Notes on PRB.67.092408

1 spin field effect transistor

A type of quantum field effect transistor that operates purely on the flow of spin current in the absence of charge current. The rotating field induces a time-independent dc spin current, and at the same time generates no charge cur- rent. The physical principle of our SFET is due to a spin flip mecha- nism provided by the field.

2 Hamiltonian

A rotating magnetic field is

$$B_x = B_0 \text{sin}\theta\ \cos(\omega \text{t}) \tag{1}$$

$$B_y = B_0 \text{sin}\theta\ \sin(\omega \text{t}) \tag{2}$$

$$B_z = B_0 \text{cos}\theta. \tag{3}$$

The Hamiltonian of system is

$$\begin{aligned} H = &\sum_{k,\sigma,\alpha=L,R} \epsilon_k C^{+}_{k\alpha\sigma} C_{k\alpha\sigma} + \sum_{n\sigma} \left[\epsilon_n + \sigma B_0 \cos\theta\right] d^{+}_{n\sigma} d_{n\sigma} \\ &+ H'(t) + \sum_{k,n,\sigma,\alpha=L,R} \left[T_{k\alpha n} C^{+}_{k\alpha\sigma} d_{n\sigma} + \text{c.c.}\right] \end{aligned} \tag{4}$$

We assume that there are multiple orbits in the scattering region, which is different from the original paper, in which only one orbit is considered. Energy level of lead $\epsilon_{Lk} = \epsilon_{Rk} = \epsilon_k$.

A counterclock-wise rotating field allows a spin-down electron to absorb a photon and flip to spin-up, and it does not allow a spin-up electron to absorb a photon and flip to spin-down.

$$H'(t) = \sum_n \gamma \left[\exp(-i\omega t) d^{+}_{n\uparrow} d_{n\downarrow} + \exp(i\omega t) d^{+}_{n\downarrow} d_{n\uparrow}\right] \tag{5}$$

$$\gamma = B_0 \sin\theta \tag{6}$$

The scattering region is characterized by an energy level $\epsilon_n = \epsilon^0_n - qV_g$, controlled by the gate voltage V_g.

We solve the transport properties (charge and spin currents) of the model in both adiabatic and nonadiabatic regimes using the standard Keldysh nonequilibrium Green's function technique.

3 Operator evolution

EoM of $d^{\dagger}_{\sigma}$ is

$$\partial_{t'} d^{\dagger}_{n\sigma}(t') = i[H, d^{\dagger}_{n\sigma}] \tag{7}$$

central dot part:

$$[\sum_{n'\sigma'}[\epsilon_{n'}+\sigma B_0\cos\theta]\,d^\dagger_{n'\sigma'}d_{n'\sigma'},d^\dagger_{n\sigma}]=(\epsilon_n+\sigma B_0\cos\theta)d^\dagger_{n\sigma} \tag{8}$$

rotating magnetic field part:

$$\begin{aligned}[d^\dagger_{n'\uparrow}d_{n'\downarrow},d^\dagger_{n\sigma}]&=d^\dagger_{n'\uparrow}\{d_{n'\downarrow},d^\dagger_{n\sigma}\}-\{d^\dagger_{n'\uparrow},d^\dagger_{n\sigma}\}d_{n'\downarrow}\\&=d^\dagger_{\uparrow}\delta_{nn'}\delta_{\sigma\downarrow},\end{aligned} \tag{9}$$

$$\begin{aligned}[d^\dagger_{n'\downarrow}d_{n'\uparrow},d^\dagger_{n\sigma}]&=d^\dagger_{n'\downarrow}\{d_{n'\uparrow},d^\dagger_{n\sigma}\}-\{d^\dagger_{n'\downarrow},d^\dagger_{n\sigma}\}d_{n'\uparrow}\\&=d^\dagger_{n'\uparrow}\delta_{nn'}\delta_{\sigma\uparrow}.\end{aligned} \tag{10}$$

Then

$$[H'(t),d^\dagger_{n\sigma}]=\gamma(e^{-i\omega t}d^\dagger_{n\uparrow}\delta_{\sigma\downarrow}+e^{i\omega t}d^\dagger_{n\downarrow}\delta_{\sigma\uparrow}) \tag{11}$$

Coupling part

$$\sum_{k,n',\sigma',\alpha=L,R}\Big[T_{n'k\alpha}C^\dagger_{k\alpha\sigma'}d_{n'\sigma'}+\text{c.c.}\ ,d^\dagger_{n\sigma}]=\sum_{k,\alpha=L,R}T_{k\alpha n}C^\dagger_{k\alpha\sigma} \tag{12}$$

Substitute these three parts into Eq. (7), we get

$$\partial_{t'}d^\dagger_{n\sigma}(t')=i[(\epsilon^0_n+\sigma B_0\cos\theta)d^\dagger_{n\sigma}+\gamma(e^{-i\omega t}d^\dagger_{n\uparrow}\delta_{\sigma\downarrow}+e^{i\omega t}d^\dagger_{n\downarrow}\delta_{\sigma\uparrow})+\sum_{k,\alpha=L,R}T_{k\alpha n}C^\dagger_{k\alpha\sigma}] \tag{13}$$

Equation of motion of particle operator $\hat{N}_{\alpha k\sigma}$ in the lead α is

$$\begin{aligned}\frac{d}{dt}\hat{N}_{\alpha k\sigma}=\frac{i}{\hbar}[H,C^\dagger_{\alpha k\sigma}C_{\alpha k\sigma}]&=\left[\sum_{k',\sigma',\alpha'=L,R}\left[T_{k'\alpha'}C^{+}_{k'\alpha'\sigma'}d_{\sigma'}+\text{c.c.}\right],C^\dagger_{\alpha k\sigma}C_{\alpha k\sigma}\right]\\&=\frac{i}{\hbar}\sum_{k',\sigma',\alpha'=L,R}\left[-T_{k'\alpha'}C^{+}_{k'\alpha'\sigma'}d_{\sigma'}\delta_{\alpha\alpha'}\delta_{kk'}\delta_{\sigma\sigma'}+\text{c.c.}\right]\\&=\frac{i}{\hbar}[-T_{k\alpha}C^\dagger_{k\alpha\sigma}d_\sigma+T^*_{k\alpha}d^\dagger_\sigma C_{k\alpha\sigma}]\end{aligned} \tag{14}$$

4 Charge current

So, the charge current due to L(R) lead with spin σ is given by

$$\begin{aligned}I_{L\sigma}(t)&=e\langle\frac{d}{dt}\hat{N}_\sigma(t)\rangle\\&=\frac{ie}{\hbar}\sum_{kn\alpha\in L}(\langle-T_{k\alpha n}C^\dagger_{k\alpha\sigma}(t)d_{n\sigma}(t)\rangle+\langle T^*_{k\alpha n}d^\dagger_{n\sigma}(t)C_{k\alpha\sigma}(t)\rangle)\end{aligned} \tag{15}$$

Define the lesser Green's function

$$G^<_{n\sigma',k\alpha\sigma}(\tau,\tau')=i\langle C^\dagger_{k\alpha\sigma}(\tau')d_{n\sigma'}(\tau)\rangle, \tag{16}$$

the current becomes

$$I_{L\sigma}(t)=\frac{-e}{\hbar}\sum_{kn\alpha\in L}(T_{k\alpha n}G^<_{n,k\alpha\sigma}(t,t)-T^*_{k\alpha n}G^<_{k\alpha,n}(t,t)\rangle) \tag{17}$$

More generally, we define the contour Green's function

$$G_{n\sigma',k\alpha\sigma}(\tau,\tau') = -i\langle d_{n\sigma'}(\tau)C^{\dagger}_{k\alpha\sigma}(\tau')\rangle. \tag{18}$$

EoM of operator $C^{\dagger}_{k\alpha\sigma}$ is

$$\partial_{t'}C^{\dagger}_{k\alpha\sigma}(t') = i[H, C^{\dagger}_{k\alpha\sigma}] = i(\varepsilon_k C^{\dagger}_{k\alpha\sigma} + \sum_n T_{k\alpha n} d^{\dagger}_{n\sigma}) \tag{19}$$

The equation-of-motion for the time-ordered Green function

$$\begin{aligned}
-i\frac{\partial}{\partial t'}G^{t}_{n\sigma',k\alpha\sigma}\left(t,t'\right) &= \delta(t-t')\langle\{d_{n\sigma'}, C^{\dagger}_{k\alpha\sigma}\}\rangle - \langle T_c d_{n\sigma'}\partial_{t'}C^{\dagger}_{k\alpha\sigma}\rangle \\
&= \varepsilon_k G^{t}_{n\sigma',k\alpha\sigma}\left(t,t'\right) + \sum_m T^{*}_{k\alpha m}G^{t}_{n\sigma',m\sigma}(t,t')
\end{aligned} \tag{20}$$

So, we have

$$(-i\frac{\partial}{\partial t'} - \varepsilon_k)G^{t}_{n\sigma',k\alpha\sigma}\left(t,t'\right) = \sum_m T^{*}_{k\alpha m}G^{t}_{n\sigma',m\sigma}(t,t') \tag{21}$$

in which

$$G^{t}_{n\sigma',m\sigma}(t,t') = -i\langle T_c d_{n\sigma'}(t)d^{\dagger}_{m\sigma}(t')\rangle. \tag{22}$$

Similarly, we evaluate the EoM for free Green's function $g^{t}_{k\alpha\sigma}(t,t')$ in lead α (note that $H = \sum_{k\sigma\alpha}\epsilon_k C^{\dagger}_{k\alpha\sigma}C_{k\alpha\sigma}$).

$$\begin{aligned}
-i\frac{\partial}{\partial t'}g^{t}_{k\alpha\sigma}\left(t,t'\right) &= \delta(t-t')\langle\{C_{k\alpha\sigma}, C^{\dagger}_{k\alpha\sigma}\}\rangle - \langle T_c C_{k\alpha\sigma}\partial_{t'}C^{\dagger}_{k\alpha\sigma}\rangle \\
&= \delta(t-t') + \varepsilon_k g^{t}_{k\alpha\sigma}\left(t,t'\right),
\end{aligned} \tag{23}$$

we have

$$(-i\frac{\partial}{\partial t'} - \varepsilon_k)g^{t}_{k\alpha\sigma}\left(t,t'\right) = \delta(t-t'). \tag{24}$$

Substitute Eq. (24) into Eq. (21) and integrate on both sides, we get an equation analogous to Jauho's notation [2],

$$G_{n,k\alpha}(\tau,\tau') = \sum_m \int d\tau_1 G_{nm}\left(\tau,\tau_1\right)t^{*}_{k\alpha m}g_{k\alpha}\left(\tau_1,\tau'\right),$$

we have

$$G^{t}_{n\sigma',k\alpha\sigma}(t,t') = \sum_m \int dt_1 G_{n\sigma',m\sigma}\left(t,t_1\right)T^{*}_{k\alpha m}g^{t}_{k\alpha\sigma}\left(\tau_1,\tau'\right). \tag{25}$$

When there is only one orbit presents, this equation reduces to

$$G^{t}_{\sigma',k\alpha\sigma}(t,t') = \int dt_1 G_{\sigma',\sigma}\left(t,t_1\right)T^{*}_{k\alpha}g^{t}_{k\alpha\sigma}\left(\tau_1,\tau'\right). \tag{26}$$

Since the contour Green's function has the same structure as real-time Green's function, the we have relation

$$G_{n\sigma',k\alpha\sigma}(\tau,\tau') = \sum_m \int d\tau_1 G_{n\sigma',m\sigma}\left(\tau,\tau_1\right)T^{*}_{k\alpha m}g_{k\alpha\sigma}\left(\tau_1,\tau'\right) \tag{27}$$

where $G_{n\sigma',k\alpha\sigma}(\tau,\tau')$ is contour Green's function defined in Eq. (16), and similarly the contour Green's function for non-interacting lead is defined as

$$g_{k\alpha\sigma}(\tau,\tau') = -i\langle T_c C_{k\alpha\sigma}(\tau)C^{\dagger}_{k\alpha\sigma}(\tau')\rangle \tag{28}$$

After analytic continuation, the current is formulated as

$$I_{\alpha\sigma}(t) = -\frac{e}{\hbar}\int dt_1 \,\mathrm{Tr}\left[G^r\left(t,t_1\right)\Sigma_\alpha^<\left(t_1,t\right) \right. \\ \left. +G^<\left(t,t_1\right)\Sigma_\alpha^a\left(t_1,t\right)\right] + h.c. \tag{29}$$

Following Eq. (224) in WangJ's note, its Fourier transformation is

$$I_{\alpha\sigma}(\omega) = -\frac{e}{\hbar}\int \frac{dE}{2\pi}\frac{dE'}{2\pi}\mathrm{Tr}\left[G^r\left(E+\omega,E'\right)\Sigma_\alpha^<\left(E',E\right) \right. \\ \left. + G^<(E+\omega,E')\Sigma_\alpha^a(E',E)\right] + c.c. \tag{30}$$

Here, $G^{r,<} \equiv G^{r,<}_{n\sigma',m\sigma}$, and matrix element

$$\Sigma^\gamma_{\alpha,mn}(t_1,t_2) = \sum_k T^*_{k\alpha m}(t_1) g^\gamma_{k\alpha}(t_1,t_2) T_{k\alpha n}(t_2), \tag{31}$$

in which, $g_{k\sigma}$ is the free propagator of lead. From Dyson equation, we have (different from Eq. (77) in Chap. II ?)

$$G^< = G^r\Sigma^<G^a \tag{32}$$

whose Fourier transformation gives

$$G^<(E_1,E_2) = \int \frac{dE}{2\pi}\frac{dE'}{2\pi} G^r(E_1,E)\Sigma^<(E,E')G^a(E',E_2) \tag{33}$$

For G^r, the analytic continuation gives

$$G^r = g^r + g^r H' G^r \tag{34}$$

$$G^r(E_1,E_2) = g^r(E_1,E_2) + \int \frac{dE}{2\pi}\frac{dE'}{2\pi} g^r(E_1,E)H'(E,E')G^r(E',E_2) \tag{35}$$

in which, g^r is the free propagator of central dot(?). Using Eq. (8), we have

$$\begin{aligned} g^r_{n\sigma}(t,t') =& - i\theta(t-t')\langle\{d_{n\sigma}(t), d^\dagger_{n\sigma}(t')\}\rangle_0 \\ =& - i\theta(t-t')e^{-i(\epsilon_n+\sigma B_0\cos\theta)(t-t')} \end{aligned} \tag{36}$$

note ϵ_n is time-dependent, $g^r(t,t')$ depends only on time difference, so

$$g^r\left(E_1,E_2\right) = 2\pi g^r\left(E_1\right)\delta(E_1-E_2) \tag{37}$$

and

$$g^r_{n\sigma}(E) = \frac{1}{E-(\epsilon_n+\sigma B_0\cos\theta)+i0^+} \tag{38}$$

In the spin space $\{d^\dagger_\uparrow,\, d^\dagger_\downarrow;\, d_\uparrow,\, d_\uparrow\}$,

$$G^r \equiv \begin{pmatrix} G^r_{n\uparrow,m\uparrow} & G^r_{n\uparrow,m\downarrow} \\ G^r_{n\downarrow,m\uparrow} & G^r_{n\downarrow,m\downarrow} \end{pmatrix} \tag{39}$$

$$g^r(E) = \begin{pmatrix} g^r_{n\uparrow}(E) & 0 \\ 0 & g^r_{n\downarrow}(E) \end{pmatrix}. \tag{40}$$

H' is given by

$$H' = \begin{pmatrix} 0 & \gamma e^{-i\omega t} \\ \gamma e^{i\omega t} & 0 \end{pmatrix} \tag{41}$$

we have

$$H'(E,E') = \begin{pmatrix} 0 & \gamma\delta(E-\omega) \\ \gamma\delta(E+\omega) & 0 \end{pmatrix} 2\pi\delta(E-E'). \tag{42}$$

Substitute these equations into Eq. (35), we get

$$\begin{aligned}
\begin{pmatrix} G^r_{n\uparrow,m\uparrow}(E_1,E_2) & G^r_{n\uparrow,m\downarrow}(E_1,E_2) \\ G^r_{n\downarrow,m\uparrow}(E_1,E_2) & G^r_{n\downarrow,m\downarrow}(E_1,E_2) \end{pmatrix} &= \begin{pmatrix} g^r_{n\uparrow}(E_1) & 0 \\ 0 & g^r_{n\downarrow}(E_1) \end{pmatrix} 2\pi\delta(E_1-E_2) \\
&+ \int \frac{dE}{2\pi}\frac{dE'}{2\pi} \begin{pmatrix} g^r_{n\uparrow}(E_1) & 0 \\ 0 & g^r_{n\downarrow}(E_1) \end{pmatrix} 2\pi\delta(E_1-E)2\pi\delta(E-E') \\
&\times \begin{pmatrix} 0 & \gamma\delta(E-\omega) \\ \gamma\delta(E+\omega) & 0 \end{pmatrix} \begin{pmatrix} G^r_{n\uparrow,m\uparrow} & G^r_{n\uparrow,m\downarrow} \\ G^r_{n\downarrow,m\uparrow} & G^r_{n\downarrow,m\downarrow} \end{pmatrix} \\
&= \begin{pmatrix} g^r_{n\uparrow}(E_1) & 0 \\ 0 & g^r_{n\downarrow}(E_1) \end{pmatrix} 2\pi\delta(E_1-E_2) \\
&+ \begin{pmatrix} G_1(E_1,E_2) & G_2(E_1,E_2) \\ G_3(E_1,E_2) & G_4(E_1,E_2) \end{pmatrix}.
\end{aligned} \tag{43}$$

In which G_1, G_2, G_3, G_4 are

$$G_1(E_1,E_2) = \int dEdE' g^r_{n\uparrow}(E_1)\,\delta(E_1-E)\gamma\delta(E-\omega)\delta(E-E')G^r_{n\downarrow,m\uparrow}(E',E_2) \tag{44}$$

$$G_2(E_1,E_2) = \int dEdE' g^r_{n\uparrow}(E_1)\,\delta(E_1-E)\gamma\delta(E-\omega)\delta(E-E')G^r_{n\downarrow,m\downarrow}(E',E_2) \tag{45}$$

$$G_3(E_1,E_2) = \int dEdE' g^r_{n\downarrow}(E_1)\,\delta(E_1-E)\gamma\delta(E+\omega)\delta(E-E')G^r_{n\uparrow,m\uparrow}(E',E_2) \tag{46}$$

$$G_4(E_1,E_2) = \int dEdE' g^r_{n\uparrow}(E_1)\,\delta(E_1-E)\gamma\delta(E+\omega)\delta(E-E')G^r_{n\uparrow,m\downarrow}(E',E_2) \tag{47}$$

So, the G_1 term

$$\begin{aligned}
G^r_{n\uparrow,m\uparrow}(E_1,E_2) &= g^r_{n\uparrow}(E_1)\,2\pi\delta\,(E_1-E_2) + G_1\,(E_1,E_2) \\
&= 2\pi g^r_{n\uparrow}(E_1)\,\delta\,(E_1-E_2) + \gamma g^r_{n\uparrow}(E_1)\,\delta(E_1-\omega)G^r_{n\downarrow,m\uparrow}(E_1,E_2)
\end{aligned} \tag{48}$$

The G_2 term

$$\begin{aligned}
G^r_{n\uparrow,m\downarrow}(E_1,E_2) =& G_2\,(E_1,E_2) \\
=& \int dE'\gamma g^r_{n\uparrow}(E_1)\,G^r_{n\downarrow,m\downarrow}(E',E_2)\left[\int dE\delta(E_1-E)\delta(E-\omega)\delta(E-E')\right] \\
=& \int dE'\gamma g^r_{n\uparrow}(E_1)\,G^r_{n\downarrow,m\downarrow}(E',E_2)\left[\delta(E_1-E')\delta(E'-\omega)\right] \\
=& \gamma g^r_{n\uparrow}(E_1)\,\delta(E_1-\omega)G^r_{n\downarrow,m\downarrow}(E_1,E_2)
\end{aligned} \tag{49}$$

The G_3 term

$$\begin{aligned}
G^r_{n\downarrow,m\uparrow}(E_1,E_2) &= G_3\,(E_1,E_2) \\
&= \gamma g^r_{n\downarrow}(E_1)\,\delta(E_1+\omega)G^r_{n\uparrow,m\uparrow}(E_1,E_2)
\end{aligned} \tag{50}$$

and the G_4 term

$$\begin{aligned} G^r_{n\downarrow,m\downarrow}\left(E_1,E_2\right) &= g^r_{n\downarrow}\left(E_1\right) 2\pi\delta\left(E_1-E_2\right)+G_4\left(E_1,E_2\right) \\ &= 2\pi g^r_{n\downarrow}\left(E_1\right)\delta\left(E_1-E_2\right)+\gamma g^r_{n\uparrow}(E_1)\delta(E_1+\omega)G^r_{n\uparrow,m\downarrow}(E_1,E_2) \end{aligned} \tag{51}$$

After collection terms, we get

$$\begin{aligned} G^r_{n\uparrow,m\uparrow}(E_1,E_2) =& 2\pi g^r_{n\uparrow}\left(E_1\right)\delta\left(E_1-E_2\right)+\gamma g^r_{n\uparrow}\left(E_1\right)\delta(E_1-\omega) \\ & \times\gamma g^r_{n\downarrow}\left(E_1\right)\delta(E_1+\omega)G^r_{n\uparrow,m\uparrow}(E_1,E_2) \\ =& \frac{2\pi g^r_{n\uparrow}\left(E_1\right)\delta\left(E_1-E_2\right)}{1-\gamma^2 g^r_{n\uparrow}\left(E_1\right)g^r_{n\downarrow}\left(E_1\right)\delta(E_1-\omega)\delta(E_1+\omega)} \end{aligned} \tag{52}$$

and

$$G^r_{n\downarrow,m\downarrow}(E_1,E_2) = \frac{2\pi g^r_{n\downarrow}\left(E_1\right)\delta\left(E_1-E_2\right)}{1-\gamma^2 g^r_{n\uparrow}\left(E_1\right)g^r_{n\uparrow}\left(E_1\right)\delta(E_1-\omega)\delta(E_1+\omega)} \tag{53}$$

and

$$\begin{aligned} G^r_{n\uparrow,m\downarrow}\left(E_1,E_2\right) &= \gamma g^r_{n\uparrow}\left(E_1\right)\delta(E_1-\omega)G^r_{n\downarrow,m\downarrow}(E_1,E_2) \\ &= \frac{2\pi\gamma g^r_{n\uparrow}\left(E_1\right)g^r_{n\downarrow}\left(E_1\right)\delta(E_1-\omega)\delta\left(E_1-E_2\right)}{1-\gamma^2 g^r_{n\uparrow}\left(E_1\right)g^r_{n\uparrow}\left(E_1\right)\delta(E_1-\omega)\delta(E_1+\omega)} \end{aligned} \tag{54}$$

$$\begin{aligned} G^r_{n\downarrow,m\uparrow}\left(E_1,E_2\right) &= \gamma g^r_{n\downarrow}\left(E_1\right)\delta(E_1+\omega)G^r_{n\uparrow,m\uparrow}(E_1,E_2) \\ &= \frac{2\pi\gamma g^r_{n\uparrow}\left(E_1\right)g^r_{n\downarrow}\left(E_1\right)\delta(E_1+\omega)\delta\left(E_1-E_2\right)}{1-\gamma^2 g^r_{n\uparrow}\left(E_1\right)g^r_{n\downarrow}\left(E_1\right)\delta(E_1-\omega)\delta(E_1+\omega)} \end{aligned} \tag{55}$$

5 Appendix

5.1 Double Fourier transformation

Note the double Fourier transformation, if

$$F(t) = \int dt_1 G_1\left(t,t_1\right)G_2\left(t_1,t\right) \tag{56}$$

then

$$F(\omega) = \int \frac{dE}{2\pi}\frac{dE'}{2\pi}G_1\left(E+\omega,E'\right)G_2\left(E',E\right). \tag{57}$$

If

$$F\left(t_1,t_2\right) = \int dt G_1\left(t_1,t\right)G_2\left(t,t_2\right), \tag{58}$$

then

$$F\left(E_1,E_2\right) = \int \frac{dE}{2\pi}G_1\left(E_1,E\right)G_2\left(E,E_2\right). \tag{59}$$

If $F(t_1,t_2)$ depends only on t_1-t_2,

$$F\left(E_1,E_2\right) = 2\pi F\left(E_1\right)\delta(E_1-E_2) \tag{60}$$

Note that

$$\int_{-\infty}^{\infty}\delta(x)dx = 1. \tag{61}$$

$$\int_{a-\epsilon}^{a+\epsilon} f(x)\delta(x-a)dx = f(a) \tag{62}$$

6 Adiabatic regime(ω is small)

So, the charge current is given by (why? DC?)

$$dQ_{\alpha\sigma}(t)/dt = q\int \frac{dE}{2\pi}\left(-\partial_E f\right)\left[\Gamma_\alpha \mathbf{G}^r(t)\boldsymbol{\Delta}\mathbf{G}^a(t)\right]_{\sigma\sigma} \tag{63}$$

References

[1] Y, K, Kato. Observation of the Spin Hall Effect in Semiconductors[J]. Science, 2004.

[2] Antti-Pekka Jauho, Quantum Kinetics in Transport and Optics of Semiconductors, P188.