CS & IT ENGINEERING

Data Structure

Arrays

Chapter -2

Lec- 04

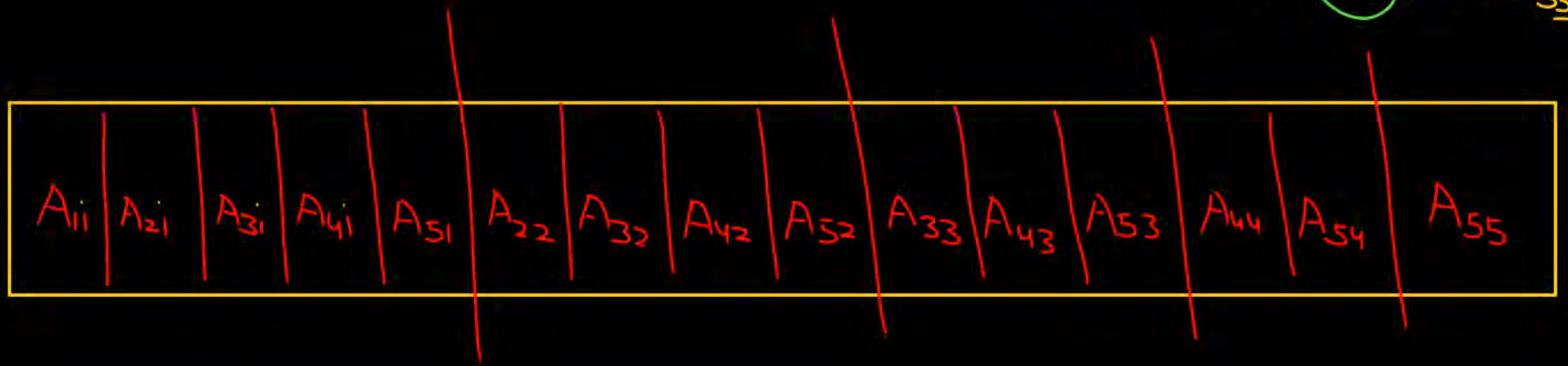
By- Pankaj Sharma sir

Arrays-4

Lower Triangular Matrix

add (a_{53})

	1	2	3	4	5
1	A_{11}	0	0	0	0
2	A_{21}	A_{22}	0	0	0
3	A_{31}	A_{32}	A_{33}	0	0
4	A_{41}	A_{42}	A_{43}	A_{44}	0
5	A_{51}	A_{52}	A_{53}	A_{54}	A_{55}



Lower Triangular Matrix

add (a_{53})

within 3rd col, ele before (a_{53})

= 5 - 3

= 2

cols already filled = col1, col2

(5 + 4) = 9

Total 11 elements are filled before (a_{53})

	1	2	3	4	5
1	A_{11}	0	0	0	0
2	A_{21}	A_{22}	0	0	0
3	A_{31}	A_{32}	A_{33}	0	0
4	A_{41}	A_{42}	A_{43}	A_{44}	0
5	A_{51}	A_{52}	A_{53}	A_{54}	A_{55}

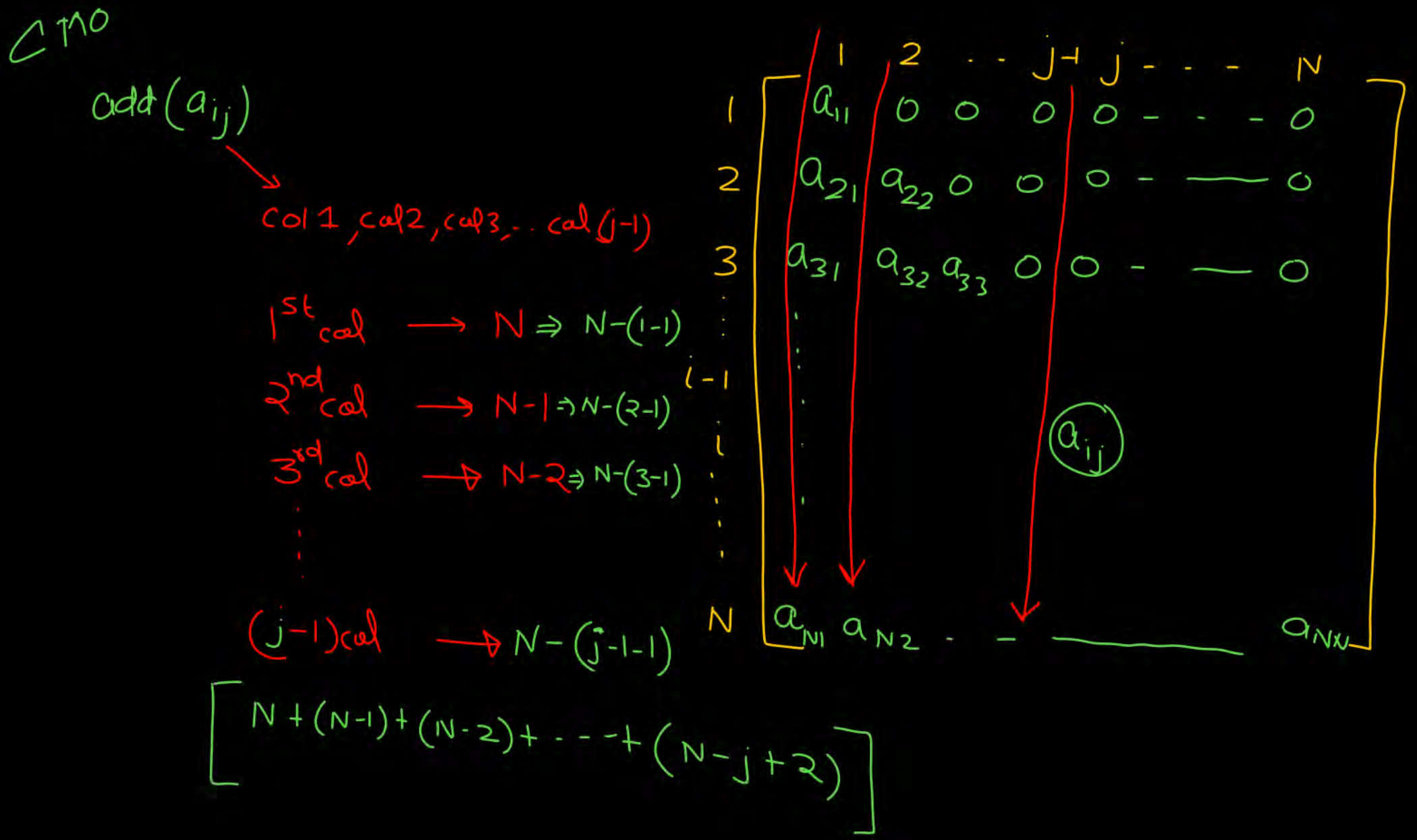
A_{ii} → 0

$A_{i+1,i}$

$A_{i+2,i}$

$A_{i+3,i}$

A_{11}	A_{21}	A_{31}	A_{41}	A_{51}	A_{22}	A_{32}	A_{42}	A_{52}	A_{33}	A_{43}	A_{53}	A_{44}	A_{54}	A_{55}


add(a_{ij})
col 1, col 2, col 3, . . col (j-1)
1st col → N ⇒ N-(1-1)
2nd col → N-1 ⇒ N-(2-1)
3rd col → N-2 ⇒ N-(3-1)
(j-1) col → N-(j-1-1)
1 2 . . j-1 j . . . N
1 a_{11} 0 0 0 0 . . . 0
2 a_{21} a_{22} 0 0 0 . . . 0
3 a_{31} a_{32} a_{33} 0 0 . . . 0
i-1
i
N a_{N1} a_{N2} . . . a_{NN}
a_{ij}
[N + (N-1) + (N-2) + . . . + (N-j+2)]

$S_n = \frac{n}{2}(a+l)$

C110

add(a_{ij})

col 1, col 2, col 3, .. col (j-1)

1st col → N ⇒ N-(1-1)

2nd col → N-1 ⇒ N-(2-1)

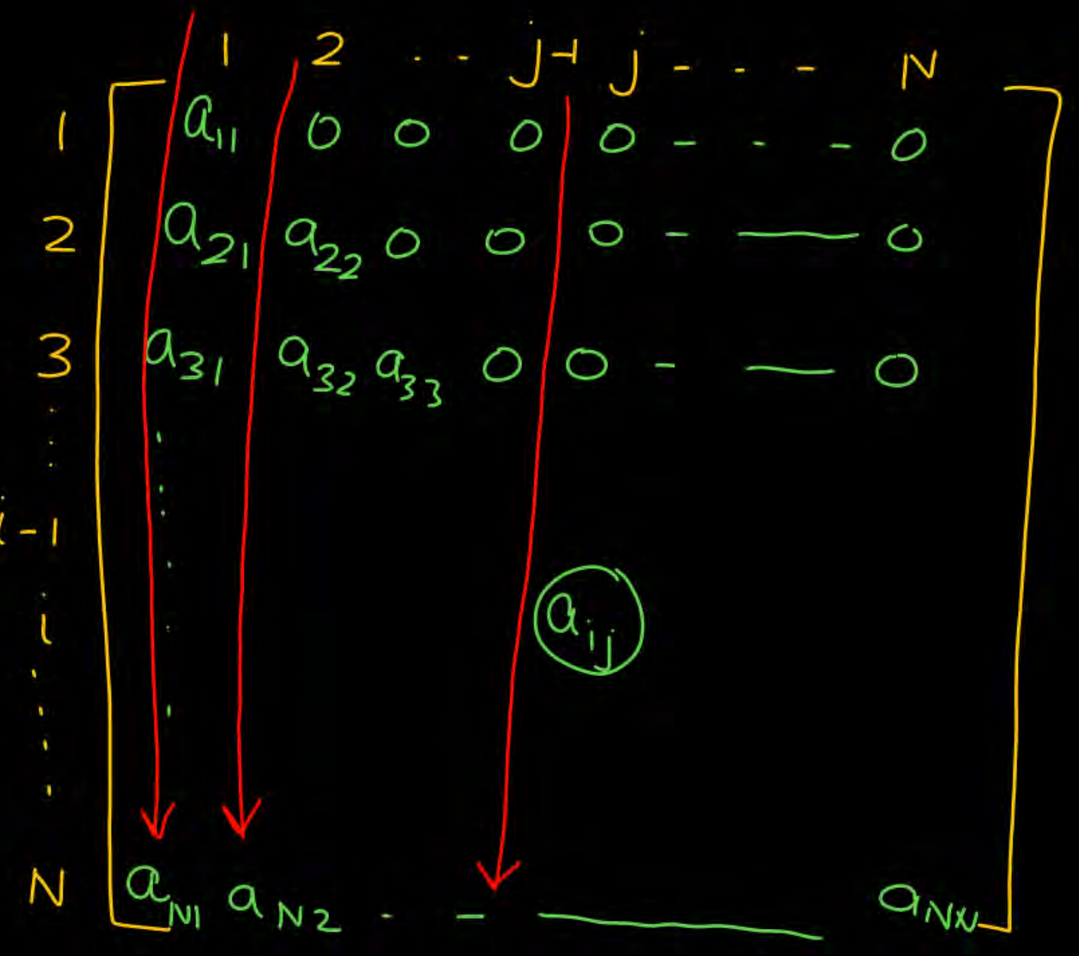
3rd col → N-2 ⇒ N-(3-1)

(j-1) col → N-(j-1-1)

$\frac{(j-1)}{2}\left[N + N-(j-2)\right]$

$\left[N + (N-1) + (N-2) + \cdots + (N-j+2)\right]$

=



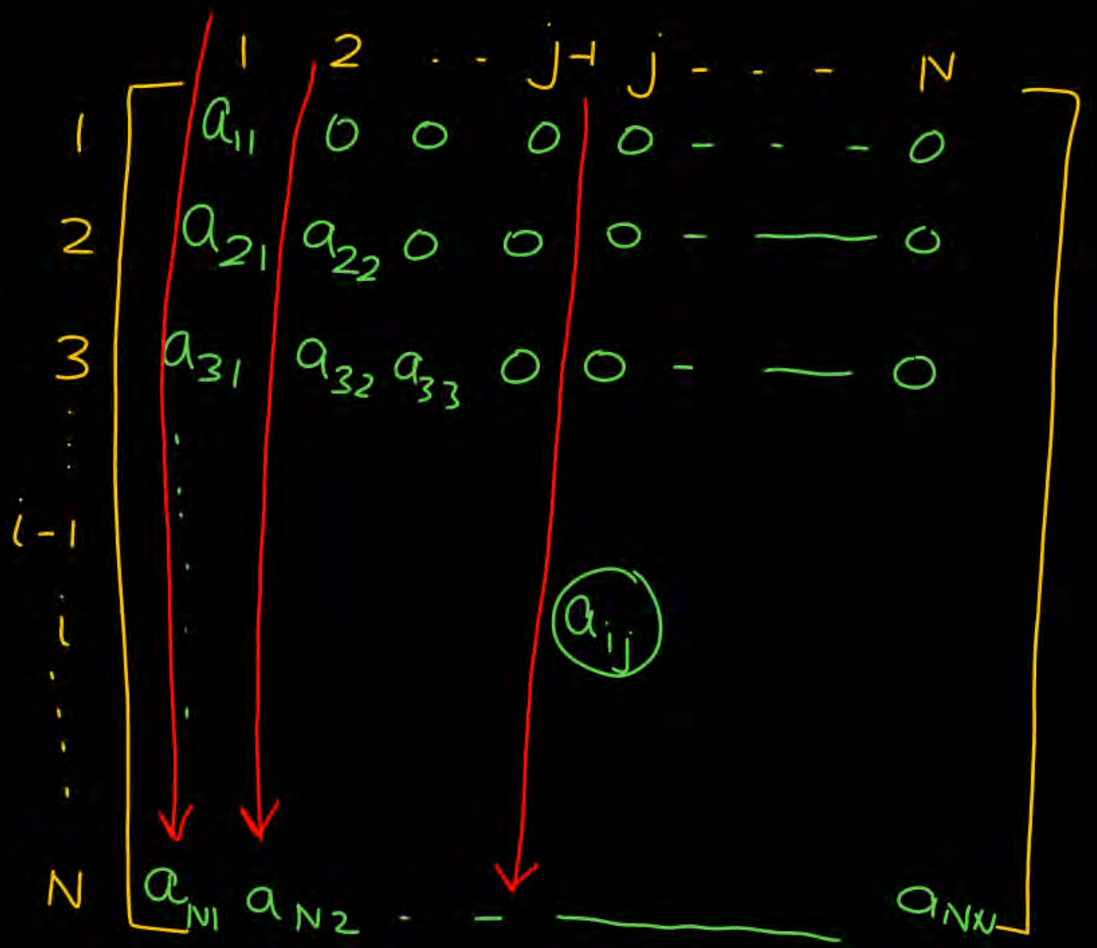
$S_n = \frac{n}{2}(a+l)$

$\subset$ TMO

add(a_{ij})

col 1, col 2, col 3, .. col (j-1)

$$\frac{(j-1)}{2}\left[N + N - (j-2)\right]$$



Within jth col, ele before A_{ij}

$\Rightarrow (i-j)$

Total elem. already filled before a_{ij} $= (i-j) + \frac{(j-1)}{2}\left[2N - (j-2)\right]$

$= (i-j) + (j-1).N - \frac{(j-1)(j-2)}{2}$

CMO

$S_n = \frac{n}{2}(a + l)$

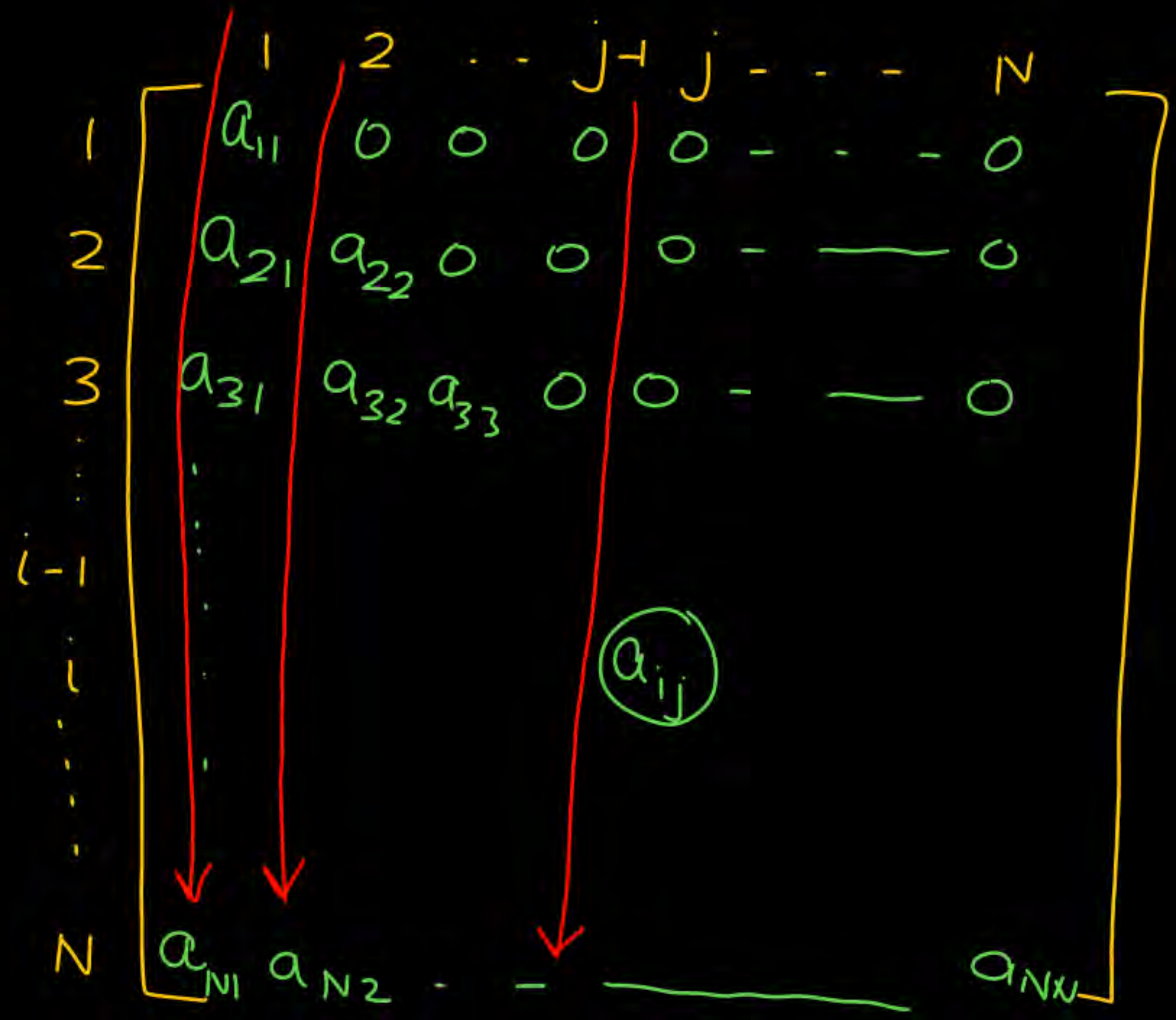
add(a_{ij})

col 1, col2, col3, .. col(j-1)

⇓

$$\frac{(j-1)}{2}\left[N + N - (j-2)\right]$$

रटना नहीं है



within j^{th} col, ele before A_{ij}

$\Rightarrow (i-j)$

Total elem. already filled before $a_{ij} = (i-j) + \frac{(j-1)}{2}\left[2N - (j-2)\right]$

$$= (i-j) + (j-1).N - \frac{(j-1)(j-2)}{2}$$

9

LTM A[1..100][1..100]

CMO

BA = 1000

w = 2 byte

add (A[50][43])

1 — 100 100 − (1−1)

2 — 99 100 − (2−1)

⋮

42 — 100 − (42−1)

100 − 41

59

cal filled 1, 2, 3, ... 42

(100 + 99 + 98 + ... 59)

$\frac{42}{2}[100 + 59] = 21 \times 159$

= 3339 elements

Within Target cal ⇒ (50, 43)

= 7 elements

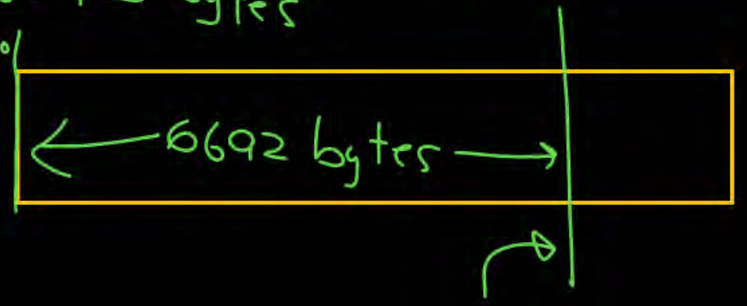
159 × 21 = 159 + 3180 = 3339

Total ele already filled = 3339 + 7 = 3346 elements

Memory already = 3346 × 2 = 6692 bytes

add(A[50,43]) = 1000 + 6692

= 7692



20 - (-20) + 1 = 41

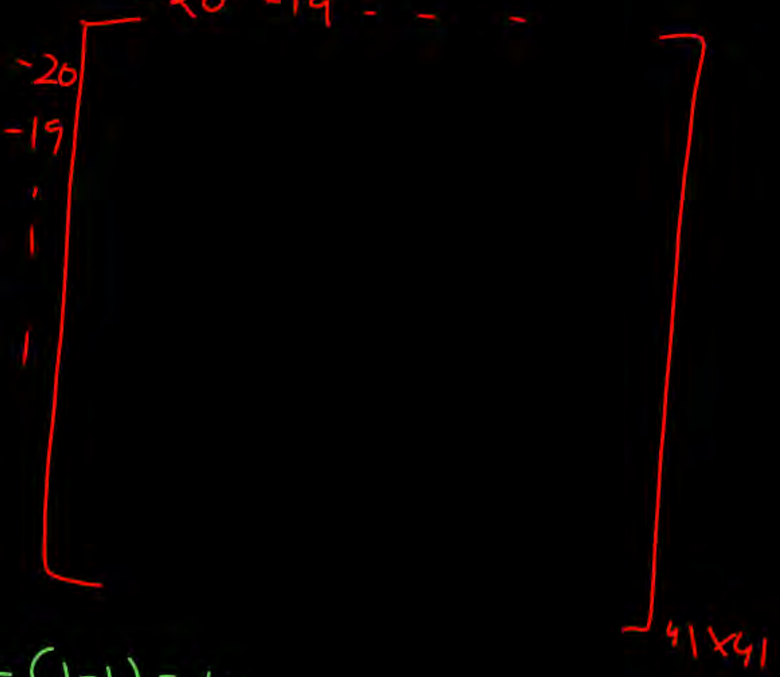
41

Q

LTM A[-20..20][-20..20]

CMO, w = 1 byte BA = 1000

add(A[-2][-6])



within (-6) index col

-2 - (-6)
= -2 + 6
= 4 elem.

-7 - (-20) + 1
-7 + 21
14 cols

1st → 41 - (1-1) = 41
2nd → 41 - (2-1) = 40
⋮
14th → 41 - (14-1) = 41 - 13 = 28

Total ⇒ 487 elements [41 + 40 + ... - 28]
= 14/2 (41 + 28) = 7 x 69 = 483 elem.

Memory = 487 x 1 = 487 bytes

Final ans = 1000 + 487
= 1487

Q LTM

A[-3..3][-5..1]

-3-(-5)=2 घटाना पड़ेगा

w = 2 bytes

BA = 1000

add($a_{3,-1}$)

Ans 1048

-5 to -2

-2-(-5)+1

-2+6=4 col

7+6+5+4

22 elem

within col index -1

⇒ 3-(-1)=4

formula निपटा देगा

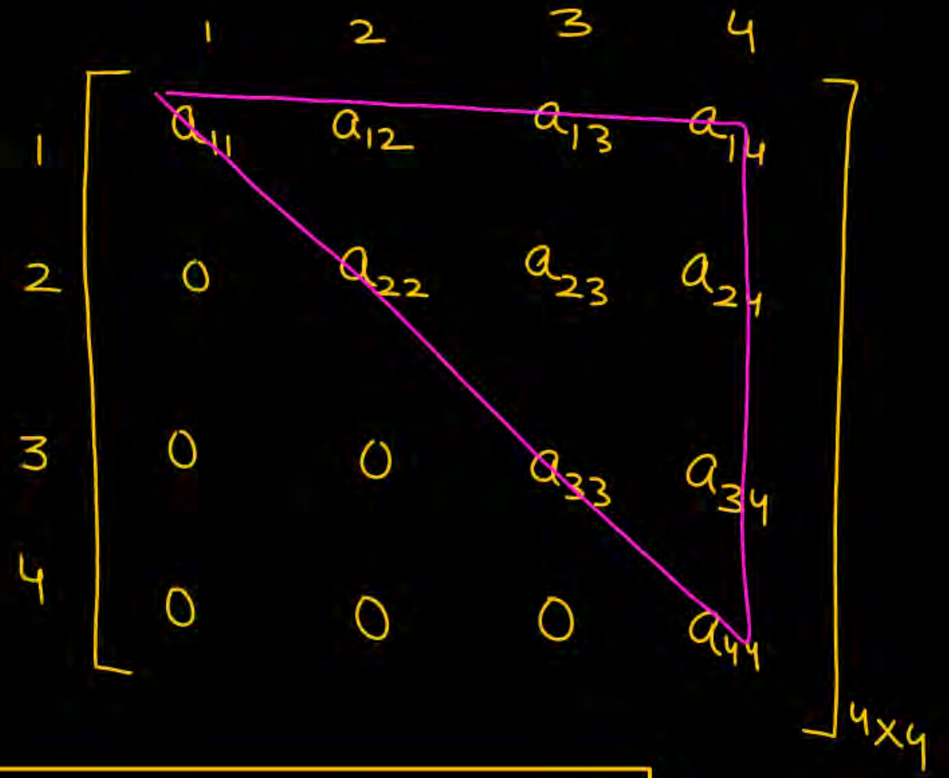
4-2 = 2

	-5	-4	-3	-2	-1	0	1
-3	X	0	0	0	0	0	0
-2	X	X	0	0	0	0	0
-1	X	X	X	0	0	0	0
0	X	X	X	X	0	0	0
1	X	X	X	X	X	0	0
2	X	X	X	X	X	X	0
3	X	X	X	X	X	X	X

7x7

RMO

Upper triangular Matrix



Upper triangular Matrix

RMO

add(a_{34})

#of ele before(a_{34})

	1	2	3	4
1	a_{11}	a_{12}	a_{13}	a_{14}
2	0	a_{22}	a_{23}	a_{24}
3	0	0	a_{33}	a_{34}
4	0	0	0	a_{44}

4x4

Upper triangular Matrix

RMO

a_{34}

row1, row2 → (4 + 3) = 7 elements

within row (with index 3) → elements before a_{34} → (4-3) = 1 elem

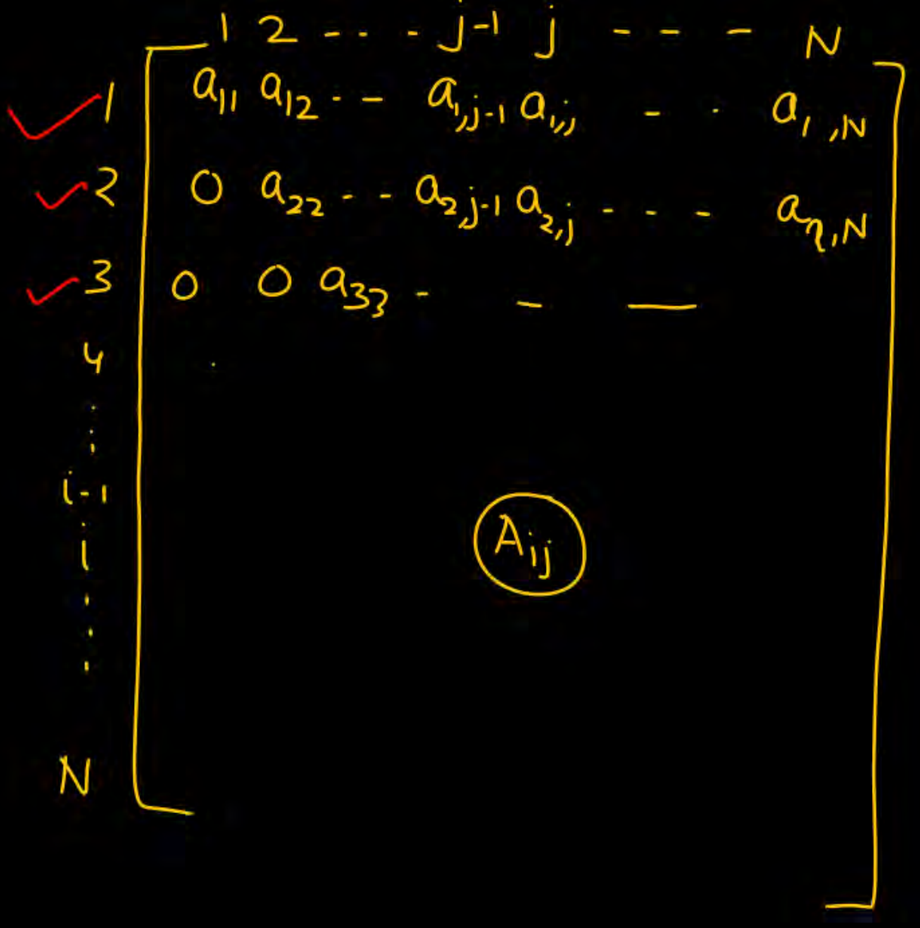
Total ele before a_{34} = 8 elements

	1	2	3	4
1	a_{11}	a_{12}	a_{13}	a_{14}
2	0	a_{22}	a_{23}	a_{24}
3	0	0	a_{33}	a_{34}
4	0	0	0	a_{44}

4x4

a_{11}	a_{12}	a_{13}	a_{14}	a_{22}	a_{23}	a_{24}	a_{33}	a_{34}	a_{44}

add (A_{ij})



rows already filled

$1, 2, 3, \ldots (i-1)$

1^{st} row $\longrightarrow N \Rightarrow N-(1-1)$

2^{nd} row $\longrightarrow (N-1) \Rightarrow N-(2-1)$

3^{rd} row $\longrightarrow (N-2) \Rightarrow N-(3-1)$

$\vdots$

$(i-1)^{th}$ row $\Rightarrow N-(i-1-1)$

$$N+(N-1)+(N-2)+\cdots+(N-i+2)$$

$$= \frac{(i-1)}{2}\left[N+(N-i+2)\right] = \frac{i-1}{2}\left[2N-(i-2)\right] = \boxed{(i-1)\cdot N - \frac{(i-1)(i-2)}{2}}$$

UTM in RMO

add (a_{ij})

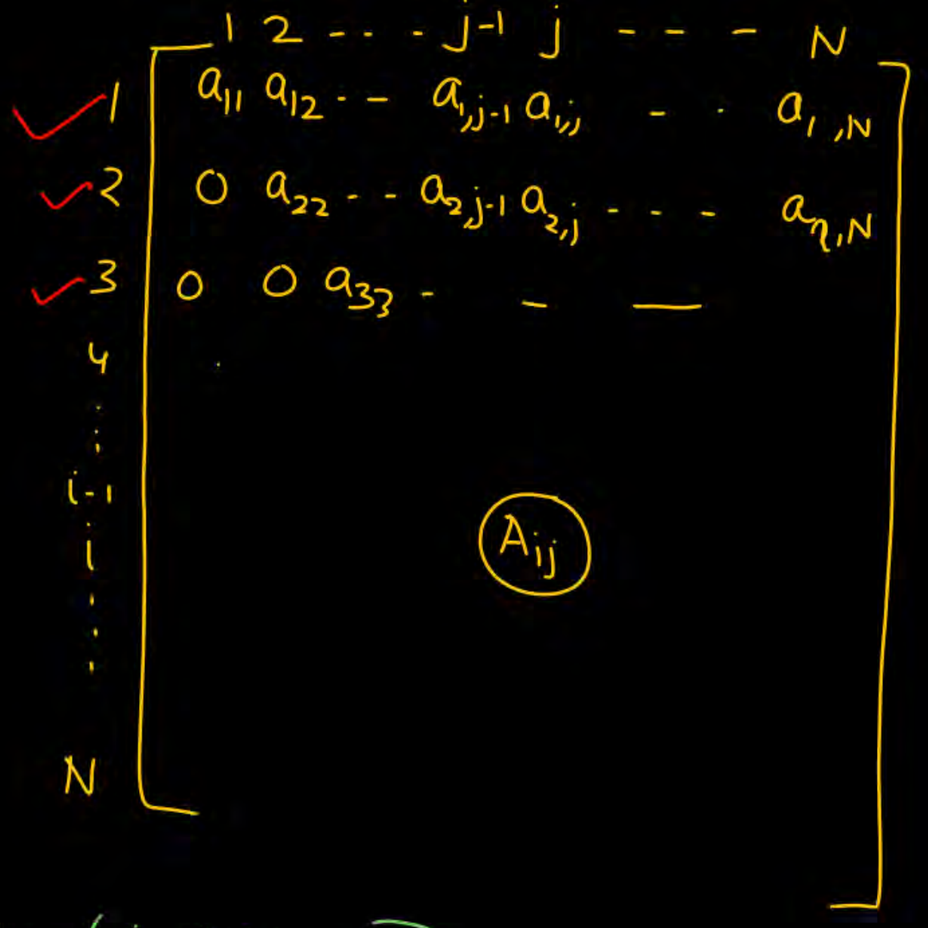
rows already filled
$1, 2, 3, \ldots (i-1)$

within i^{th} row ele before a_{ij}
$= (j-i)$

$$\#\text{ of ele} = \boxed{(i-1)\cdot N - \frac{(i-1)(i-2)}{2}}$$

Total ele already filled before a_{ij}

$$= (j-1) + \left[(i-1)N - \frac{(i-1)(i-2)}{2}\right]$$



UTM in RMO

add (a_{ij})

22 ना नहीं है

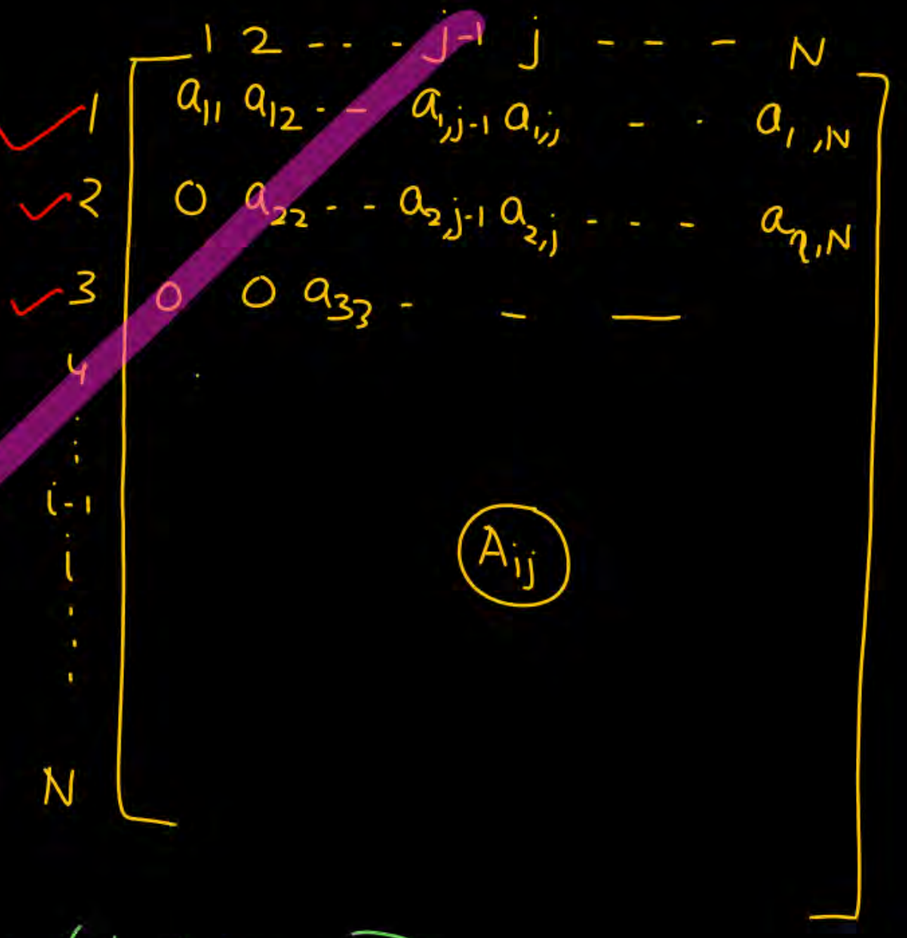
rows already filled
1, 2, 3, ... (i-1)

within ith row ele before a_{ij}
$= (j-i)$

# of ele $= \boxed{(i-1)\cdot N - \frac{(i-1)(i-2)}{2}}$

Total ele already filled before a_{ij}

$$= (j-1) + \left[(i-1)N - \frac{(i-1)(i-2)}{2}\right]$$



Q RMO

UTM $A[-12..12][-12..12]$

$w = 2$ bytes, $BA = 1000$

add($A_{0,3}$)

25×25

rows already filled

-12 to -1

$= -1-(-12)+1 = 12$ rows

1st 2nd ... 12th row

↓

25 24 ... 14

$(25+24+\cdots+14)$

$= \frac{12}{2}[25+14] = 6 \times 39$

$= 234$

within 0^{th} index row

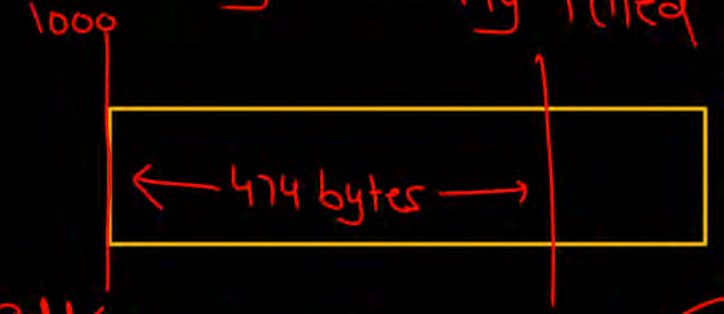
ele before $A_{0,3}$

$= 3-0 = 3$ elements

Total ele already filled = 237 elements

Memory already filled = 237×2

= 474 bytes



add($A_{0,3}$) = 1000 + 474 = 1474

Upper triangular Matrix

(CMO)

add(a_{34})

?

	1	2	3	4
1	a_{11}	a_{12}	a_{13}	a_{14}
2	0	a_{22}	a_{23}	a_{24}
3	0	0	a_{33}	a_{34}
4	0	0	0	a_{44}

4x4

a_{11}	a_{12}	a_{22}	a_{13}	a_{23}	a_{33}	a_{14}	a_{24}	a_{34}	a_{44}

Upper triangular Matrix

(CMO)

add(a_{34})

cols already filled
Col 1, col 2, col 3

1 + 2 + 3

6 elements

Within 4th col
ele already
filled before a_{34}

= 2 ele ⇒ Total = 8 elements

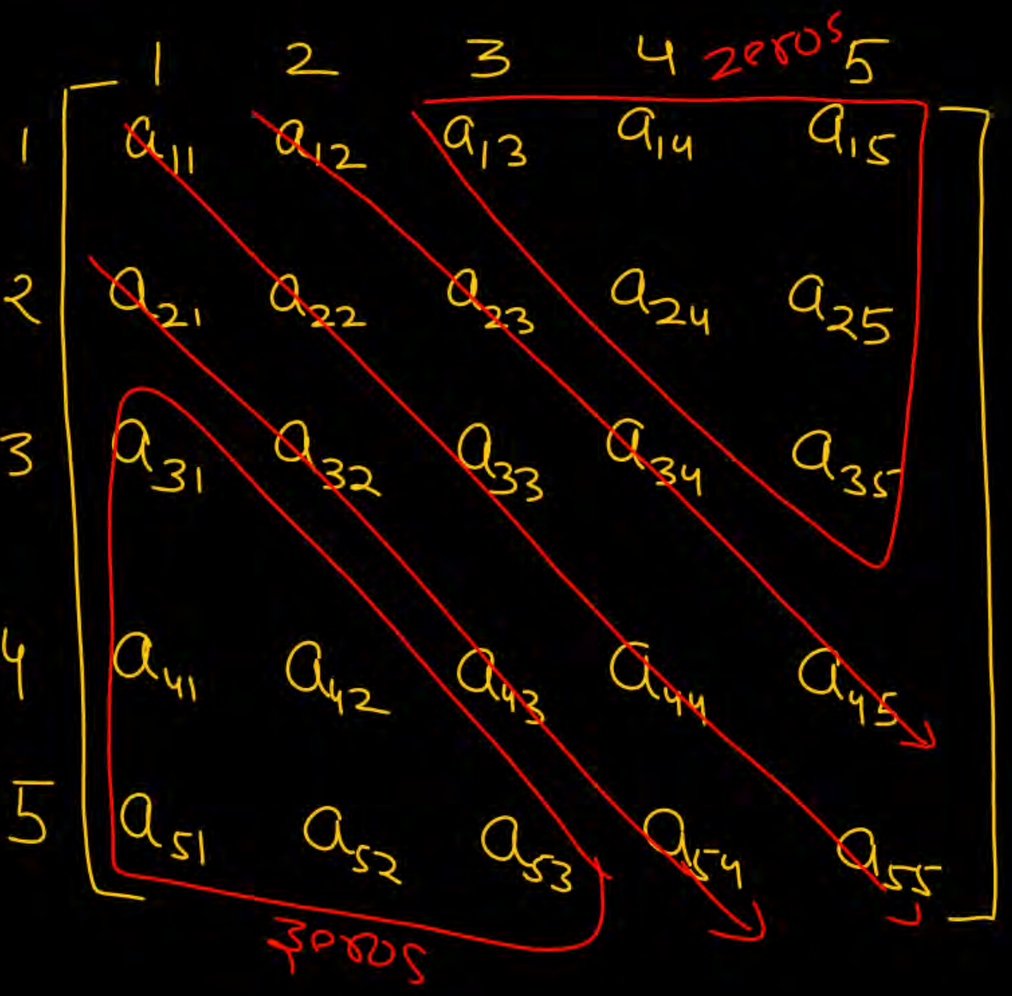
	1	2	3	4
1	a_{11}	a_{12}	a_{13}	a_{14}
2	0	a_{22}	a_{23}	a_{24}
3	0	0	a_{33}	a_{34}
4	0	0	0	a_{44}

4x4

1,2									
a_{11}	a_{12}	a_{22}	a_{13}	a_{23}	a_{33}	a_{14}	a_{24}	a_{34}	a_{44}

Tri-diagonal Matrix

1. Square matrix
2. Main diagonal
   diagonal just above main diagonal
   diagonal just before main diagonal



Tri-diagonal Matrix

1. Square matrix
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   diagonal just above main diagonal
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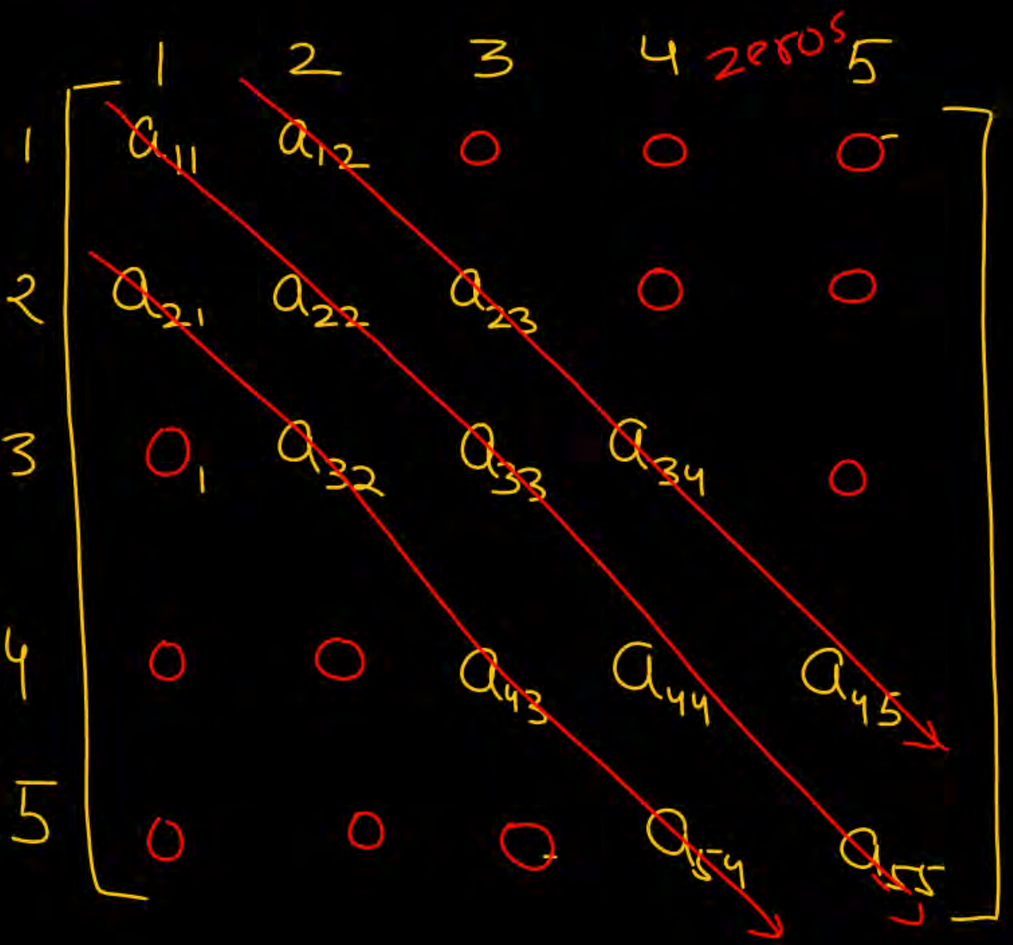


Tri-diagonal Matrix

of elements in 1st row = 2

" " " last " = 2

" " " other rows = 3

$n \times n$ Tri-diagonal matrix,

total no. of elements

= 1st + last + (n-2) rows

= 2 + 2 + 3(n-2)

= 4 + 3n - 6

 # no. of elements = 3n-2

$n = 5$

(13)

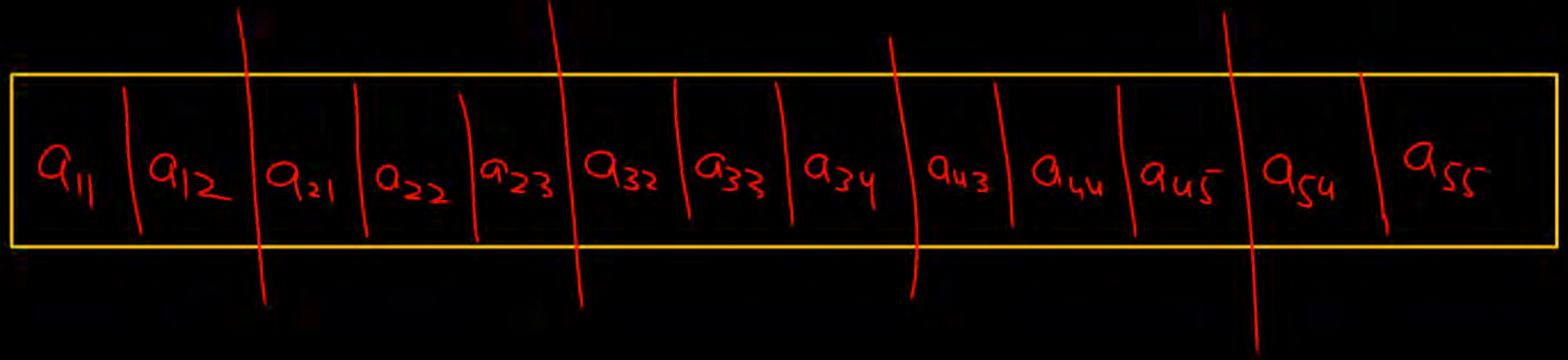
	1	2	3	4	zeros 5
1	a_{11}	a_{12}	0	0	0
2	a_{21}	a_{22}	a_{23}	0	0
3	0	a_{32}	a_{33}	a_{34}	0
4	0	0	a_{43}	a_{44}	a_{45}
5	0	0	0	a_{54}	a_{55}

Tri-diagonal Matrix

RMO

add(a_{45})

	1	2	3	4	5 (zeros)
1	a_{11}	a_{12}	0	0	0
2	a_{21}	a_{22}	a_{23}	0	0
3	0	a_{32}	a_{33}	a_{34}	0
4	0	0	a_{43}	a_{44}	a_{45}
5	0	0	0	a_{54}	a_{55}



Tri-diagonal Matrix

Total elem. before a_{45} = 10 elements

RMO

add(a_{45})

3 rows already filled

Out of these 3

1st row ⇒ 2
2nd row ⇒ 3
3rd row ⇒ 3

} 8 elements

within 4th row ele, before a_{45} (5-4+1) = 2

	1	2	3	4	zeros 5
1	a_{11}	a_{12}	0	0	0
2	a_{21}	a_{22}	a_{23}	0	0
3	0	a_{32}	a_{33}	a_{34}	0
4	0	0	a_{43}	a_{44}	a_{45}
5	0	0	0	a_{54}	a_{55}

5-4+1

a_{11}	a_{12}	a_{21}	a_{22}	a_{23}	a_{32}	a_{33}	a_{34}	a_{43}	a_{44}	a_{45}	a_{54}	a_{55}

Tri-diagonal Matrix

RMO

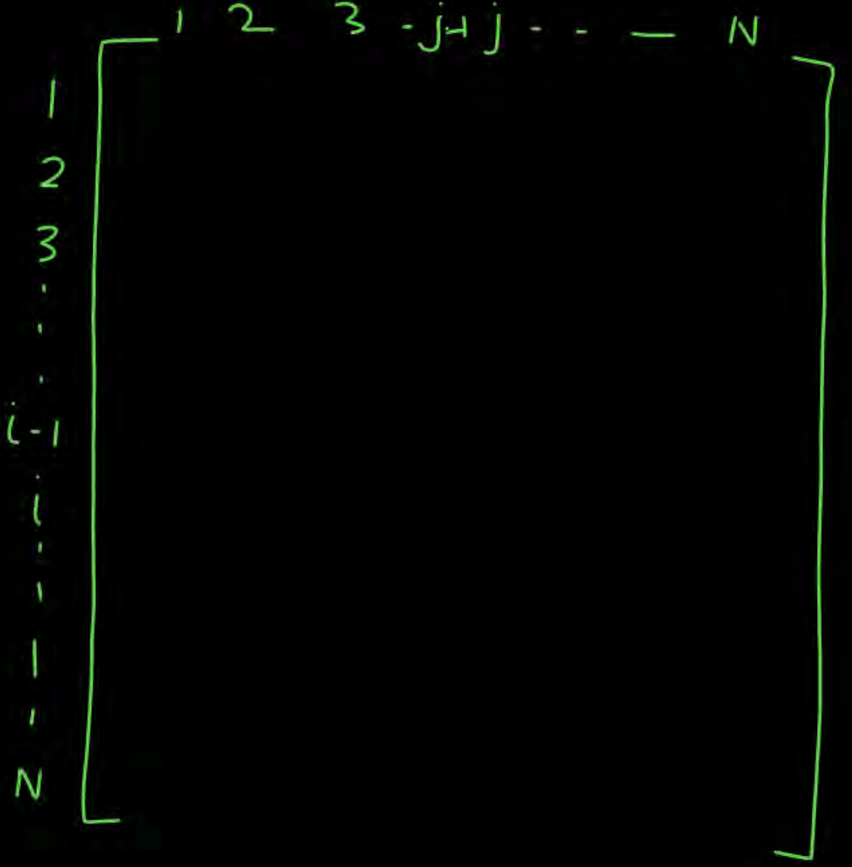
add(a_{ij})

rows already filled
1, 2, - - i-1
(i-1) rows

1st row ⇒ 2
rem. (i-2) rows ⇒ 3

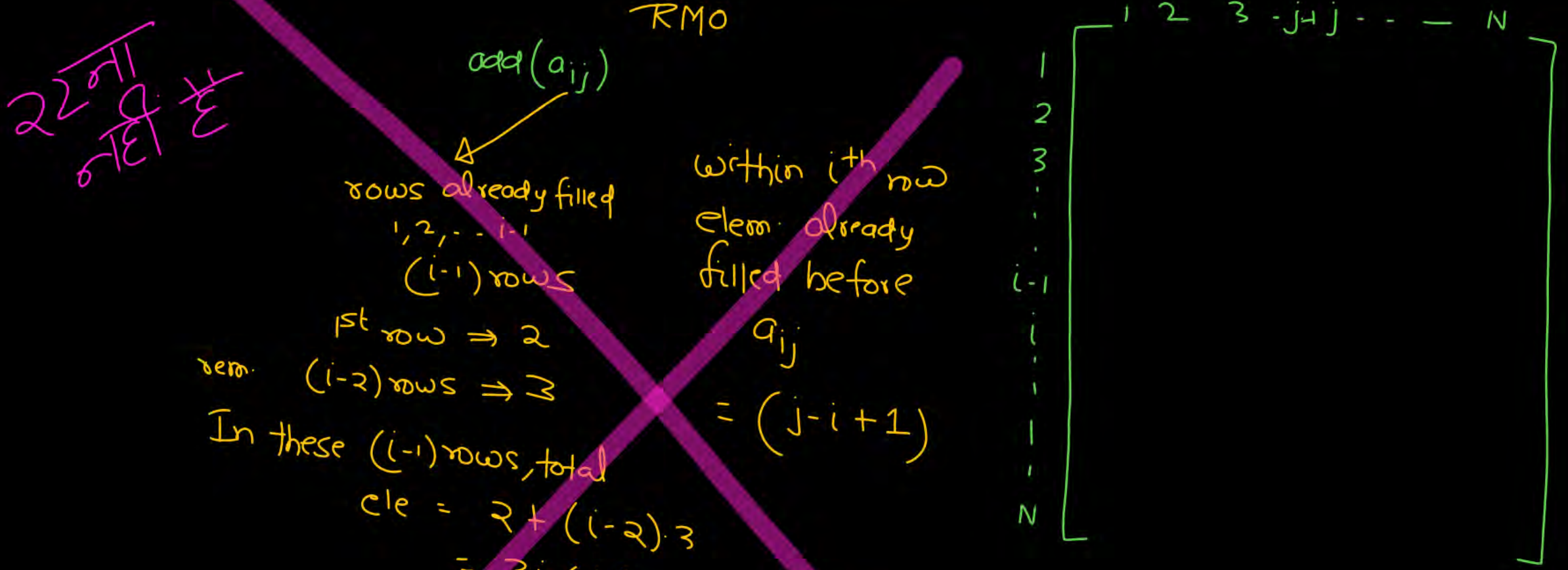
In these (i-1) rows, total
ele = $2 + (i-2)\cdot 3$
$= 3i - 6 + 2$
$= 3i - 4$

within i^{th} row
elem. already
filled before
a_{ij}
$= (j - i + 1)$

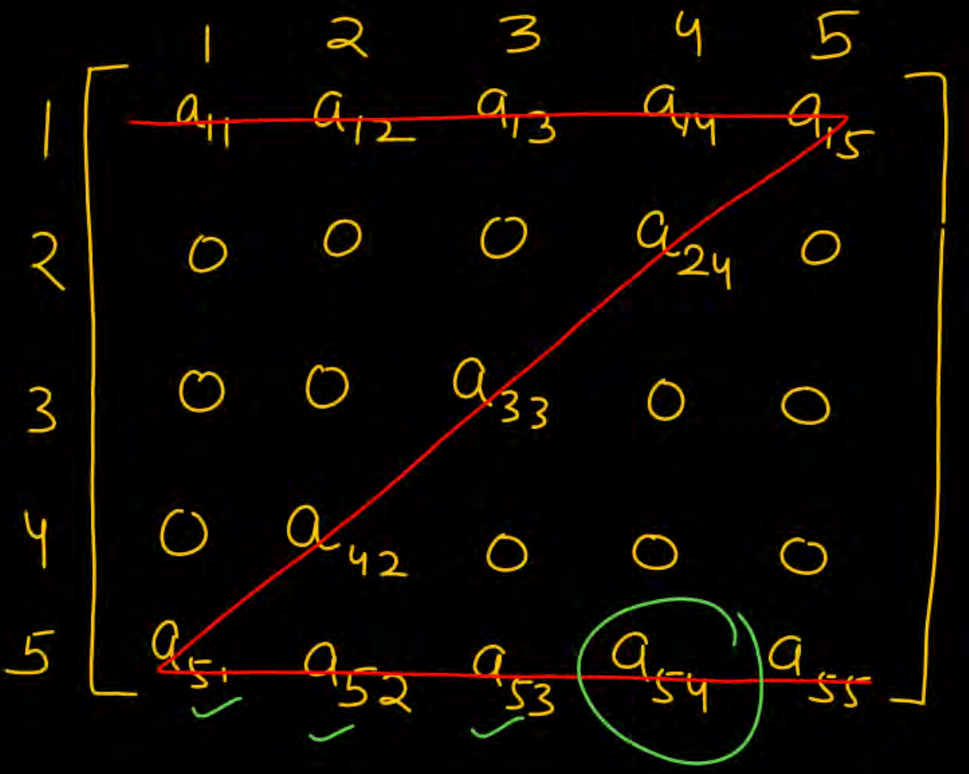


Total ele. already filled before a_{ij} : $3i - 4 + j - i + 1 = \boxed{2i + j - 3}$

Tri-diagonal Matrix



Total ele. already filled before a_{ij} : $3i-4+j-i+1 = \boxed{2i+j-3}$



RMO

1) 1st row — N
2) (n-2) rows — 1
3) last row — N

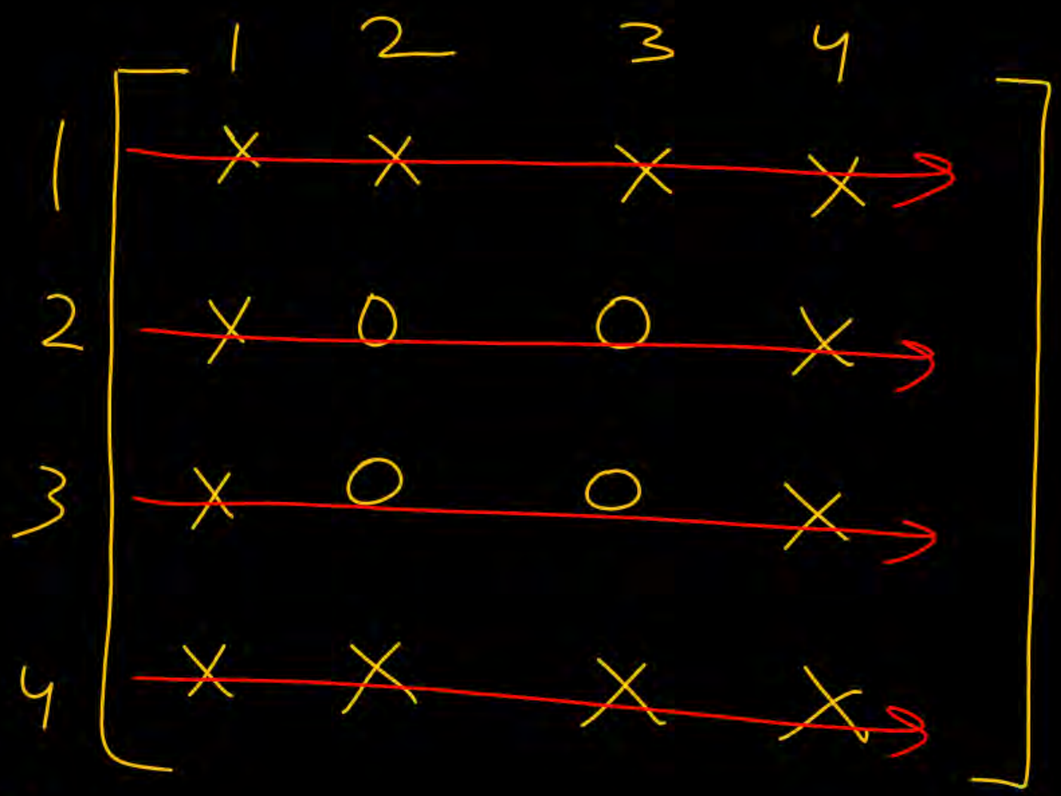
add(a_{54})

4 rows filled

1st row = 1 5

rem. 3 rows = 3+1 = 3] 8 elements

within 5th row ele. already filled before a_{54} = col1, col2, col3 = 3



1st → N

last → N

rem. rows ⇒ 2


THANK YOU GW
SOLDIERS !