

1 Crowd dynamics

1.1 Parameters

$$\langle \mathbf{x}, \mathbf{x} \rangle = \mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}, \mathbf{x}\|^2$$

\mathbf{x}_i = Centre of mass of agent i

\mathbf{v}_i = Velocity of agent i

r_i = Radius of agent i

m_i = Mass of agent i

$$\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$$

$$\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$$

$$r_{ij} = r_i + r_j$$

$$d_{ij} = \|\mathbf{x}_{ij}\|$$

$$a = \mathbf{v}_{ij} \cdot \mathbf{v}_{ij}$$

$$b = -\mathbf{x}_{ij} \cdot \mathbf{v}_{ij}$$

$$c = d_{ij}^2 - r_{ij}^2$$

$$d = \sqrt{b^2 - ac}$$

$$\tau = \frac{b - \sqrt{d}}{a} > 0$$

$$h_{iw} = r_i - d_{iw}$$

$$h_{ij} = r_i - d_{ij}$$

$$\mathbf{n}_{ij} = \frac{\mathbf{x}_{ij}}{d_{ij}}$$

$$\mathbf{t}_{ij} = R(45^\circ) \cdot \mathbf{n}_{ij}$$

1.2 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} (\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^c) + \sum_w (\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^c) + \boldsymbol{\xi}_i$$

- i) Force adjusting agent's movement towards desired in some characteristic time τ_i^{adj}

$$\begin{aligned} \mathbf{f}_i^{adj} &= \frac{m_i}{\tau_i^{adj}} (\mathbf{v}_i^0 - \mathbf{v}_i) \\ &= \frac{m_i}{\tau_i^{adj}} (\|v_i^0\| \mathbf{e}_i - \mathbf{v}_i) \end{aligned}$$

- ii) Psychological tendency to keep a certain distance to other agents

$$\mathbf{f}_{ij}^{soc} = \begin{cases} \mathbf{f}_{ij}^{pow} & d_{ij} \leq sight \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^{soc} = A_i \exp\left(\frac{h_{iw}}{B_i}\right) \mathbf{n}_{iw}$$

- iii) Physical contact forces with other agents

$$\mathbf{f}_{ij}^c = \begin{cases} h_{ij} (\mu \cdot \mathbf{n}_{ij} - \kappa \cdot (\mathbf{v}_{ji} \cdot \mathbf{t}_{ij}) \mathbf{t}_{ij}) & h_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^c = \begin{cases} h_{iw} (\mu \cdot \mathbf{n}_{iw} - \kappa \cdot (\mathbf{v}_i \cdot \mathbf{t}_{iw}) \mathbf{t}_{iw}) & h_{iw} > 0 \\ 0 & \text{otherwise} \end{cases}$$

- iv) Uniformly distributed random fluctuation force

$$\boldsymbol{\xi}_i = \mathcal{U}(-1, 1).$$

1.3 Universal power law governing pedestrian interactions

Interaction force between agents

$$\begin{aligned} \mathbf{f}_{ij}^{pow} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) = -\nabla_{\mathbf{x}_{ij}} \left(\frac{k}{\tau^2} \exp\left(-\frac{\tau}{\tau_0}\right) \right) \\ &= -\left(\frac{k}{a\tau^2} \right) \left(\frac{2}{\tau} + \frac{1}{\tau_0} \right) \exp\left(-\frac{\tau}{\tau_0}\right) \left(\mathbf{v}_{ij} - \frac{a\mathbf{x}_{ij} + b\mathbf{v}_{ij}}{d} \right) \end{aligned}$$