ENGINEERING MATHEMATICS

ALL BRANCHES

Vector Calculus
Line, surface & Volume
Integral, Stokes, Green & Gauss
Divergence Theorem

DPP-04 Solution

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Question - 01

The line integral $\int \bar{V} \cdot d\bar{r}$ of the vector $\vec{V} = 2xyz\hat{i} + x^2z\hat{j} + x^2y\hat{k}$ from the origin of the point (1, 1, 1)

A is 1

B is zero

C is zero

D cannot be determined without specifying the path

$$\int \vec{V} \cdot \vec{dr} = \int_{(0,0,0)}^{(1,1,1)} 2xyz\,dx + x^2z\,dy + x^2y\,dz$$

$$\int d(x^2yz) = [x^2yz]_{(0,0,0)}^{(1,1,1)}$$

$$= 1^2.1.1 - 0.0.0$$

$$= 1$$

Question - 02

Value of the integral $\oint_C (xy\,dy - y^2 dx)$, where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$ will be (use Green's theorem to change the line integral into double integral)

A. 1/2

B. 1

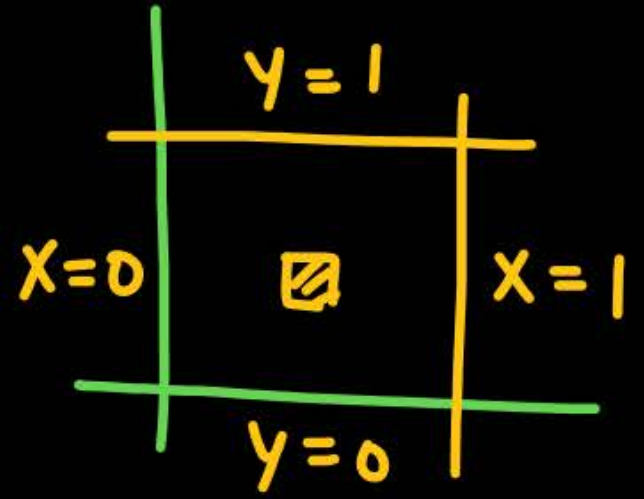
C. 3/2

D. 5/3

$$\oint M dx + N dy = \iint \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dx\, dy$$

$$\oint -y^2 dx + xy\, dy = \int_0^1\int_0^1 y - (-2y)\, dx\, dy$$

$$\int_0^1 3\left[\frac{y^2}{2}\right]_0^1 dx = \frac{3}{2}[x]_0^1 = \frac{3}{2}$$



Question - 03

Consider points P and Q in the x-y plane, with $P = (1, 0)$ and $Q = (0, 1)$. The line integral $2\int_P^Q (xdx + ydy)$ along the semicircle with the line segment PQ as its diameter

A is -1

B is 0

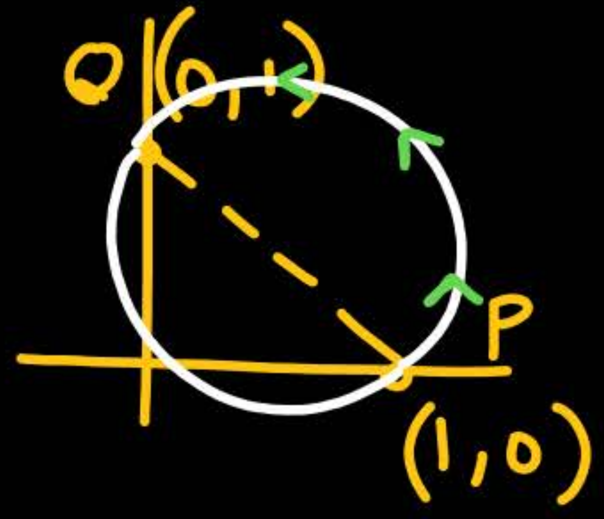
C is 1

D depends on the direction (clockwise or anti-clockwise of the semicircle)

$$I = 2\int_P^Q x\,dx + y\,dy$$

$$= 2\left[\int_1^0 x\,dx + \int_0^1 y\,dy\right]$$

$$= 2\left(\left[\frac{x^2}{2}\right]_1^0 + \left[\frac{y^2}{2}\right]_0^1\right) = 0$$



Question - 04

If $\overline{r}$ is the position vector of any point on a closed surface S that encloses the volume V then $\iint_S (\overline{r} \cdot d\overline{s})$ is equal to

A $\frac{1}{2}V$

B V

C 2V

D 3V

Apply G.D.T. :-

$$\iint_S \vec{r} \cdot \vec{ds} = \iiint_V \mathrm{div}\, \vec{r}\, dV$$

$$= 3 \iiint dV$$

$$= 3V$$

Question - 05

$F(x,y) = (x^2+xy)\hat{a}_x + (y^2+xy)\hat{a}_y$. It's line integral over the straight line from $(x,y) = (0,2)$ to $(2,0)$ evaluate to

A -8

B 4

C 8

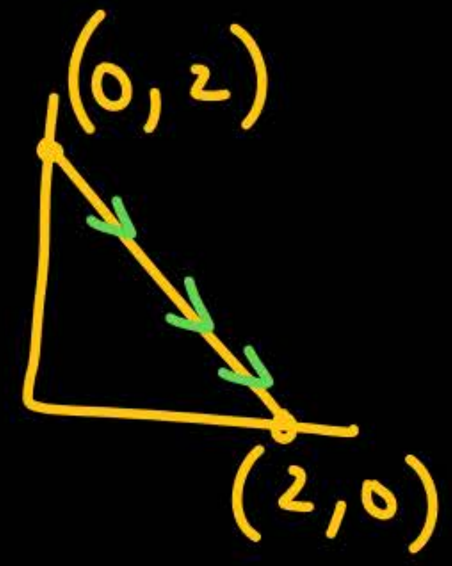
D 0

$$I = \int_{(0,2)}^{(2,0)} (x^2+xy)\,dx + (y^2+xy)\,dy$$

$$= \int_0^2 x^2 + x(2-x)\,dx + \int_2^0 y^2 + (2-y)y\,dy$$

$$= [x^2]_0^2 + [y^2]_2^0$$

$$= (4-0) + (0-4) = 0$$



$$\frac{x}{2} + \frac{y}{2} = 1$$

$$x + y = 2$$

Question - 06

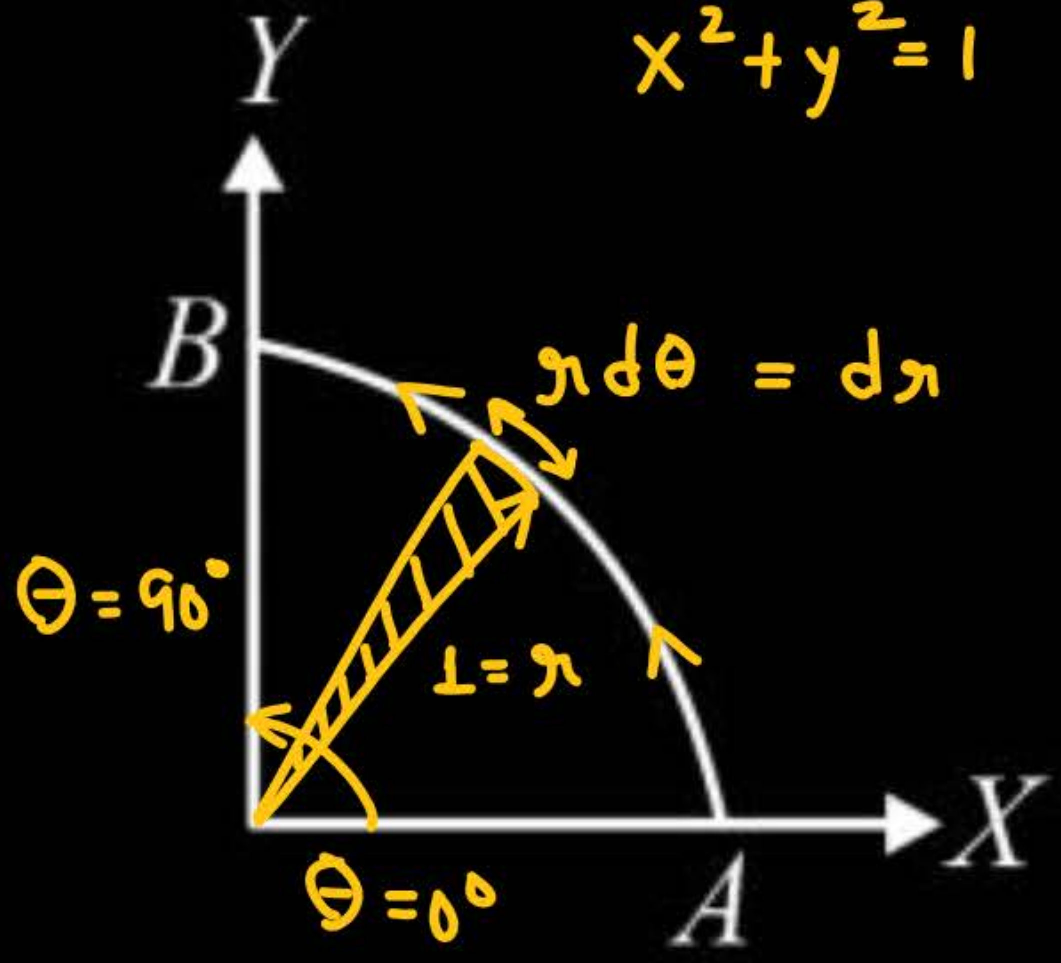
A path AB in the form of one quarter of a circle of unit radius is shown in the figure. Integration of $(x + y)^2$ on path AB traversed in counter-clockwise sense is

A $\frac{\pi}{2} - 1$

B $\frac{\pi}{2} + 1$

C $\frac{\pi}{2}$

D 1



$$\int_C (x+y)^2\, dr = \int_C (\cos\theta + \sin\theta)^2\, r\, d\theta$$

$$= \int 1 + \sin 2\theta \quad d\theta$$

Let $x = r\cos\theta$

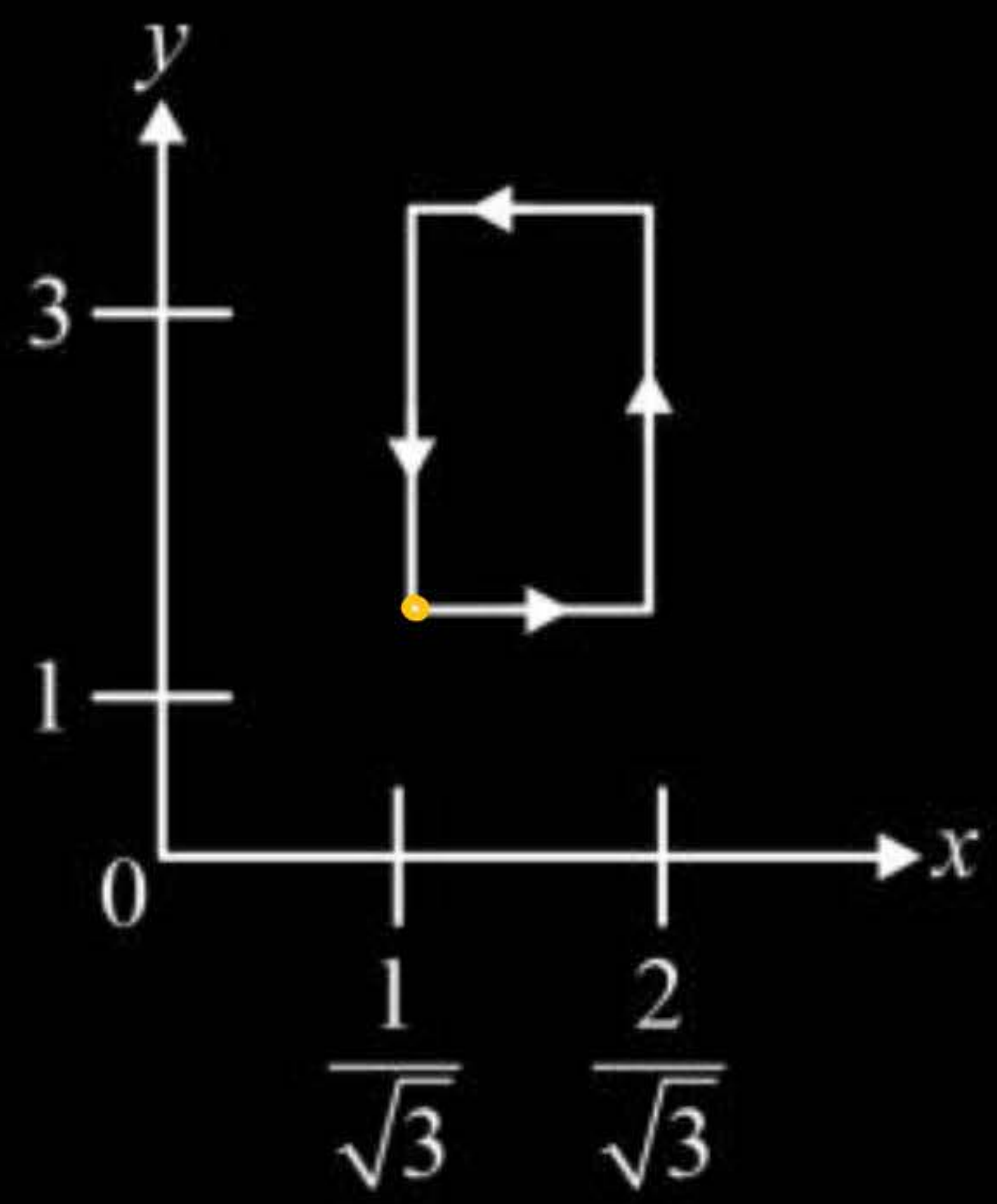
$y = r\sin\theta$

$$\left[\theta - \frac{\cos 2\theta}{2}\right]_0^{\pi/2} = \left(\frac{\pi}{2} + \frac{1}{2}\right) - \left(0 - \frac{1}{2}\right)$$

$$= \pi/2 + 1$$

Question - 07

If $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$, $\oint_C \vec{A}\cdot\vec{dl}$ over the path shown in the figure is



A. 0

B. $\frac{2}{\sqrt{3}}$

C. 1

D. $2\sqrt{3}$

$$\oint M\,dx + N\,dy = \iint \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dx\,dy$$

$$\oint \underbrace{xy}_{M}\,dx + \underbrace{x^2}_{N}\,dy = \int_1^3 \int_{1/\sqrt{3}}^{2/\sqrt{3}} (2x - x)\,dx\,dy$$

$$\int_1^3 \left[\frac{x^2}{2}\right]_{1/\sqrt{3}}^{2/\sqrt{3}} dy$$

$$\int_1^3 \frac{\left(2/\sqrt{3}\right)^2 - \left(1/\sqrt{3}\right)^2}{2}\,dy$$

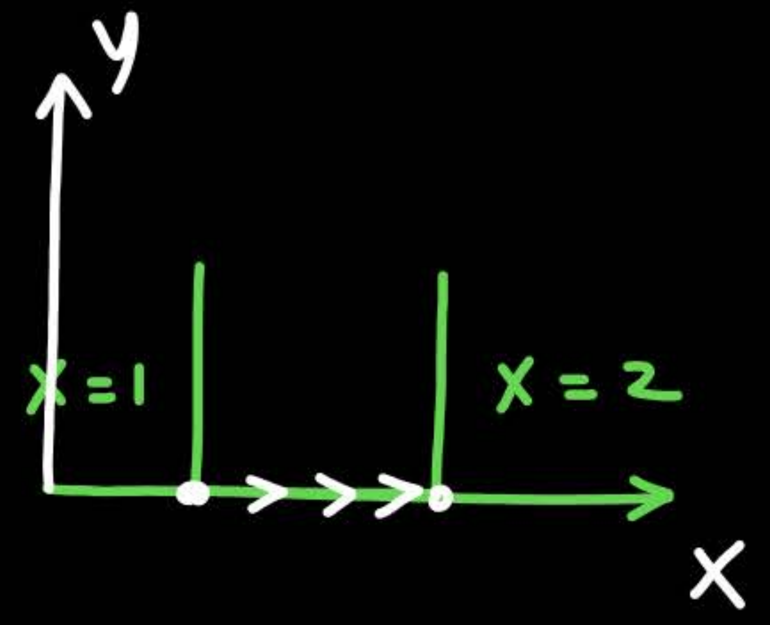
$$\frac{1}{2}[y]_1^3 = \frac{2}{2} = 1$$

Question - 08

The line integral of the vector function $\bar{F} = 2x\hat{i} + x^2\hat{j}$ along the x-axis from $x = 1$ to $x = 2$ is

$$\int_{x=1}^{x=2} 2x\,dx + x^2\,dy$$

$$[x^2]_1^2 = 3$$



$y = 0$

$dy = 0$

A 0

B 2.33

C 3

D 5.33

Question - 09

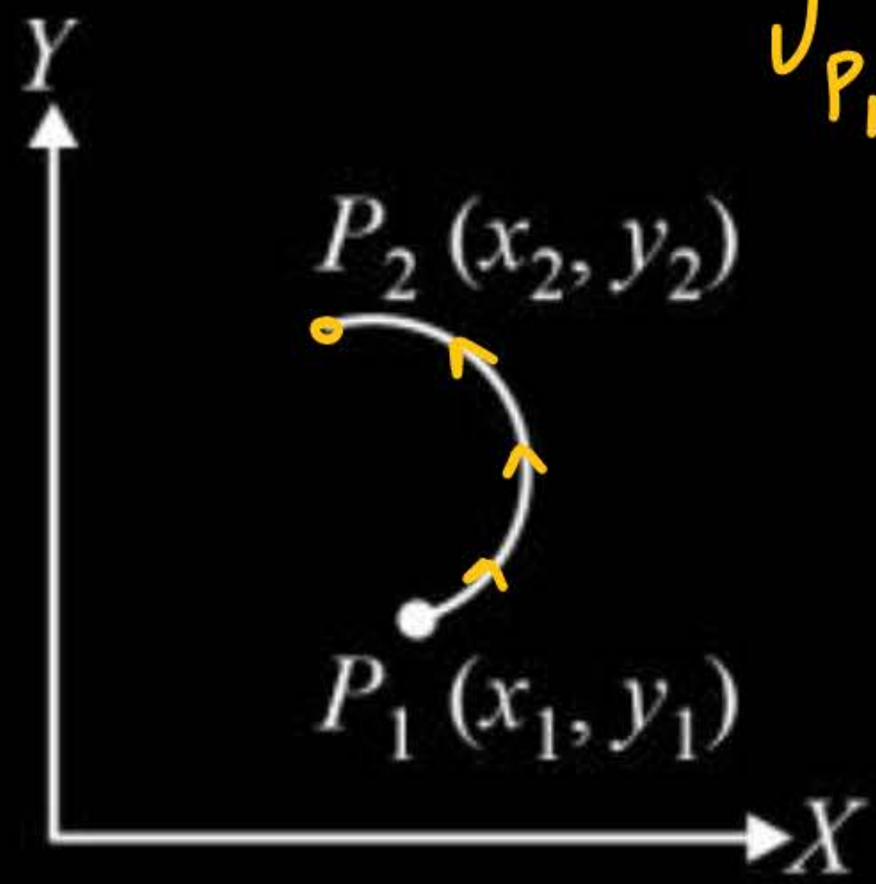
The line integral $\int_{P_1}^{P_2}(y\,dx + x\,dy)$ from $P_1\,(x_1, y_1)$ to $P_2(x_2, y_2)$ along the semi-circle P_1P_2 shown in the figure is

A $x_2 y_2 - x_1 y_1$

B $(y_2{}^2 - y_1{}^2) + (x_2{}^2 - x_1{}^2)$

C $(x_2 - x_1)\,(y_2 - y_1)$

D $(y_2 - y_1)^2 + (x_2 - x_1)^2$



$$\int_{P_1}^{P_2} d(xy)$$

$$[xy]_{P_1}^{P_2}$$

$$x_2y_2 - x_1y_1$$

Question - 10

The area of the triangle formed by the tips of vectors $\overline{a}, \overline{b}$ and $\overline{c}$ is

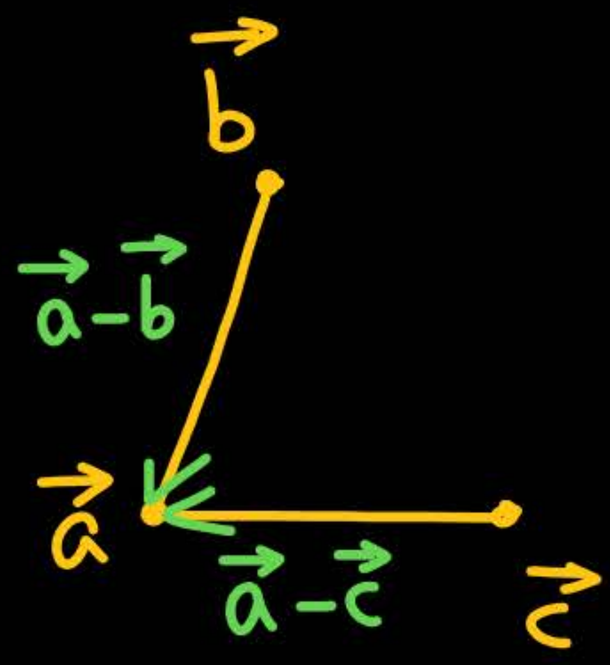
A $\frac{1}{2}(a-b)\cdot(a-c)$

B $\frac{1}{2}|(a-b)\times(a-c)|$

C $\frac{1}{2}|a\times b\times c|$

D $\frac{1}{2}(a\times b)\cdot c$

$\frac{1}{2}|(a-b)\times(a-c)|$



Question - 11

Consider a close surface S surrounding volume V. If $\vec{r}$ is the position vector of a point inside S, with the unit normal on S the value of the integral $\iint_S 5\vec{r}\cdot\hat{n}\,dS$ is

A 3 V

B 5 V

C 10 V

D 15 V

Apply G.D.T.

$$\iint_S 5\vec{r}\cdot\hat{n}\,dS = \iiint_V \text{div}\, 5\vec{r}\,dV$$

$$= 5\times 3\iiint dV$$

$$= 15V$$

Question - 12

The line integral of function $F = yz\hat{\imath}$, in the counter clockwise direction, along the circle $x^2 + y^2 = 1$ at $z = 1$

A -2π

B $-\pi$

C π

D 2π

$$\int_C \vec{F}\cdot\vec{dr}$$

$$\int_C yz\,dx$$

$$\int_0^{2\pi} \sin\theta \cdot 1\,(-\sin\theta)\,d\theta$$

$$-\int_0^{2\pi} \sin^2\theta = -4\int_0^{\pi/2} \sin^2\theta$$

$$= -4 \cdot \frac{1}{2} \cdot \frac{\pi}{2} = -\pi$$

$x = r\cos\theta$

$y = r\sin\theta$

$z = 1$

Thank you

GW Soldiers !