac{dE'}{2\pi} \text{Tr} [G^r(E + \omega, E') \Sigma_{\alpha}^{<}(E', E) + G^{<}(E + \omega, E') \Sigma_{\alpha}^a(E', E)] + c.c. \quad (30)$$

Here, $G^{r,<} \equiv G_{n\sigma',m\sigma}^{r,<}$, and matrix element

$$\Sigma_{\alpha,mn}^{\gamma}(t_1, t_2) = \sum_k T_{k\alpha m}^*(t_1) g_{k\alpha}^{\gamma}(t_1, t_2) T_{k\alpha n}(t_2), \quad (31)$$

in which, $g_{k\sigma}$ is the free propagator of lead. From Dyson equation, we have (different from Eq. (77) in Chap. II ?)

$$G^{<} = G^r \Sigma^{<} G^a \quad (32)$$

whose Fourier transformation gives

$$G^{<}(E_1, E_2) = \int \frac{dE}{2\pi} \frac{dE'}{2\pi} G^r(E_1, E) \Sigma^{<}(E, E') G^a(E', E_2) \quad (33)$$

For G^r , the analytic continuation gives

$$G^r = g^r + g^r H' G^r \quad (34)$$

$$G^r(E_1, E_2) = g^r(E_1, E_2) + \int \frac{dE}{2\pi} \frac{dE'}{2\pi} g^r(E_1, E) H'(E, E') G^r(E', E_2) \quad (35)$$

in which, g^r is the free propagator of central dot(?). Using Eq. (8), we have

$$\begin{aligned} g_{n\sigma}^r(t, t') &= -i\theta(t - t') \langle \{d_{n\sigma}(t), d_{n\sigma}^{\dagger}(t')\} \rangle_0 \\ &= -i\theta(t - t') e^{-i(\epsilon_n + \sigma B_0 \cos \theta)(t - t')} \end{aligned} \quad (36)$$

note ϵ_n is time-dependent, $g^r(t, t')$ depends only on time difference, so

$$g^r(E_1, E_2) = 2\pi g^r(E_1) \delta(E_1 - E_2) \quad (37)$$

and

$$g_{n\sigma}^r(E) = \frac{1}{E - (\epsilon_n + \sigma B_0 \cos \theta) + i0^+} \quad (38)$$

In the spin space $\{d_{\uparrow}^{\dagger}, d_{\downarrow}^{\dagger}; d_{\uparrow}, d_{\downarrow}\}$,

$$G^r \equiv \begin{pmatrix} G_{n\uparrow, m\uparrow}^r & G_{n\uparrow, m\downarrow}^r \\ G_{n\downarrow, m\uparrow}^r & G_{n\downarrow, m\downarrow}^r \end{pmatrix} \quad (39)$$

$$g^r(E) = \begin{pmatrix} g_{n\uparrow}^r(E) & 0 \\ 0 & g_{n\downarrow}^r(E) \end{pmatrix}. \quad (40)$$

H' is given by

$$H' = \begin{pmatrix} 0 & \gamma e^{-i\omega t} \\ \gamma e^{i\omega t} & 0 \end{pmatrix} \quad (41)$$

we have

$$H'(E, E') = \begin{pmatrix} 0 & \gamma \delta(E - \omega) \\ \gamma \delta(E + \omega) & 0 \end{pmatrix} 2\pi \delta(E - E'). \quad (42)$$

Substitute these equations into Eq. (35), we get

$$\begin{aligned} \begin{pmatrix} G_{n\uparrow, m\uparrow}^r(E_1, E_2) & G_{n\uparrow, m\downarrow}^r(E_1, E_2) \\ G_{n\downarrow, m\uparrow}^r(E_1, E_2) & G_{n\downarrow, m\downarrow}^r(E_1, E_2) \end{pmatrix} &= \begin{pmatrix} g_{n\uparrow}^r(E_1) & 0 \\ 0 & g_{n\downarrow}^r(E_1) \end{pmatrix} 2\pi \delta(E_1 - E_2) \\ &+ \int \frac{dE}{2\pi} \frac{dE'}{2\pi} \begin{pmatrix} g_{n\uparrow}^r(E_1) & 0 \\ 0 & g_{n\downarrow}^r(E_1) \end{pmatrix} 2\pi \delta(E_1 - E) 2\pi \delta(E - E') \\ &\times \begin{pmatrix} 0 & \gamma \delta(E - \omega) \\ \gamma \delta(E + \omega) & 0 \end{pmatrix} \begin{pmatrix} G_{n\uparrow, m\uparrow}^r & G_{n\uparrow, m\downarrow}^r \\ G_{n\downarrow, m\uparrow}^r & G_{n\downarrow, m\downarrow}^r \end{pmatrix} \\ &= \begin{pmatrix} g_{n\uparrow}^r(E_1) & 0 \\ 0 & g_{n\downarrow}^r(E_1) \end{pmatrix} 2\pi \delta(E_1 - E_2) \\ &+ \begin{pmatrix} G_1(E_1, E_2) & G_2(E_1, E_2) \\ G_3(E_1, E_2) & G_4(E_1, E_2) \end{pmatrix}. \end{aligned} \quad (43)$$

In which G_1, G_2, G_3, G_4 are

$$G_1(E_1, E_2) = \int dE dE' g_{n\uparrow}^r(E_1) \delta(E_1 - E) \gamma \delta(E - \omega) \delta(E - E') G_{n\downarrow, m\uparrow}^r(E', E_2) \quad (44)$$

$$G_2(E_1, E_2) = \int dE dE' g_{n\uparrow}^r(E_1) \delta(E_1 - E) \gamma \delta(E - \omega) \delta(E - E') G_{n\downarrow, m\downarrow}^r(E', E_2) \quad (45)$$

$$G_3(E_1, E_2) = \int dE dE' g_{n\downarrow}^r(E_1) \delta(E_1 - E) \gamma \delta(E + \omega) \delta(E - E') G_{n\uparrow, m\uparrow}^r(E', E_2) \quad (46)$$

$$G_4(E_1, E_2) = \int dE dE' g_{n\uparrow}^r(E_1) \delta(E_1 - E) \gamma \delta(E + \omega) \delta(E - E') G_{n\uparrow, m\downarrow}^r(E', E_2) \quad (47)$$

So,

$$G_{n\downarrow, m\uparrow}^r(E', E_2) = g_{n\uparrow}^r(E_1) 2\pi \delta(E_1 - E_2) + G_1(E_1, E_2) \quad (48)$$

$$G_{n\uparrow, m\downarrow}^r(E_1, E_2) = G_2(E_1, E_2) \quad (49)$$

$$G_{n\downarrow, m\uparrow}^r(E_1, E_2) = G_3(E_1, E_2) \quad (50)$$

$$G_{n\downarrow, m\downarrow}^r(E_1, E_2) = g_{n\downarrow}^r(E_1) 2\pi \delta(E_1 - E_2) + G_4(E_1, E_2) \quad (51)$$

5 Appendix

5.1 Double Fourier transformation

Note the double Fourier transformation, if

$$F(t) = \int dt_1 G_1(t, t_1) G_2(t_1, t) \quad (52)$$

then

$$F(\omega) = \int \frac{dE}{2\pi} \frac{dE'}{2\pi} G_1(E + \omega, E') G_2(E', E). \quad (53)$$

If

$$F(t_1, t_2) = \int dt G_1(t_1, t) G_2(t, t_2), \quad (54)$$

then

$$F(E_1, E_2) = \int \frac{dE}{2\pi} G_1(E_1, E) G_2(E, E_2). \quad (55)$$

If $F(t_1, t_2)$ depends only on $t_1 - t_2$,

$$F(E_1, E_2) = 2\pi F(E_1) \delta(E_1 - E_2) \quad (56)$$

Note that

$$\int_{-\infty}^{\infty} \delta(x) dx = 1. \quad (57)$$

$$\int_{a-\epsilon}^{a+\epsilon} f(x) \delta(x-a) dx = f(a) \quad (58)$$

6 Adiabatic regime(ω is small)

So, the charge current is given by (why? DC?)

$$dQ_{\alpha\sigma}(t)/dt = q \int \frac{dE}{2\pi} (-\partial_E f) [\Gamma_{\alpha} \mathbf{G}^r(t) \mathbf{\Delta G}^a(t)]_{\sigma\sigma} \quad (59)$$

References

- [1] Y, K, Kato. Observation of the Spin Hall Effect in Semiconductors[J]. Science, 2004.
- [2] Antti-Pekka Jauho, Quantum Kinetics in Transport and Optics of Semiconductors, P188.