1 Constants

Table 1

Symbol	Unit	Value	Explanation
			System
dt	s		Discrete timestep used to update the differential system
			Constants
τ_{adj}	s	0.5	Characteristic time in which agent adjusts its movement
k	N	1.5	Social force scaling constant
τ_0	s	3.0	Max interaction range 2 - 4, aka interaction time horizon
μ	$\frac{\mathrm{kg}}{\mathrm{s}^2}$	$1.2e+05$	Compression counteraction constant
κ	$\frac{\mathrm{kg}}{\mathrm{m\,s}}$	$2.4e+05$	Sliding friction constant
A	N	$2.0e+03$	Scaling coefficient for social force between wall and agent
B	m	0.08	Coefficient for social force between wall and agent
			Limits
f_{max}	N		Forces that are greater will be truncated to max force

2 Parameters

2.1 Agent

Properties of an agent i

r_i	Radius	Constant
m_i	Mass	Constant
v_i^0	Goal velocity	Variable/Constant
$\mathbf{x}_i$	Position	Variable
$\mathbf{v}_i$	Velocity	Variable
$\hat{\mathbf{e}}_i$	Goal direction	Variable
$\mathbf{p}_i$	Herding tendency	Variable/Constant

Relative properties

$\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$	Relative position between two agents
$\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$	Relative velocity between two agents
$r_{ij} = r_i + r_j$	Total radius
$d_{ij} = \lVert \mathbf{x}_{ij} \rVert$	Distance between agents
$h_{ij} = r_{ij} - d_{ij}$	Relative distance between agents
$\hat{\mathbf{n}}_{ij} = \mathbf{x}_{ij}/d_{ij}$	Normal vector
$\hat{\mathbf{t}}_{ij} = R(-90^\circ) \cdot \hat{\mathbf{n}}_{ij}$	Tangent vector

2.2 Wall

Properties of linear wall w

$\mathbf{p}_0$	Starting point of linear wall
$\mathbf{p}_1$	End point of linear wall

Relative properties

$$h_{iw} = r_i - d_{iw}$$
$$l_w = \lVert \mathbf{p}_1 - \mathbf{p}_0 \rVert$$
$$\hat{\mathbf{t}}_w = (\mathbf{p}_1 - \mathbf{p}_0)/l_w$$
$$\hat{\mathbf{n}}_w = R(90^\circ) \cdot \hat{\mathbf{t}}_w$$

3 Crowd dynamics

3.1 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c} \right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c} \right) + \boldsymbol{\xi}_i$$

3.2 Adjusting Force

Force adjusting agent's movement towards desired in some characteristic time

$$\mathbf{f}_i^{adj} = \frac{m_i}{\tau_i^{adj}} (v_i^0 \cdot \hat{\mathbf{e}}_i - \mathbf{v}_i)$$

Herding behavior

$$\mathbf{e}_i = (1 - p_i) \hat{\mathbf{e}}_i^0 + p_i \left\langle \hat{\mathbf{e}}_j^0 \right\rangle_i$$

3.3 Agent-Agent

Psychological tendency to keep a certain distance to other agents

$$\mathbf{f}_{ij}^{soc} = \mathbf{f}_{ij}^{pow}, \quad d_{ij} \leq \text{sight}$$

Physical contact forces with other agents

$$\mathbf{f}_{ij}^{c} = h_{ij} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{ij} - \kappa \cdot \left(\mathbf{v}_{ji} \cdot \hat{\mathbf{t}}_{ij} \right) \hat{\mathbf{t}}_{ij} \right), \quad h_{ij} > 0$$

3.4 Agent-Wall

Psychological tendency to keep a certain distance to walls

$$\mathbf{f}_{iw}^{soc} = A_i \exp \left(\frac{h_{iw}}{B_i} \right) \hat{\mathbf{n}}_{iw}, \quad d_{iw} \leq \text{sight}$$

Physical contact forces with walls

$$\mathbf{f}_{iw}^{c} = h_{iw} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{iw} - \kappa \cdot (\mathbf{v}_i \cdot \hat{\mathbf{t}}_{iw}) \hat{\mathbf{t}}_{iw} \right), \quad h_{iw} > 0$$

3.5 Random Fluctuation

Uniformly distributed random fluctuation force

$$\boldsymbol{\xi}_i = \boldsymbol{\mathcal{U}}(-1, 1).$$

3.6 Universal power law governing pedestrian interactions

Interaction force between agents

$$\begin{aligned}
\mathbf{f}_{ij}^{pow} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) \\
&= -\nabla_{\mathbf{x}_{ij}} \left(\frac{k}{\tau^2} \exp \left(-\frac{\tau}{\tau_0} \right) \right) \\
&= -\left(\frac{k}{a \tau^2} \right) \left(\frac{2}{\tau} + \frac{1}{\tau_0} \right) \exp \left(-\frac{\tau}{\tau_0} \right) \left(\mathbf{v}_{ij} - \frac{a \mathbf{x}_{ij} + b \mathbf{v}_{ij}}{d} \right),
\end{aligned}$$

where

$$\begin{aligned}
a &= \mathbf{v}_{ij} \cdot \mathbf{v}_{ij} \\
b &= -\mathbf{x}_{ij} \cdot \mathbf{v}_{ij} \\
c &= \mathbf{x}_{ij} \cdot \mathbf{x}_{ij} - r_{ij}^2 \\
d &= \sqrt{b^2 - ac} \\
\tau &= \frac{b - d}{a} > 0.
\end{aligned}$$

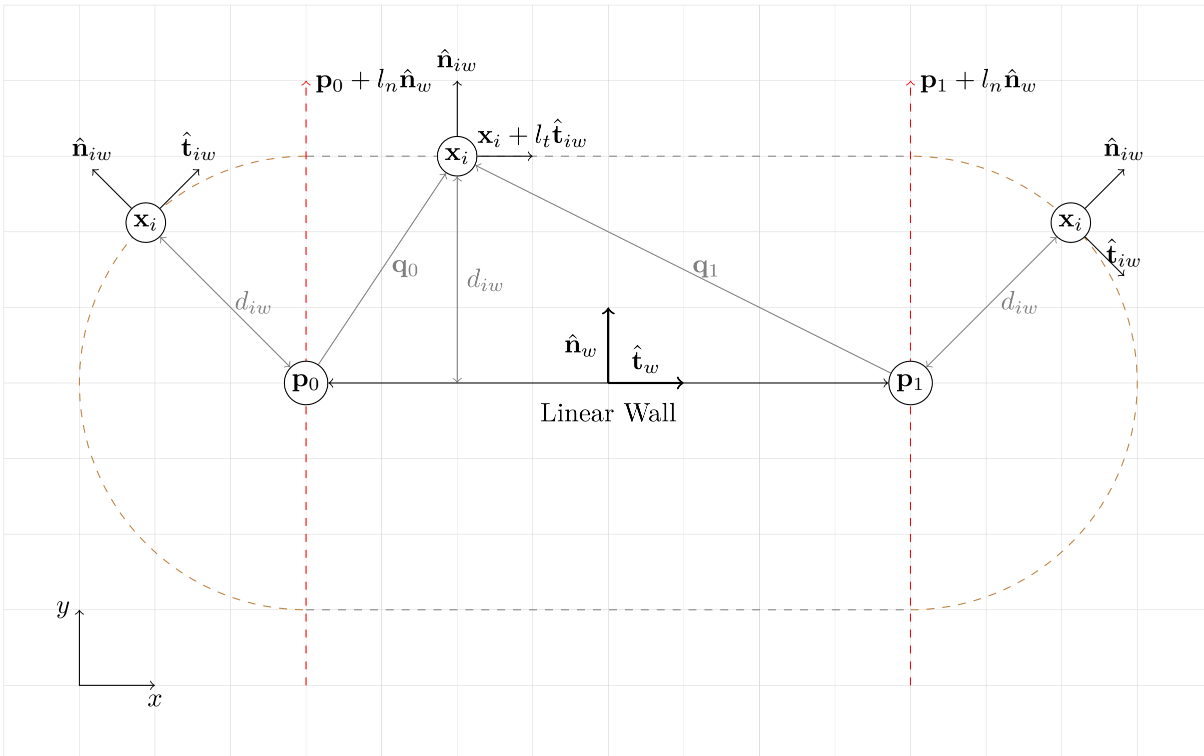

Figure 1: Linear wall

4 Walls

4.1 Round

4.2 Linear

Solving linear system of equations determining the position of the agent $\mathbf{x}_i$ in relation to wall

$$\begin{cases} \mathbf{p}_0 + l_{n_0}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_0}\hat{\mathbf{t}}_w \\ \mathbf{p}_1 + l_{n_1}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_1}\hat{\mathbf{t}}_w \end{cases}$$

$$\begin{cases} l_{n_0}\hat{\mathbf{n}}_w - l_{t_0}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_0 = \mathbf{q}_0 \\ l_{n_1}\hat{\mathbf{n}}_w - l_{t_1}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_1 = \mathbf{q}_1 \end{cases}$$

In matrix form

$$\begin{bmatrix} l_{n_0} & l_{n_1} \\ l_{t_0} & l_{t_1} \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \mathbf{q}_0 & \mathbf{q}_1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} \hat{\mathbf{n}}_w & -\hat{\mathbf{t}}_w \end{bmatrix} = \begin{bmatrix} -t_1 & -t_0 \\ t_0 & -t_1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{A}^{-1} &= \frac{1}{t_0^2 + t_1^2} \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} = \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} \\ &= \begin{bmatrix} \hat{\mathbf{n}}_w \\ -\hat{\mathbf{t}}_w \end{bmatrix} = \mathbf{A}^T \end{aligned}$$

Conditions

$$\begin{aligned} l_n &= l_{n_0} \vee l_{n_1} = \hat{\mathbf{n}}_w \cdot \mathbf{q}_0 \vee \hat{\mathbf{n}}_w \cdot \mathbf{q}_1 \\ l_t &= l_{t_1} + l_{t_0} = -\hat{\mathbf{t}}_w \cdot \mathbf{q}_1 - \hat{\mathbf{t}}_w \cdot \mathbf{q}_0 \end{aligned}$$

Distance between agent and linear wall

$$d_{iw} = \begin{cases} \|\mathbf{q}_0\| & l_t > l_w \\ |l_n| & \text{otherwise} \\ \|\mathbf{q}_1\| & l_t < -l_w \end{cases}$$

Normal vector away from the wall

$$\hat{\mathbf{n}}_{iw} = \begin{cases} \hat{\mathbf{q}}_0 & l_t > l_w \\ \operatorname{sign}(l_n)\hat{\mathbf{n}}_w & \text{otherwise} \\ \hat{\mathbf{q}}_1 & l_t < -l_w \end{cases}$$

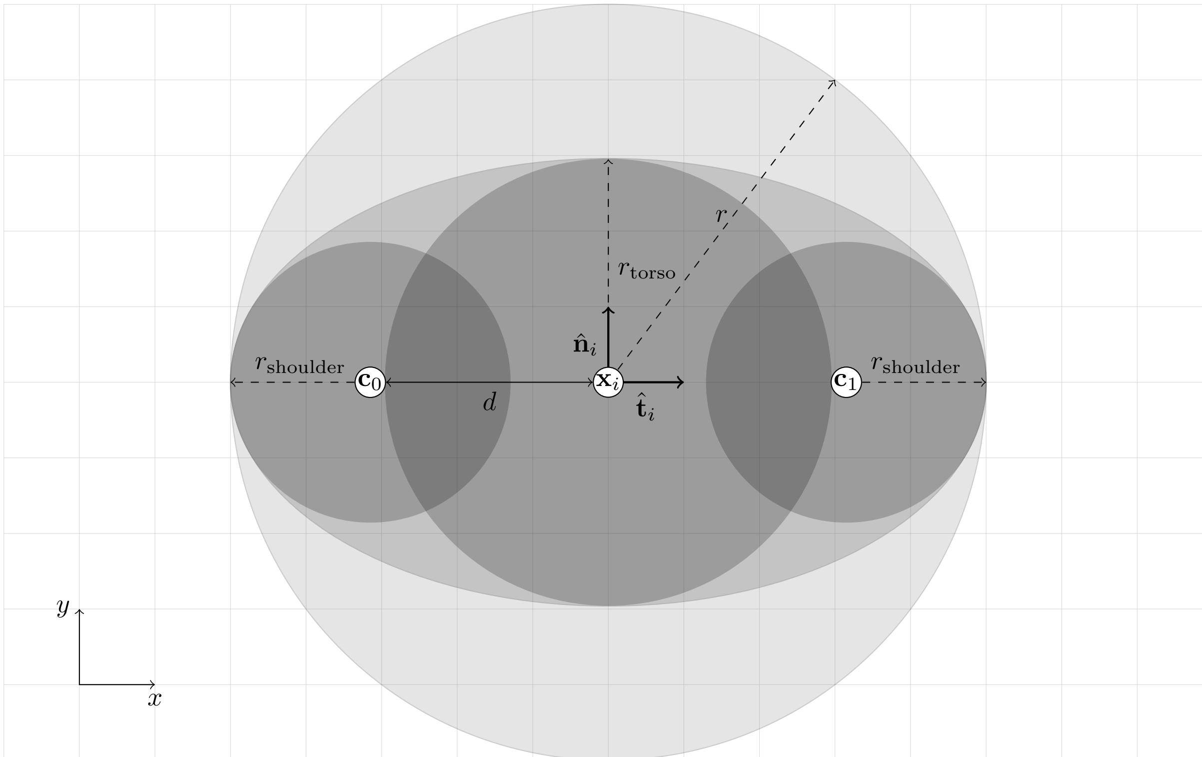

Figure 2: Circle, Ellipse and Three circle representations of an agent.

5 Agents

Table 2: Agent body properties

	r (m)	$\pm$	r_{torso}/r	r_{shoulder}/r	d/r	speed (m/s)	$\pm$
adult	0.255	0.035	0.5882	0.3725	0.6275	1.25	0.3
child	0.21	0.015	0.5714	0.3333	0.6667	0.9	0.3
eldery	0.25	0.02	0.6	0.36	0.64	0.8	0.3
female	0.24	0.02	0.5833	0.375	0.625	1.15	0.2
male	0.27	0.02	0.5926	0.3704	0.6296	1.35	0.2

6 Differential system

Acceleration on an agent i

$$a_i(t) = \frac{\mathbf{f}_i(t)}{m_i}$$

Updating velocity using discrete time step Δt

$$\Delta \mathbf{v} = a(t_k)\Delta t$$
$$\mathbf{v}(t_{k+1}) = \mathbf{v}(t_k) + \Delta \mathbf{v}$$

Updating position

$$\Delta \mathbf{x} = \mathbf{v}(t_{k+1})\Delta t$$
$$\mathbf{x}(t_{k+1}) = \mathbf{x}(t_k) + \Delta \mathbf{x}$$