1 Constants

Symbol	Unit	Value	Explanation
Δt	s	$0.01 - 0.001$	Timestep
τ_{adj}	s	0.5	Characteristic time in which agent adjusts its movement.
k	N	1.5	Social force scaling constant.
τ_0	s	3.0	Interaction time horizon.
μ	kg s^{-2}	$1.2e+05$	Compression counteraction constant.
κ	kg m^{-1} s^{-1}	$2.4e+05$	Sliding friction constant.
A	N	$2.0e+03$	Scaling coefficient for social force.
B	m	0.08	Coefficient for social force.
$\Vert\mathbf{f}_{max}\Vert$	N		Force magnitude limit.

	Total		Torso	Shoulder		Velocity	
	r (m)	$\pm$	r_t/r	r_s/r	r_{t-s}/r	v (m/s)	$\pm$
adult	0.255	0.035	0.5882	0.3725	0.6275	1.25	0.3
child	0.210	0.015	0.5714	0.3333	0.6667	0.90	0.3
eldery	0.250	0.020	0.6000	0.3600	0.6400	0.80	0.3
female	0.240	0.020	0.5833	0.3750	0.6250	1.15	0.2
male	0.270	0.020	0.5926	0.3704	0.6296	1.35	0.2

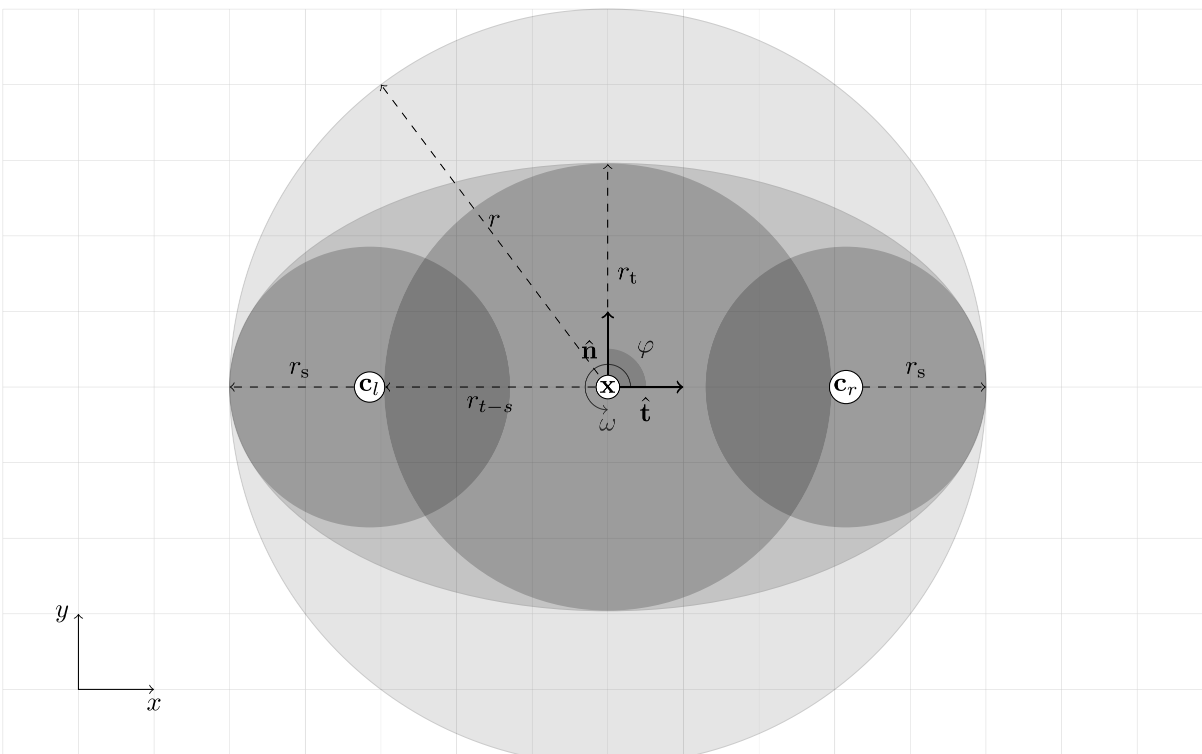

Figure 1: Circle, ellipse and three circle representations of an agent.

2 Agents

2.1 Properties

r	m		Radius
m	kg	80	Mass
I	kg m^2	4.0	Moment of inertia
$\mathbf{x}$			Position
$\mathbf{v}$			Velocity
v_0			Goal velocity
$\hat{\mathbf{e}}_0$			Goal direction
$\hat{\mathbf{e}}$			Target direction
φ			Body angle
ω			Angular velocity
φ_0			Target angle
ω_0	s^{-1}	4π	Max angular velocity
p		$0-1$	Herding tendency

Relative

$\tilde{\mathbf{x}} = \mathbf{x}_i - \mathbf{x}_j$	Relative position
$\tilde{\mathbf{v}} = \mathbf{v}_i - \mathbf{v}_j$	Relative velocity
$d = \lVert\tilde{\mathbf{x}}\rVert$	Distance
$\hat{\mathbf{n}} = \tilde{\mathbf{x}}/d$	Normal vector
$\hat{\mathbf{t}} = R(-90^\circ)\cdot\hat{\mathbf{n}}$	Tangent vector

2.2 Circular agent

Total radius and relative distance

$$\begin{aligned}\tilde{r} &= r_i + r_j \\ h &= \tilde{r} - d\end{aligned}$$

2.3 Three circles

2.4 Rotational equation

Rotational equation of motion

$$I\frac{d^2}{dt^2}\varphi(t) = M(t) + \eta(t),$$

where $\eta(t)$ is small random fluctuation torque, and $M(t)$ is total torque, which is the sum of contact, social and motivational torque

$$M_i(t) = M_i^c + M_i^{soc} + M_i^\tau$$

Torque from contact forces

$$\mathbf{M}_i^c = \sum_{j\neq i}\left(\mathbf{R}_i^c \times \mathbf{f}_{ij}^c\right)$$

and from social forces

$$\mathbf{M}_i^{soc} = \sum_{j\neq i}\left(\mathbf{R}_i^{soc} \times \mathbf{f}_{ij}^{soc}\right)$$

Motivational torque

$$M_i^\tau = \frac{I_i}{\tau_i}\left((\varphi_i(t) - \varphi_i^0)\omega^0 - \omega(t)\right)$$

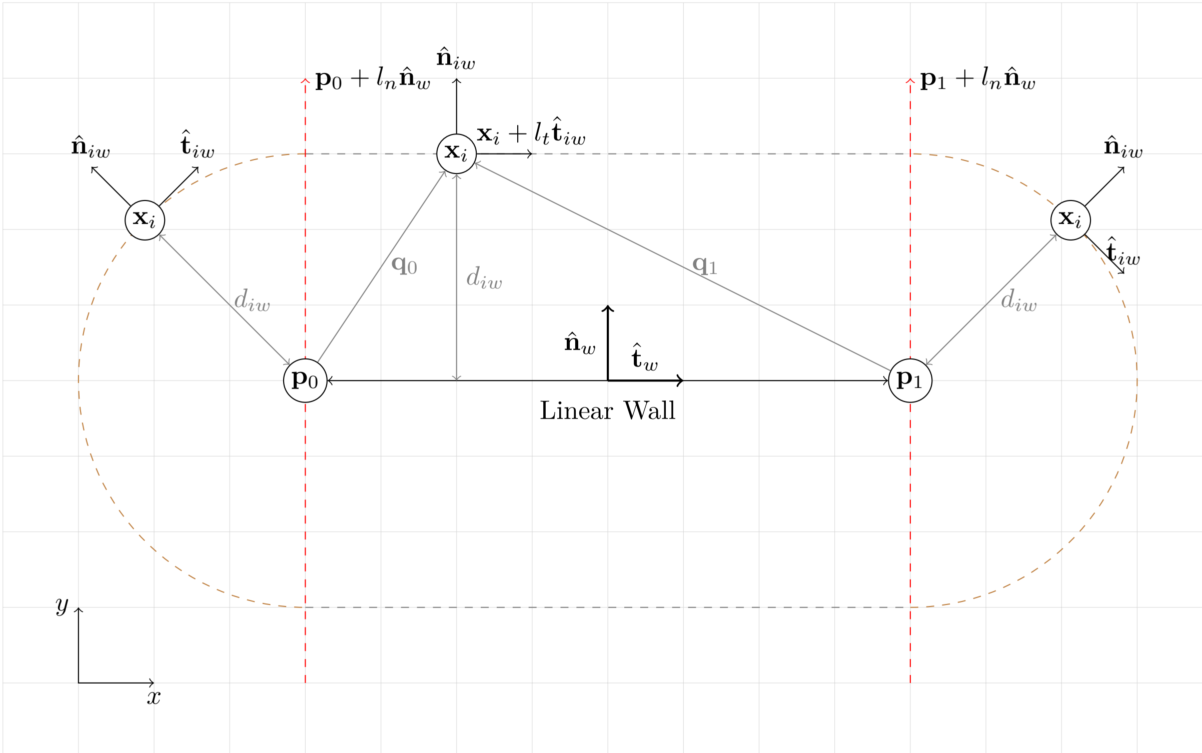

Figure 2: Absolute distance from a linear wall.

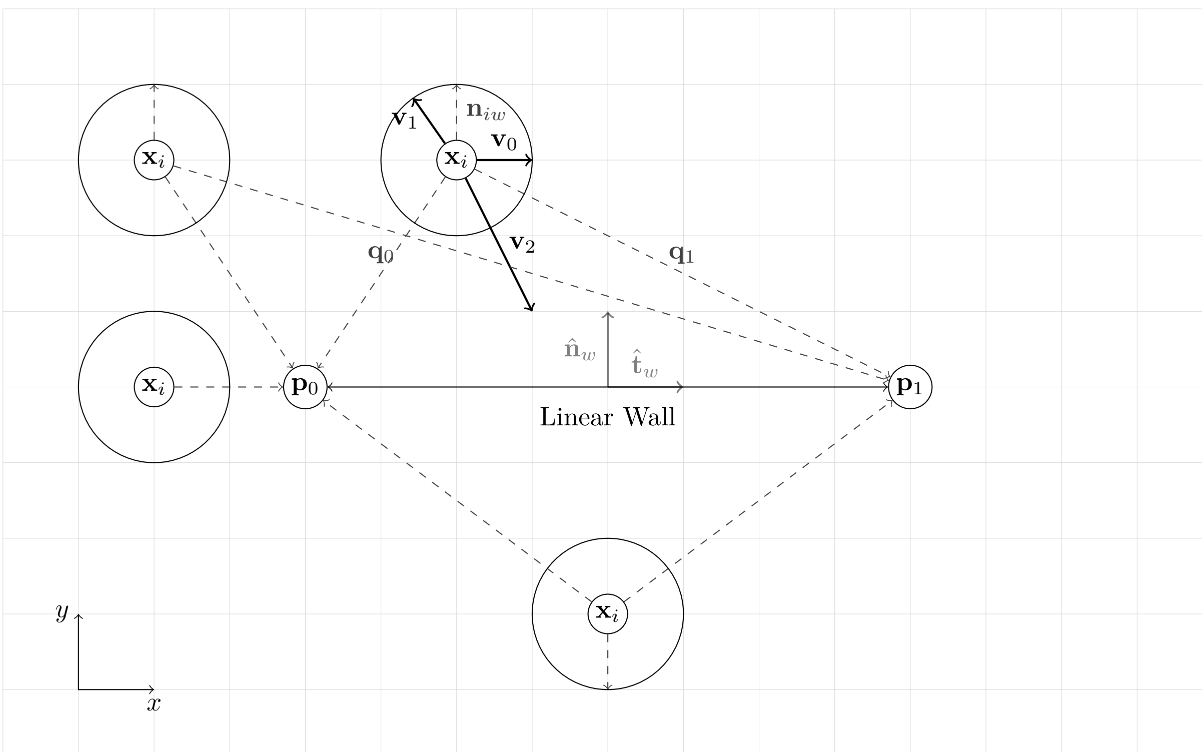

Figure 3: Velocity dependent distance from a linear wall.

3 Linear wall

3.1 Properties

$\mathbf{p}_0$	Start point
$\mathbf{p}_1$	End point

Relative

$$\begin{aligned} h_{iw} &= r_i - d_{iw} \\ l_w &= \|\mathbf{p}_1 - \mathbf{p}_0\| \\ \hat{\mathbf{t}}_w &= (\mathbf{p}_1 - \mathbf{p}_0)/l_w \\ \hat{\mathbf{n}}_w &= R(90^\circ) \cdot \hat{\mathbf{t}}_w \end{aligned}$$

3.2 Absolute distance

Solving linear system of equations determining the position of the agent $\mathbf{x}_i$ in relation to wall

$$\begin{cases} \mathbf{p}_0 + l_{n_0}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_0}\hat{\mathbf{t}}_w \\ \mathbf{p}_1 + l_{n_1}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_1}\hat{\mathbf{t}}_w \end{cases}$$

$$\begin{cases} l_{n_0}\hat{\mathbf{n}}_w - l_{t_0}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_0 = \mathbf{q}_0 \\ l_{n_1}\hat{\mathbf{n}}_w - l_{t_1}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_1 = \mathbf{q}_1 \end{cases}$$

In matrix form

$$\begin{bmatrix} l_{n_0} & l_{n_1} \\ l_{t_0} & l_{t_1} \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \mathbf{q}_0 & \mathbf{q}_1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \hat{\mathbf{n}}_w & -\hat{\mathbf{t}}_w \end{bmatrix} = \begin{bmatrix} -t_1 & -t_0 \\ t_0 & -t_1 \end{bmatrix} \\ \mathbf{A}^{-1} &= \frac{1}{t_0^2 + t_1^2} \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} = \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} \\ &= \begin{bmatrix} \hat{\mathbf{n}}_w \\ -\hat{\mathbf{t}}_w \end{bmatrix} = \mathbf{A}^T \end{aligned}$$

Conditions

$$l_n = l_{n_0} \vee l_{n_1} = \hat{\mathbf{n}}_w \cdot \mathbf{q}_0 \vee \hat{\mathbf{n}}_w \cdot \mathbf{q}_1$$

$$l_t = l_{t_1} + l_{t_0} = -\hat{\mathbf{t}}_w \cdot \mathbf{q}_1 - \hat{\mathbf{t}}_w \cdot \mathbf{q}_0$$

Distance between agent and linear wall

$$d_{iw} = \begin{cases} \|\mathbf{q}_0\| & l_t > l_w \\ |l_n| & \text{otherwise} \\ \|\mathbf{q}_1\| & l_t < -l_w \end{cases}$$

Normal vector away from the wall

$$\hat{\mathbf{n}}_{iw} = \begin{cases} \hat{\mathbf{q}}_0 & l_t > l_w \\ \operatorname{sign}(l_n)\hat{\mathbf{n}}_w & \text{otherwise} \\ \hat{\mathbf{q}}_1 & l_t < -l_w \end{cases}$$

3.3 Velocity relative distance

$\tilde{\mathbf{x}} = \mathbf{x}_{iw}$	Relative position
$\tilde{\mathbf{v}} = \mathbf{v}_{iw} = \mathbf{v}_i$	Relative velocity
$\tilde{r} = r_{iw}$	Total radius
$d = \|\tilde{\mathbf{x}}\|$	Distance
$h = \tilde{r} - d$	Relative distance

Dividing vectors

$$\begin{aligned} \mathbf{q}_0 &= \mathbf{p}_0 - \mathbf{x} \\ \mathbf{q}_1 &= \mathbf{p}_1 - \mathbf{x} \\ \hat{\mathbf{n}}_{iw} &= -\operatorname{sign}(\hat{\mathbf{n}}_w \cdot \mathbf{q}_0)\hat{\mathbf{n}}_w \end{aligned}$$

Angle of 2D vector is found using `https://en.wikipedia.org/wiki/Atan2` where angle is between $[-\pi, \pi]$

$$\boldsymbol{\alpha} = [\operatorname{angle}(\mathbf{q}_0), \operatorname{angle}(\mathbf{q}_1), \operatorname{angle}(\hat{\mathbf{n}}_{iw})]$$

$$\varphi = \operatorname{angle}(\mathbf{v})$$

4 Crowd dynamics

4.1 Social force model

Total force exerted on the agent is the sum of movement adjusting, social and contact forces between other agents and wall.

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c} \right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c} \right) + \boldsymbol{\xi}_i$$

4.2 Adjusting force

Force adjusting agent's movement towards desired in some characteristic time

$$\mathbf{f}^{adj} = \frac{m}{\tau^{adj}} (v_0 \cdot \hat{\mathbf{e}} - \mathbf{v})$$

4.3 Social force

Psychological force for collision avoidance

4.3.1 Velocity independent

$$\mathbf{f}^{soc} = A \exp\left(\frac{h}{B}\right) \hat{\mathbf{n}}$$

4.3.2 Velocity dependent

$$\begin{aligned}
\mathbf{f}^{soc} &= -\nabla_{\tilde{\mathbf{x}}} E(\tau) \\
&= -\nabla_{\tilde{\mathbf{x}}} \left(\frac{k}{\tau^2} \exp\left(-\frac{\tau}{\tau_0} \right) \right) \\
&= -\left(\frac{k}{a\tau^2} \right) \left(\frac{2}{\tau} + \frac{1}{\tau_0} \right) \exp\left(-\frac{\tau}{\tau_0} \right) \left(\tilde{\mathbf{v}} - \frac{a\tilde{\mathbf{x}} + b\tilde{\mathbf{v}}}{d} \right),
\end{aligned}$$

where

$$\begin{aligned}
a &= \tilde{\mathbf{v}} \cdot \tilde{\mathbf{v}} \\
b &= -\tilde{\mathbf{x}} \cdot \tilde{\mathbf{v}} \\
c &= \tilde{\mathbf{x}} \cdot \tilde{\mathbf{x}} - \tilde{r}^2 \\
d &= \sqrt{b^2 - ac}, \quad b^2 - ac > 0 \\
\tau &= \frac{b - d}{a} > 0.
\end{aligned}$$

4.4 Contact force

Physical contact force

$$\mathbf{f}^c = h \cdot \left(\mu \cdot \hat{\mathbf{n}} - \kappa \cdot (\mathbf{v} \cdot \hat{\mathbf{t}}) \hat{\mathbf{t}} \right), \quad h > 0$$

4.5 Random Fluctuation

Uniformly distributed random fluctuation force

$$\boldsymbol{\xi} = f \cdot \left[\cos(\varphi), \quad \sin(\varphi) \right],$$

where

$$f \in [0, f_{max}], \quad \varphi \in [0, 2\pi)$$

4.6 Target direction

Herding behavior

$$\mathbf{e}_i = (1 - p_i) \hat{\mathbf{e}}_i^0 + p_i \left\langle \hat{\mathbf{e}}_j^0 \right\rangle_i$$

5 Integrators

5.1 Differential systems

Angle and angular velocity

$$I \frac{d^2}{dt^2} \varphi(t) = M(t)$$

Position and velocity

$$m \frac{d^2}{dt^2} \mathbf{x}(t) = \mathbf{f}(t)$$

5.2 Numerical methods

Updating using discrete time step Δt

$$\begin{aligned}
t_0 &= 0 \\
t_1 &= t_0 + \Delta t \\
&\vdots \\
t_k &= t_{k-1} + \Delta t
\end{aligned}$$

5.3 Excplicit Euler Method

Angular acceleration

$$\begin{aligned}
\alpha_k &= M_k / I \\
\omega_{k+1} &= \omega_k + \alpha_k \Delta t \\
\varphi_{k+1} &= \varphi_k + \omega_{k+1} \Delta t
\end{aligned}$$

Acceleration on an agent

$$\begin{aligned}
a_k &= \mathbf{f}_k / m \\
\mathbf{v}_{k+1} &= \mathbf{v}_k + a_k \Delta t \\
\mathbf{x}_{k+1} &= \mathbf{x}_k + \mathbf{v}_{k+1} \Delta t
\end{aligned}$$

5.4 Velocity verlet

Velocity verlet algorithm

$$
\begin{aligned}
\mathbf{v}_{k+\frac{1}{2}} &= \mathbf{v}_k + \frac{1}{2} a_k \Delta t \\
\mathbf{x}_{k+1} &= \mathbf{x}_k + \mathbf{v}_{k+\frac{1}{2}} \Delta t \\
\mathbf{v}_{k+1} &= \mathbf{v}_{k+\frac{1}{2}} + \frac{1}{2} a_{k+1} \Delta t
\end{aligned}
$$

or more simply

$$
\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{x}_k + \mathbf{v}_k \Delta t + \frac{1}{2} a_k \Delta t^2 \\
\mathbf{v}_{k+1} &= \mathbf{v}_k + \frac{1}{2} \left(a_k + a_{k+1} \right) \Delta t
\end{aligned}
$$