ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-05

Probability

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Topics to be Covered

- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM
- STATISTICS – I (PROBABILITY DISTRIBUTIONS)
- STATISTICS – II (CORRELATION AND REGRESSION)

Ex:- The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain today or tomorrow is 0.7. What is the probability that it will rain today & tomorrow?

A. 0.3 B. 0.25

C. 0.35 D. 0.4

$P(\text{Today}) = 0.5$ $p(\text{Tomorrow}) = 0.6$

$p(\text{Today} \cup \text{Tom.}) = 0.7$

$p(\text{Today} \cap \text{Tom.}) = ?$

$\rightarrow p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$0.7 = 0.5 + 0.6 - p(A \cup B)$

$p(A \cap B) = 1.1 - 0.7 = 0.4$

$p(A \cap B) = p(A) . p(B)$

$= 0.5 \times 0.6$

$= 0.3$

$\neq 0.4$

Q. A box contains 5 black balls and 3 red balls. A total of three balls are picked from the box one after another, without replacing them back. The probability of getting two black balls and one red ball is

A. 3/8

B. 2/15

C. 15/28

D. 1/2

$\boxed{\begin{matrix} 5B \\ 3R \end{matrix}}$

$$P(2B \cap 1R) = {}^3C_1\left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}\right) \quad \left\{\begin{matrix} B\,B\,R \\ B\,R\,B \\ R\,B\,B \end{matrix}\right\}$$

$$\frac{3!}{2!\,1!}$$

Q5. Two dices are thrown. What is the probability that the sum of the numbers on the two dice is eight?

A. 5/36

B. 5/18

C. ¼

D. 1/3

Sum = 7 (6)

$$\left.\begin{array}{l}\text{Sum} = 8 \\ \text{Sum} = 6\end{array}\right\} (5)$$

$$p(\text{Sum} = 8) = \frac{5}{36}$$

Q. A fair die is rolled twice. The probability that an odd number will follow an even number is

A. ½

B. 1/6

C. 1/3

D. 1/4

$$P(\text{Even no.} \cap \text{Odd no.})$$

$$= P(E) \cdot P(O)$$

$$= \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$$

Q. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be kings, if first card is NOT replaced?

A. 1/26

B. 1/52

C. 1/169

D. 1/221

$$P(K_1 \cap K_2) = P(K_1) \cdot P(K_2/K_1)$$

$$= \frac{4}{52} \times \frac{3}{51}$$

$$= \frac{{}^{4}C_2}{{}^{52}C_2}$$

$$4K \downarrow 2K.$$

Q. A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads?

A. $(1/2)^2$

B. $(1/2)^{10}$

C. ${}^{10}C_2(1/2)^2$

D. ${}^{10}C_2(1/2)^{10}$

HH T T T T T T T T

$$= \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8$$

Exactly 2H out of 10

$$= {}^{10}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8$$

Q. A fair unbiased coin was tossed in succession 4 times and resulted in following outcomes i) Head ii)I Head iii) Head iv) Head. The probability of obtaining a tail when the coins is tossed again is

A. 0

B. ½

C. 4/5

D. 1/3

$$\underbrace{HHHH}\longrightarrow \begin{cases} H \\ T \end{cases}$$

$$1\times1\times1\times1\times\frac{1}{2}=\frac{1}{2}$$

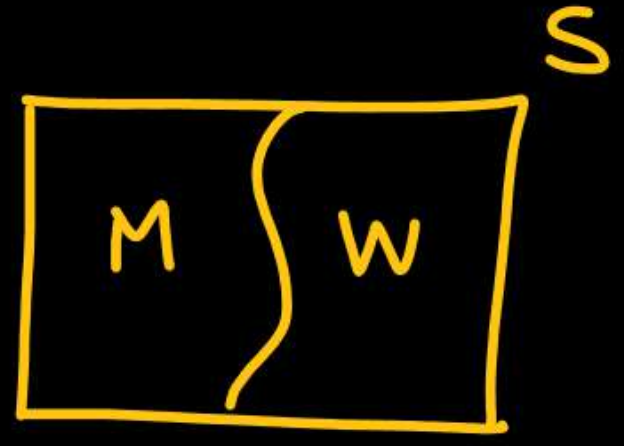
$$\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times\frac{1}{2}=\frac{1}{32}$$

Q. A group consists of equal number of men and women. Of this group 20% men and 50% women are unemployed. If a person is selected at random from this group, the probability of the selected group person being employed is ....

Group

50% men — 20% UE, 80% E

50% women — 50% UE, 50% E



Prob. of being employed

$$P(E) = P(M) \cdot P(E/M) + P(W) \cdot P(E/W)$$

$$= \frac{1}{2} \times \frac{80}{100} + \frac{1}{2} \times \frac{50}{100} = \frac{130}{200} = 0.65$$

Q. Four fair coins are tossed simultaneously. The probability that at least one heads and at least one tails turn up is

A. 1/16

B. 1/8

C. 7/8

D. 15/16

$$1 - p(\text{no head}) - p(\text{no tail})$$

$$1 - \left(\overset{T\ T\ T\ T}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}\right) - \left(\overset{H\ H\ H\ H}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}\right)$$

$$\frac{14}{16} = \frac{7}{8}$$

Q. If P and Q are two random events, then the following is TRUE

A. Independence of P and Q implies that probability $\underbrace{(P\cap Q) = 0}$ → Mutually exclusive.

B. Probability $(P\cup Q) \geq$ Probability (P) + Probability (Q)

C. If P and Q are mutually exclusive, then they must be independent

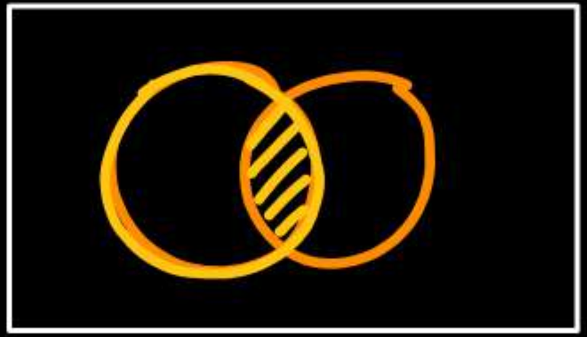
D. Probability $(P\cap Q) \leq$ Probability (P)

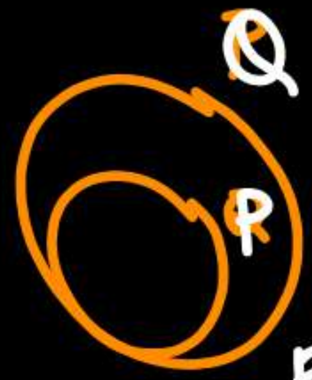
$$p(P\cap Q) = p(P)\cdot p(Q)$$

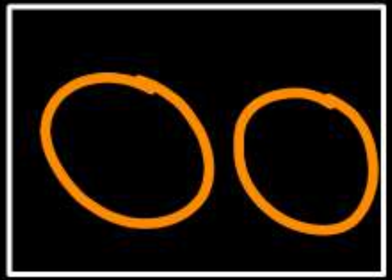
$$p(P\cup Q) = p(P) + p(Q) - p(P\cap Q)$$

$$p(P\cup Q) \leq p(P) + p(Q)$$

$$p(P\cap Q) \leq p(P)$$
$$\leq p(Q)$$







$$p(P\cap Q) = p(Q)$$
$$= p(P)$$

$P = \{1,3,5\}$

$Q = \{1,3,4\}$

$P\cap Q = \{1,3\}$

$P\cup Q = \{1,3,4,5\}$

Q. There are 25 calculators in a box. Two of them are defective. Suppose 5 calculators are randomly picked for inspection(i.e., each has the same chance of being selected), what is the probability that only one of the defective calculators will be include in the inspection?

A. 1/2

B. 1/3

C. 1/4

D. 1/5

25 cal.

2D 23 ND

5 cal.

1D 4N.D.

$$p(E) = \frac{n(E)}{n(S)} = \frac{{}^{2}C_{1} \times {}^{23}C_{4}}{{}^{25}C_{5}}$$

Q. A person on a trip has a choice between private car and public transport. The probability of using a private car is 0.45. While using the public transport further choices available are bus and metro, out of which the probability of commuting by a bus is 0.55. In such a situation, the probability (rounded up to two decimals) of using a car, bus and metro, respectively would be

A. 0.45, 0.30 and 0.25

B. 0.45, 0.25 and 0.30

C. 0.45, 0.55 and 0.00

D. 0.45, 0.35 and 0.20

$$\begin{tikzcd} & \text{P. Car } (0.45) & \\ {} \arrow[ur] \arrow[r] & \text{P. Transport } (0.55) \arrow[r] \arrow[dr] & \text{Bus } (0.55) \\ & & \text{Metro } (0.45) \end{tikzcd}$$

$p(\text{Car}) = 0.45$

$p(\text{Bus}) = 0.55 \times 0.55 = 0.3025 \approx 0.30$

$p(\text{Metro}) = 0.55 \times 0.45 = 0.2475 \approx 0.25$

Q. Two players A and B alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is

[GATE]

A. 5/11

B. ½

C. 7/13

D. 6/11 ✓

$p = 1/6$

$q = 5/6$

$A, \bar{A}\bar{B}A, \bar{A}\bar{B}\bar{A}\bar{B}A, \ldots\ldots$ infinite

$p(A) + p(\bar{A}\bar{B}A) + p(\bar{A}\bar{B}\bar{A}\bar{B}A)$

$$\frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}$$

$$\frac{1}{6}\left\{1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \cdots \infty\right\}$$

$r = \frac{25}{36}$

$$p(A\text{ wins}) = \frac{1}{6}\left\{\frac{1}{1 - 25/36}\right\} = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

$$p(B\text{ wins}) = 5/11 \quad \{\bar{A}B, \bar{A}\bar{B}\bar{A}B, \bar{A}\bar{B}\bar{A}\bar{B}\bar{A}B, \ldots\}$$

Q. The probability that a student knows the answer to a MCQ is 2/3. If the student does not know the answer, then the student guesses it. The probability of the guessed answer being correct is ¼. Given that student has answered the question correctly, the conditional probability that the student knows the correct answer is

A. 2/3

B. ¾

C. 5/6

D. 8/9

$P(\text{Correct}\overset{\text{Knows}}{\text{ Answer}}) = 2/3$ $\quad$ $P(\text{He does not correct answer}) = 1/3$

$P(E_1) = 2/3$ $\quad$ $P(E_2) = 1/3$

C → Correct answer.

$P(C/E_1) = 1$ $\quad$ $P(C/E_2) = 1/4$

$$P(E_1/C) = \frac{P(E_1 \cap C)}{P(C)} = \frac{P(E_1).P(C/E_1)}{P(E_1).P(C/E_1) + P(E_2).P(C/E_2)} = \frac{\frac{2}{3}\times 1}{\frac{2}{3}\times 1 + \frac{1}{3}\times\frac{1}{4}}$$

$$= 8/9$$

Q. A dice is tossed two times. The probability that the second toss results in a value that is higher than the first toss is

A. 2/36

B. 2/6

C. 5/12

D. 1/2

(1,2) (1,3) (1,4) (1,5) (1,6)

(2,3) (2,4) (2,5) (2,6)

(3,4) (3,5) (3,6)

(4,5) (4,6)

(5,6)

$$= \frac{15}{36} = \frac{5}{12}$$

Q. A coin is tossed 4 times independently. The probability that "the number of times heads show up is more than the number of times tails show up is"

A. 1/16

B. 1/8

C. ¼

D. 5/16

_ _ _ _

$(3H, 1T) + (4H, 0T)$

$= {}^4C_3 \cdot \left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} + {}^4C_4 \left(\frac{1}{2}\right)^4$

$= 4 \times \frac{1}{16} + 1 \times \frac{1}{16} = \frac{5}{16}$

$$\begin{array}{ccccc} 0H & 1H & 2H & 3H & 4H \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 4 & 6 & 4 & 1 \end{array} = 16$$

(2^4)

Q. Three vendors are asked to supply a component. The respective probabilities of their meeting the design specifications are 0.8, 0.7 and 0.5. Each vendor supplies one component. The probability that out of total three components supplied by the vendors at least one will meet the design specifications is ....

$$
\begin{aligned}
p(\text{at least one}) &= 1 - p(\text{None}) \\
&= 1 - p(\overline{V_1}) \cdot p(\overline{V_2}) \cdot p(\overline{V_3}) \\
&= 1 - (1-0\cdot8)(1-0\cdot7)(1-0\cdot2) \\
&=
\end{aligned}
$$

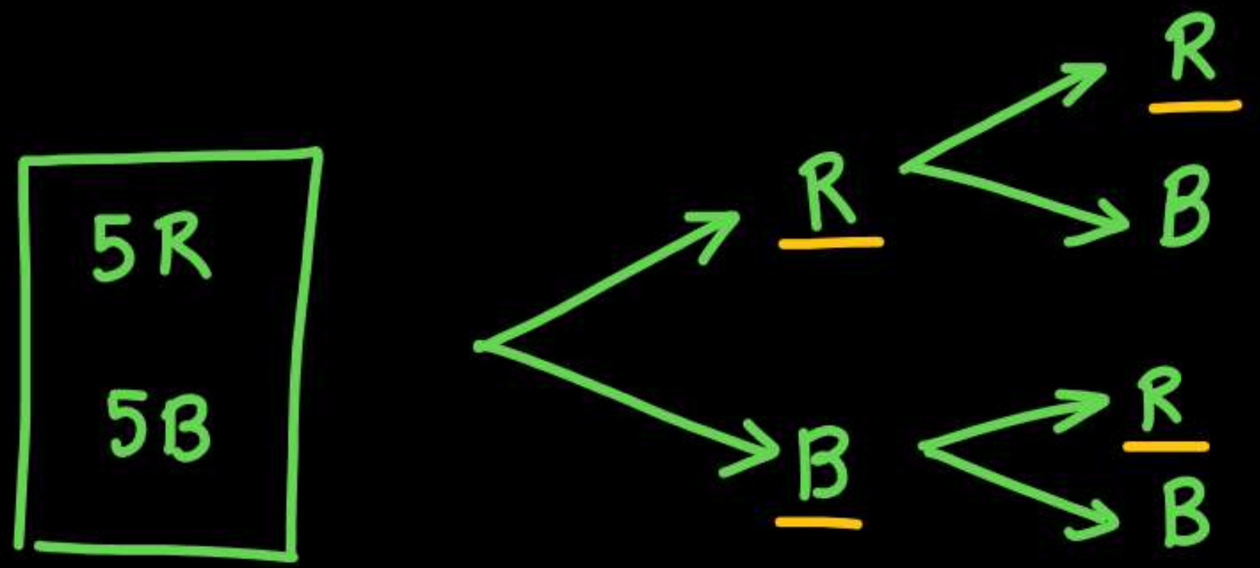
Q. An urn contains 5 red balls and 5 black balls. In the first draw, one ball is picked at random and discarded without noticing its colour. The probability to get a red ball in the second draw is

A. ½

B. 4/9

C. 5/9

D. 6/9



$$P(R \cap R) + P(B \cap R)$$

$$\frac{5}{10} \times \frac{4}{9} + \frac{5}{10} \times \frac{5}{9}$$

Q. The number of integers between 1 and 500(both inclusive) that are divisible by 3 or 5 or 7 is ___________.

$n(3) = 166$

$n(5) = 100$

$n(7) = 71$

$n(3 \cap 5) = 500/3\times5 = 33$

$n(5 \cap 7) = 500/5\times7 = 14$

$n(3 \cap 7) = 500/3\times7 = 23$

$n(3 \cap 5 \cap 7) = 500/3\times5\times7 = 4$

$n(3 \cup 5 \cup 7) =$

Q. A insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0.01,0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Q. Find the Rank of the word "SACHIN" = $6!$

→ Starting with A = $5! = 120$
→ Starting with C = $5! = 120$
→ Starting with H = $5! = 120$
→ " " I = $5! = 120$
→ " " N = $5! = 120$

Rank of SACHIN = 601

Q. Find rank of word 'VIKAS'.
'SANJAY'.

A CHINS	— 1
A CHISN	— 2
A	⋮
A	— 120
CAHINS	— 121
C ⋮	⋮
C	— 240
H ⋮	— 360
T ⋮	— 480
N ⋮	— 600
SACHIN	— 601

Q. A fair dice is tossed eight times. The probability that a third six is observed on the eighth throw is

$p = \frac{1}{6} \quad q = \frac{5}{6}$

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_}_{7 \text{ throws } (2 \text{ six})} \ \_$

3^{rd} six

8^{th} throw

$$\left\{ {}^7C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5 \right\} \times \frac{1}{6}$$

Random Experiment: An experiment whose outcomes are predicted with some certainity.

Random Variable: If we assign each outcome by some variable qty. X that qty. is Known as Random variable.

Ex:- Two coins are tossed together X denotes the number of head appearing.

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

$\{HH, HT, TH, TT\}$

(TT) (HT, TH) (HH)

X	0 Head	1 Head	2 Head
P(X)	¼	½	¼

Probability distribution

$P(X=0) = \frac{1}{4}$

$P(X=1) = \frac{2}{4}$

$P(X=2) = \frac{1}{4}$

Discrete Random Variable

→ A random variable that takes finite values

$$P(a) = P(X = a)$$ Probability mass function

→ Sum of probability mass function for various values of X is __1__.

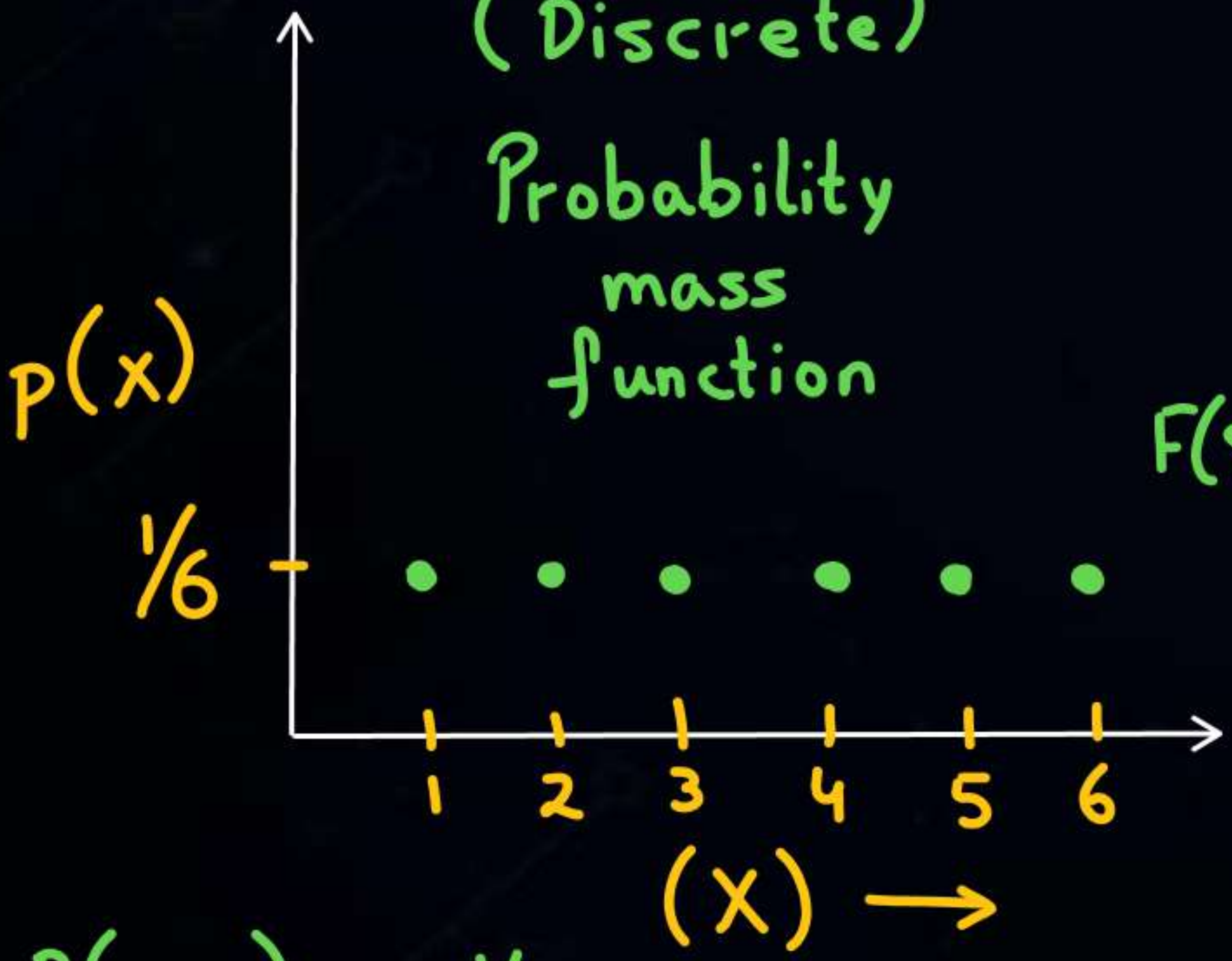
$$\boxed{\sum_{i=1}^{\infty} p(x=i) = 1}$$

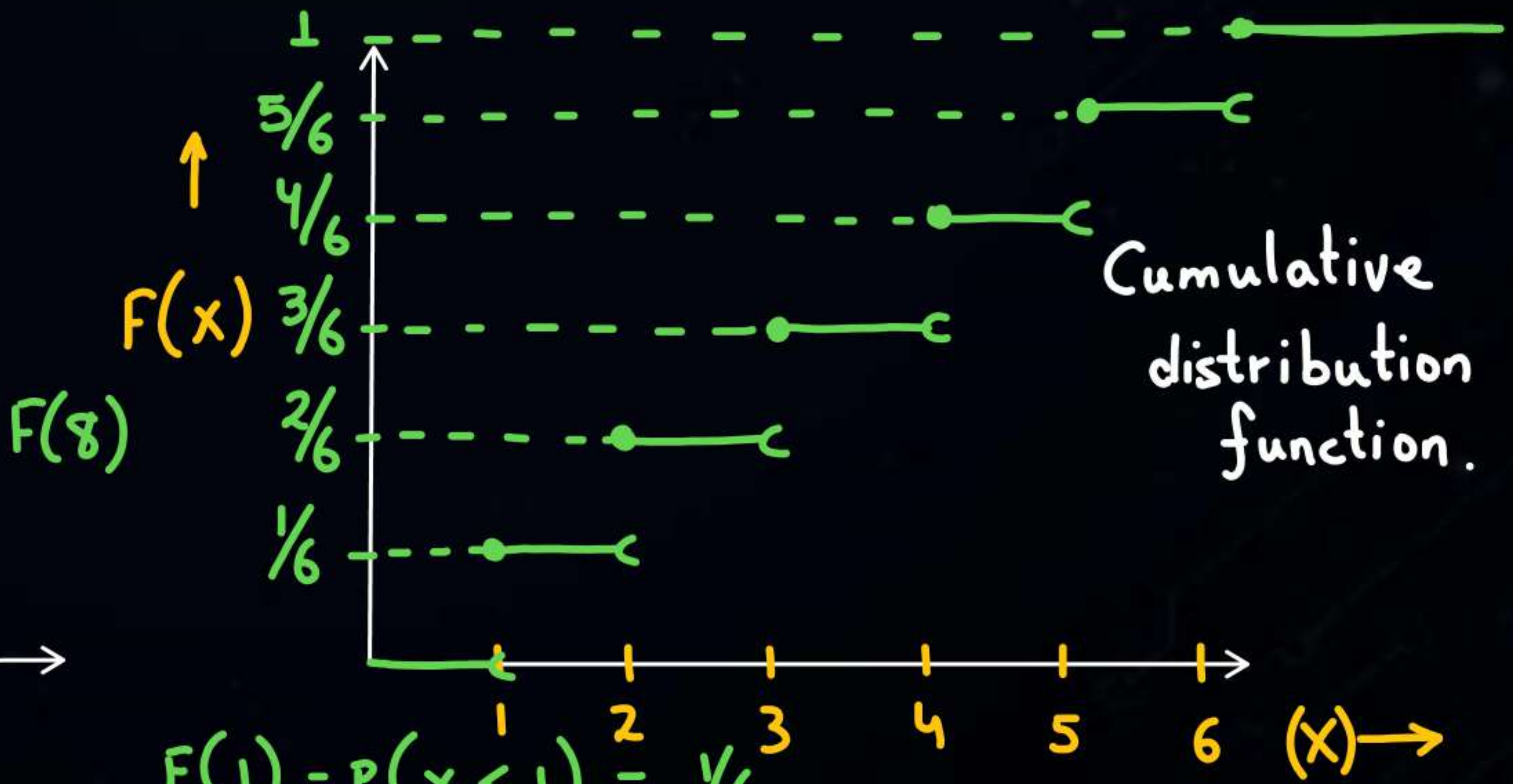
→ Cumulative distribution function (C.D.F.)

$$\boxed{F(a) = \sum_{i=a}^{x=a} p(x_i) = P(x \le a) \quad \text{for all } x_i \le a}$$

$X \rightarrow$ Outcome of dice.

X	1	2	3	4	5	6
$p(x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$





$P(x=1) \Rightarrow 1/6$

$P(x=2) \Rightarrow 1/6$

$F(1) = P(x \leq 1) = 1/6$

$F(2) = P(x \leq 2) = P(x=1) + P(x=2) = 2/6$

$F(3) = P(x \leq 3) = P(x=1) + P(x=2) + P(x=3) = 3/6$

[PROBABILITY BASICS]

Ex:- Consider a dice with property that the probability of a face with n dots showing up is proportional to n. The probability of the face with 3 dots showing up is _______.

Thank you

GW Soldiers !