1 Crowd dynamics

1.1 Parameters

$$\langle \mathbf{x}, \mathbf{x} \rangle = \mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}, \mathbf{x}\|^2$$

$$\begin{aligned}
\mathbf{x}_i &= \text{Centre of mass of agent } i \\
\mathbf{v}_i &= \text{Velocity of agent } i \\
r_i &= \text{Radius of agent } i \\
m_i &= \text{Mass of agent } i
\end{aligned}$$

$$\begin{aligned}
\mathbf{x}_{ij} &= \mathbf{x}_i - \mathbf{x}_j \\
\mathbf{v}_{ij} &= \mathbf{v}_i - \mathbf{v}_j \\
r_{ij} &= r_i + r_j \\
d_{ij} &= \|\mathbf{x}_{ij}\|
\end{aligned}$$

$$\begin{aligned}
h_{iw} &= r_i - d_{iw} \\
h_{ij} &= r_{ij} - d_{ij}
\end{aligned}$$

$$\begin{aligned}
\mathbf{n}_{ij} &= \frac{\mathbf{x}_{ij}}{d_{ij}} \\
\mathbf{t}_{ij} &= R(45^\circ) \cdot \mathbf{n}_{ij} \\
\Delta \mathbf{v}_{ji}^t &= \mathbf{v}_{ji} \cdot \mathbf{t}_{ij}
\end{aligned}$$

1.2 Social force model

Total force on the agent i

$$\mathbf{f}_i(t) = \mathbf{f}_i^{adjust} + \sum_{j \neq i} \left(\mathbf{f}_{ij}^{soc} + \mathbf{f}_{ij}^{c} \right) + \sum_{w} \left(\mathbf{f}_{iw}^{soc} + \mathbf{f}_{iw}^{c} \right) + \boldsymbol{\xi}_i$$

i) Force adjusting pedestrian movement towards desired in characteristic time τ_i

$$\begin{aligned}
\mathbf{f}_i^{adjust} &= \frac{m_i}{\tau_i} (\mathbf{v}_i^0 - \mathbf{v}_i) \\
&= \frac{m_i}{\tau_i} (\|v_i^0\| \, \mathbf{e}_i - \mathbf{v}_i)
\end{aligned}$$

ii) Psychological tendency to keep a certain distance to other pedestrians

$$\mathbf{f}_{ij}^{soc} \to \text{power law}$$

and walls

$$\mathbf{f}_{iw}^{soc} = A_i \exp\left(\frac{h_{iw}}{B_i}\right) \mathbf{n}_{iw}$$

iii) Physical contact forces with other pedestrians

$$\mathbf{f}_{ij}^{c} = \begin{cases} h_{ij} \left(k \cdot \mathbf{n}_{ij} - \kappa \cdot \Delta \mathbf{v}_{ij}^t \mathbf{t}_{ij} \right) & h_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

and walls

$$\mathbf{f}_{iw}^{c} = \begin{cases} h_{iw} \left(k \cdot \mathbf{n}_{iw} - \kappa \cdot (\mathbf{v}_i \cdot \mathbf{t}_{iw}) \mathbf{t}_{iw} \right) & h_{iw} > 0 \\ 0 & \text{otherwise} \end{cases}$$

iv) Uniformly distributed random fluctuation force

$$\boldsymbol{\xi}_i = \boldsymbol{\mathcal{U}}(-1, 1).$$

1.3 Power Law

Interaction force between agents

$$\begin{aligned}
\mathbf{F}_{ij} &= -\nabla_{\mathbf{x}_{ij}} E(\tau) \\
&= -\nabla_{\mathbf{x}_{ij}} \left(k \tau^{-2} e^{-\tau/\tau_0} \right) \\
&= -\left[\frac{k e^{-\tau/\tau_0}}{\|\mathbf{v}_{ij}\|^2 \tau^2} \left(\frac{2}{\tau} + \frac{1}{\tau_0} \right) \right] \\
&\left[\mathbf{v}_{ij} - \frac{\|\mathbf{v}_{ij}\|^2 \mathbf{x}_{ij} - (\mathbf{x}_{ij} \cdot \mathbf{v}_{ij}) \mathbf{v}_{ij}}{\sqrt{(\mathbf{x}_{ij} \cdot \mathbf{v}_{ij})^2 - \|\mathbf{v}_{ij}\|^2 \left(\|\mathbf{x}_{ij}\|^2 - r_{ij}^2 \right)}} \right]
\end{aligned}$$