ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-02

Numerical Methods

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Topics to Be Covered

- Graphical method
- BISECTION METHOD
- REGULA FALSI METHOD
- SECANT METHOD
- NEWTON RAPHSON METHOD

NUMERICAL INTEGRATION

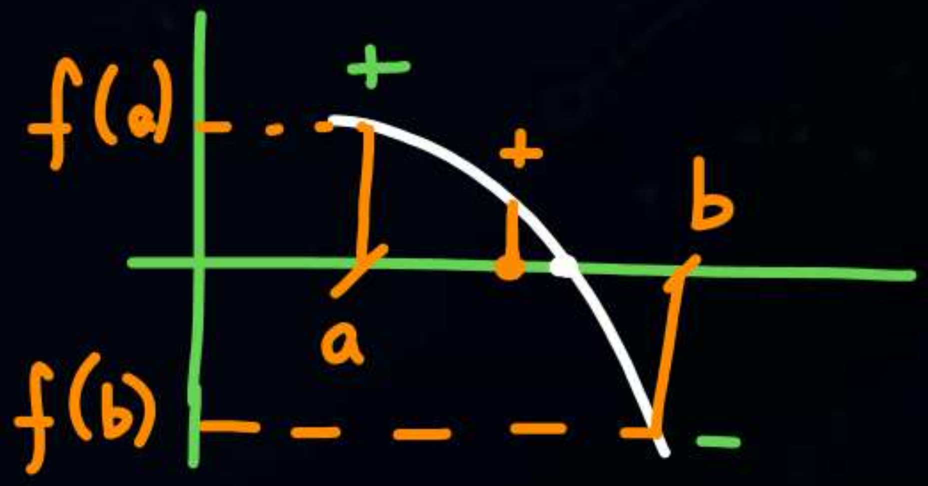
- TRAPEZOIDAL RULE
- SIMPSON'S 1/3RD RULE
- SIMPSON'S 3/8TH RULE

1). Bisection method/Bolzano method/half interval method

Let $f(x) = 0$ real, continuous in (a, b)

Assume initial roots $x = a$ & $x = b$ such that $\rightarrow f(a).f(b) < 0$

$$\boxed{x_n = \frac{a+b}{2}}$$



$\rightarrow x = a;\ \underset{(+)}{f(a)}$

$f(a)f(b) < 0$

$x_1 = \frac{a+b}{2};\ \underset{(+)}{f\left(\frac{a+b}{2}\right)}$

$f(a).f(b) < 0$ $\rightarrow x_2 = \frac{x_1+b}{2};\ \underset{(-)}{f(x_2)}$

Stop when $f(x_n) = 0$

$\rightarrow x = b;\ \overset{(-)}{f(b)}$

Q. Find the value real root of $\underline{x - \cos x} = 0$

b/w 0 and 1.

$0.625 < \text{Root} < 0.75$

$\text{Root} = 0.739375$

$x_7 = 0.734$

$x = 0 \; ; \; f(0) = -1$ $(-)$

$f(a) \cdot f(b) < 0$

$x_1 = \frac{0+1}{2} \; ; \; f(0.5) = -0.377$ $(-)$

(0.5)

$x_2 = \frac{0.5+1}{2} \; ; \; f(0.75) = 0.018$ $(+)$

(0.75)

$x_3 = \frac{0.5+0.75}{2} \; ; \; f(0.625) = -0.18$ $(-)$

0.625

x_4

$x = 1 \; ; \; f(1) = 0.45$ $(+)$

2).Regular falsi method/method of false position/ method of chords

Let $f(x) = 0$

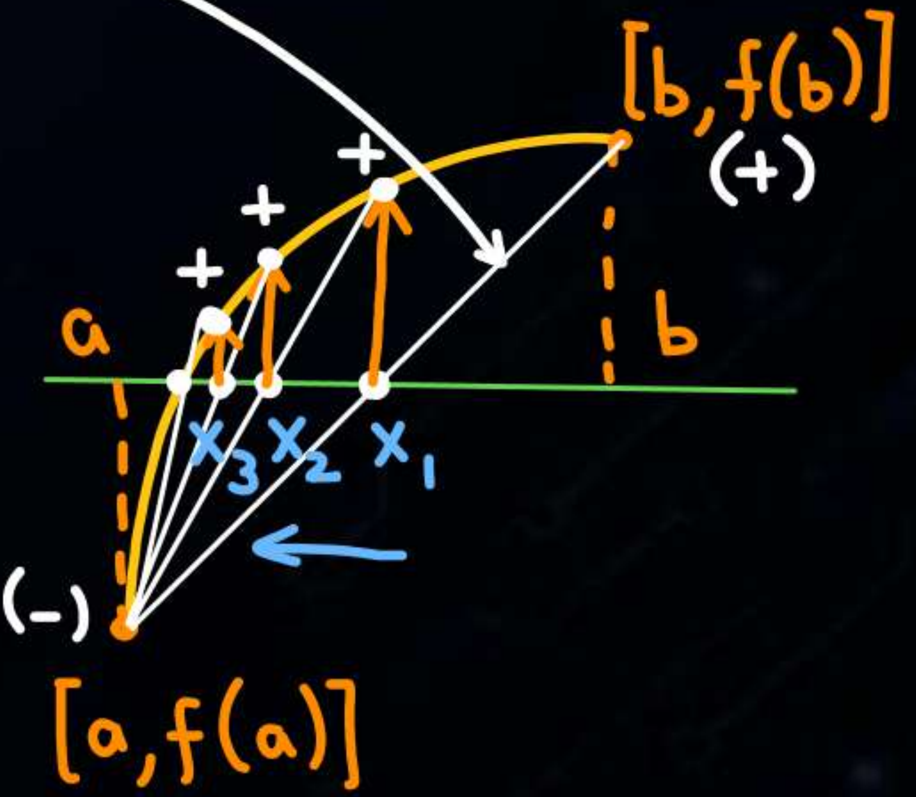
Assume 2 initial roots $x = a$ and $x = b$ such that $f(a) \cdot f(b) < 0$.

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

Put $y = 0$, pt. of intersection with x-axis

$$0 - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

$$\boxed{x_n = \frac{a f(b) - b f(a)}{f(b) - f(a)}}$$



3). Secant method

This is an improvement over Regula falsi method.

* We do not apply $f(a) \cdot f(b) < 0$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

⋮

$x_2 =$ 〃 Root (x_1, b)

⋮

$x_3 =$ 〃 Root (x_2, b)

Ex:- Find the root of $f(x) = x - \cos x$ after 2^{nd} approximation using Regula falsi method.

Soln:- $a = 0 \quad f(a) = \underset{(-)}{-1} \quad ; \quad b = 1 \quad f(b) = \underset{(+)}{+0.45}$

1st trial $\rightarrow$ $$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{0 \times 0.45 - 1 \times (-1)}{0.45 - (-1)} = 0.69$$

$$f(0.69) = -0.081 \; (-)$$

$$a = 0.69, \; f(0.69) = -0.081; \quad b = 1, \; f(b) = +0.45$$

2^{nd} trial $\rightarrow$ $$x_2 = \frac{0.69 \times 0.45 - 1 \times (-0.081)}{0.45 - (-0.081)} = \boxed{0.737}$$

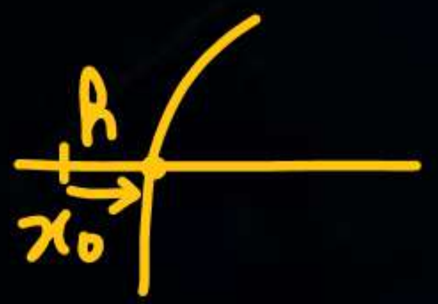
4). Newton Raphson method : method of tangents

Let $f(x) = 0$

Let initial root be x_0.

Thus $f(x_0 + h) = 0$

$$f(x_0) + h\, f'(x_0) + \frac{h^2}{2!} f''(x_0) + \frac{h^3}{3!} f'''(x_0) + \cdots = 0$$



Neglecting h^2 and higher powers of h.

$$f(x_0) + h\, f'(x_0) = 0$$

$$h = -\frac{f(x_0)}{f'(x_0)}$$

4).Newton Raphson method : method of tangents

Thus, new approximated root;

$$x_1 = x_0 + h = x_0 - \frac{f(x_0)}{f'(x_0)}$$

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$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Stop when $x_{n+1} = x_n$

Q. By N.R method; find root at $\boxed{x^4 - x - 10 = 0}$ which is nearer to $x = 2$

$f(x) = x^4 - x - 10$

$f'(x) = 4x^3 - 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^4 - x_n - 10}{4x_n^3 - 1}$$

$$\boxed{x_{n+1} = \frac{3x_n^4 + 10}{4x_n^3 - 1}}$$

Iterative/Recursive eqn. by N.R. method.

$$x_1 = \frac{3x_0^4 + 10}{4x_0^3 - 1} = \frac{3 \times 2^4 + 10}{4 \times 2^3 - 1} = 1.871$$

$$x_2 = \frac{3x_1^4 + 10}{4x_1^3 - 1} = \frac{3 \times 1.871^4 + 10}{4 \times 1.871^3 - 1} = 1.856$$

$$x_3 = \frac{3x_2^4 + 10}{4x_2^3 - 1} = \frac{3 \times 1.856^4 + 10}{4 \times 1.856^3 - 1} = 1.856$$

Since $x_2 = x_3$, required root is 1.856.

Q. Find real root of equation $x = e^{-x}$ using N.R. method

$f(x) = xe^{x} - 1 \qquad f'(x) = xe^{x} + e^{x}$

Assume $x_0 = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1e^{1} - 1}{1e^{1} + e^{1}} = 1 - \frac{e-1}{2e} = 0.6839$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.6839 - \frac{0.6839\,e^{0.6839} - 1}{0.6839\,e^{0.6839} + e^{0.6839}} = 0.57745$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.5671$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.5671$$

Ex:- Find square root of a number.

$$x = \sqrt{N} \Rightarrow x^2 - N = 0$$

$$x = \sqrt[3]{N} \Rightarrow x^3 - N = 0$$

$$x = N^{0.25} \Rightarrow x^4 - N = 0$$

✓ $$x = N^{0.2} \Rightarrow x^5 - N = 0$$

$$X_{i+1} = x_i - \frac{x_i^5 - N}{5x_i^4}$$

$(1000)^{0.2}$

$$= x_i - \frac{x_i}{5} + \frac{N}{5x_i^4}$$

$$X_{i+1} = \frac{4}{5}x_i + \frac{N}{5x_i^4} = \frac{1}{5}\left(4x_i + \frac{N}{x_i^4}\right)$$

Ex:- Find reciprocal of a number.

$$x = \frac{1}{N}$$

$$x - \frac{1}{N} = 0 \quad (\times)$$

$$\frac{1}{x} - N = 0 \quad (\checkmark)$$

$$X_{i+1} = x_i - \frac{x_i - \frac{1}{N}}{1}$$

$$X_{i+1} = x_i - \frac{\frac{1}{x_i} - N}{-\frac{1}{x_i^2}}$$

Numerical Solution of Algebric & Transcendental Equations.

	Order at convergence	Remark	Formula
Bisection method	1	Assume 2 initial roots f(a). f(b) < 0	$x_i = \frac{a+b}{2}$
Regula - Falsi method	1	Assume 2 initial roots f(a). f(b) < 0	$x_i = \frac{af(b) - bf(a)}{f(b) - f(a)}$

secant method	1.618 ≈1.62	Assume 2 initial roots [redacted]	$x_i = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$
Newton Raphson	2	Assume only 1 root	$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$\boxed{\text{Error} \propto \frac{1}{n\,(\text{No. of iterations})}}$$

Ex:- Find no. of iterations when $a=0$, $b=1$ such that error is of magnitude 0.001

$$\boxed{\frac{|b-a|}{2^n} < \varepsilon}$$

$$\frac{|1-0|}{2^n} < 0.001$$

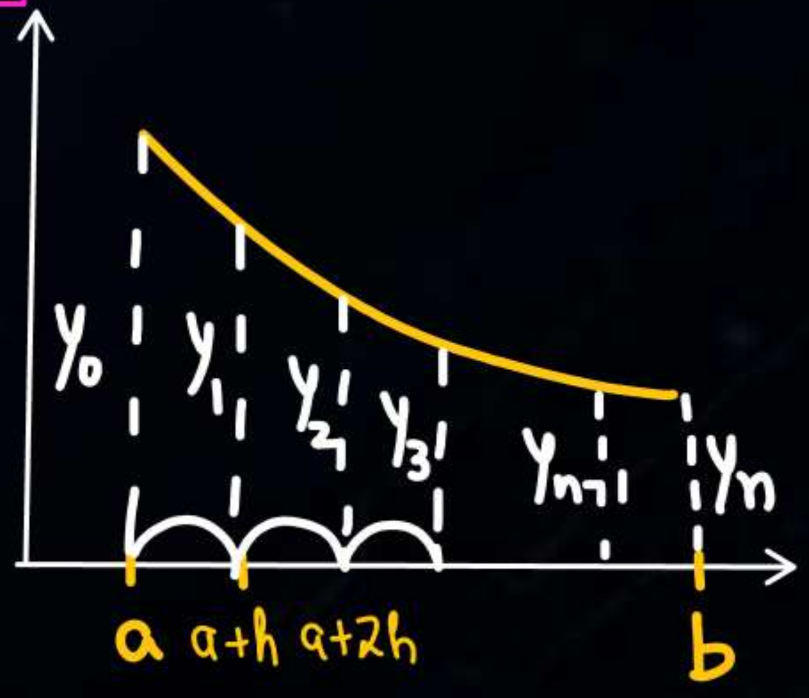
$$1000 < 2^n \Rightarrow \boxed{n = 10}$$

NUMERICAL INTEGRATION

$$b = a + nh$$
$$h = \frac{b-a}{n}$$

Trapezoidal Formula

$$\boxed{\int_a^b f(x)\,dx = \frac{h}{2}\left\{ (y_0 + y_n) + 2(y_1 + y_2 + y_3 + \cdots y_{n-1}) \right\}}$$



- This formula will give no error / is suitable for LINEAR function.

Cumulative error → $\boxed{|E| < \frac{(b-a)}{12} h^2 M}$

$M \rightarrow \max(y_0'', y_1'', y_2'' \ldots y_{n-1}'')$

Thank you

GW Soldiers !