1 Crowd dynamics

1.1 Total Force on Agent

Total force on the agent

$$\begin{aligned}\mathbf{F}_i &= m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{d^2\mathbf{x}_i}{dt^2} \\ &= \mathbf{f}_i(t) + \boldsymbol{\xi}_i\end{aligned}$$

1. Force on the agent i

$$\mathbf{f}_i(t) = \frac{m_i}{\tau_i}(\mathbf{v}_i^0 - \mathbf{v}_i) + \sum_{i \neq j} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c}\right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c}\right)$$

   (a) Force adjusting pedestrian movement towards desired in characteristic time τ_i

$$\frac{m_i}{\tau_i}(\mathbf{v}_i^0 - \mathbf{v}_i)$$

   (b) Psychological tendency to keep a certain distance to other pedestrians $\mathbf{f}_{ij}^{soc}$ and walls $\mathbf{f}_{iw}^{soc}$.

   (c) Physical contact forces with other pedestrians $\mathbf{f}_{ij}^{c}$ and walls $\mathbf{f}_{iw}^{c}$.

2. Random fluctuation force $\boldsymbol{\xi}_i$.

1.2 Power Law

Interaction force between agents

$$\begin{aligned}\mathbf{F}_{ij} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) \\ &= -\nabla_{\mathbf{x}_{ij}} \left(k\tau^{-2} e^{-\tau/\tau_0}\right) \\ &= -\left[\frac{k e^{-\tau/\tau_0}}{\|\mathbf{v}_{ij}\|^2 \tau^2}\left(\frac{2}{\tau} + \frac{1}{\tau_0}\right)\right] \\ &\left[\mathbf{v}_{ij} - \frac{\|\mathbf{v}_{ij}\|^2 \mathbf{x}_{ij} - (\mathbf{x}_{ij} \cdot \mathbf{v}_{ij})\,\mathbf{v}_{ij}}{\sqrt{(\mathbf{x}_{ij} \cdot \mathbf{v}_{ij})^2 - \|\mathbf{v}_{ij}\|^2 \left(\|\mathbf{x}_{ij}\|^2 - (r_i + r_j)^2\right)}}\right]\end{aligned}$$

$$\begin{aligned}\mathbf{x}_{ij} &= \mathbf{x}_i - \mathbf{x}_j \\ \mathbf{v}_{ij} &= \mathbf{v}_i - \mathbf{v}_j\end{aligned}$$

1.3 Properties

Indexing

1. Number of agents $N \in \mathbb{N}$
   (a) Current agent i
   (b) Other agent j

   $i, j \in \{0, \ldots, N\}$ and $i \neq j$
2. Wall w

Agent

1. Mass m_i
2. Shape
   (a) Circle, radius r_i
   (b) Ellipse
3. Velocity
   (a) Desired velocity v_i^0
   (b) Current velocity v_i
4. Characteristic time
   (a) τ_0
   (b) τ_i