Notes on PRB.67.092408

1 spin field effect transistor

A type of quantum field effect transistor that operates purely on the flow of spin current in the absence of charge current. The rotating field induces a time-independent dc spin current, and at the same time generates no charge cur- rent. The physical principle of our SFET is due to a spin flip mecha- nism provided by the field.

2 Hamiltonian

A rotating magnetic field is

$$B_x = B_0 \sin\theta \, \cos(\omega t) \tag{1}$$

$$B_y = B_0 \sin\theta \, \sin(\omega t) \tag{2}$$

$$B_z = B_0 \cos\theta. \tag{3}$$

The Hamiltonian of system is

$$
\begin{aligned}
H = & \sum_{k,\sigma,\alpha=L,R} \epsilon_k C^+_{k\alpha\sigma} C_{k\alpha\sigma} + \sum_{n\sigma} \left[\epsilon_n + \sigma B_0 \cos\theta \right] d^+_{n\sigma} d_{n\sigma} \\
& + H'(t) + \sum_{k,n,\sigma,\alpha=L,R} \left[T_{k\alpha n} C^+_{k\alpha\sigma} d_{n\sigma} + \text{c.c.} \right]
\end{aligned}
\tag{4}
$$

We assume that there are multiple orbits in the scattering region, which is different from the original paper, in which only one orbit is considered. Energy level of lead $\epsilon_{Lk} = \epsilon_{Rk} = \epsilon_k$.

A counterclock-wise rotating field allows a spin-down electron to absorb a photon and flip to spin-up, and it does not allow a spin-up electron to absorb a photon and flip to spin-down.

$$H'(t) = \sum_n \gamma \left[\exp(-i\omega t) d^+_{n\uparrow} d_{n\downarrow} + \exp(i\omega t) d^+_{n\downarrow} d_{n\uparrow} \right] \tag{5}$$

$$\gamma = B_0 \sin\theta \tag{6}$$

The scattering region is characterized by an energy level $\epsilon_n = \epsilon_n^0 - qV_g$, controlled by the gate voltage V_g.

We solve the transport properties (charge and spin currents) of the model in both adiabatic and nonadiabatic regimes using the standard Keldysh nonequilibrium Green's function technique.

3 Operator evolution

EoM of $d^\dagger_\sigma$ is

$$\partial_{t'} d^\dagger_{n\sigma}(t') = i[H, d^\dagger_{n\sigma}] \tag{7}$$

central dot part:

$$[\sum_{n'\sigma'}[\epsilon_{n'} + \sigma B_0 \cos\theta] d^\dagger_{n'\sigma'} d_{n'\sigma'}, d^\dagger_{n\sigma}] = (\epsilon_n + \sigma B_0 \cos\theta) d^\dagger_{n\sigma} \tag{8}$$

rotating magnetic field part:

$$[d^\dagger_{n'\uparrow} d_{n'\downarrow}, d^\dagger_{n\sigma}] = d^\dagger_{n'\uparrow} \{d_{n'\downarrow}, d^\dagger_{n\sigma}\} - \{d^\dagger_{n'\uparrow}, d^\dagger_{n\sigma}\} d_{n'\downarrow}$$
$$= d^\dagger_\uparrow \delta_{nn'} \delta_{\sigma\downarrow}, \tag{9}$$

$$[d^\dagger_{n'\downarrow} d_{n'\uparrow}, d^\dagger_{n\sigma}] = d^\dagger_{n'\downarrow} \{d_{n'\uparrow}, d^\dagger_{n\sigma}\} - \{d^\dagger_{n'\downarrow}, d^\dagger_{n\sigma}\} d_{n'\uparrow}$$
$$= d^\dagger_{n'\uparrow} \delta_{nn'} \delta_{\sigma\uparrow}. \tag{10}$$

Then

$$[H'(t), d^\dagger_{n\sigma}] = \gamma(e^{-i\omega t} d^\dagger_{n\uparrow} \delta_{\sigma\downarrow} + e^{i\omega t} d^\dagger_{n\downarrow} \delta_{\sigma\uparrow}) \tag{11}$$

Coupling part

$$\sum_{k,n',\sigma',\alpha=L,R} \left[T_{n'k\alpha} C^\dagger_{k\alpha\sigma'} d_{n'\sigma'} + \text{c.c.}, d^\dagger_{n\sigma}\right] = \sum_{k,\alpha=L,R} T_{k\alpha n} C^\dagger_{k\alpha\sigma} \tag{12}$$

Substitute these three parts into Eq. (7), we get

$$\partial_{t'} d^\dagger_{n\sigma}(t') = i[(\epsilon^0_n + \sigma B_0 \cos\theta) d^\dagger_{n\sigma} + \gamma(e^{-i\omega t} d^\dagger_{n\uparrow} \delta_{\sigma\downarrow} + e^{i\omega t} d^\dagger_{n\downarrow} \delta_{\sigma\uparrow}) + \sum_{k,\alpha=L,R} T_{k\alpha n} C^\dagger_{k\alpha\sigma}] \tag{13}$$

Equation of motion of particle operator $\hat{N}_{\alpha k\sigma}$ in the lead α is

$$\frac{d}{dt}\hat{N}_{\alpha k\sigma} = \frac{i}{\hbar}[H, C^\dagger_{\alpha k\sigma} C_{\alpha k\sigma}] = \left[\sum_{k',\sigma',\alpha'=L,R} [T_{k'\alpha'} C^+_{k'\alpha'\sigma'} d_{\sigma'} + \text{c.c.}], C^\dagger_{\alpha k\sigma} C_{\alpha k\sigma}\right]$$
$$= \frac{i}{\hbar} \sum_{k',\sigma',\alpha'=L,R} [-T_{k'\alpha'} C^+_{k'\alpha'\sigma'} d_{\sigma'} \delta_{\alpha\alpha'} \delta_{kk'} \delta_{\sigma\sigma'} + \text{c.c.}]$$
$$= \frac{i}{\hbar}[-T_{k\alpha} C^\dagger_{k\alpha\sigma} d_\sigma + T^*_{k\alpha} d^\dagger_\sigma C_{k\alpha\sigma}] \tag{14}$$

4 Charge current

So, the charge current due to L(R) lead with spin σ is given by

$$I_{L\sigma}(t) = e\langle \frac{d}{dt}\hat{N}_\sigma(t)\rangle$$
$$= \frac{ie}{\hbar} \sum_{kn\alpha\in L} (\langle -T_{k\alpha n} C^\dagger_{k\alpha\sigma}(t) d_{n\sigma}(t)\rangle + \langle T^*_{k\alpha n} d^\dagger_{n\sigma}(t) C_{k\alpha\sigma}(t)\rangle) \tag{15}$$

Define the lesser Green's function

$$G^<_{n\sigma',k\alpha\sigma}(\tau,\tau') = i\langle C^\dagger_{k\alpha\sigma}(\tau') d_{n\sigma'}(\tau)\rangle, \tag{16}$$

the current becomes

$$I_{L\sigma}(t) = \frac{-e}{\hbar} \sum_{kn\alpha\in L} (T_{k\alpha n} G^<_{n,k\alpha\sigma}(t,t) - T^*_{k\alpha n} G^<_{k\alpha,n}(t,t)\rangle) \tag{17}$$

More generally, we define the contour Green's function

$$G_{n\sigma',k\alpha\sigma}(\tau,\tau') = -i\langle d_{n\sigma'}(\tau)C_{k\alpha\sigma}^\dagger(\tau')\rangle. \tag{18}$$

EoM of operator $C_{k\alpha\sigma}^\dagger$ is

$$\partial_{t'}C_{k\alpha\sigma}^\dagger(t') = i[H,C_{k\alpha\sigma}^\dagger] = i(\varepsilon_k C_{k\alpha\sigma}^\dagger + \sum_n T_{k\alpha n}d_{n\sigma}^\dagger) \tag{19}$$

The equation-of-motion for the time-ordered Green function

$$\begin{aligned}
-i\frac{\partial}{\partial t'}G_{n\sigma',k\alpha\sigma}^t\left(t,t'\right) &= \delta(t-t')\langle\{d_{n\sigma'},C_{k\alpha\sigma}^\dagger\}\rangle - \langle T_c d_{n\sigma'}\partial_{t'}C_{k\alpha\sigma}^\dagger\rangle \\
&= \varepsilon_k G_{n\sigma',k\alpha\sigma}^t\left(t,t'\right) + \sum_m T_{k\alpha m}^* G_{n\sigma',m\sigma}^t(t,t')
\end{aligned} \tag{20}$$

So, we have

$$(-i\frac{\partial}{\partial t'} - \varepsilon_k)G_{n\sigma',k\alpha\sigma}^t\left(t,t'\right) = \sum_m T_{k\alpha m}^* G_{n\sigma',m\sigma}^t(t,t') \tag{21}$$

in which

$$G_{n\sigma',m\sigma}^t(t,t') = -i\langle T_c d_{n\sigma'}(t)d_{m\sigma}^\dagger(t')\rangle. \tag{22}$$

Similarly, we evaluate the EoM for free Green's function $g_{k\alpha\sigma}^t(t,t')$ in lead α (note that $H = \sum_{k\sigma\alpha}\epsilon_k C_{k\alpha\sigma}^\dagger C_{k\alpha\sigma}$).

$$\begin{aligned}
-i\frac{\partial}{\partial t'}g_{k\alpha\sigma}^t\left(t,t'\right) &= \delta(t-t')\langle\{C_{k\alpha\sigma},C_{k\alpha\sigma}^\dagger\}\rangle - \langle T_c C_{k\alpha\sigma}\partial_{t'}C_{k\alpha\sigma}^\dagger\rangle \\
&= \delta(t-t') + \varepsilon_k g_{k\alpha\sigma}^t\left(t,t'\right),
\end{aligned} \tag{23}$$

we have

$$(-i\frac{\partial}{\partial t'} - \varepsilon_k)g_{k\alpha\sigma}^t\left(t,t'\right) = \delta(t-t'). \tag{24}$$

Substitute Eq. (24) into Eq. (21) and integrate on both sides, we get an equation analogous to Jauho's notation [2],

$$G_{n,k\alpha}(\tau,\tau') = \sum_m\int d\tau_1 G_{nm}\left(\tau,\tau_1\right)t_{k\alpha m}^* g_{k\alpha}\left(\tau_1,\tau'\right),$$

we have

$$G_{n\sigma',k\alpha\sigma}^t(t,t') = \sum_m\int dt_1 G_{n\sigma',m\sigma}\left(t,t_1\right)T_{k\alpha m}^* g_{k\alpha\sigma}^t\left(\tau_1,\tau'\right). \tag{25}$$

When there is only one orbit presents, this equation reduces to

$$G_{\sigma',k\alpha\sigma}^t(t,t') = \int dt_1 G_{\sigma',\sigma}\left(t,t_1\right)T_{k\alpha}^* g_{k\alpha\sigma}^t\left(\tau_1,\tau'\right). \tag{26}$$

Since the contour Green's function has the same structure as real-time Green's function, the we have relation

$$G_{n\sigma',k\alpha\sigma}(\tau,\tau') = \sum_m\int d\tau_1 G_{n\sigma',m\sigma}\left(\tau,\tau_1\right)T_{k\alpha m}^* g_{k\alpha\sigma}\left(\tau_1,\tau'\right) \tag{27}$$

where $G_{n\sigma',k\alpha\sigma}(\tau,\tau')$ is contour Green's function defined in Eq. (16), and similarly the contour Green's function for non-interacting lead is defined as

$$g_{k\alpha\sigma}(\tau,\tau') = -i\langle T_c C_{k\alpha\sigma}(\tau)C_{k\alpha\sigma}^\dagger(\tau')\rangle \tag{28}$$

3

After analytic continuation, the current is formulated as

$$I_{\alpha\sigma}(t) = -\frac{e}{\hbar} \int dt_1 \operatorname{Tr} \left[G^r(t, t_1) \Sigma_\alpha^< (t_1, t) \right.$$
$$\left. + G^<(t, t_1) \Sigma_\alpha^a (t_1, t) \right] + h.c. \tag{29}$$

Following Eq. (224) in WangJ's note, its Fourier transformation is

$$I_{\alpha\sigma}(\omega) = -\frac{e}{\hbar} \int \frac{dE}{2\pi} \frac{dE'}{2\pi} \operatorname{Tr} \left[G^r(E + \omega, E') \Sigma_\alpha^< (E', E) \right.$$
$$\left. + G^<(E + \omega, E') \Sigma_\alpha^a (E', E) \right] + c.c. \tag{30}$$

Here, $G^{r,<} \equiv G_{n\sigma',m\sigma}^{r,<}$, and matrix element

$$\Sigma_{\alpha,mn}^\gamma (t_1, t_2) = \sum_k T_{k\alpha m}^*(t_1) g_{k\alpha}^\gamma(t_1, t_2) T_{k\alpha n}(t_2), \tag{31}$$

in which, $g_{k\sigma}$ is the free propagator of lead, and γ is $\{>, <, r, a\}$. From Dyson equation, we have (different from Eq. (77) in Chap. II ?)

$$G^< = G^r \Sigma^< G^a \tag{32}$$

whose Fourier transformation gives

$$G^<(E_1, E_2) = \int \frac{dE}{2\pi} \frac{dE'}{2\pi} G^r(E_1, E) \Sigma^<(E, E') G^a(E', E_2) \tag{33}$$

5   Calculate G^r

For G^r, the Dyson equation gives (refer to [3])

$$G^r(t_1, t_2) = G^{0r}(t_1 - t_2) + \int dt \, G^{0r}(t_1 - t) H'(t) G^r(t, t_2) \tag{34}$$

in which, G^{0r} is not the free propagator of central dot, but the equilibrium Green's function when the pumping potential H is set to zero, i.e. the hamiltonian is $H - H'$, not $\varepsilon_n d_{n\sigma}^\dagger d_{n\sigma}$. Multiply $e^{iE_1 t_1 - iE_2 t_2}$ and integrate on both sides, we have

$$G^r(E_1, E_2) \equiv \int dt_1 dt_2 e^{iE_1 t_1 - iE_2 t_2} G^r(t_1, t_2)$$
$$= 2\pi G^{0r}(E_1) \delta(E_1 - E_2)$$
$$+ \int dt_1 G^{0r}(t_1 - t) e^{iE_1(t_1 - t)} H'(t) e^{i(E_1 - E)t} \iint dt dt_2 G^r(t, t_2) e^{iEt - E_2 t_2} \tag{35}$$
$$= 2\pi G^{0r}(E_1) \delta(E_1 - E_2) + \int \frac{dE}{2\pi} G^{0r}(E_1) H'(E_1 - E) G^r(E, E_2),$$

in which, we inserted the inverse Fourier transformation

$$H'(t) = \int \frac{dE}{2\pi} e^{-i(E_1 - E)t} H'(E_1 - E). \tag{36}$$

4

5.1 Calcualte G^{0r}

Using Eq. (8) and Eq. (12), we have

$$\partial_t d_{n\sigma}^\dagger = i(\epsilon_n + \sigma B_0 \cos\theta)d_{n\sigma}^\dagger + i \sum_{k,\alpha=L,R} T_{k\alpha n}C_{k\alpha\sigma}^\dagger \tag{37}$$

$$\partial_t d_{n\sigma} = -i(\epsilon_n + \sigma B_0 \cos\theta)d_{n\sigma} - i \sum_{k,\alpha=L,R} T_{k\alpha n}^* C_{k\alpha\sigma}^\dagger \tag{38}$$

then

$$d_{n\sigma}^\dagger(t) = d_{n\sigma}^\dagger(0)e^{i(\epsilon_n + \sigma B_0 \cos\theta)t} + it \sum_{k,\alpha=L,R} T_{k\alpha n}C_{k\alpha\sigma}^\dagger \tag{39}$$

$$d_{n\sigma}(t) = d_{n\sigma}(0)e^{-i(\epsilon_n + \sigma B_0 \cos\theta)} - it \sum_{k,\alpha=L,R} T_{k\alpha n}^* C_{k\alpha\sigma} \tag{40}$$

Note ϵ_n is time-dependent, $G^{0r}(t,t')$ depends only on time difference.

$$G_{n\sigma}^{0r}(t,t') = -i\theta(t-t')\langle\{d_{n\sigma}(t), d_{n\sigma}^\dagger(t')\}\rangle \tag{41}$$

The hamiltonian $H - H'$ cannot flip spin, so in the spin space $\{d_\uparrow^\dagger, d_\downarrow^\dagger; d_\uparrow, d_\uparrow\}$, G^{0r} is diagonal

$$G^{0r}(E) = \begin{pmatrix} G_{n\uparrow}^{0r}(E) & 0 \\ 0 & G_{n\downarrow}^{0r}(E) \end{pmatrix}. \tag{42}$$

5.2 Calcualte G^r

In spin space

$$G^r \equiv \begin{pmatrix} G_{n\uparrow,m\uparrow}^r & G_{n\uparrow,m\downarrow}^r \\ G_{n\downarrow,m\uparrow}^r & G_{n\downarrow,m\downarrow}^r \end{pmatrix} \tag{43}$$

H' in Eq. (34)is given by

$$H' = \begin{pmatrix} 0 & \gamma e^{-i\omega t} \\ \gamma e^{i\omega t} & 0 \end{pmatrix} \tag{44}$$

we have

$$\begin{aligned} H'(E_1 - E) &= \int dt e^{i(E_1-E)t} H'(t) \\ &= \begin{pmatrix} 0 & \gamma \int dt e^{i(E_1-E-\omega)t} \\ \gamma \int dt e^{i(E_1-E+\omega)t} & 0 \end{pmatrix} \\ &= 2\pi\gamma \begin{pmatrix} 0 & \delta(E_1 - E - \omega) \\ \delta(E_1 - E + \omega) & 0 \end{pmatrix} \end{aligned} \tag{45}$$

5

Substitute these equations into Eq. (35), we get

$$
\begin{pmatrix} G^r_{n\uparrow,m\uparrow}(E_1,E_2) & G^r_{n\uparrow,m\downarrow}(E_1,E_2) \\ G^r_{n\downarrow,m\uparrow}(E_1,E_2) & G^r_{n\downarrow,m\downarrow}(E_1,E_2) \end{pmatrix} = \begin{pmatrix} G^{0r}_{n\uparrow}(E_1) & 0 \\ 0 & G^{0r}_{n\downarrow}(E_1) \end{pmatrix} 2\pi\delta(E_1-E_2)
$$
$$
+ \gamma \int dE \begin{pmatrix} G^{0r}_{n\uparrow}(E_1) & 0 \\ 0 & G^{0r}_{n\downarrow}(E_1) \end{pmatrix}
$$
$$
\times \begin{pmatrix} 0 & \delta(E_1-E-\omega) \\ \delta(E_1-E+\omega) & 0 \end{pmatrix} \begin{pmatrix} G^r_{n\uparrow,m\uparrow} & G^r_{n\uparrow,m\downarrow} \\ G^r_{n\downarrow,m\uparrow} & G^r_{n\downarrow,m\downarrow} \end{pmatrix}
$$
$$
= \begin{pmatrix} G^{0r}_{n\uparrow}(E_1) & 0 \\ 0 & G^{0r}_{n\downarrow}(E_1) \end{pmatrix} 2\pi\delta(E_1-E_2)
$$
$$
+ \begin{pmatrix} G_1(E_1,E_2) & G_2(E_1,E_2) \\ G_3(E_1,E_2) & G_4(E_1,E_2) \end{pmatrix}.
$$

(46)

where we have omitted the independent variables (E,E_2) of $G^r_{n\sigma,m\sigma'}$ for sake of convenience. In the above equations G_1,G_2,G_3,G_4 are

$$
G_1(E_1,E_2) = \gamma \int dE G^{0r}_{n\uparrow}(E_1)\,\delta(E_1-E-\omega)G^r_{n\downarrow,m\uparrow}(E,E_2)
$$

(47)

$$
G_2(E_1,E_2) = \gamma \int dE G^{0r}_{n\uparrow}(E_1)\,\delta(E_1-E-\omega)G^r_{n\downarrow,m\downarrow}(E,E_2)
$$

(48)

$$
G_3(E_1,E_2) = \gamma \int dE G^{0r}_{n\downarrow}(E_1)\,\delta(E_1-E+\omega)G^r_{n\uparrow,m\uparrow}(E,E_2)
$$

(49)

$$
G_4(E_1,E_2) = \gamma \int dE G^{0r}_{n\uparrow}(E_1)\,\delta(E_1-E+\omega)G^r_{n\uparrow,m\downarrow}(E,E_2)
$$

(50)

So, the G_1 term

$$
\begin{aligned}
G^r_{n\uparrow,m\uparrow}(E_1,E_2) &= G^{0r}_{n\uparrow}(E_1)\,2\pi\delta(E_1-E_2) + G_1(E_1,E_2) \\
&= 2\pi G^{0r}_{n\uparrow}(E_1)\,\delta(E_1-E_2) + \gamma G^{0r}_{n\uparrow}(E_1)\,G^r_{n\downarrow,m\uparrow}(E_1-\omega,E_2)
\end{aligned}
$$

(51)

The G_2 term

$$
\begin{aligned}
G^r_{n\uparrow,m\downarrow}(E_1,E_2) &= G_2(E_1,E_2) \\
&= \gamma G^{0r}_{n\uparrow}(E_1)\,G^r_{n\downarrow,m\downarrow}(E_1-\omega,E_2)
\end{aligned}
$$

(52)

The G_3 term

$$
\begin{aligned}
G^r_{n\downarrow,m\uparrow}(E_1,E_2) &= G_3(E_1,E_2) \\
&= \gamma G^{0r}_{n\downarrow}(E_1)\,G^r_{n\uparrow,m\uparrow}(E_1+\omega,E_2)
\end{aligned}
$$

(53)

and the G_4 term

$$
\begin{aligned}
G^r_{n\downarrow,m\downarrow}(E_1,E_2) &= g^r_{n\downarrow}(E_1)\,2\pi\delta(E_1-E_2) + G_4(E_1,E_2) \\
&= 2\pi G^{0r}_{n\downarrow}(E_1)\,\delta(E_1-E_2) + \gamma G^{0r}_{n\uparrow}(E_1)G^r_{n\uparrow,m\downarrow}(E_1+\omega,E_2)
\end{aligned}
$$

(54)

After collecting terms, we get

$$
\begin{aligned}
G^r_{n\uparrow,m\uparrow}(E_1,E_2) &= 2\pi G^{0r}_{n\uparrow}(E_1)\,\delta(E_1-E_2) + \gamma G^{0r}_{n\uparrow}(E_1)\,\gamma G^{0r}_{n\downarrow}(E_1-\omega)G^r_{n\uparrow,m\uparrow}(E_1,E_2) \\
&= \frac{2\pi G^{0r}_{n\uparrow}(E_1)\,\delta(E_1-E_2)}{1-\gamma^2 G^{0r}_{n\uparrow}(E_1)\,G^{0r}_{n\downarrow}(E_1-\omega)}
\end{aligned}
$$

(55)

6

and

$$G^r_{n\downarrow,m\downarrow}(E_1, E_2) = \frac{2\pi G^{0r}_{n\downarrow}(E_1)\,\delta(E_1 - E_2)}{1 - \gamma^2 G^{0r}_{n\uparrow}(E_1)\,G^{0r}_{n\uparrow}(E_1 + \omega)} \tag{56}$$

and

$$G^r_{n\uparrow,m\downarrow}(E_1, E_2) = \gamma G^{0r}_{n\uparrow}(E_1)\,G^r_{n\downarrow,m\downarrow}(E_1 - \omega, E_2)$$
$$= \frac{2\pi\gamma G^{0r}_{n\uparrow}(E_1)\,G^{0r}_{n\downarrow}(E_1 - \omega)\,\delta(E_1 - \omega - E_2)}{1 - \gamma^2 G^{0r}_{n\uparrow}(E_1 - \omega)\,G^{0r}_{n\uparrow}(E_1)} \tag{57}$$

$$G^r_{n\downarrow,m\uparrow}(E_1, E_2) = \gamma G^{0r}_{n\downarrow}(E_1)\,G^r_{n\uparrow,m\uparrow}(E_1 + \omega, E_2)$$
$$= \frac{2\pi\gamma G^{0r}_{n\uparrow}(E_1 + \omega)\,G^{0r}_{n\downarrow}(E_1)\,\delta(E_1 + \omega - E_2)}{1 - \gamma^2 G^{0r}_{n\uparrow}(E_1 + \omega)\,G^{0r}_{n\downarrow}(E_1)} \tag{58}$$

6   Calcualte $\Sigma_{\alpha,mn}$

In Eq. (**??**)

7   Appendix

7.1   Double Fourier transformation

Note the double Fourier transformation, if

$$F(t) = \int dt_1 G_1(t, t_1)\,G_2(t_1, t) \tag{59}$$

then

$$F(\omega) = \int \frac{dE}{2\pi}\frac{dE'}{2\pi} G_1\left(E + \omega, E'\right) G_2\left(E', E\right). \tag{60}$$

If

$$F(t_1, t_2) = \int dt\, G_1(t_1, t)\,G_2(t, t_2), \tag{61}$$

then

$$F(E_1, E_2) = \int \frac{dE}{2\pi} G_1(E_1, E)\,G_2(E, E_2). \tag{62}$$

If $F(t_1, t_2)$ depends only on $t_1 - t_2$,

$$F(E_1, E_2) = 2\pi F(E_1)\,\delta(E_1 - E_2) \tag{63}$$

Note that

$$\int_{-\infty}^{\infty} \delta(x)dx = 1. \tag{64}$$

$$\int_{a-\epsilon}^{a+\epsilon} f(x)\delta(x - a)dx = f(a) \tag{65}$$

8   Adiabatic regime(ω is small)

So, the charge current is given by (why? DC?)

$$dQ_{\alpha\sigma}(t)/dt = q \int \frac{dE}{2\pi} \left(-\partial_E f\right) \left[\Gamma_\alpha \mathbf{G}^r(t)\mathbf{\Delta G}^a(t)\right]_{\sigma\sigma} \tag{66}$$

References

[1] Y, K, Kato. Observation of the Spin Hall Effect in Semiconductors[J]. Science, 2004.

[2] Antti-Pekka Jauho, Quantum Kinetics in Transport and Optics of Semiconductors, P188.

[3] PRB 67, 092408 (2003).