CS & IT ENGINEERING

DISCRETE MATHS

Mathematical Logic

Lecture No. 08

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TOPICS

01 English to logic expression

02 Nested quantifier

03 TYPE 5

Quantifier with I.R.:

Universal specification:

ALL

$P(a)$

$a = 1, 2, 3, 4.$

$$\underbrace{\forall x\, P(x)}_{T} \rightarrow P(a) \quad \left(\text{a is for all elements}\right)$$

$P(x): x^2 \leq 9.$

$\forall x\, P(x)$

$D: \{1, 2, 3\}.$

$\underbrace{\forall x\,(x^2 \leq 9)}_{T.} \rightarrow P(1)\ (1^2 \leq 9)$

$\rightarrow P(2)\ (2^2 \leq 9)$

$\rightarrow P(3)\ (3^2 \leq 9)$

$\forall x\, P(x) \rightarrow \left(P(a)\right.$ a is forall$\left.\right)$

Existential specification:

$$\exists x\, p(x) \rightarrow p(a)$$

[a is specific]

D: $\{1, 2, 3\}$.

$p(x): x^2 = 4$.

$$\exists x\, p(x)$$

$$\exists x\, (x^2 = 4) \rightarrow p(1)$$

$T = p(2)$

$p(3)$

Universal Generalization

$p(a) \rightarrow \forall x\, p(x)$

a is for all

Existential Generalization

$p(a)$ $a = 0$

(a is fixed) $\rightarrow \exists x\, p(x)$

→ All the students in class will go to IIT $\forall x\,[C(x) \rightarrow IIT(x)]$

→ Some students in class are not solving Questions $\exists x\,[C(x) \wedge \neg Q(x)]$

→ Some students are not solving question will go to IIT $\exists x\,[\neg Q(x) \wedge IIT(x)]$

$$\left[\forall x\,[C(x) \rightarrow \| T(x)] \wedge \exists x\,[C(x) \wedge \neg Q(x)]\right] \rightarrow \exists x\left[\neg Q(x) \wedge \| T(x)\right]$$

1) $\forall x\,[C(x) \rightarrow \| T(x)]$ (Given) (T)

2) $C(a) \rightarrow \| T(a)$ [a is for all U.S] $a = 1, 2, 3 \ldots 10$
$(a = 1 \ldots 10)$

5) $C(a)$ [4(A), a=5]
$(a = 5)$

6) $\| T(a)$ [a = 5]

3) $\exists x\,[C(x) \wedge \neg Q(x)]$ Given (T)
$(a = 5)$

4) $C(a) \wedge \neg Q(a)$ [a is fixed]

4A) $C(a)$ [4, simplification Rule]
$a = 5$

4B) $\neg Q(a)$ [4, simplification]

7) $\neg Q(a) \wedge \| T(a)$ [a = 5]

8) $\exists x\,[\neg Q(x) \wedge \| T(x)]$ (E.G)

Conjunction:

$$\begin{array}{c} p_1 \\ \wedge \\ p_2 \\ \wedge \\ p_3 \\ \wedge \\ \vdots \\ \hline \end{array}$$

$$\begin{array}{c} p_1 \\ \wedge \\ p_2 \\ \hline p_1 \wedge p_2. \end{array}$$

$$(p_1) \wedge (p_2) \longrightarrow p_1 \wedge p_2$$

Q.1: $\left(\forall x P(x) \wedge \forall x[P(x) \rightarrow Q(x)]\right) \rightarrow \forall x Q(x)$

1) $\forall x[P(x) \rightarrow Q(x)]$ [Given/T]
2) $P(a) \rightarrow Q(a)$ [U.S/a is Forall]
3) $\forall x P(x)$ (Given]
4) $P(a)$ [a is Forall]
5) $P(a)$ [4]

6) $Q(a)$ [a is all/m.p]
7) $\forall x Q(x)$ [6, U.G]

Q.2: $\left(\forall x\,[p(x) \rightarrow Q(x)] \wedge \exists x\, p(x)\right) \rightarrow \exists x\, Q(x)$

1) $\forall x\,(p(x) \rightarrow Q(x))$

2) $\exists x\, p(x)$

1A) $p(a) \rightarrow Q(a)$ [a = all / U.S]

3) $p(a)$ [a = Fixed]

4) $p(a)$ ([a = Fixed] / 3)

4A) $Q(a)$ [a = Fixed]

4B) $\exists x\, Q(x)$ [Existential generalisation]

$\exists x\,[P(x) \wedge Q(x)] \longrightarrow \exists x P(x) \wedge \exists x Q(x)$.

1) $\exists x\,[P(x) \wedge Q(x)]$

2) $P(a) \wedge Q(a)$ [a is fixed $a = 3$]

3) $P(a)$ (2, simpl)   4) $Q(a)$ (2, simpl)

$\exists x P(x)$ (E.G) $\exists x Q(x)$ (E.G)

$\exists x P(x) \wedge \exists x Q(x)$

$$\exists x P(x) \wedge \exists x Q(x) \rightarrow \exists x [P(x) \wedge Q(x)]$$

1) $\exists x P(x)$

2) $P(a)$ [a is fixed)

3) $\exists x Q(x)$

4) $Q(b)$ [b is fixed]

$$\exists x P(x) \wedge \exists x Q(x) \not\rightarrow \exists x [P(x) \wedge Q(x)]$$

$\boxed{T \vee F}$ $\quad$ $\boxed{F \vee T}$

$\boxed{T \wedge F}$ F

$\vee$

$\boxed{F \wedge T}$ F

$$\left[\exists x\,[P(x) \to Q(x)] \wedge \exists x\, P(x)\right] \longrightarrow \exists x\, Q(x)$$

(Invalid)

1) $\exists x\,[P(x) \to Q(x)]$

3) $\exists x\, P(x)$

2) $P(a) \longrightarrow Q(a)$ [a is fixed]
(a = 5)

4) $P(a)$ [a is fixed]
(a = 3)

all → all
all
———
all

all → all
some
———
some

some → some
some
———
X

$P(x): x+1=4 \quad Q(x): x+1=3 \quad D: \{1, 2, 3\}$

$x=1 \quad 1+1 \neq 4 \qquad x=1 \quad 1+1 \neq 3$

$x=2 \quad 2+1 \neq 4 \qquad x=2 \quad 2+1 = 3 (T)$

$x=3 \quad 3+1 = 4 (T)$

$$\exists x P(x) \wedge \exists x Q(x) \not\Rightarrow \exists x (P(x) \wedge Q(x))$$

$\forall x\,[p(x) \lor q(x)]$

$\exists x\, \neg p(x)$

$\forall x\,[\neg q(x) \lor r(x)]$

$\forall x\,[s(x) \rightarrow \neg r(x)]$

$\therefore \exists x\, \neg s(x)$

$p(a) \lor q(a)$ [a = all]

$\neg p(a)$ [a = Fixed]

$q(a)$ [a = Fixed]

$\neg q(a) \lor R(a)$ [a = all]

$R(a)$ [Fixed]

$\neg S(a) \lor \neg R(a)$ [all]

$\neg S(a)$ (fixed)

$\forall x\,[p(x) \lor q(x)]$

$\forall x\,[(\neg p(x) \land q(x)) \rightarrow r(x)]$

$\therefore \forall x\,[\neg r(x) \rightarrow p(x)]$

$\forall x\,[p(x) \rightarrow (q(x) \land r(x))]$

$\forall x\,[p(x) \land s(x)]$

$\therefore \forall x\,[r(x) \land s(x)]$

No mothers are male.
Some males are politians.

Some politians are not mother.

$$mL(a) \to \neg mT(a)$$

$$\cancel{mL(a)} \quad \neg mT(a)$$

$$\forall x\,[mT(x) \to \neg mL(x)]$$

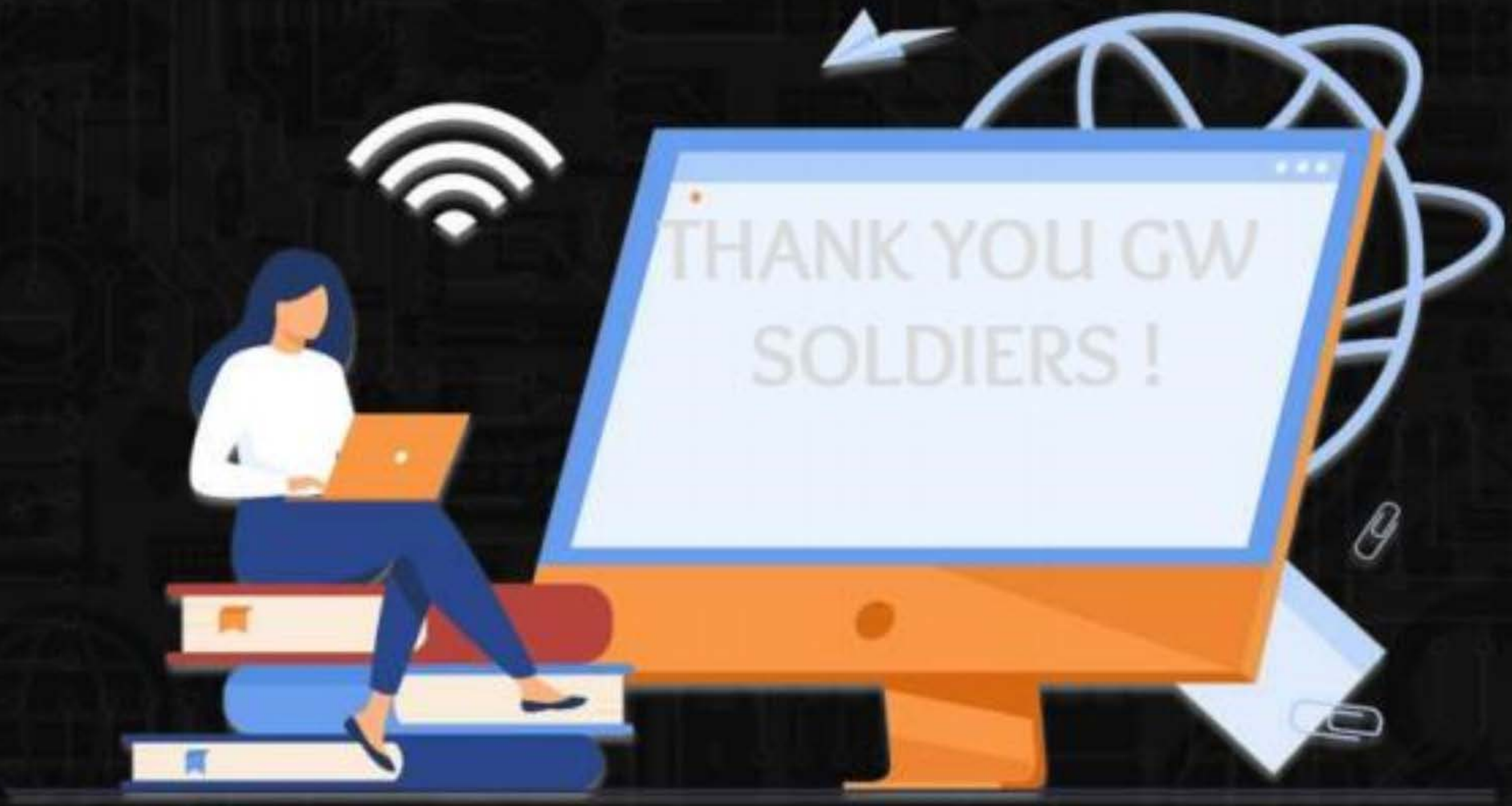
$$\exists x\,[mL(x) \wedge pL(x)]$$

$$\overline{\exists x\,[pL(x) \wedge \neg mT(x)]}$$

$$\neg mT(a) \vee \neg mL(a)$$

$$mL(a)$$

$$\overline{\neg mT(a)}$$


THANK YOU GW
SOLDIERS !