ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-06

Probability

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Topics to be Covered

- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM
- STATISTICS – I (PROBABILITY DISTRIBUTIONS)
- STATISTICS – II (CORRELATION AND REGRESSION)

Ex:- Consider a dice with property that the probability of a face with n dots showing up is proportional to n. The probability of the face with 3 dots showing up is ______.

$$P(n \text{ dots}) \propto n$$

$$P(n \text{ dots}) = Kn$$

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$

$$K + 2K + 3K + 4K + 5K + 6K = 1$$

$$21K = 1$$

$$\boxed{K = 1/21}$$

$$P(3) = 3K = 3\left(\frac{1}{21}\right) = \frac{1}{7}$$

Types of Discrete Random Variable

1. Bernoulli Random Variable

In an experiment, outcome
(1 trials)

FAILURE ($X=0$)

SUCCESS ($X=1$) a

X	0	1
$p(x)$	$1-a$	a

Parameter $\rightarrow$ a

Mean $\rightarrow$ a

Variance $\rightarrow$

Types of Discrete Random Variable

1 trial $\begin{cases} \to p \\ \to q \end{cases}$

$\{p + q = 1\}$

2. ** Binomial Random Variable

n independent trials, $\begin{cases} \to r \text{ success} \\ \to n-r \text{ failure} \end{cases}$

$$\boxed{P(X = r) = {}^nC_r\, p^r q^{n-r}}$$

$$\boxed{\sum_{r=0}^{r=n} {}^nC_r\, p^r q^{n-r} = 1}$$

n+1 cases

	$r=0$	$r=1$	$r=2$		$r=n$
X	0	1	2	. . . .	n
P(x)	${}^nC_0 p^0 q^n$	${}^nC_1 p^1 q^{n-1}$	${}^nC_2 p^2 q^{n-2}$		${}^nC_n p^n q^{n-n}$

Parameters $\begin{cases} \to n \\ \to p \end{cases}$

- Mean $= np$
- Variance $= npq$
- $\sigma = \sqrt{npq}$

Ex:- A coin is tossed is 10 times

i) Find the probability of getting exactly 3 Heads, 7 Tails

ii) Find the probability of obtaining at least 2 Heads

Soln :-

_ _ _ _ _ _ _ _ _ _

$n = 10$

$\underbrace{p = 1/2}_{\text{Success}}, \underbrace{q = 1/2}_{\text{Failure}}$

i) $P(x=3) = {}^{10}C_3\, p^3 q^7 = {}^{10}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^7$

ii) $P(x \geq 2) = 1 - P(x=0) - P(x=1)$

$= 1 - {}^{10}C_0\, p^0 q^{10-0} - {}^{10}C_1\, p^1 q^{10-1}$

Types of Discrete Random Variable

$\begin{cases} p \\ q \end{cases}$ $p + q = 1$

3. Geometric Random Variable

Suppose n independent trials (each trial having success probability p) are performed until a success occurs.

X	1	2	3	4	n
$p(x)$	p	qp	q^2p	q^3p	$q^{n-1}p$

$$P(X=r) = q^{r-1}p$$

Parameter $\rightarrow$ p

- Mean
- Variance

$\rightarrow$ **Question** $p = \frac{1}{10}$

$\rightarrow P(X=5) \Rightarrow q^4 p^1$

$= (0.9)^4 (0.1)$

Types of Discrete Random Variable

4. ** Poisson Random Variable

n independent trials $\begin{cases} \to r \text{ success} \\ \to n-r \text{ failures} \end{cases}$

1 trial $\begin{cases} \to p \\ \to q \end{cases}$

STATISTICS –I (PROBABILITY DISTRIBUTIONS)

Limiting case of Binomial distribution under the condition

i) $n \to \infty$ (Very large)
ii) $P \to 0$ (Very small)
iii) $np \to \lambda$ (Constant)

$$P(X=r) = \frac{e^{-np}(np)^r}{r!} = \boxed{\frac{e^{-\lambda}\cdot\lambda^r}{r!}}$$

	$r=0$	$r=1$	$r=2$	r
X	0	1	2	r
$p(x)$	$\frac{e^{-\lambda}\lambda^0}{0!}$	$\frac{e^{-\lambda}\lambda^1}{1!}$	$\frac{e^{-\lambda}\lambda^2}{2!}$	$\frac{e^{-\lambda}\lambda^r}{r!}$

Parameters $\to n$, $\to p$

- Mean $\to \lambda = np$
- Variance $\to np$

Ex:- On an average, T.I takes bribe 6 times a day Find the probability that he will take bribe. The probability is poison distributed

i) 3 times a day

ii) He takes no bribe on a day

iii) At least once a day

$\lambda \to 6/\text{day}$

i) $P(X=3) = \dfrac{e^{-\lambda}\lambda^{r}}{r!} = \dfrac{e^{-6}\cdot(6)^{3}}{3!} = 0.089 = 8.9\%$

ii) $P(X=0) = \dfrac{e^{-6}\cdot(6)^{0}}{0!} = 0.0025 = 0.25\%$

iii) $P(X\geq 1) = 1 - P(X=0) = 1 - \dfrac{e^{-6}\,6^{0}}{0!} = 0.9976$

Ex:- Probability of motorist in F-1 race being killed in accidents during a year is $\frac{1}{2400}$. What is the probability that there will be at least one fatal accident in a year when 200 motorists take part in a race?

$n = 200 \qquad p = \frac{1}{2400}$

$\lambda = np = 200 \times \frac{1}{2400} = \frac{1}{12}$

$P(x \geq 1) = 1 - P(x=0)$

$P(\text{At least one accident}) = 1 - P(\text{No accident})$

$= 1 - \frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-1/12}(1/12)^0}{0!}$

$= 0.08$

Ex:- In a factory, there is a small chance of 1 in 500 tyres to be defective. Tyres are supplied in lots of 10. Calculate the approximate number of lots containing

i) No defective in a consignment of 10000

ii) 1 defective in a consignment of 10000

iii) 2 defective in a consignment of 10000

9802 → 0 def. 196 → 1 def. 2 → 2 def

$n = 10$ $p = \frac{1}{500}$ $\lambda = np = 10 \times \frac{1}{500} = 0.02$

i) $P(x=0) = e^{-0.02}(0.02)^0/0! = 0.9802 \times 10000 = 9802$ lots

ii) $P(x=1) = e^{-0.02}(0.02)^1/1! = 0.0196 \times 10000 = 196$ lots

iii) $P(x=2) = e^{-0.02}(0.02)^2/2! = 0.000196 \times 10000 = 1.96 \simeq 2$ lots

10000 lots

Continuous Random Variable

When a variable X takes every value in an interval, it gives rise to continuous random variable x.

$f(x)$ is probability density function.

$$f(x) = \begin{cases} 0 & x < 0 \\ f(x) & a \le x \le b \\ 0 & x > b \end{cases}$$

Range → Finite, Infinite

Properties :-

i) P.d.f. is always positive i.e. $f(x) > 0$

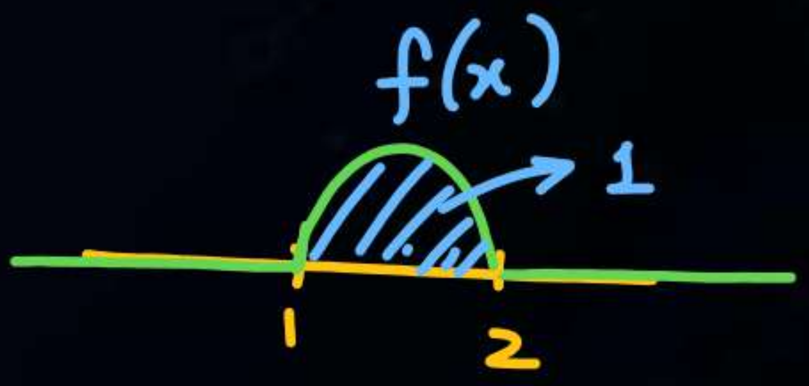
ii) ** $\boxed{\int_{-\infty}^{+\infty} f(x)\,dx = 1}$

	DISCRETE R.V.	CONTINUOUS R.V.
	Outcome → Finite, discontinuous values.	Outcome → Infinite, continuous values
	$\sum_{i=1}^{n} p(x_i) = 1$	$\int_{-\infty}^{\infty} f(x)\,dx = 1$
C.D.F.	$F(a) = \sum p(x_i)$ for all $x_i \le a$	$F(a) = \int_{-\infty}^{a} f(x)\,dx$ for all $x_i \le a$

Ex:- Find the value of λ such that fn. f (x) is a valid probability density function

$f(x) = \lambda(x-1)(2-x)$, For $1 \leq x \leq 2$

$= \quad 0 \quad$ Otherwise



$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Rightarrow \int_{1}^{2} \lambda(x-1)(2-x)\,dx = 1$$

$$\int_{1}^{2} \lambda(2x - x^2 - 2 + x)\,dx = \lambda \int_{1}^{2} (-x^2 + 3x - 2)\,dx = 1$$

$$\lambda\left[-\frac{x^3}{3} + 3\frac{x^2}{2} - 2x\right]_{1}^{2} = 1 \Rightarrow \lambda\left(\frac{1}{6}\right) = 1$$

$$\boxed{\lambda = 6}$$

Thank you

GW

Soldiers !