CS & IT ENGINEERING

Discrete maths
Mathematical logic

Lecture No 04

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01 Open stmnt

02 predicate variable

03 Predicate logic

04 Quantifier

05 Practice

predicate variable.

predicate logic → T / F

$P(x)$: x is even no. (open stmt)

$P(0)$: 0 is even no. (Tvue) → simple

$P(1)$: 1 is even no. (false) – Simple.

$x^2 \geq 0$

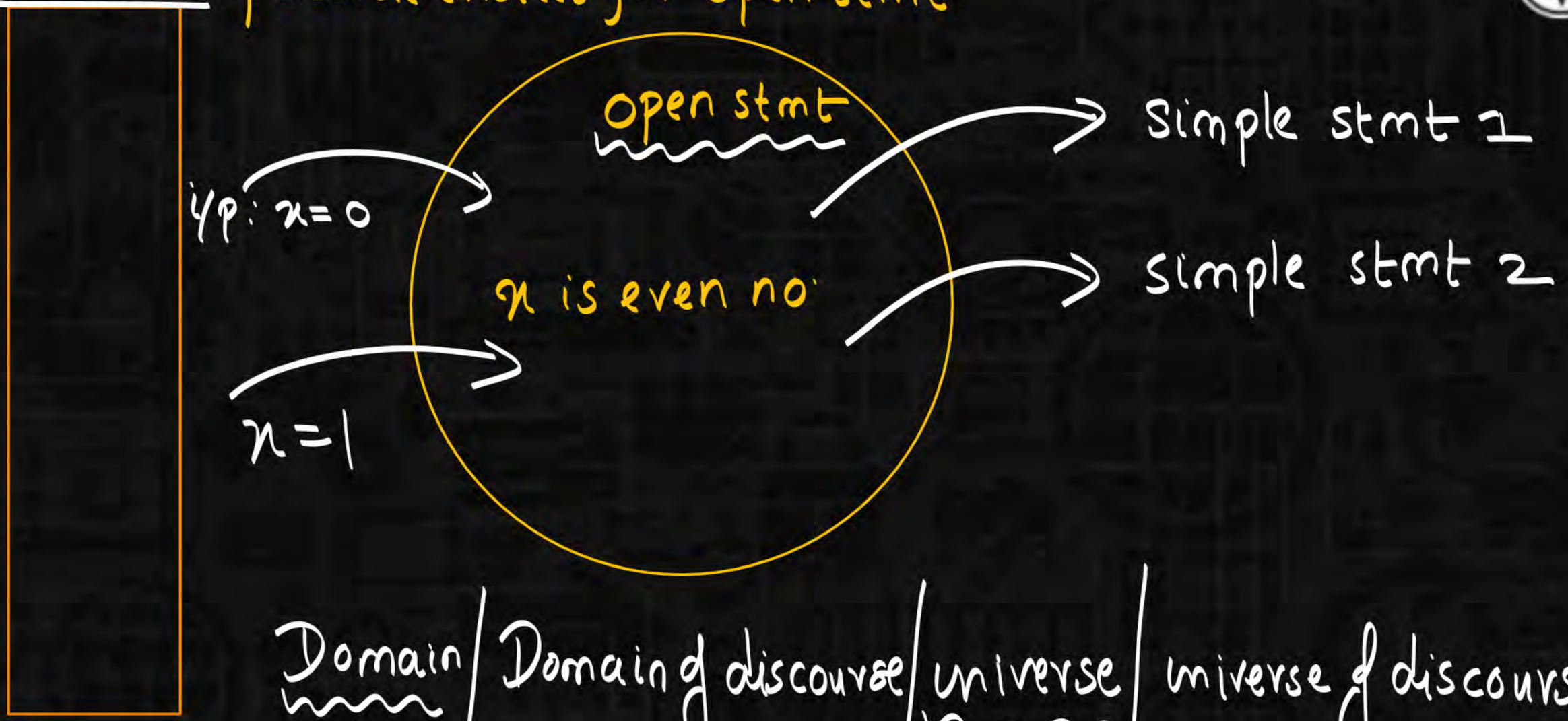
$x^2 + y^2 \leq 9$.

open stmt:

it is a type of stmt where we can not directly define truth value.

once we provide input it changes, simple stmt.

Domain: possible choices for open stmt.



Domain / Domain of discourse / universe / universe of discourse.

Domain:

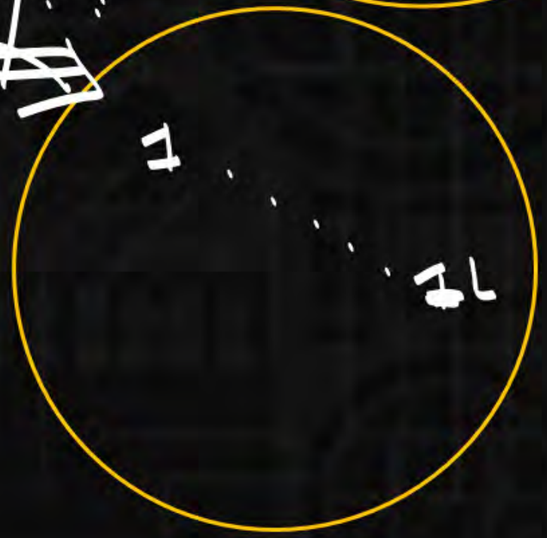
x is even no.

1 is even no(F)

Quantity

ALL

SOME



Quantity → Truth value.

Tool :

→ Tool, which used to find out truth value in terms of quantity (ALL)

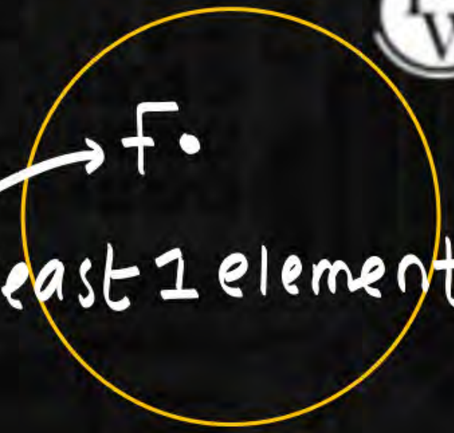
ALL (universal quantifier) $(\forall x)$

$(\forall x)$ Universal quantifier (ALL)

$\forall x\, p(x)$:

for all value of x, such that it satisfies $p(x)$

$\forall x\, p(x)$ → True



$\neg p(x)$ → false

> for all of x, such that $p(x)$
>
> for every value of x, such that $p(x)$

$D: \{1, 2, 3\}$.

$P(x): x^2 \leq 9$.

$\forall x\, P(x)$:

OR.

$\forall x (x^2 \leq 9)$

$x = 1 \quad 1^2 \leq 9$ (T)

$2^2 \leq 9$ (T)

$3^2 \leq 9$ (T)

True.

$D: \{1, 2, 3, \ldots\}$.

$P(x): x^2 \leq 9$.

1: T, 2: T, 3: T, 9: F.

$\forall x\, P(x)$ → false.

$D: \mathbb{Z}$

$P(x): x^2 \geq 0$.

$\forall x (x^2 \geq 0)$

True.

$\forall x P(x) \rightarrow$ True

When it is True for all the elements in the Domain.

$\forall x P(x)$

↳ false.

→ atleast 1 element is false.

Existential quantifier: ($\exists x$)

→ Tool, which is used to find out truth in terms of quantity.

(some)
(atleast)

$\exists x P(x)$

there exist x . which satisfies $P(x)$

there exist x , such that $P(x)$

some of x , such that $P(x)$

atleast 1 value of x , such that $P(x)$

True.

at least 1 value of x

$\exists x P(x) \rightarrow$ True

false

all are false.

$\exists x P(x) \rightarrow$ false.

$D: \{1, 2, 3 \ldots 10\}$.

$P(x): x^2 = 4$.

$\exists x P(x)$

OR

$\exists x (x^2 = 4)$

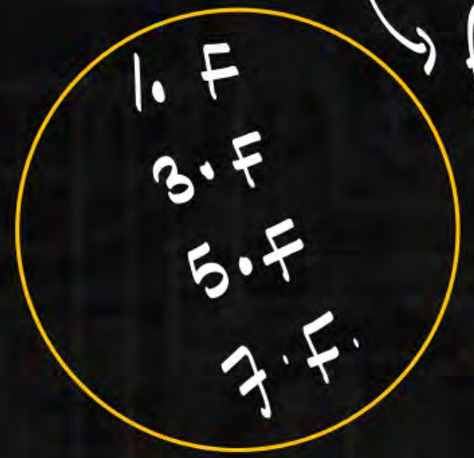
↳ True.

$D: \{1, 3, 5, 7\}$

$P(x): x^2 = 4$.

$\exists x (x^2 = 4)$

↳ false



$D: \mathbb{Z}$.

$P(x): x^2 \geq 0$.

$\forall x\, P(x)$

$\forall x (x^2 \geq 0)$

True

$D: \mathbb{Z}$.

$P(x): x^2 \geq 0$.

$\exists x\, P(x)$

$\exists x (x^2 \geq 0)$

$\forall x (x^2 \geq 0) \rightarrow \exists x (x^2 \geq 0)$

True True

$\forall x\, P(x) \rightarrow \exists x\, P(x)$ ✓

True True

Negation of quantifiers:

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

$$\therefore \neg \exists x P(x) \equiv \forall x \neg P(x)$$

$D: \{1, 2, 3\}$.

$$\neg \forall x P(x) \equiv \neg [P(1) \wedge P(2) \wedge P(3)]$$

$$\equiv \neg P(1) \vee \neg P(2) \vee \neg P(3)$$

$$\equiv \exists x \neg P(x).$$

$D: \{1, 2, 3\}$

$P(x): x+1=4$

$x=3 \rightarrow P(3)$

$x=1 \quad 1+1=4 \ (F)$

$x=2 \quad 2+1=4 \ (F)$

$x=3 \quad 3+1=4 \ (T)$

$D: \{1, 2, 3\}$

$Q(x): 2x+1=7$

$x=3$

$x=1 \quad 2(1)+1=7 \ (F)$

$x=2 \quad 2(2)+1=7 \ (F)$

$x=3 \quad 2(3)+1=7 \ (T)$

$D: \{1, 2, 3\}$

$\exists x\,[P(x) \wedge Q(x)]$

$x=1$

$P(1) \wedge Q(1)$ — F

$\vee$

$P(2) \wedge Q(2)$ — F

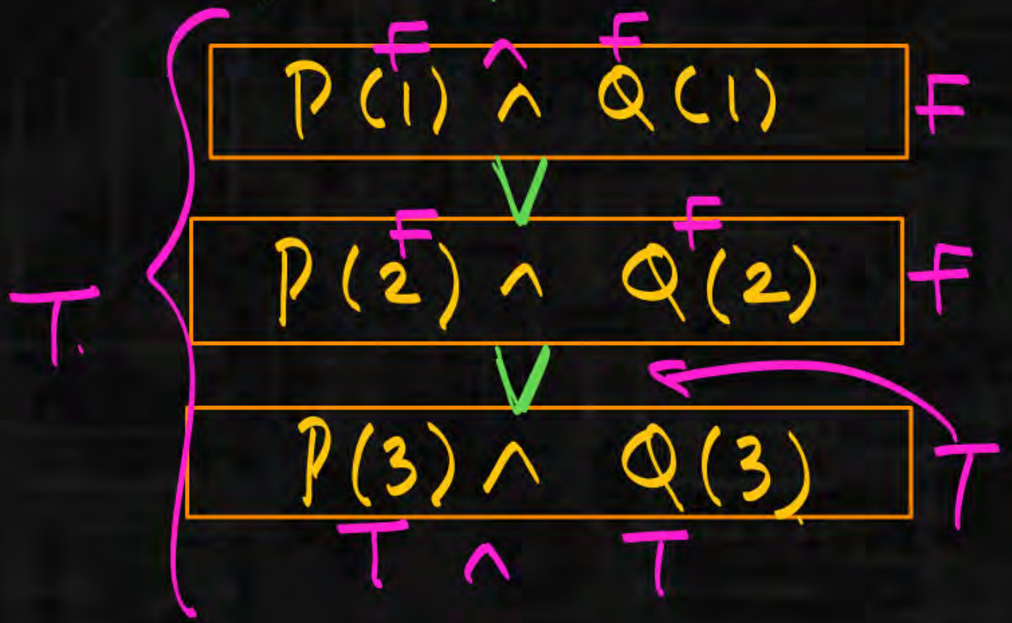
$\vee$

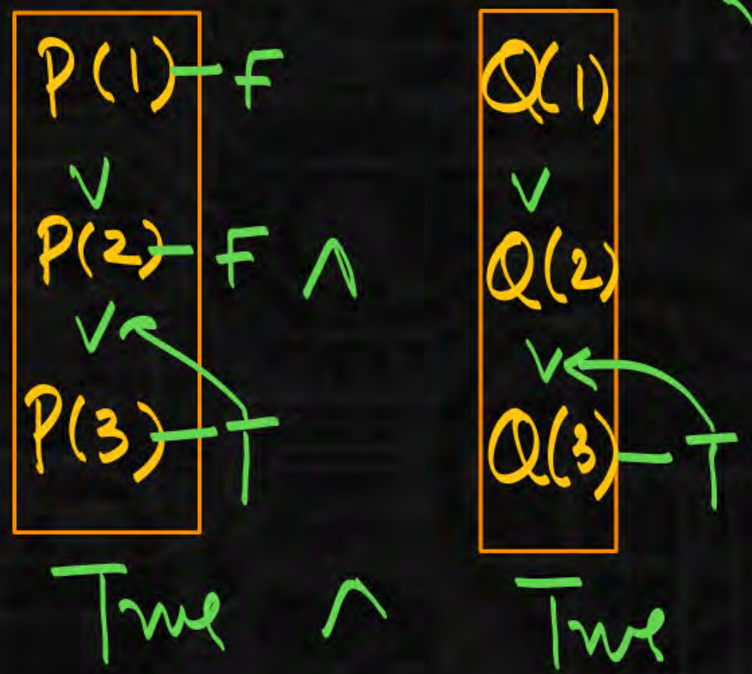
$P(3) \wedge Q(3)$ — T

True

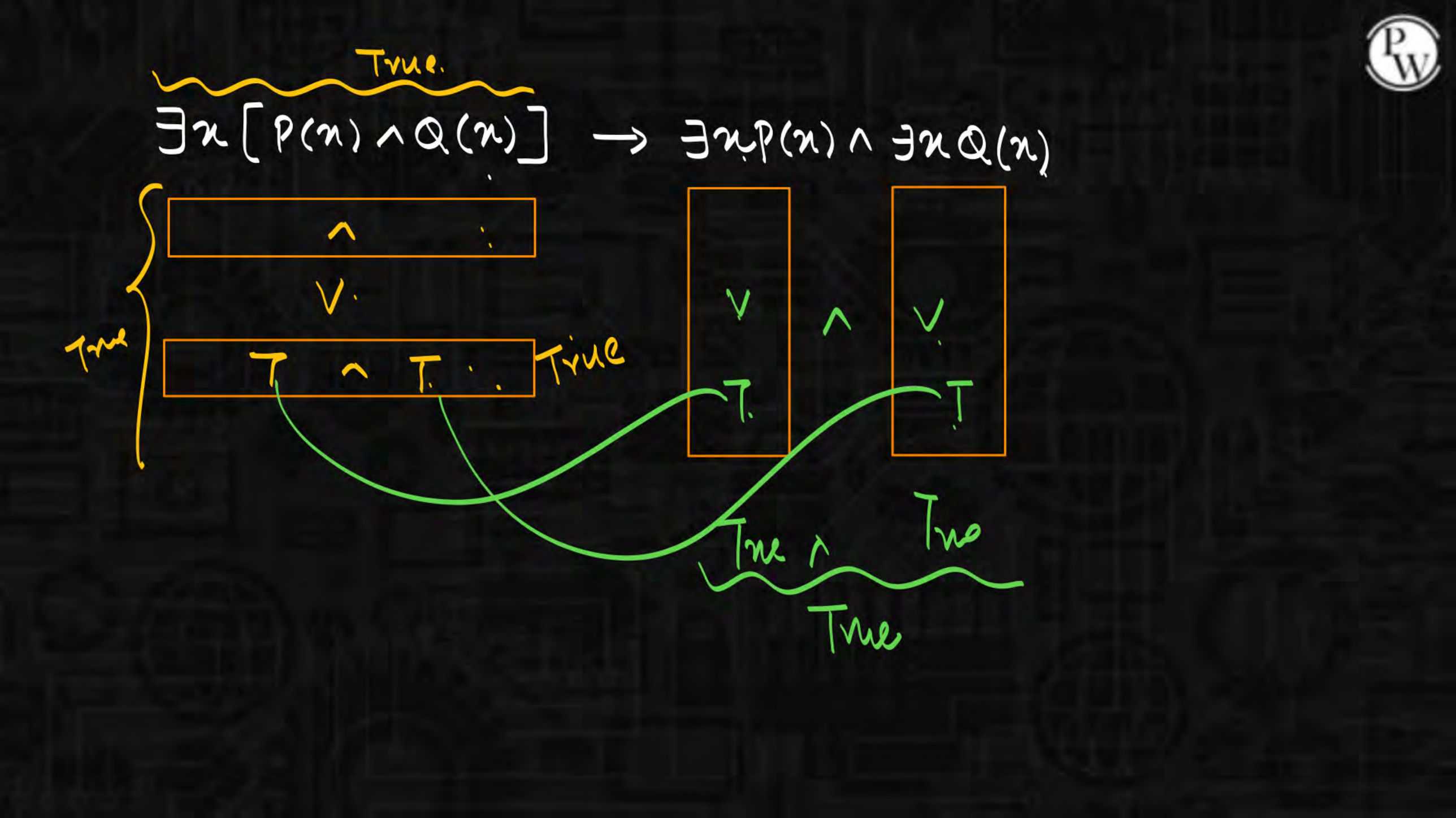
D: 1, 2, 3.

$\exists x[P(x) \wedge Q(x)] \rightarrow \exists x P(x) \wedge \exists x Q(x)$ D: {1, 2, 3}





Thm.


True.
$\exists x [P(x) \wedge Q(x)] \rightarrow \exists x P(x) \wedge \exists x Q(x)$
True
$\wedge$
$\vee$
T $\wedge$ T
True
$\vee$
T
$\wedge$
$\vee$
T
True $\wedge$ True
True

True

$\exists x P(x) \wedge \exists x Q(x) \not\rightarrow \exists x [P(x) \wedge Q(x)]$

T
V
F

True

$\wedge$

F
V
T

True

T $\wedge$ F	F
V	V
F $\wedge$ T	F

False.

1. T → T
2. T → T
3. T → F

X.

$\forall x\,[P(x) \wedge Q(x)] \equiv \forall x\,P(x) \wedge \forall x\,Q(x)$

$\forall x\,[P(x) \vee Q(x)]$ $\forall x\,P(x) \vee \forall x\,Q(x)$

$\exists x\,[P(x) \wedge Q(x)] \rightarrow \exists x\,P(x) \wedge \exists x\,Q(x)$

$\exists x\,[P(x) \vee Q(x)]$ $\exists x\,[P(x) \vee \exists x\,Q(x)]$

$\forall x\,[P(x) \rightarrow Q(x)]$ $\forall x\,P(x) \rightarrow \forall x\,Q(x)$

$\forall x\,[P(x) \Leftrightarrow Q(x)]$ $\forall x\,P(x) \Leftrightarrow \forall x\,Q(x)$


THANK YOU GW
SOLDIERS !