1 Crowd dynamics

1.1 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adjust} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c} \right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c} \right) + \boldsymbol{\xi}_i$$

i) Force adjusting pedestrian movement towards desired in characteristic time τ_i

$$\begin{aligned} \mathbf{f}_i^{adjust} &= \frac{m_i}{\tau_i} (\mathbf{v}_i^0 - \mathbf{v}_i) \\ &= \frac{m_i}{\tau_i} (\left\| v_i^0 \right\| \mathbf{e}_i - \mathbf{v}_i) \end{aligned}$$

ii) Psychological tendency to keep a certain distance to other pedestrians

$$\mathbf{f}_{ij}^{soc} \to \text{power law}$$

and walls

$$\mathbf{f}_{iw}^{soc} = a_i \exp\left(\frac{r_i - d_{iw}}{b_i} \right)$$

iii) Physical contact forces with other pedestrians

$$\mathbf{f}_{ij}^{c} = k \cdot (r_{ij} - d_{ij}) \mathbf{n}_{ij} - \kappa \cdot (r_{ij} - d_{ij}) \Delta \mathbf{v}_{ij}^{t} \mathbf{t}_{ij}$$

$$\begin{aligned} r_{ij} &= r_i + r_j \\ d_{ij} &= \left\| \mathbf{x}_i - \mathbf{x}_j \right\| \\ \mathbf{n}_{ij} &= \frac{\mathbf{x}_i - \mathbf{x}_j}{d_{ij}} \\ \mathbf{t}_{ij} &= R(\theta) \mathbf{n}_{ij} \\ R(\theta) &= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \\ \Delta \mathbf{v}_{ji}^{t} &= (\mathbf{v}_j - \mathbf{v}_i) \cdot \mathbf{t}_{ij} \end{aligned}$$

and walls

$$\mathbf{f}_{iw}^{c} = k \cdot (r_i - d_{iw}) \mathbf{n}_{iw} - \kappa \cdot (r_i - d_{iw}) (\mathbf{v}_i \cdot \mathbf{t}_{iw}) \mathbf{t}_{iw}$$

iv) Random fluctuation force

$$\boldsymbol{\xi}_i.$$

Uniformly distributed.

1.2 Power Law

Interaction force between agents

$$\begin{aligned} \mathbf{F}_{ij} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) \\ &= -\nabla_{\mathbf{x}_{ij}} \left(k\tau^{-2} e^{-\tau/\tau_0} \right) \\ &= -\left[\frac{k e^{-\tau/\tau_0}}{\left\| \mathbf{v}_{ij} \right\|^2 \tau^2} \left(\frac{2}{\tau} + \frac{1}{\tau_0} \right) \right] \\ &\left[\mathbf{v}_{ij} - \frac{\left\| \mathbf{v}_{ij} \right\|^2 \mathbf{x}_{ij} - (\mathbf{x}_{ij} \cdot \mathbf{v}_{ij}) \mathbf{v}_{ij}}{\sqrt{(\mathbf{x}_{ij} \cdot \mathbf{v}_{ij})^2 - \left\| \mathbf{v}_{ij} \right\|^2 \left(\left\| \mathbf{x}_{ij} \right\|^2 - (r_i + r_j)^2 \right)}} \right] \end{aligned}$$

$$\begin{aligned} \mathbf{x}_{ij} &= \mathbf{x}_i - \mathbf{x}_j \\ \mathbf{v}_{ij} &= \mathbf{v}_i - \mathbf{v}_j \end{aligned}$$