1 Constants

Symbol	Unit	Explanation
τ_{adj}	s	Characteristic time in which agent adjusts its movement
k	N	Social force scaling constant
τ_0	s	Max interaction range 2 - 4, aka interaction time horizon
$sight$	m	Max distance between agents for interaction to occur
f_{max}	N	Forces that are greater will be truncated to max force
μ	$\frac{\mathrm{kg}}{\mathrm{s}^2}$	Compression counteraction constant
κ	$\frac{\mathrm{kg}}{\mathrm{m\,s}}$	Sliding friction constant
A	N	Scaling coefficient for social force between wall and agent
B	m	Coefficient for social force between wall and agent

2 Definitions

Vector norm and inner product

$$\langle \mathbf{x}, \mathbf{x} \rangle = \mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}\|^2$$

Unit vector

$$\hat{\mathbf{e}} = \frac{\mathbf{e}}{\|\mathbf{e}\|}$$

Rotation matrix to counterclockwise direction

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$R(90^\circ) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad R(-90^\circ) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

3 Parameters

Properties of an agent i

$$\begin{aligned} \mathbf{x}_i &= \text{Centre} \\ \mathbf{v}_i &= \text{Velocity} \\ r_i &= \text{Radius} \\ m_i &= \text{Mass} \end{aligned}$$

$$\begin{aligned} \mathbf{x}_{ij} &= \mathbf{x}_i - \mathbf{x}_j \\ \mathbf{v}_{ij} &= \mathbf{v}_i - \mathbf{v}_j \\ r_{ij} &= r_i + r_j \end{aligned}$$

$$\begin{aligned} d_{ij} &= \|\mathbf{x}_{ij}\| \\ h_{ij} &= r_{ij} - d_{ij} \\ h_{iw} &= r_i - d_{iw} \end{aligned}$$

$$\begin{aligned} \hat{\mathbf{n}}_{ij} &= \frac{\mathbf{x}_{ij}}{d_{ij}} \\ \hat{\mathbf{t}}_{ij} &= R(-90^\circ) \cdot \mathbf{n}_{ij} \end{aligned}$$

4 Crowd dynamics

4.1 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c} \right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c} \right) + \boldsymbol{\xi}_i$$

i) Force adjusting agent's movement towards desired in some characteristic time τ_i^{adj}

$$\mathbf{f}_i^{adj} = \frac{m_i}{\tau_i^{adj}} (\mathbf{v}_i^0 - \mathbf{v}_i)$$

Target velocity can be broken down

$$\mathbf{v}_i^0 = v_i^0 \cdot \hat{\mathbf{e}}_i^0$$

Target direction

$$\begin{aligned} \mathbf{e}_i^0(t) &= (1 - p_i)\mathbf{e}_i + p_i \left\langle \mathbf{e}_j^0(t) \right\rangle_i \\ \hat{\mathbf{e}}_i^0(t) &= \frac{\mathbf{e}_i^0(t)}{\|\mathbf{e}_i^0(t)\|} \end{aligned}$$

ii) Psychological tendency to keep a certain distance to other agents

$$\mathbf{f}_{ij}^{soc} = \begin{cases} \mathbf{f}_{ij}^{pow} & d_{ij} \leq \text{sight} \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^{soc} = \begin{cases} A_i \exp\left(\frac{h_{iw}}{B_i}\right) \hat{\mathbf{n}}_{iw} & d_{iw} \leq \text{sight} \\ 0 & \text{otherwise} \end{cases}$$

iii) Physical contact forces with other agents

$$\mathbf{f}_{ij}^{c} = \begin{cases} h_{ij} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{ij} - \kappa \cdot \left(\mathbf{v}_{ji} \cdot \hat{\mathbf{t}}_{ij}\right) \hat{\mathbf{t}}_{ij}\right) & h_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^{c} = \begin{cases} h_{iw} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{iw} - \kappa \cdot (\mathbf{v}_{i} \cdot \hat{\mathbf{t}}_{iw}) \hat{\mathbf{t}}_{iw}\right) & h_{iw} > 0 \\ 0 & \text{otherwise} \end{cases}$$

iv) Uniformly distributed random fluctuation force

$$\boldsymbol{\xi}_i = \boldsymbol{\mathcal{U}}(-1, 1).$$

4.2 Universal power law governing pedestrian interactions

Interaction force between agents

$$\begin{aligned} \mathbf{f}_{ij}^{pow} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) = -\nabla_{\mathbf{x}_{ij}} \left(\frac{k}{\tau^2} \exp\left(-\frac{\tau}{\tau_0}\right)\right) \\ &= -\left(\frac{k}{a\tau^2}\right)\left(\frac{2}{\tau} + \frac{1}{\tau_0}\right) \exp\left(-\frac{\tau}{\tau_0}\right)\left(\mathbf{v}_{ij} - \frac{a\mathbf{x}_{ij} + b\mathbf{v}_{ij}}{d}\right), \end{aligned}$$

where

$$\begin{aligned} a &= \mathbf{v}_{ij} \cdot \mathbf{v}_{ij} \\ b &= -\mathbf{x}_{ij} \cdot \mathbf{v}_{ij} \\ c &= \mathbf{x}_{ij} \cdot \mathbf{x}_{ij} - r_{ij}^2 \\ d &= \sqrt{b^2 - ac} \\ \tau &= \frac{b - d}{a} > 0. \end{aligned}$$

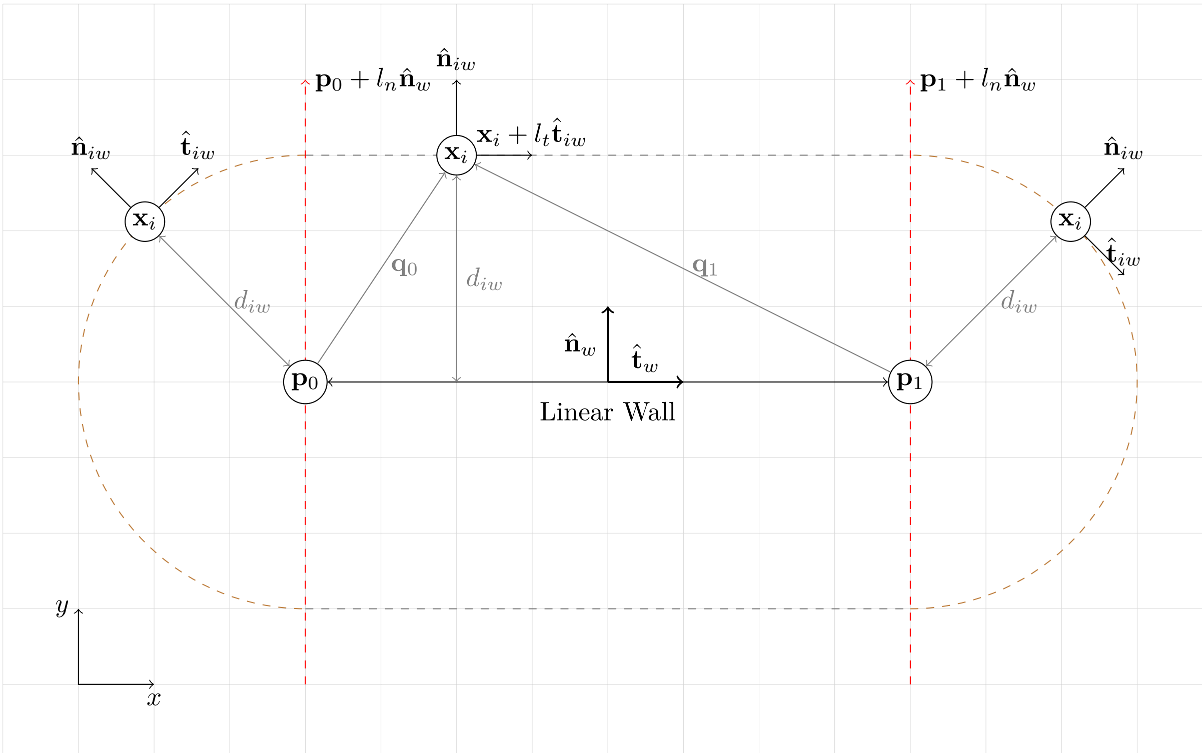

Figure 1: Linear wall

5 Field

5.1 Walls

5.1.1 Round

5.1.2 Linear

Properties

$$l_w = \|\mathbf{p}_1 - \mathbf{p}_0\|, \quad \hat{\mathbf{t}}_w = \frac{\mathbf{p}_1 - \mathbf{p}_0}{l_w}, \quad \hat{\mathbf{n}}_w = R(90°) \cdot \hat{\mathbf{t}}_w$$

Solving linear system of equations determining the position of the agent $\mathbf{x}_i$ in relation to wall

$$\begin{cases} \mathbf{p}_0 + l_{n_0} \hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_0} \hat{\mathbf{t}}_w \\ \mathbf{p}_1 + l_{n_1} \hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_1} \hat{\mathbf{t}}_w \end{cases}$$

$$\begin{cases} l_{n_0} \hat{\mathbf{n}}_w - l_{t_0} \hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_0 = \mathbf{q}_0 \\ l_{n_1} \hat{\mathbf{n}}_w - l_{t_1} \hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_1 = \mathbf{q}_1 \end{cases}$$

In matrix form

$$\begin{bmatrix} l_{n_0} & l_{n_1} \\ l_{t_0} & l_{t_1} \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \mathbf{q}_0 & \mathbf{q}_1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \hat{\mathbf{n}}_w & -\hat{\mathbf{t}}_w \end{bmatrix} = \begin{bmatrix} -t_1 & -t_0 \\ t_0 & -t_1 \end{bmatrix} \\ \mathbf{A}^{-1} &= \frac{1}{t_0^2 + t_1^2} \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} = \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} \\ &= \begin{bmatrix} \hat{\mathbf{n}}_w \\ -\hat{\mathbf{t}}_w \end{bmatrix} = \mathbf{A}^T \end{aligned}$$

$$\begin{aligned} l_n &= l_{n_0} \lor l_{n_1} = \hat{\mathbf{n}}_w \cdot \mathbf{q}_0 \lor \hat{\mathbf{n}}_w \cdot \mathbf{q}_1 \\ l_t &= l_{t_1} + l_{t_0} = -\hat{\mathbf{t}}_w \cdot \mathbf{q}_1 - \hat{\mathbf{t}}_w \cdot \mathbf{q}_0 \end{aligned}$$

$$d_{iw} = \begin{cases} \|\mathbf{q}_0\| & l_t > l_w \\ |l_n| & \text{otherwise} \\ \|\mathbf{q}_1\| & l_t < -l_w \end{cases}$$

$$\hat{\mathbf{n}}_{iw} = \begin{cases} \hat{\mathbf{q}}_0 & l_t > l_w \\ \operatorname{sign}(l_n)\hat{\mathbf{n}}_w & \text{otherwise} \\ \hat{\mathbf{q}}_1 & l_t < -l_w \end{cases}$$

5.2 Agents

5.2.1 Round

5.2.2 Elliptical

6 Differential system

Acceleration on an agent i

$$a_i(t) = \frac{\mathbf{f}_i(t)}{m_i}$$

Updating velocity using discrete time step Δt

$$\begin{aligned} \Delta\mathbf{v} &= a(t_k)\Delta t \\ \mathbf{v}(t_{k+1}) &= \mathbf{v}(t_k) + \Delta\mathbf{v} \end{aligned}$$

Updating position

$$\begin{aligned} \Delta\mathbf{x} &= \mathbf{v}(t_{k+1})\Delta t \\ \mathbf{x}(t_{k+1}) &= \mathbf{x}(t_k) + \Delta\mathbf{x} \end{aligned}$$