CS & IT ENGINEERING

DISCRETE MATHS GRAPH THEORY

Lecture No. 14

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TOPICS

01 INDEPENDCE NO.

02 DOMINATION NO.

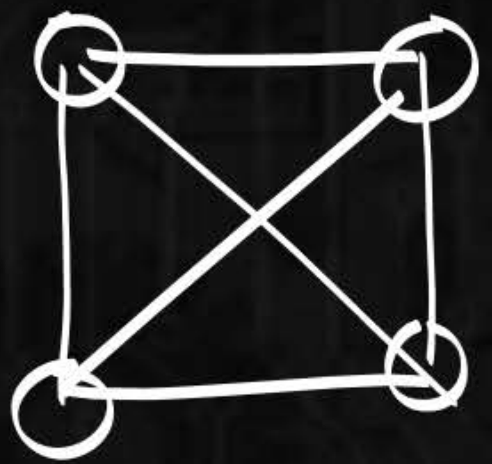
03 MATCHING NO .

Planar Graph :

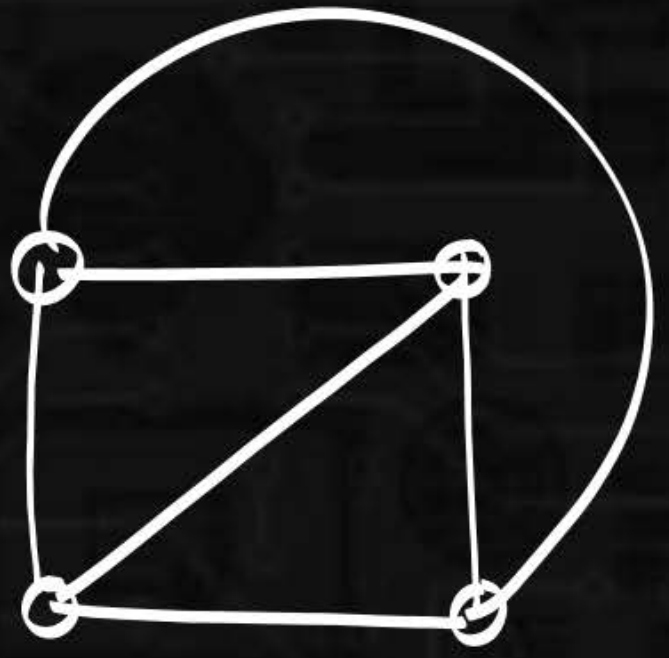
Graph

planar:

nonplanar:



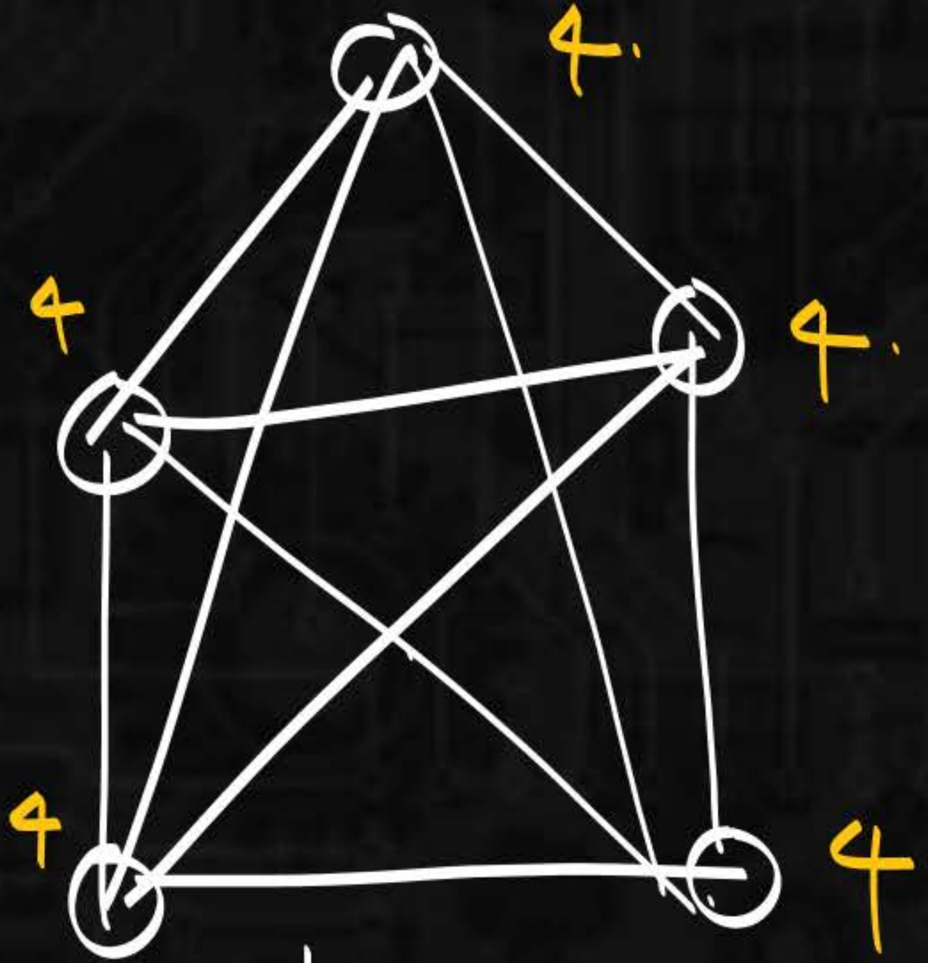
K4.



Planar Graph:

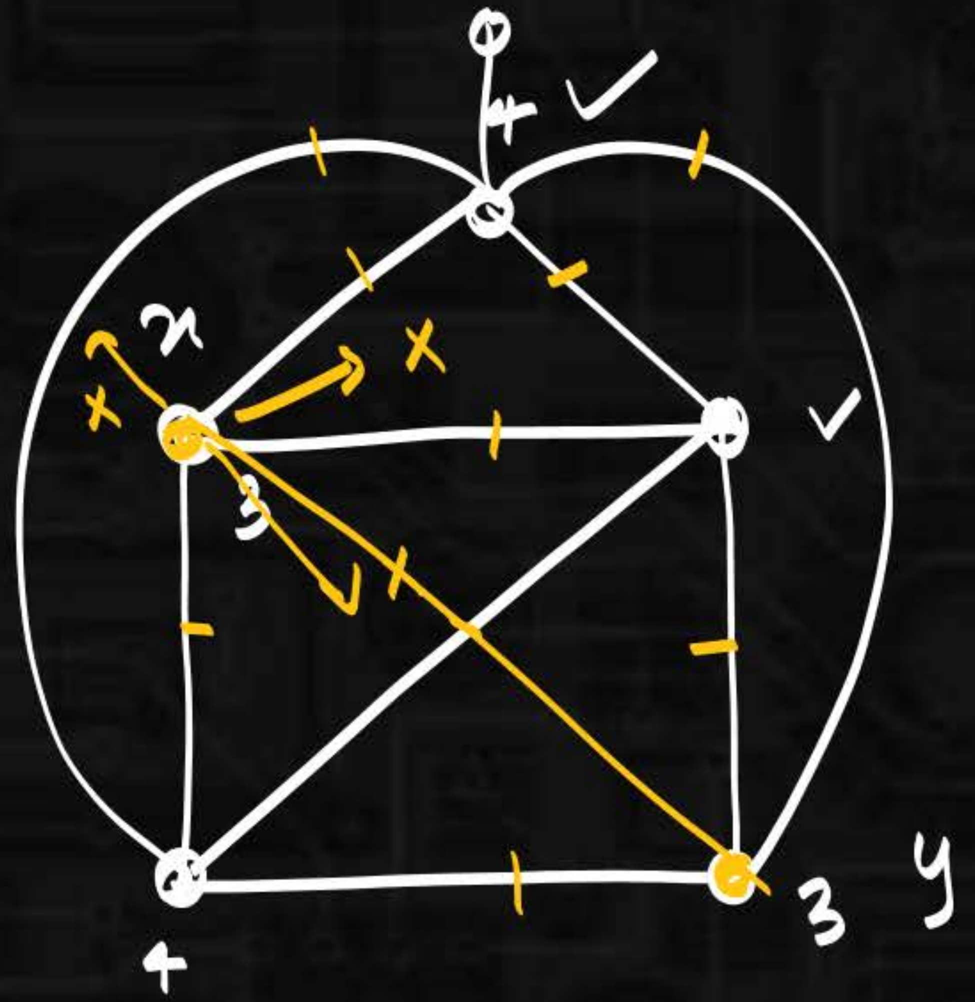
if we can draw a graph on a plane without intersection of its edges.

Otherwise it is non planar.



K5

$n = 5$
$e = 10$



planar

$n = 5$
$e = 9$

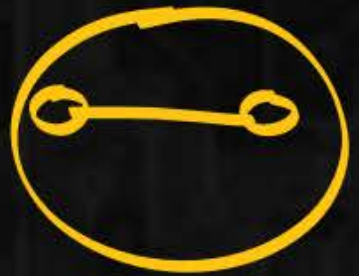
$\boxed{K5 - e}$

K 5 is nonplanar

Kurawtoski's 1st G.

K 5 is nonplanar Graph.

$n = 1$ 

$n = 2$  

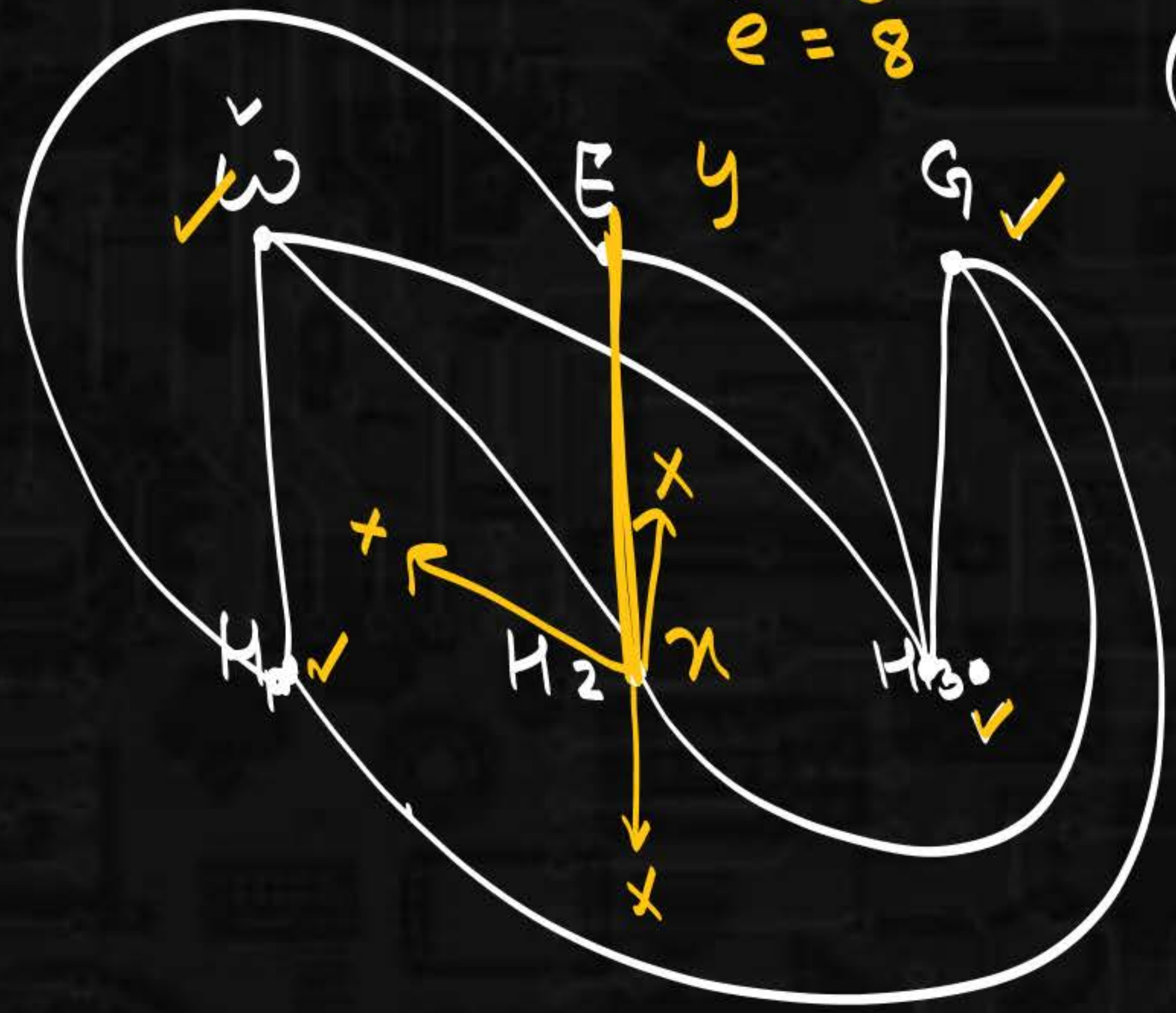
$n = 3$

$n = 4$ . . . . $K_4 \rightarrow$ planar.

$n = 5$ . . . . $n = 5$, $e = 9$. $\{K_5 - e\}$ ↑ planar.

$K_5 \rightarrow$ nonplanar.

$n = 6$
$e = 8$



$(K_{3,3})$

$\begin{cases} n = 6 \\ e = 9. \end{cases}$

$\delta(G) = \frac{2e}{n} = \Delta(G) = 3.$

Degree of each vertex will be 3.

$\rightarrow$ $K_{3,3}$ is nonplanar.

$\{K_{3,3} - e\}$

note:

→ both graphs are $\boxed{\text{nonplanar}}$, removal single edge from both the graphs will make $\boxed{\text{planar Graph.}}$

→ both graphs are Regular Graphs.

→

nonplanar

min.

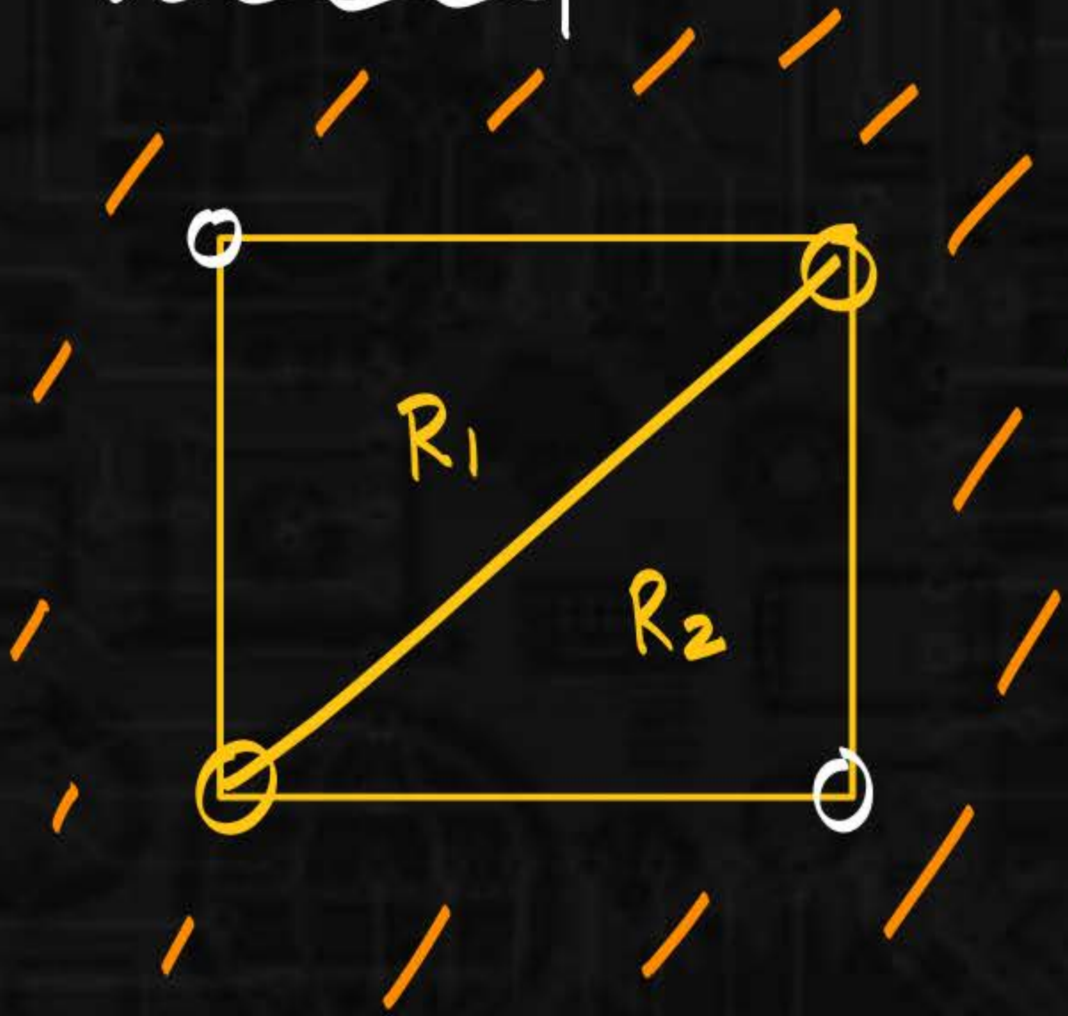
K_5	$n = 5$	$e = 10$
$K_{3,3}$	$n = 6$	$e = 9$ ← min.

→ K_5 is nonplanar graph with min no. of vertices.

→ $K_{3,3}$ is nonplanar graph with min no. of edges.

Draw a planar Graph on a plane. it creates or make.

embedding:



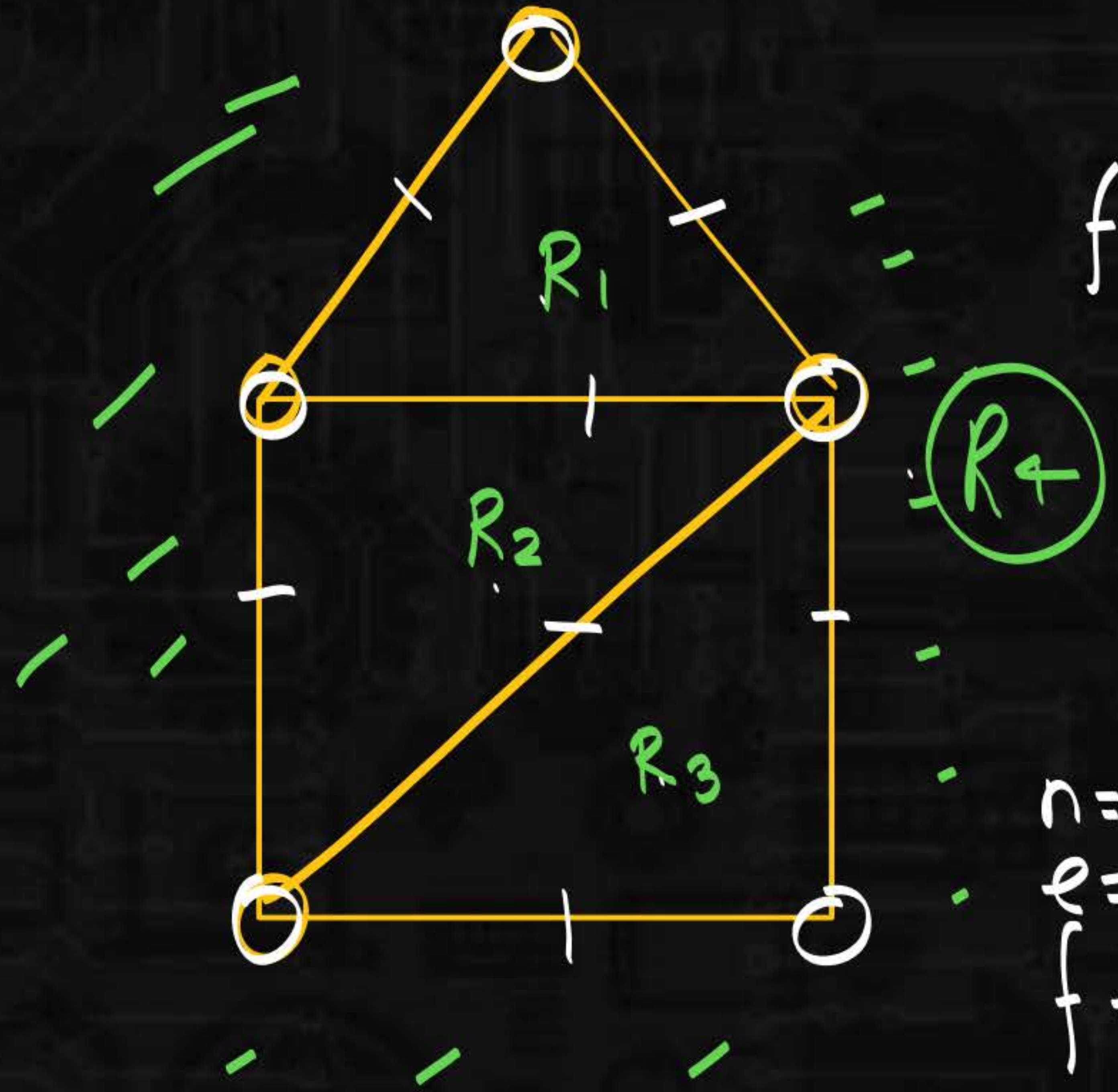
$$n - e + f = 2.$$

$n = 4$.
$e = 5$
$f = 3$.

$4 - 5 + 3$
$-1 + 3 = 2$.

region / faces (r/f)

- closed, bounded, finite: R1, R2
- open, unbounded, infinite: eg: R3.



$f = 4.$

$n = 5.$
$e = 7$
$f = 4.$

$n \to$ Total vertices.
$e \to$ no of edges.
$f \to$ no. of faces. (Planar)

Euler's Thm:

$$\boxed{n - e + f = 2.}$$

Planar

Consider a graph having 10 vertices & 15 edges.
what will be no. of faces?

(GATE-12)

$n = 10 \quad e = 15$

$n - e + f = 2$

$10 - 15 + f = 2$

$\boxed{f = 7}$

Q2: what will be bounded region?

Total = bounded + 1.

bounded = 7 - 1 = 6.

$n-e+f=2.$

Case 1:

$n=2$

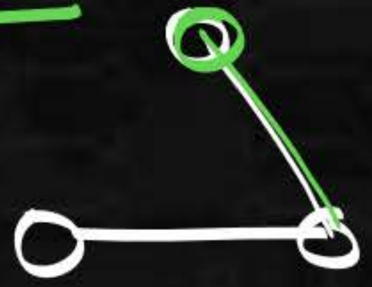
$e=1.$

$n-e+f=2.$

$2-1+f=2$

$f=1$

Case 2



$n+$

$e+$

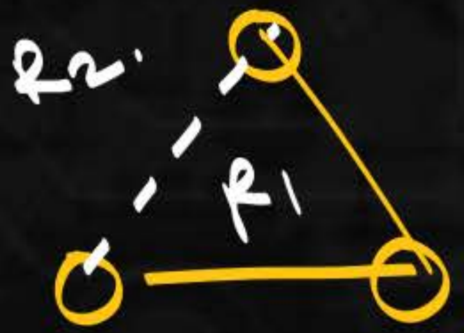
$n=3 \quad e=2$

$n-e+f=2.$

$3-2+f=2.$

$f=1$

Case 3:



$e+.$

$f+.$

$(n \geq 2)$

$n-e+f=2$

$3-3+f=2.$

$f=2.$

2

$$\frac{n\uparrow}{e\uparrow}$$

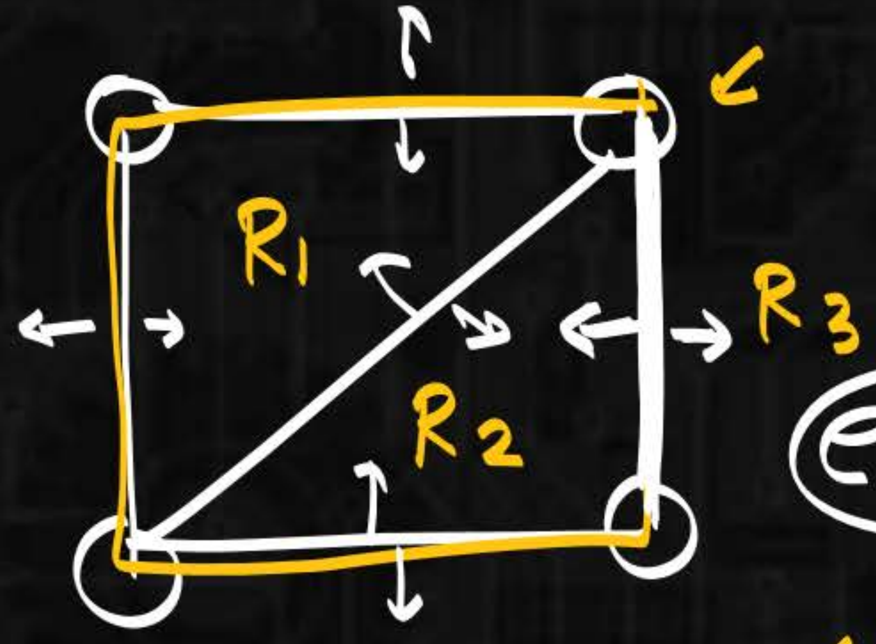
3

$e\uparrow$
$f\uparrow$

$$\boxed{n - e + f = 2}$$

$n = 10$
$e = 9$

$n - e + f = 2$
$10 - 9 + f = 2$
$f = 1$



$e = 5$

Degree of regions = no. of edges involved into this.

$\deg(R_1) = 3$

$\deg(R_2) = 3$

$\deg(R_3) = 4.$

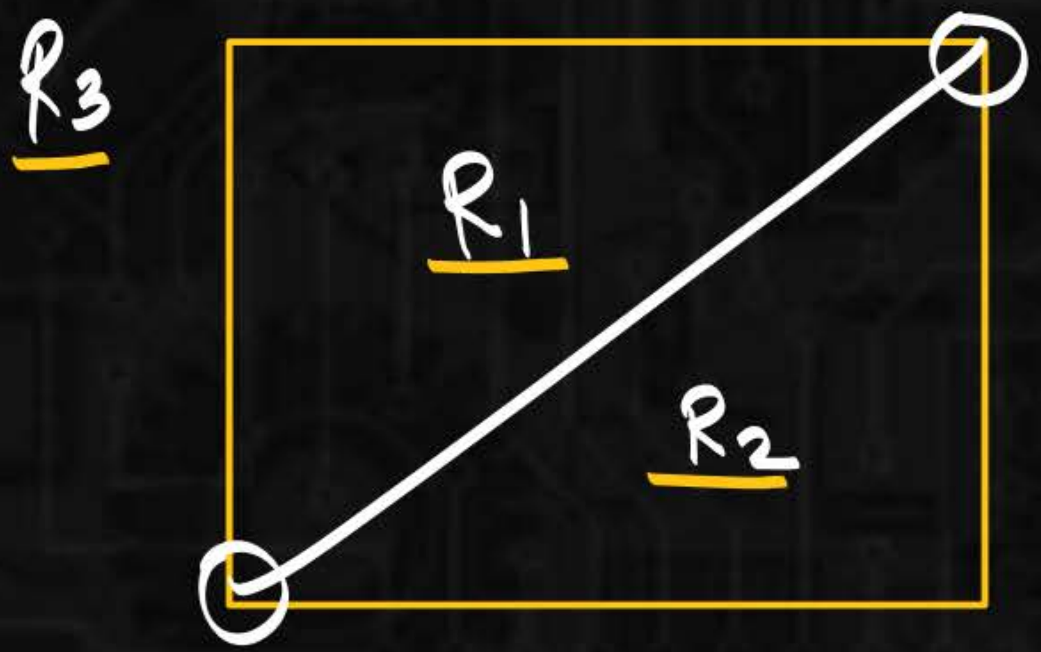
$\deg(R_1) + \deg(R_2) + \deg(R_3) = 3 + 3 + 4.$

$\sum d(R_i) = 10 = 2(5)$

no. of edges

$$\boxed{\sum d(R_i) = 2e.}$$

Sum of degrees of all regions is equals to twice the no. of edges.



$$\begin{cases} \deg(R_1) = 3 \\ \deg(R_2) = 3 \\ \deg(R_3) = 4. \end{cases}$$

{ all region is made up by atleast 3 edges.

($n \geq 3$)

$\deg(R_1) \geq 3$

$\deg(R_2) \geq 3$

$\deg(R_3) \geq 3$

$\deg(R_1) + \deg(R_2) + \deg(R_3) \geq 3+3+3$

$\geq 3 \cdot 3 \to$ no of regions.

$\deg(R_1)+\deg(R_2)+\deg(R_3) \geq 3 \cdot 3 \rightarrow$ no of faces.

$\sum d(R_i) \geq 3 \cdot f$

$$\boxed{2e \geq 3f}$$

$n - e + f = 2$

$$\boxed{f = 2 + e - n}$$

($n \geq 3$ all regions are made by atleast 3 edges)

$2e \geq 3(2+e-n)$

$2e \geq 6 + 3e - 3n$

$3n - 6 \geq 3e - 2e$

$3n - 6 \geq e$

$e \leq 3n - 6$

<u>Thm</u>: if Graph is planar then $e \leq 3n-6$.

Viceversa is not True.

Thm: if Graph is planar then $e \leq 3n-6$. $(n \geq 3)$.

$\neg(e \leq 3n-6)$

$e > 3n-6$.

viceversa is not True.

if $e \leq 3n-6$ then G is Planar (false)

eg: $K_{3,3}$

$e = 9 \quad n = 6$.

$e \leq 3n-6$.

$9 \leq 3(6)-6$.

$9 \leq 12$ (True)

Contrapositive:

if not $(e \leq 3n-6)$ then G is not planar

***

$\boxed{\text{if } e > 3n-6 \text{ then G is nonplanar.}}$

$p \to q$ **Contrapositive:**

$\neg q \to \neg p$.

G is connected planar with 10 vertices. if the number of edges on each face is 3 then no. of edges in G.

$$e = 3n - 6$$

$$e = 3(10) - 6$$

$$e = 24$$

Q.

formula.

Concept.

level-1.

Q: level-2

Concept

Concept

$e \le 3n-6$

$\delta(G) \le \frac{2e}{n} \le \Delta(G)$

Refer

Q. level-3.

Subjects

Subjects


THANK YOU GW
SOLDIERS !