

CS & IT ENGINEERING

DISCRETE MATHS

Mathematical Logic



Lecture No. 09



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TOPICS

01 Connectives

02 Type 1

03 Type 2

$$[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$$

- | | |
|--|---|
| a) $\neg(p \vee \neg q) \rightarrow \neg p$ | b) $p \rightarrow (q \rightarrow r)$ |
| c) $(p \rightarrow q) \rightarrow r$ | d) $(p \rightarrow q) \rightarrow (q \rightarrow p)$ |
| e) $[p \wedge (p \rightarrow q)] \rightarrow q$ | f) $(p \wedge q) \rightarrow p$ |
| g) $q \leftrightarrow (\neg p \vee \neg q)$ | |
| h) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ | |

- a)** $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
b) $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$
c) $[(p \vee q) \wedge \neg p] \rightarrow q$
d) $[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$

- a)** $[p \wedge (p \rightarrow q) \wedge r] \rightarrow [(p \vee q) \rightarrow r]$
b) $[[p \wedge q] \rightarrow r] \wedge \neg q \wedge (p \rightarrow \neg r) \rightarrow (\neg p \vee \neg q)$
c) $[p \vee (q \vee r)] \wedge \neg q \rightarrow (p \vee r)$

$$\neg(\neg p \wedge \neg q) \wedge (q \rightarrow (p \rightarrow R)) \rightarrow \neg R \quad \text{F}$$

$$\neg p \wedge \neg q$$

$$\downarrow$$

$$q$$

$$q \rightarrow (p \rightarrow R)$$

$$\neg q \vee (\neg p \vee R)$$

$$q$$

$$\neg p \vee R$$

$$\wedge$$

$$\neg p$$

$$\neg p \wedge (\neg p \vee R)$$

$$\neg p$$

Type-2:

$$\leftrightarrow \equiv$$

$$\begin{cases} T \leftrightarrow T \checkmark \\ F \leftrightarrow F \checkmark \end{cases}$$

$$q \leftrightarrow (\neg p \vee \neg q)$$

$$T \leftrightarrow (F \vee F)$$

$$T \leftrightarrow F \equiv \textcircled{F}$$

7. For the universe of all integers, let $p(x)$, $q(x)$, $r(x)$, $s(x)$, and $t(x)$ be the following open statements.

$p(x)$: $x > 0$

$q(x)$: x is even

$r(x)$: x is a perfect square

$s(x)$: x is (exactly) divisible by 4

$t(x)$: x is (exactly) divisible by 5

a) Write the following statements in symbolic form.

- i) At least one integer is even. $\exists x q(x)$
- ii) There exists a positive integer that is even.
- iii) If x is even, then x is not divisible by 5.
- iv) No even integer is divisible by 5.
- v) There exists an even integer divisible by 5.
- vi) If x is even and x is a perfect square, then x is divisible by 4.

no/none

$\forall x [(q(x) \wedge r(x)) \rightarrow s(x)] \quad \forall x [\quad \rightarrow \neg \quad]$

$\exists x [p(x) \wedge q(x)]$

$\forall x [q(x) \rightarrow \neg t(x)]$

English \rightarrow conversion.

Graph is connected. $\rightarrow \forall x [G(x) \rightarrow (x \rightarrow (x))]$

Gold and Silver ornaments are precious.

$$\forall x (G(x) \wedge S(x) \rightarrow P(x))$$

$$\forall x (G(x) \vee S(x) \rightarrow P(x))$$

$$\top \vee$$

$$\vee \top$$

