CS & IT ENGINEERING

DISCRETE MATHS

Mathematical Logic

Lecture No. 07

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TOPICS

01 English to logic expression

02 Nested quantifier

03 TYPE 5

13. Consider the open statement

$$p(x, y): \quad y - x = y + x^2$$

where the universe for each of the variables x, y comprises all integers. Determine the truth value for each of the following statements.

a) $p(0, 0)$ (T)
b) $p(1, 1)$ (F)
c) $p(0, 1)$ (T)
d) $\forall y\, p(0, y)$ (T)
e) $\exists y\, p(1, y)$ (F)
f) $\forall x\, \exists y\, p(x, y)$ (F)
g) $\exists y\, \forall x\, p(x, y)$ (F)
h) $\forall y\, \exists x\, p(x, y)$

14. Determine whether each of the following statements is true or false. If false, provide a counterexample. The universe comprises all integers.

a) $\forall x\, \exists y\, \exists z\, (x = 7y + 5z)$
b) $\forall x\, \exists y\, \exists z\, (x = 4y + 6z)$

$p(x, y):\ \cancel{y} - x = \cancel{y} + x^2.$

$x^2 + x = 0$

$x(x+1) = 0$

$\boxed{x = 0 \quad x = -1.}$

f) $\forall x\, \exists y\, p(x,y)$

$\exists x \exists y (xy = 1)$ – T.

$x = 1 \quad y = 3$

$\exists x \exists y ((2x + y = 5) \wedge (x - 3y = -8))$ True.

$\exists x \exists y ((3x - y = 7) \wedge (2x + 4y = 3)) \Rightarrow$ <u>False</u>.

$x = \frac{31}{14} \quad y =$

$x \notin Z$

$y \notin Z$.

$$\forall x \forall y \left[(x > y) \rightarrow (x - y > 0) \right]$$

$$\neg \forall x \forall y \left[\neg (x > y) \vee (x - y > 0) \right]$$

$$\exists x \exists y \left[\neg\neg (x > y) \wedge \neg (x - y > 0) \right]$$

$$\exists x \exists y \left[(x > y) \wedge (x - y \leq 0) \right]$$

$$\forall x \forall y \left[(x < y) \rightarrow \exists z \left(x < z < y \right) \right]$$

$$\neg \forall x \forall y \left[\neg (x < y) \vee \exists z \left(x < z < y \right) \right] \quad \neg \left(x < z \wedge z < y \right)$$

$$\left(x \geq z \vee z \geq y \right)$$

$$\exists x \exists y \left[\neg\neg \left(x < y \right) \wedge \neg \exists z \left(x < z < y \right) \right]$$

$$\forall z \left(x \geq z \text{ OR } z \geq y \right) \Big]$$

$$\neg \forall x \forall y \left[\left(|x| = |y| \right) \rightarrow \left(y = \pm x \right) \right]$$

$$\neg \forall x \forall y \left[\neg \left(|x| = |y| \right) \vee \left(y = \pm x \right) \right]$$

$$\exists x \exists y \left[\neg\neg \left(|x| = |y| \right) \wedge \neg \left(y = \pm x \right) \right]$$

$$\exists x \exists y \left(|x| = |y| \wedge \left(y \neq \pm x \right) \right)$$

$y = x$

$\boxed{y \neq x}$

$$\left[\forall x,\ \alpha \rightarrow \left(\exists y,\ \beta \rightarrow \left(\forall u,\ \exists v,\ \gamma\right)\right)\right]$$ (GATE-08)

a) $\exists x,\ \alpha \rightarrow (\forall y,\ \beta \rightarrow (\exists u, \forall v, \gamma))$ ✗

b) $\exists x\ \alpha \rightarrow (\forall y,\ \beta \rightarrow (\exists u, \forall v, \neg\gamma))$ ✗

c) $\forall x\ \neg\alpha \rightarrow (\forall y,\ \neg\beta \rightarrow (\forall u, \forall v, \neg\gamma))$

d) $\left[\exists x,\ \alpha \wedge \left(\forall y,\ \beta \wedge \left(\exists u, \forall v, \neg\gamma\right)\right)\right]$

(MSQ)

$P(x, y)$: x is divisor of y.

$D: Z$.

$P(x, y): \frac{y}{x}$.

$S_1: \forall x \exists y\, P(x, y)$ (False)

$S_1: \forall x \exists y \left(\frac{y}{x} \in Z \right)$

$\frac{y}{\cdots\cdots 0}$

$S_2: \forall y \exists x\, P(x, y)$

$S_3: \exists x \forall y\, P(x, y)$

$S_4: \exists y \forall x\, P(x, y)$

x $\quad$ y $\quad$ $x = 0$

0

$\forall x \exists y \left(\frac{y}{0} \right.$

$1 \longrightarrow$

$2 \longrightarrow$

$3 \longrightarrow$

(MSQ) $D : \mathbb{Z}$.

$P(x, y)$: x is divisor of y.

S_1: $\forall x \exists y\, P(x, y)$
(False)

S_2: $\forall y \exists x\, P(x, y)$

S_3: $\exists x \forall y\, P(x, y)$

S_4: $\exists y \forall x\, P(x, y)$

$\forall x \exists y \left(\frac{y}{x} \right)$ $\left(\frac{y}{\cdots\cdots 0} \right.$

all x, there exist y.

$$\begin{array}{ccc} x & & y \\ 2 & \longrightarrow & 4 \\ 3 & \longrightarrow & 6 \\ 4 & \longrightarrow & 8 \\ 0 & & \end{array}$$

$\frac{2}{0}$

(MSQ)

$D : \mathbb{Z}$.

$P(x, y)$: x is divisor of y.

S_1: $\forall x \exists y \, P(x, y)$ (False)

S_2: $\forall y \exists x \, P(x, y)$ (True)

S_3: $\exists x \forall y \, P(x, y)$ (True)

S_4: $\exists y \forall x \, P(x, y)$ (False)

S_2: $\forall y \exists x \left(\frac{y}{x} \right)$

all the integers are ÷ by an element called as 1.

$\frac{\cdots\cdots\cdots\cdots}{1}$

D: Z

$S_1: \forall n \exists m (n+m=5)$

$S_2: \exists n \forall m (n \cdot m = m)$

$S_3: \forall m \exists n (m \cdot n = 1)$

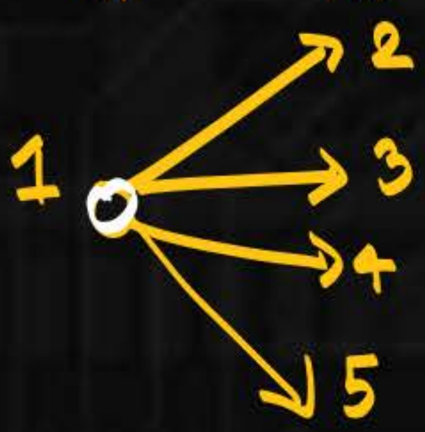
$S_4. \exists m \forall n (m+n=0)$

$\underline{S_1}: \overrightarrow{\forall n \exists m} (n+m=5)$ (True)

n	m
$1 \longrightarrow$	4
$2 \longrightarrow$	3
$4 \longrightarrow$	1
$0 \longrightarrow$	0
$0 \longrightarrow$	0

$S_2. \exists n \forall m (n \cdot m = m)$ (True)

n m



$1 \cdot m = m$

$1 \cdot 2 = 2$

$1 \cdot 3 = 3$

$1 \cdot 4 = 4$

$1 \cdot 5 = 5$

$S_3: \forall m \exists n (m \cdot n = 1)$

$2 \to \times$

$m = 2$

$2 \cdot n = 1$

$n = 1/2$

$S_4. \exists m \forall n (m+n=0)$ (False)

m n

$D: \mathbb{Z}$

True.

$S_1: \forall n \, \exists m \, (n+m=5)$

$S_2: \exists n \, \forall m \, (n \cdot m = m)$

$S_3: \forall m \, \exists n \, (m \cdot n = 1)$

$S_4. \exists m \, \forall n \, (m+n=0)$

$S_1: \forall n \, \exists m \, (n+m=5)$

n	m
1	4
2	3
3	2
0	5

$n+m=5$

$1+m=5$

$m=5-1=4.$

$D: \mathbb{Z}$

True.

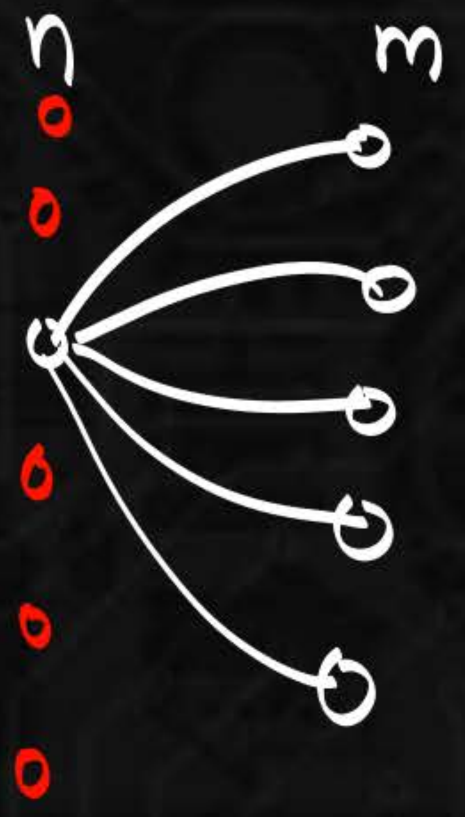
$S_1: \forall n \exists m (n+m=5)$

$S_2: \exists n \forall m (n \cdot m = m)$

$S_3: \forall m \exists n (m \cdot n = 1)$

$S_4. \exists m \forall n (m+n=0)$

$S_2 \cdot \exists n \forall m (n \cdot m = m)$



$n \cdot m = m$

$n = 1$

$1 \cdot x = x$

$1 \cdot 5 = 5$

$1 \cdot 6 = 6$

$D : \mathbb{Z}$

True.

$S_1 : \forall n \exists m \, (n + m = 5)$

$S_2 : \exists n \forall m \, (n \cdot m = m)$

$S_3 : \forall m \exists n \, (m \cdot n = 1)$

$S_4 . \exists m \forall n \, (m + n = 0)$

$\overrightarrow{\forall m \exists n \, (m \cdot n = 1)}$

m		n
1 o	$\longrightarrow$	o
2 o	$\longrightarrow$	o
3 o	$\longrightarrow$	o
⋮		

arrow does not exist

$m = 2$

$m \cdot n = 1.$

$2 \cdot n = 1$

$n = \frac{1}{2}.$

2 does not have a f

$\boxed{D : \mathbb{Z}}$

$S_1 : \forall n \exists m (n + m = 5)$ True.

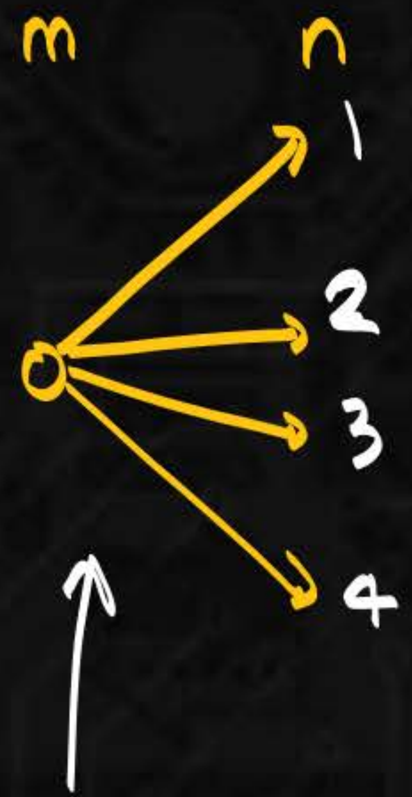
$S_2 : \exists n \forall m (n \cdot m = m)$ True

$S_3 : \forall m \exists n (m \cdot n = 1)$ False

$S_4 . \exists m \forall n (m + n = 0)$ False.

$\exists m \forall n (m + n = 0)$

$\forall m \exists n (m+n=0)$ (True)



$m + n = 0$

$n = 2.$

$m + 2 = 0$

$m = -2.$

$n = 3$

$m = -3$

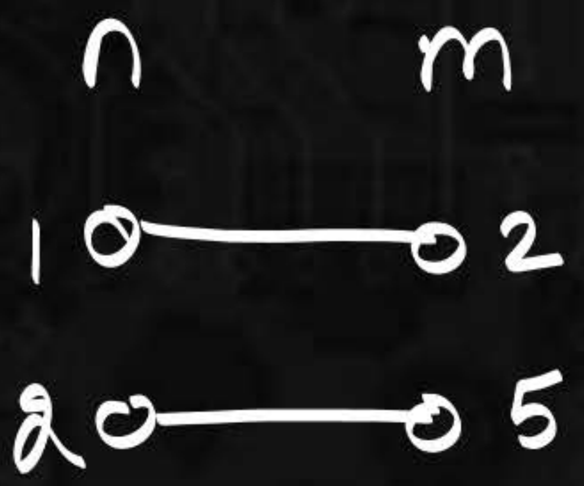
True

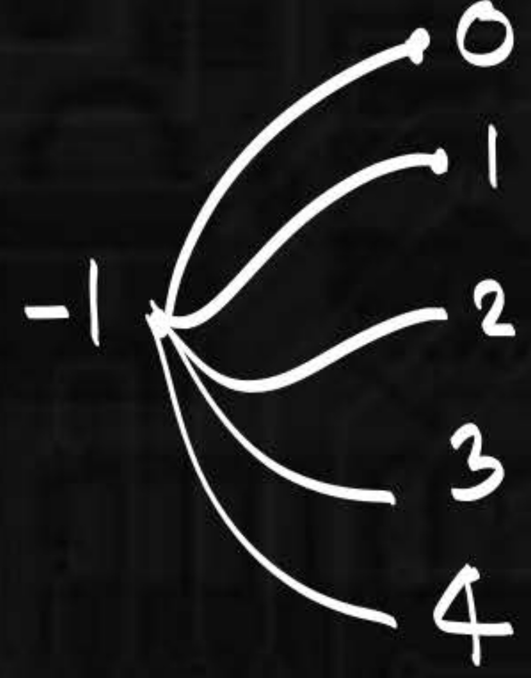
$\forall n \exists m (n^2 < m)$

True

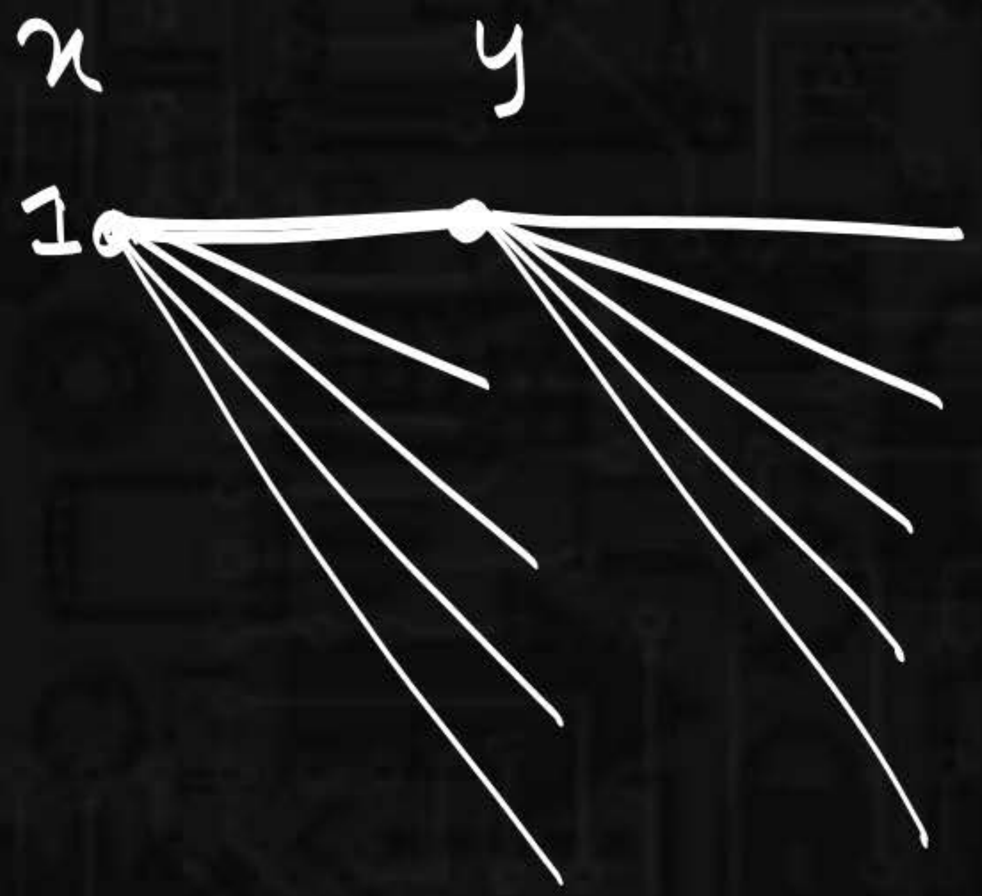
$\exists n \forall m (n < m^2)$

there exist element which is less than square of any int.



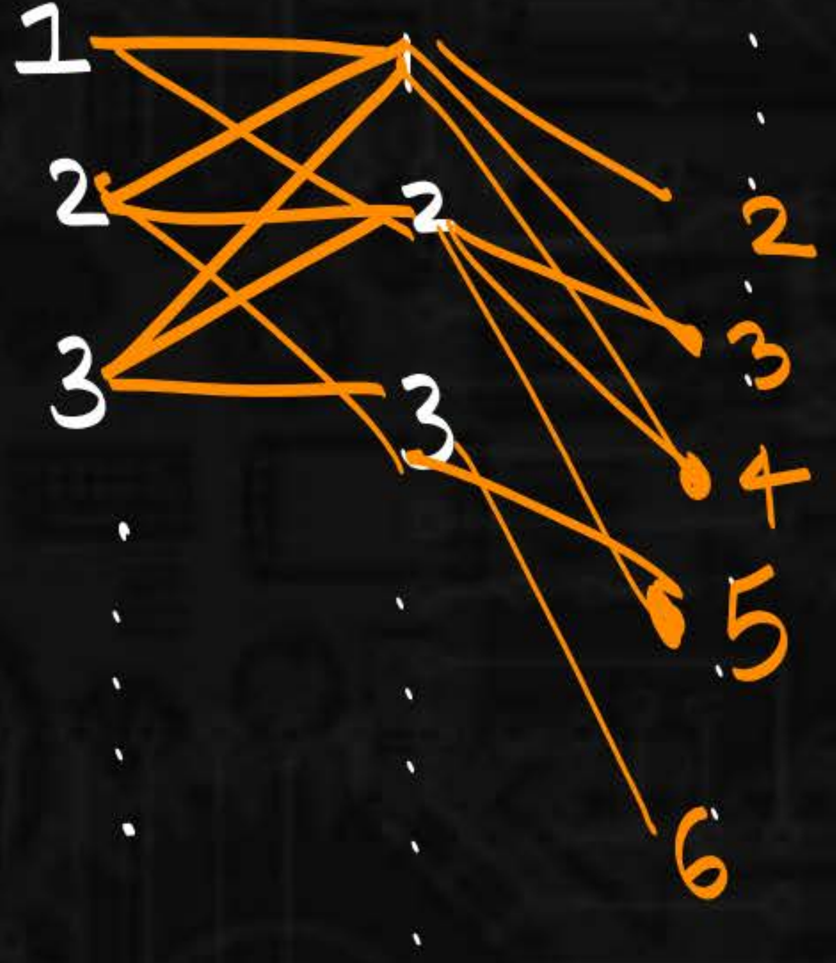


$$\forall x \forall y \ \forall z \ (x + y = z)$$

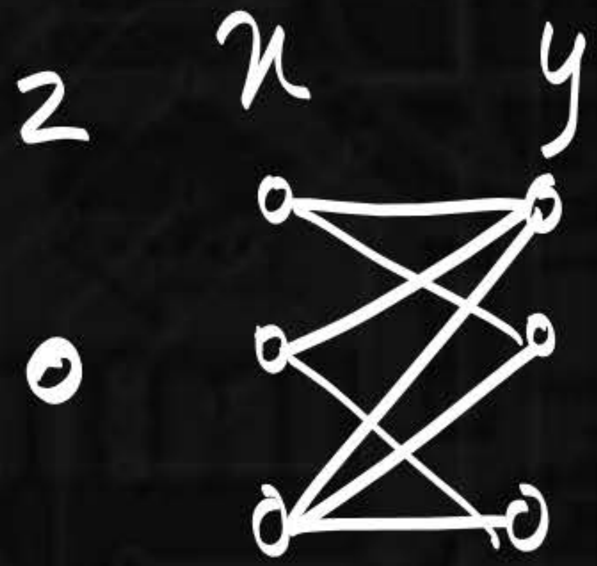


$D : \mathbb{Z}$

$\forall x\, \forall y\, \exists z\, (x + y = z)$



$\exists z\, \forall x\, \forall y\, (x + y = z)$



D: $\mathbb{Z}$.

$$\forall x \forall y \exists z \left(z = \frac{x+y}{2} \right)$$

x y $\quad$ $\mathbb{Z}$.

1 $\quad$ 2

$\exists x \, \forall y$

Some boys are taller than all girls

taller(y, x) : x is taller y

a) $\exists x \big(boy(x) \rightarrow \forall y \, (girl(y) \wedge taller(y, x)) \big)$

b) $\exists x \big(boy(x) \wedge \forall y \, (girl(y) \wedge taller(y, x)) \big)$

$\exists x \big(boy(x) \wedge \forall y \, (girl(y) \rightarrow taller(y, x)) \big)$ ✓

$\exists x \, \forall y \, \big(boy(x) \wedge (girl(y) \rightarrow taller(y, x)) \big)$


THANK YOU GW
SOLDIERS !