CS & IT ENGINEERING

DISCRETE MATHS GRAPH THEORY

Lecture No. 16

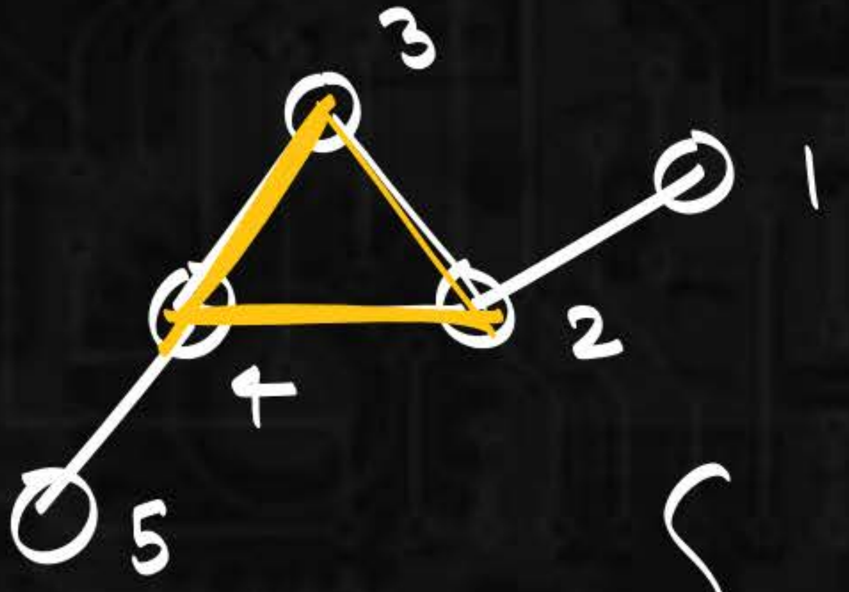
By- SATISH YADAV SIR

TOPICS

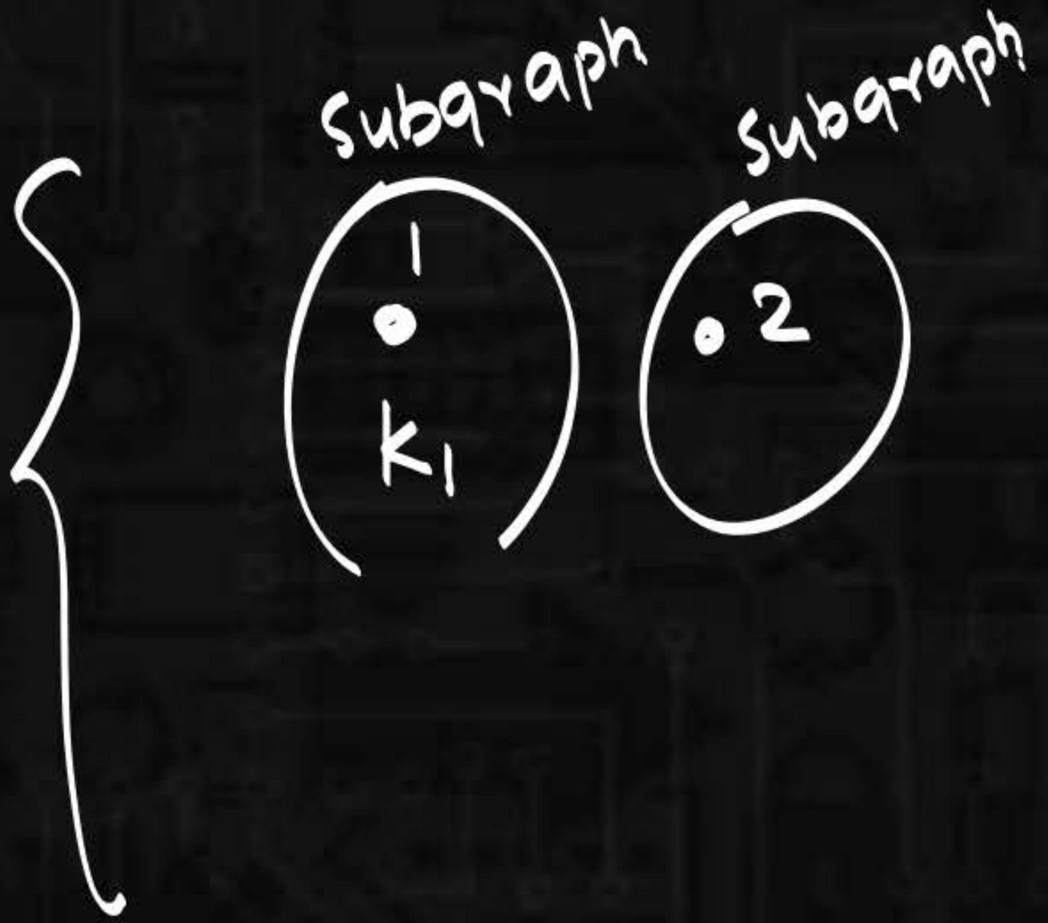
01 INDEPENDCE NO.

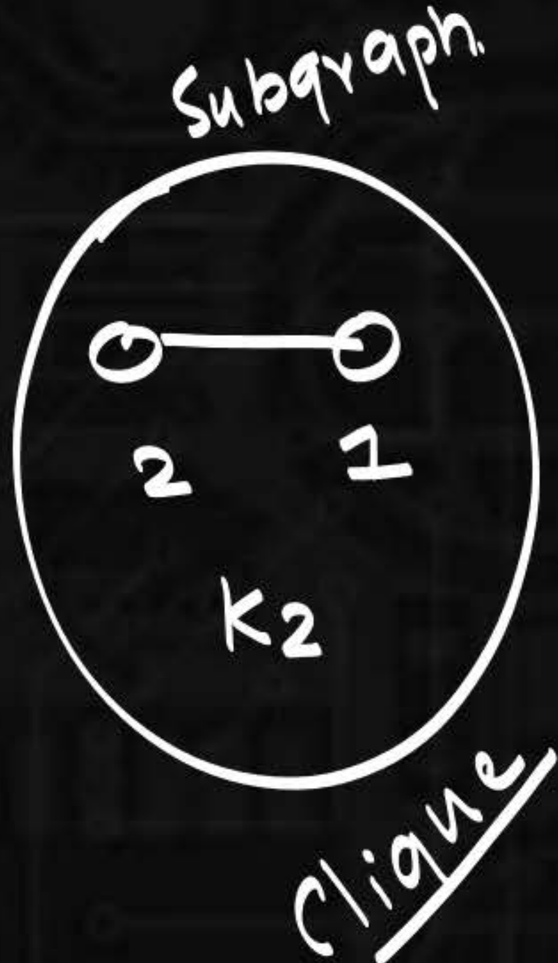
02 DOMINATION NO.

03 MATCHING NO .



Clique :

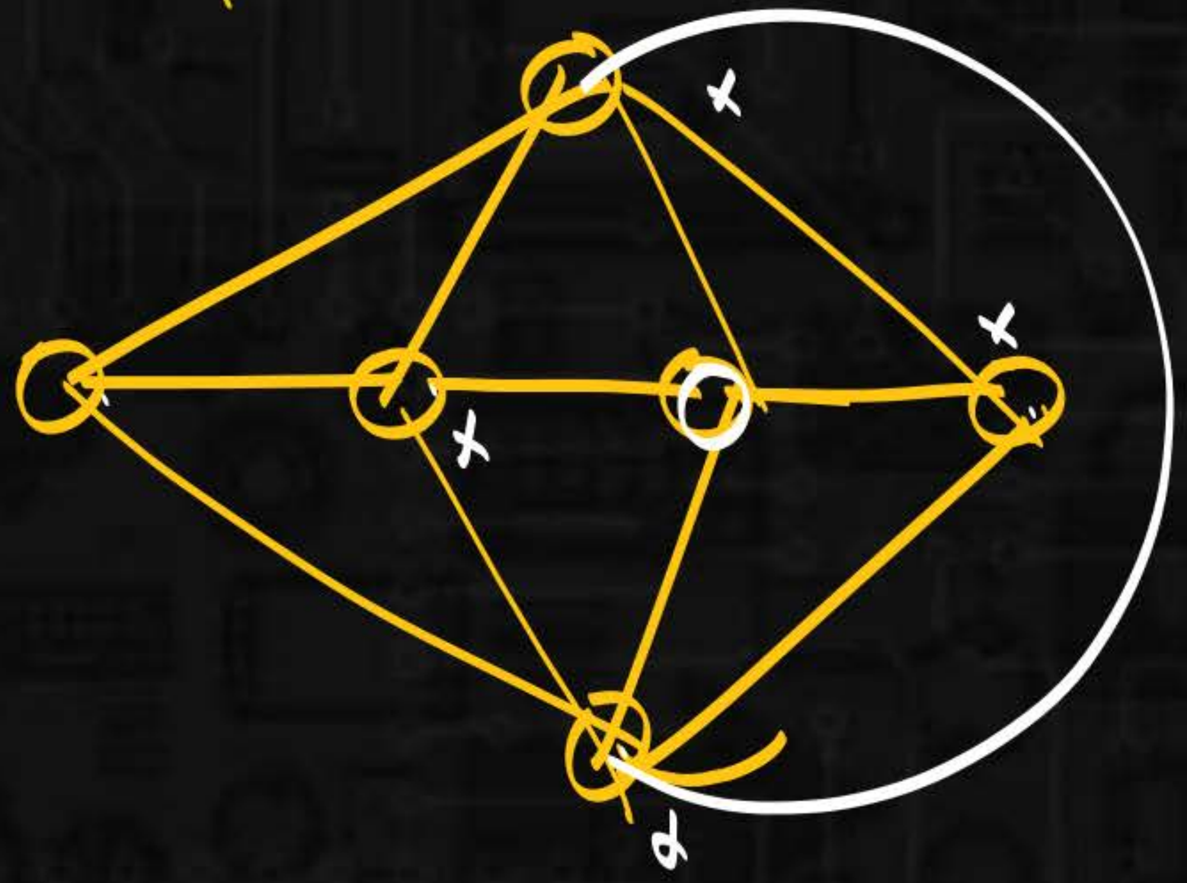


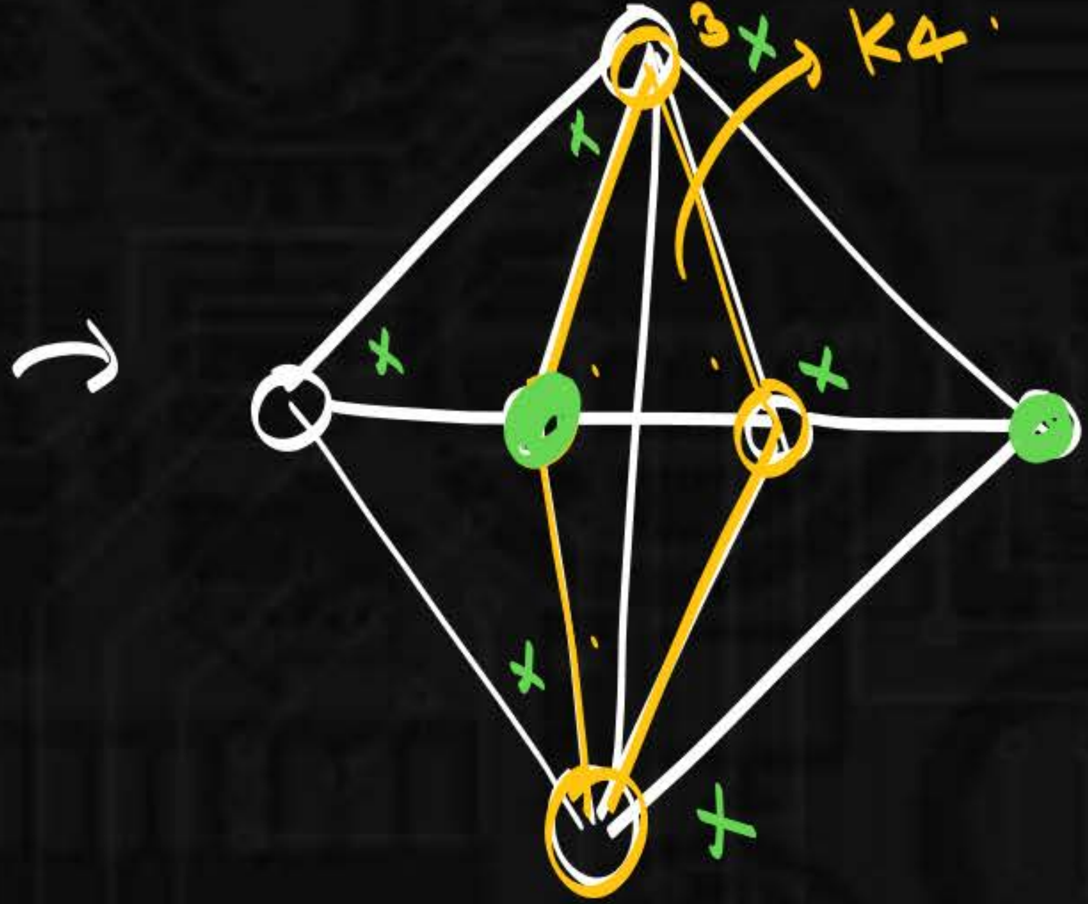


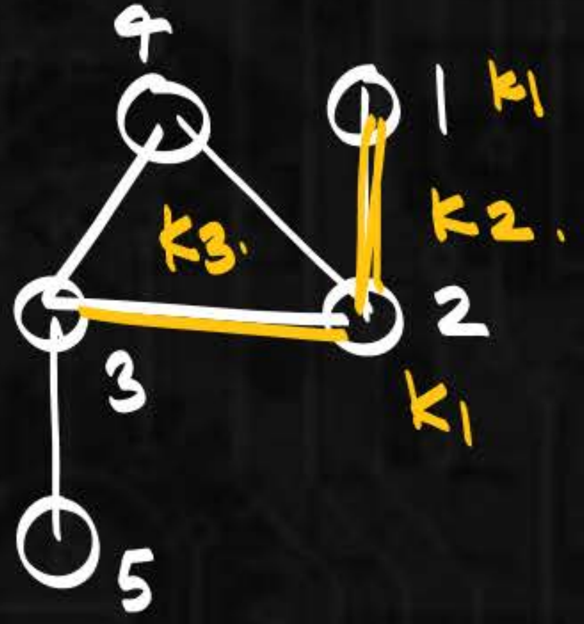
Clique no : $\boxed{\omega(G) = 3}$

$\xrightarrow{} 4.$ $\xrightarrow{} 2.$

maximum size of clique. / maximum size of Independent set







Subgraphs:

$n = 1$.

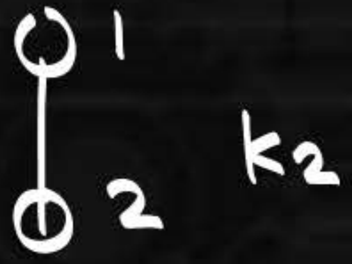
1

2

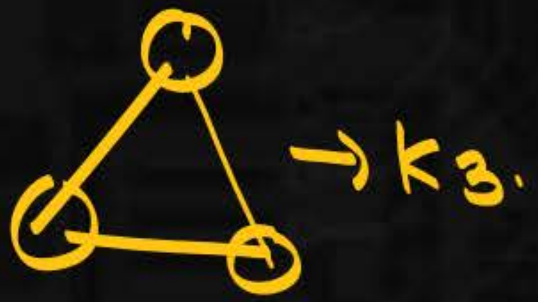
3

4

5



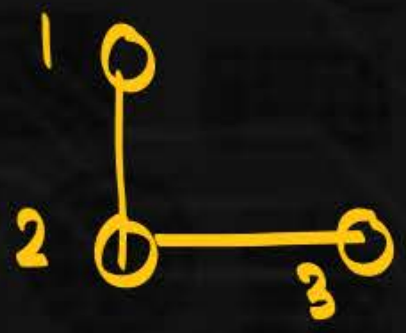
$\omega(h) = 3$.



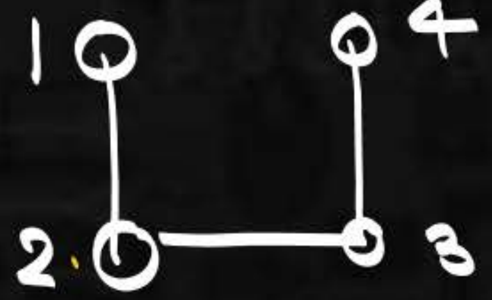
Graph operation :

1) union ($\cup$)
2) Intersection ($\cap$)
3) Ring sum ($\oplus$)

G1



G2.

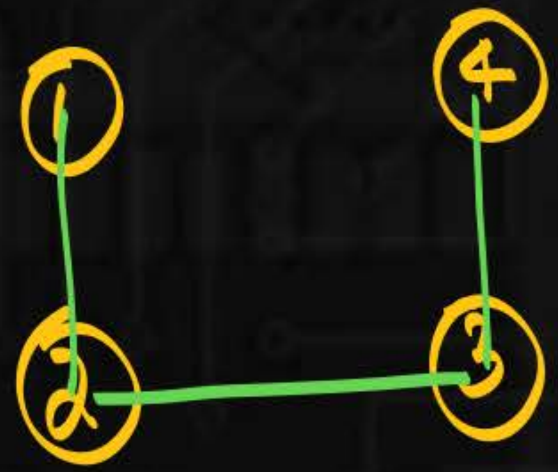


$G_3 = (V_3, E_3)$

$G_1 = (V_1, E_1)$ $\qquad$ $G_2 = (V_2, E_2)$

$V_1 = \{1, 2, 3\} \cup V_2 = \{1, 2, 3, 4\} = V_3$

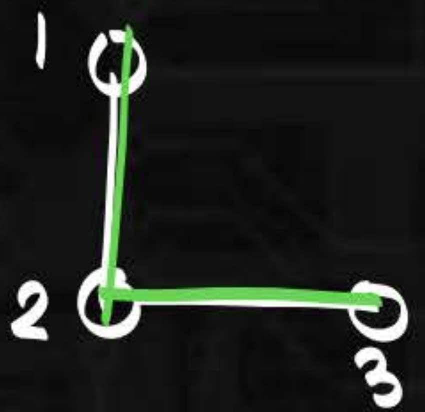
$E_1 = \{12, 23\} \cup E_2 = \{12, 23, 34\} = E_3$



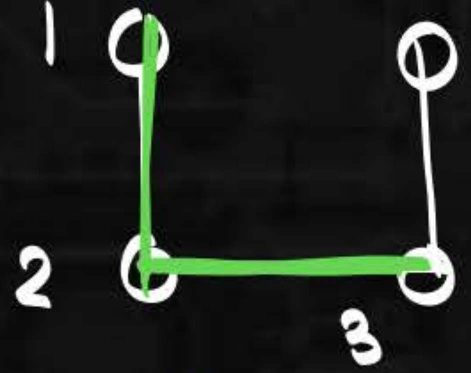
$G_3 = (V_3 . E_3)$

Intersection ($\cap$)

$G_1 = (V_1, E_1)$ $\qquad$ $G_2 = (V_2, E_2)$



$\cap$



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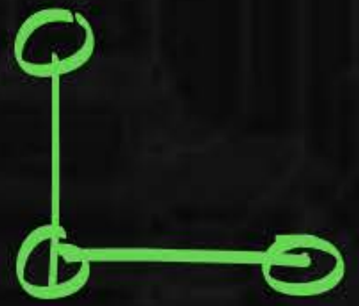


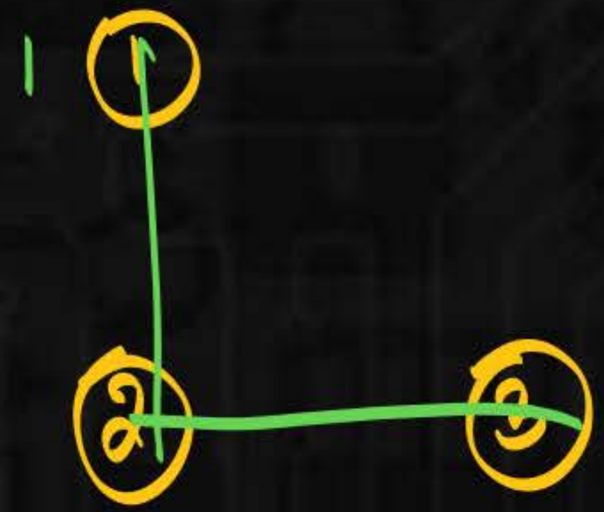
$G_4 = (V_4, E_4)$

$V_4 = V_1 \cap V_2$

$E_4 = E_1 \cap E_2$

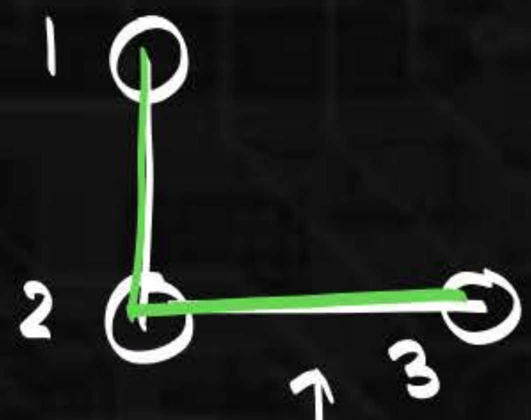
$V_1 = \{1, 2, 3\} \cap V_2 = \{1, 2, 3, 4\} = V_4 = \{1, 2, 3\}$

$E_1 = \{12, 23\} \cap E_2 = \{12, 23, 34\} = \{12, 23\}$

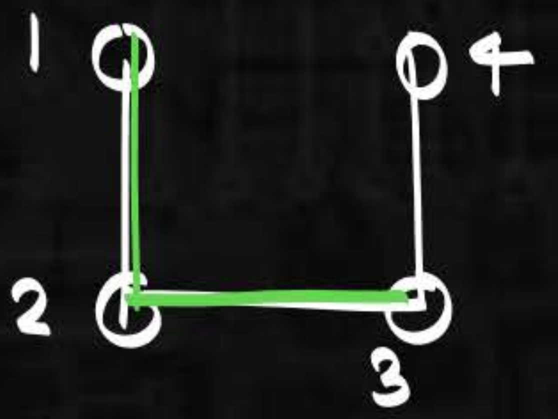


Ringsum ($\oplus$)

$G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$



G_1



G_2

$G_6 = (V_6, E_6)$

$V_6 = V_1 \cup V_2$.

$E_6 = (E_1 \cup E_2) - (E_1 \cap E_2)$

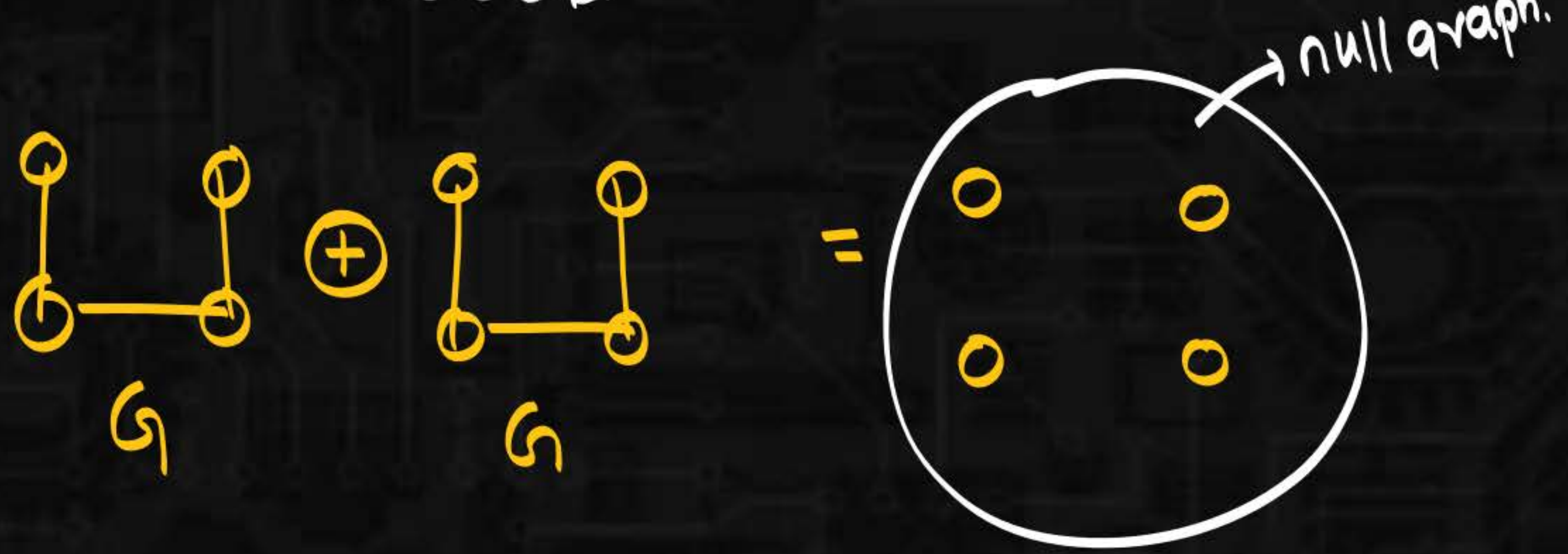
$G_6 = (V_6, E_6)$

→ edges in G_1 OR G_2 but not in both.

$G \oplus G$ = null graph

$G \oplus G =$ null graph.

G



null graph.

set of isolated vertices.

Graph representation : $n \times n$ - symmetric binary matrix.

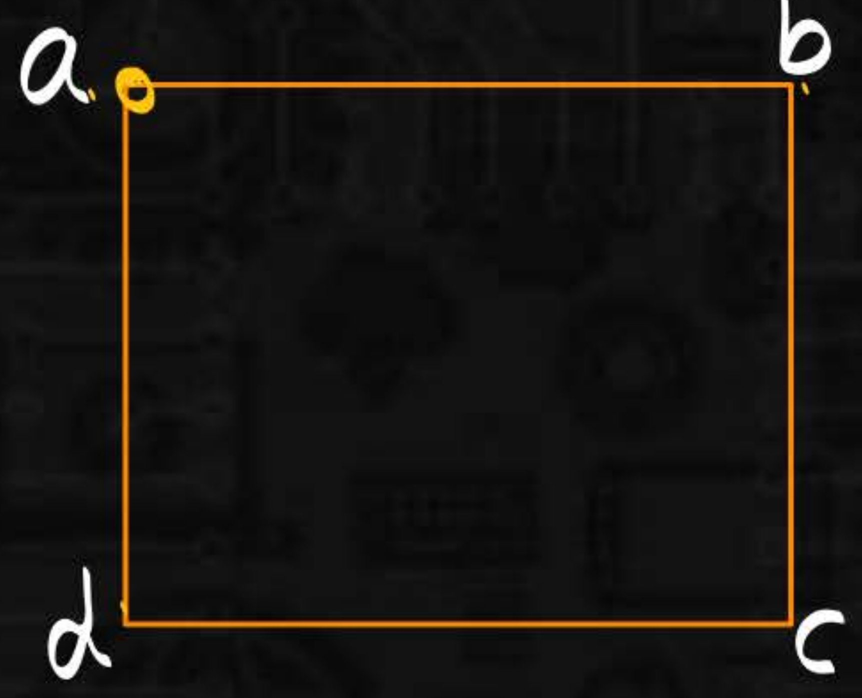
adjacency matrix / connection matrix.

$A(G)$

$(n \times n)$ - binary matrix

$x_{ij} = 1$ connection betn vertex i & j

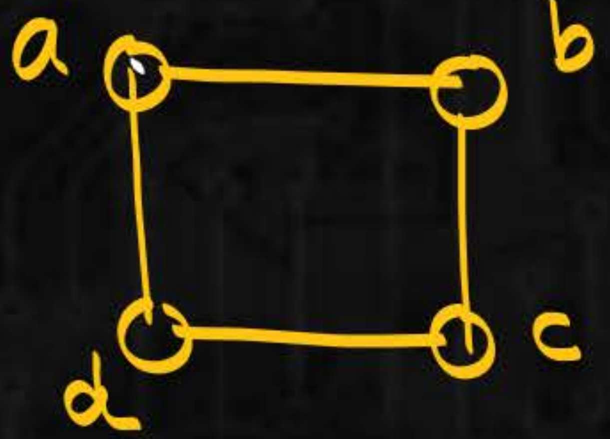
$= 0$ (else)



$$A(G) = \begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}_{4 \times 4} \end{array}$$

$x_{ij} = x_{ji}$

$x_{ab} = x_{ba}$



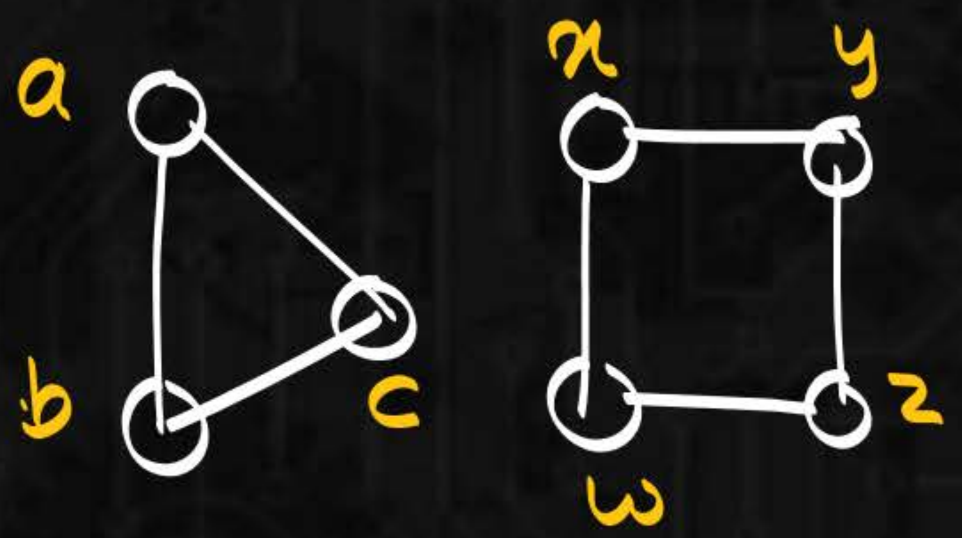
$$\begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \hline \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}_{4 \times 4} \end{array}$$

1.

→ no self loop.

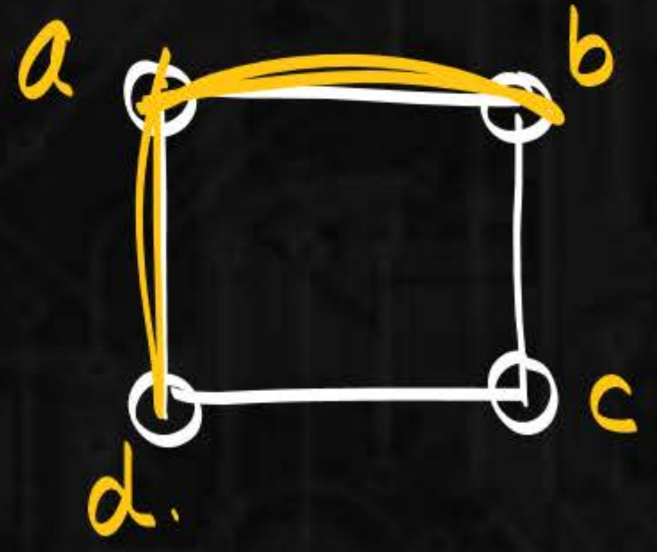
→ Diagonal elements will be zero.

→ no. of 1's are present in row/column will give degree.



$$A(G) = \left[\begin{array}{c|c} A(G_1) & \\ \hline & A(G_2) \end{array}\right]$$

$$\begin{array}{c|ccc|cccc}
 & a & b & c & x & y & z & w \\
\hline
a & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
b & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
c & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
\hline
x & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
y & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot \\
z & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot \\
w & 0 & 0 & 0 & \cdot & \cdot & \cdot &
\end{array}$$



$$A = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 0 & 1 & 0 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$

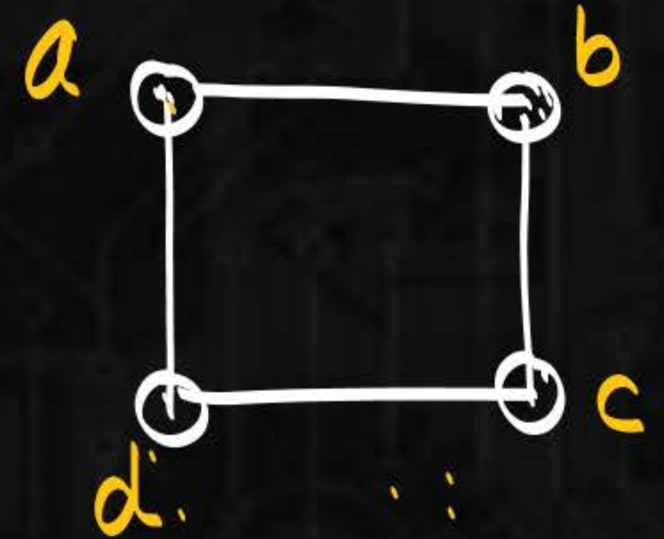
$A^2(i,i)$ = degree of vertex v_i

$0\times0 + 1\times1 + 0\times0 + 1\times1 = 2.$

$$A^2 = A \times A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & \boxed{2} & 0 & 2 & 0 \\ b & 0 & 2 & 0 & 2 \\ c & 2 & 0 & 2 & 0 \\ d & 0 & 2 & 0 & 2 \end{array}$$

$a \to b$ (length 3) walk



$$A^2 = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

$A^3(a,b) = \{$ 4 walks of length 3 from a, b.

* $A^n(i,j)$ = no. of walks from i. j of length n.

$A^3(a,b)$: $a \to b$

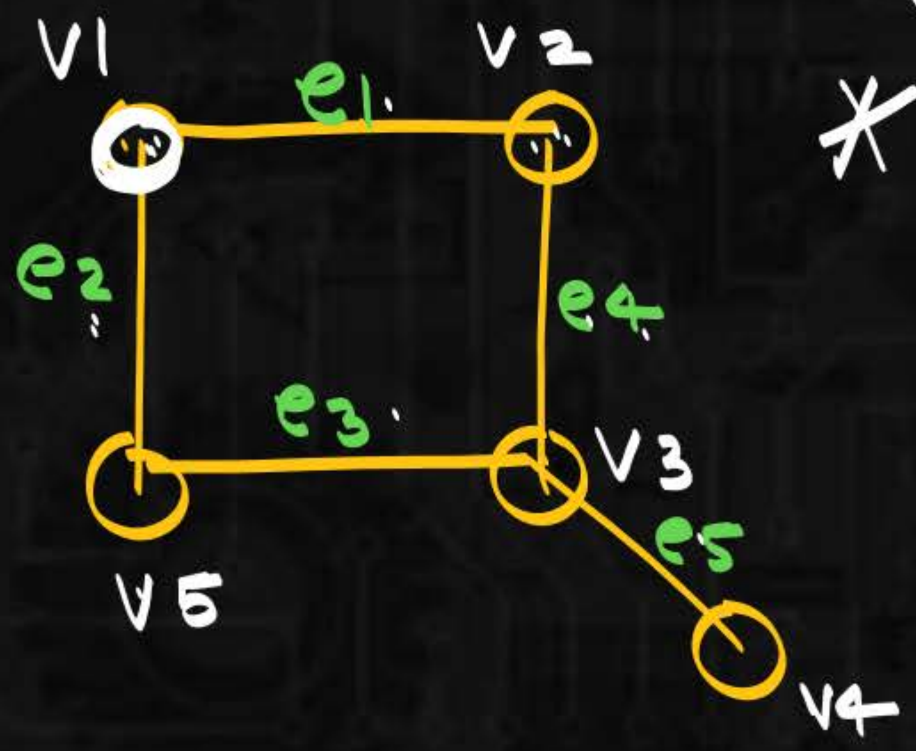
4 walks of length 3.

- a – d – a – b
- a – d – c – b
- a – b – a – b
- a – b – c – b

$$A^3 = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$A^3 : \begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \hline \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 4 & 0 & 4 \\ 4 & 0 & 4 & 0 \\ 0 & 4 & 0 & 4 \\ 4 & 0 & 4 & 0 \end{bmatrix} \end{array}$$

incident matrix (m(G)):



*

$$
\begin{array}{c|ccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 \\
\hline
V_1 & 1 & 1 & 0 & 0 & 0 \\
V_2 & 1 & 0 & 0 & 1 & 0 \\
V_3 & 0 & 0 & 1 & 1 & 1 \\
V_4 & 0 & 0 & 0 & 0 & 1 \\
V_5 & 0 & 1 & 1 & 0 & 0
\end{array}
$$

*

rank(m(G)) = n-1.

- → Every column will have only 2 ones.
- → every row will give degree of a vertex.


THANK YOU GW
SOLDIERS !