CS & IT ENGINEERING

Algorithm

Heap Algorithms

Lecture No.- 03

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Topics to be Covered

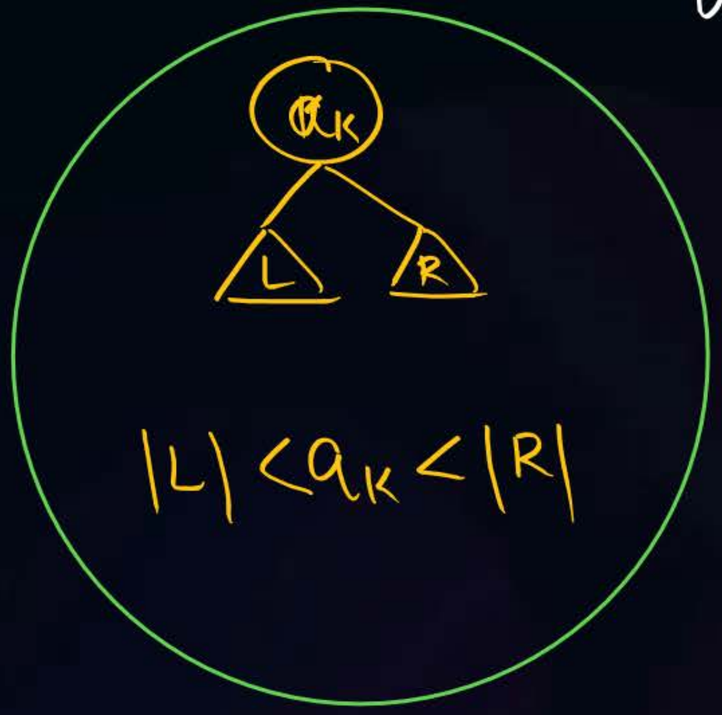
Topic

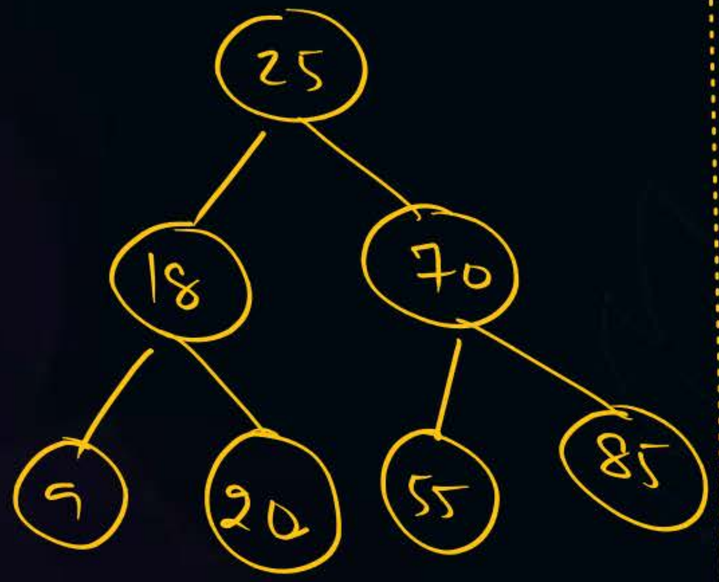
Cost of BST

Construction of BST

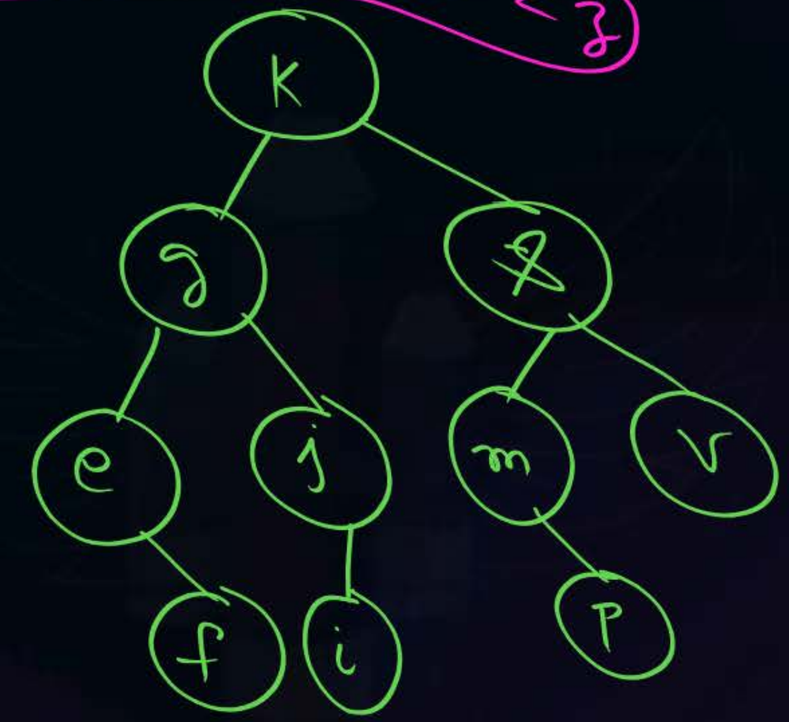
Optimal Cost Binary Search Tree <OBST>/D.P

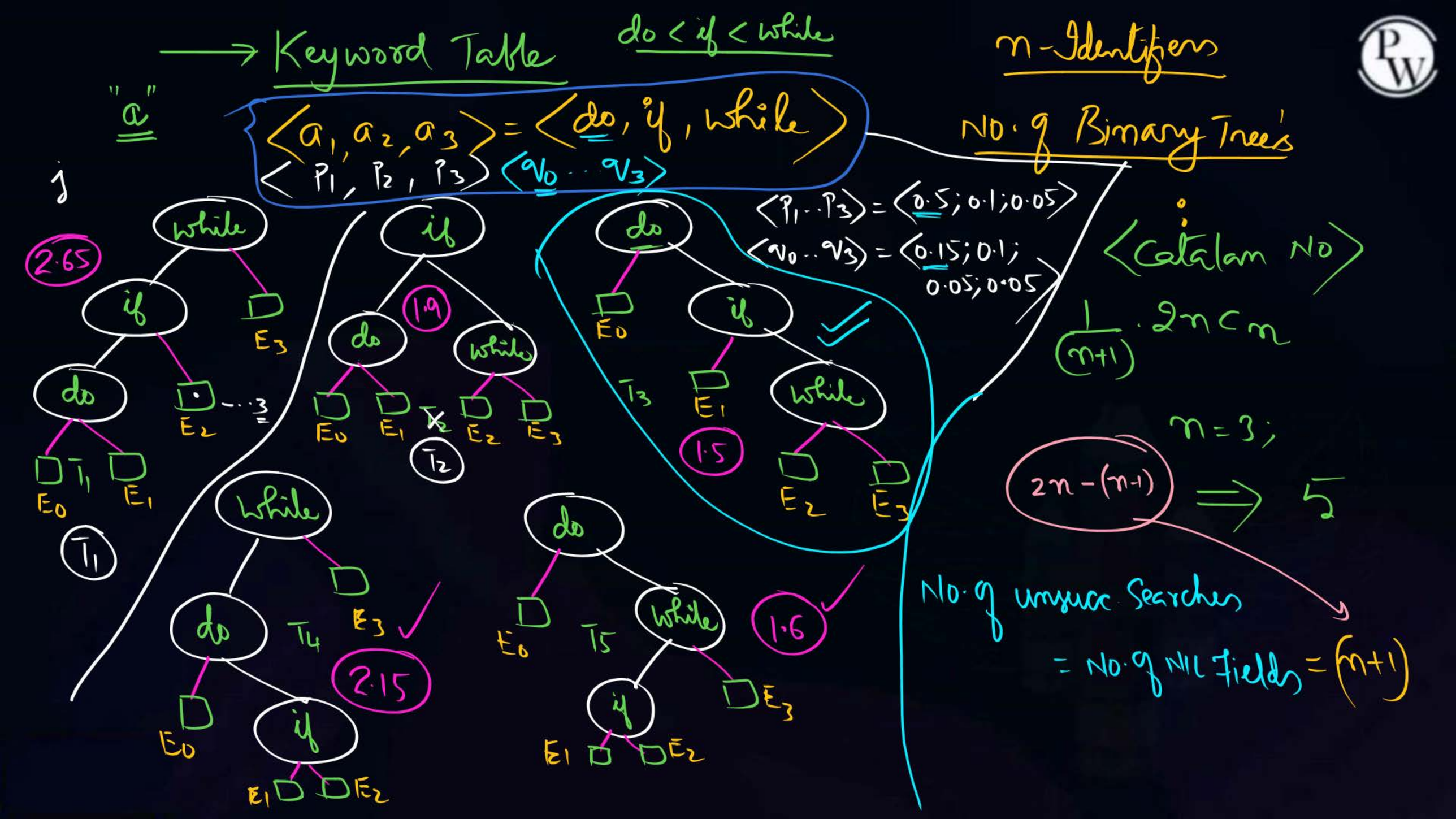
Defn: B.S.T: is a Binary Tree, with the property that, the value @ each Node is greater than the values of left children & less than the values of right children;





$a < b < c < d \ldots \ldots < z$




Keyword Table
do < if < while
n-Identifiers
"a"
$\langle a_1, a_2, a_3 \rangle = \langle do, if, while \rangle$
$\langle p_1, p_2, p_3 \rangle$ $\langle q_0 \cdots q_3 \rangle$
No. of Binary Trees
$\langle p_1 \cdots p_3 \rangle = \langle 0.5; 0.1; 0.05 \rangle$
$\langle q_0 \cdots q_3 \rangle = \langle 0.15; 0.1; 0.05; 0.05 \rangle$
<Catalan NO>
$\frac{1}{(n+1)} \cdot 2nCn$
n = 3;
2n - (n-1) ⟹ 5
No. of unsucc Searches
= No. of NIL fields = (n+1)
j
2.65
1.9
1.5
2.15
1.6
while
if
do
E_0
E_1
E_2
E_3
T_1
T_2
T_3
T_4
T_5

Procedure to determine Cost of B.S.T (T)

1) $Cost(T) = Cost(\text{Succ Searches}) + Cost(\text{unsuccessful Searches})$

2) $Cost(\text{Succ Searches}) = \sum_{i=1}^{n} Cost(a_i) = \sum_{i=1}^{n} P_i * level(a_i)$

3) $Cost(a_i) \propto level(a_i)$

$Cost(a_i) = P_i * level(a_i)$

P_i = Prob of using 'a_i' in appl.

4) $Cost(\text{unsucc. Searches}) = \sum_{i=0}^{n} Cost(E_i)$

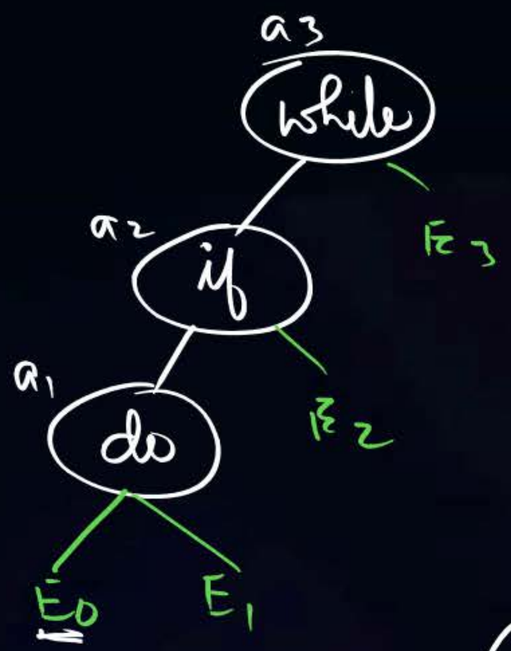
5) $Cost(E_i) \propto (level(E_i) - 1)$

$= (q_i * (level(E_i) - 1))$

q_i = Prob of using the identifiers in the set 'E_i'

6) $Total(\text{unsucc. Searches}) = \sum_{i=0}^{n} q_i * (level(E_i) - 1)$

int , for , break



C-problem to read three no's

-for-loops

```
main()
{
    int a, b, c;
    int i, j, k;
```

unsucc. search

$$Cost(T_1) = \left(0.5*3 + 0.1*2 + 0.05*1\right) + \left(0.15*3 + 0.1*3 + 0.05*2 + 0.05*1\right) = 2.65$$

$$Cost(T) = \sum_{i=1}^{n} P_i * level(a_i) + \sum_{i=0}^{n} q_i * (level(E_i) - 1)$$

Note: The Cost of B.S.T is dependent on both the height (levels) & also Probabilities;

II. Construction of OBST:

I/P $\begin{cases} \langle a_1, a_2, \ldots, a_n \rangle \\ \langle p_1, p_2, \ldots, p_n \rangle \\ \langle q_0, q_1, \ldots, q_n \rangle \end{cases}$ [Keywords] Σn $n = 200$

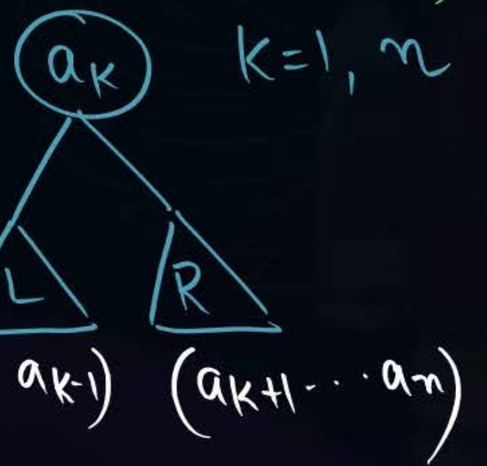
$\frac{400C_{200}}{201}$

I) Brute-Force :

II. DP based Soln

$\langle a_1, a_2, a_3, \ldots, a_n \rangle$

$T_{1,n}$



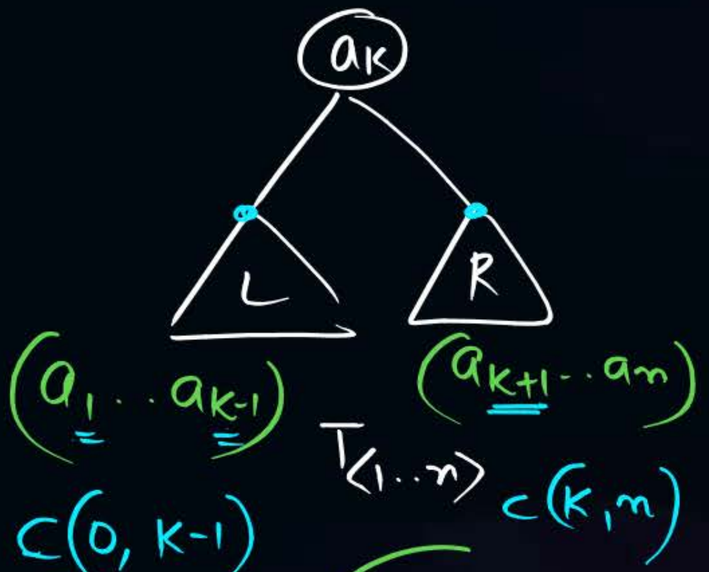
$$Cost(T) = Cost(a_k) + Cost(L) + Cost(R)$$

$$Cost(T) = P_k + Cost(L) + Cost(R)$$

Let $c(0, n)$ be the cost of $T_{1..n}$

$$c(0,n) = \min_{1 \le k \le n} \{ P_k + \underbrace{c(0,k-1)}_{L} + c(k,n) \} + w(0,n)$$

Const



$$c(i,j) = \min_{i < k \le j} \{ P_k + c(i,k-1) + c(k,j) \} + w(i,j)$$

(1)

$w(0,n)$ = wt that is added to the L.S.T & R.S.T, to balance it w.r.to root

$c(i,i) = 0$

$R(i,j)$ = 'k' that minimizes eq (1)

Time : $O(n^3)$

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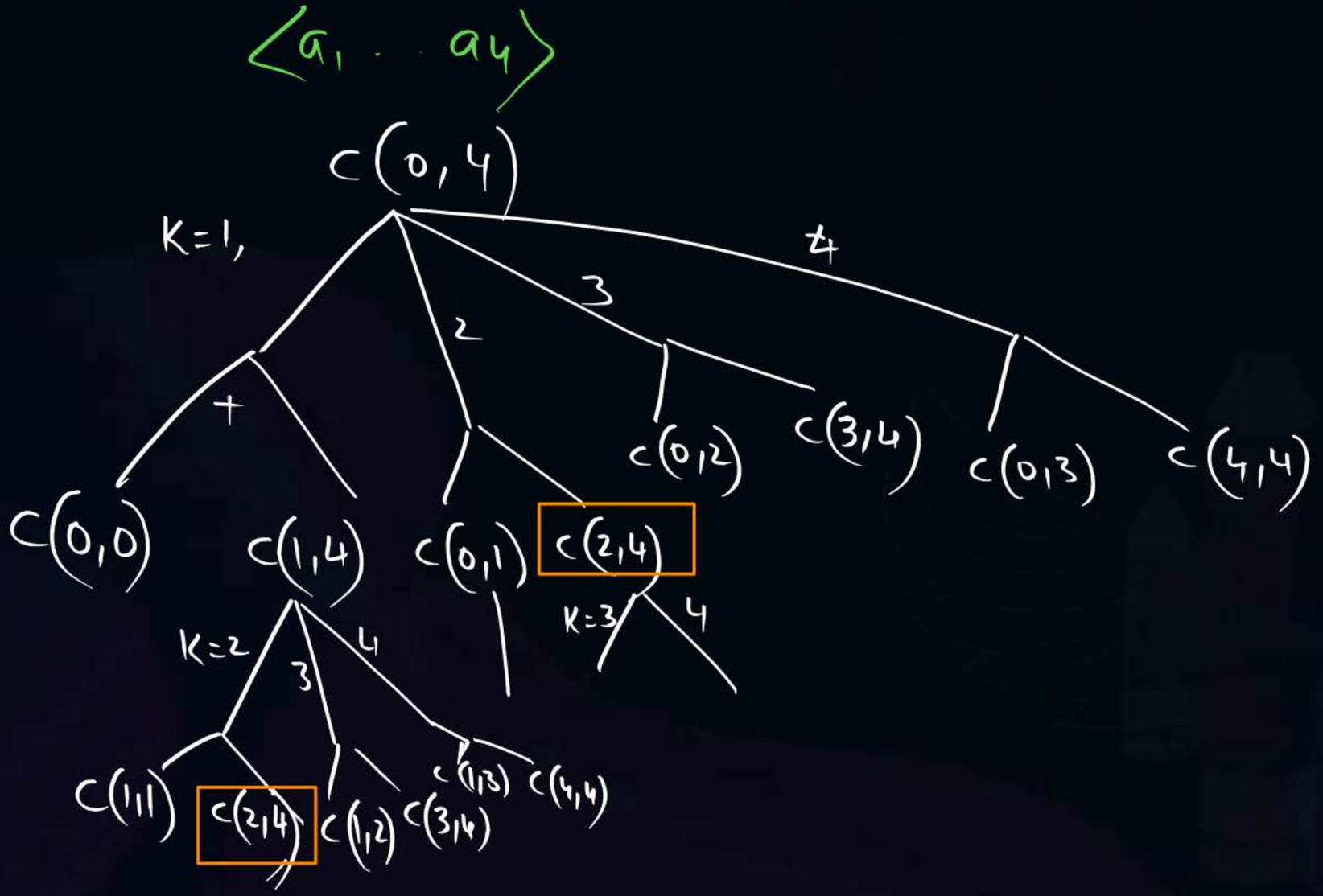
$$c(0,n) = \sum_{i=1}^{n} P_i * level(a_i) + \sum_{i=0}^{n} q_i * (level(E_i) - 1)$$



Induction

1) $T(n) = 2 \cdot T(n/2 + 17) + n$

$T(n) = 2T(n/2) + n$

$\rightarrow O(n \cdot \log n)$

2) $T(n) = T(n-1) + T(n/2) + n$

i) $T(n/2) < T(n-1)$

$T(n) < T(n-1) + T(n-1) + n < 2 \cdot T(n-1) + n$

$\rightarrow O(d \cdot 2^n)$

$T(n) > 2 \cdot T(n/2) + n \Rightarrow n \log n$

THANK - YOU