CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 08

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TOPICS

01 Homogenous equation

02 Non Homogenous equation

3 Exercise

Recurrence relation:

In colony, there is 5 bacterias which is present at time 0 bacterias are increasing 2 times as the previous what will be total bacterias at time 100?

a_0: no. of bacterias at time 0 → 5

a_1:

a_{100}: no. of bacterias at time 100 → ?

Initial Condition

$a_0 \quad a_1 \quad a_2 \quad a_3 \ldots a_{99} \quad a_{100}$

$a_0 \downarrow 5$

General Solution:

$a_1 = 2 a_0$

$a_2 = 2 a_1$

$a_2 = 2 (2 \cdot a_0)$

$a_2 = 2^2 \cdot a_0$

$a_3 = 2 a_2$

$= 2 (2^2 \cdot a_0)$

$a_3 = 2^3 \cdot a_0$

$a_4 = 2^4 \cdot a_0$

$\vdots$

$a_{100} = 2^{100} \cdot a_0$

$a_n = 2^n a_0 \rightarrow O(2^n)$

Type-I :

$$a_n = \alpha \cdot a_{n-1}$$

a_0 = initial condition.

$$\boxed{a_n = \alpha^n \cdot a_0} \rightarrow \underline{\underline{\text{Soltn}}}$$

1 pair → $\boxed{mf}$

new pair → mature → after 2 months.

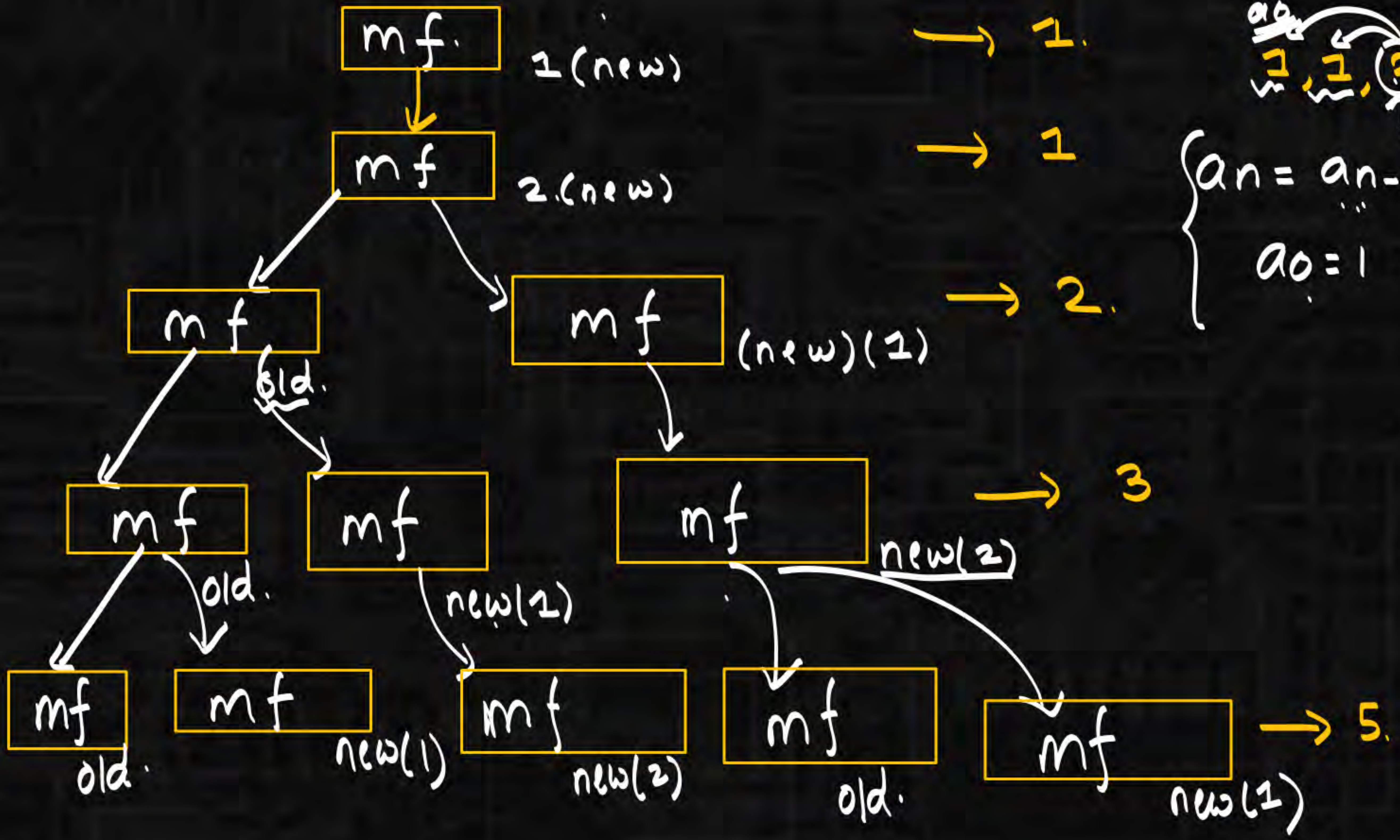
↓

1 pair ($\boxed{mf}$)

Old pair → Every month.

↳ new pair.

$\boxed{mf}$ Rabits.



a_0

1, 1, 2, 3, 5, 8

$$\begin{cases} a_n = a_{n-1} + a_{n-2} & (n \geq 2) \\ a_0 = 1 \quad a_1 = 1 \end{cases}$$

Example :

$$a_n = 6a_{n-1} + a_{n-2} \quad \boxed{(n \geq 2)}$$

$$\begin{cases} a_0 = 2 \\ a_1 = 3 \end{cases}$$

$$a_{n+1} = 6a_n + a_{n-1} \quad (n \geq 1)$$

$a_0 = 2$

$a_1 = 3$

$$a_{n+2} = 6a_{n+1} + a_n \quad (n \geq 0)$$

$a_0 = 2$

$a_1 = 3$

$$(a_{n+2}) = (3a_{n+1} - 2a_n) \quad (n \geq 0) \quad \begin{array}{l} a_0 = 0 \\ a_1 = 1. \end{array}$$

$$\sum a_{n+2} \cdot x^{n+2} = 3 \cdot \sum a_{n+1} \cdot x^{n+2} - 2 \sum a_n \cdot x^{n+2}.$$

$$\sum_{n \geq 0} a_{n+2} \cdot x^{n+2} = 3x \sum_{n \geq 0} a_{n+1} \cdot x^{n+1} - 2x^2 \boxed{\sum_{n \geq 0} a_n \cdot x^n}$$

$$G(x) - a_0 x^0 - a_1 x^1 = 3x\left(\underline{G(x) - a_0 x^0}\right) - \underline{2x}^2 G(x)$$

$$G(x) - 0 \cdot x^0 - x = \underline{3x}\left(\underline{G(x)}\right) - 2x^2 \underline{G(x)}$$

$$G(x) - x = (3x - 2x^2)\underline{G(x)}$$

$$G(x) - (3x - 2x^2) G(x) = x$$

$$G(x)\left[1 - 3x + 2x^2\right] = x.$$

$$G(x) = \frac{x}{1 - 3x + 2x^2}.$$

$$G(x) = \sum_{i \geq 0} a_i x^i$$

$$G(x) = \sum_{n \geq 0} a_n \cdot x^n.$$

$$G(x) = a_0 x^0 + a_1 x^1 + \boxed{a_2 x^2 + a_3 x^3 + a_4 x^4 \ldots\ldots}$$

$$G(x) = a_0 x^0 + a_1 x^1 + \sum_{n \geq 0} a_{n+2} \cdot x^{n+2}$$

$$G(x) - a_0 x^0 - a_1 x^1 = .$$

$$G(x) = \frac{x}{1-3x+2x^2} = \frac{x}{(1-x)(1-2x)} = \left\{\frac{A}{1-x} + \frac{B}{1-2x}\right\} = \frac{\ldots}{(1-x)(1-2x)}$$

$$= A(1-2x) + B(1-x)$$

$$= A - 2Ax + B - Bx$$

$1-3x+2x^2$

$1-x-2x+2x^2$

$(1-x)-2x[1-x]$

$[1-x][1-2x]$

$$\frac{x}{(1-x)(1-2x)} = \frac{(-2Ax-Bx)+(A+B)}{(1-x)(1-2x)}$$

$-2A-B=1 \qquad A+B=0$

$A=-1 \qquad B=1$

$\downarrow \qquad \downarrow$

$c_1 \qquad c_2$

$$G(x) = \frac{A}{1-x} + \frac{B}{1-2x} \qquad A = -1 \quad B = 1.$$

$$= \frac{-1}{1-x} + \frac{1}{1-2x}$$

$$\boxed{G(x) = \left(\frac{1}{1-2x}\right) - \left(\frac{1}{1-x}\right)}$$

$$\sum_{n\geq 0} a_n x^n = \sum_{n\geq 0} 2^n x^n - \sum_{n\geq 0} 1^n x^n$$

$$\boxed{a_n = 2^n - 1.}$$

$$\frac{1}{1-2x} = 1 + (2x) + (2x)^2 + (2x)^3 + (2x)^4 \cdots$$

$$= \sum_{n\geq 0} 2^n x^n$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 \cdots$$

$$= \sum_{n\geq 0} 1^n x^n$$

$a_n = 2^n - 1.$ $\qquad G(x)$

$\sum a_n \cdot x^n = \sum 2^n \cdot x^n - \sum 1 \cdot x^n$

$$\boxed{G(x) = \frac{1}{1-2x} - \frac{1}{1-x}}$$

$a_n = 2n + 3.$

$\sum a_n x^n = 2 \sum n \cdot x^n + 3 \sum x^n \left(\times \sum x^n \right)$

$$G(x) = 2 \frac{x}{(1-x)^2} + \frac{3}{(1-x)}$$

$$\sum_{n \geq 0} n \cdot x^n$$

$$\frac{x}{(1-x)^2} \qquad 0x^0 + 1 \cdot x + 2x^2 + 3x^3$$

$$x + 2x^2 + 3x^3 + 4x^4$$

$$x\left(1 + 2x + 3x^2 + 4x^3\right)$$

$a_n = 2n + 3$

$\left(\sum x^n\right.$

$$\sum a_n x^n = 2\sum n\cdot x^n + 3\sum x^n$$

$\sum n\cdot x^n$

$1\cdot x + 2x^2 + 3x^3 + 4x^4$

$x + 2x^2 + 3x^3 + 4x^4$

$x\left(1 + 2x + 3x^2 + 4x^3 + \dots\right)$

$$= \frac{2x}{(1-x)^2} + \frac{3}{(1-x)}\left(\frac{1-x}{1-x}\right)$$

$$= \frac{2x + 3 - 3x}{(1-x)^2} = \frac{3-x}{(1-x)^2}$$

$$\frac{x}{(1-x)^2}$$

$$a_{n+2} = 3a_{n+1} - 2a_n$$

$a_0 = 0 \quad a_1 = 1.$

Roots

CE

$n = 0 \quad n = 1.$

$1 \cdot x^2 = 3x - 2.$

$x^2 - 3x + 2 = 0$

Roots : 2, 1.

Characterist eqtn : $a_n = 2^n c_1 + 1^n c_2.$

$n = 0$

$a_0 = 2^0 c_1 + 1^0 c_2 \qquad c_1 + c_2 = 0$

$2c_1 + c_2 = 1.$

$-c_1 + 0 = -1$

$n = 1.$

$a_1 = 2c_1 + c_2$

$$\boxed{\begin{aligned} c_1 &= +1. \\ c_2 &= -1. \end{aligned}}$$

$a_n = 2^n \cdot 1 - 1^n.$

$a_n = 2^n - 1$

$$a_n = 5a_{n-1} - 6\underline{a_{n-2}}.$$

$$a_0 = 2 \qquad a_1 = 3.$$

$$x^2 = 5x - 6.$$

$n = 0$ $\qquad$ $n = 1.$

$$x^2 - 5x + 6.$$

Roots: $3, 2.$

$c_1 =$ $\qquad$ $c_2 = .$

CE: $a_n = 3^n c_1 + 2^n c_2.$

Type-2:

$$a_n = X a_{n-1} + Y a_{n-2}$$

Roots R_1, R_2.

CE: $a_n = (R_1)^n c_1 + (R_2)^n c_2$.

$n = 0$ $\qquad$ $n = 1$

Type-3:

$$a_n = X a_{n-1} + Y a_{n-2}.$$

Roots: R, R.

CE: $a_n = R^n c_1 + \underline{n \cdot R^n c_2}$.

$a_n = a_{n-1} + a_{n-2}$ $\boxed{a_0 = 0 \quad a_1 = 1.}$

$x^2 = x + 1$

$x^2 - x - 1 = 0$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{1 \pm \sqrt{5}}{2}$

CE: $a_n = \left(\dfrac{1+\sqrt{5}}{2}\right)^n c_1 + \left(\dfrac{1-\sqrt{5}}{2}\right)^n c_2.$

$c_1 = \dfrac{1}{\sqrt{5}} \quad c_2 = -\dfrac{1}{\sqrt{5}}.$

$a_n = \dfrac{1}{\sqrt{5}}\left(\dfrac{1+\sqrt{5}}{2}\right)^n - \dfrac{1}{\sqrt{5}}\left(\dfrac{1-\sqrt{5}}{2}\right)^n.$

$a_n = 6a_{n-1} - 9a_{n-2}$. $a_0 = 1$ $a_1 = 2$.

Roots: R. R.

$a_n = R^n c_1 + n \cdot R^n c_2$

CE: $a_n = 3^n c_1 + n \cdot 3^n c_2$. $\longrightarrow$ $\boxed{a_n = 3^n + n \cdot 3^n \left(-\frac{1}{3}\right)}$

$n = 0$

$a_0 = 3^0 c_1 + \underline{0 \cdot 3^0 \cdot c_2}$.

$1 = 1 \cdot c_1$

$c_1 = 1$.

$n = 1$.

$a_1 = 3^1 c_1 + 1 \cdot 3^1 c_2$.

$2 = 3 \cdot 1 + 3c_2$.

$-1 = 3c_2$. $c_2 = -1/3$.

Type-4 :

$a_n = X a_{n-1} + Y a_{n-2} + Z a_{n-3}$. $a_0 =$ $a_1 =$ $a_2 =$.

Roots : R_1, R_2, R_3. CE: $a_n = R_1^n c_1 + R_2^n c_2 + R_3^n c_3$.

Roots : $\boxed{R, R,}$ R_1. CE: $a_n = R^n c_1 + n.R^n c_2 + R_1^n c_3$.

Roots : $\boxed{R, R, R}$ CE $a_n = R^n c_1 + n.R^n c_2 + n^2 R^n c_3$.


THANK YOU GW
SOLDIERS !