CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 1

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TOPICS

01 product rule

02 Sum rule

3 Combination with repetation

Product Rule :

```
k = 0
for i = 1 to n1.
    for j = 1 to n2.
        k = k+1
```

$K = n_1 \times n_2$

$i=1 \quad j=1-n_2$

$i=2 \quad j=1-n_2$

$i=3 \quad j=1-n_3$

$\vdots$

$i=n_1 \quad j=1-n_2$

Task T.

$n_1 \rightarrow T_1 \qquad T_2. \leftarrow n_2$

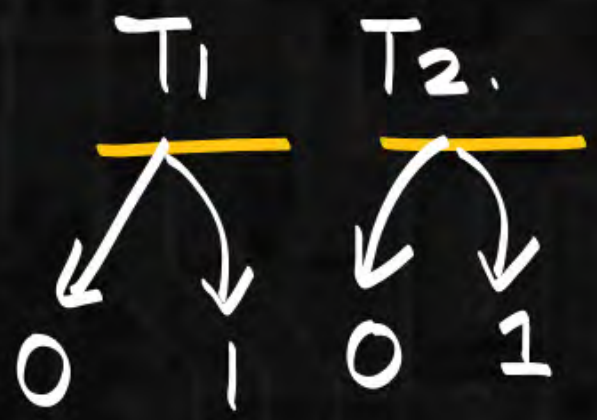
n_1 ways to do task T_1.

n_2 ways to do task T_2.

each of n_1 ways we can perform n_2.

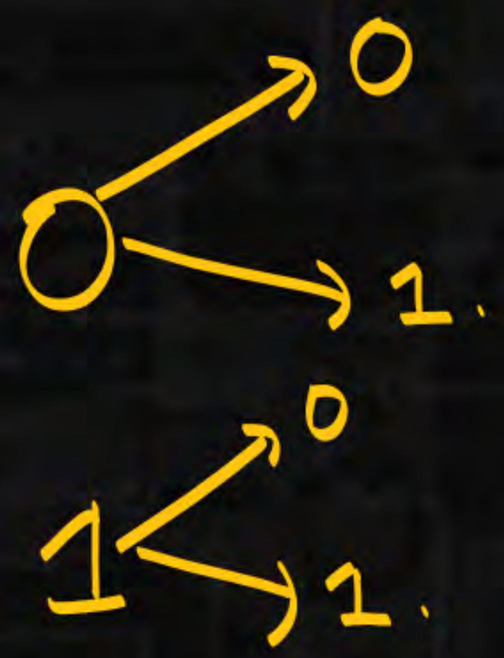
both the task are happening Simultaneously.

How many code we can generate if we have 2 bits?



Task → code

00
01
10
11
.



$T \to T_1, T_2, T_3, \ldots, T_K$

T_1 T_2 T_3 $\ldots$ T_K.

n_1 n_2 n_3 n_K

Simultaneously

$K = 0$

for $x_1 = 1$ to n_1

for $x_2 = 1$ to n_2

$\vdots$

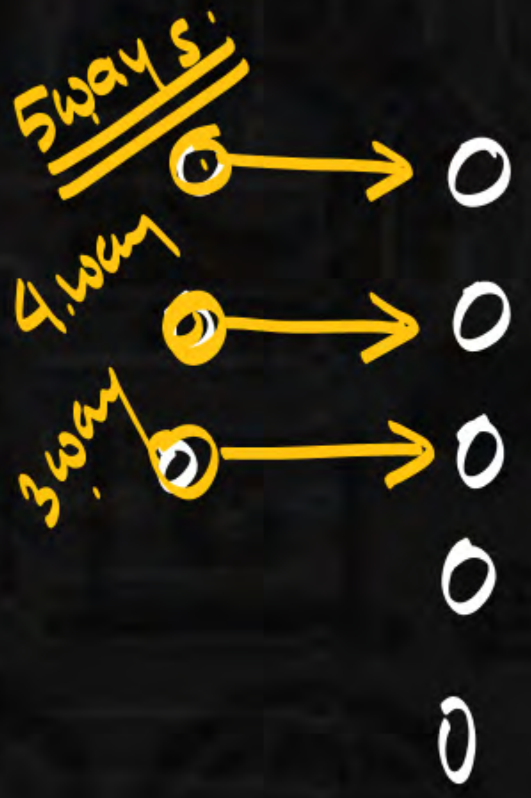
for $x_K = 1$ to n_K

$K = K + 1$.

$K = n_1 \cdot n_2 \cdot n_3 \cdots n_K$

$f : A \to B$

$|A| = 3 \quad |B| = 5$



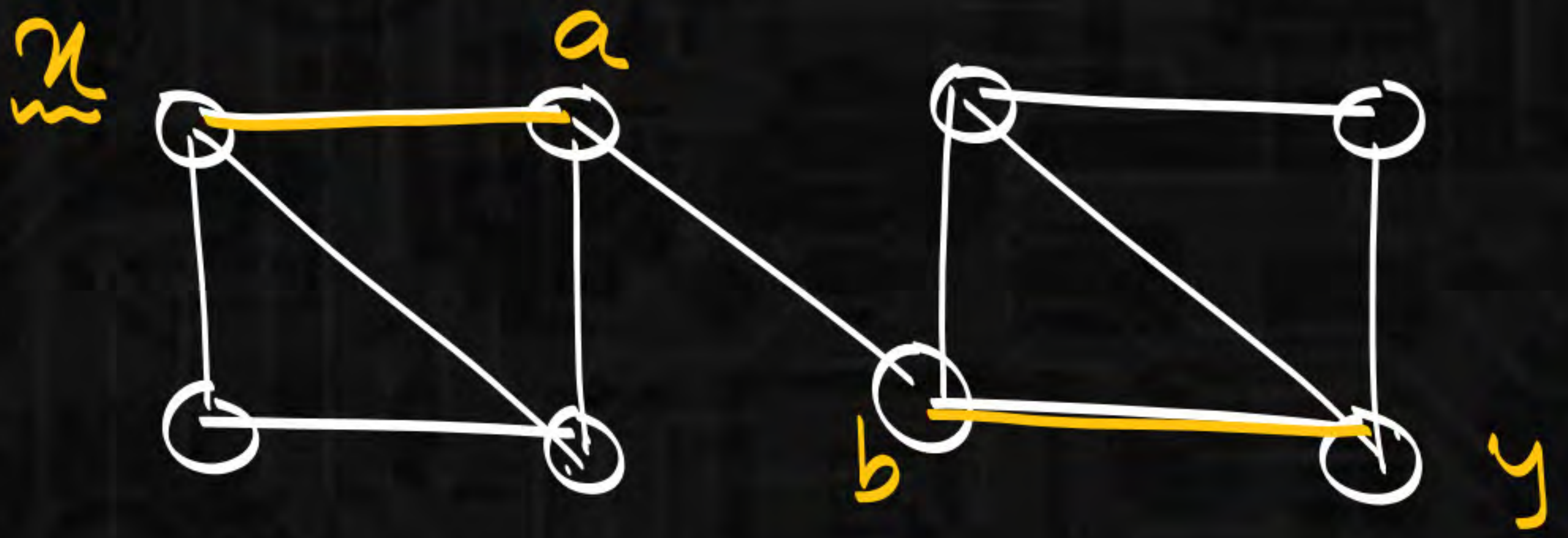
$= 5 \cdot 4 \cdot 3$

How many number plate we can generate.
if 4 characters followed by 2 digit.

$\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \Big|\ \underline{\ \ }\ \underline{\ \ }$

$26 \times 26 \times 26 \times 26 \ \Big|\ 10 \times 10$

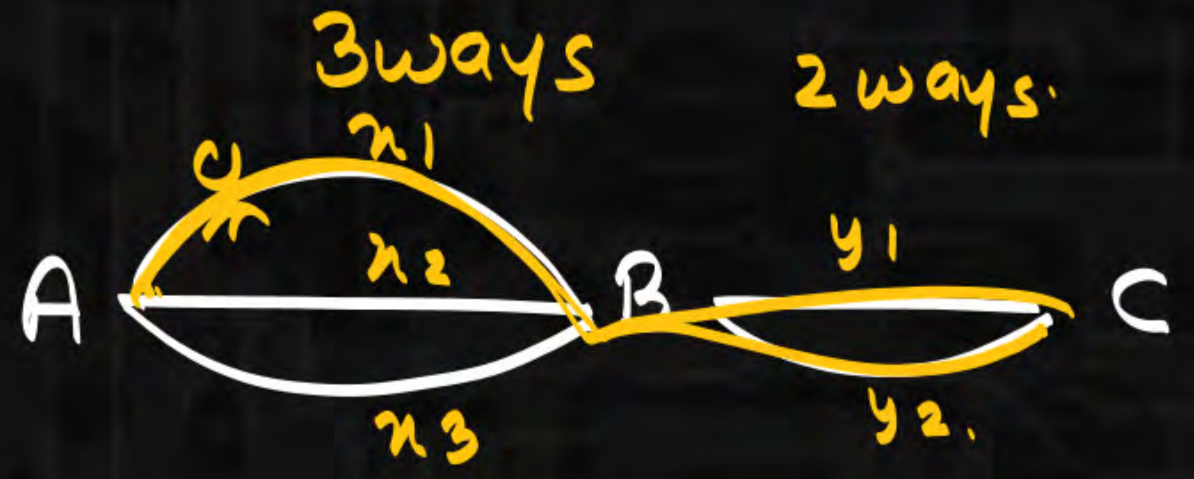
$\rightarrow 26^4 \cdot 10^2$.

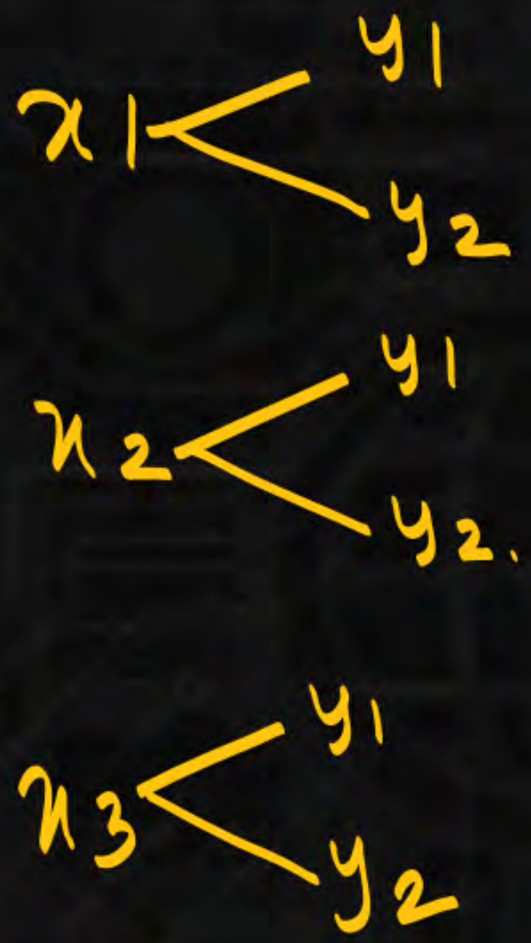


9 paths:

$x - a - b - y$

3 ways $\times$ 1 $\times$ 3 ways





A —— C

$3 \times 2 = 6$ ways.

Sum Rule:

$$\begin{tikzcd}
 & \text{Task } T \arrow[dl] \arrow[dr] & \\
T_1 & & T_2 \\
n_1 \text{ ways} \arrow[u] & & n_2 \text{ ways} \arrow[u]
\end{tikzcd}$$

both tasks are not happening simultaneously.

```
k = 0
for i = 1 to n1
    k = k + 1
for j = 1 to n2.
    k = k + 1.
```

$k = n_1 + n_2$.

Delhi
- → chair
- → Table.
- → cupboard

mumbai
- → pen
- → pencil.

$3 + 2 = \underline{\underline{5 \text{ ways}}}$.

How many number plates we can generate if 4 characters followed by 2 digit or 3 digit?

_ _ _ _ | _ _ OR _ _ _ _ | _ _ _

$$26^4 \cdot 10^2 + 26^4 \cdot 10^3 = 26^4(10^2 + 10^3)$$

$$(26 + 26^2 + 26^3 + 26^4)(10 + 10^2 + 10^3 + 10^4)$$

number plates we can generate if 1 or 2 or 3 or 4 characters followed by 1 or 2 or 3 or 4 digit?

$$26(10 + 10^2 + 10^3 + 10^4) + 26^2(10 + 10^2 + 10^3 + 10^4) + 26^3(\ldots) + 26^4(1\ldots)$$

$\underline{1\ c}$ $\longrightarrow$

OR

2 c

OR

3 c

OR

4 c

$$\left\{\begin{array}{l} 1 \\ \text{OR} \\ 2 \\ \text{OR} \\ 3 \\ \text{OR} \\ 4 \end{array}\right.$$

$$26(10+10^2+10^3+10^4)$$

$$+$$

$$26^2(10+10^2+10^3+10^4)$$

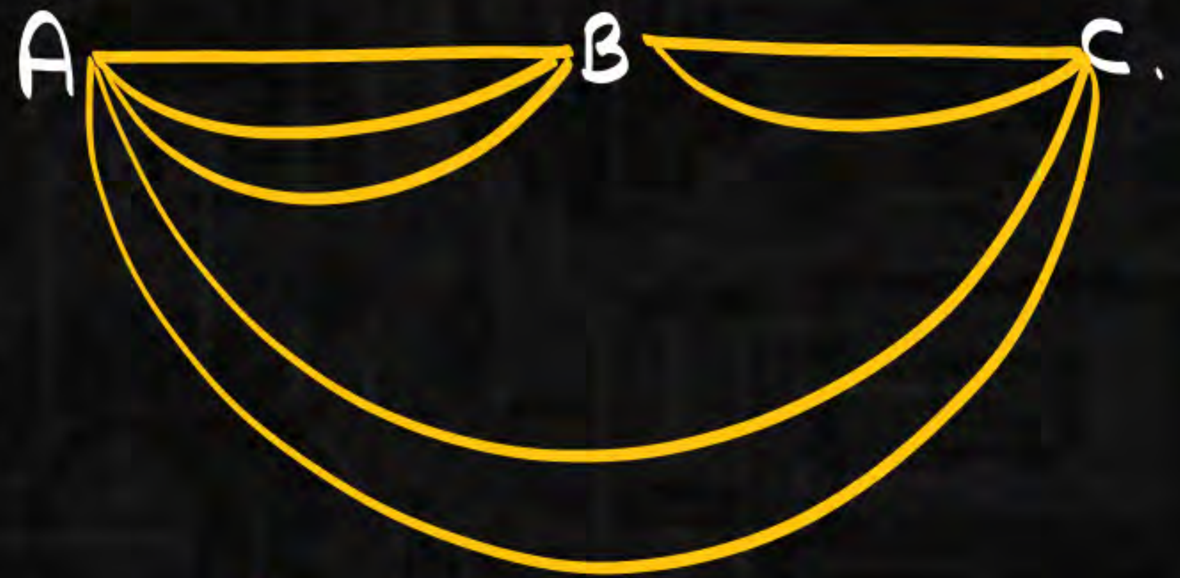
$$+$$

$$26^3(10+10^2+10^3+10^4)$$

$$+$$

$$26^4(10+10^2+10^3+10^4)$$

$$\left(26+26^2+26^3+26^4\right)\times\left(10+10^2+10^3+10^4\right)$$



1) A — B — C.

3 ways × 2 ways = 6 ways.

2) A — C.

via B.   Direct

6 ways + 2 ways.

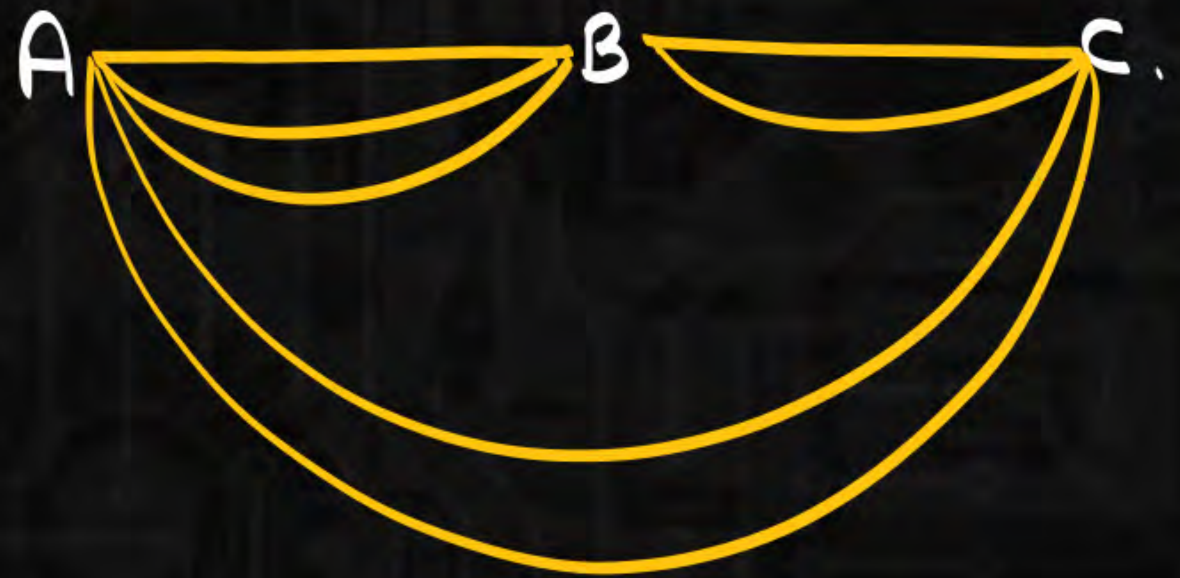
(A → C → A) (via B)

3) A — B — C — B — A

3 × 2 × 2 × 3.

6 × 6.

4) A — C — A



A — C

$(3\times 2+2)$

$(3\times 2+2)\times(3\times 2+2)$

$8\times 8 = 64$ ways.

How many 4 digit integer contains exactly one 6.

(0 − 9) = 10

$\underline{6}\ \underline{9\text{way}} \times \underline{9} \times \underline{9} + \underset{8\text{ways}}{\underline{\neq 0}}\ \underline{6}\ \underset{9\text{way}}{\underline{\ \ }}\ \underset{9\text{ways}}{\underline{\ \ }} + \overset{8}{\underline{\neq 0}}\ \overset{9}{\underline{\ \ }}\ \underline{6}\ \overset{9}{\underline{\ \ }} + \underset{8}{\underline{\neq 0}}\ \underset{9\text{w}}{\underline{\ \ }}\ \underset{9}{\underline{\ \ }}\ \underline{6}$

$9 \cdot 9 \cdot 9 + 3 \cdot (8 \cdot 9 \cdot 9)$

$\underline{6}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$

9 way $\times 9 \times 9$.

0
1
2
3
4
5
$\cancel{6}$
7
8
9.

$+$ $\neq 6$ $\neq 0$ $\underline{\quad}\ \underline{6}\ \underline{\quad}\ \underline{\quad}$

0 allowed.
6 not allowed

8 ways $\qquad 9 \times 9$

$(\cancel{0}\, \underline{1\,2\,3\,4\,5}\, \cancel{6}\, \underline{7\,8\,9})$

$+$ $\neq 0$ $\neq 6$ $\qquad$ 0 allowed. $\neq 6$ $\qquad$ 0 allowed. $\neq 6$.

$\underline{\quad}\ \underline{\quad}\ \underline{6}\ \underline{\quad}$

8 ways $\quad 9 \quad 9$.

$9\cdot9\cdot9 + 8\cdot9\cdot9 + 8\cdot9\cdot9 + 8\cdot9\cdot9$.

10 awards → 3 places

100 awards → 3 places

10 ways × 9 × 8

100 × 99 × 98

$10 \times (10-1) \times (10-2)$

$100 \times (100-1) \times (100-2)$

3 places

n objects $\longrightarrow$ r places

$$\underset{\uparrow}{n} \times \underset{\uparrow}{(n-1)} \cdots\cdots \times \underset{\uparrow}{(n-R+1)} \left(\frac{(n-R)!}{(n-R)!}\right)$$

$$= \frac{n!}{(n-R)!} = {}^{n}P_{R}$$

$\{a, b, c\}$

$\{a, b\}$

$\{b, c\}$

$\{a, c\}$

$\{S_1, S_2, S_3 \ldots S_{10}\}$

$\{$

$^{10}C_6$

how many ways we can select 6 students in a class of 10?

^nP_R

how many ways we can arrang 3 letters at 2 places.

$\{a, b, c\}$

^nC_R

$\{a, b\}$ → ab, ba

$\{b, c\}$ → bc, cb

$\{a, c\}$ → ac, ca

$R!$

$^3P_2$

product Rule.

3 ways × 2 ways

ab
ac
ba
bc
ca
cb

$$^nC_R = \frac{n!}{R!\,(n-R)!}$$

$$R! \times {^nC_R} = {^nP_R}$$

$$^{n}C_{R} = \frac{n!}{R! \times (n-R)!}$$

$$R! \times {}^{n}C_{R} = {}^{n}P_{R}$$

A store carries eight styles of pants. For each style, there are 12 different possible waist sizes, five pants lengths, and four color choices. How many different types of pants could the store have?

$8 \times 12 \times 5 \times 4.$

Given 10 different English books, six different French books, and four different German books,

(a) How many ways are there to select one book?

(b) How many ways are there to select three books, one of each language?

(a) $10+6+4,$

(b) $10 \times 6 \times 4.$

(a) How many different 6-digit numbers are there (leading zeros, e.g., 00174, not allowed)?

(b) How many even 6-digit numbers are there?

(c) How many 6-digit numbers are there with exactly one 3?

(d) How many 6-digit palindromic numbers (numbers that are the same when the order of their digits is inverted, e.g., 137731) are there?

$9 \times 10 \times 10 \times 10 \times 10 \times 10.$

$9 \times 10 \times 10 \times 10 \times 10 \times 5.$

$9^5 + 5 \times (8 \times 9^4).$

$9 \times 10 \times 10$

A rumor is spread randomly among a group of 10 people by successively having one person call someone, who calls someone, and so on. A person can pass the rumor on to anyone except the individual who just called.

(a) By how many different paths can a rumor travel through the group in three calls? In n calls?

(b) What is the probability that if A starts the rumor, A receives the third calls?

(a) $10 \times 9 \times 8 \times 8,\ 10 \times 9 \times 8^{n-2},$

(b) $9 \times 8 \times 1/9 \times 8 \times 8,$

How many integers between 1,000 and 10,000 are there with (make sure to avoid sequences of digits with leading 0s):

(a) Distinct digits?

$9 \times 9 \times 8 \times 7,$

There are 50 cards numbered from 1 to 50. Two different cards are chosen at random. What is the probability that one number is twice the other number?

$2 \times 25/50 \times 49.$

37. Sixteen people are to be seated at two circular tables, one of which seats 10 while the other seats six. How many different seating arrangements are possible?

We can select the 10 people to be seated at the table for 10 in $\binom{16}{10}$ ways. For each such selection there are 9! ways of arranging the 10 people around the table. The remaining six people can be seated around the other table in 5! ways. Consequently, there are $\binom{16}{10}9!5!$ ways to seat the 16 people around the two given tables.

38. A committee of 15 — nine women and six men — is to be seated at a circular table (with 15 seats). In how many ways can the seats be assigned so that no two men are seated next to each other?

$(8!)\binom{9}{6}6!$

a) In how many ways can seven people be arranged about a circular table?

b) If two of the people insist on sitting next to each other, how many arrangements are possible?

(a) 6! (b) Let A,B denote the two people who insist on sitting next to each other. Then there are 5! (A to the right of B) + 5! (B to the right of A) = 2(5!) seating arrangements.

26. How many different paths in the xy-plane are there from (0, 0) to (7, 7) if a path proceeds one step at a time by going either one space to the right (R) or one space upward (U)? How many such paths are there from (2, 7) to (9, 14)? Can any general statement be made that incorporates these two results?

Any such path from (0,0) to (7,7) or from (2,7) to (9,14) is an arrangement of 7 R's and 7 U's. There are (14!)/(7!7!) such arrangements.
In general, for m, n nonnegative integers, and any real numbers a, b, the number of such paths from (a, b) to $(a+m, b+n)$ is $(m+n)!/(m!n!)$.

(b) How many numbers can be formed using the digits 1, 3, 4, 5, 6, 8, and 9 if no repetitions are allowed?

The events of forming one-digit numbers, two-digit numbers, three-digit numbers, etc., are mutually exclusive events so we apply the sum rule to see that there are $7 + 7 \cdot 6 + 7 \cdot 6 \cdot 5 + 7 \cdot 6 \cdot 5 \cdot 4 + 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 + 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 + 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ different numbers we can form under the restrictions of this problem.

How many three-digit numbers are there which are even and have no repeated digits?

$9 \cdot 8 + 8 \cdot 8 \cdot 4$

14. There are five different roads from City A to City B, three different roads from City B to City C, and three different roads that go directly from A to C.
    (a) How many different ways are there to go from A to C via B?
    (b) How many different ways are there from A to C altogether?
    (c) How many different ways are there from A to C and then back to A?
    (d) How many different trips are there from A to C and back again to A that visit B both going and coming?
    (e) How many different trips are there that go from A to C via B and return directly from C to A?
    (f) How many different trips are there that go directly from A to C and return to A via B?
    (g) How many different trips are there from A to C and back to A that visit B at least once?
    (h) Suppose that once a road is used it is closed and cannot be used again. Then how many different trips are there from A to C via B and back to A again via B?
    (i) Using the assumption in (h) how many different trips are there from A to C and back to A again?

14. (a) $5 \cdot 3 = 15$.
    (b) $15 + 3 = 18$.
    (c) 18^2.
    (d) 15^2.
    (e) $15 \cdot 3 = 45$.
    (f) $3 \cdot 15 = 45$.
    (g) $15 \cdot 3 + 3 \cdot 15 + 15^2 = 18^2 - 3^2 = 15 \cdot 18 + 3 \cdot 15$.
    (h) $15 \cdot 8$.
    (i) $15 \cdot 8 + 15 \cdot 3 + 3 \cdot 15 + 3 \cdot 2$.

A newborn child can be given 1, 2, or 3 names. In how many ways can a child be named if we can choose from 300 names (and no name can be repeated)?

$300 + 300 \cdot 299 + 300 \cdot 299 \cdot 298$.


THANK YOU GW
SOLDIERS !