# CS & IT ENGINEERING

## DISCRETE MATHS
## SET THEORY

**Lecture No. 13**

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# TOPICS TO BE COVERED

01 Algebraic structures

02 Semi Group

03 Monoid

04 Group

# GROUP THEORY

$(G, *)$ is Group.

1. closed.

$a \in G,\ b \in G \quad a * b \in G.$

2. Associative.

$a * (b * c) = (a * b) * c.$

3. identity

$$a * \underbrace{e}_{\text{identity}} = a$$

$(G, *)$

1) Closed.

2) Associative

3) identity:

$a \qquad = a.$

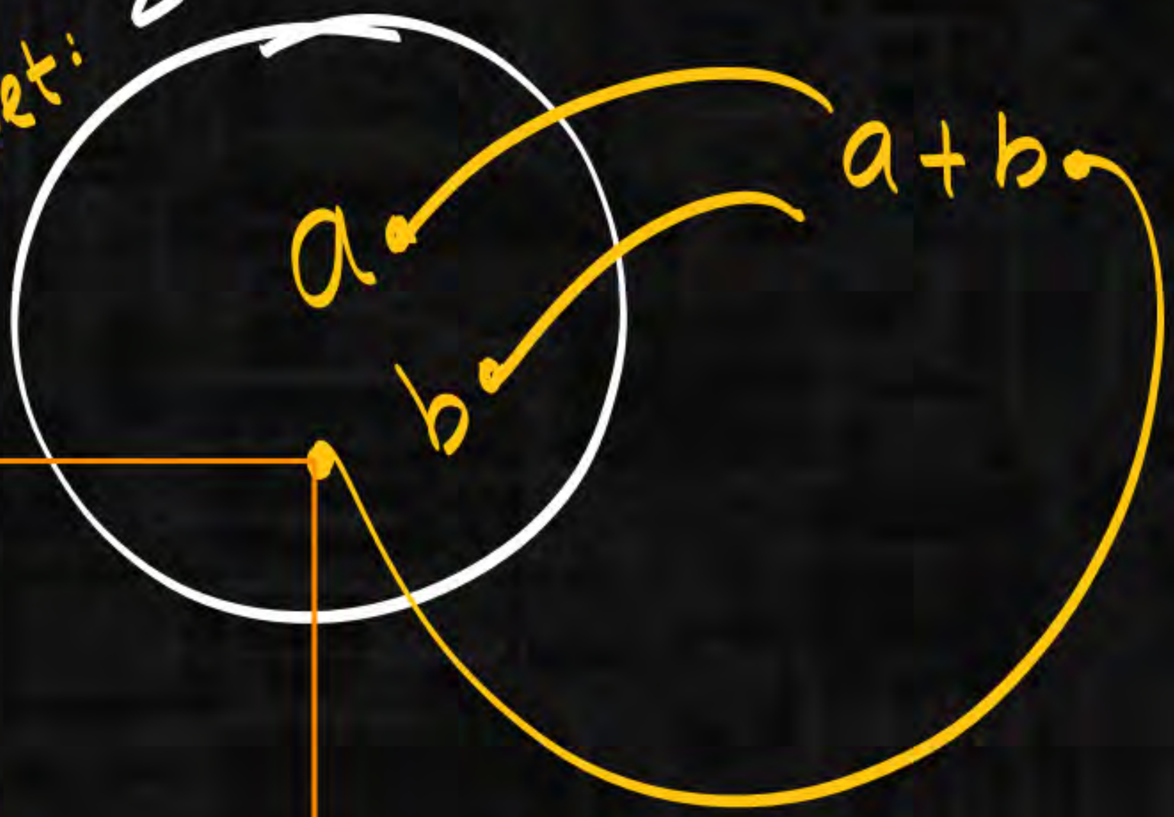
# GROUP THEORY

**Closed:**

$(G, *) \rightarrow a \in G,\ b \in G \quad a * b \in G.$

$(Z, +) \rightarrow a \in Z,\ b \in Z. \quad a + b \in Z.$

set: Z:




$(Z, +)$

$a + 0 = a$

↳ identity

**Associative.**

$a + (b + c) = (a + b) + c$ ✓

# GROUP THEORY

$(D_{12}, a|b)$ $(D_n, |)$

(set, operation)

$(\{1, 2, 3, 4, 6, 12\}, a|b) \rightarrow$ Associative.

$(\{1, 2, 3, 4, 6, 12\}, \gcd)$

$a \in D_{12}$, $b \in D_{12}$.

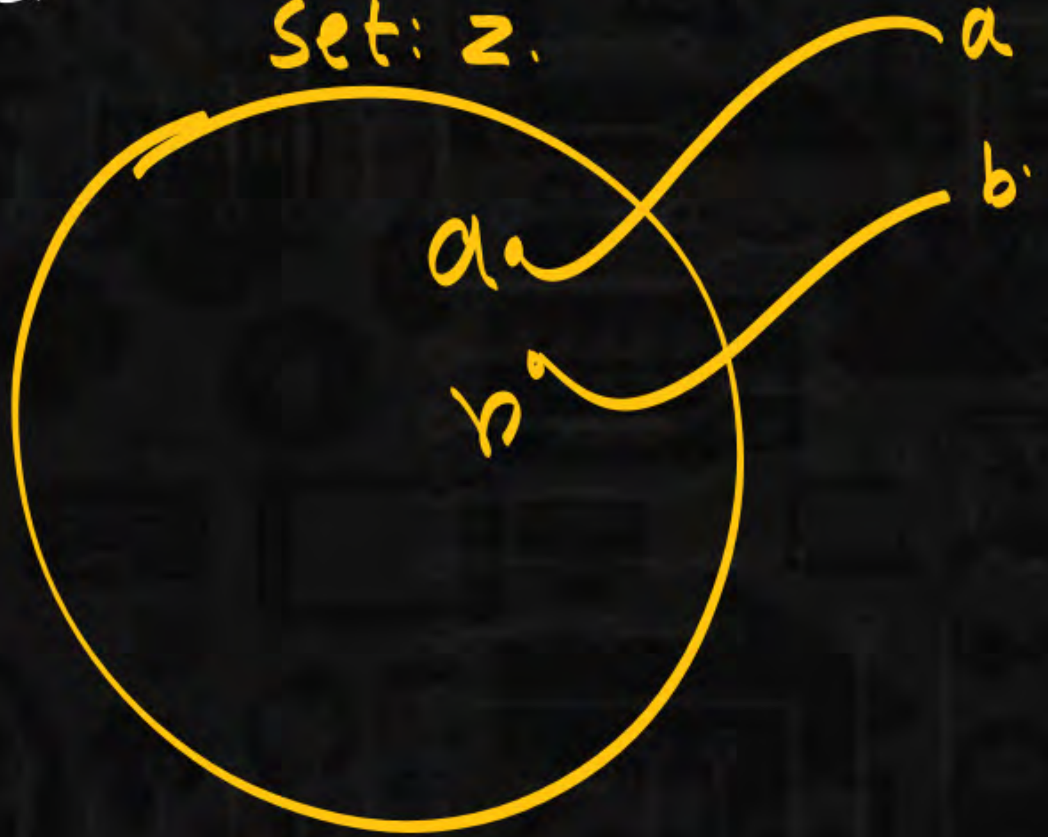
$\gcd(a, b) \in D_{12}$.

# GROUP THEORY

$(G, *)$

$(z, |)$

set: z.




$a|b \notin z.$

# GROUP THEORY

$(G, *)$

$(A, \times)$

$A = \{2^n \mid n \in z\}$

**Associative..**

$2^a \cdot (2^b \cdot 2^c) = (2^a \cdot 2^b) \cdot 2^c$

$2^a \cdot (2^{b+c})$

$2^{a+b+c} = 2^{a+b+c}$

$$\boxed{2^a \times 2^0 = 2^a}$$

$2^0 \rightarrow$ identity

A.

$2^a \mid a \in z$

$2^b \mid b \in z$

$2^a \times 2^b$

$\{2^{a+b} \mid a \in z,\ b \in z \quad a+b \in z\}$

# GROUP THEORY

$(R, *)$ $\quad \underset{L.S}{a} * \underset{R.S}{b} = \frac{ab}{2}$ $\quad$ L.S/R $\frac{\ }{2}$ <u>Associative:</u>

$$a * (b * c) = (a * b) * c$$

$$a * \left(\frac{bc}{2}\right) \qquad \left(\frac{ab}{2}\right) * c$$

$$\frac{(a)\left(\frac{bc}{2}\right)}{2} \qquad \frac{\left(\frac{ab}{2}\right)c}{2}$$

$$a * e = a$$

$$\frac{\cancel{a}e}{2} = \cancel{a}$$

$$\boxed{e = 2}$$

$\rightarrow$ unique

# GROUP THEORY

$(A, \times)$

$A = \{1, \omega, \omega^2\}$

$$\boxed{\begin{aligned} \omega \times 1 &= \omega \\ \omega^2 \times 1 &= \omega^2 \end{aligned}}$$

$\omega^3 = 1$

$$1 \times (\omega \times \omega^2) = (1 \times \omega) \times \omega^2$$

$$1 \times \omega^3 = \omega \times \omega^2$$

$$1 \times 1 = \omega^3$$

$$1 = 1$$

# GROUP THEORY

$(A, \times)$

$i^2 = -1.$

$A = \{1, i, -i, -1\}$

Closed.

Associative

$i \in A \quad -i \in A \quad i \times -i$

$-i^2.$

$-(-1)$

$1 \in A$

$$\boxed{i \times 1 = i}$$

$1 \times (i \times -i) = (1 \times i) -i$

$1 \times (-i^2) = i \times -i$

$1 \times (-(-1)) = -i^2$

$1 = -(-1)$

$1 = 1.$

# GROUP THEORY

(Set, operation)

$\downarrow$

$(M_{2\times 2}, \times)$

$\left(\left\{[\ ]_{2\times 2}, [\ ]_{2\times 2} \cdots\cdots\right\}, \times\right)$

$A = [\ ]_{2\times 2}$ $B = [\ ]_{2\times 2}$ $\boxed{A\times B \neq B\times A}$

$A\times B = [\ \ ]_{2\times 2} \in M_{2\times 2}.$

$A\cdot(B\cdot C) = (A\cdot B)\cdot C.$

$\boxed{A \times I = A.}$

# GROUP THEORY

Operation: addition mod 6.

$\oplus_6$

$(\text{Set}, *)$

$(\{0,1,2,3,4,5\}, \oplus_6)$

$a \oplus_6 b = (a+b) \bmod 6$

$2 \oplus_6 3 = (2+3) \bmod 6 = 5 \bmod 6 = \underline{\underline{5}}$

$3 \oplus_6 0 = 3$

$3 \oplus_6 (4 \oplus_6 5) = (3 \oplus_6 4) \oplus_6 5$

$3 \oplus_6 (3) \quad = (1) \oplus_6 5$ (Associative)

$0 \quad = 0$

# GROUP THEORY

$$A \oplus B = (A \cup B) - (A \cap B)$$

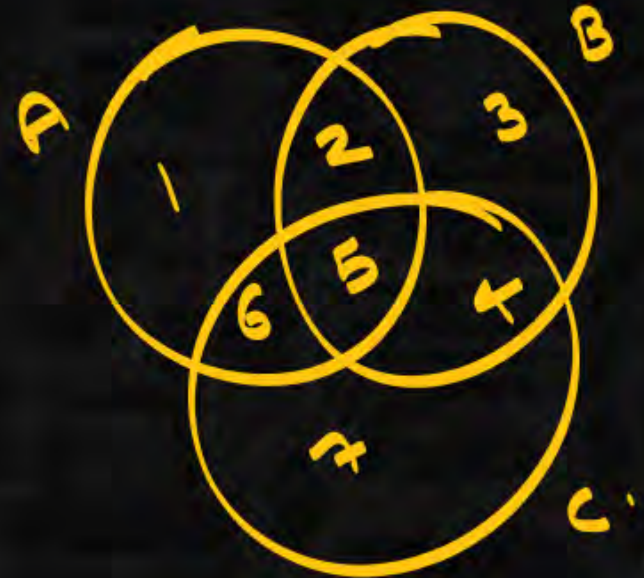
$$A = \{1, 2, 3\}$$

$$A \Delta (B \Delta C) = (A \Delta B) \Delta C$$

$(P(A), \Delta)$ OR $(2^A, \Delta)$

OR OR

$(P(A), \oplus)$ $(2^A, \oplus)$




$$A \oplus \phi = A$$

$$(A \cup \phi) - (A \cap \phi)$$

$$= A$$

$A \oplus B$

$(A \cup B) - (A \cap B)$

# GROUP THEORY

$A = \{1, 2, 3\}$.

(set, operation)

$(\{f_1, f_2, f_3, f_4, f_5, f_6\}, \circ)$

| $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ |
|---|---|---|---|---|---|
| $1 \to 1$ | $1 \to 1$ | $1 \to 2$ | $1 \to 2$ | $1 \to 3$ | $1 \to 3$ |
| $2 \to 2$ | $2 \to 3$ | $2 \to 1$ | $2 \to 3$ | $2 \to 1$ | $2 \to 2$ |
| $3 \to 3$ | $3 \to 2$ | $3 \to 3$ | $3 \to 1$ | $3 \to 2$ | $3 \to 1$ |

closed.

$f_2 \in A$

$f_3 \in A$

$f_2 \circ f_3 = f_5$

$$\begin{array}{ccccc} 1 & \to & 2 & \to & 3 \\ 2 & \to & 1 & \to & 1 \\ 3 & \to & 3 & \to & 2 \\ & f_3 & & f_2 & \end{array}$$

# GROUP THEORY

$A = \{1, 2, 3\}$.

| $f_1$ | $f_2$ | $f_3$ | $f_4$ |
|---|---|---|---|
| $1 \to 1$ | $1 \to 1$ | $1 \to 2$ | $1 \to 2$ |
| $2 \to 2$ | $2 \to 3$ | $2 \to 1$ | $2 \to 3$ |
| $3 \to 3$ | $3 \to 2$ | $3 \to 3$ | $3 \to 1$ |

| $f_5$ | $f_6$ |
|---|---|
| $1 \to 3$ | $1 \to 3$ |
| $2 \to 1$ | $2 \to 2$ |
| $3 \to 2$ | $3 \to 1$ |

$f_2 \circ (f_3 \circ f_5) = (f_2 \circ f_3) \circ f_5$

1 – 3 – 3
2 – 1 – 2
3 – 2 – 1

$(\ ) \circ f_5$

$f_2 \circ f_6$

1 – 3 – 2
2 – 2 – 3
3 – 1 – 1

$f_4$

# GROUP THEORY

$A = \{1, 2, 3\}$.

$1 \to 1$, $2 \to 2$, $3 \to 3$ : $f_1$.

$1 \to 1$, $2 \to 3$, $3 \to 2$ : $f_2$.

$1 \to 2$, $2 \to 1$, $3 \to 3$ : $f_3$

$1 \to 2$, $2 \to 3$, $3 \to 1$ : $f_4$.

$1 \to 3$, $2 \to 1$, $3 \to 2$ : $f_5$

$1 \to 3$, $2 \to 2$, $3 \to 1$ : $f_6$.

(Set, operation)

$((f_1, f_2, f_3, f_4, f_5, f_6), \circ)$

$f_5 \circ f_1 = f_5$.


THANK YOU GW
SOLDIERS !