1 Constants

Table 1: Constants and limits

Symbol	Unit	Value	Explanation
dt	s		Discrete timestep used to update the differential system
τ_{adj}	s	0.5	Characteristic time in which agent adjusts its movement
k	N	1.5	Social force scaling constant
τ_0	s	3.0	Max interaction range 2 - 4, aka interaction time horizon
μ	$\frac{\mathrm{kg}}{\mathrm{s}^2}$	$1.2e+05$	Compression counteraction constant
κ	$\frac{\mathrm{kg}}{\mathrm{m\,s}}$	$2.4e+05$	Sliding friction constant
A	N	$2.0e+03$	Scaling coefficient for social force between wall and agent
B	m	0.08	Coefficient for social force between wall and agent
f_{max}	N		Forces that are greater will be truncated to max force

Table 2: Agent body properties

	Total		Torso	Shoulder		Velocity	
	r (m)	$\pm$	r_t/r	r_s/r	r_{t-s}/r	v (m/s)	$\pm$
adult	0.255	0.035	0.5882	0.3725	0.6275	1.25	0.3
child	0.210	0.015	0.5714	0.3333	0.6667	0.90	0.3
eldery	0.250	0.020	0.6000	0.3600	0.6400	0.80	0.3
female	0.240	0.020	0.5833	0.3750	0.6250	1.15	0.2
male	0.270	0.020	0.5926	0.3704	0.6296	1.35	0.2

2 Agents

r	Radius
m	Mass
v_0	Goal velocity
$\mathbf{x}$	Position
$\mathbf{v}$	Velocity
$\hat{\mathbf{e}}_0$	Goal direction
$\hat{\mathbf{e}}$	Target direction
p	Herding tendency
I_z	Moment of inertia
ω	current angular velocity
φ	current body angle
ω_0	max angular velocity
φ_0	target angle
$\tilde{\omega}_0$	target angular velocity

$\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$	Relative position between two agents
$\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$	Relative velocity between two agents
$r_{ij} = r_i + r_j$	Total radius
$d_{ij} = \lVert \mathbf{x}_{ij} \rVert$	Distance between agents
$h_{ij} = r_{ij} - d_{ij}$	Relative distance between agents
$\hat{\mathbf{n}}_{ij} = \mathbf{x}_{ij}/d_{ij}$	Normal vector
$\hat{\mathbf{t}}_{ij} = R(-90^\circ) \cdot \hat{\mathbf{n}}_{ij}$	Tangent vector

Rotational equation of motion

$$I_i^z \frac{d^2}{dt^2} \varphi_i(t) = M_i^z(t) + \eta_i^z(t),$$

where $\varphi_i(t)$ is the angle of the agent i, I_i^z is moment of inertia, $\eta_i^z(t)$ is small random fluctuation torque, and $M_i^z(t)$ is total torque, which is the sum of contact, social and motivational torque

$$M_i^z(t) = M_i^c + M_i^{soc} + M_i^{\tau}$$

Torque from contact forces

$$\mathbf{M}_i^c = \sum_{j \neq i} \left(\mathbf{R}_i^c \times \mathbf{f}_{ij}^c \right)$$

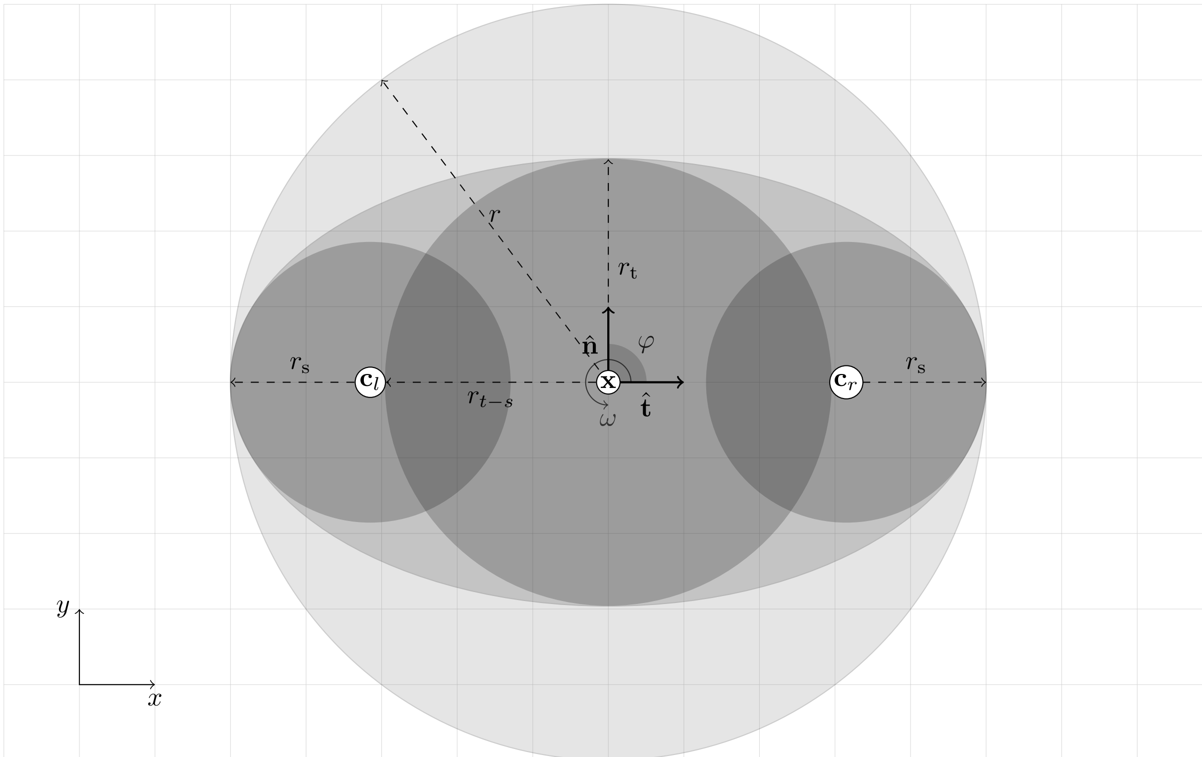

Figure 1: Circle, ellipse and three circle representations of an agent.

and from social forces

$$\mathbf{M}_i^{soc} = \sum_{j \neq i} \left(\mathbf{R}_i^{soc} \times \mathbf{f}_{ij}^{soc} \right)$$

Motivational torque

$$\begin{aligned} M_i^{\tau} &= \frac{I_i^z}{\tau_i^z} \left((\varphi_i(t) - \varphi_i^0) \omega^0 - \omega(t) \right) \\ &= \frac{I_i^z}{\tau_i^z} \left(\tilde{\omega}_i^0 - \omega(t) \right) \end{aligned}$$

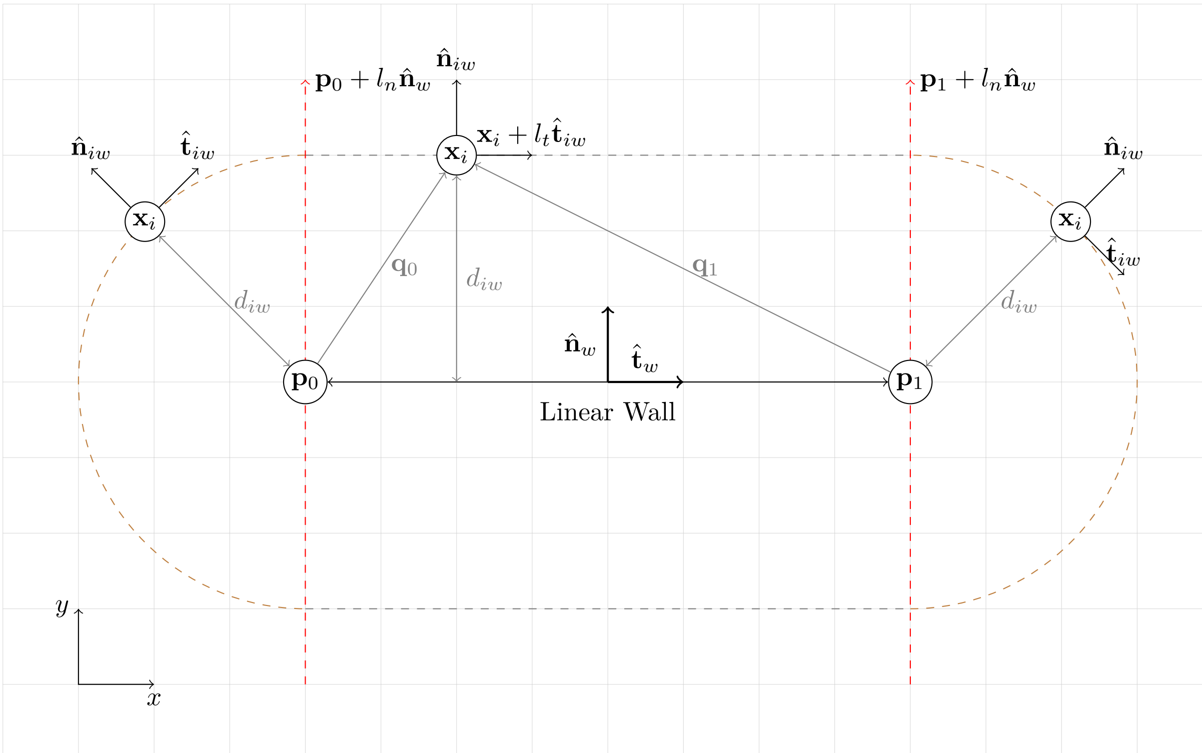

Figure 2: Absolute distance from a linear wall.

3 Linear wall

3.1 Properties

$\mathbf{p}_0$	Start point
$\mathbf{p}_1$	End point

Relative

$$
\begin{aligned}
&h_{iw} = r_i - d_{iw} \\
&l_w = \|\mathbf{p}_1 - \mathbf{p}_0\| \\
&\hat{\mathbf{t}}_w = (\mathbf{p}_1 - \mathbf{p}_0) / l_w \\
&\hat{\mathbf{n}}_w = R(90°) \cdot \hat{\mathbf{t}}_w
\end{aligned}
$$

3.2 Absolute distance

Solving linear system of equations determining the position of the agent $\mathbf{x}_i$ in relation to wall

$$
\begin{cases}
\mathbf{p}_0 + l_{n_0} \hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_0} \hat{\mathbf{t}}_w \\
\mathbf{p}_1 + l_{n_1} \hat{\mathbf{n}}_w = \mathbf{x}_i + l_{t_1} \hat{\mathbf{t}}_w
\end{cases}
$$

$$
\begin{cases}
l_{n_0} \hat{\mathbf{n}}_w - l_{t_0} \hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_0 = \mathbf{q}_0 \\
l_{n_1} \hat{\mathbf{n}}_w - l_{t_1} \hat{\mathbf{t}}_w = \mathbf{x}_i - \mathbf{p}_1 = \mathbf{q}_1
\end{cases}
$$

In matrix form

$$
\begin{bmatrix} l_{n_0} & l_{n_1} \\ l_{t_0} & l_{t_1} \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \mathbf{q}_0 & \mathbf{q}_1 \end{bmatrix}
$$

$$
\begin{aligned}
\mathbf{A} &= \begin{bmatrix} \hat{\mathbf{n}}_w & -\hat{\mathbf{t}}_w \end{bmatrix} = \begin{bmatrix} -t_1 & -t_0 \\ t_0 & -t_1 \end{bmatrix} \\
\mathbf{A}^{-1} &= \frac{1}{t_0^2 + t_1^2} \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} = \begin{bmatrix} -t_1 & t_0 \\ -t_0 & -t_1 \end{bmatrix} \\
&= \begin{bmatrix} \hat{\mathbf{n}}_w \\ -\hat{\mathbf{t}}_w \end{bmatrix} = \mathbf{A}^T
\end{aligned}
$$

Conditions

$$
\begin{aligned}
l_n &= l_{n_0} \vee l_{n_1} = \hat{\mathbf{n}}_w \cdot \mathbf{q}_0 \vee \hat{\mathbf{n}}_w \cdot \mathbf{q}_1 \\
l_t &= l_{t_1} + l_{t_0} = -\hat{\mathbf{t}}_w \cdot \mathbf{q}_1 - \hat{\mathbf{t}}_w \cdot \mathbf{q}_0
\end{aligned}
$$

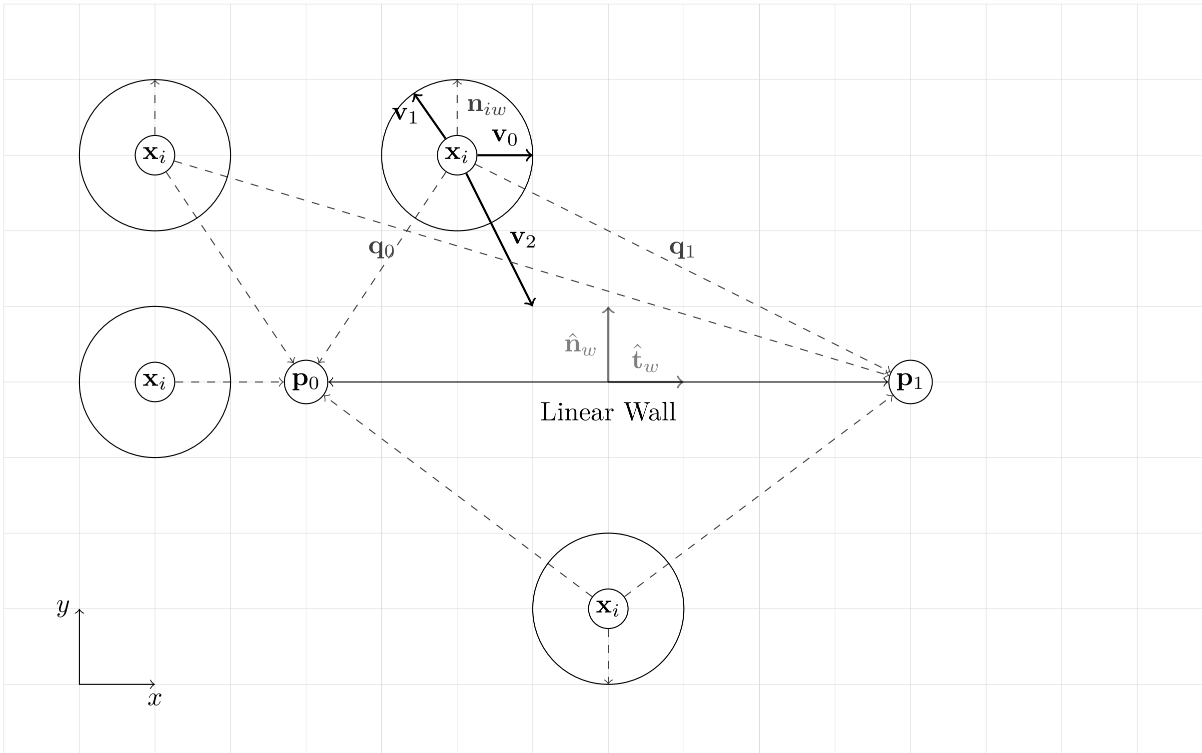

Figure 3: Velocity dependent distance from a linear wall.

Distance between agent and linear wall

$$d_{iw} = \begin{cases} \|\mathbf{q}_0\| & l_t > l_w \\ |l_n| & \text{otherwise} \\ \|\mathbf{q}_1\| & l_t < -l_w \end{cases}$$

Normal vector away from the wall

$$\hat{\mathbf{n}}_{iw} = \begin{cases} \hat{\mathbf{q}}_0 & l_t > l_w \\ \operatorname{sign}(l_n)\hat{\mathbf{n}}_w & \text{otherwise} \\ \hat{\mathbf{q}}_1 & l_t < -l_w \end{cases}$$

3.3 Velocity relative distance

4 Crowd dynamics

4.1 Social force model

Total force exerted on the agent is the sum of movement adjusting, social and contact forces between other agents and wall.

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adj} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c}\right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c}\right) + \boldsymbol{\xi}_i$$

4.2 Adjusting force

Force adjusting agent's movement towards desired in some characteristic time

$$\mathbf{f}^{adj} = \frac{m}{\tau^{adj}}(v_0 \cdot \hat{\mathbf{e}} - \mathbf{v})$$

4.3 Social force

Psychological force for collision avoidance

4.3.1 Velocity independent

$$\mathbf{f}^{soc} = A \exp\left(\frac{h}{B}\right) \hat{\mathbf{n}}$$

4.3.2 Velocity dependent

$$\begin{aligned}
\mathbf{f}_{ij}^{soc} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) \\
&= -\nabla_{\mathbf{x}_{ij}} \left(\frac{k}{\tau^2} \exp\left(-\frac{\tau}{\tau_0}\right)\right) \\
&= -\left(\frac{k}{a\tau^2}\right)\left(\frac{2}{\tau} + \frac{1}{\tau_0}\right) \exp\left(-\frac{\tau}{\tau_0}\right)\left(\mathbf{v}_{ij} - \frac{a\mathbf{x}_{ij} + b\mathbf{v}_{ij}}{d}\right),
\end{aligned}$$

where

$$\begin{aligned}
a &= \mathbf{v}_{ij} \cdot \mathbf{v}_{ij} \\
b &= -\mathbf{x}_{ij} \cdot \mathbf{v}_{ij} \\
c &= \mathbf{x}_{ij} \cdot \mathbf{x}_{ij} - r_{ij}^2 \\
d &= \sqrt{b^2 - ac} \\
\tau &= \frac{b - d}{a} > 0.
\end{aligned}$$

4.4 Contact force

Physical contact force

$$\mathbf{f}^c = h \cdot \left(\mu \cdot \hat{\mathbf{n}} - \kappa \cdot (\mathbf{v} \cdot \hat{\mathbf{t}})\hat{\mathbf{t}}\right), \quad h > 0$$

4.5 Random Fluctuation

Uniformly distributed random fluctuation force

$$\boldsymbol{\xi} = f \cdot \left[\cos(\varphi), \quad \sin(\varphi)\right],$$

where

$$f \in [0, f_{max}], \quad \varphi \in [0, 2\pi)$$

4.6 Target direction

Herding behavior

$$\mathbf{e}_i = (1 - p_i)\hat{\mathbf{e}}_i^0 + p_i \left\langle \hat{\mathbf{e}}_j^0 \right\rangle_i$$

5 Differential system

Acceleration on an agent

$$a(t) = \frac{\mathbf{f}(t)}{m}$$

Updating velocity using discrete time step Δt

$$\begin{aligned}
t_0 &= 0 \\
t_1 &= t_0 + \Delta t \\
t_2 &= t_1 + \Delta t \\
&\ldots
\end{aligned}$$

$$\begin{aligned}
\mathbf{v}_{k+1} &= \mathbf{v}_k + \Delta \mathbf{v}, \quad \Delta \mathbf{v} = a_k \Delta t \\
\mathbf{x}_{k+1} &= \mathbf{x}_k + \Delta \mathbf{x}, \quad \Delta \mathbf{x} = \mathbf{v}_{k+1} \Delta t
\end{aligned}$$