ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-01

Differential equations

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Topics to be Covered

- DEFINITION & TYPES
- ORDER & DEGREE OF DE
- SOLUTION OF DE
- FORMATION OF DE
- WRONSKIAN & LD/LI SOLUTIONS
- METHODS OF SOLVING DE
- PARTIAL DIFFERENTIAL EQUATIONS

[STOKES' THEOREM]

Let $\vec{F}$ be a continuous vector fn. & has continuous first partial derivative in a region of space which S in its interior. S is open surface bounded by simple closed C. Then

Simple closed curve C $\rightarrow$ Open surface S

$$\boxed{\begin{aligned}\oint \vec{F}\cdot\vec{dr} &= \iint_S \text{curl } F \cdot \hat{n}\, dS \\ &= \iint_S (\vec{\nabla}\times\vec{F})\cdot\hat{n}\, dS\end{aligned}}$$

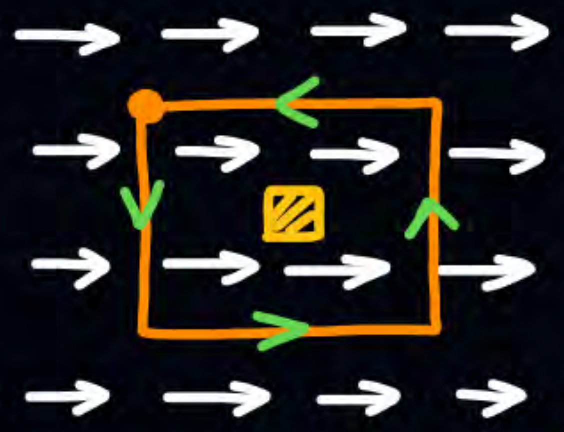
Vector form

$$\boxed{\oint F_1 dx + F_2 dy + F_3 dz = \iint \left(\frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z}\right)\hat{i} + \left(\frac{\partial F_1}{\partial z}-\frac{\partial F_3}{\partial x}\right)\hat{j} + \left(\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right)\hat{k}\cdot\hat{n}\, dS}$$

Cartesian form

STOKES' THEOREM

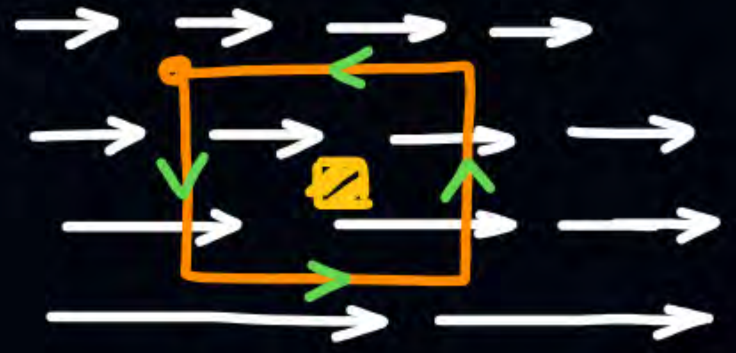
Note: It is applicable only when simple closed curve encloses open surface.



Irrotational field

curl $\vec{F} = 0$

$\oint F \cdot dr = 0$



Rotational field

curl $\vec{F} \neq 0$

$\oint F \cdot dr \neq 0$

GREEN'S THEOREM

Stokes theorem in plane is referred as Green's theorem.

Let $\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$, then Green's theorem

$$\boxed{\oint F \cdot \vec{dr} = \iint_S \text{curl}\,\vec{F} \cdot \hat{n}\, dS}$$

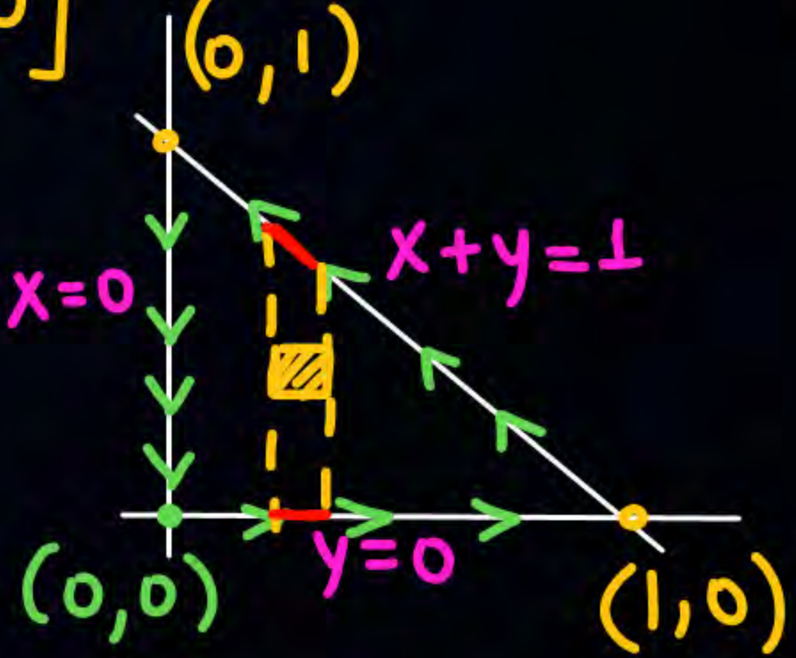
$$\oint F_1\,dx + F_2\,dy = \iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\hat{k} \cdot \hat{k}\, dx\,dy$$

$$\boxed{\oint F_1\,dx + F_2\,dy = \iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy}$$

S is any plane bounded by closed curve C.

Q. Find the volume of $\int_c (3x - 8y^2)\, dx + (4y - 6xy)dy$.
Where C is boundary of region bounded by x=0, y=0 &
x + y=1

Soln :- $\vec{F} = \underbrace{(3x-8y^2)}_{F_1}\hat{i} + \underbrace{(4y-6xy)}_{F_2}\hat{j}$ $[\text{curl } \vec{F} \neq 0]$



Apply Green's theorem ;

$$\int_c F_1 dx + F_2 dy = \iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$

$$= \int_{x=0}^{x=1}\int_{y=0}^{y=1-x} -6y - (-16y)\; dy\; dx$$

$x + y = 1$

$\oint \vec{F}\cdot\vec{dr}$

$$\int_0^1 10\left[\frac{y^2}{2}\right]_0^{1-x} dx = 5\int_0^1 (1-x)^2 dx = -5\left[\frac{(1-x)^3}{3}\right]_0^1$$

$$= 5/3$$

Q. If $\vec{F} = \underbrace{a}x\hat{\imath} + \underbrace{b}x\hat{\jmath} + \underbrace{c}z\hat{k}$. Find $\int_S \vec{F} \cdot \hat{n}\, ds$ **S is surface of unit sphere.**

$$\iint_S \vec{F}\cdot\hat{n}\, dS = \iiint_V \operatorname{div} F\, dV \qquad [\text{By G.D.T.}]$$

$$(a + 0 + c) \boxed{\iiint dV}$$

$$(a+c)\left[\frac{4}{3}\pi\right]$$

$$= \frac{4}{3}(a+c)\pi$$

$r = 1$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi\, 1^3$$

[DEFINTION & TYPES]

Differential Equations

→ An eqn. which consists of dependent variable, independent variable & differential coefficient of dependent variable w.r.t. independent variable.

$y \rightarrow$ Dependent variable

$x \rightarrow$ Independent variable

$$\frac{d^2y}{dx^2} + xy + y^2 = 0$$

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^2 + 6y = x^3$$

[DEFINTION & TYPES]

(ODE)

Ordinary Differential Equations: If there is only one independent variable in a D.E.

$y \to f(\underbrace{x}_{\text{Independent variable}})$

Ex:- $\frac{dy}{dx} + x = x^2$

Ex:- $5xy\,dy - x^3 y\,dx = 0$

$5xy\frac{dy}{dx} - x^3 y = 0$

(PDE)

Partial Differential Equations: If there is more than one independent variable in a D.E.

$z \to f(\underbrace{x, y}_{\text{Independent variable}})$

Ex:- $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial y}\right)^2 + 5y = z$

Order → The order of highest derivative in a D.E.

Degree → It is exponent/power of highest derivative when it is made free from fractional notations & radical signs.

① $$\frac{\sqrt{1+\left(\frac{dy}{dx}\right)^{7/2}}}{xy} = \frac{d^2y}{dx^2}$$

② $$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 6y = 5$$

Order = 2

Degree = 1

[ORDER & DEGREE]

Find order and degree of DE

1) $$y = x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^2$$

Order = 1

Degree = 2

[ORDER & DEGREE]

Find order and degree of DE

2) $$\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 0$$

$$\frac{d^2y}{dx^2} = -\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1+\left(\frac{dy}{dx}\right)^2$$

Order = 2

Degree = 2

Eqn. is first made free from roots

[ORDER & DEGREE]

Find order and degree of DE

3) $$S=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$S\,\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

$$S^2\left(\frac{d^2y}{dx^2}\right)^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{\cancel{2}}\times\cancel{2}}$$

Order = 2

Degree = 2

Find order and degree of DE

4) $\left(\frac{d^3y}{dx^3}\right)^4 - 6x^2\left(\frac{dy}{dx}\right)^8 = 0$

Order = 3
Degree = 4

ORDER & DEGREE

$y \to f(x)$

Ordinary Non- Linear Differential Equation :-

Any D.E. is said to be non-linear if

1) Degree is more than 1.
2) Exponent of dependent variable (i.e. y) is more than 1.
3) Exponent of any differential coefficient is more than 1.
4) Eqn. containing product of dependent variable & differential coefficient.

$yy', \quad yy'', \quad y^2y'$

Linear D.E. :- Any eqn. not following above properties is linear.

[ORDER & DEGREE]

Identify DE and find order & degree

1) $$x^2\left(\frac{d^2y}{dx^2}\right)^2 + x\frac{dy}{dx} + 2 = 0$$

(Non-linear) 1, 3

Order = 2
Degree = 2

2) $$\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + 2y + 5 = 0$$

(Non-linear) 3

Order = 2
Degree = 1

3) $$y''' + yy' + 2x^2 = 7$$

$y\frac{dy}{dx}$

(Non-linear) 4

Order = 3
Degree = 1

[ORDER & DEGREE]

Identify DE and find order & degree

4) $y''' + (y'')^2 + y^2 x = \sin x$ (Non-linear) ② , ③

Order = 3

Degree = 1

5) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} = \log x$ Linear

Order = 3

Degree = 1

SOLUTION OF DIFFERENTIAL EQUATION

Soln → Relation b/w dependent & independent variable which satisfies the given D.E.

Types of Solution

1) General Solution: -

If a solution contains same no. of arbitrary constants as the order of D.E.

Ex:- $x^2 = t + c$

$2x \frac{dx}{dt} = 1$

$\boxed{\text{Order} = 1}$

Degree = 1

Non-Linear

Ex:- $y = C_1 \cos 5x + C_2 \sin 5x$

↳ Soln. of 2nd order D.E.

Types of Solution

2) Particular Solution:-

If we assign a particular value to arbitrary constants using initial & boundary conditions.

Ex:- $x^2 = t + 1$

Ex:- $y = -\cos 5x + 6 \sin 5x$

FORMATION OF DIFFERENTIAL EQUATION

General Solution $\longrightarrow$ Differential Eqn.
(Some arbitrary constants)

By eliminating the given number of arbitrary constants from general solution, we can obtain differential eqn.

FORMATION OF DIFFERENTIAL EQUATION

Ex:- Find DE of solution $y = e^x (A\cos x + B\sin x)$

General soln.
(A, B)

$$y' = e^x(-A\sin x + B\cos x) + e^x(A\cos x + B\sin x)$$

$$y' = e^x(-A\sin x + B\cos x) + y$$

$$y'' = e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x) + y'$$

$$y'' = -e^x(A\cos x + B\sin x) + y' - y + y'$$

$$y'' = -y + 2y' - y$$

$$\boxed{y'' - 2y' + 2y = 0}$$

$$\boxed{\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0}$$

Order = 2

Ex:- Find DE for Family of all parabolas $y^2 = 4a(x+a)$

$$y^2 = 4a(x+a)$$

General soln.

(a)

$$2y\frac{dy}{dx} = 4a$$

$$a = \frac{2y}{4}y' = \frac{yy'}{2}$$

$$y^2 = 4\frac{yy'}{2}\left(x + \frac{yy'}{2}\right)$$

D.E. Order = 1

Thank you

GW

Soldiers !