Notes on PRB.67.092408

1 spin field effect transistor

A type of quantum field effect transistor that operates purely on the flow of spin current in the absence of charge current. The rotating field induces a time-independent dc spin current, and at the same time generates no charge cur- rent. The physical principle of our SFET is due to a spin flip mecha- nism provided by the field.

2 Hamiltonian

A rotating magnetic field is

$$B_x = B_0 \mathrm{sin}\theta\ \cos(\omega \mathrm{t}) \tag{1}$$

$$B_y = B_0 \mathrm{sin}\theta\ \sin(\omega \mathrm{t}) \tag{2}$$

$$B_z = B_0 \mathrm{cos}\theta. \tag{3}$$

The Hamiltonian of system is

$$\begin{aligned} H = & \sum_{k,\sigma,\alpha=L,R} \epsilon_k C^+_{k\alpha\sigma} C_{k\alpha\sigma} + \sum_\sigma \left[\epsilon + \sigma B_0 \cos\theta\right] d^+_\sigma d_\sigma \\ & + H'(t) + \sum_{k,\sigma,\alpha=L,R} \left[T_{k\alpha} C^+_{k\alpha\sigma} d_\sigma + \mathrm{c.c.}\right] \end{aligned} \tag{4}$$

We assume that there is only one orbit in the scattering region.

$$\epsilon_{Lk} = \epsilon_{Rk} = \epsilon_k.$$

A counterclock-wise rotating field allows a spin-down electron to absorb a photon and flip to spin-up, and it does not allow a spin-up electron to absorb a photon and flip to spin-down.

$$H'(t) = \gamma \left[\exp(-i\omega t) d^+_\uparrow d_\downarrow + \exp(i\omega t) d^+_\downarrow d_\uparrow\right] \tag{5}$$

$$\gamma = B_0 \sin\theta \tag{6}$$

The scattering region is characterized by an energy level$\epsilon = \epsilon_0 - qV_g$, controlled by the gate voltage V_g.

We solve the transport properties (charge and spin currents) of the model in both adiabatic and nonadiabatic regimes using the stan- dard Keldysh nonequilibrium Green's function technique.

3 Operator evolution

$$\begin{aligned}\frac{d}{dt}\hat{N}_{\alpha k\sigma} &= \frac{i}{\hbar}[H, C^{\dagger}_{\alpha k\sigma}C_{\alpha k\sigma}] = \left[\sum_{k',\sigma',\alpha'=L,R}\left[T_{k'\alpha'}C^{+}_{k'\alpha'\sigma'}d_{\sigma'} + \text{c.c.}\right], C^{\dagger}_{\alpha k\sigma}C_{\alpha k\sigma}\right] \\ &= \frac{i}{\hbar}\sum_{k',\sigma',\alpha'=L,R}\left[-T_{k'\alpha'}C^{+}_{k'\alpha'\sigma'}d_{\sigma'}\delta_{\alpha\alpha'}\delta_{kk'}\delta_{\sigma\sigma'} + \text{c.c.}\right] \\ &= \frac{i}{\hbar}[-T_{k\alpha}C^{\dagger}_{k\alpha\sigma}d_{\sigma} + T^{*}_{k\alpha}d^{\dagger}_{\sigma}C_{k\alpha\sigma}]\end{aligned} \tag{7}$$

So, the charge current is given by

$$\begin{aligned}I_{\alpha\sigma}(t) &= e\langle\frac{d}{dt}\hat{N}_{\alpha k\sigma}(t)\rangle \\ &= \frac{ie}{\hbar}(\langle -T_{k\alpha}C^{\dagger}_{k\alpha\sigma}(t)d_{\sigma}(t)\rangle + \langle T^{*}_{k\alpha}d^{\dagger}_{\sigma}(t)C_{k\alpha\sigma}(t)\rangle)\end{aligned} \tag{8}$$

4 Adiabatic regime(ω is small)

Equation of motion of particle operator $\hat{N}_{\alpha k\sigma}$ in the lead α is

$$\begin{aligned}\frac{d}{dt}\hat{N}_{\alpha k\sigma} &= \frac{i}{\hbar}[H, c^{\dagger}_{\alpha k\sigma}c_{\alpha k\sigma}] = \left[\sum_{k',\sigma',\alpha'=L,R}\left[t_{k'\alpha'}c^{\dagger}_{k'\alpha'\sigma'}d_{\sigma'} + \text{c.c.}\right], c^{\dagger}_{\alpha k\sigma}c_{\alpha k\sigma}\right] \\ &= \frac{i}{\hbar}\sum_{k',\sigma',\alpha'=L,R}\left[-t_{k'\alpha'}c^{\dagger}_{k'\alpha'\sigma'}d_{\sigma'}\delta_{\alpha\alpha'}\delta_{kk'}\delta_{\sigma\sigma'} + \text{c.c.}\right] \\ &= \frac{i}{\hbar}[-t_{k\alpha}c^{\dagger}_{k\alpha\sigma}d_{\sigma} + t^{*}_{k\alpha}d^{\dagger}_{\sigma}c_{k\alpha\sigma}]\end{aligned} \tag{9}$$

So, the charge current is given by

$$\begin{aligned}I_{\alpha\sigma}(t) &= e\langle\frac{d}{dt}\hat{N}_{\alpha k\sigma}(t)\rangle \\ &= \frac{ie}{\hbar}(\langle -t_{k\alpha}c^{\dagger}_{k\alpha\sigma}(t)d_{\sigma}(t)\rangle + \langle t^{*}_{k\alpha}d^{\dagger}_{\sigma}(t)c_{k\alpha\sigma}(t)\rangle)\end{aligned} \tag{10}$$

Define the lesser Green's function

$$G^{<}_{\sigma',k\alpha\sigma}(\tau,\tau') = i\langle C^{\dagger}_{k\alpha\sigma}(\tau')d_{\sigma'}(\tau)\rangle. \tag{11}$$

More generally, we define the contour Green's function

$$G_{\sigma',k\alpha\sigma}(\tau,\tau') = -i\langle d_{\sigma'}(\tau)C^{\dagger}_{k\alpha\sigma}(\tau')\rangle. \tag{12}$$

Following Jauho's notation [2]

$$G_{n,k\alpha}(\tau,\tau') = \sum_{m}\int d\tau_1 G_{nm}\left(\tau,\tau_1\right)t^{*}_{k\alpha m}g_{k\alpha}\left(\tau_1,\tau'\right) \tag{13}$$

we have???

$$G_{n,k\alpha}(\tau,\tau') = \sum_{m}\int d\tau_1 G_{nm}\left(\tau,\tau_1\right)t^{*}_{k\alpha m}g_{k\alpha}\left(\tau_1,\tau'\right) \tag{14}$$

So, the charge current is given by (why? DC?)

$$dQ_{\alpha\sigma}(t)/dt = q\int\frac{dE}{2\pi}\left(-\partial_E f\right)\left[\Gamma_{\alpha}\mathbf{G}^{r}(t)\mathbf{\Delta G}^{a}(t)\right]_{\sigma\sigma} \tag{15}$$

References

[1] Y, K, Kato. Observation of the Spin Hall Effect in Semiconductors[J]. Science, 2004.

[2] Antti-Pekka Jauho, Quantum Kinetics in Transport and Optics of Semiconductors, P188.