1 Constants

Table 1: Constants and limits

Symbol	Unit	Value	Explanation
dt	s		Discrete timestep used to update the differential system
τ_{adj}	s	0.5	Characteristic time in which agent adjusts its movement
k	N	1.5	Social force scaling constant
τ_0	s	3.0	Max interaction range 2 - 4, aka interaction time horizon
μ	$\frac{\mathrm{kg}}{\mathrm{s}^2}$	$1.2e+05$	Compression counteraction constant
κ	$\frac{\mathrm{kg}}{\mathrm{m\,s}}$	$2.4e+05$	Sliding friction constant
A	N	$2.0e+03$	Scaling coefficient for social force between wall and agent
B	m	0.08	Coefficient for social force between wall and agent
f_{max}	N		Forces that are greater will be truncated to max force

Table 2: Agent body properties

	r (m)	$\pm$	r_{torso}/r	r_{shoulder}/r	d/r	v (m/s)	$\pm$
adult	0.255	0.035	0.5882	0.3725	0.6275	1.25	0.3
child	0.210	0.015	0.5714	0.3333	0.6667	0.90	0.3
eldery	0.250	0.020	0.6000	0.3600	0.6400	0.80	0.3
female	0.240	0.020	0.5833	0.3750	0.6250	1.15	0.2
male	0.270	0.020	0.5926	0.3704	0.6296	1.35	0.2

2 Agents

r_i	Radius
m_i	Mass
v_i^0	Goal velocity
$\mathbf{x}_i$	Position
$\mathbf{v}_i$	Velocity
$\hat{\mathbf{e}}_i^0$	Goal direction
$\hat{\mathbf{e}}_i$	Target direction
$\mathbf{p}_i$	Herding tendency
I_i^z	Moment of inertia
ω_i^0	max target angular velocity
ω_i	current angular velocity
φ_i	current body angle
φ_i^0	target angle
$\tilde{\omega}_i^0$	target angular speed

$\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$	Relative position between two agents
$\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$	Relative velocity between two agents
$r_{ij} = r_i + r_j$	Total radius
$d_{ij} = \lVert \mathbf{x}_{ij} \rVert$	Distance between agents
$h_{ij} = r_{ij} - d_{ij}$	Relative distance between agents
$\hat{\mathbf{n}}_{ij} = \mathbf{x}_{ij}/d_{ij}$	Normal vector
$\hat{\mathbf{t}}_{ij} = R(-90^\circ) \cdot \hat{\mathbf{n}}_{ij}$	Tangent vector

Rotational equation of motion

$$I_i^z \frac{d^2\varphi_i(t)}{dt^2} = M_i^z(t) + \eta_i^z(t),$$

where $\varphi_i(t)$ is the angle of the agent i, I_i^z is moment of inertia, $\eta_i^z(t)$ is small random fluctuation torque, and $M_i^z(t)$ is total torque, which is the sum of contact, social and motivational torque

$$M_i^z(t) = M_i^c + M_i^{soc} + M_i^{\tau}$$

Torque from contact forces

$$\mathbf{M}_i^c = \sum_{j \neq i} \left(\mathbf{R}_i^c \times \mathbf{f}_{ij}^c \right)$$

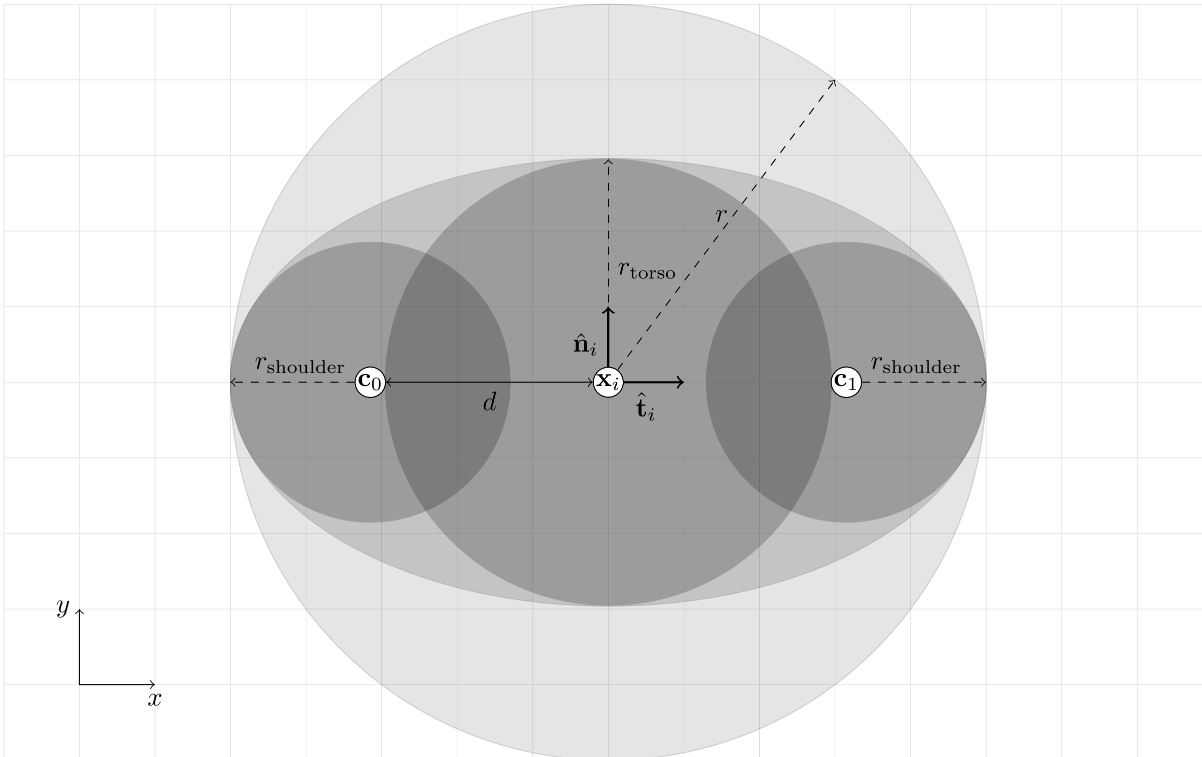

Figure 1: Circle, ellipse and three circle representations of an agent.

and from social forces

$$\mathbf{M}_i^{soc} = \sum_{j \neq i} \left(\mathbf{R}_i^{soc} \times \mathbf{f}_{ij}^{soc} \right)$$

Motivational torque

$$\begin{aligned} M_i^{\tau} &= \frac{I_i^z}{\tau_i^z} \left((\varphi_i(t) - \varphi_i^0)\omega^0 - \omega(t) \right) \\ &= \frac{I_i^z}{\tau_i^z} \left(\tilde{\omega}_i^0 - \omega(t) \right) \end{aligned}$$

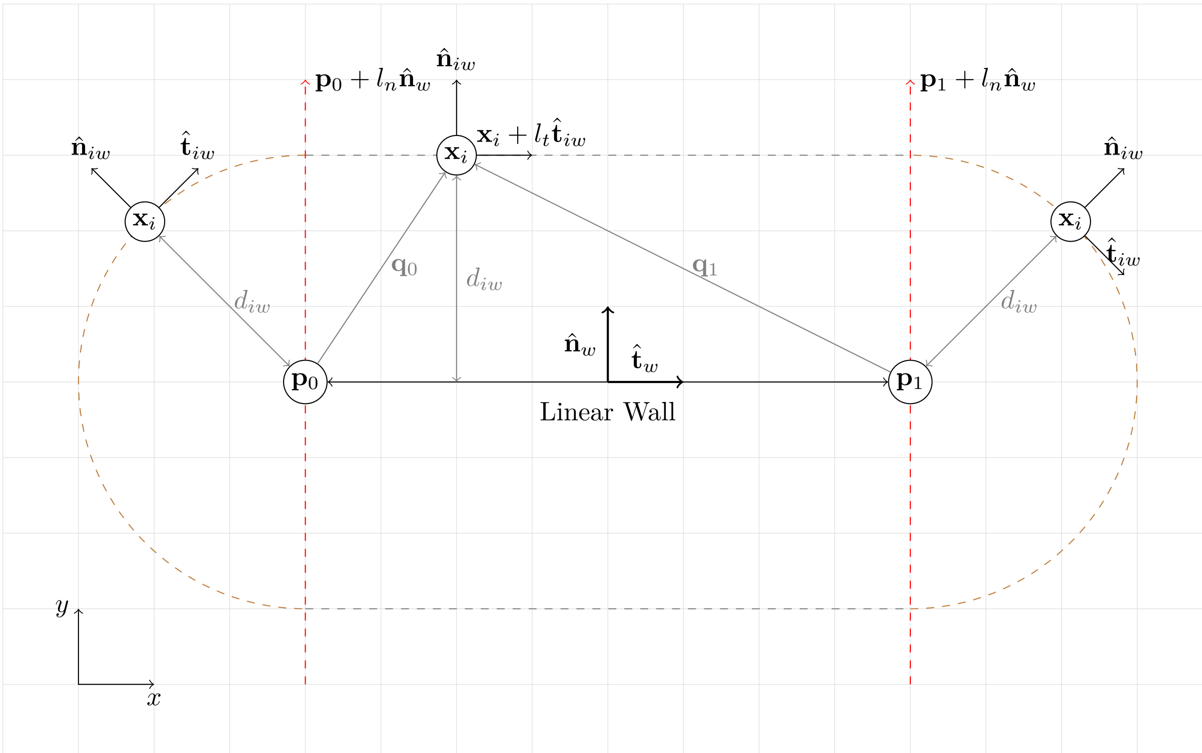

Figure 2: Absolute distance from a linear wall.

3 Linear wall

3.1 Properties

$\mathbf{p}_0$	Starting point of linear wall
$\mathbf{p}_1$	End point of linear wall

Relative

$$\begin{aligned} h_{iw} &= r_i - d_{iw} \\ l_w &= \|\mathbf{p}_1 - \mathbf{p}_0\| \\ \hat{\mathbf{t}}_w &= (\mathbf{p}_1 - \mathbf{p}_0)/l_w \\ \hat{\mathbf{n}}_w &= R(90^\circ) \cdot \hat{\mathbf{t}}_w \end{aligned}$$

3.2 Absolute distance

Solving linear system of equations determining the position of the agent $\mathbf{x}_i$ in relation to wall

$$\begin{cases} \mathbf{p}_0 + l_{n_0}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_0}\hat{\mathbf{t}}_w \\ \mathbf{p}_1 + l_{n_1}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_1}\hat{\mathbf{t}}_w \end{cases}$$

$$\begin{cases} l_{n_0}\hat{\mathbf{n}}_w - l_{t_0}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_0 = \mathbf{q}_0 \\ l_{n_1}\hat{\mathbf{n}}_w - l_{t_1}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_1 = \mathbf{q}_1 \end{cases}$$

In matrix form

$$\begin{bmatrix} l_{n_0} & l_{n_1} \\ l_{t_0} & l_{t_1} \end{bmatrix} = \mathbf{A}^{-1}\begin{bmatrix} \mathbf{q}_0 & \mathbf{q}_1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \hat{\mathbf{n}}_w & -\hat{\mathbf{t}}_w \end{bmatrix} = \begin{bmatrix} -t_1 & -t_0 \\ t_0 & -t_1 \end{bmatrix} \\ \mathbf{A}^{-1} &= \frac{1}{t_0^2 + t_1^2}\begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} = \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} \\ &= \begin{bmatrix} \hat{\mathbf{n}}_w \\ -\hat{\mathbf{t}}_w \end{bmatrix} = \mathbf{A}^T \end{aligned}$$

Conditions

$$\begin{aligned} l_n &= l_{n_0} \vee l_{n_1} = \hat{\mathbf{n}}_w \cdot \mathbf{q}_0 \vee \hat{\mathbf{n}}_w \cdot \mathbf{q}_1 \\ l_t &= l_{t_1} + l_{t_0} = -\hat{\mathbf{t}}_w \cdot \mathbf{q}_1 - \hat{\mathbf{t}}_w \cdot \mathbf{q}_0 \end{aligned}$$

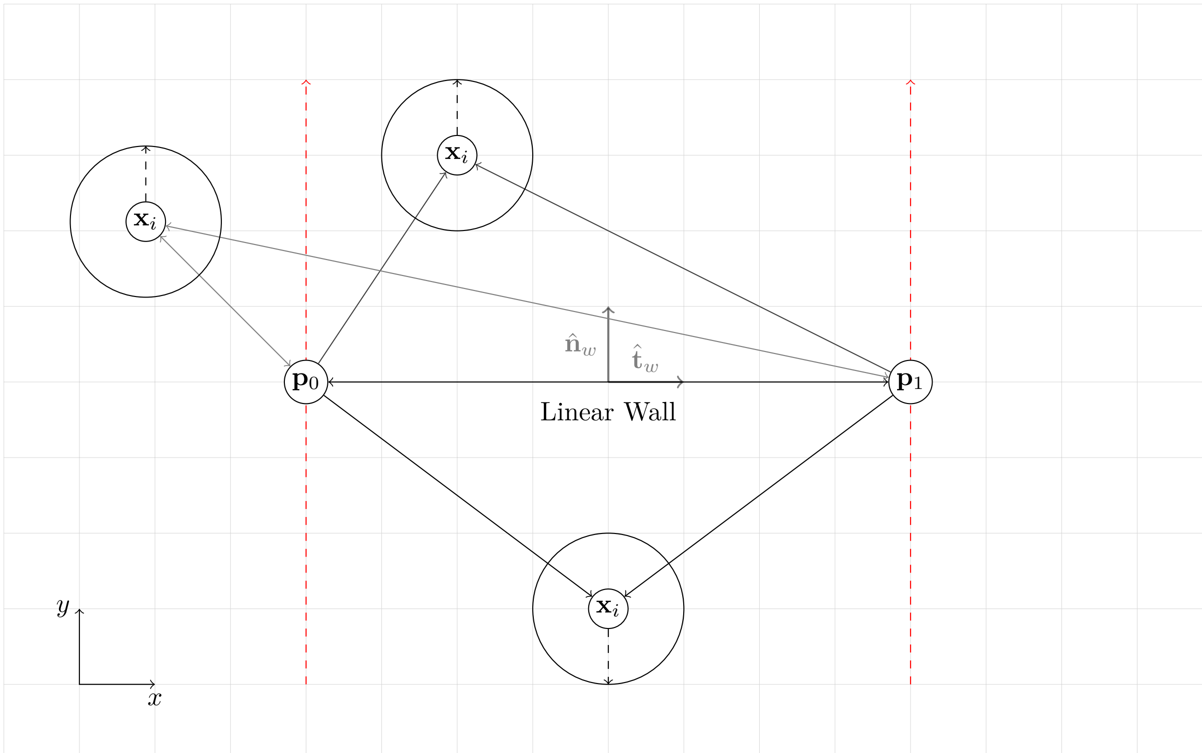

Figure 3: Velocity dependent distance from a linear wall.

Distance between agent and linear wall

$$d_{iw} = \begin{cases} \|\mathbf{q}_0\| & l_t > l_w \\ |l_n| & \text{otherwise} \\ \|\mathbf{q}_1\| & l_t < -l_w \end{cases}$$

Normal vector away from the wall

$$\hat{\mathbf{n}}_{iw} = \begin{cases} \hat{\mathbf{q}}_0 & l_t > l_w \\ \operatorname{sign}(l_n)\hat{\mathbf{n}}_w & \text{otherwise} \\ \hat{\mathbf{q}}_1 & l_t < -l_w \end{cases}$$

3.3 Velocity relative distance

Angle between two vectors

$$\cos(\sphericalangle(\mathbf{v}, \mathbf{u})) = \frac{\mathbf{v} \cdot \mathbf{u}}{\|\mathbf{v}\| \, \|\mathbf{u}\|} \in [0, \pi)$$

4 Crowd dynamics

4.1 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c} \right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c} \right) + \boldsymbol{\xi}_i$$

4.2 Adjusting Force

Force adjusting agent's movement towards desired in some characteristic time

$$\mathbf{f}_i^{adj} = \frac{m_i}{\tau_i^{adj}} (v_i^0 \cdot \hat{\mathbf{e}}_i - \mathbf{v}_i)$$

4.3 Agent-Agent

Psychological tendency to keep a certain distance to other agents

$$\mathbf{f}_{ij}^{soc}, \quad d_{ij} \leq \text{sight}$$

Physical contact forces with other agents

$$\mathbf{f}_{ij}^{c} = h_{ij} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{ij} - \kappa \cdot \left(\mathbf{v}_{ji} \cdot \hat{\mathbf{t}}_{ij} \right) \hat{\mathbf{t}}_{ij} \right), \quad h_{ij} > 0$$

4.4 Agent-Wall

Psychological tendency to keep a certain distance to walls

$$\mathbf{f}_{iw}^{soc} = A_i \exp\left(\frac{h_{iw}}{B_i} \right) \hat{\mathbf{n}}_{iw}, \quad d_{iw} \leq \text{sight}$$

Physical contact forces with walls

$$\mathbf{f}_{iw}^{c} = h_{iw} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{iw} - \kappa \cdot (\mathbf{v}_i \cdot \hat{\mathbf{t}}_{iw}) \hat{\mathbf{t}}_{iw} \right), \quad h_{iw} > 0$$

4.5 Random Fluctuation

Uniformly distributed random fluctuation force

$$\boldsymbol{\xi}_i = c \cdot \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \end{bmatrix}, \quad c \in [0, f_{max}], \varphi \in [0, 2\pi)$$

4.6 Universal power law governing pedestrian interactions

Interaction force between agents

$$\begin{aligned}
\mathbf{f}_{ij}^{soc} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) \\
&= -\nabla_{\mathbf{x}_{ij}} \left(\frac{k}{\tau^2} \exp\left(-\frac{\tau}{\tau_0} \right) \right) \\
&= -\left(\frac{k}{a\tau^2} \right) \left(\frac{2}{\tau} + \frac{1}{\tau_0} \right) \exp\left(-\frac{\tau}{\tau_0} \right) \left(\mathbf{v}_{ij} - \frac{a\mathbf{x}_{ij} + b\mathbf{v}_{ij}}{d} \right),
\end{aligned}$$

where

$$\begin{aligned}
a &= \mathbf{v}_{ij} \cdot \mathbf{v}_{ij} \\
b &= -\mathbf{x}_{ij} \cdot \mathbf{v}_{ij} \\
c &= \mathbf{x}_{ij} \cdot \mathbf{x}_{ij} - r_{ij}^2 \\
d &= \sqrt{b^2 - ac} \\
\tau &= \frac{b - d}{a} > 0.
\end{aligned}$$

4.7 Target direction

Herding behavior

$$\mathbf{e}_i = (1 - p_i)\hat{\mathbf{e}}_i^0 + p_i \left\langle \hat{\mathbf{e}}_j^0 \right\rangle_i$$

5 Differential system

Acceleration on an agent i

$$a_i(t) = \frac{\mathbf{f}_i(t)}{m_i}$$

Updating velocity using discrete time step Δt

$$\begin{aligned}
\Delta \mathbf{v} &= a(t_k) \Delta t \\
\mathbf{v}(t_{k+1}) &= \mathbf{v}(t_k) + \Delta \mathbf{v}
\end{aligned}$$

Updating position

$$\begin{aligned}
\Delta \mathbf{x} &= \mathbf{v}(t_{k+1}) \Delta t \\
\mathbf{x}(t_{k+1}) &= \mathbf{x}(t_k) + \Delta \mathbf{x}
\end{aligned}$$