1 Crowd dynamics

1.1 Parameters

Vector norm and inner product

$$\langle \mathbf{x}, \mathbf{x} \rangle = \mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}\|^2$$

Unit vector

$$\hat{\mathbf{e}} = \frac{\mathbf{e}}{\|\mathbf{e}\|}$$

$$\begin{aligned} \mathbf{x}_i &= \text{Centre of mass of agent } i \\ \mathbf{v}_i &= \text{Velocity of agent } i \\ r_i &= \text{Radius of agent } i \\ m_i &= \text{Mass of agent } i \end{aligned}$$

$$\begin{aligned} \mathbf{x}_{ij} &= \mathbf{x}_i - \mathbf{x}_j \\ \mathbf{v}_{ij} &= \mathbf{v}_i - \mathbf{v}_j \\ r_{ij} &= r_i + r_j \\ d_{ij} &= \|\mathbf{x}_{ij}\| \end{aligned}$$

$$\begin{aligned} a &= \mathbf{v}_{ij} \cdot \mathbf{v}_{ij} \\ b &= -\mathbf{x}_{ij} \cdot \mathbf{v}_{ij} \\ c &= d_{ij}^2 - r_{ij}^2 \\ d &= \sqrt{b^2 - ac} \\ \tau &= \frac{b - \sqrt{d}}{a} > 0 \end{aligned}$$

$$\begin{aligned} h_{ij} &= r_{ij} - d_{ij} \\ h_{iw} &= r_i - d_{iw} \end{aligned}$$

$$\begin{aligned} \mathbf{n}_{ij} &= \frac{\mathbf{x}_{ij}}{d_{ij}} \\ \mathbf{t}_{ij} &= R(-45^\circ) \cdot \mathbf{n}_{ij} \end{aligned}$$

1.2 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c} \right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c} \right) + \boldsymbol{\xi}_i$$

i) Force adjusting agent's movement towards desired in some characteristic time τ_i^{adj}

$$\mathbf{f}_i^{adj} = \frac{m_i}{\tau_i^{adj}} (\mathbf{v}_i^0 - \mathbf{v}_i)$$

Target velocity can be broken down

$$\mathbf{v}_i^0 = v_i^0 \cdot \hat{\mathbf{e}}_i^0$$

Target direction

$$\begin{aligned} \mathbf{e}_i^0(t) &= (1 - p_i)\mathbf{e}_i + p_i \left\langle \mathbf{e}_j^0(t) \right\rangle_i \\ \hat{\mathbf{e}}_i^0(t) &= \frac{\mathbf{e}_i^0(t)}{\|\mathbf{e}_i^0(t)\|} \end{aligned}$$

ii) Psychological tendency to keep a certain distance to other agents

$$\mathbf{f}_{ij}^{soc} = \begin{cases} \mathbf{f}_{ij}^{pow} & d_{ij} \leq sight \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^{soc} = A_i \exp\left(\frac{h_{iw}}{B_i} \right) \mathbf{n}_{iw}$$

iii) Physical contact forces with other agents

$$\mathbf{f}_{ij}^{c} = \begin{cases} h_{ij} \left(\mu \cdot \mathbf{n}_{ij} - \kappa \cdot (\mathbf{v}_{ji} \cdot \mathbf{t}_{ij}) \, \mathbf{t}_{ij} \right) & h_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^{c} = \begin{cases} h_{iw} \left(\mu \cdot \mathbf{n}_{iw} - \kappa \cdot (\mathbf{v}_{i} \cdot \mathbf{t}_{iw}) \mathbf{t}_{iw} \right) & h_{iw} > 0 \\ 0 & \text{otherwise} \end{cases}$$

iv) Uniformly distributed random fluctuation force

$$\boldsymbol{\xi}_i = \boldsymbol{\mathcal{U}}(-1, 1).$$

1.3 Universal power law governing pedestrian interactions

Interaction force between agents

$$\mathbf{f}_{ij}^{pow} = -\nabla_{\mathbf{x}_{ij}} E(\tau) = -\nabla_{\mathbf{x}_{ij}} \left(\frac{k}{\tau^2} \exp\left(-\frac{\tau}{\tau_0} \right) \right)$$
$$= -\left(\frac{k}{a\tau^2} \right) \left(\frac{2}{\tau} + \frac{1}{\tau_0} \right) \exp\left(-\frac{\tau}{\tau_0} \right) \left(\mathbf{v}_{ij} - \frac{a\mathbf{x}_{ij} + b\mathbf{v}_{ij}}{d} \right)$$