Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics

Set Theory

DPP-03

[MSQ]

1. Let $f, g : R \to R$, where $g(x) = 1 - x + x^2$ and $f(x) = ax + b$. If $(g \circ f)(x) = 9x^2 - 9x + 3$, determine a, b.

(a) a = –1, b = 3 (b) a = 3, b = –1
(c) a = –3, b = 2 (d) a = 2, b = –3

[NAT]

2. Let $A = \{1,2,3,4,5\}$ and $B = \{6,7,8,9,10,11,12\}$. How many functions $f : A \to B$ are such that $f^{-1}(\{6,7,8\}) = \{1,2\}$?

[NAT]

3. How many functions are 1:1 and onto both?

(a) $f(x) = x + 7$

(b) $f(x) = 2x - 3$

(c) $f(x) = -x + 5$

(d) $f(x) = x^2$

(e) $f(x) = x^2 + x$

(f) $f(x) = x^3$

[MCQ]

4. A chemist who has five assistants is engaged in a research project that calls for nine compounds that must be synthesized. In how many ways can the chemist assign these syntheses to the five assistants so that each is working on at least one synthesis?

(a) $5!S(9,5)$ (b) (4!) $S(7,4)$
(c) (3!) $\dot{S}(6,3)$ (d) None of the above

[MCQ]

5. Let A = {1, 2, 3, 4} and B = {*x*, *y*, *z*}, now let x is the number of functions from A to B and y be the number of one-to-one functions from A to B. Then the value of *x*/*y* is?

(a) 81 (b) 0
(c) undefined (d) 81/4

[NAT]

6. If there exists 2187 functions f: A→B and the cardinality of set B is 3. What is the cardinality of set A?

[MCQ]

7. If A = {1, 2, 3, 4, 5} and there are 6720 injective functions f: A→B, what is |B| ?

(a) 16 (b) 17
(c) 18 (d) None of these

[MCQ]

8. Let f: A→B, with $A_1, A_2 \subseteq A$. Then choose the correct option from the following regarding the given statements.

S_1: $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

S_2: $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$

S_3: $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ when f is one to one

(a) Only S_2 and S_3 are true.
(b) Only S_1 and S_3 are true.
(c) Only S_1 and S_2 are true.
(d) All S_1, S_2 and S_3 are true.

Answer Key

1. (b, c)
2. (576)
3. (2)
4. (a)
5. (c)
6. (7)
7. (d)
8. (d)

Hints and solutions

1. **(b, c)**

$9x^2 - 9x + 3 = g(f(x))$

$= g(f(x))$

$= 1 - (ax+b) + (ax+b)^2$

$= a^2x^2 + (2ab - a)x + (b^2 - b + 1)$

By comparing coefficients on like powers of $x, a = 3, b = -1$ or $a = -3, b = 2$.

2. **(576)**

Since $f^{-1}(\{6,7,8\}) = \{1,2\}$ there are three choices for each of $f(1)$ and $f(2)$ - namely, 6 , 7 or 8. Furthermore $3,4,5 \notin f^{-1}(\{6,7,8\})$ so $3,4,5 \in f^{-1}(\{9,10,11,12\})$ and we have four choices for each of $f(3), f(4)$, and $f(5)$.

Therefore, it follows by the rule of product that there are $3^2 \cdot 4^3 = 576$ functions $f : A \to B$ where $f^{-1}(\{6,7,8\}) = \{1,2\}$.

3. **(2)**

(a) One-to-one and onto.

(b) One-to-one but not onto. The range consists of all the odd integers.

(c) One-to-one and onto.

(d) Since $f(-1) = f(1), f$ is not one-to-one.

Also f is not onto. The range of $f = \{0,1,4,9,16,\ldots\}$.

(e) Since $f(0) = f(-1), f$ is not one-to-one.

Also f is not onto. The range of $f = \{0,2,6,12,20,\ldots\}$.

(f) One-to-one but not onto. The range of $f = \{\ldots,-64,-27,-8,-1,0,1,8,27,\ldots\}$

4. **(a)**

Let A be the set of compounds and B the set of assistants. Then the number of assignments with no idle assistants is the number of onto functions from set A to set B. There are $5!S(9,5)$ such functions.

5. **(c)**

If a set A has m elements and set B has n elements then the number of functions possible from A to B is n^m

Here, |A|= 4, |B| = 3.

Number of possible functions $\Rightarrow 3^4 \Rightarrow 9 \times 9 = 81 = x$.

The number of one-to-one functions from A to B with m and n elements respectively is ${}^nC_m$ condition being $n \geq m$.

In the given set A and B have 4 and 3 elements respectively. We can observe that m > n, therefore total number of one-one function from A to B is 0.

$y = 0$

$x = 81$

$\therefore x/y = 81/0 \Rightarrow$ undefined.

6. **(7)**

We know that number of functions from A→B with m and n elements respectively are n^m.

Hence, $n^m = 2187$

$3^m = 2187$

$3^7 = 2187$

So, the cardinality of A is 7.

7. **(d)**

Number of injective functions (one to one functions)⇒ ${}^nC_m$ where n is the number of elements in set B and m is the number of elements in A for a functions f: A→B

Here, |A| = 5 (m), so |B| = n

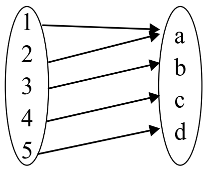
$n \geq 5$

There exists no value of n that generates number of one-to-one functions as 6720.

8. (d)

All the given statements are true.

Consider two set A and B as follows:



$A_1 = \{1, 2\}$ $\quad$ $A_2 = \{2, 3, 4\}$

S_1: $f(A_1 \cup A_2) = f(a \cup a, b, c) = (a, b, c)$

$f(A_1) \cup f(A_2) \Rightarrow \{a\} \cup \{a, b, c\} = \{a, b, c\}$

So, S_1 is true.

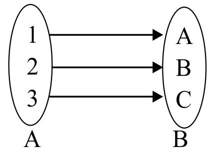
S_2: $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$

$(\{a\} \cap \{a, b, c\}) \subseteq \{a\} \cap \{a, b, c\}$

$a \subseteq a$, true.

So, S_2 is also true.

S_3: $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ when f is one-to-one consider following sets A and B.



$A_1 = \{1, 2\}$ $\quad$ $A_2 = \{2, 3\}$

$f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$

$B = \{B\} \cap \{B, C\}$

$B = B$, true

So, S_3 is also true.