Homework 6

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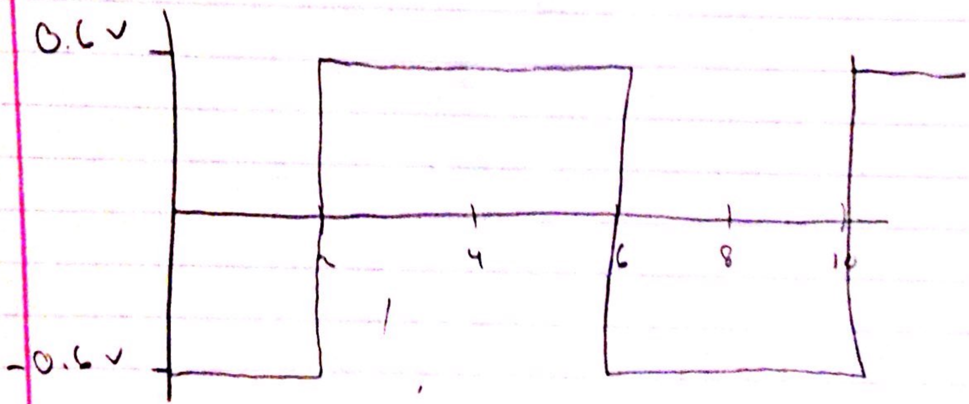
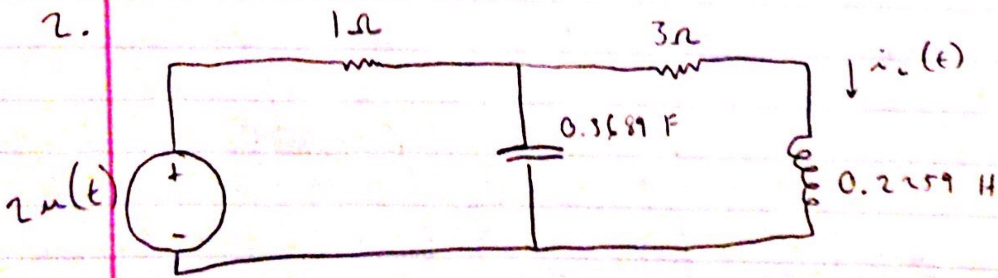
1. $RC = (50\ k\Omega)(2\ \mu F) = 0.1$

$$V_{out} = -RC\frac{dv_i}{dt} = -0.1 \times 6 = \boxed{-0.6\ V}$$

Slope from $0 \le t \le 8$

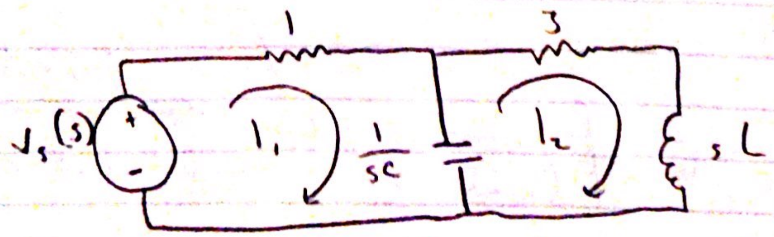


2.



to s domain

→



$$\mathcal{L}[V(t)] = 2\mathcal{L}(u(t)) = 2/s$$

e f g

$V_s = 2/s$

$-V_s + 1(I_1) + \frac{1}{0.3689s}(I_1 - I_2) = 0$

$3I_2 + 0.2259\, sI_2 + \frac{1}{0.3689s}(I_2 - I_1) = 0$

$-V_s + I_1 + \frac{1}{0.3689s} I_1 - \frac{1}{0.3689s} I_2 = 0$

$3I_2 + \frac{1}{0.3689s} I_2 + 0.2259s - \frac{1}{0.3689s} I_1 = 0$

Eq 1: $-\frac{2}{s} + I_1\left(1 + \frac{1}{0.3689s}\right) - \frac{1}{0.3689s} I_2 = 0$

Eq 2: $-I_1\left(\frac{1}{0.3689s}\right) + I_2\left(3 + 0.2259s + \frac{1}{0.3689s}\right) = 0$

$\frac{2}{s} = \left(1 + \frac{1}{0.3689s}\right) I_1 - \left(\frac{1}{0.3689s}\right) I_2$

$I_2 = \left(\frac{\frac{1}{0.3689s}}{3 + 0.2259s + \frac{1}{0.3689s}}\right) I_1$

Solving for I_2

$\longrightarrow$

$\frac{2}{s} = \left(1 + \frac{1}{0.3689s}\right) I_1 - \left(\frac{1}{0.3689s}\right)\left(\frac{\frac{1}{0.3689s}}{3 + 0.2259s + \frac{1}{0.3689s}}\right) I_1$

multiply by $\left(\frac{0.3689s}{0.3689s}\right)$

$\frac{2}{s} = \left(\frac{0.3689s + 1}{0.3689s}\right) I_1 - \left[\frac{1}{(0.3689s)^2}\right]\left(\frac{0.3689s}{(3 + 0.2259s)(0.3689s) + 1}\right) I_1$

$$\frac{2}{s} = \left[0.3689s + 1 - \frac{1}{(3+0.2259s)(0.3689s)+1}\right]\frac{I_1}{0.3689s}$$

$$\frac{I_1}{0.3689s} = \frac{2}{s}\left[0.3689s + 1 - \frac{1}{(3+0.2259s)(0.3689s)+1}\right]$$

$$I_1 = 0.7378\left[0.3689s + 1 - \frac{1}{(3+0.2259s)(0.3689s)+1}\right] \star$$

$$= \left(\frac{\frac{1}{0.3689s}}{3+0.2259s+\frac{1}{0.3689s}}\right)\left(0.7378\left[0.3689s + 1 - \frac{1}{(3+0.2259s)(0.3689s)+1}\right]\right)$$

$$I_2 = \left(\frac{\frac{1}{0.3689s}}{\frac{(3+0.2259s)(0.3689s)+1}{0.3689s}}\right)\left(\frac{0.7378s(1.1067s+0.0833s^2+1)}{s(0.0307s^2+0.4915s+1.4756)}\right)$$

$$\left(\frac{1}{\cancel{0.3689s}} \cdot \cancel{0.3689s}\right)$$

$$I_2 = \left(\frac{1}{(3+0.2259s)(0.3689s)+1}\right)\left(\frac{0.7378s\,\cancel{(1.1067s+0.0833s^2+1)}}{s(0.0307s^2+0.4915s+1.4756)}\right)$$

$$\cancel{1.1067s+0.0833s^2+1}$$

$$I_2 = \frac{0.7378}{s(0.0307s^2+0.4915s+1.4756)} \star$$

$$V_c(s) = \frac{1}{0.3689s}(I_1 - I_2)$$

$$V_c(s) = \frac{1}{0.3689\,s}\left(\frac{0.7378(1.1067s + 0.0833s^2 + 1)}{s(0.0307s^2 + 0.4915s + 1.4756)} - \frac{0.7378}{s(0.0307s^2 + 0.4915s + 1.4756)}\right)$$

$$V_c(s) = \frac{1}{0.3689\,s}\left(\frac{0.8165s + 0.0615s^2 + 0.7378 - 0.7378}{s(0.0307s^2 + 0.4915s + 1.4756)}\right)$$

$$V_c(s) = \frac{0.8165s + 0.0615s^2}{0.3689s^2(0.0307s^2 + 0.4915s + 1.4756)}$$

$$\frac{\cancel{s}}{\cancel{s}}\ \frac{(0.8165 + 0.0615s)}{0.3689\,s(0.0307s^2 + 0.4915s + 1.4256)}$$

$$= \frac{\frac{0.8165}{0.3689} + \frac{0.0615s}{0.3689}}{s(0.0307s^2 + 0.4915s + 1.4756)} = \frac{2.21 + 0.1667s}{} \rightarrow$$

$$0.0307s\left(s^2 + \frac{0.4915}{0.0307}s + \frac{1.4756}{0.0307}\right)$$

$$0.0307s(s^2 + 16s + 48)$$

$$V_c(s) = \frac{72.1 + 5.426s}{s(s+12)(s+4)}$$

$$V_c(s) = \frac{A_1}{s} + \frac{A_2}{s+12} + \frac{A_3}{s+4} \rightarrow \text{Partial Fraction}$$

make s undefined

$$s=0 \rightarrow \frac{72.1 + 0}{(12)(4)} \quad A_1 = 1.5$$

make s+12 undefined

$$s=-12 \quad \frac{72.1 + 5.426\cancel{(-12)}}{\cancel{(-12)}(-12+4)} = A_2 = 0.0728$$

make s+4 undefined

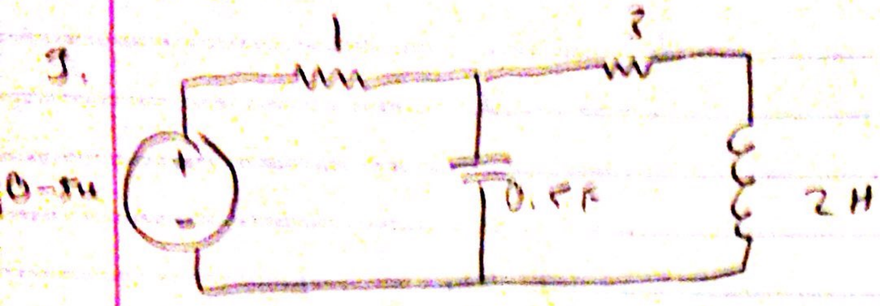
$$s=-4 \quad \frac{72.1 + 5.426(-4)}{(-4)(-4+12)} = -1.575 \quad A_3 =$$

$$V_c(s) = \frac{1.5}{s} + \frac{0.072}{s+12} - \frac{1.57}{s+4}$$

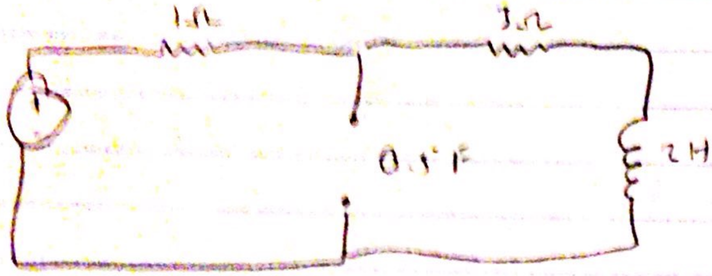
$$V = 1.5\,\mathcal{L}^{-1}\left[\frac{1}{s}\right] + 0.072\,\mathcal{L}^{-1}\left[\frac{1}{s+12}\right] + -1.57\,\mathcal{L}^{-1}\left[\frac{1}{s+4}\right]$$

$$\frac{1}{s-(-12)} \quad \boxed{V(t) = \left[1.5 + 0.072e^{-12t} - 1.57e^{-4t}\right]u(t)}$$

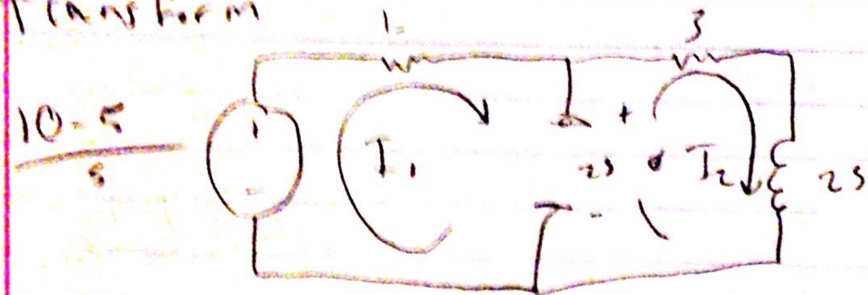
3.



Modified Circuit



$\mathcal{L}$ Transform



$$-\left(\frac{10-5}{s}\right) + 1 + 2s(I_1 - I_2) = 0$$

$$2S(I_1 - I_1) + 3 + 2S = 0$$

$$\left[\frac{10+5}{S} + 1 + 2SI_1 - 2SI_2 = 0\right] \times S$$

$$-10 + 5 + S + 2S^2 I_1 - 2S^2 I_2$$

$$-5 + S + 2S^2 I_1 - 2S^2 I_2 = 0$$

$$2S^2 I_1 - 2S^2 I_2 = 5 - S$$

$$2S^2(\cancel{x}I_1 - \cancel{x}I_2) = 5 - S$$

$$I_1 - I_2 = \frac{5-S}{2S^2}$$

$$I_1 = \frac{5-S}{2S^2} + I_2$$

$$2s\Big(I_2 - \overbrace{\left(\frac{5-s}{2s^2} + I_2\right)}^{I_1 \text{ from previos eq.}}\Big) + 3 + 2s = 0$$

$$2sI_2 - 2s\left(\frac{5-s}{2s^2} + I_2\right) + 3 + 2s = 0$$

$$2sI_2 - \frac{5+s}{s} + 2sI_2 + 3 + 2s$$

$$4sI_2 - \frac{5+s}{s} + 3 + 2s = 0$$

$$4sI_2 - \frac{5}{s} + 4 + 2s = 0$$

$$4sI_2 = \frac{5}{s} - 4 - 2s$$

$$\underbrace{I_2 = \frac{5}{4s^2} - \frac{1}{s} - \frac{1}{2}}$$

$$\mathcal{L}^{-1}(I_2) = \frac{5}{4}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{2}$$

$$\frac{5}{4}(t) - 1 - \frac{1}{2}$$

$$I_2 = 5/4\,t - 3/2$$

$$I_2 = \boxed{i_2 = \frac{5}{4}t - \frac{3}{2}}$$

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