ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-1

Calculus

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Topics to be Covered

- FUNCTIONS
- TYPES OF FUNCTIONS
- DOMAIN AND RANGE OF FUNCTIONS
- GRAPH OF FUNCTIONS
- GRAPH TRANSFORMATION

[LU DECOMPOSITION] / Factorization method

Set of linear equations can be solved by :-

Do-Little method

$$AX = B$$

$$A = LU$$

$$\begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{21} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & -1 \\ 3 & 0 & 2 \\ 6 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$l_{ii} = 1$$

Crout's method

$$AX = B$$

$$A = LU$$

$$\begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & U_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & -1 \\ 3 & 0 & 2 \\ 6 & 1 & 1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$U_{ii} = 1$$

LU DECOMPOSITION

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

$\Rightarrow \boxed{u_{11} = 1}$, $\boxed{u_{12} = 3}$, $\boxed{u_{13} = 8}$, $l_{21}u_{11} = 1$ $\boxed{l_{21} = 1}$, $l_{31}u_{11} = 1$ $\boxed{l_{31} = 1}$

$\Rightarrow l_{21}u_{12} + u_{22} = 4 \Rightarrow \boxed{u_{22} = 1}$

$1 \times 3 + u_{22} = 4$

$\Rightarrow l_{21}u_{13} + u_{23} = 3 \Rightarrow \boxed{u_{23} = -5}$

$1 \times 8 + u_{23} = 3$

$\Rightarrow l_{31}u_{12} + l_{32}u_{22} = 3 \Rightarrow \boxed{l_{32} = 0}$

$1 \times 3 + l_{32} \times 1 = 3$

$\Rightarrow l_{31}u_{13} + l_{32}u_{23} + u_{33} = 4$

$1 \times 8 + 0 \times -5 + u_{33} = 4$

$\boxed{u_{33} = -4}$

LU DECOMPOSITION

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} = \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}} \overset{U}{\begin{bmatrix} 1 & 3 & 8 \\ 0 & 1 & -5 \\ 0 & 0 & -4 \end{bmatrix}}$$

Ex:- Solve the system of equations:-

$$\begin{aligned} x + 3y + 8z &= 4 \\ x + 4y + 3z &= -2 \\ x + 3y + 4z &= 1 \end{aligned} \qquad \begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

$$AX = B$$

$$LUX = B \qquad \boxed{\therefore UX = Y}$$

$$LY = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} y_1 &= 4 \\ y_2 &= -6 \\ y_3 &= -3 \end{aligned} \qquad \begin{aligned} y_1 + y_2 &= -2 \\ y_1 + y_3 &= 1 \end{aligned}$$

LU DECOMPOSITION

$$UX = Y$$

$$\begin{bmatrix} 1 & 3 & 8 \\ 0 & 1 & -5 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -3 \end{bmatrix}$$

$$x + 3y + 8z = 4$$

$$y - 5z = -6$$

$$-4z = -3$$

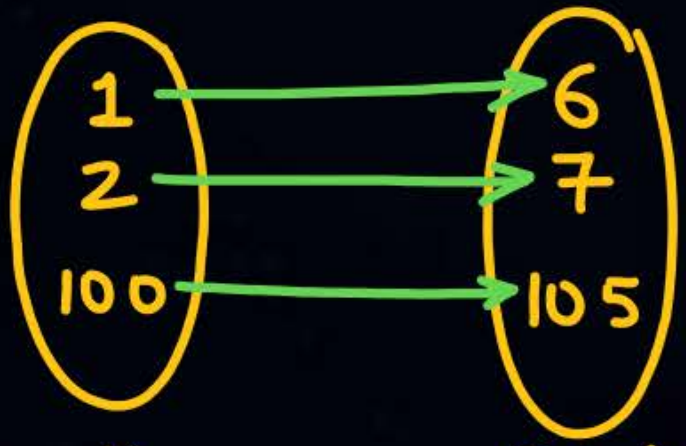
$$z = \frac{3}{4}, \quad y = -\frac{9}{4}, \quad x = \frac{19}{4}$$

FUNCTIONS

A relation R from set A to B is said to be a function (f) if every element of set A has one and only one image in set B:

$y = f(x)$

$x+5$



x $\quad$ $f(x)$

Domain $\quad$ y Co-domain Range

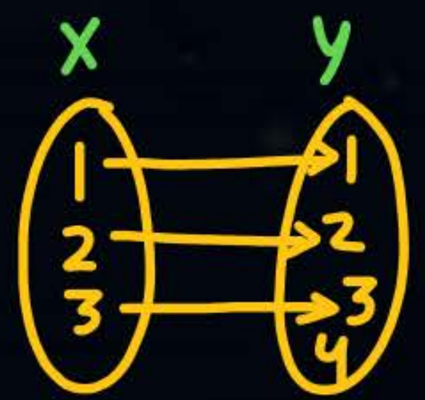
Input → Output

1) One-one and onto (Bijective)
2) one-one and into
3) Many-one and onto
4) Many-one and into

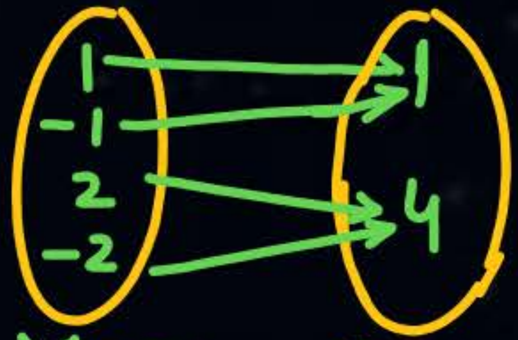
TYPES OF FUNCTIONS

I). One-One function (Injective):

$y = x$



One - one
$y = x$

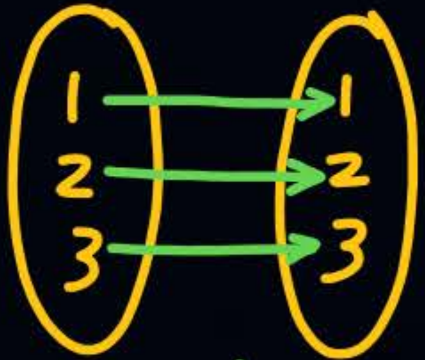


Many - One
$y = x^2$

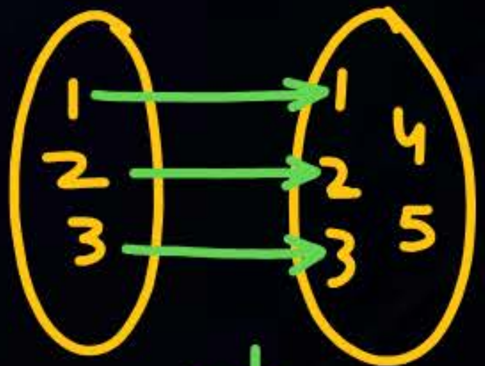
II). Onto function (Surjective):

↳ Entire range is covered
(ONTO) → Surjective

↳ If entire range is not covered
(INTO) → Not surjective.



onto

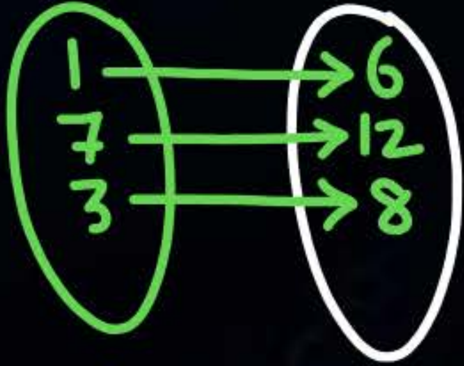


into

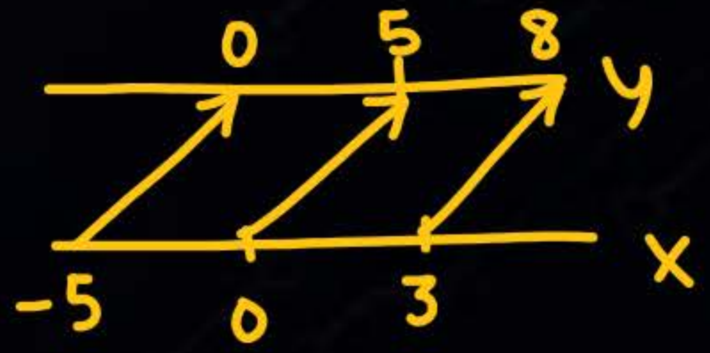
III). Bijective function:

↳ function is both one-one and onto

$f(x) = x + 5$ → $x \in R$
→ $y \in R$

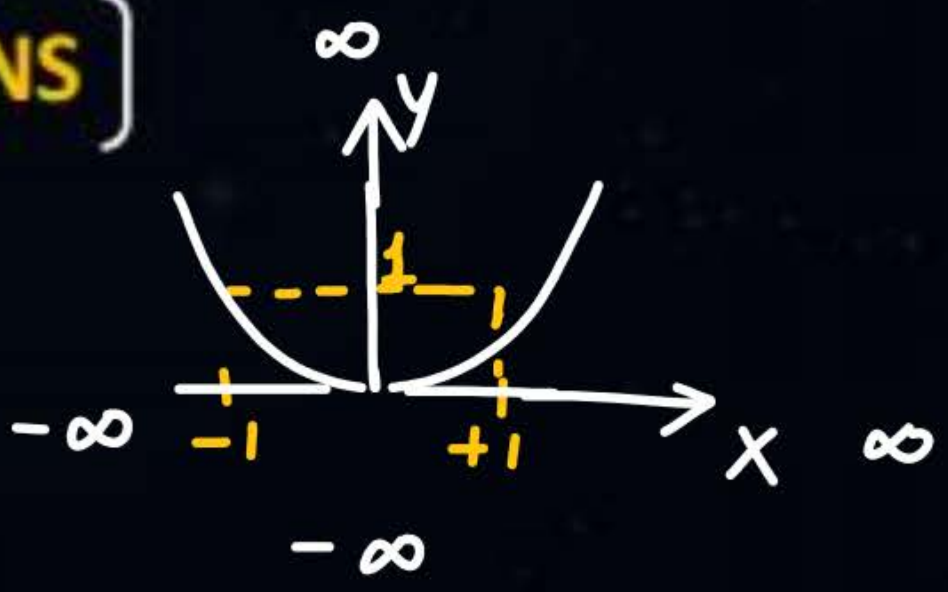


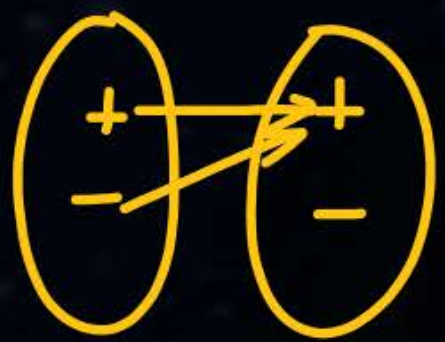
one-one and onto



DOMAIN AND RANGE OF FUNCTIONS

Ex :- Is x^2 bijective ?





INTO

① $x \in \{-\infty, \infty\}$

$y \in \{-\infty, \infty\}$

$y = x^2$

y can only be mapped to positive real numbers (R^+).

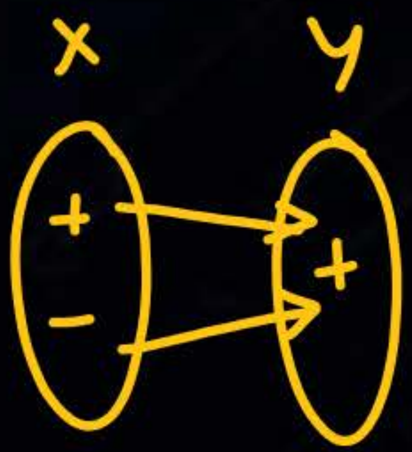
x^2 is not bijective

one-one ✗ many-one ✓

onto. ✗ into ✓

② $x \in \{-\infty, \infty\}$

$y \in \{0, \infty\}$



ONTO

one-one ✗ many-one ✓

onto ✓

x^2 is not bijective.

③ $y = 5x + 6$ Bijective?

DOMAIN AND RANGE OF FUNCTIONS

$X \to N^+ \{1, 2, 3 \ldots\ldots\}$

$Y \to N^+ \{1, 2, 3 \ldots\ldots\}$

$y = 5x + 6$

into ✓

one-one ✓

(not bijective)

Ex:-

$y = x^2$

$y = x^3 + 5$

$x \in R \quad y = 5x + 6$

$y \in R \quad y = \log x$

$y = e^{-x}$

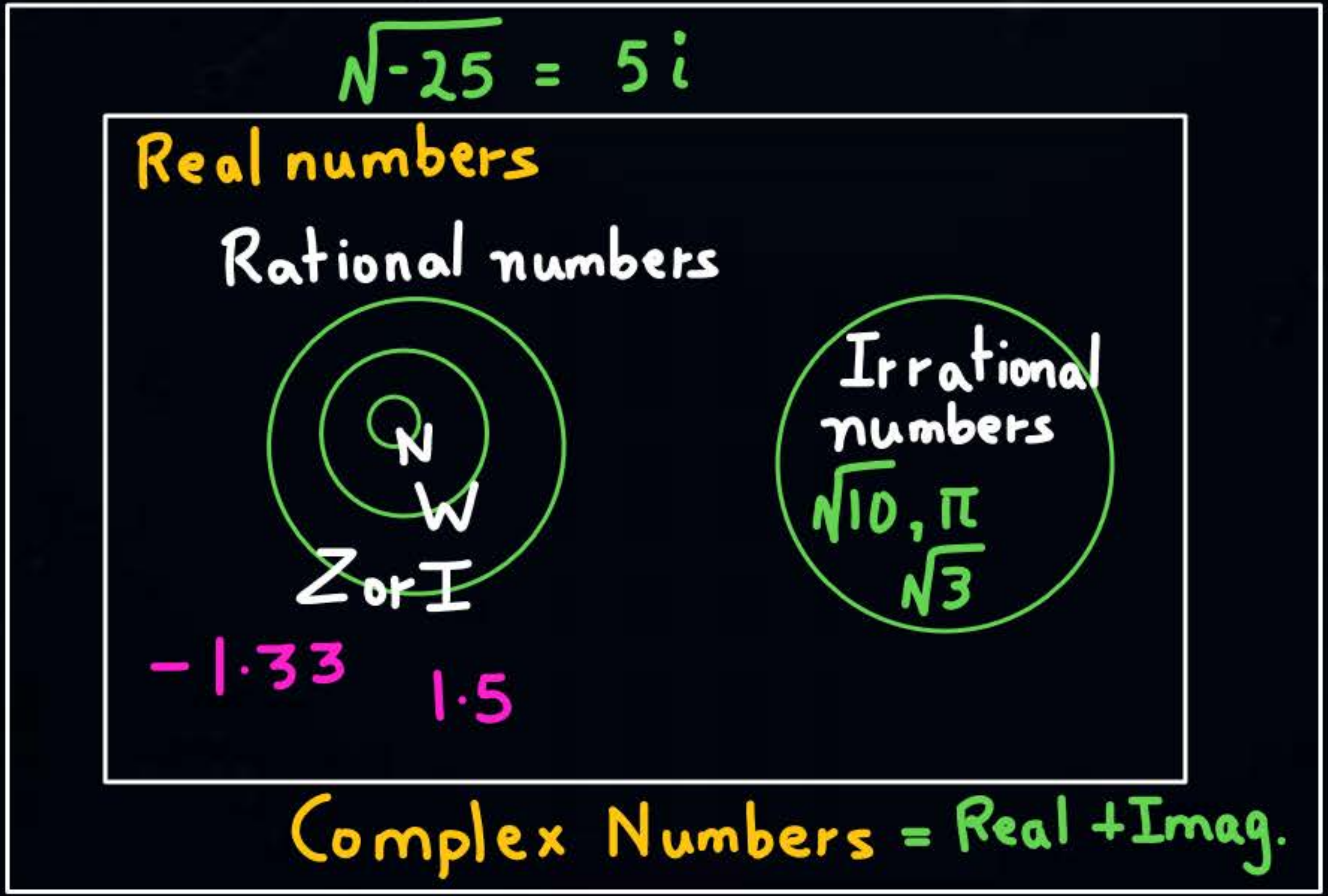
$y = \sin x$

GRAPH OF FUNCTIONS

(x^3+5x^2+x+6)

- FUNCTIONS
  - Algebraic
    - Polynomial
    - Rational
    - Irrational
    - Piecewise
      - Modulus
      - Signum
      - Greatest integer function
      - Fractional part function
      - Least integer function
  - Transcendental
    - Trigonometric
    - Exponential
    - Logarithmic/Inverse of exponential
    - Inverse trigonometric curves

GRAPH OF FUNCTIONS



Irrational numbers
(are those numbers which can't be expressed as fraction)
(Non-recurring & non-terminating)

→ Terminating
→ Non-terminating (Recurring)

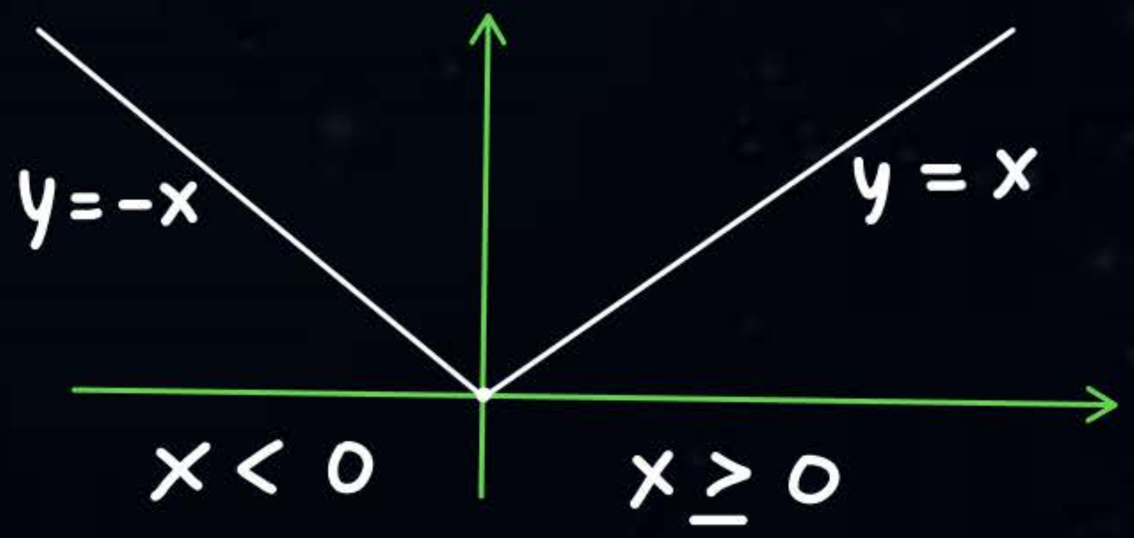
Rational numbers → $\frac{4}{3}, \frac{22}{7}, \frac{3}{2}$
(can be expressed as fraction)

Natural numbers → 1, 2, 3 . . .
Whole numbers → 0, 1, 2, 3 . . .
Integers → −5, −4, . . 0, 1, 2 . . .

GRAPH OF FUNCTIONS

1) Modulus function :-

$$f(x) = |x| = \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases}$$

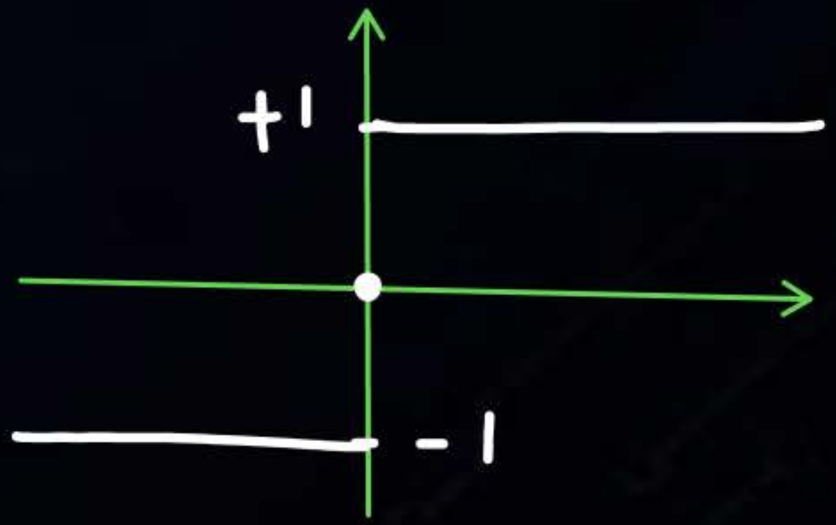


$x \in R$
$y \in R$ $\rightarrow$ into and many one

2) Signum function :-

$$f(x) = \frac{|x|}{x} \quad ; x \neq 0$$
$$= 0 \quad ; x = 0$$

$$\Rightarrow \begin{cases} 1 & ; x > 0 \\ -1 & ; x < 0 \\ 0 & ; x = 0 \end{cases}$$



GRAPH OF FUNCTIONS

$[-4.5] = -5$

$[1.0] = 1$

$[1.5] = 1$

3) Greatest integer function.

$x = [x] + \{x\}$

G.I.F. + Fractional part

$$f(x) = \begin{cases} -2 & -2 \le x < 1 \\ -1 & -1 \le x < 0 \\ 0 & 0 \le x < 1 \\ 1 & 1 \le x < 2 \end{cases}$$

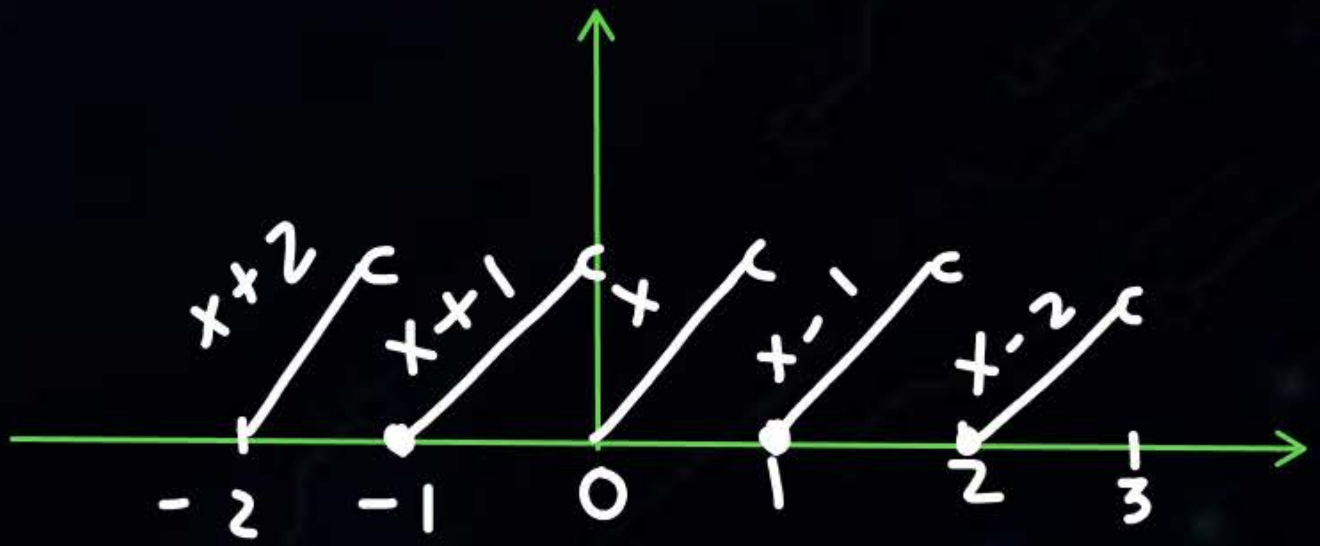
$x - 1$

$5.6 = 5 + 0.6$

$x = [x] + \{x\}$

4) Fractional part $\{x\} = x - [x]$

$$f(x) = \begin{cases} x+2 & -2 \le x \le 1 \\ x+1 & -1 \le x < 0 \\ x & 0 \le x < 1 \\ x-1 & 1 \le x < 2 \\ x-2 & \end{cases}$$



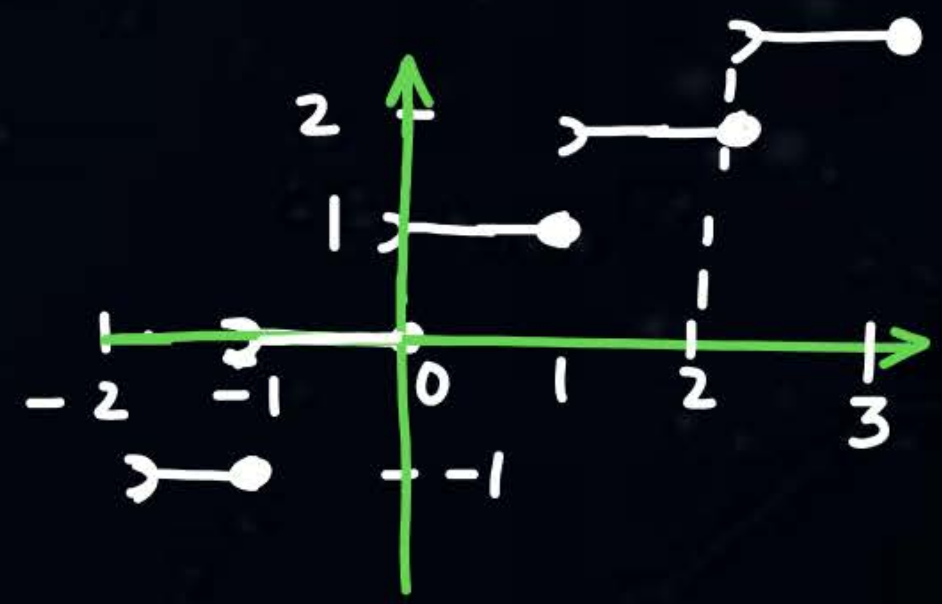
GRAPH OF FUNCTIONS

5) Least Integer Function.

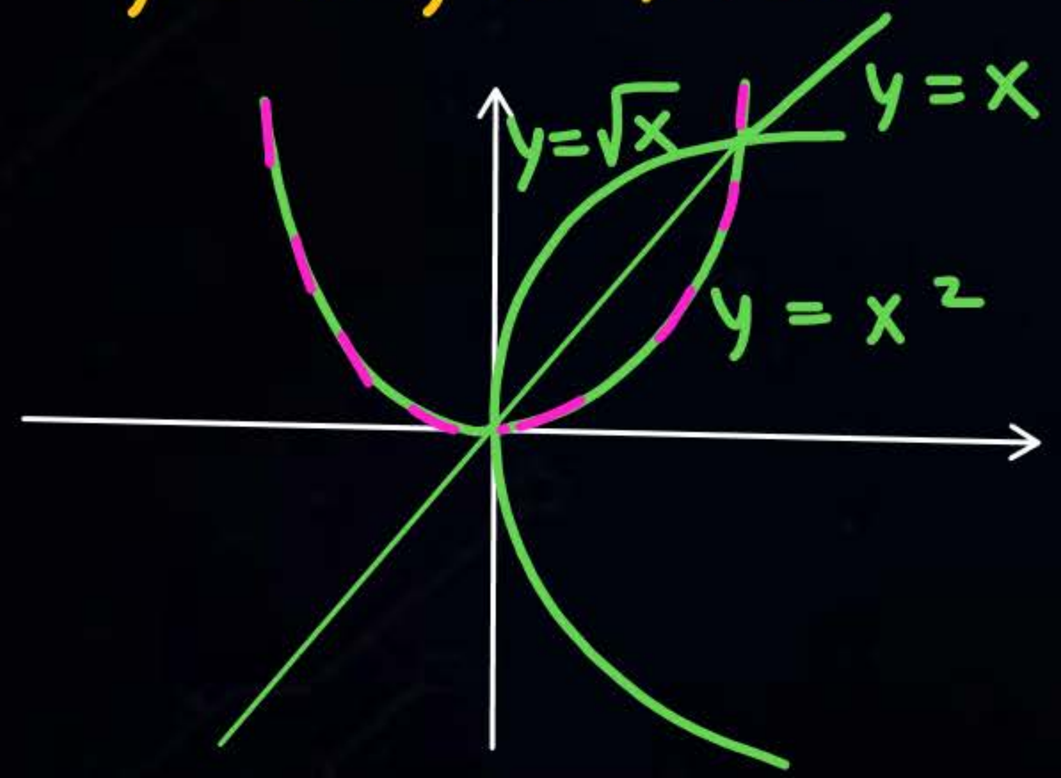
$[5.6] = 6$ $\quad$ $[1] = 1$

$[0.7] = 1$ $\quad$ $[1.5] = 2$

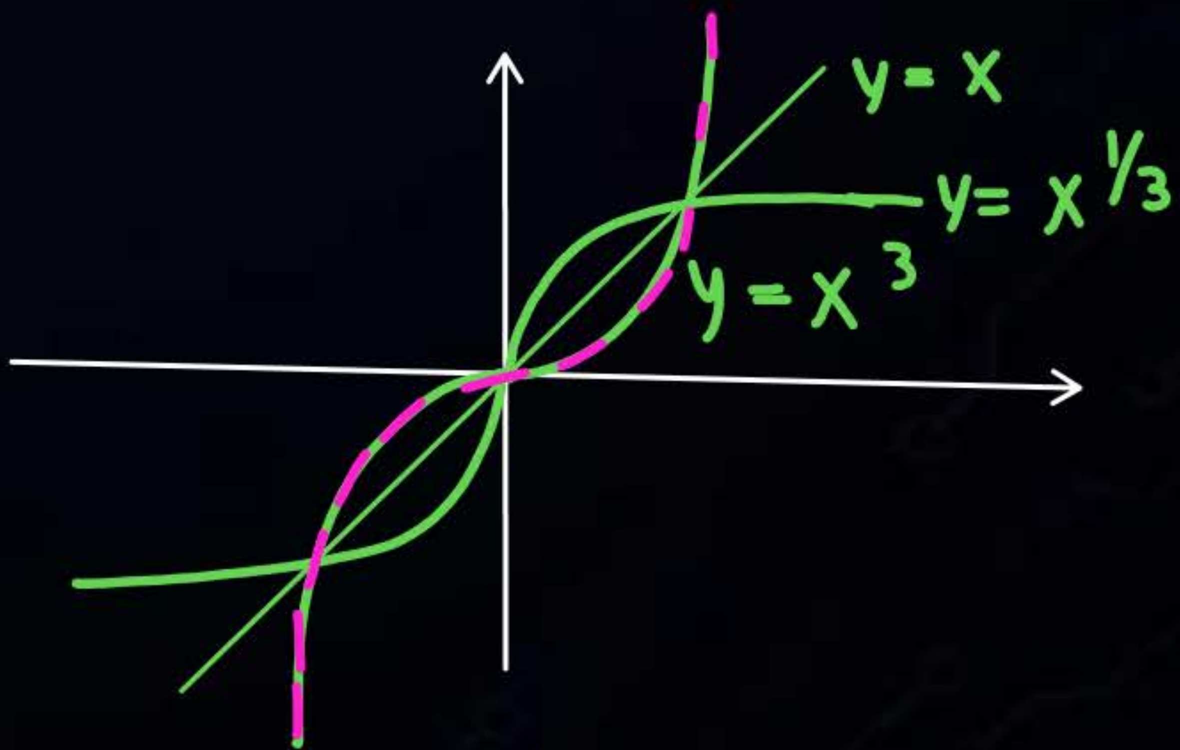
$[2] = 2$



$y = x, x^2, \sqrt{x}$

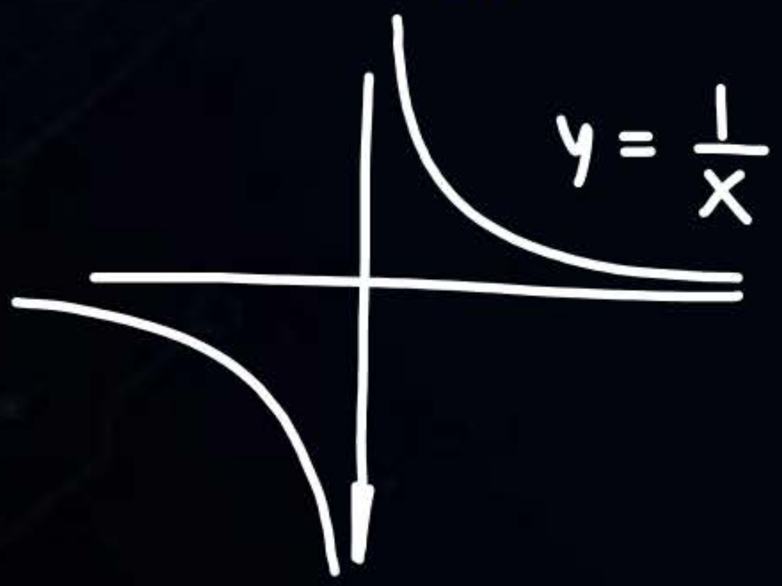


$y = x, x^3, x^{1/3}$

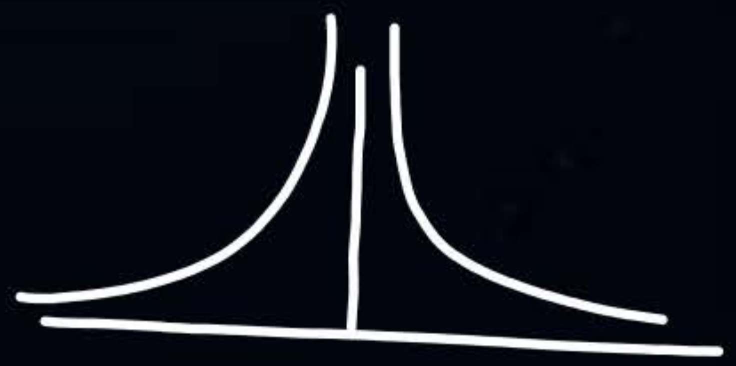


GRAPH OF FUNCTIONS

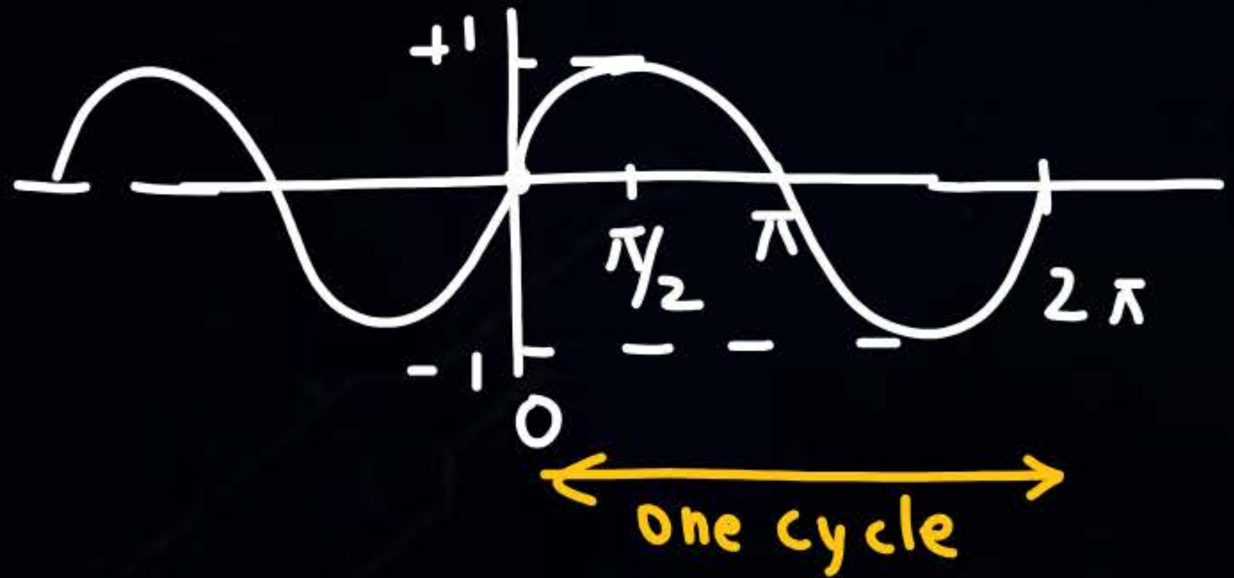
$y = \frac{1}{x}$



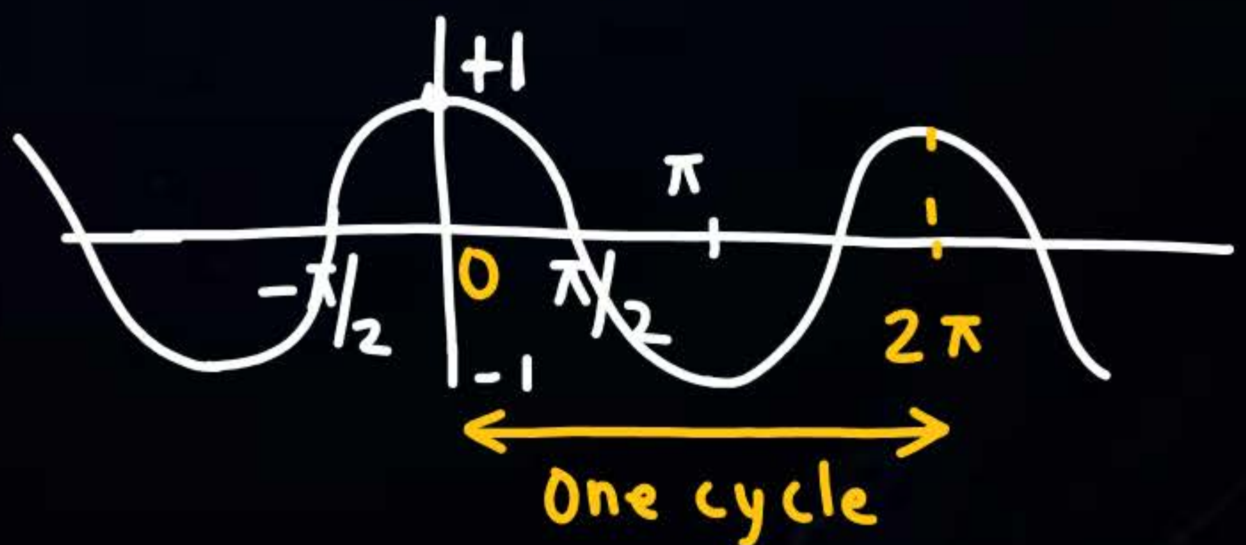
$y = \frac{1}{x^2}$



sin x



cos x



GRAPH OF FUNCTIONS

$\log x$

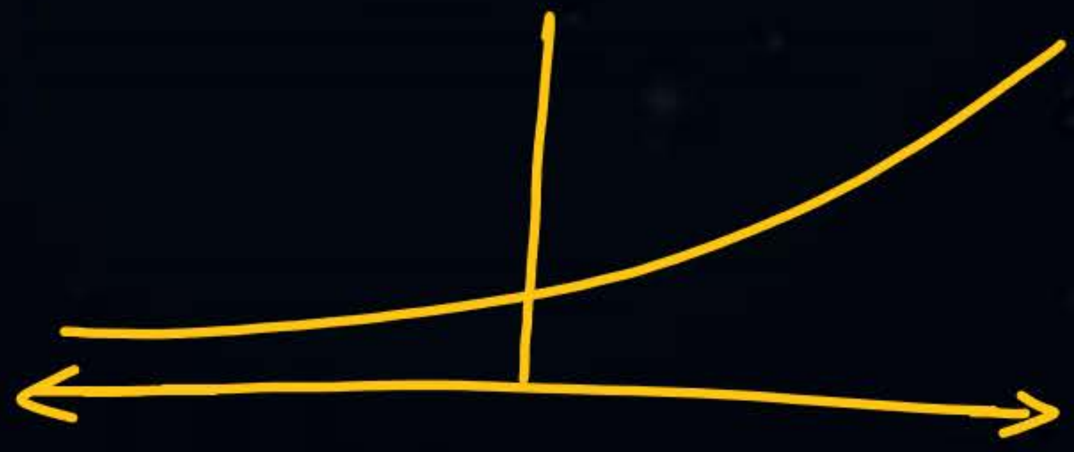
$x \to$ can't be $-$ve.

$\log 0 \to -\infty$

$\log 0 \to 1$

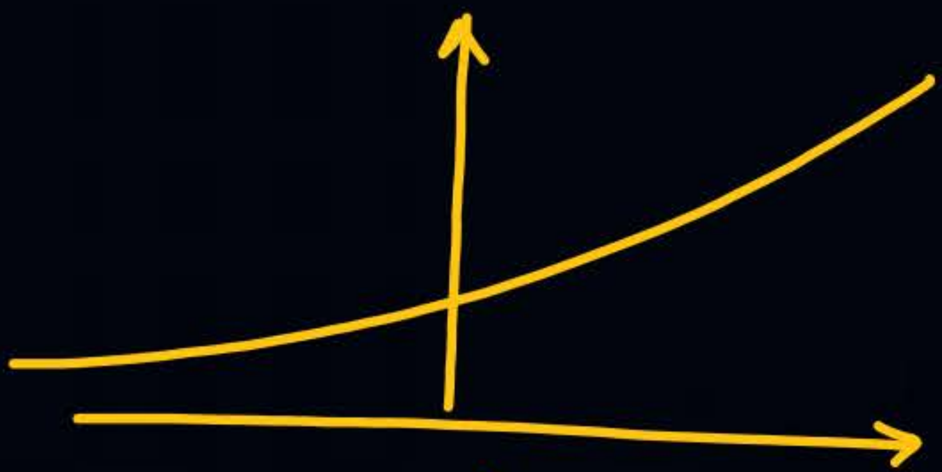
$\log 10 \to 1$

$\log 100 \to 2$

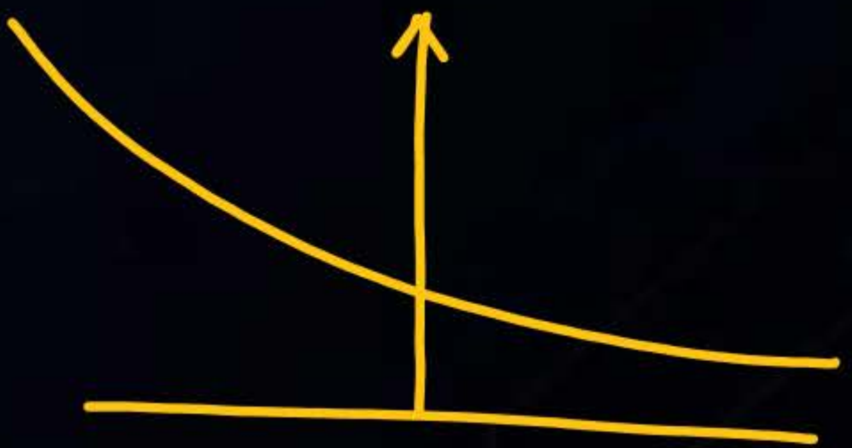


e^{x}

e^{-x}



a^{x}



a^{-x}

$(a, e > 1)$

GRAPH TRANSFORMATION

1. (i) $f(x) + a \rightarrow$ shift the graph of $f(x)$ upward by a units.

   (ii) $f(x) - a \rightarrow$ shift the graph of $f(x)$ downward by a units.

2. (i) $f(x + a) \rightarrow$ shift the graph of $f(x)$ leftward by a units.

   (ii) $f(x - a) \rightarrow$ shift the graph of $f(x)$ rightward by a units.

3. (i) $af(x) \rightarrow$ stretch the graph of $f(x)$, a times along y axis. [Ex:- $y = x^2$
   $y = 5x^2$

   (ii) $\frac{1}{a}f(x) \rightarrow$ shrink the graph of $f(x)$, a times along y axis. Ex:- $y = x^2$
   $y = x^2/5$

[GRAPH TRANSFORMATION]

$y = 5x + 3$

4. (i) f(ax) → stretch the graph of f(x), a times along x axis. $y(5x) \rightarrow 25x+3$

   (ii) $f\left(\frac{x}{a}\right)$ → stretch the graph of f(x), a times along x axis. $y\left(\frac{x}{5}\right) \rightarrow x+3$

5. (i) f(–x) → Take the mirror image of f(x), about x axis.

   (ii) –f(x) → Take the mirror image of f(x) about x axis.

   (iii) –f(–x) → First take the mirror image about y axis and take the mirror image of new graph about x axis.

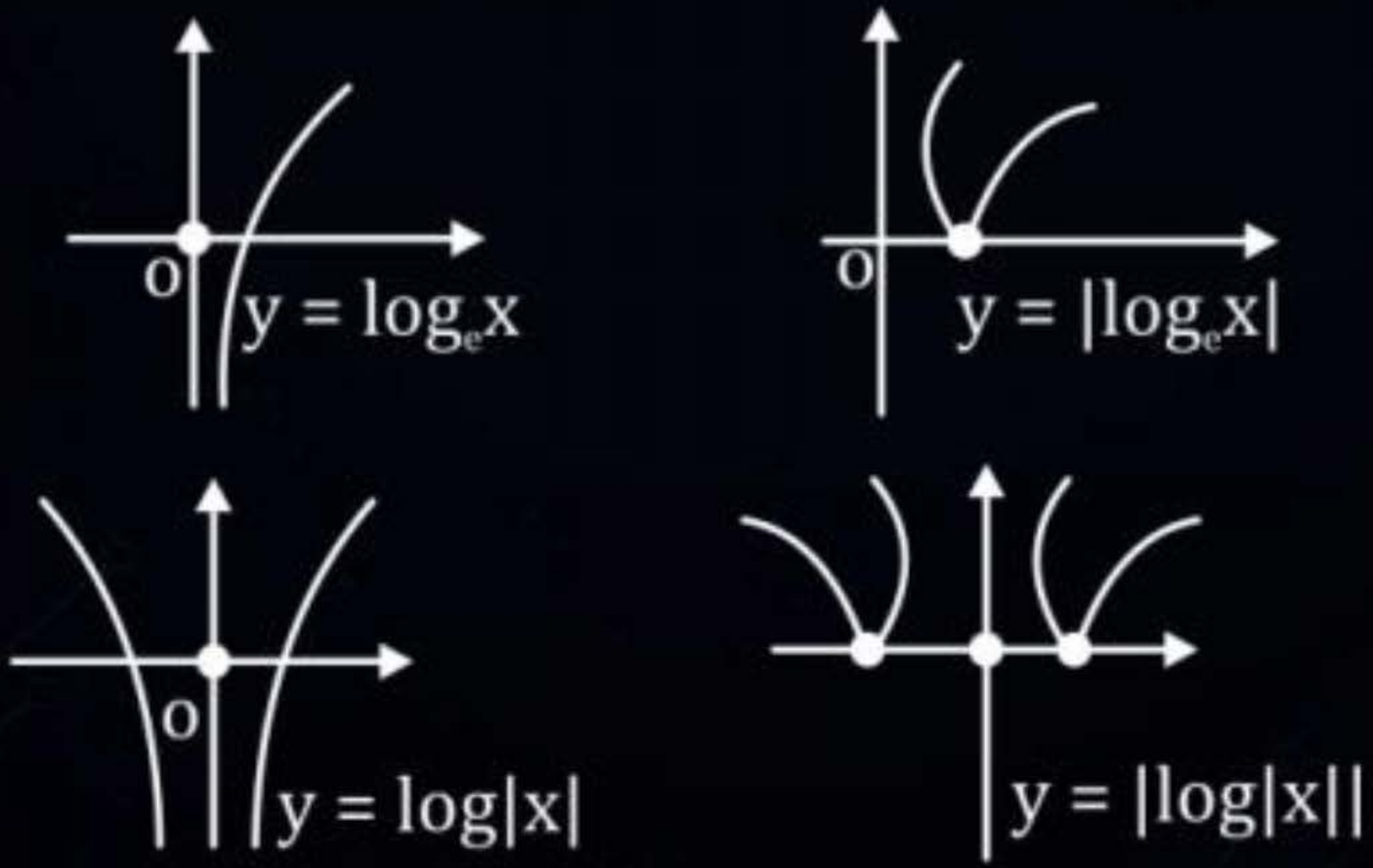
GRAPH TRANSFORMATION

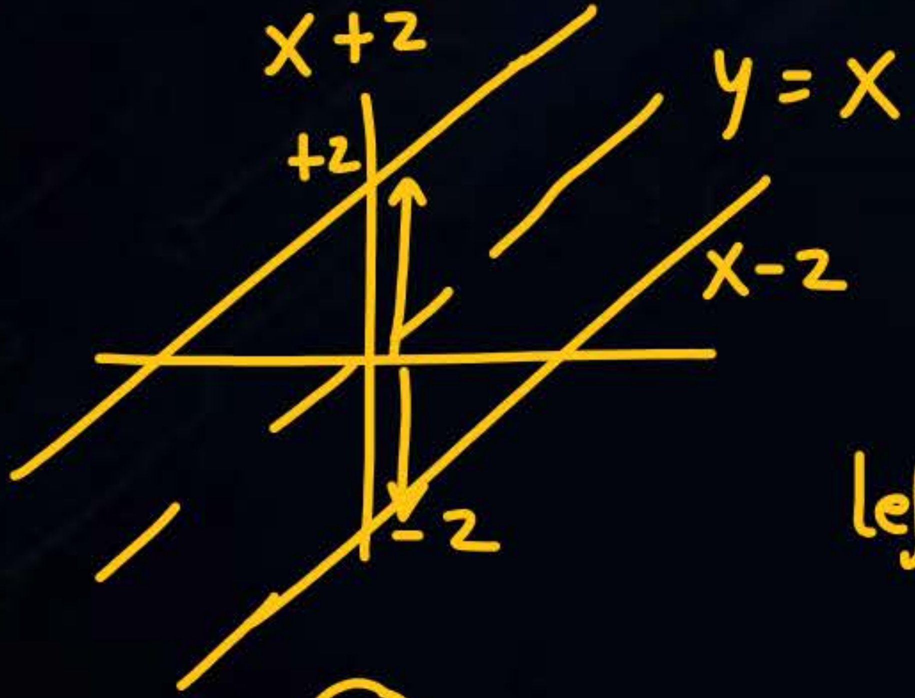
6. (i) $|f(x)| \rightarrow$ Take the mirror image about x axis, of that portion of graph, which lies below x axis. While graph that lies above x axis remains as it is.

(ii) $f(|x|) \rightarrow$ First unit that portion of graph which lies in the left side of y axis, and then take the mirror image about y axis of the remaining portion of the graph.

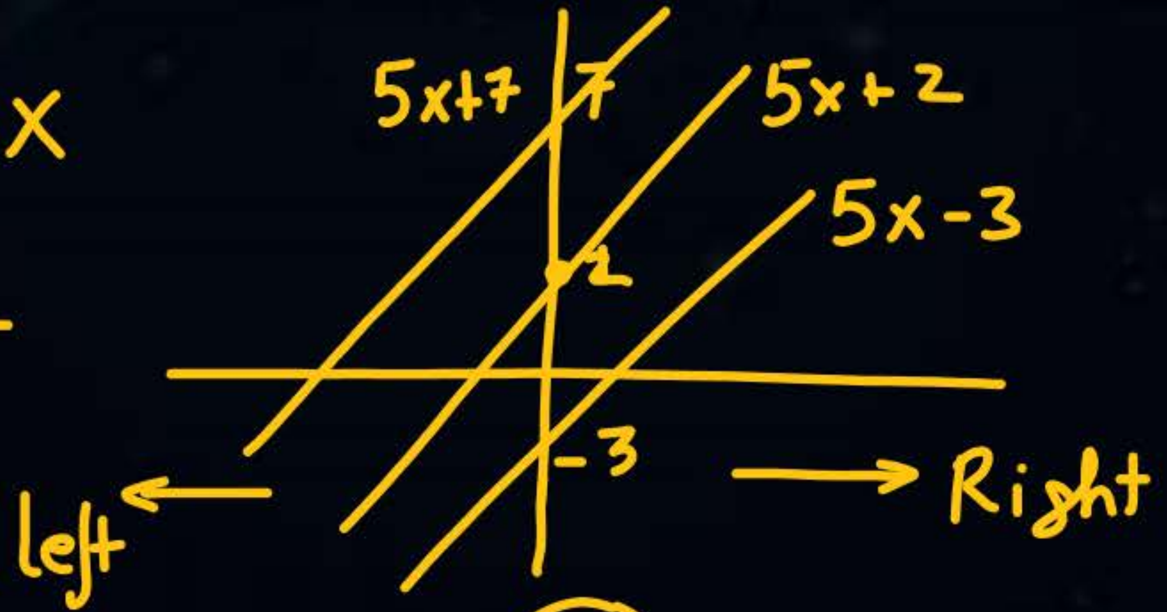
[GRAPH TRANSFORMATION]

(iii) $|f(|x|)| \rightarrow$ First form the graph of $|f(x)|$ using part (i) and then from the graph of $|f(x)|$ using part (ii).





1



2

$y = 5x + 2$

$f(x-1) = 5(x-1)+2 = 5x-3 \quad (x-a) \rightarrow \text{Right}$

$f(x+1) = 5(x+1)+2 = 5x+7 \quad (x+a) \rightarrow \text{Left}$

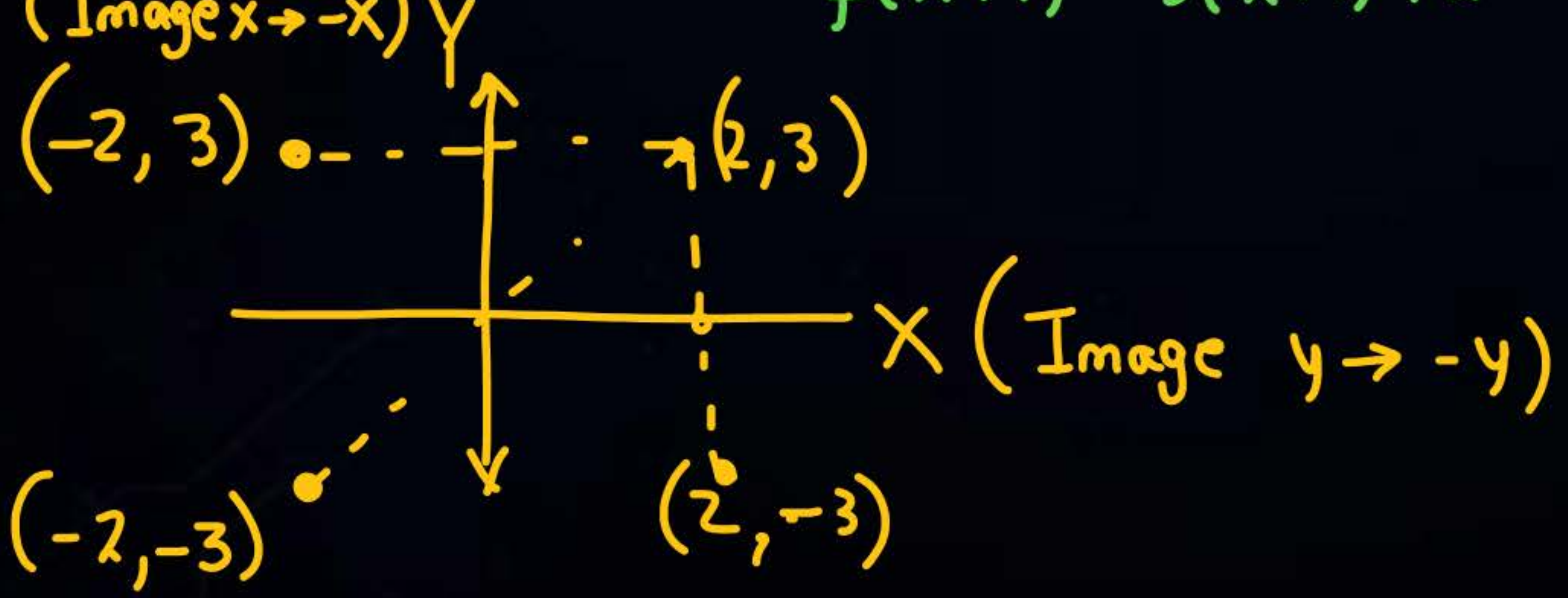


Image about

i) x axis → $y \rightarrow -y$

ii) y axis → $x \rightarrow -x$

iii) Origin → $x \rightarrow -x,\ y \rightarrow -y$

Q.

The curve given by the equation $x^2 + y^2 = 3axy$ is

(a) Symmetrical about x-axis

(b) Symmetrical about y-axis

(c) Symmetrical about the line $y = x$

(d) Tangential to $x = y = a/3$

$$x^2 + y^2 = 3axy$$

$\rightarrow$ $x \leftrightarrow y$

$$y^2 + x^2 = 3ayx$$

NOTE :-

$\rightarrow$ If fn. is symmetrical about

i) X-axis $\rightarrow$ $y \rightarrow -y$
same fn.

ii) y-axis $\rightarrow$ $x \rightarrow -x$
same fn.

iii) origin $\rightarrow$ $x \rightarrow -x, y \rightarrow -y$
same fn.

Thank you

GW Soldiers !