1 Definitions

Vector norm and inner product

$$\langle \mathbf{x}, \mathbf{x} \rangle = \mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}\|^2$$

Unit vector

$$\hat{\mathbf{e}} = \frac{\mathbf{e}}{\|\mathbf{e}\|}$$

Rotation matrix to counterclockwise direction

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$R(90°) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad R(-90°) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

2 Parameters

Properties of an agent i

$$\mathbf{x}_i = \text{Centre}$$
$$\mathbf{v}_i = \text{Velocity}$$
$$r_i = \text{Radius}$$
$$m_i = \text{Mass}$$

$$\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$$
$$\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$$
$$r_{ij} = r_i + r_j$$
$$d_{ij} = \|\mathbf{x}_{ij}\|$$
$$h_{ij} = r_{ij} - d_{ij}$$
$$h_{iw} = r_i - d_{iw}$$

$$a = \mathbf{v}_{ij} \cdot \mathbf{v}_{ij}$$
$$b = -\mathbf{x}_{ij} \cdot \mathbf{v}_{ij}$$
$$c = \mathbf{x}_{ij} \cdot \mathbf{x}_{ij} - r_{ij}^2$$
$$d = \sqrt{b^2 - ac}$$
$$\tau = \frac{b - \sqrt{d}}{a} > 0$$

$$\hat{\mathbf{n}}_{ij} = \frac{\mathbf{x}_{ij}}{d_{ij}}$$
$$\hat{\mathbf{t}}_{ij} = R(-90°) \cdot \mathbf{n}_{ij}$$

$$\tau^{adj} = 0.5$$
$$\tau_0 = 3.0$$
$$\text{sight} = 7$$
$$\mathbf{f}_{max} = 5.0$$
$$\mu = 1.2 \cdot 10^5$$
$$\kappa = 2.4 \cdot 10^5$$
$$A = 2 \cdot 10^3$$
$$B = 0.08$$

3 Crowd dynamics

3.1 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^c \right) + \sum_w \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^c \right) + \boldsymbol{\xi}_i$$

i) Force adjusting agent's movement towards desired in some characteristic time τ_i^{adj}

$$\mathbf{f}_i^{adj} = \frac{m_i}{\tau_i^{adj}} (\mathbf{v}_i^0 - \mathbf{v}_i)$$

Target velocity can be broken down

$$\mathbf{v}_i^0 = v_i^0 \cdot \hat{\mathbf{e}}_i^0$$

Target direction

$$\mathbf{e}_i^0(t) = (1 - p_i)\mathbf{e}_i + p_i \left\langle \mathbf{e}_j^0(t) \right\rangle_i$$
$$\hat{\mathbf{e}}_i^0(t) = \frac{\mathbf{e}_i^0(t)}{\|\mathbf{e}_i^0(t)\|}$$

ii) Psychological tendency to keep a certain distance to other agents

$$\mathbf{f}_{ij}^{soc} = \begin{cases} \mathbf{f}_{ij}^{pow} & d_{ij} \leq \text{sight} \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^{soc} = \begin{cases} A_i \exp\left(\frac{h_{iw}}{B_i}\right) \hat{\mathbf{n}}_{iw} & d_{iw} \leq \text{sight} \\ 0 & \text{otherwise} \end{cases}$$

iii) Physical contact forces with other agents

$$\mathbf{f}_{ij}^c = \begin{cases} h_{ij} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{ij} - \kappa \cdot \left(\mathbf{v}_{ji} \cdot \hat{\mathbf{t}}_{ij} \right) \hat{\mathbf{t}}_{ij} \right) & h_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^c = \begin{cases} h_{iw} \cdot \left(\mu \cdot \hat{\mathbf{n}}_{iw} - \kappa \cdot (\mathbf{v}_i \cdot \hat{\mathbf{t}}_{iw}) \hat{\mathbf{t}}_{iw} \right) & h_{iw} > 0 \\ 0 & \text{otherwise} \end{cases}$$

iv) Uniformly distributed random fluctuation force

$$\boldsymbol{\xi}_i = \mathcal{U}(-1, 1).$$

3.2 Universal power law governing pedestrian interactions

Interaction force between agents

$$\mathbf{f}_{ij}^{pow} = -\nabla_{\mathbf{x}_{ij}} E(\tau) = -\nabla_{\mathbf{x}_{ij}} \left(\frac{k}{\tau^2} \exp\left(-\frac{\tau}{\tau_0} \right) \right)$$

$$= -\left(\frac{k}{a\tau^2} \right) \left(\frac{2}{\tau} + \frac{1}{\tau_0} \right) \exp\left(-\frac{\tau}{\tau_0} \right) \left(\mathbf{v}_{ij} - \frac{a\mathbf{x}_{ij} + b\mathbf{v}_{ij}}{d} \right)$$
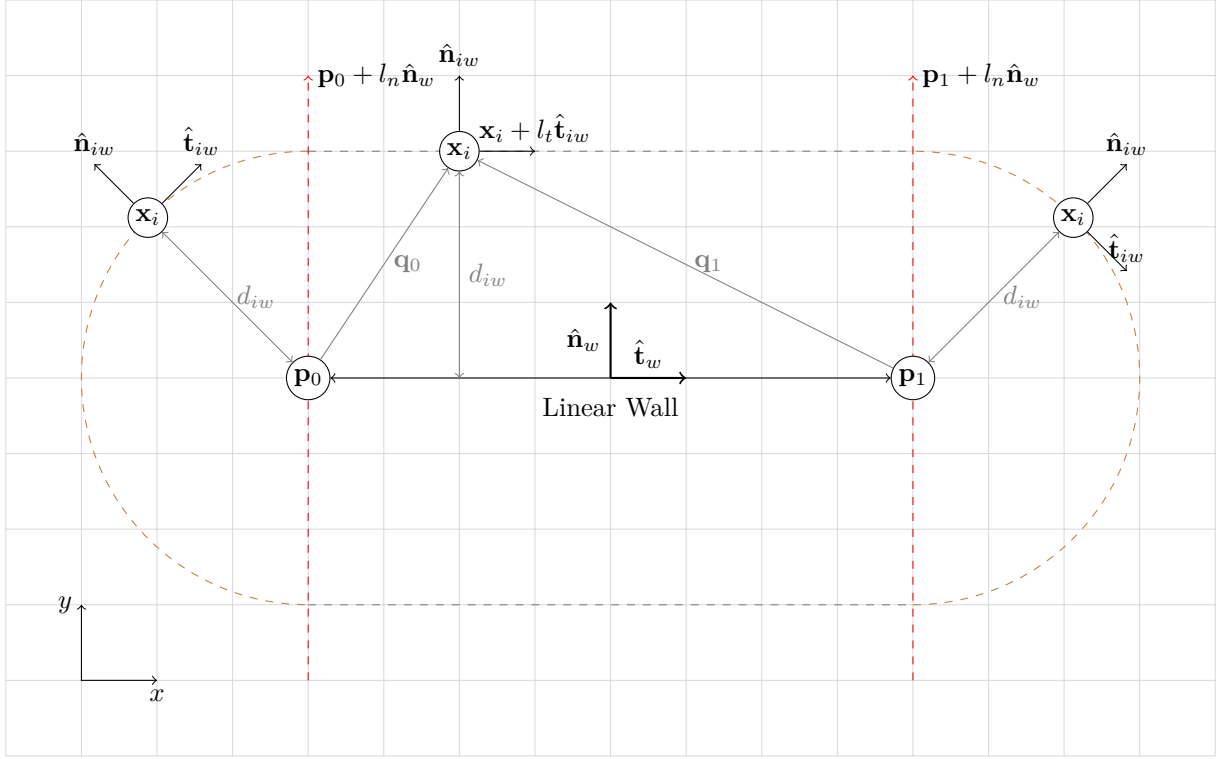
Figure 1: Linear wall

4 Field

4.1 Walls

4.1.1 Round

4.1.2 Linear

Properties

$$l_w = \|\mathbf{p}_1 - \mathbf{p}_0\|, \quad \hat{\mathbf{t}}_w = \frac{\mathbf{p}_1 - \mathbf{p}_0}{l_w}, \quad \hat{\mathbf{n}}_w = R(90°) \cdot \hat{\mathbf{t}}_w$$

Solving linear system of equations determining the position of the agent $\mathbf{x}_i$ in relation to wall

$$\begin{cases} \mathbf{p}_0 + l_{n_0}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_0}\hat{\mathbf{t}}_w \\ \mathbf{p}_1 + l_{n_1}\hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_1}\hat{\mathbf{t}}_w \end{cases}$$

$$\begin{cases} l_{n_0}\hat{\mathbf{n}}_w - l_{t_0}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_0 = \mathbf{q}_0 \\ l_{n_1}\hat{\mathbf{n}}_w - l_{t_1}\hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_1 = \mathbf{q}_1 \end{cases}$$

In matrix form

$$\begin{bmatrix} l_{n_0} & l_{n_1} \\ l_{t_0} & l_{t_1} \end{bmatrix} = \mathbf{A}^{-1}\begin{bmatrix} \mathbf{q}_0 & \mathbf{q}_1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} \hat{\mathbf{n}}_w & -\hat{\mathbf{t}}_w \end{bmatrix} = \begin{bmatrix} -t_1 & -t_0 \\ t_0 & -t_1 \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{t_0^2 + t_1^2}\begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} = \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\mathbf{n}}_w \\ -\hat{\mathbf{t}}_w \end{bmatrix} = \mathbf{A}^T$$

$$l_n = l_{n_0} \vee l_{n_1} = \hat{\mathbf{n}}_w \cdot \mathbf{q}_0 \vee \hat{\mathbf{n}}_w \cdot \mathbf{q}_1$$
$$l_t = l_{t_1} + l_{t_0} = -\hat{\mathbf{t}}_w \cdot \mathbf{q}_1 - \hat{\mathbf{t}}_w \cdot \mathbf{q}_0$$

$$d_{iw} = \begin{cases} \|\mathbf{q}_0\| & l_t > l_w \\ |l_n| & \text{otherwise} \\ \|\mathbf{q}_1\| & l_t < -l_w \end{cases}$$

3

$$\hat{\mathbf{n}}_{iw} = \begin{cases} \hat{\mathbf{q}}_0 & l_t > l_w \\ \text{sign}(l_n)\hat{\mathbf{n}}_w & \text{otherwise} \\ \hat{\mathbf{q}}_1 & l_t < -l_w \end{cases}$$

4.2   Agents

4.2.1   Round

4.2.2   Elliptical

5   Differential system

Acceleration on an agent i

$$a_i(t) = \frac{\mathbf{f}_i(t)}{m_i}$$

Updating velocity using discrete time step Δt

$$\Delta \mathbf{v} = a(t_k)\Delta t$$
$$\mathbf{v}(t_{k+1}) = \mathbf{v}(t_k) + \Delta \mathbf{v}$$

Updating position

$$\Delta \mathbf{x} = \mathbf{v}(t_{k+1})\Delta t$$
$$\mathbf{x}(t_{k+1}) = \mathbf{x}(t_k) + \Delta \mathbf{x}$$