ENGINEERING MATHEMATICS

ALL BRANCHES

Properties of Eigen Values & Vectors

Linear Algebra

DPP-09 Solution

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Question 1

The eigen values of the matrix $\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -1 & 4 \end{bmatrix}$ are

A $2, -2, 1, -1$

B $2, 3, -2, 4$

C $2, 3, 1, 4$

D None

Characterstic eqn. $|A-\lambda I|=0$

$$\begin{vmatrix} 2-\lambda & -1 & 0 & 0 \\ 0 & 3-\lambda & 0 & 0 \\ 0 & 0 & -2-\lambda & 0 \\ 0 & 0 & -1 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\begin{vmatrix} 3-\lambda & 0 & 0 \\ 0 & -2-\lambda & 0 \\ 0 & -1 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda)\{(-2-\lambda)(4-\lambda)\} = 0$$

$$\lambda = 2, 3, -2, 4$$

Question 2

The necessary condition to diagonalize a matrix is that

A its all eigen values should be distinct

B its eigen vectors should be independent

C its eigen values should be real

D the matrix is non-singular

$$D = P^{-1} A P$$

Modal matrix

$P = [X_1 \; X_2 \; X_3]$

$X_1, X_2, X_3 \rightarrow$ Eigen vectors

Necessary cond^n.
- $A \rightarrow$ square matrix
- $|A| \neq 0$, Non-singular

Sufficient cond^n.
- $A_{n \times n}$ is diagonalizable if and only if it has n linearly independent eigen vectors.

Question 3

Obtain the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 34 & 49 \\ 0 & 2 & 43 & 94 \\ 0 & 0 & -2 & 104 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

A $1, 2, -2, -1$

B $-1, -2, -1, -2$

Since it is UTM $\therefore$ eigen values are diagonal elements.

C $1, 2, 2, 1$

D None

Question 4

For the matrix $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Since it UTM;
$\therefore$ Eigen values $\rightarrow 3, -2, 1$

one of the eigen value is equal to –2. Which of the following is an eigen vector?

A $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

B $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

C $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

D $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$ ✓

Eigen value problem $(A-\lambda I)X = 0$

$$\begin{bmatrix} 3-\lambda & -2 & 2 \\ 0 & -2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix} X = 0$$

let $\vec{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

For $\lambda = -2$

$$\begin{bmatrix} 5 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 5x - 2y + 2z = 0 \quad -1)$

$\Rightarrow z = 0 \quad -2)$

Let $x = K$, $y = \frac{5}{2}K$ and $z = 0$

Eigen vector $\begin{bmatrix} K \\ \frac{5}{2}K \\ 0 \end{bmatrix}$ for $K = 2$; $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

Question 5

The minimum and the maximum eigen values of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

are –2 and 6, respectively. What is the other eigen value?

A 5

B 3

C 1

D –1

Sum of eigen values = Trace of matrix

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 + 5 + 1$$

$$(-2) + (6) + \lambda_3 = 7$$

$$\boxed{\lambda_3 = 3}$$

Question 6

Given that

$$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

the value A^3 is

A $15A + 12I$

B $19A + 30I$

C $17A + 15I$

D $17A + 21I$

$$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$$

Charecterstic eqn. $|A - \lambda I| = 0$

$$\begin{vmatrix} -5-\lambda & -3 \\ 2 & 0-\lambda \end{vmatrix} = 0$$

$$5\lambda + \lambda^2 - (-6) = 0$$

$$\lambda^2 + 5\lambda + 6 = 0$$

By Cayley Hamilton Theorem $\longrightarrow$ $A^2 + 5A + 6I = 0$

$$\Rightarrow A^2 = -5A - 6I$$

$$A^3 = A^2 \cdot A = (-5A - 6I)A = -5A^2 - 6A$$

$$= -5(-5A - 6I) - 6A = 25A + 30I - 6A$$

$$A^3 = 19A + 30I$$

Question 7

Suppose that the eigen values of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is $\underline{\ 1/8\ }$ (0.125).

$$A \rightarrow \lambda_1, \lambda_2, \lambda_3$$
$$\rightarrow 1, 2, 4$$

Theorem :- $|A|$ = Product of eigen values

$$|A| = 1 \times 2 \times 4 = 8$$

$$|(A^{-1})^T| = |A^{-1}| = \frac{1}{|A|} = \frac{1}{8}$$

Question 8

Let

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

and $B = A^3 - A^2 - 4A + 5I$, where I is the 3×3 identity matrix. The determinant of B is $\underline{\;1.0\;}$ (up to 1 decimal place).

Characterstic eqn. $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)(2-\lambda)(-2-\lambda) - 1(0) = 0$$

$$\Rightarrow (\lambda^2 - 3\lambda + 2)(-2-\lambda) = 0$$

$$-2\lambda^2+6\lambda-4-\lambda^3+3\lambda^2-2\lambda=0$$

$$\lambda^3-\lambda^2-4\lambda+4=0$$

By C.H.T.

$\lambda \to A$

$$A^3-A^2-4A+4I=0$$

$$B=A^3-A^2-4A+5I$$

$$B=(A^3-A^2-4A+4I)+I$$

$$B=0+I$$

$$B=I$$

$$|B|=|I|$$

$$|B|=1$$

Thank you

GW

Soldiers !