

1 Crowd dynamics

1.1 Total Force on Agent

Total force on the agent

$$\begin{aligned}\mathbf{F}_i &= m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{d^2\mathbf{x}_i}{dt^2} \\ &= \mathbf{f}_i(t) + \boldsymbol{\xi}_i\end{aligned}$$

1. Force on the agent i

$$\mathbf{f}_i(t) = \frac{m_i}{\tau_i}(\mathbf{v}_i^0 - \mathbf{v}_i) + \sum_{i \neq j} (\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^c) + \sum_w (\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^c)$$

- (a) Force adjusting pedestrian movement towards desired in characteristic time τ_i

$$\frac{m_i}{\tau_i}(\mathbf{v}_i^0 - \mathbf{v}_i)$$

- (b) Psychological tendency to keep a certain distance to other pedestrians \mathbf{f}_{ij}^{soc} and walls \mathbf{f}_{iw}^{soc} .

- (c) Physical contact forces with other pedestrians \mathbf{f}_{ij}^c and walls \mathbf{f}_{iw}^c .

2. Random fluctuation force $\boldsymbol{\xi}_i$.

1.2 Power Law

Interaction force between agents

$$\begin{aligned}\mathbf{F}_{ij} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) \\ &= -\nabla_{\mathbf{x}_{ij}} \left(k\tau^{-2} e^{-\tau/\tau_0} \right) \\ &= - \left[\frac{ke^{-\tau/\tau_0}}{\|\mathbf{v}_{ij}\|^2 \tau^2} \left(\frac{2}{\tau} + \frac{1}{\tau_0} \right) \right] \\ &\quad \left[\mathbf{v}_{ij} - \frac{\|\mathbf{v}_{ij}\|^2 \mathbf{x}_{ij} - (\mathbf{x}_{ij} \cdot \mathbf{v}_{ij}) \mathbf{v}_{ij}}{\sqrt{(\mathbf{x}_{ij} \cdot \mathbf{v}_{ij})^2 - \|\mathbf{v}_{ij}\|^2 (\|\mathbf{x}_{ij}\|^2 - (r_i + r_j)^2)}} \right]\end{aligned}$$

$$\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$$

$$\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$$

1.3 Symbols

1. Number of agents $N \in \mathbb{N}$
 - (a) Current agent i
 - (b) Other agent j $i, j \in \{0, \dots, N\}$ and $i \neq j$
2. Wall w
3. Mass of an agent m_i
4. Radius of an agent r_i
5. Velocity of an agent
 - (a) Desired velocity v_i^0
 - (b) Current velocity v_i
6. Characteristic time
 - (a) τ_0
 - (b) τ_i
7. k