1 Parameters

Vector norm and inner product

$$\langle \mathbf{x}, \mathbf{x} \rangle = \mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}\|^2$$

Unit vector

$$\hat{\mathbf{e}} = \frac{\mathbf{e}}{\|\mathbf{e}\|}$$

Rotation matrix to counterclockwise direction

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$R(90^\circ) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad R(-90^\circ) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Properties of an agent i

$$\begin{aligned} \mathbf{x}_i &= \text{Centre} \\ \mathbf{v}_i &= \text{Velocity} \\ r_i &= \text{Radius} \\ m_i &= \text{Mass} \end{aligned}$$

$$\begin{aligned} \mathbf{x}_{ij} &= \mathbf{x}_i - \mathbf{x}_j \\ \mathbf{v}_{ij} &= \mathbf{v}_i - \mathbf{v}_j \\ r_{ij} &= r_i + r_j \\ d_{ij} &= \|\mathbf{x}_{ij}\| \\ h_{ij} &= r_{ij} - d_{ij} \\ h_{iw} &= r_i - d_{iw} \end{aligned}$$

$$\begin{aligned} a &= \mathbf{v}_{ij} \cdot \mathbf{v}_{ij} \\ b &= -\mathbf{x}_{ij} \cdot \mathbf{v}_{ij} \\ c &= d_{ij}^2 - r_{ij}^2 \\ d &= \sqrt{b^2 - ac} \\ \tau &= \frac{b - \sqrt{d}}{a} > 0 \end{aligned}$$

$$\begin{aligned} \hat{\mathbf{n}}_{ij} &= \frac{\mathbf{x}_{ij}}{d_{ij}} \\ \hat{\mathbf{t}}_{ij} &= R(-90^\circ) \cdot \mathbf{n}_{ij} \end{aligned}$$

2 Crowd dynamics

2.1 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c}\right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c}\right) + \boldsymbol{\xi}_i$$

i) Force adjusting agent's movement towards desired in some characteristic time τ_i^{adj}

$$\mathbf{f}_i^{adj} = \frac{m_i}{\tau_i^{adj}} (\mathbf{v}_i^0 - \mathbf{v}_i)$$

Target velocity can be broken down

$$\mathbf{v}_i^0 = v_i^0 \cdot \hat{\mathbf{e}}_i^0$$

Target direction

$$\begin{aligned} \mathbf{e}_i^0(t) &= (1 - p_i)\mathbf{e}_i + p_i \left\langle \mathbf{e}_j^0(t) \right\rangle_i \\ \hat{\mathbf{e}}_i^0(t) &= \frac{\mathbf{e}_i^0(t)}{\|\mathbf{e}_i^0(t)\|} \end{aligned}$$

ii) Psychological tendency to keep a certain distance to other agents

$$\mathbf{f}_{ij}^{soc} = \begin{cases} \mathbf{f}_{ij}^{pow} & d_{ij} \leq \text{sight} \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^{soc} = \begin{cases} A_i \exp\left(\frac{h_{iw}}{B_i}\right) \hat{\mathbf{n}}_{iw} & d_{iw} \leq \text{sight} \\ 0 & \text{otherwise} \end{cases}$$

iii) Physical contact forces with other agents

$$\mathbf{f}_{ij}^{c} = \begin{cases} h_{ij} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{ij} - \kappa \cdot \left(\mathbf{v}_{ji} \cdot \hat{\mathbf{t}}_{ij}\right) \hat{\mathbf{t}}_{ij}\right) & h_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^{c} = \begin{cases} h_{iw} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{iw} - \kappa \cdot (\mathbf{v}_i \cdot \hat{\mathbf{t}}_{iw})\hat{\mathbf{t}}_{iw}\right) & h_{iw} > 0 \\ 0 & \text{otherwise} \end{cases}$$

iv) Uniformly distributed random fluctuation force

$$\boldsymbol{\xi}_i = \boldsymbol{\mathcal{U}}(-1, 1).$$

2.2 Universal power law governing pedestrian interactions

Interaction force between agents

$$\mathbf{f}_{ij}^{pow} = -\nabla_{\mathbf{x}_{ij}} E(\tau) = -\nabla_{\mathbf{x}_{ij}} \left(\frac{k}{\tau^2} \exp\left(-\frac{\tau}{\tau_0} \right) \right)$$

$$= -\left(\frac{k}{a\tau^2}\right)\left(\frac{2}{\tau} + \frac{1}{\tau_0}\right) \exp\left(-\frac{\tau}{\tau_0}\right)\left(\mathbf{v}_{ij} - \frac{a\mathbf{x}_{ij} + b\mathbf{v}_{ij}}{d}\right)$$

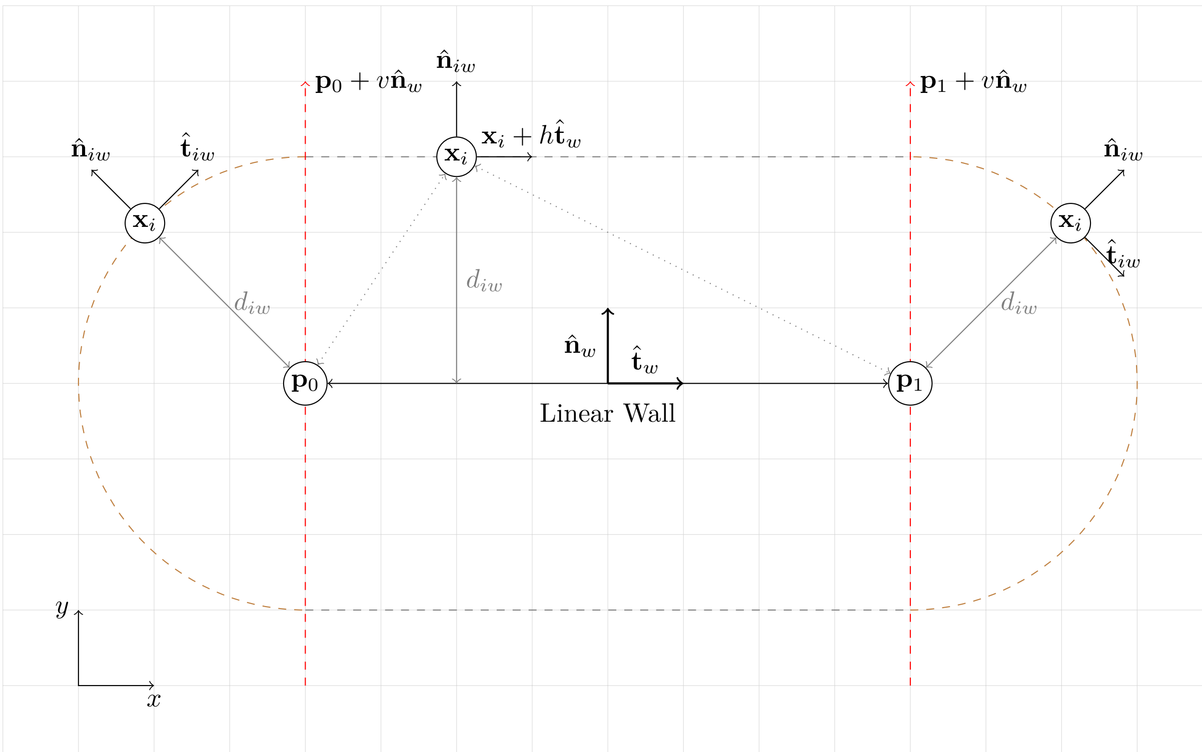

Figure 1: Linear wall

3 Field

3.1 Walls

3.1.1 Round

3.1.2 Linear

Properties

$$l_w = \|\mathbf{p}_1 - \mathbf{p}_0\|, \quad \hat{\mathbf{t}}_w = \frac{\mathbf{p}_1 - \mathbf{p}_0}{l_w}, \quad \hat{\mathbf{n}}_w = R(90°) \cdot \hat{\mathbf{t}}_w$$

Solving linear system of equations determining the position of the agent $\mathbf{x}_i$ in relation to wall

$$\begin{cases} \mathbf{p}_0 + v_0\hat{\mathbf{n}}_w = \mathbf{x}_i + h_0\hat{\mathbf{t}}_w \\ \mathbf{p}_1 + v_1\hat{\mathbf{n}}_w = \mathbf{x}_i + h_1\hat{\mathbf{t}}_w \end{cases}$$

$$\begin{cases} v_0\hat{\mathbf{n}}_w - h_0\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_0 = \mathbf{q}_0 \\ v_1\hat{\mathbf{n}}_w - h_1\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_1 = \mathbf{q}_1 \end{cases}$$

In matrix form

$$\begin{bmatrix} v_0 & v_1 \\ h_0 & h_1 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \mathbf{q}_0 & \mathbf{q}_1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} \hat{\mathbf{n}}_w & -\hat{\mathbf{t}}_w \end{bmatrix} = \begin{bmatrix} -t_1 & -t_0 \\ t_0 & -t_1 \end{bmatrix}$$

$$
\begin{aligned}
\mathbf{A}^{-1} &= \frac{1}{t_0^2 + t_1^2} \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} = \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} \\
&= -\begin{bmatrix} \hat{\mathbf{t}}_w & \hat{\mathbf{n}}_w \end{bmatrix} = \mathbf{A}^T
\end{aligned}
$$

$$
\begin{aligned}
h &= h_1 - h_0 \\
v &= v_0 \wedge v_1, \quad (v_0 = v_1)
\end{aligned}
$$

$$
d_{iw} = \begin{cases} \|\mathbf{q}_0\| & h > l_w \\ |v| & h = l_w \\ \|\mathbf{q}_1\| & h < l_w \end{cases}
$$

$$
\hat{\mathbf{n}}_{iw} = \begin{cases} \hat{\mathbf{q}}_0 & h > l_w \\ \operatorname{sign}(v)\hat{\mathbf{n}}_w & h = l_w \\ \hat{\mathbf{q}}_1 & h < l_w \end{cases}
$$

3.2 Agents

3.2.1 Round

3.2.2 Elliptical

4 Differential system

Acceleration on an agent i

$$
a_i(t) = \frac{\mathbf{f}_i(t)}{m_i}
$$

Updating velocity using discrete time step Δt

$$
\begin{aligned}
\Delta \mathbf{v} &= a_i(t_k)\Delta t \\
\mathbf{v}_i(t_{k+1}) &= \mathbf{v}_i(t_k) + \Delta \mathbf{v}
\end{aligned}
$$

Updating position

$$
\begin{aligned}
\Delta \mathbf{x} &= \mathbf{v}_i(t_{k+1})\Delta t \\
\mathbf{x}_i(t_{k+1}) &= \mathbf{x}_i(t_k) + \Delta \mathbf{x}_i
\end{aligned}
$$