1 Constants

Table 1

Symbol	Unit	Value	Explanation
			System
dt	s		Discrete timestep used to update the differential system
			Constants
τ_{adj}	s	0.5	Characteristic time in which agent adjusts its movement
k	N	1.5	Social force scaling constant
τ_0	s	3.0	Max interaction range 2 - 4, aka interaction time horizon
μ	$\frac{\mathrm{kg}}{\mathrm{s}^2}$	$1.2e+05$	Compression counteraction constant
κ	$\frac{\mathrm{kg}}{\mathrm{m\,s}}$	$2.4e+05$	Sliding friction constant
A	N	$2.0e+03$	Scaling coefficient for social force between wall and agent
B	m	0.08	Coefficient for social force between wall and agent
			Limits
f_{max}	N		Forces that are greater will be truncated to max force

2 Parameters

2.1 Agent

Properties of an agent i

r_i	Radius	Constant
m_i	Mass	Constant
v_i^0	Goal velocity	Variable/Constant
$\mathbf{x}_i$	Position	Variable
$\mathbf{v}_i$	Velocity	Variable
$\hat{\mathbf{e}}_i$	Goal direction	Variable
$\mathbf{p}_i$	Herding tendency	Variable/Constant

Relative properties

$\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$	Relative position between two agents
$\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$	Relative velocity between two agents
$r_{ij} = r_i + r_j$	Total radius
$d_{ij} = \lVert\mathbf{x}_{ij}\rVert$	Distance between agents
$h_{ij} = r_{ij} - d_{ij}$	Relative distance between agents
$\hat{\mathbf{n}}_{ij} = \mathbf{x}_{ij}/d_{ij}$	Normal vector
$\hat{\mathbf{t}}_{ij} = R(-90°) \cdot \hat{\mathbf{n}}_{ij}$	Tangent vector

2.2 Wall

Properties of linear wall w

$\mathbf{p}_0$	Starting point of linear wall
$\mathbf{p}_1$	End point of linear wall

Relative properties

$$h_{iw} = r_i - d_{iw}$$
$$l_w = \lVert\mathbf{p}_1 - \mathbf{p}_0\rVert$$
$$\hat{\mathbf{t}}_w = (\mathbf{p}_1 - \mathbf{p}_0)/l_w$$
$$\hat{\mathbf{n}}_w = R(90°) \cdot \hat{\mathbf{t}}_w$$

3 Crowd dynamics

3.1 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c}\right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c}\right) + \boldsymbol{\xi}_i$$

3.2 Adjusting Force

Force adjusting agent's movement towards desired in some characteristic time

$$\mathbf{f}_i^{adj} = \frac{m_i}{\tau_i^{adj}} (v_i^0 \cdot \hat{\mathbf{e}}_i - \mathbf{v}_i)$$

Herding behavior

$$\mathbf{e}_i = (1 - p_i)\hat{\mathbf{e}}_i^0 + p_i \left\langle \hat{\mathbf{e}}_j^0 \right\rangle_i$$

3.3 Agent-Agent

Psychological tendency to keep a certain distance to other agents

$$\mathbf{f}_{ij}^{soc} = \mathbf{f}_{ij}^{pow}, \quad d_{ij} \leq \text{sight}$$

Physical contact forces with other agents

$$\mathbf{f}_{ij}^{c} = h_{ij} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{ij} - \kappa \cdot \left(\mathbf{v}_{ji} \cdot \hat{\mathbf{t}}_{ij}\right) \hat{\mathbf{t}}_{ij}\right), \quad h_{ij} > 0$$

3.4 Agent-Wall

Psychological tendency to keep a certain distance to walls

$$\mathbf{f}_{iw}^{soc} = A_i \exp\left(\frac{h_{iw}}{B_i}\right) \hat{\mathbf{n}}_{iw}, \quad d_{iw} \leq \text{sight}$$

Physical contact forces with walls

$$\mathbf{f}_{iw}^{c} = h_{iw} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{iw} - \kappa \cdot (\mathbf{v}_i \cdot \hat{\mathbf{t}}_{iw})\hat{\mathbf{t}}_{iw}\right), \quad h_{iw} > 0$$

3.5 Random Fluctuation

Uniformly distributed random fluctuation force

$$\boldsymbol{\xi}_i = \boldsymbol{\mathcal{U}}(-1, 1).$$

3.6 Universal power law governing pedestrian interactions

Interaction force between agents

$$\begin{aligned}
\mathbf{f}_{ij}^{pow} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) \\
&= -\nabla_{\mathbf{x}_{ij}} \left(\frac{k}{\tau^2} \exp\left(-\frac{\tau}{\tau_0}\right)\right) \\
&= -\left(\frac{k}{a\tau^2}\right) \left(\frac{2}{\tau} + \frac{1}{\tau_0}\right) \exp\left(-\frac{\tau}{\tau_0}\right) \left(\mathbf{v}_{ij} - \frac{a\mathbf{x}_{ij} + b\mathbf{v}_{ij}}{d}\right),
\end{aligned}$$

where

$$\begin{aligned}
a &= \mathbf{v}_{ij} \cdot \mathbf{v}_{ij} \\
b &= -\mathbf{x}_{ij} \cdot \mathbf{v}_{ij} \\
c &= \mathbf{x}_{ij} \cdot \mathbf{x}_{ij} - r_{ij}^2 \\
d &= \sqrt{b^2 - ac} \\
\tau &= \frac{b - d}{a} > 0.
\end{aligned}$$

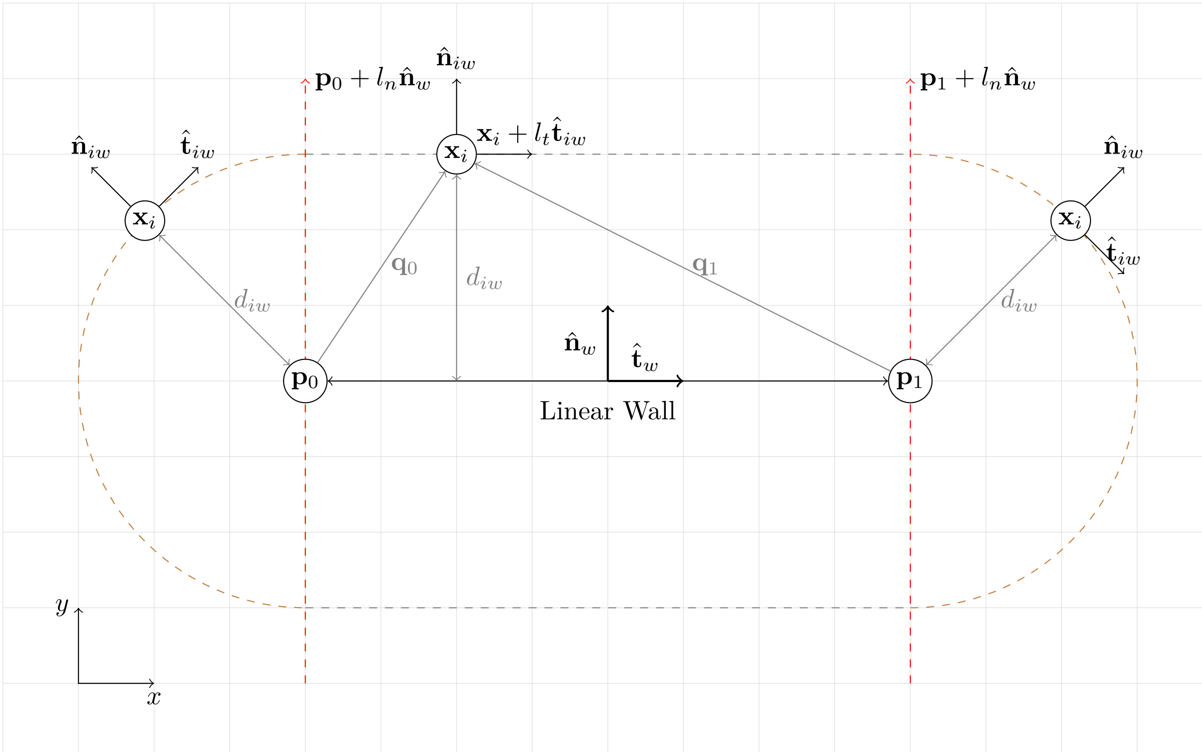

Figure 1: Linear wall

4 Field

4.1 Walls

4.1.1 Round

4.1.2 Linear

Solving linear system of equations determining the position of the agent $\mathbf{x}_i$ in relation to wall

$$\begin{cases} \mathbf{p}_0 + l_{n_0}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_0}\hat{\mathbf{t}}_w \\ \mathbf{p}_1 + l_{n_1}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_1}\hat{\mathbf{t}}_w \end{cases}$$

$$\begin{cases} l_{n_0}\hat{\mathbf{n}}_w - l_{t_0}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_0 = \mathbf{q}_0 \\ l_{n_1}\hat{\mathbf{n}}_w - l_{t_1}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_1 = \mathbf{q}_1 \end{cases}$$

In matrix form

$$\begin{bmatrix} l_{n_0} & l_{n_1} \\ l_{t_0} & l_{t_1} \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \mathbf{q}_0 & \mathbf{q}_1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} \hat{\mathbf{n}}_w & -\hat{\mathbf{t}}_w \end{bmatrix} = \begin{bmatrix} -t_1 & -t_0 \\ t_0 & -t_1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A}^{-1} &= \frac{1}{t_0^2 + t_1^2} \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} = \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} \\ &= \begin{bmatrix} \hat{\mathbf{n}}_w \\ -\hat{\mathbf{t}}_w \end{bmatrix} = \mathbf{A}^T \end{aligned}$$

Conditions

$$\begin{aligned} l_n &= l_{n_0} \vee l_{n_1} = \hat{\mathbf{n}}_w \cdot \mathbf{q}_0 \vee \hat{\mathbf{n}}_w \cdot \mathbf{q}_1 \\ l_t &= l_{t_1} + l_{t_0} = -\hat{\mathbf{t}}_w \cdot \mathbf{q}_1 - \hat{\mathbf{t}}_w \cdot \mathbf{q}_0 \end{aligned}$$

Distance between agent and linear wall

$$d_{iw} = \begin{cases} \|\mathbf{q}_0\| & l_t > l_w \\ |l_n| & \text{otherwise} \\ \|\mathbf{q}_1\| & l_t < -l_w \end{cases}$$

Normal vector away from the wall

$$\hat{\mathbf{n}}_{iw} = \begin{cases} \hat{\mathbf{q}}_0 & l_t > l_w \\ \operatorname{sign}(l_n)\hat{\mathbf{n}}_w & \text{otherwise} \\ \hat{\mathbf{q}}_1 & l_t < -l_w \end{cases}$$

4.2 Agents

4.2.1 Round

4.2.2 Elliptical

5 Differential system

Acceleration on an agent i

$$a_i(t) = \frac{\mathbf{f}_i(t)}{m_i}$$

Updating velocity using discrete time step Δt

$$\begin{aligned}\Delta \mathbf{v} &= a(t_k)\Delta t \\ \mathbf{v}(t_{k+1}) &= \mathbf{v}(t_k) + \Delta \mathbf{v}\end{aligned}$$

Updating position

$$\begin{aligned}\Delta \mathbf{x} &= \mathbf{v}(t_{k+1})\Delta t \\ \mathbf{x}(t_{k+1}) &= \mathbf{x}(t_k) + \Delta \mathbf{x}\end{aligned}$$