Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics

Set Theory

DPP-10

[MSQ]

1. The set of all positive rational numbers forms an abelian group under the composition * defined by a * b = (ab)/2.

Which of the following is/are TRUE?

(a) The identity element is 2

(b) The inverse of a is 4/a.

(c) The inverse of 4 is 1

(d) the identity element is 1

[MSQ]

2. Let r be the set of all real numbers and * is a binary operation defined by

a * b = a + b + ab.

Which of the following is TRUE?

(a) Identity element is 0.

(b) the inverse of –1 is 1.

(c) The inverse of a is –a/(a + 1).

(d) R is not a group.

[MCQ]

3. The set G = {0, 1, 2, 3, 4, 5} is a group with respect to addition modulo 6.

Which of the following is false?

(a) The inverse of 2 is 4

(b) The inverse of 3 is 3

(c) The inverse of 5 is 2

(d) The inverse of 1 is 5

[NAT]

4. G = {1, –1, i, –i}is a group w.r.t multiplication. The order –i is

[MCQ]

5. If G is a group of order p, where p is a prime number. Then the number of sub groups of G is____.

(a) 1 (b) 2

(c) p –1 (d) p

Answer Key

1. **(a, b, c)**
2. **(a, c, d)**
3. **(c)**
4. **(4)**
5. **(b)**

Hints and Solutions

1. (a, b, c)

Let e be the identity element.

$\therefore$ a * e = a

$\Rightarrow$ (ae/2) = a

$\Rightarrow$ e = 2

$\therefore$ Option (a) is true and option (d) is false.

Let a^{-1} = inverse of a

$a * a^{-1} = e$

$$\Rightarrow \frac{a \times a^{-1}}{2} = 2$$

$$\Rightarrow a^{-1} = \frac{4}{a}$$

Inverse of $4 = \frac{4}{4}$

$\therefore$ Option (b) and (c) are true.

2. (a, c, d)

Let e be the identity element.

$\therefore$ a * e = a

$\Rightarrow$ a + e + a. e = a

$\Rightarrow$ e = 0

Let a^{-1} = inverse of a

$a * a^{-1} = e$

$a + a^{-1} + aa^{-1} = 0$ ($\because$ 0 is identity element)

$$\Rightarrow a^{-1} = \frac{-a}{a+1}$$

$\therefore$ Inverse of –1 does not exist.

Hence, Option (b) is false.

3. (c)

$5 \oplus_6 2 = 1$

$\Rightarrow$ Inverse of 5 is not 2.

4. (4)

Order of (–i) = 4, because the smallest integer n such that $(-i)^n = 1$ is n = 4

5. (b)

Let (H, *) be a subgroup of order n, By Lagrange's theorem,

$\Rightarrow$ n is a divisor of p

$\Rightarrow$ n = 1 or n = p

$\Rightarrow$ H = {e} or H = G

$\therefore$ G has only 2 trivial subgroups