```
%% Derivations
% Symbolic derivations of some quantities
syms y h L rho_0 A B C D mu
E = [A B C D];
nu = E*[exp(y/h*L); exp(-y/h*L); y/h; 1];
rho = rho_0*nu
```

rho =

$$\rho_0 \left(\mathrm{D} + A\,\mathrm{e}^{\frac{L\,y}{h}} + B\,\mathrm{e}^{-\frac{L\,y}{h}} + \frac{C\,y}{h} \right)$$

```
d_rho = diff(rho, y)
```

d_rho =

$$\rho_0 \left(\frac{C}{h} + \frac{A\,L\,\mathrm{e}^{\frac{L\,y}{h}}}{h} - \frac{B\,L\,\mathrm{e}^{-\frac{L\,y}{h}}}{h} \right)$$

```
D_rho = int(rho, y) + h/L*(B-A)*rho_0 % Integral of rho
```

D_rho =

$$\frac{\rho_0\,(C\,y^2 + 2\,\mathrm{D}\,h\,y)}{2\,h} + \frac{\rho_0 \left(2\,A\,h^2\,\mathrm{e}^{\frac{L\,y}{h}} - 2\,B\,h^2\,\mathrm{e}^{-\frac{L\,y}{h}} \right)}{2\,L\,h} - \frac{h\,\rho_0\,(A - B)}{L}$$

```
DD_rho = int(D_rho, y) % Integral of integral of rho
```

DD_rho =

$$\frac{\rho_0 \left(6\,A\,h^3\,\mathrm{e}^{\frac{L\,y}{h}} + 6\,B\,h^3\,\mathrm{e}^{-\frac{L\,y}{h}} \right)}{6} - \frac{L\,\rho_0\,(6\,A\,h^2\,y - 6\,B\,h^2\,y)}{6} + \frac{\rho_0\,(C\,y^3 + 3\,\mathrm{D}\,h\,y^2)}{6\,h}$$

$$L^2\,h$$

```
drho2_dy = diff(d_rho^2, y) % d((d(rho)/dy)^2)/dy
```

drho2_dy =

$$2\,{\rho_0}^2 \left(\frac{A\,L^2\,\mathrm{e}^{\frac{L\,y}{h}}}{h^2} + \frac{B\,L^2\,\mathrm{e}^{-\frac{L\,y}{h}}}{h^2} \right) \left(\frac{C}{h} + \frac{A\,L\,\mathrm{e}^{\frac{L\,y}{h}}}{h} - \frac{B\,L\,\mathrm{e}^{-\frac{L\,y}{h}}}{h} \right)$$

```
expand(drho2_dy)
```

ans =

$$\frac{2\,A^2\,L^3\,{\rho_0}^2\,e^{\sigma_1}}{h^3}-\frac{2\,B^2\,L^3\,{\rho_0}^2\,e^{-\sigma_1}}{h^3}+\frac{2\,A\,C\,L^2\,{\rho_0}^2\,e^{\frac{L\,y}{h}}}{h^3}+\frac{2\,B\,C\,L^2\,{\rho_0}^2\,e^{-\frac{L\,y}{h}}}{h^3}$$

where

$$\sigma_1=\frac{2\,L\,y}{h}$$