1 Crowd dynamics

1.1 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adjust} + \sum_{i \neq j} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c}\right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c}\right) + \boldsymbol{\xi}_i$$

i) Force adjusting pedestrian movement towards desired in characteristic time τ_i

$$\begin{aligned} \mathbf{f}_i^{adjust} &= \frac{m_i}{\tau_i}(\mathbf{v}_i^0 - \mathbf{v}_i) \\ &= \frac{m_i}{\tau_i}(\left\|v_i^0\right\| \mathbf{e}_i - \mathbf{v}_i) \end{aligned}$$

ii) Psychological tendency to keep a certain distance to other pedestrians

$$\mathbf{f}_{ij}^{soc}$$

and walls

$$\mathbf{f}_{iw}^{soc}.$$

iii) Physical contact forces with other pedestrians

$$\mathbf{f}_{ij}^{c}$$

and walls

$$\mathbf{f}_{iw}^{c}.$$

iv) Random fluctuation force

$$\boldsymbol{\xi}_i$$

1.2 Power Law

Interaction force between agents

$$\begin{aligned} \mathbf{F}_{ij} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) \\ &= -\nabla_{\mathbf{x}_{ij}} \left(k\tau^{-2} e^{-\tau/\tau_0}\right) \\ &= -\left[\frac{k e^{-\tau/\tau_0}}{\left\|\mathbf{v}_{ij}\right\|^2 \tau^2}\left(\frac{2}{\tau} + \frac{1}{\tau_0}\right)\right] \\ &\left[\mathbf{v}_{ij} - \frac{\left\|\mathbf{v}_{ij}\right\|^2 \mathbf{x}_{ij} - (\mathbf{x}_{ij} \cdot \mathbf{v}_{ij})\,\mathbf{v}_{ij}}{\sqrt{(\mathbf{x}_{ij} \cdot \mathbf{v}_{ij})^2 - \left\|\mathbf{v}_{ij}\right\|^2 \left(\left\|\mathbf{x}_{ij}\right\|^2 - (r_i + r_j)^2\right)}}\right] \end{aligned}$$

$$\begin{aligned} \mathbf{x}_{ij} &= \mathbf{x}_i - \mathbf{x}_j \\ \mathbf{v}_{ij} &= \mathbf{v}_i - \mathbf{v}_j \end{aligned}$$

1.3 Properties

Indexing

1. Number of agents $N \in \mathbb{N}$
   (a) Current agent i
   (b) Other agent j

   $i, j \in \{0, \ldots, N\}$ and $i \neq j$
2. Wall w

Agent

1. Mass m_i
2. Shape
   (a) Circle, radius r_i
   (b) Ellipse
3. Velocity
   (a) Desired velocity v_i^0
   (b) Current velocity v_i
4. Characteristic time
   (a) τ_0
   (b) τ_i