CS & IT ENGINEERING

DISCRETE MATHS GRAPH THEORY

Lecture No. 4

By- SATISH YADAV SIR

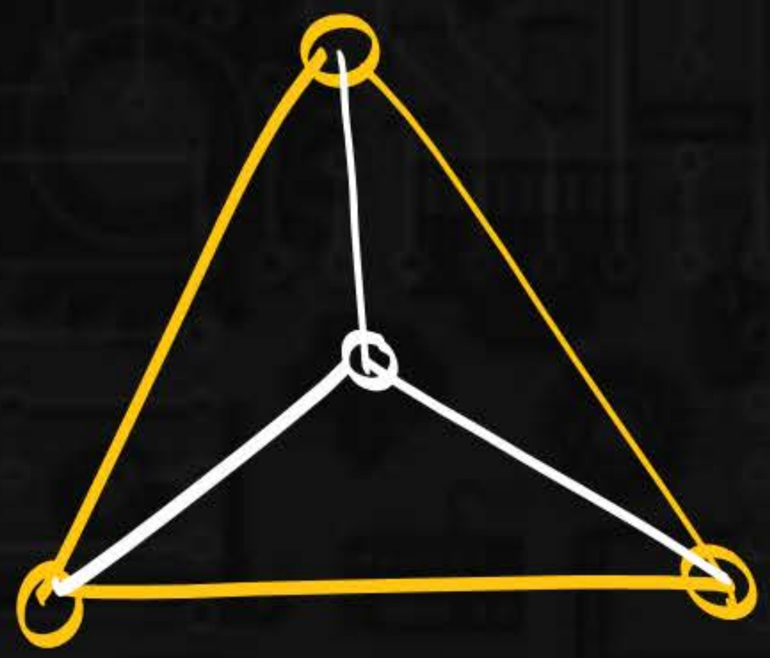
Connectivity

01 Types of Graphs -1

02 Types Of graphs -2

03 Practice

Wheel Graph (W_n) $(n \geq 4)$



$n = 5$ $\quad \overset{n-1}{4} + \overset{n-1}{4}$

W5 $\quad$ C4.

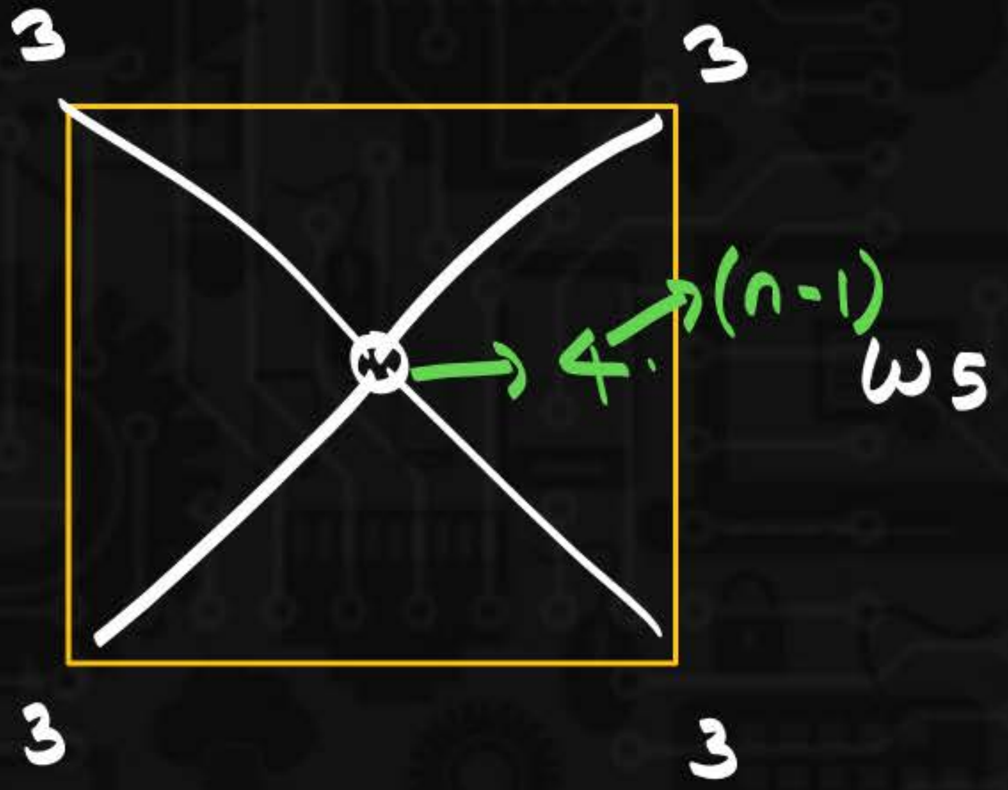
$\downarrow \quad \downarrow$

Wn $\quad$ 4 edges. + 4 edges

n-1 edges + n-1 edges.

$$e(W_n) = n-1+n-1.$$

$$\boxed{e(W_n) = 2(n-1).}$$



W5

4, 3, 3, 3, 3

W6

5 $\underbrace{3, 3, 3, 3, 3}_{\text{5 vertices}}$

W100

99, $\underbrace{3, 3, 3, 3 \ldots 3}_{\text{99 vertices}}$

W99

98,

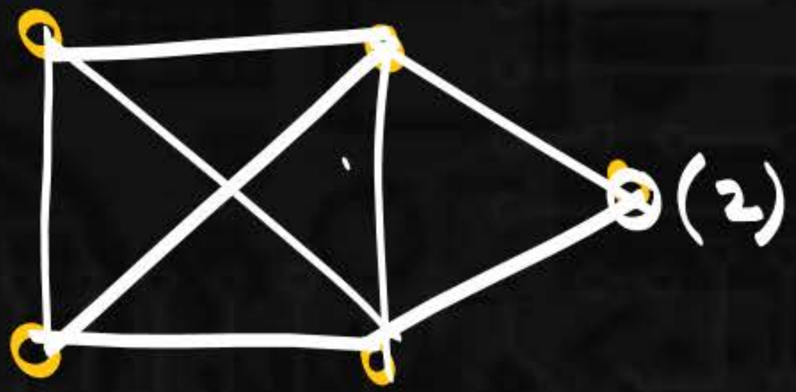
1

if G is $W_n \rightarrow e = 2(n-1)$ (True)

if G is having $e = 2(n-1) \longrightarrow G$ is wheel Graph. (False)

$n = 5 \quad e = 2(n-1)$

$= 2(4) = 8$

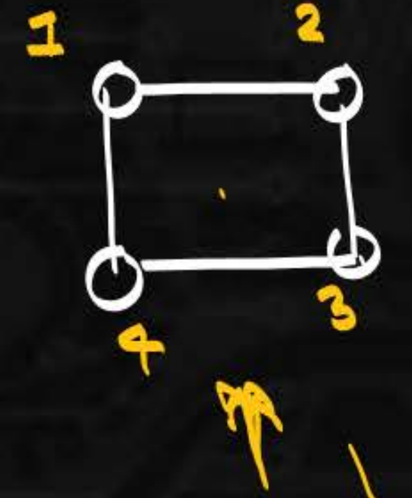


Bipartite Graph.

$G = (V, E)$

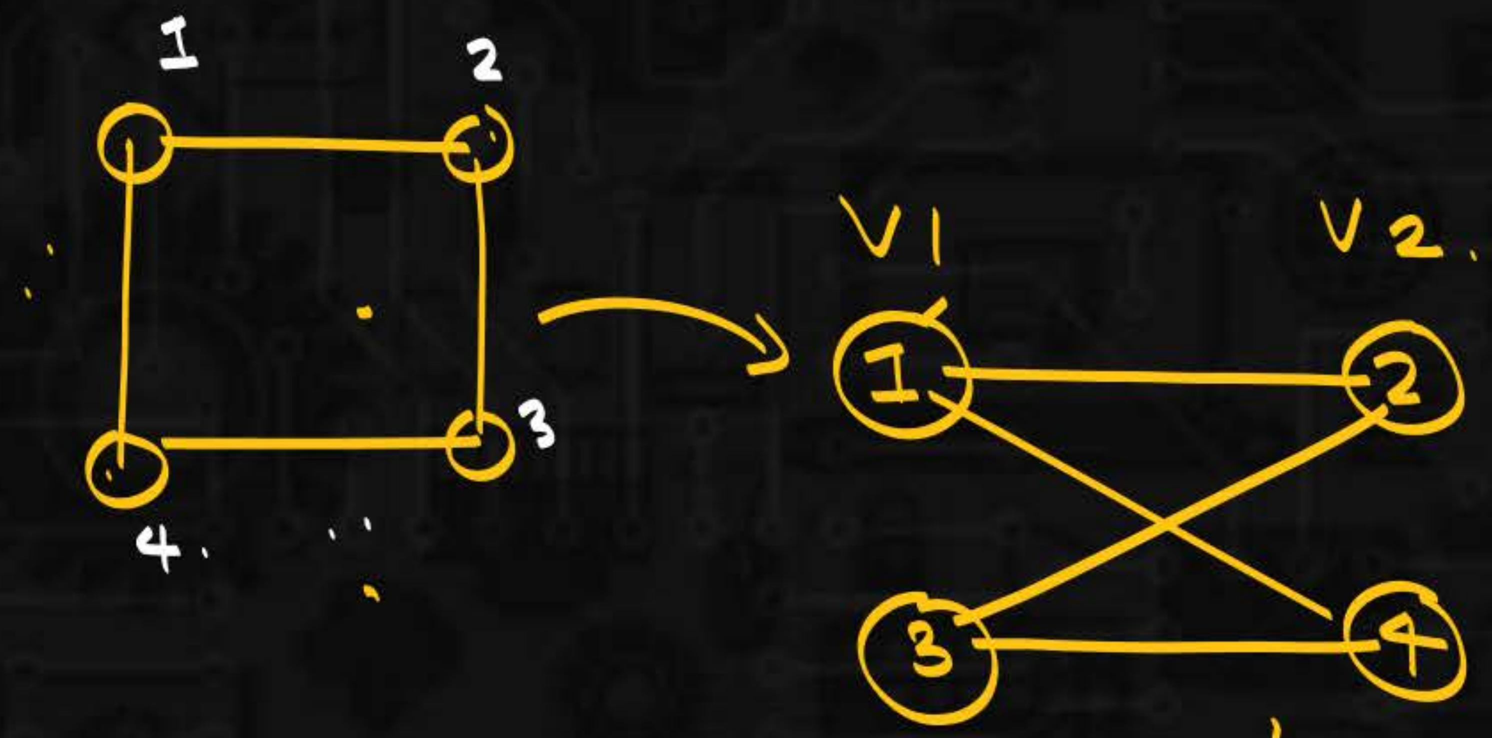
V can be divided into 2 parts V1, V2.

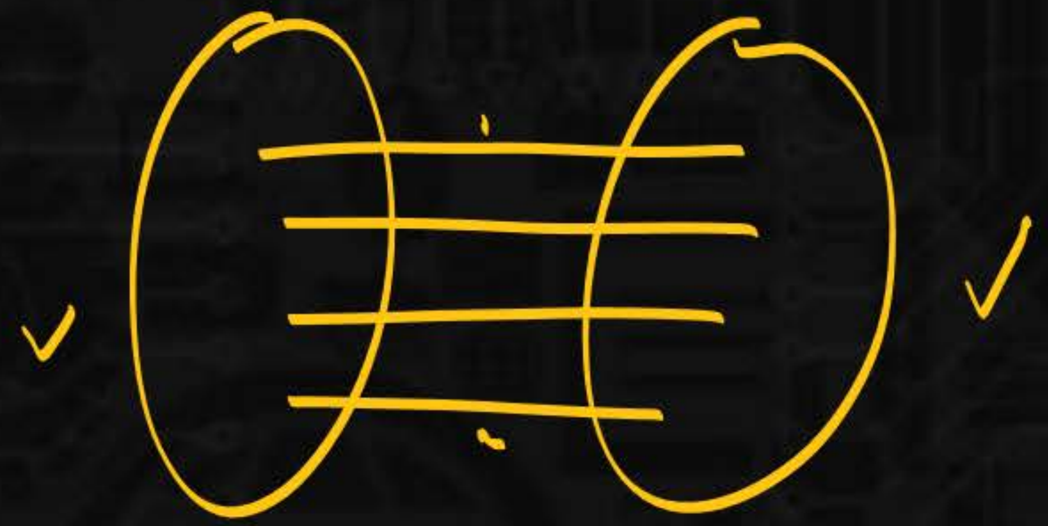
each edge must be from one set to another set but not in same set.

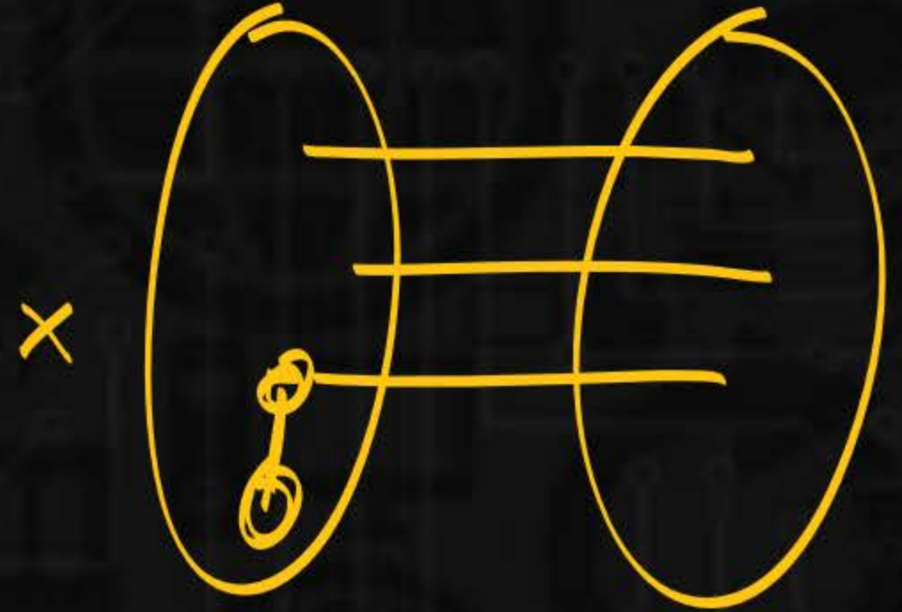


$G = (V, E)$

Bipartite Graph.

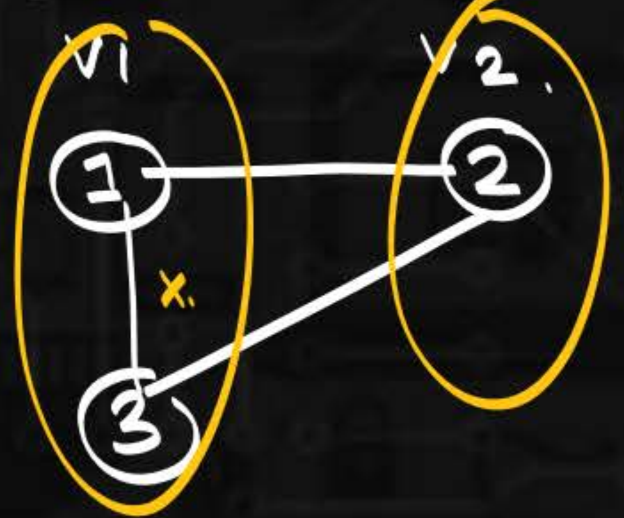

1
2
3
4
V1
V2
1
2
3
4



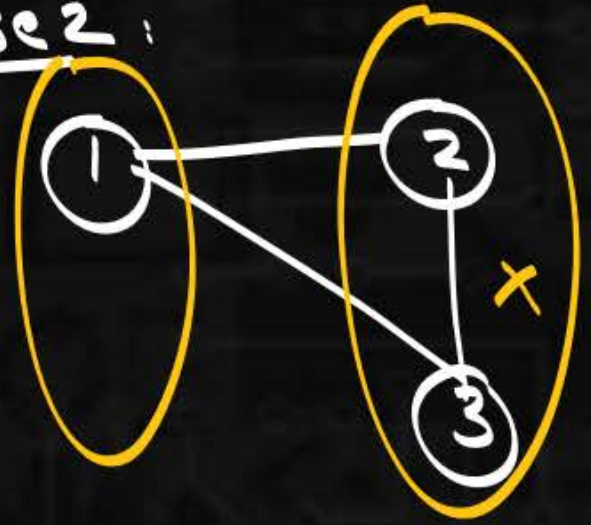

×

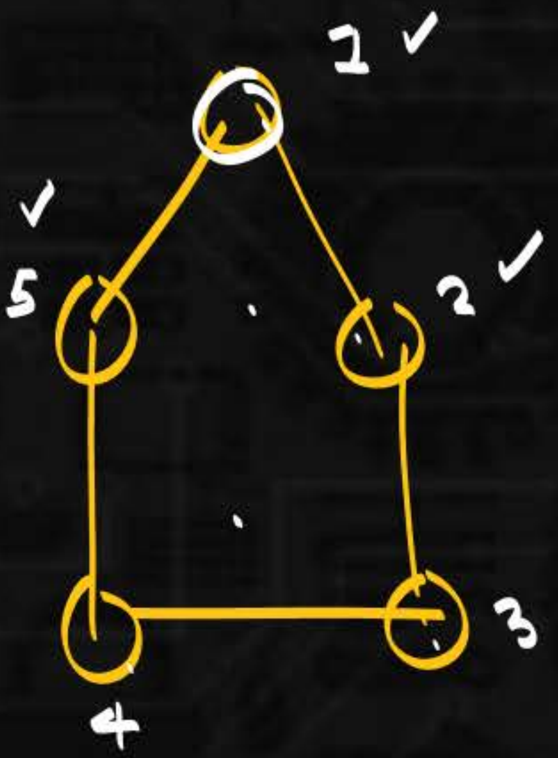
∴ not bipartite Graph.

Case 1:

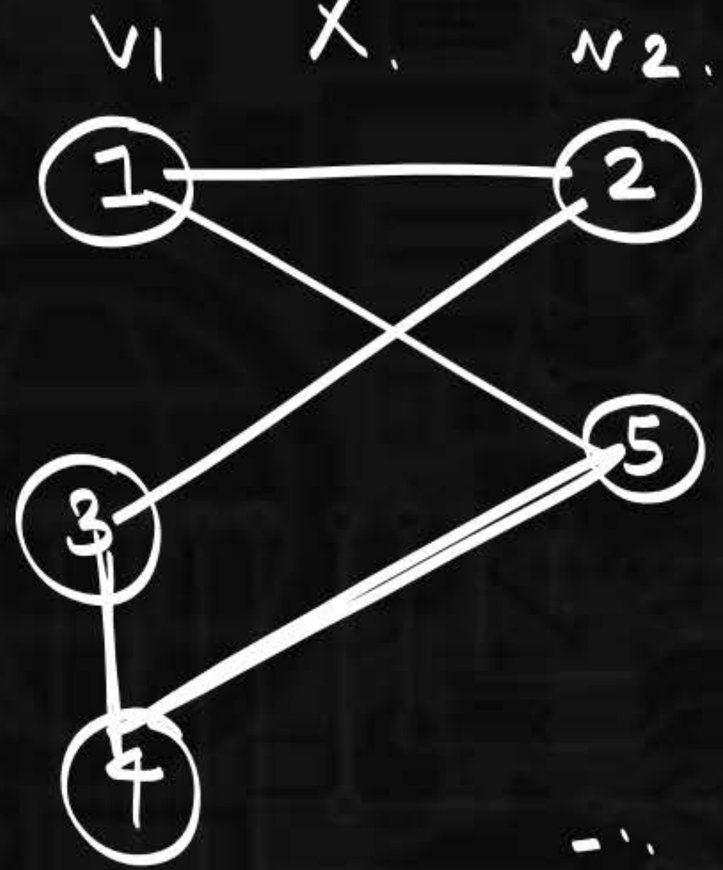


Case 2:



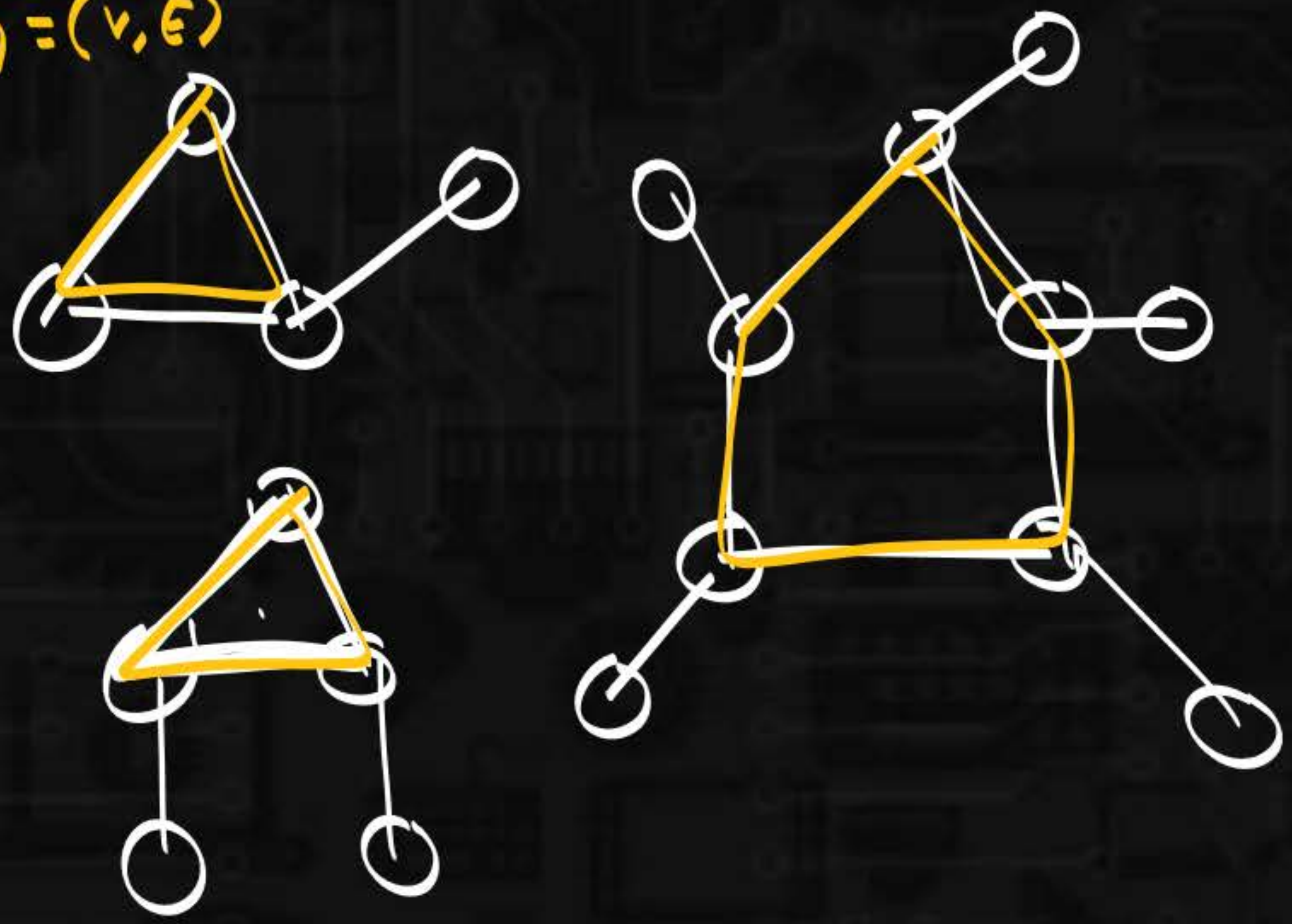


Case 1:



∴

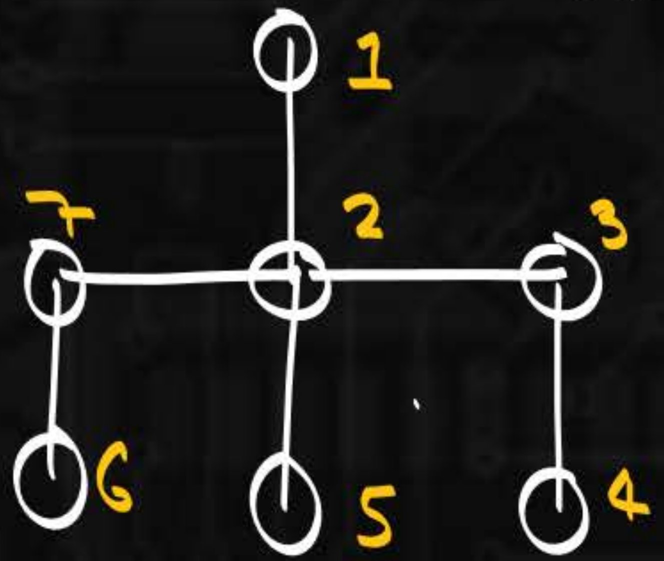
$G = (V, E)$



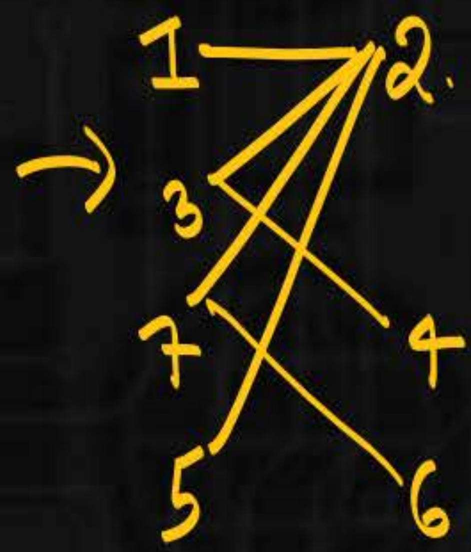
Thm 7:

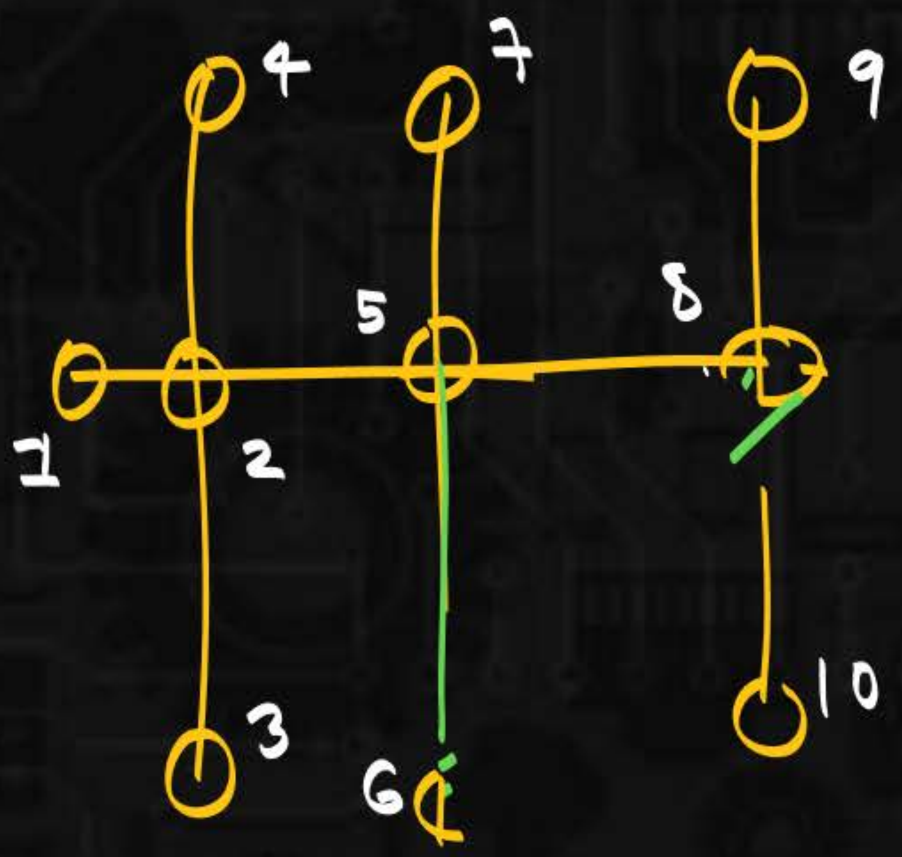
Bipartite Graph does not contains odd length cycle.

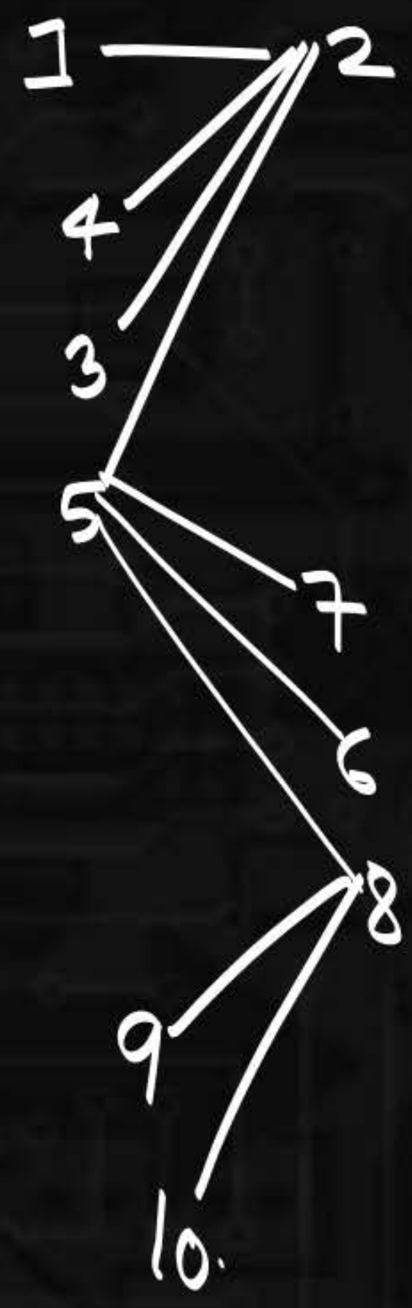
no odd length cycle
- → even length cycle ✓
- → no cycle at all ✓

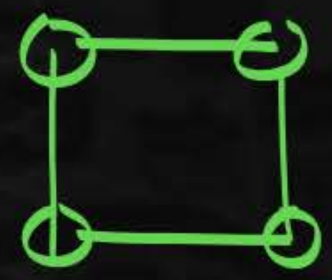


→







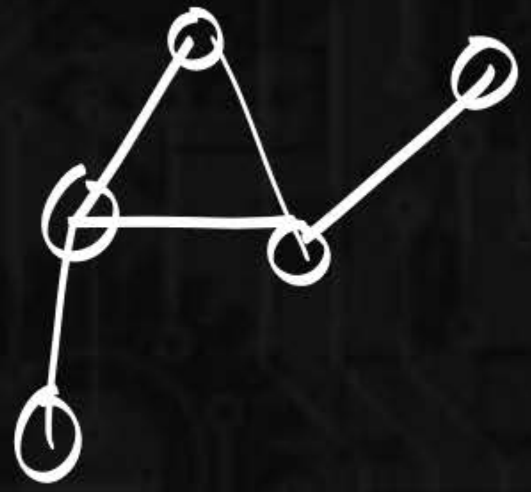


$n = e = 4.$

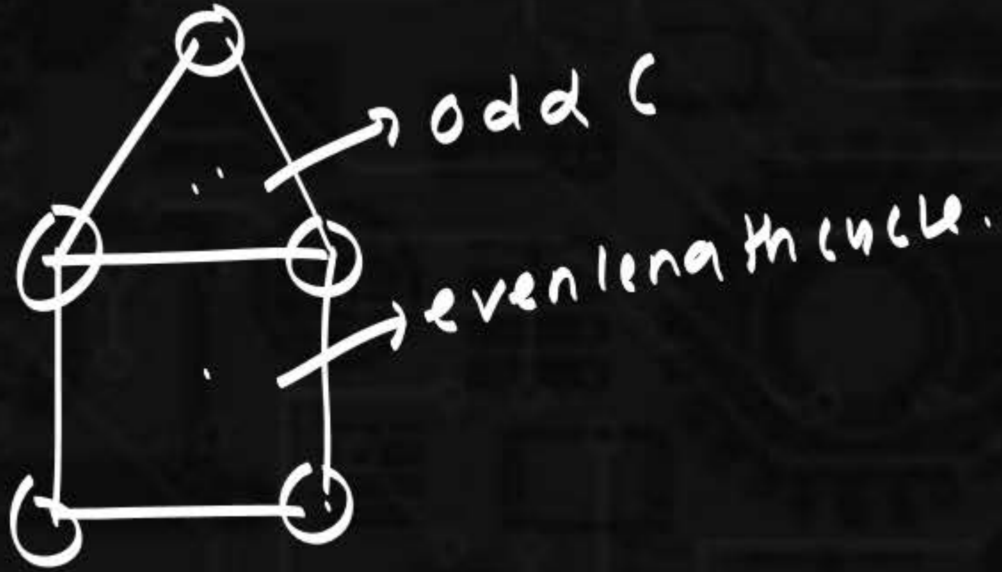
even length cycle

$n = e = 3$

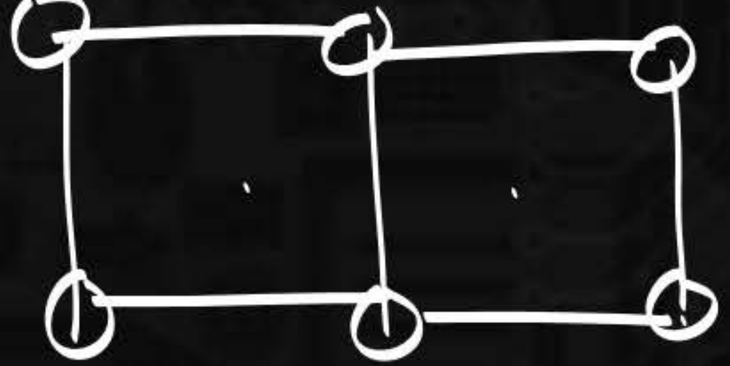
odd length cycle.



Graph contains odd length cycle (C_3)

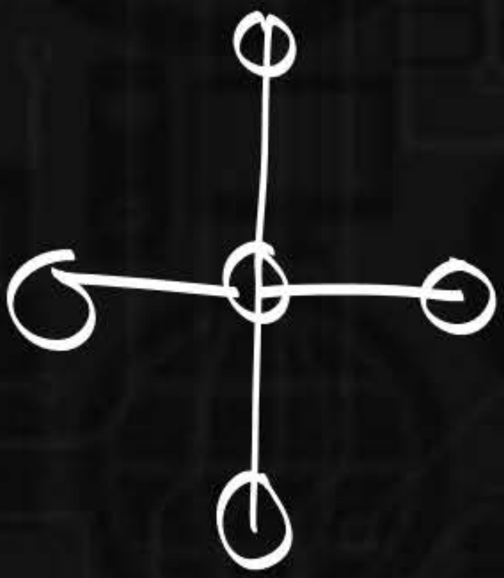


BPG ✗.



no odd length cycle.

BPG ✓



no odd length

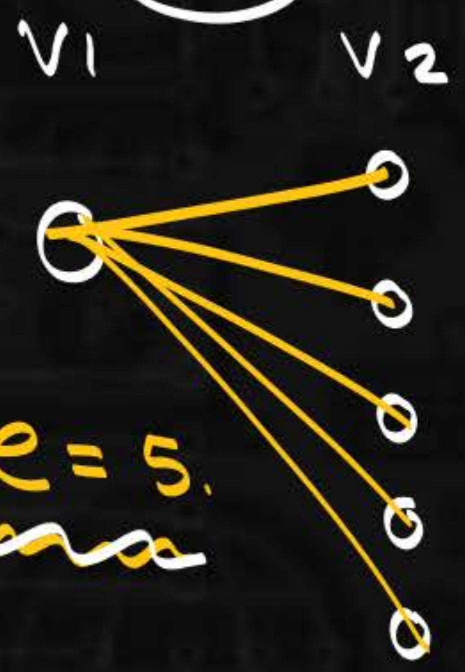
BPG ✓

$n \rightarrow$ Total vertices in Bipartite Graph.

What will be maximum no. of edges?

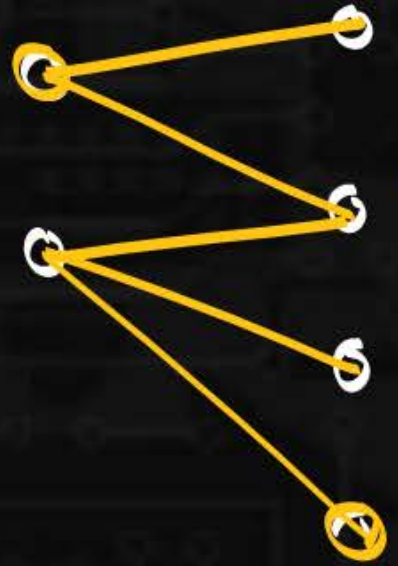
$n = 6$.

Case 1: $K_{1,5}$



$e = 5$.

Case 2: $K_{2,4}$



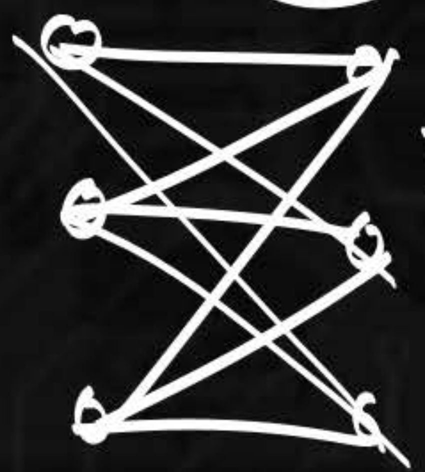
$e = 5$.

$e = 6$

$\vdots$

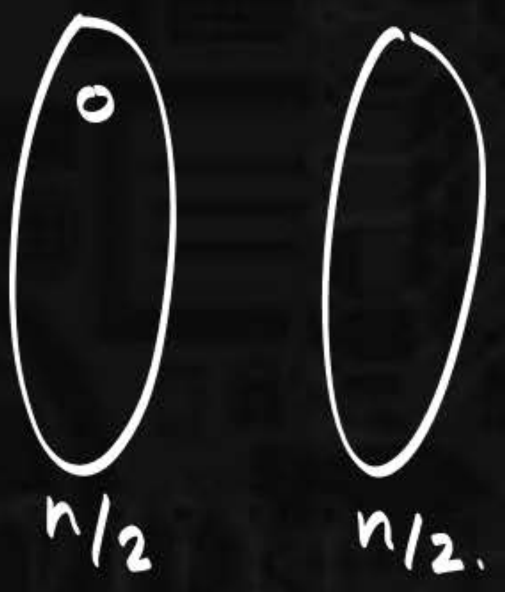
$e = 8$

Case 3: $K_{3,3}$



$e = 9$.

n = Total vertices



$e = n/2 \times n/2$.

$e = \frac{n^2}{4}$.

Thm 8 :

n = Total vertices.

maximum no. of edges in bipartite graph $e \le \frac{n^2}{4}$.

$n = 6 \quad e \le \frac{6^2}{4}$.

$e \le 9$

$n = 7 \quad e \le \frac{7^2}{4}$. $\quad e \le 12.5$

$e \le \frac{49}{4} \qquad e \le 12$

$\lceil 12.5 \rceil = 13$

$\lfloor 12.5 \rfloor = 12$

13

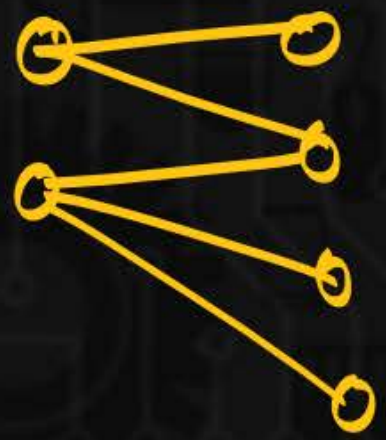
↑

12.5

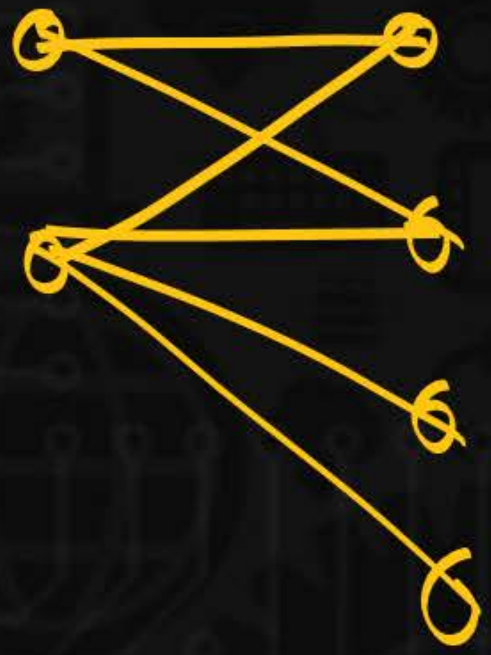
↓

12.

Case 2 :

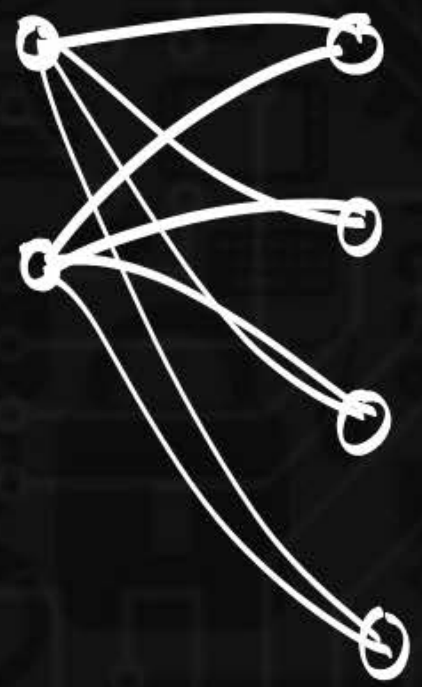


e = 5.



e = 6.

e = 8



$e \le \left\lfloor \frac{n^2}{4} \right\rfloor$ $e \le \left\lfloor \frac{7^2}{4} \right\rfloor$ $e \le \lfloor 12.5 \rfloor$

$e \le 12.$

$n = 7.$

1 6 2 5 3 4 4 3.

$e = 12$ $e = 12$

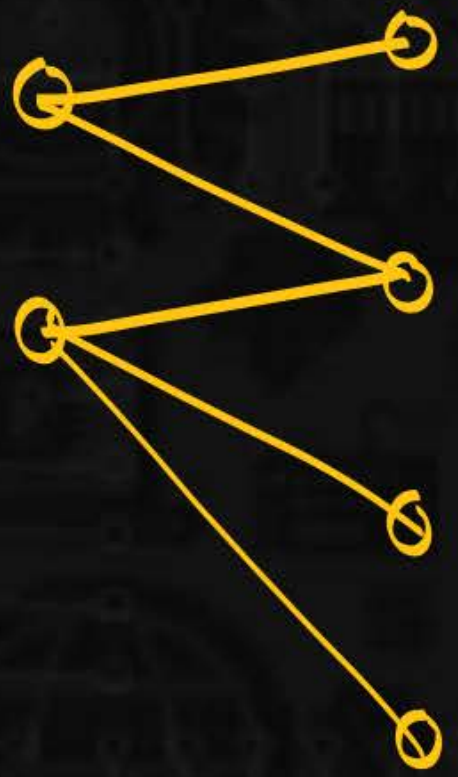
$e = \frac{n(n-1)}{2}.$

→ maximum no. of edges in bipartite graph $e \le \frac{n^2}{4}$.

→ if bipartite Graph contains more than $\frac{n^2}{4}$ edges

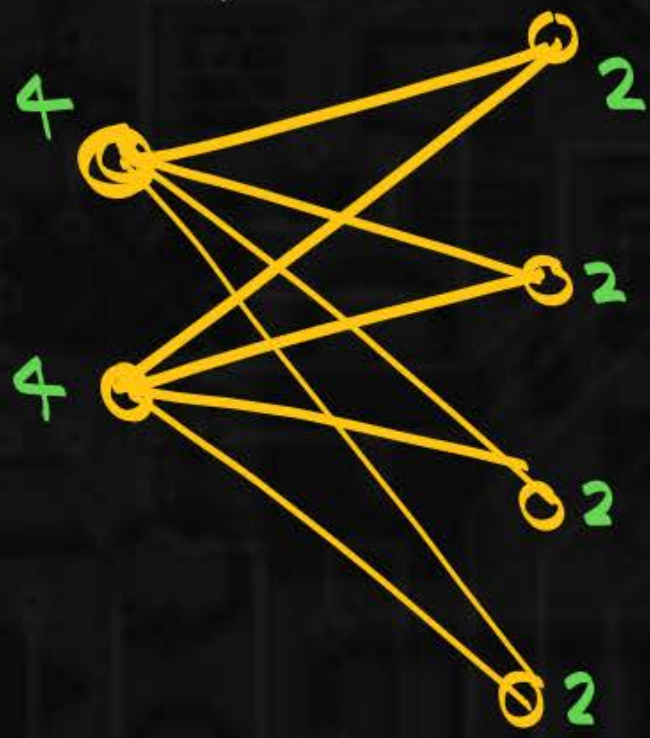
it contains odd length cycle.

Bipartite Graph.



Complete bipartite Graph ($K_{m,n}$)

$K_{2,4}$.



$K_{2,4}$.

$|V_1| = m$

$|V_2| = n$.

Total vertices $= 2+4 = 6$. $V = m+n$

Total edges $= 2 \times 4 = 8$ $\quad E = m \cdot n$

$\Delta(K_{2,4}) = 4$ $\quad \Delta(K_{m,n}) = \max(m,n)$

$\delta(K_{2,4}) = 2$. $\quad \delta(K_{m,n}) = \min(m,n)$

Complete bipartite Graph.

$\downarrow$

Star Graph ($K_{1,n-1}$)

$K_{1,n-1}$ Total vertices = n.

Total edges = n-1.

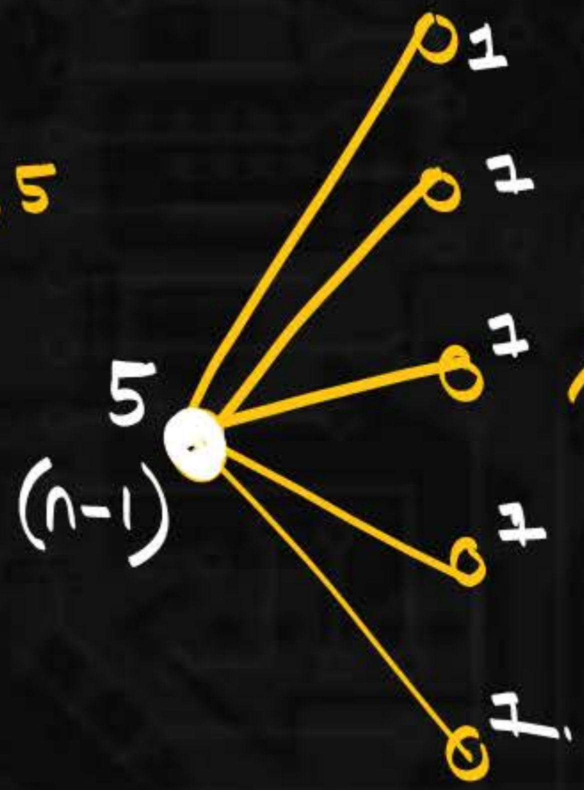
$\Delta(K_{1,n-1}) = n-1$.

$\delta(K_{1,n-1}) = 1$.

Draw star Graph of 6 vertices.

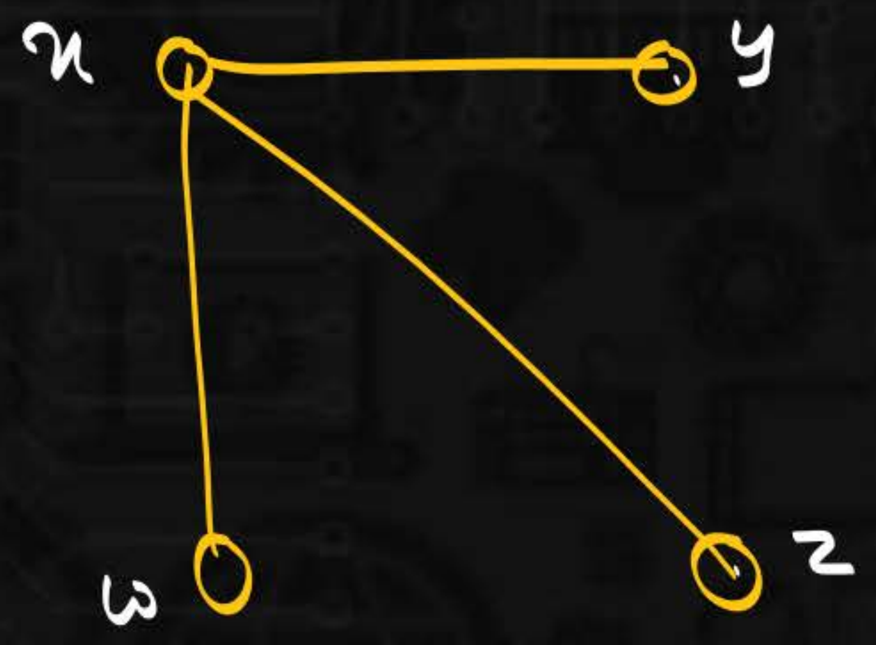
$n = 6$

$K_{1,5}$

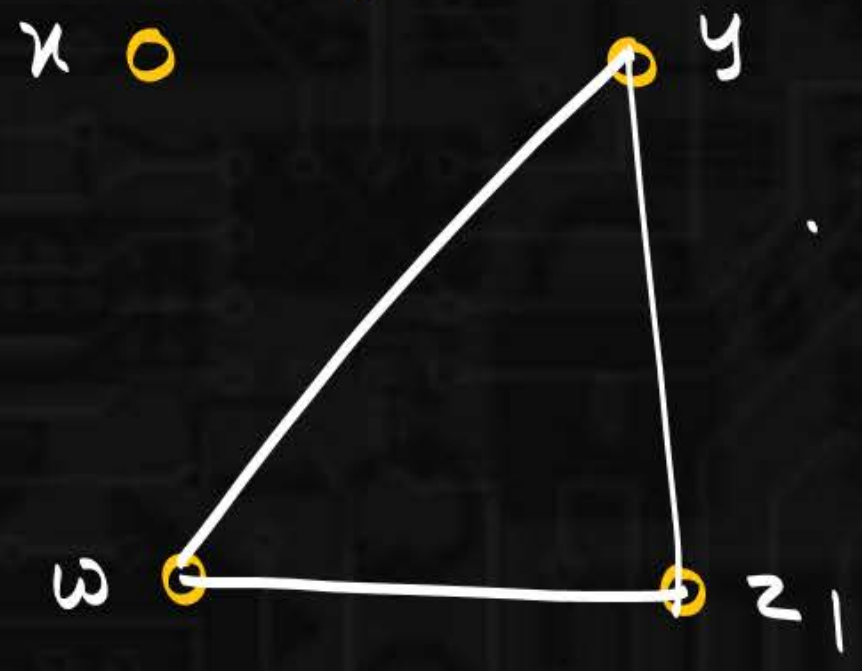


Complement Graph. ($\bar{G}$)

edges → present
edges → absent



edge → absent
edges → present



$$G + \bar{G} = K_n$$

$$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$$

$$\boxed{e(\bar{G}) = \frac{n(n-1)}{2} - e(G)}$$

y 1 edge → p. → $\overline{G}$

2 ans. $y \to 2$

K_4 3 3 3 3 $\quad n = 4$

$G \to (x)\,3,\ (y)\,1,\ 1,\ 1.$

$\to \overline{G}$ 0 2 2, 2.

Q. what will be edges in the complement of the graph having degree sequence

5, 2, 2, 2, 2, 1.

$e(\overline{G}) = ?$ $e(G) = 7$.

$G \rightarrow 5, 2, 2, 2, 2, 1.$

$e(G) + e(\overline{G}) = \frac{n(n-1)}{2}$, Total vertices $= 6$.

$\sum d(v_i) = 2e.$

$n = 6$

$7 + e(\overline{G}) = \frac{6.5}{2} = \frac{30}{2} = 15$

$5+2+2+2+2+1 = 2e$

$14 = 2e$

$\boxed{e = 7}$

$e(\overline{G}) = 15 - 7 = 8$

note:

$$\begin{array}{lllllll} K_n & n-1 & n-1 & n-1 & n-1 & & n-1 \\ G \rightarrow & d_1, & d_2, & d_3, & d_4 & \dots & d_n \\ \hline \overline{G} & n-1-d_1, & n-1-d_2, & n-1-d_3 & \dots & \dots & n-1-d_n \end{array}$$

eg: what will be no. of edges in the complement of the graph having ds
5, 2, 2, 2, 2, 1.

Total Vertices = 6.

$$\begin{array}{lllllll} K_6 & 5 & 5 & 5 & 5 & 5 & 5 \\ G & 5, & 2, & 2, & 2, & 2, & 1 \\ \hline \overline{G} & 0, & 3, & 3, & 3, & 3, & 4 \end{array}$$

$\Sigma d(v_i) = 2e$

$0+3+3+3+4 = 2e$

$16 = 2e$

$e = 8$


THANK YOU GW
SOLDIERS !