# CS & IT ENGINEERING

DISCRETE MATHS
GRAPH THEORY

Lecture No. 5

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# Connectivity

01 Types of Graphs -1

02 Types Of graphs -2

03 Practice

$G + \bar{G} = K_n$

$$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$$

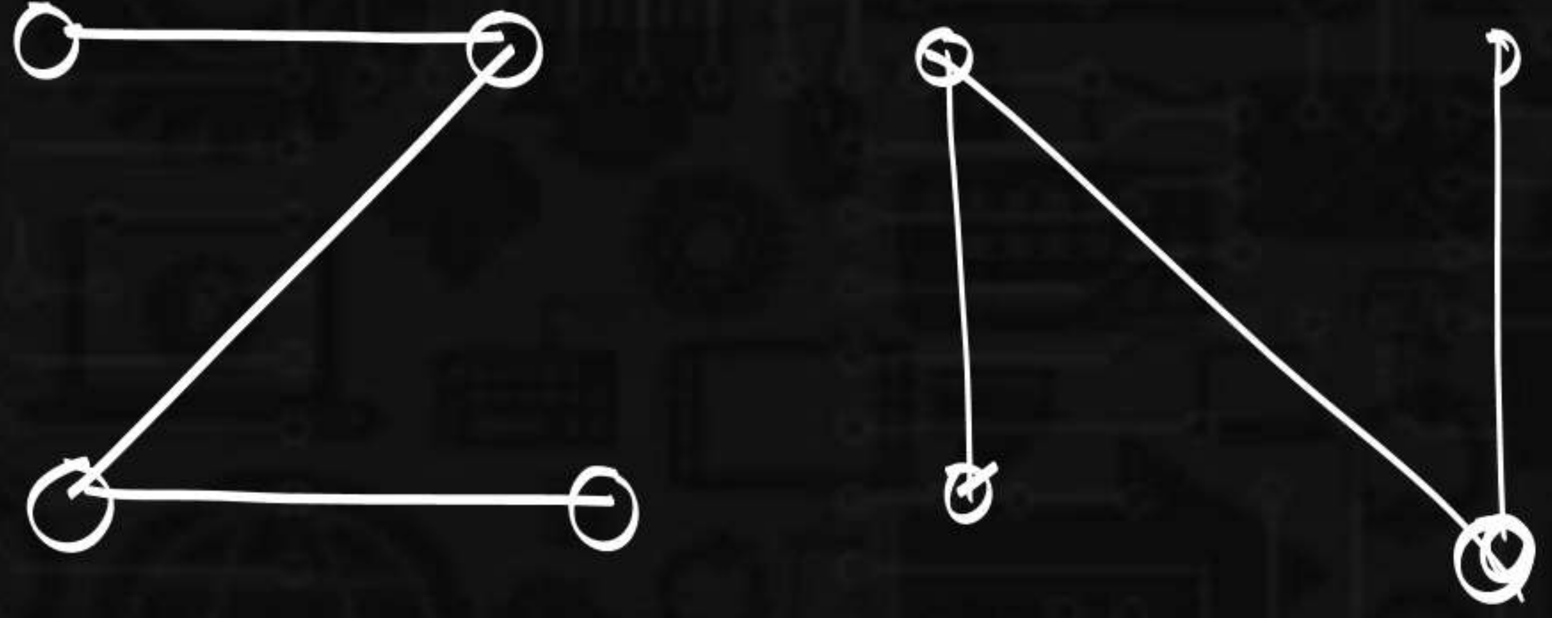
$K_n \quad n-1 \quad n-1 \quad \cdots n-1$

$G \rightarrow d_1, d_2, d_3 \cdots d_n$

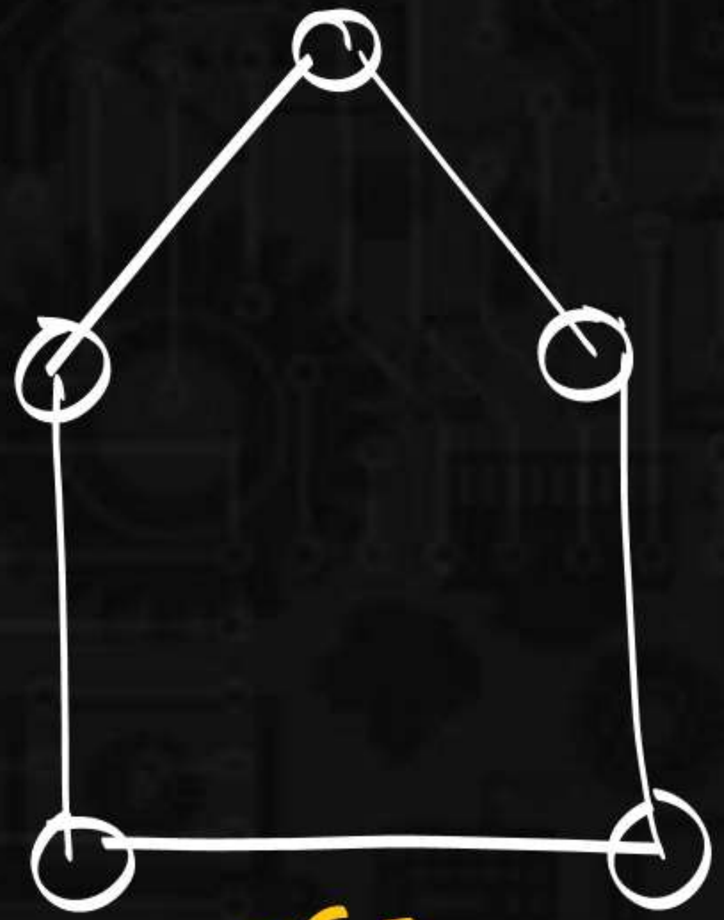
$\bar{G} \quad n-1-d_1, n-1-d_2, n-1-d_3 \cdots n-1-d_n$

Self-complement Graph ($G \equiv \bar{G}$) Graph is same as it's own complement.

$G$ $\bar{G}$

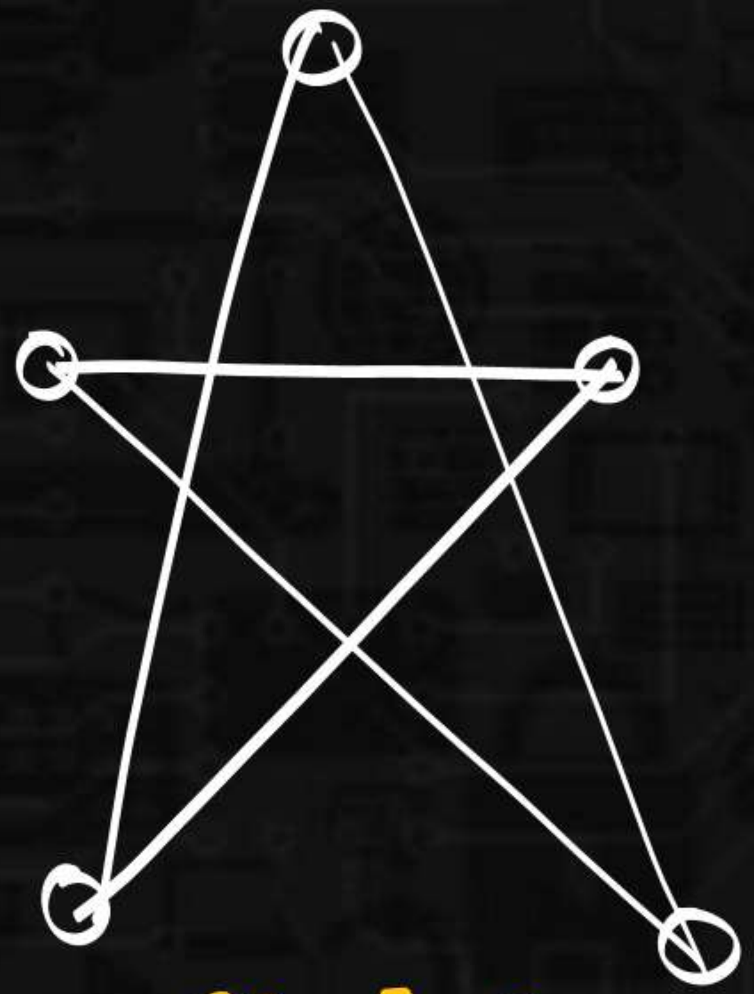


$C_5$ $G$



$\equiv$

$\bar{G}$ $C_5$



$C_5$

$n = e = 5$

$2, 2, 2, 2, 2.$

$n = 5 = e$

$2, 2, 2, 2, 2.$

$G$ is same as $\bar{G}$

$G + \bar{G} = K_n$

$\underbrace{e(G)}_{e} + \underbrace{e(\bar{G})}_{e} = \frac{n(n-1)}{2}$

$e + e = \frac{n(n-1)}{2}$

$2e = \frac{n(n-1)}{2}$

$$\boxed{e = \frac{n(n-1)}{4}}$$

$e = \frac{n(n-1)}{4}$ $\qquad \frac{n}{4}$ OR $\frac{n-1}{4}$

✓ $n = 4$ $\to 0$ $\quad e = \frac{n(n-1)}{4} = \frac{\cancel{4} \cdot 3}{\cancel{4}} = 3$ ✓

$\to 1$

$$\boxed{✓\ n = 5 \quad e = \frac{n(n-1)}{4} = \frac{5 \cdot \cancel{4}}{\cancel{4}} = 5\ ✓}$$

✗ $n = 6$ $\quad e = \frac{6 \cdot 5}{4} = \frac{15}{2} = 7.5$ ✗

✗ $n = 7$ $\quad e = \frac{7 \cdot 6}{4} = -$ ✗

✓ $n = 8$

✓ $n = 9$

$a \equiv b \pmod{n}$

$a$ & $b$ ______ , , ____________________ w.r.t $\underline{\underline{n}}$

1 & 5 are having same remainder w.r.t 4.

$$1 \equiv 5 \pmod{4}$$

$$a \equiv b \pmod{n}$$

$$\frac{a-b}{n} \in Z$$

$\frac{n}{4}$ OR $\frac{n-1}{4}$

$\frac{n-0}{4}$ OR $\frac{n-1}{4}$

$n \equiv 0 \pmod{4}$ OR $n \equiv 1 \pmod{4}$

$$\boxed{n \equiv 0 \text{ OR } 1 \pmod{4}}$$

Cycle Graph.

$C_n$

$n = 5$

C3.

C4.

C5. → self-complement e = 5

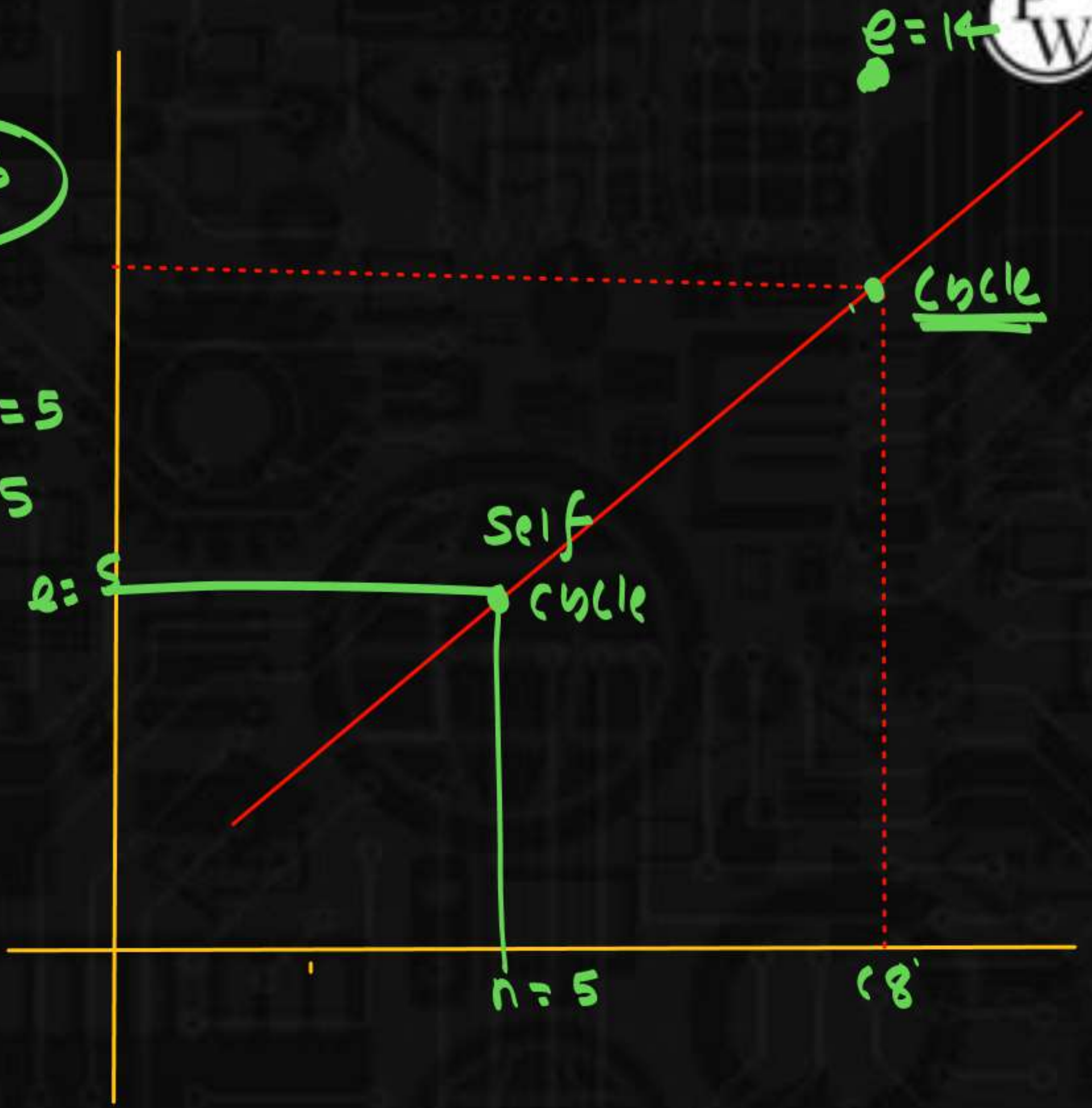
Cycle Graph e = 5

C6. ✗

C7. ✗

C8. cycle Graph e = 8.

C9 Selfcomplement 14.



for self complement Graph in $C_n$.

$n = ?$

$n = 5$

Graph vertices are represented as n-bit signal, and 2 vertices are connected if there bit position changes by 1 bit

What will be total edges?

$n = 2$ bit

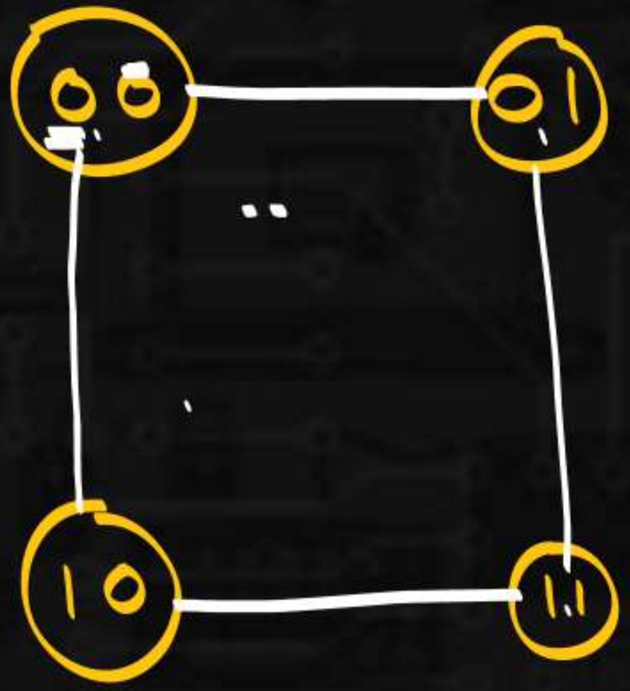
Total vertices $= 2^2$

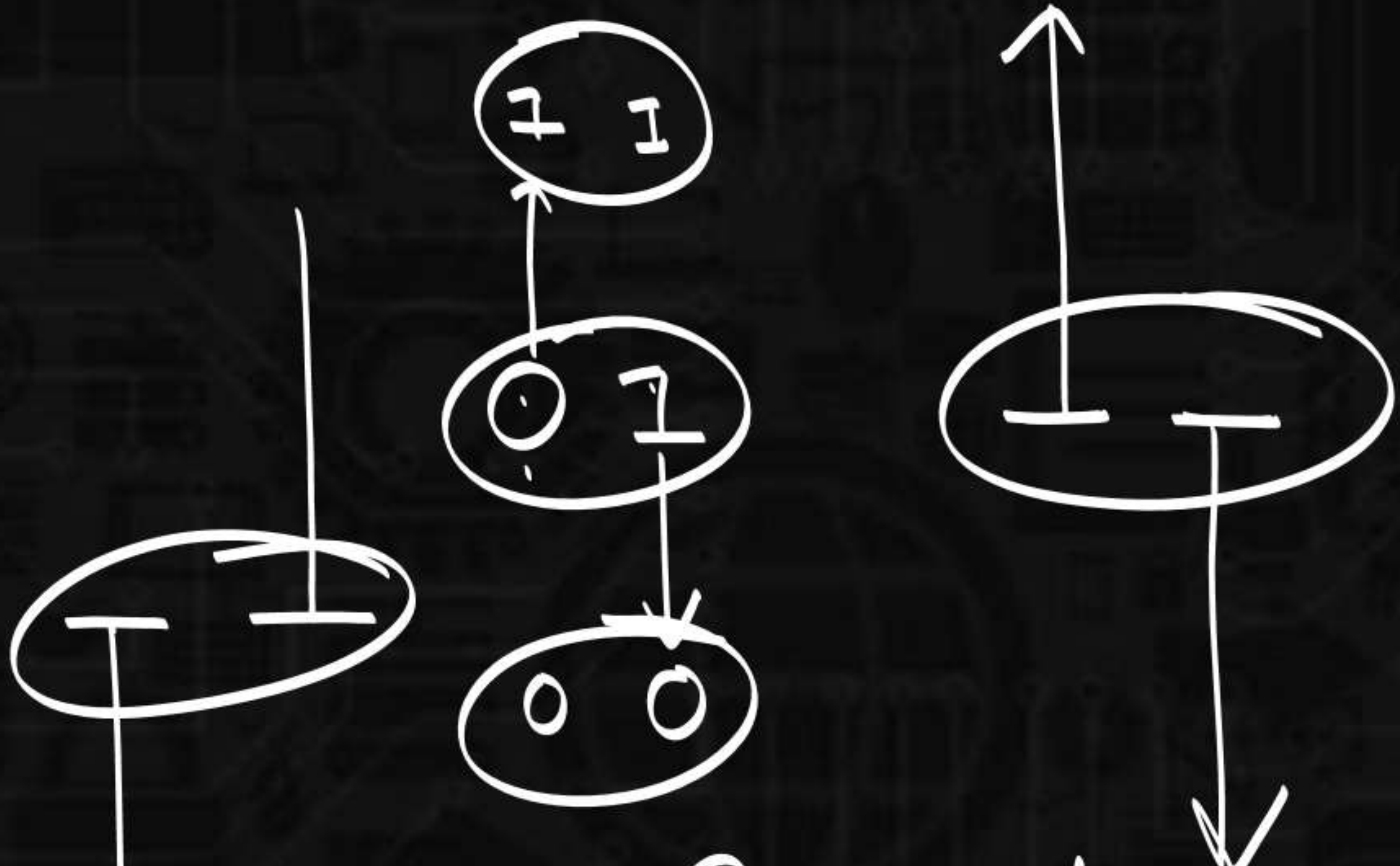
00

01

10

11.



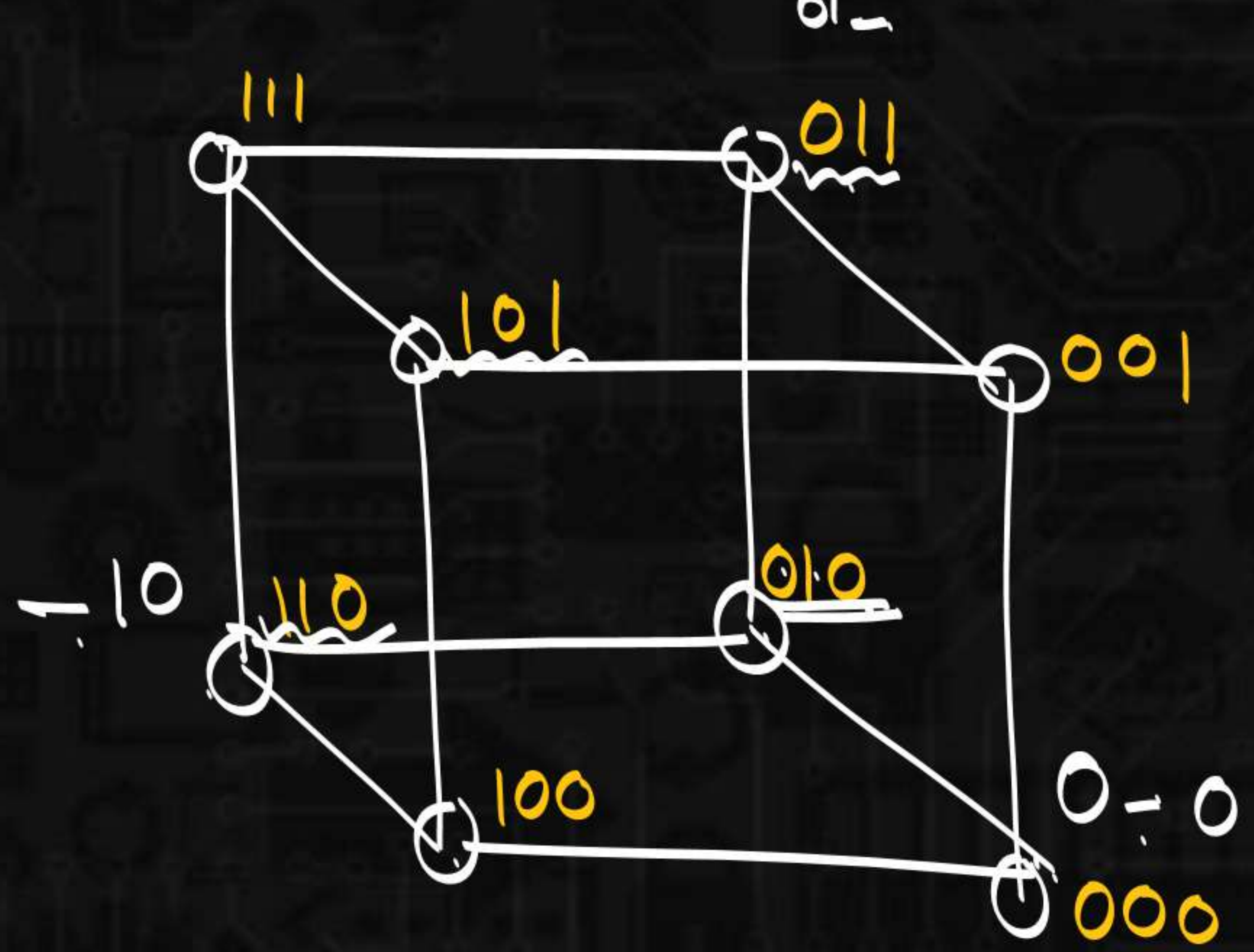

Degree of each vertex will be 2

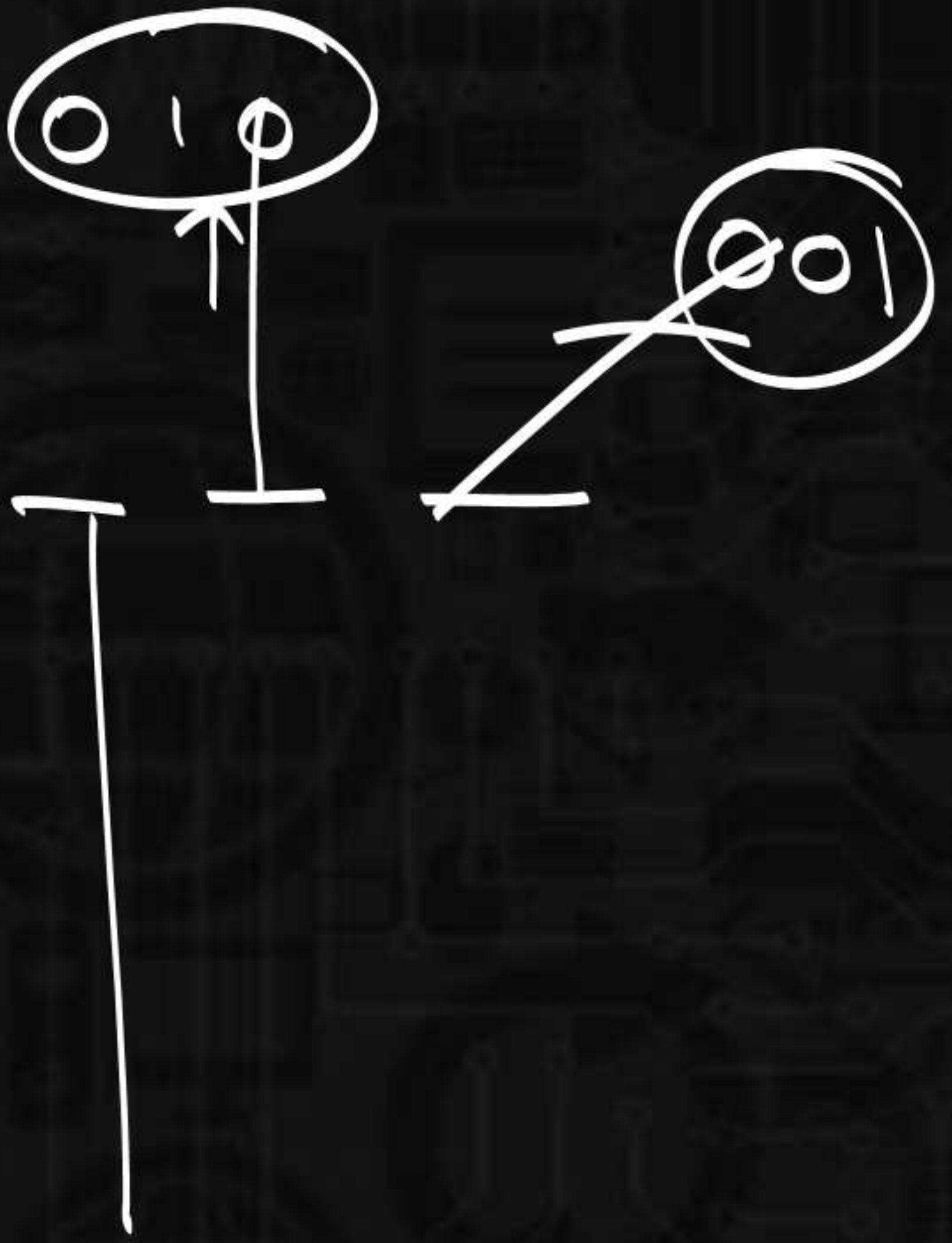
Degree of each vertex will be 3.

$n = 3$ bit

Total vertices $= 2^3$

000
001
⋮
111



$n \rightarrow$ bit signal

Total vertices $(n) = \underline{2^n}$.

Degree of each vertex $= \underline{n}$.

$\sum d(v_i) = 2e$.

$2^n \cdot n = 2e$.

$$\boxed{e = n \cdot 2^{n-1}}$$

3 bit signal

Total vertices $= 2^3$.

Degree of each vertex 3.

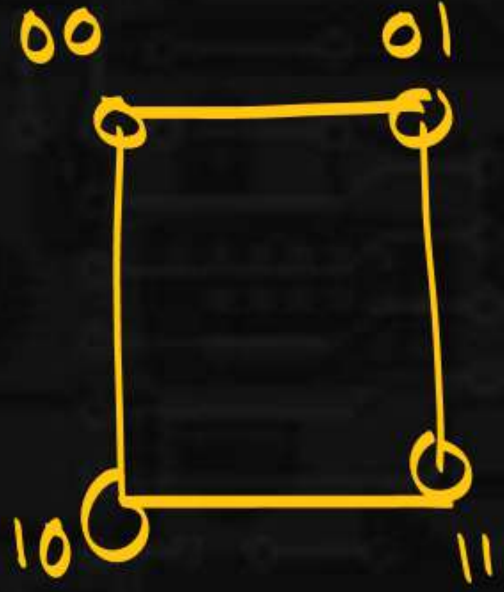
Hypercube ($Q_n$) ($n \geq 1$)

n-bit signal.

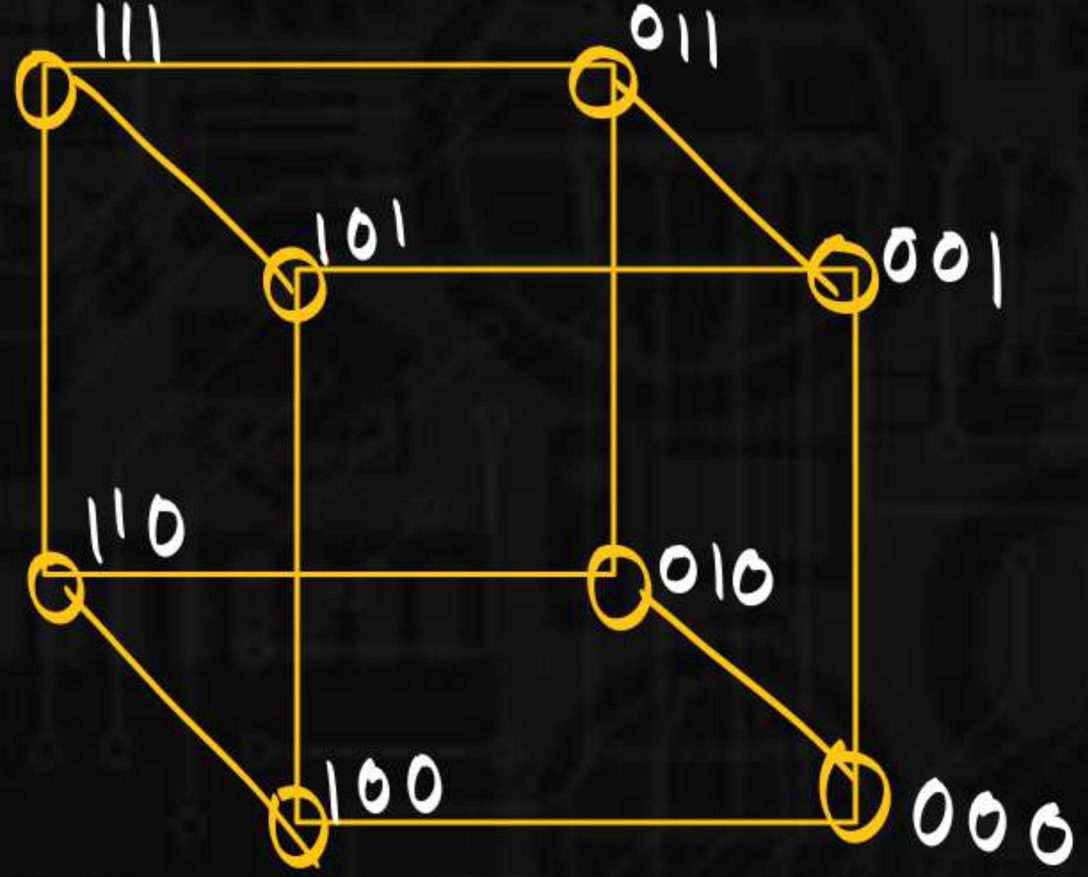
$$e = n \cdot 2^{n-1}$$

$Q_1$


$Q_2$




$Q_3$



$e(G) = n \cdot 2^{n-1}$.

Total vertices $= x = 2^n$.

$e(G) + e(\bar{G}) = K_x$.

$n \cdot 2^{n-1} + e(\bar{G}) = \frac{x(x-1)}{2}$.

$n \cdot 2^{n-1} + e(\bar{G}) = \frac{2^n(2^n-1)}{2}$.

$e(\bar{G}) = \frac{2^n(2^n-1)}{2} - n \cdot 2^{n-1}$.

$x = \text{Total vertices} = 2^n$

$K_x \quad x-1 \quad x-1 \quad x-1$

$G \rightarrow n,\ n,\ n,\ n,\ n \ldots\ldots n$

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$\bar{G} \quad x-1-n,\ x-1-n,\ x-1-n$

$2^n-1-n,\ 2^n-1-n,\ 2^n-1-n \ldots\ldots$

Consider a graph vertices are represented as a no. from

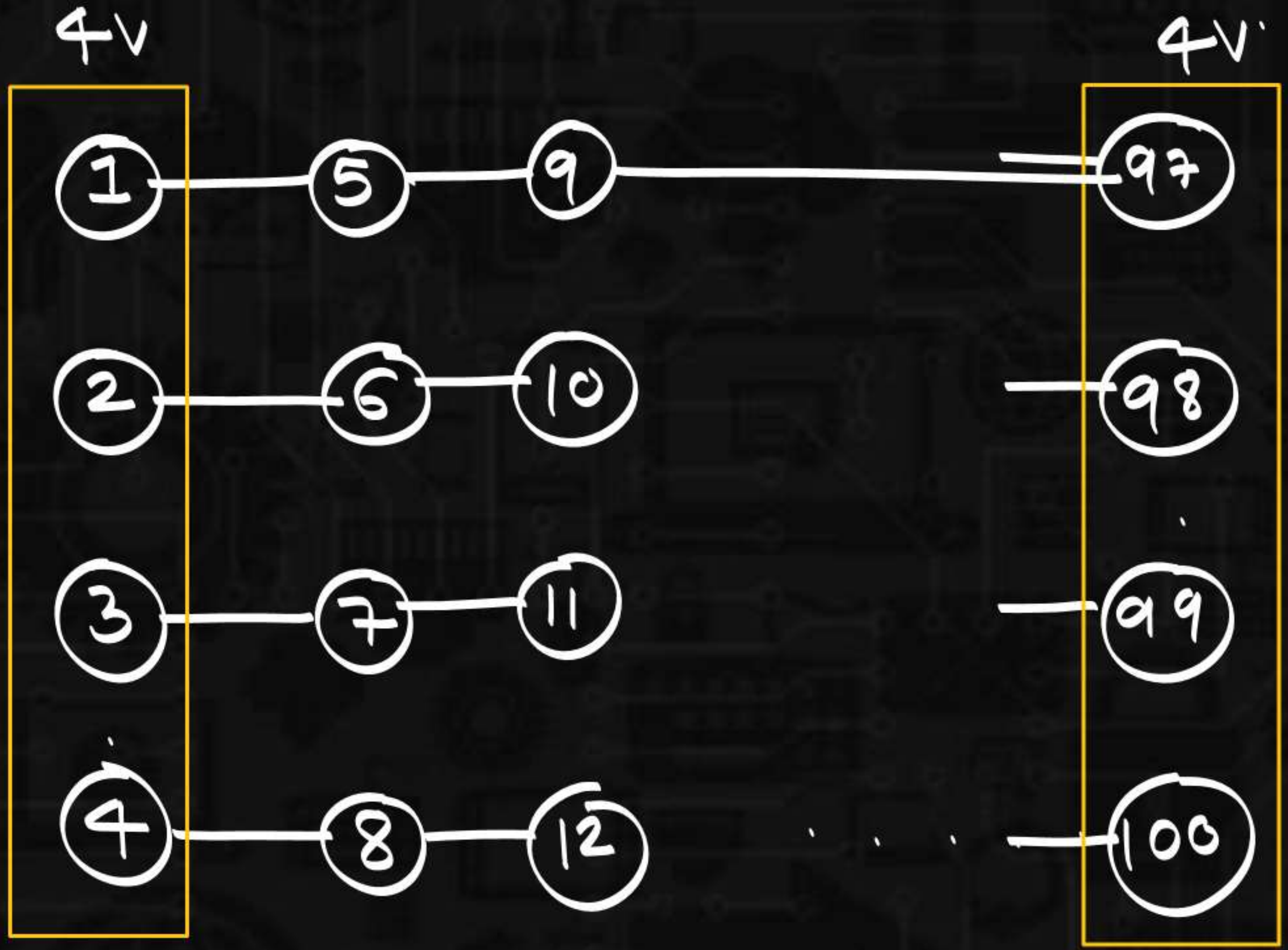
$\{1 \ldots\ldots 100\}$

two vertices are connected $|a-b|=4$ eg $|1-5|=4$ OR there difference is 4.

what will be total edges in this ?

eg $|1-5|=4$

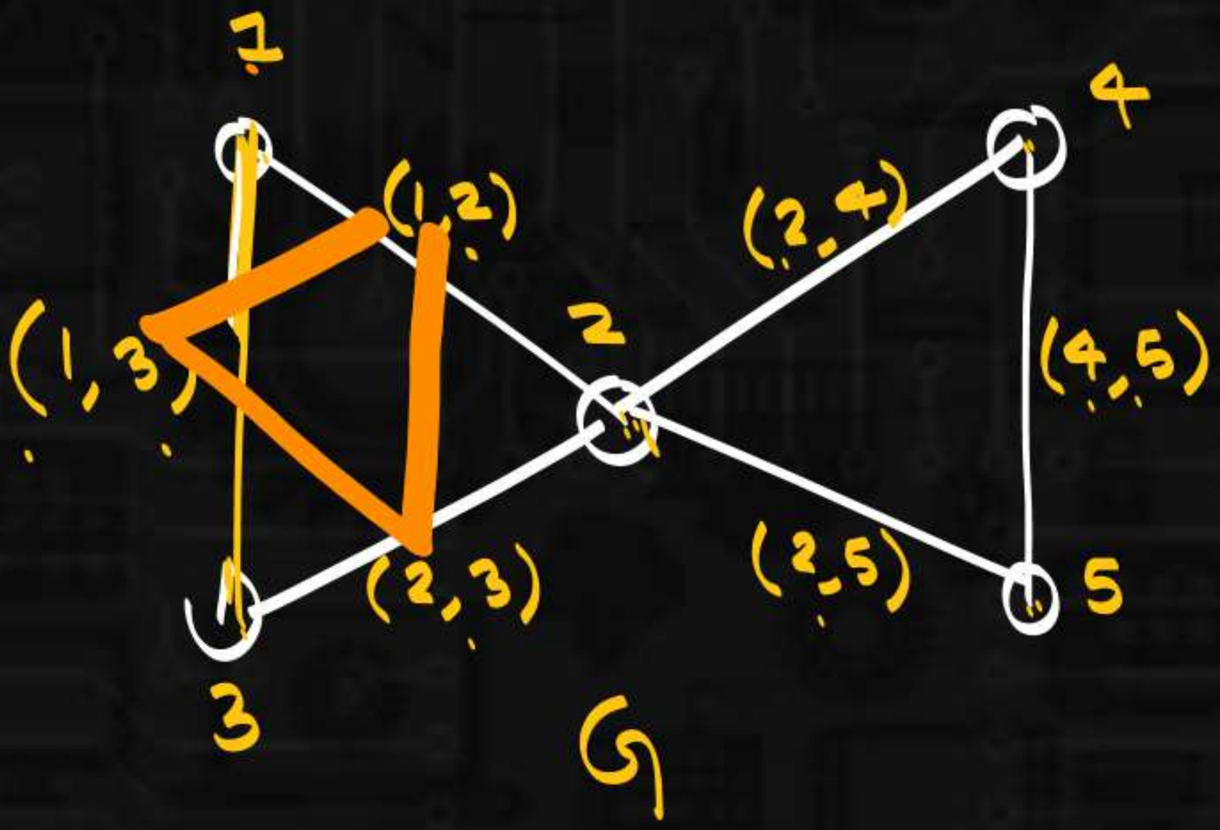
$|5-9|=4$.




$4\times1+4\times1+92\times2=2e$

$e=96$

Line Graph. L(G)

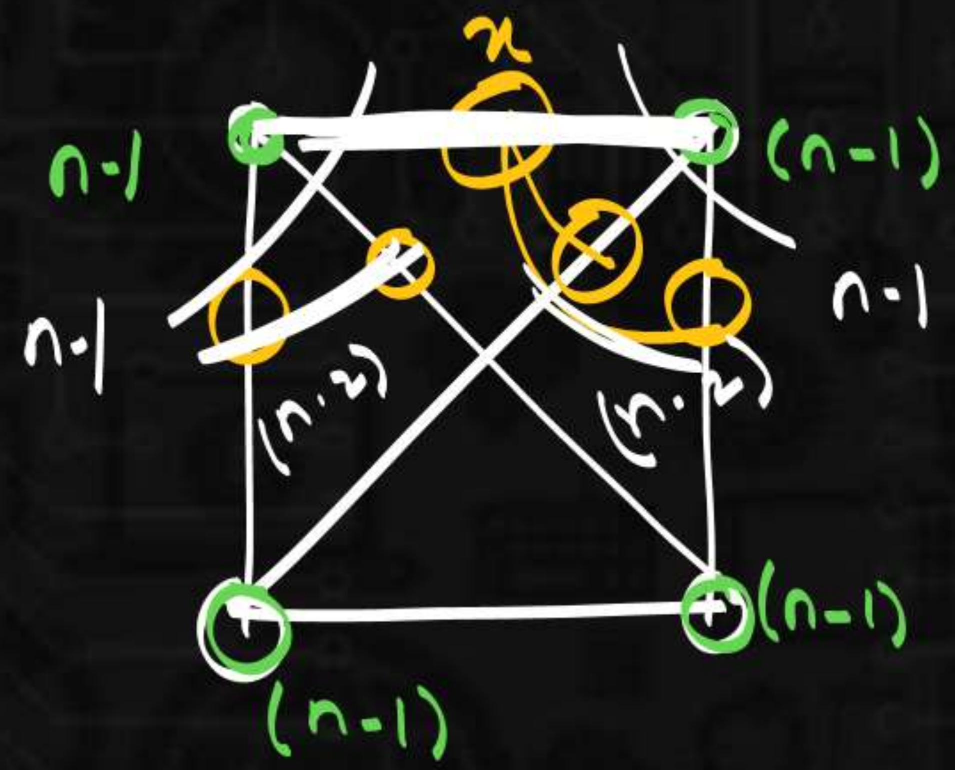


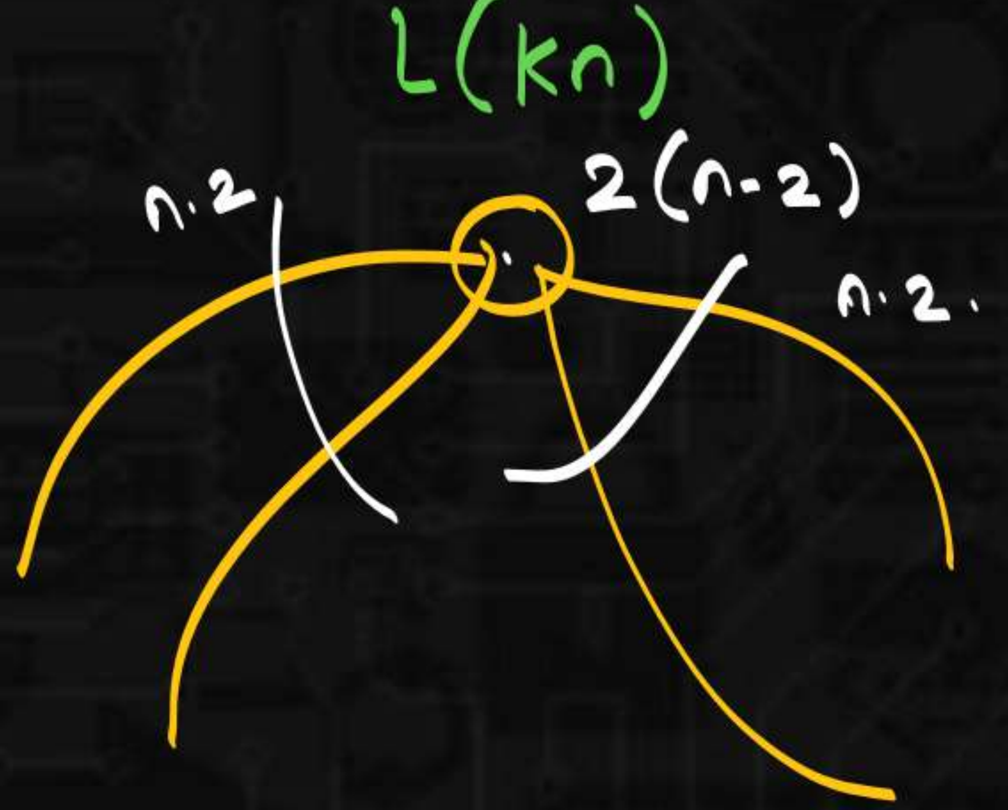


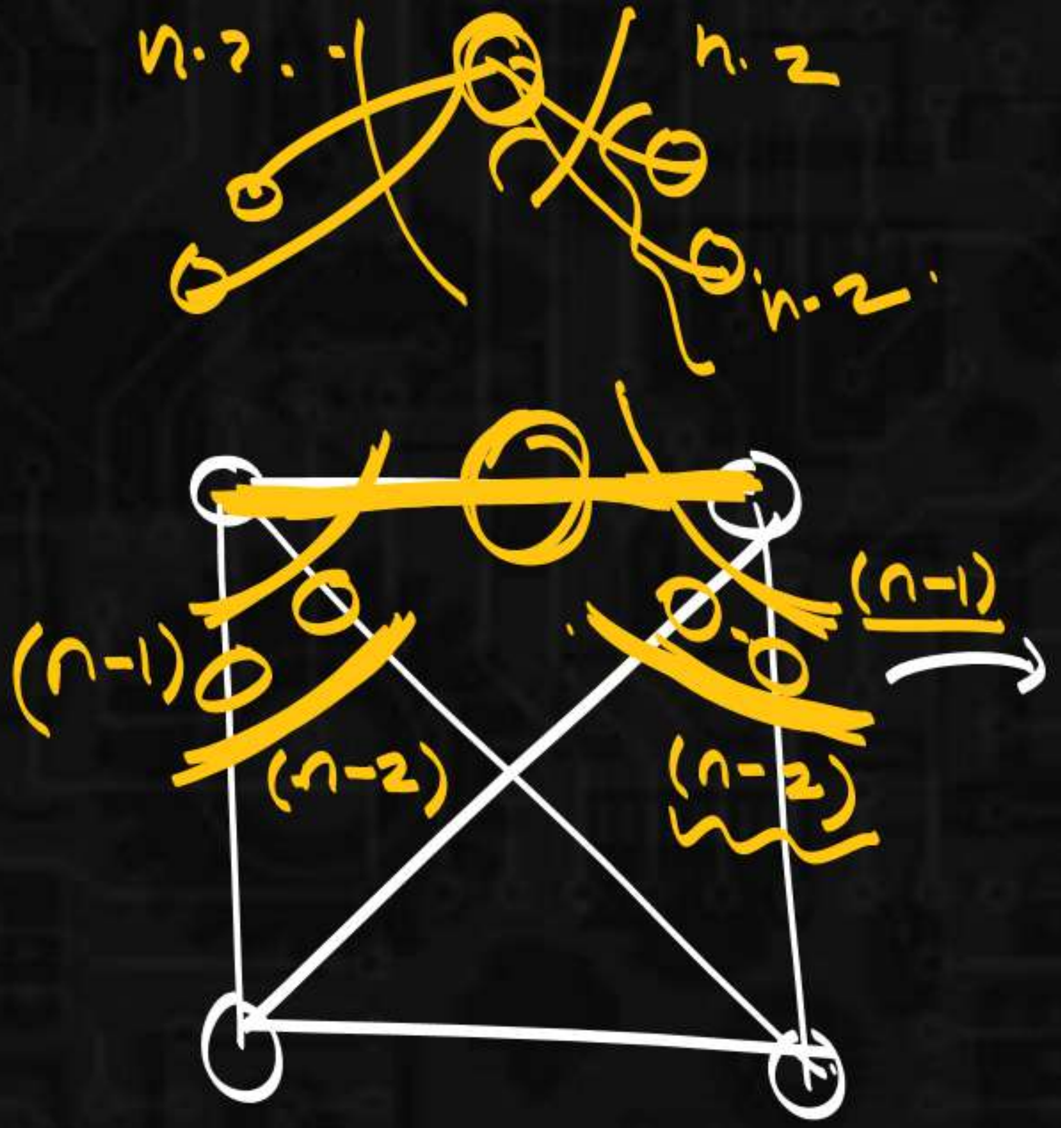
Line graph
↓
edge graph

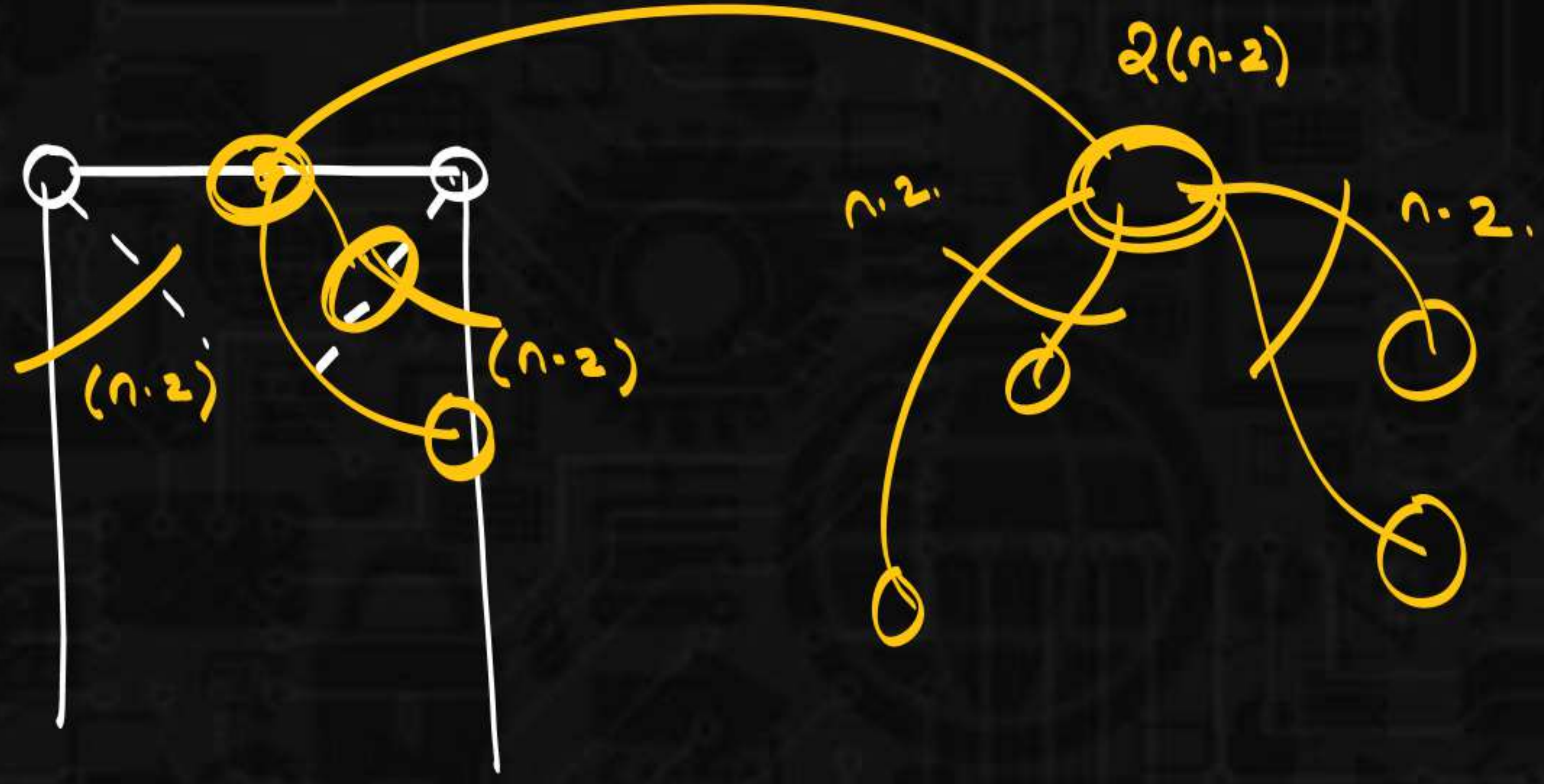
What will be degree sequence of $L(K_n)$ ?

$K_n$ / $K_4$.




n.2
n.2.
(n-1)
(n-2)
(n-1)
(n-2)


(n.2)
(n.2)
2(n-2)
n.2.
n.2.

Degree of each vertex in $L(K_n)$ is $2(n-2)$.


THANK YOU GW
SOLDIERS !