CS & IT ENGINEERING

DISCRETE MATHS

Mathematical Logic

Lecture No. 06

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TOPICS

01 English to logic expression

02 Nested quantifier

03 TYPE 5

nested / quantifier:

↳ Tool → used defined truthvalue. → quantity.

D: {1, 2, 3}

$\forall x \forall y \; (x + y = 10)$

$1 + y = 10$

$D: \{1, 2, 3\}$

$P(x, y): x \times y \leq 9$

$\forall x \forall y (x \times y \leq 9)$

for all values of x, all values of y.

for every value of x, every value of y.

$D: \{1, 2, 3\}$

$x = 1 \quad y = 1$

$(x \times y \leq 9) \quad 1 \times 1 \leq 9 \ (T)$

$x = 1 \quad y = 2 \quad 1 \times 2 \leq 9 \ (T)$

$x = 1 \quad y = 3 \quad 1 \times 3 \leq 9 \ (T)$

$x = 2 \quad y = 1 \quad 2 \times 1 \leq 9 \ (T)$

for all values of x, all values of y.

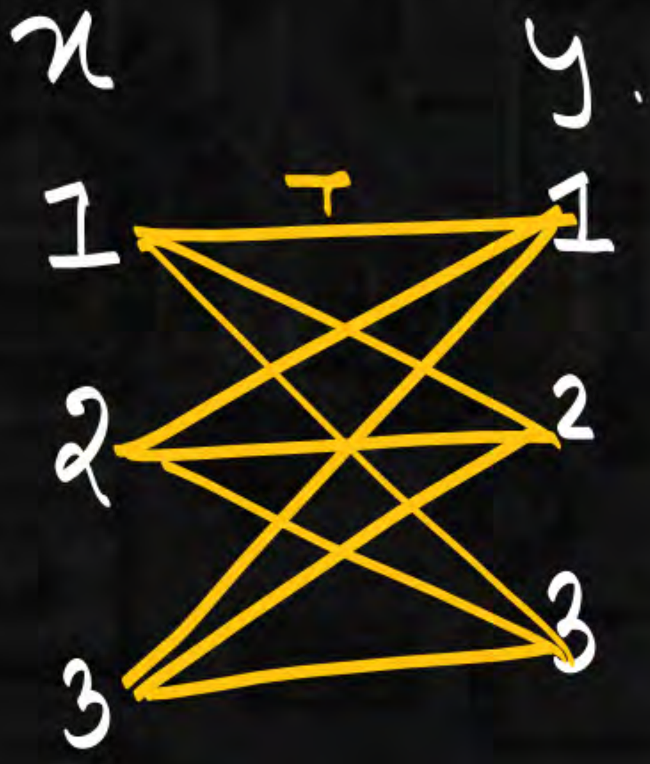
for $x = 1$ to 3 $x = 1$ $y = 1, 2, 3$

for $y = 1$ to 3 $x = 2$ $y = 1, 2, 3$

$x \times y \leq 9$ $x = 3$ $y = 1, 2, 3$

$\boxed{x=1 \quad y=1}$ $1 \times 1 \leq 9$ (T)

$x \times y \leq 9.$



$\forall x \forall y \to$ True.

when all edges are True

$\to$ pair

$\forall x \forall y \to$ False

atleast 1 edge is false.

$D: \mathbb{Z}$.

$\forall m \forall n \, (m \cdot n = n) \rightarrow$ false

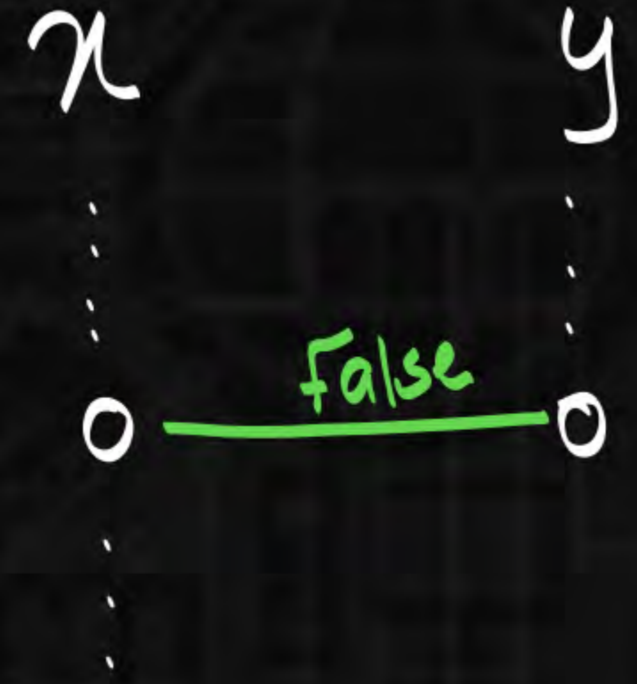
$m = 2 \quad n = 1$

$m \cdot n = n$.

$2 \cdot 1 \neq 1$.

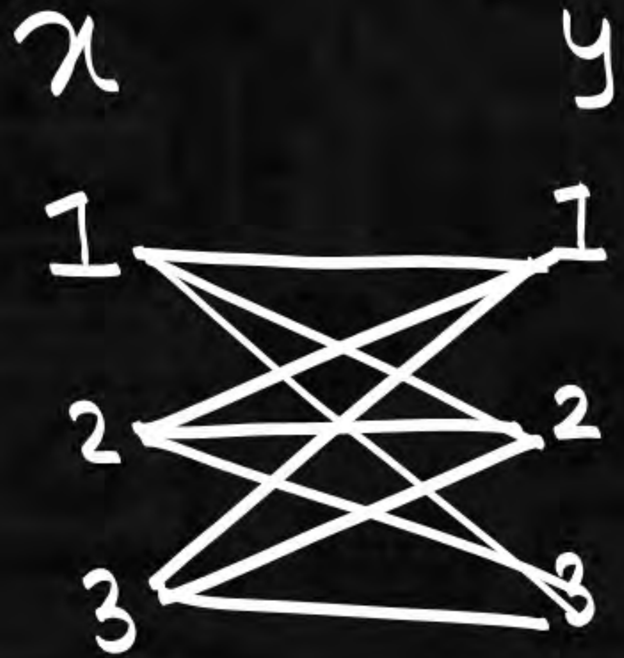
$D: \mathbb{Z}$.

$\forall x \forall y \left(\frac{x}{y} \in \mathbb{Z} \right) \rightarrow$ False.



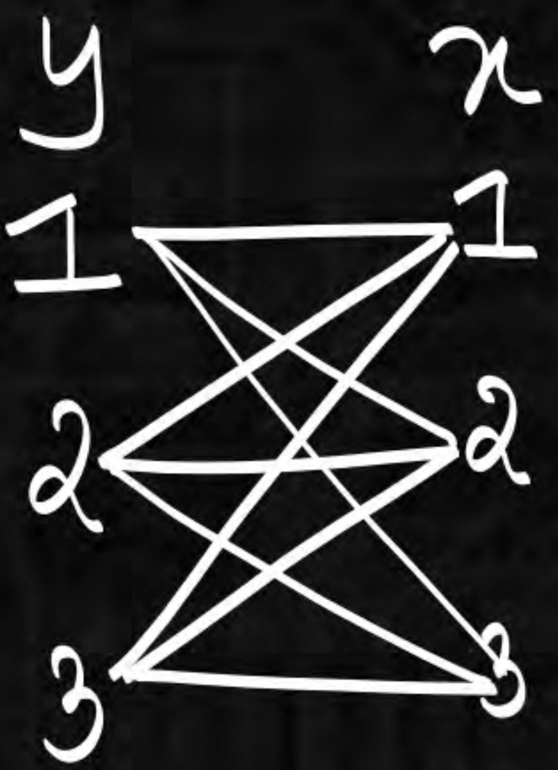
$D: \{1, 2, 3\}$

$\forall x \forall y \, (x \times y \leq 9)$



$D: \{1, 2, 3\}$

$\forall y \forall x \, (x \times y \leq 9)$



$$\boxed{\forall x \forall y \equiv \forall y \forall x}$$

$$\forall x \forall y \Leftrightarrow \forall y \forall x$$

D: {1, 2, 3}.

$\exists x \exists y \, (x \times y = 4)$

there exist x, there exist y

at least 1 value of x, at least 1 value of y.

for some value of x, some value of y.

$\exists x \exists y \, (x \times y = 4)$

$x=1 \quad y=1$

$1 \times 1 \neq 4$

$x=1 \quad y=2$

$1 \times 2 \neq 4$

$x=1 \quad y=3$

$1 \times 3 \neq 4$.

$\exists x \exists y \rightarrow$ True.

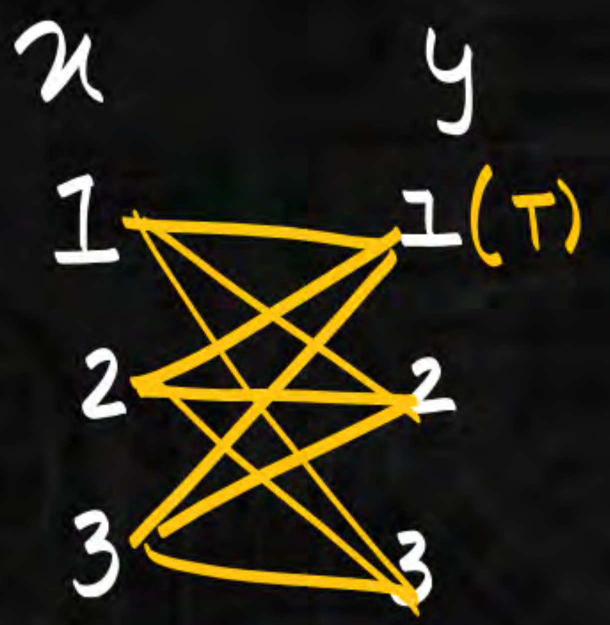
atleast 1 edge is True.

$\exists x \exists y \rightarrow$ false

when all edges are false

$D: \{1, 2, 3\}$.

$\exists x \exists y (x \times y \leq 9) \rightarrow$ True.



$x=1 \; y=1$

$1 \times 1 \leq 9$.

$D: \mathbb{Z}$

$$\exists x \exists y \left(\frac{x}{y} \in \mathbb{Z} \right)$$

$x \quad y$

$2 \xrightarrow{\text{True}} 2$

$3 \xrightarrow{\text{True}} 3$

$D: \mathbb{Z}$

$$\exists x \exists y \, (x + y = 10)$$

$x \quad y$

$1 \xrightarrow{\text{True}} 9$

$2 \xrightarrow{\text{True}} 8$

$x = 1 \quad y = 9$

$1 + 9 = 10 \; (T)$

$x = 2 \quad y = 8$

$2 + 8 = 10 \; (T)$

$D: \mathbb{Z}.$

$$\exists x \, \exists y \, (x + y = 10 \wedge 2x + y = 15)$$

x $\quad\quad$ y

$5 \overset{\text{True}}{\text{———}} 5$

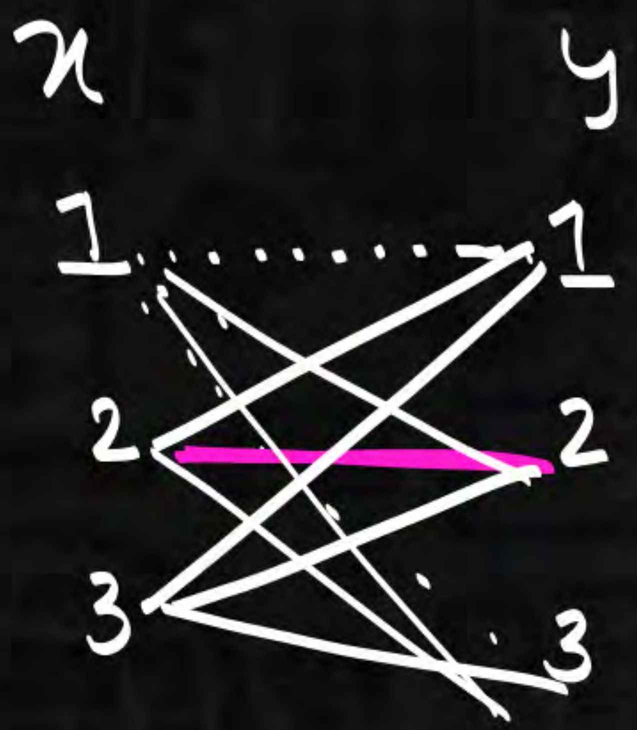
$$\begin{array}{r} x + \cancel{y} = 10 \\ \underline{-2x \mp \cancel{y} = -15} \\ -x = -5 \end{array}$$

$x = 5$

$y = 5.$

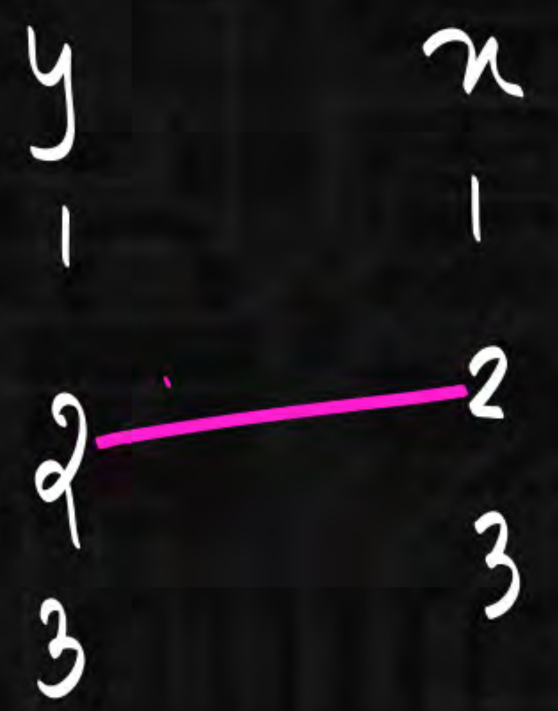
$D: \{1, 2, 3\}$

$\exists x \exists y \, (x \times y = 4)$



$D: \{1, 2, 3\}$

$\exists y \exists x \, (x \times y = 4)$



$$\boxed{\exists x \exists y \equiv \exists y \exists x}$$

$\forall x \forall y \equiv \forall y \forall x$ $\exists x \exists y \equiv \exists y \exists x$.

$$\boxed{\begin{array}{l} \forall x \forall y \rightarrow \exists x \exists y \checkmark \\ \forall y \forall x \rightarrow \exists x \exists y \\ \forall y \forall x \rightarrow \exists y \exists x \\ \forall x \forall y \rightarrow \exists y \exists x \end{array}}$$

Domain is fixed.

$D : Z$

(True)

$\forall x \exists y \, (x + y = 10)$

for all value of x, there exist y.

for every value of x, there exist y.

$$\begin{array}{ccc} x & & y \\ 1 & \longrightarrow & 9. \\ 2 & \longrightarrow & 8 \\ 3 & \longrightarrow & 7 \\ \vdots & & \vdots \end{array}$$

$x + y = 10$

$x = 1$	$x = 2$
$1 + y = 10$	$y = 8$
$y = 9.$	

D: Z

$$\forall x \exists y (x+y=10) \not\Rightarrow \exists y \forall x (x+y=10)$$

(false)

there exist y, all of x.

Forcing y to be constant.

x	y
1	9
2	8
3	7
...	...

D: Z

$x + y = 10$

$y = 10 - x$

$y = 10 - \boxed{\vdots}$

$$\exists y \forall x \, (x + y = 10)$$

there exist y, for all of x.

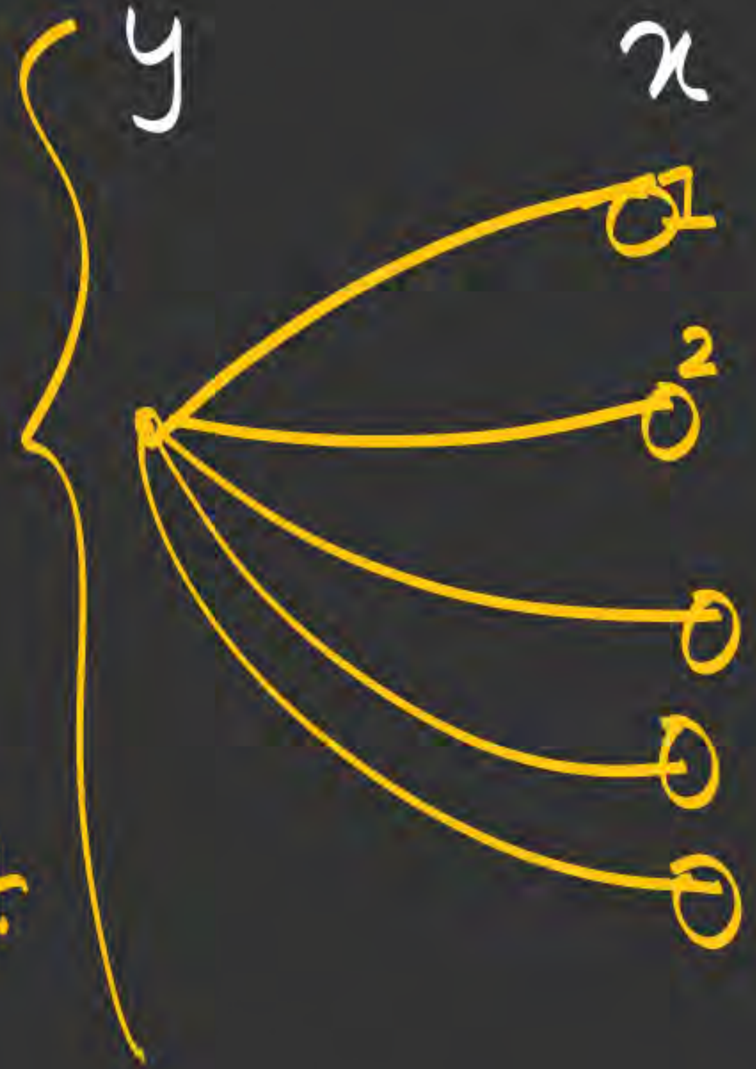
atleast 1 value of y, all of x.

Some value of y, all of x.

$\exists y \forall x$

$\downarrow$

forcing y to be constant



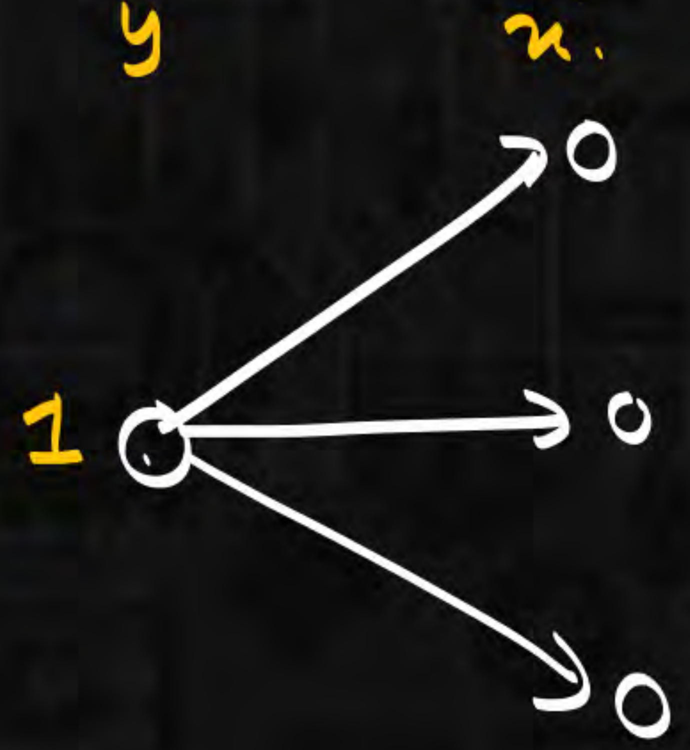
$D : \mathbb{Z}$ $\qquad\qquad$ $D : \mathbb{Z}$

$$\forall x \, \exists y \, (x+y=10) \;\not\Rightarrow\; \exists y \, \forall x \, (x+y=10)$$

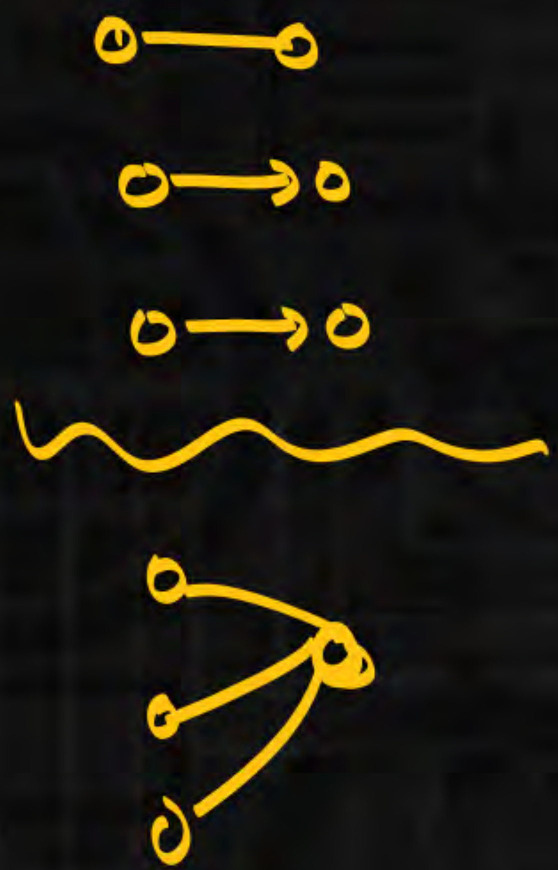
$\forall x \, \exists y \, (x+y=10)$ ↳ True

$\exists y \, \forall x \, (x+y=10)$ ↳ false

$$\overset{T.}{\exists y \forall x}\left(\frac{x}{y} \in Z\right) \longrightarrow \overset{T.}{\forall x \exists y}\left(\frac{x}{y} \in Z\right)$$



$\forall x \exists y.$



S1 S2 S3

S4: notes → micro
DPP / Test → (section)
PYQ

PYQ → 2 SOS

nano
Test - 65Q/Day.

July

Aug sep oct nov Dec Jan Feb.

6 subjects.

Dec → PYQ. micro

Feb → 65 Q → practice.
12 sub → Revision.

S1 S2 S3 | S4 S5 S6
S1 S2 S3

Running notes → 300 → 60 hrs

* micro notes → 30

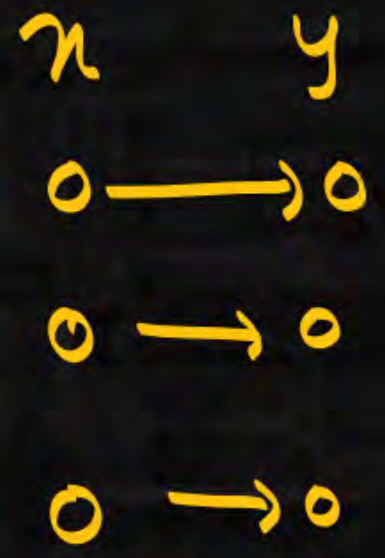
nano notes → 3

1

$\forall x \forall y$

2

$\forall x \exists y$



3

$\exists y \forall x$

4

$\exists x \exists y$

O.S Fixed.

domain : Fixed.

$1 \rightarrow \text{all}$

$\text{all} \rightarrow 4.$

$3 \rightarrow 2.$

$1 = 2$

$4 = 8$

$3 \rightarrow 4$

$4 = 8$

$3 \rightarrow 8$

$$
\begin{array}{llcll}
1. & \forall x \forall y & \equiv & \forall y \forall x & 5 \\
 & \downarrow & & & \\
2. & \exists y \forall x & & \exists x \forall y & 6 \\
 & \downarrow & & \downarrow & \\
3. & \forall x \exists y & & \forall y \exists x & 7 \\
 & \downarrow & & \downarrow & \\
4. & \exists x \exists y & \equiv & \exists y \exists x & 8.
\end{array}
$$

$1 \rightarrow$ all

$5 \rightarrow$ all.

all $\rightarrow 4, 8$

$2 \rightarrow 3$

no relation betn

2 & 6.

13. Consider the open statement

$$p(x, y): \quad y - x = y + x^2$$

where the universe for each of the variables x, y comprises all integers. Determine the truth value for each of the following statements.

a) $p(0, 0)$
b) $p(1, 1)$
c) $p(0, 1)$
d) $\forall y\, p(0, y)$
e) $\exists y\, p(1, y)$
f) $\forall x\, \exists y\, p(x, y)$
g) $\exists y\, \forall x\, p(x, y)$
h) $\forall y\, \exists x\, p(x, y)$

14. Determine whether each of the following statements is true or false. If false, provide a counterexample. The universe comprises all integers.

a) $\forall x\, \exists y\, \exists z\, (x = 7y + 5z)$
b) $\forall x\, \exists y\, \exists z\, (x = 4y + 6z)$

Truthvalue:

$\exists x \, \exists y \, [xy = 1]$

$\exists x \, \exists y \, [(2x + y = 5) \wedge (x - 3y = -8)]$
$\exists x \, \exists y \, [(3x - y = 7) \wedge (2x + 4y = 3)]$

Truthvalue.

6. Let $p(x, y), q(x, y)$ denote the following open statements.

$$p(x, y): \quad x^2 \geq y \qquad q(x, y): \quad x + 2 < y$$

If the universe for each of x, y consists of all real numbers, determine the truth value for each of the following statements.

a) $p(2, 4)$ **b)** $q(1, \pi)$

c) $p(-3, 8) \wedge q(1, 3)$ **d)** $p\left(\frac{1}{2}, \frac{1}{3}\right) \vee \neg q(-2, -3)$

e) $p(2, 2) \rightarrow q(1, 1)$ **f)** $p(1, 2) \leftrightarrow \neg q(1, 2)$

negate.

$\forall x \, \forall y \, [(x > y) \rightarrow (x - y > 0)]$
$\forall x \, \forall y \, [(x < y) \rightarrow \exists z \, (x < z < y)]$
$\forall x \, \forall y \, [(|x| = |y|) \rightarrow (y = \pm x)]$

12. a) Let $p(x, y)$ denote the open statement "x divides y," where the universe for each of the variables x, y comprises all integers. (In this context "divides" means "exactly divides" or "divides evenly.") Determine the truth value of each of the following statements; if a quantified statement is false, provide an explanation or a counterexample.

i) $p(3, 7)$ **ii)** $p(3, 27)$

iii) $\forall y\, p(1, y)$ **iv)** $\forall x\, p(x, 0)$

v) $\forall x\, p(x, x)$ **vi)** $\forall y\, \exists x\, p(x, y)$

vii) $\exists y\, \forall x\, p(x, y)$

viii) $\forall x\, \forall y\, [(p(x, y) \wedge p(y, x)) \rightarrow (x = y)]$

a) $\forall x\, \exists y\, p(x, y)$ b) $\forall y\, \exists x\, p(x, y)$

c) $\exists x\, \forall y\, p(x, y)$ d) $\exists y\, \forall x\, p(x, y)$


THANK YOU GW
SOLDIERS !