# ALL BRANCHES

# ENGINEERING MATHEMATICS

Lecture No.-07

Probability

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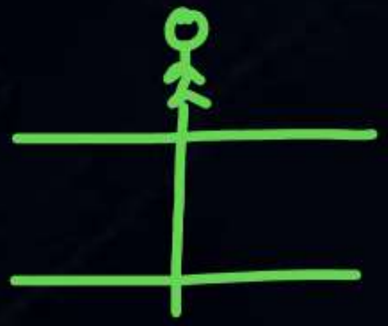
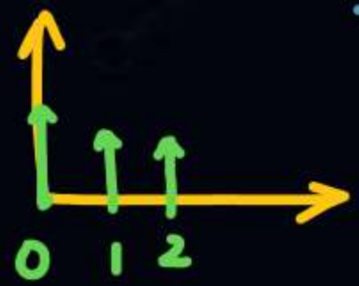
# TOPICS TO BE COVERED

- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM
- STATISTICS – I (PROBABILITY DISTRIBUTIONS)
- STATISTICS – II (CORRELATION AND REGRESSION)






$720 \frac{\text{veh}}{\text{hr}}$

$X \rightarrow$ No. of cars passing through this road section

| $p$(2 cars pass in 1hr) | $p$(2 cars pass in 1min) |
|---|---|
| $P(X = 2\,\text{car}/\text{hr})$ | $P(X = 2/\text{min})$ |
| $\lambda = 1\,\cancel{\text{hr}} \times 720\frac{\text{veh}}{\cancel{\text{hr}}}$ | $\lambda = 1\,\cancel{\text{min}} \times \frac{720\,\text{veh}}{60\,\cancel{\text{min}}}$ |
| $\lambda = 720\,\text{veh}/\text{hr}$ | $\lambda = 12\,\text{veh}/\text{min}$ |
| $P(x=2) = \frac{e^{-720}(720)^2}{2!}$ | $P(x=2) = \frac{e^{-12}(12)^2}{2!}$ |

$$P(x=2) = \frac{e^{-\lambda}\lambda^2}{2!}$$

Sample Space $\longrightarrow$ Real number

# STATISTICS – I (PROBABILITY DISTRIBUTIONS)

| Discrete Random Variable | Continuous Random Variable |
|---|---|
| → Finite/Infinite, discontinuous values. | → Infinite, continuous values |
| • $\sum_{i=1}^{n} p(x_i) = 1$ | • $\int_{-\infty}^{+\infty} f(x)\,dx = 1$ |
| C.D.F. • $F(a) = \sum p(x_i)$ for all $x_i \le a$ → p.m.f. (Prob. at single point) (impulse) | • $\int_{-\infty}^{a} f(x)\,dx$ for all $x_i \le a$ → p.d.f. (continuous fn.) |
| • $E(x) = \sum_{i=1}^{n} x_i\, p(x_i) = \mu_x$ (Mean) | • $E(x) = \int_{-\infty}^{+\infty} x\, f(x)\,dx = \mu_x$ |
| • $E(x^2) = \sum_{i=1}^{n} x_i^2\, p(x_i)$ (Mean of square value) | • $E(x^2) = \int_{-\infty}^{+\infty} x^2 f(x)\,dx$ |

# STATISTICS –I (PROBABILITY DISTRIBUTIONS)

$$\text{Variance}(x) = (S.D.)^2 = \sigma^2$$

## Discrete Random Variable

$$E[g(x)] = \sum_{i=1}^{n} g(x_i)\, p(x_i)$$

$$Var(x) = \sigma_x^2 = E(X^2) - [E(X)]^2$$

$$= \sum x^2 p(x) - \left[\sum x\, p(x)\right]^2$$

## Continuous Random Variable

$$E[g(x)] = \int_{-\infty}^{+\infty} g(x)\, f(x)\, dx$$

$$Var(x) = \sigma_x^2 = E(X^2) - [E(X)]^2$$

$$= \int_{-\infty}^{+\infty} x^2 f(x) - \left[\int_{-\infty}^{+\infty} x\, f(x)\right]^2$$

$$\text{Variance}(x) = E(X^2) - \mu_x^2 = \sigma_x^2$$

**NOTE :-** $Var(x)$ is always positive. $\{Var(x) \geq 0\}$

$\rightarrow$ If $Var(x) = 0 = E(x^2) - [E(x)]^2$

$$\Rightarrow E(x^2) = [E(x)]^2 = \mu_x^2$$

$X \rightarrow$ Outcome of dice

i) Mean $= E(x) = \mu_x$

ii) Mean of square value $= E(x^2)$

iii) $Var(x)$

iv) $\sigma_x$ (S.D.)

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $p(x)$ | $1/6$ | $1/6$ | $1/6$ | $1/6$ | $1/6$ | $1/6$ |

$\rightarrow$ i) $E(x) = \sum_{i=1}^{6} x_i p(x_i) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = 7/2$

$\rightarrow$ ii) $E(x^2) = \sum_{i=1}^{6} x_i^2 p(x_i) = 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + \dots 6^2 \times \frac{1}{6} = \frac{1}{6}[1^2 + 2^2 + \dots 6^2]$

$= 91/6$

$\rightarrow$ iii) $Var(x) = E(x^2) - [E(x)]^2 = 91/6 - (7/2)^2 = \frac{35}{12}$

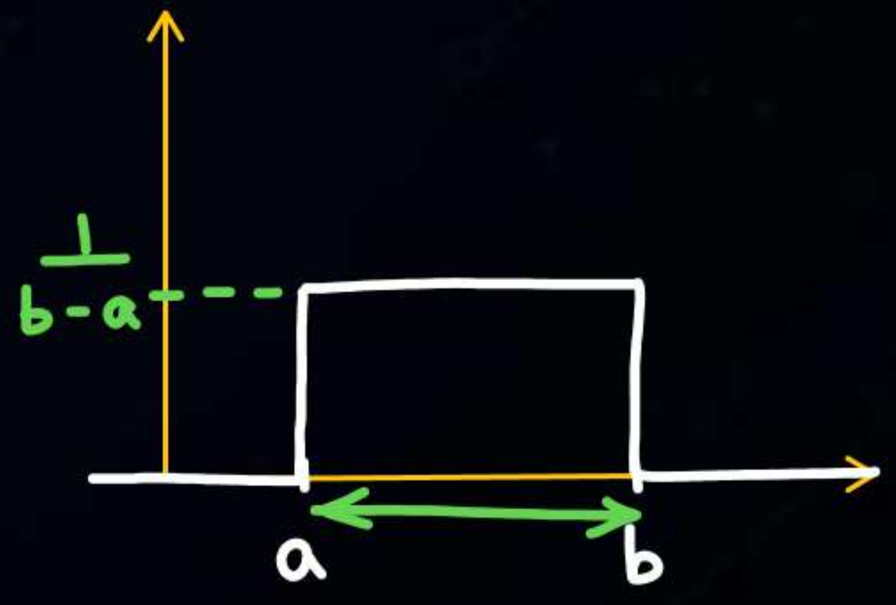
$\rightarrow$ iv) S.D.$(\sigma_x) = \sqrt{Var(x)} = \sqrt{\frac{35}{12}}$

## Types of Continuous Random Variable

### 1) Uniform Random Variable

A Random variable is uniformly distributed b/w a and b, then its p. d. function is

$$f(x) = \begin{cases} 0 & ; x < a \\ \dfrac{1}{b-a} & ; a < x < b \\ 0 & ; x > b \end{cases}$$




- Mean $= E(x) = \mu_x = \dfrac{a+b}{2}$
- $E(x^2) = \dfrac{a^2+b^2+ab}{3}$
- Variance $= \dfrac{(b-a)^2}{12}$

- $E(X) = \int_{-\infty}^{+\infty} x\, f(x) = \int_a^b x\, f(x)\, dx = \int_a^b x \cdot \frac{1}{b-a} \cdot dx$

$$= \frac{1}{b-a} \cdot \left[\frac{x^2}{2}\right]_a^b = \frac{1}{b-a} \cdot \frac{b^2-a^2}{2} = \frac{\cancel{(b-a)}(b+a)}{2\,\cancel{(b-a)}} = \boxed{\frac{a+b}{2}}$$

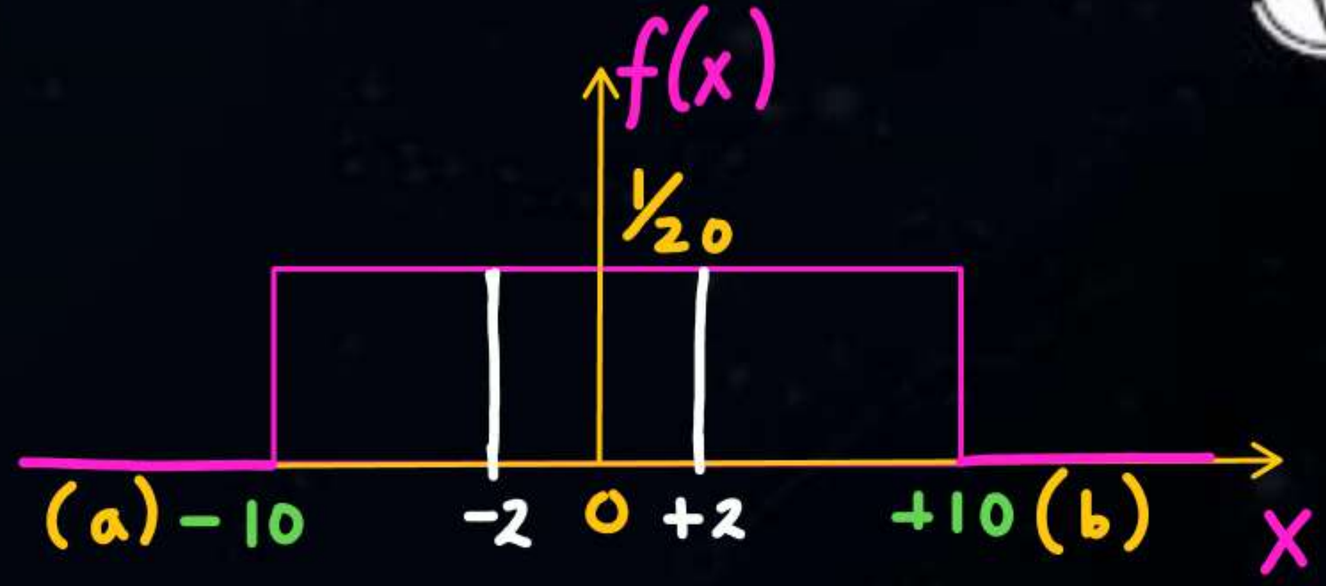
- $E(X^2) = \int_a^b x^2 f(x)\, dx = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b = \frac{1}{b-a} \cdot \frac{b^3-a^3}{3}$

$$= \frac{(b^2+a^2+ab)\cancel{(b-a)}}{3\,\cancel{(b-a)}} = \boxed{\frac{a^2+b^2+ab}{3}}$$

- $Var(X) = E(X^2) - [E(X)]^2 = \frac{a^2+b^2+ab}{3} - \left(\frac{a+b}{2}\right)^2$

$$\frac{4a^2+4b^2+4ab-3a^2-3b^2-6ab}{12} = \frac{a^2+b^2-2ab}{12} = \boxed{\frac{(b-a)^2}{12}}$$

# STATISTICS –I (PROBABILITY DISTRIBUTIONS)




Ex:- Probability density is given as

$$f(x) = \begin{cases} k : -10 < x < 10 \\ 0 : \text{otherwise} \end{cases}$$

Find

i) k

ii) $P[-10 < x < 0]$

iii) $P[x > 0]$

iv) $P[x^2 < 4]$

v) $P[1 < x^2 < 9]$

i) $f(x) = \frac{1}{b-a} = \frac{1}{10-(-10)} = \frac{1}{20}$

ii) $P(-10 < x < 0) = \int_{-10}^{0} f(x)\,dx = \int_{-10}^{0} \frac{1}{20}\,dx = \frac{10}{20} = 0.5$

iii) $P(x > 0) = \int_{0}^{10} f(x)\,dx = \int_{0}^{10} \frac{1}{20}\,dx = \frac{10}{20} = 0.5$

iv) $P(-2 < x < 2) = \int_{-2}^{2} f(x)\,dx = \int_{-2}^{2} \frac{1}{20}\,dx = \frac{4}{20} = 0.2$

v) $P[(-3 < x < -1) \cup (1 < x < 3)]$

$= \int_{-3}^{-1} f(x)\,dx + \int_{1}^{3} f(x)\,dx = \frac{2}{20} + \frac{2}{20} = \frac{4}{20} = 0.2$

vi) $E(x) = \frac{a+b}{2} = -\frac{10+10}{2} = 0$

vii) $E(x^2) = \frac{a^2+b^2+ab}{3} = \frac{10^2+(-10)^2+(10)(-10)}{3} = \frac{100}{3}$

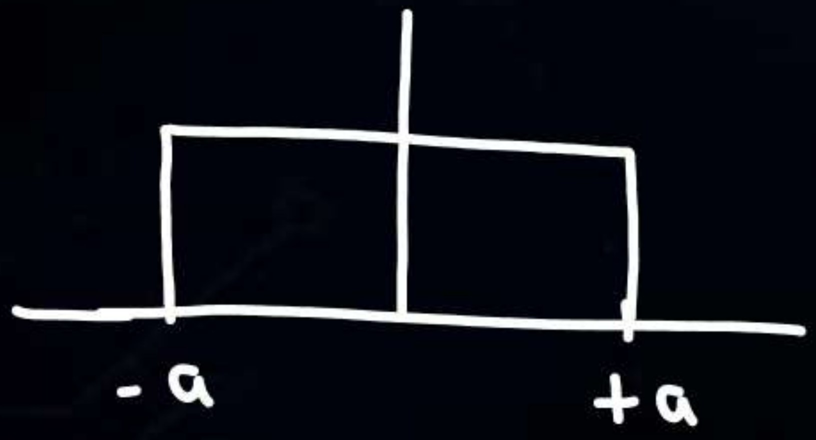
ix) $Var(x) = E(x^2) - [E(x)]^2 = \frac{100}{3} - 0^2 = \frac{100}{3}$

x) $\sigma_x = \sqrt{Var(x)} = \sqrt{\frac{100}{3}}$

$$\frac{a^2+(-a)^2+a(-a)}{3}$$

Note:- 1) $Var(x) \geq 0 \Rightarrow E(x^2) \geq [E(x)]^2$

2)




Symmetric C. Uniform R.V.

$\rightarrow E(x) = 0 \left\{ \because E(x) = x \frac{1}{b-a} \text{ is odd} \right\}$

$\rightarrow Var(x) = E(x^2) = \frac{a^2}{3}$

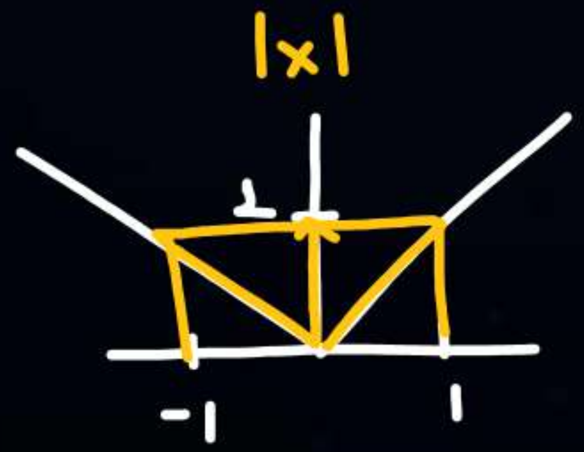
vi) $P[|x| < 1]$ $= P(-1 < X < 1) = \int_{-1}^{1} f(x)\,dx = \frac{2}{20} = 0.1$

vii) Mean = $0$

viii) Variance = $\frac{10^2}{3} = 100/3$

$|x| < a$

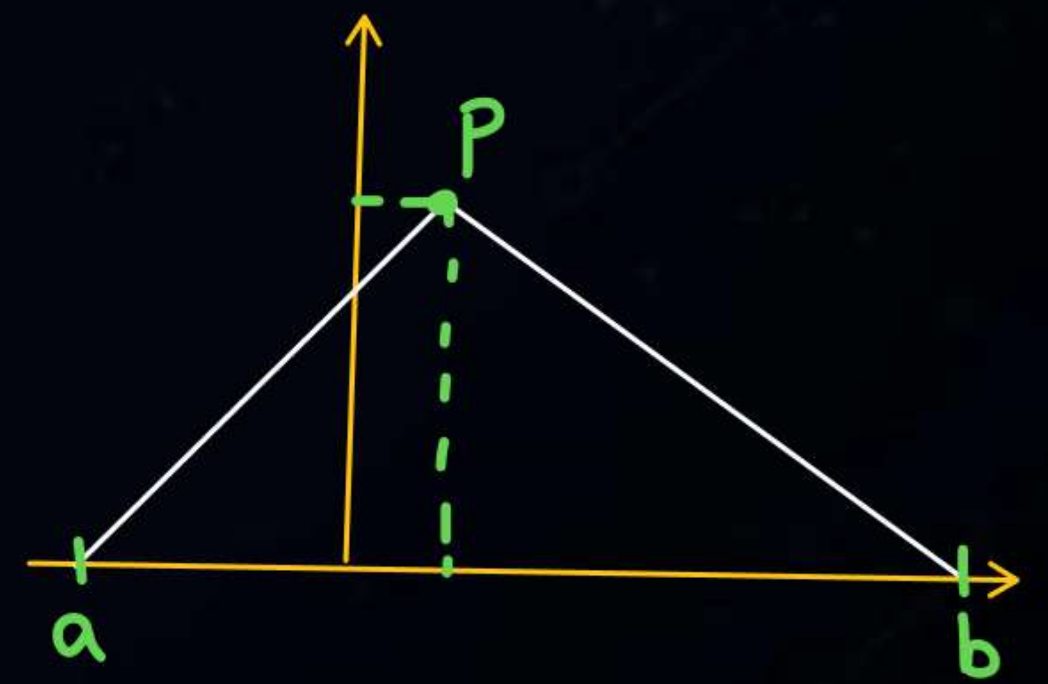
$-a < X < a$



## Types of Continuous Random Variable

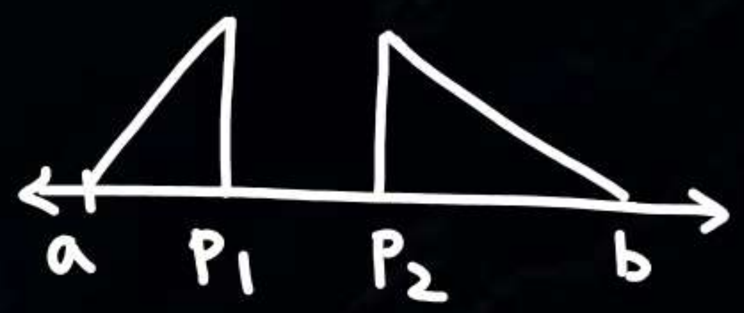
2) Triangular Random Variable.

$$f(x) = \begin{cases} m_1 x + c_1 & ; a < x < p \\ m_2 x + c_2 & ; p < x < b \end{cases}$$

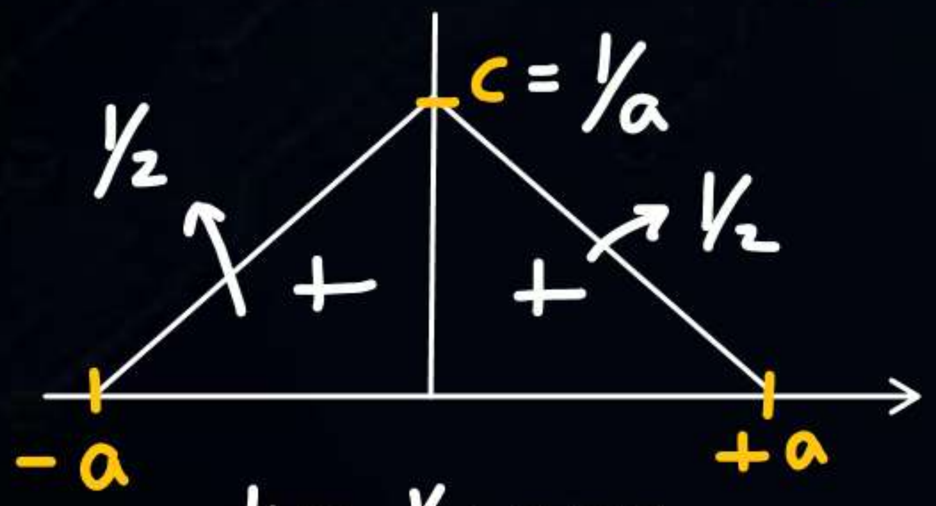



$$E(x) = \text{Mean} = \mu_x = \frac{a + p + b}{3}$$

Inclined straight lines combination



## Symmetrical triangular pdf.

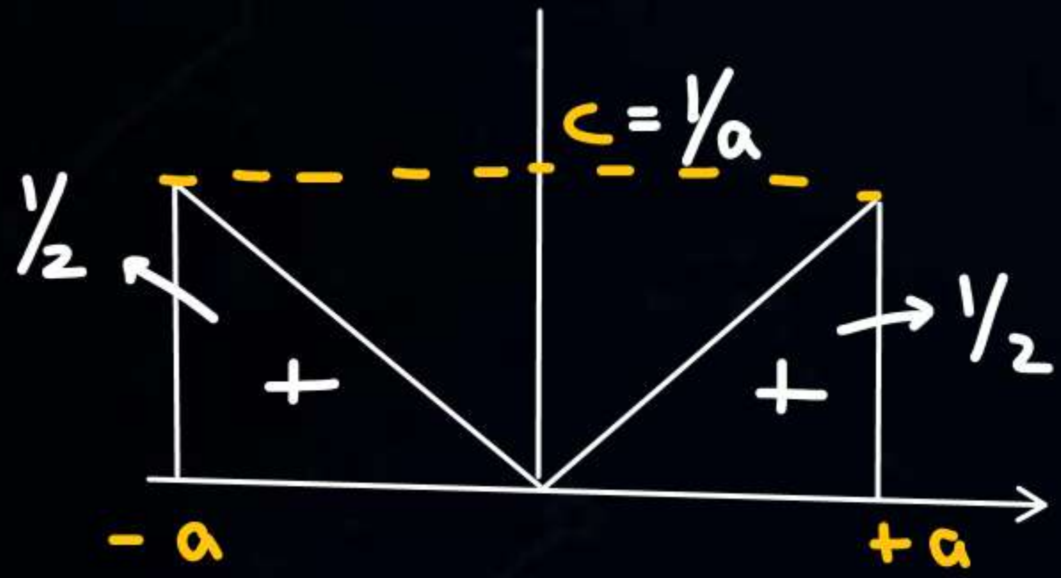



$\cancel{\tfrac{1}{2}} = \cancel{\tfrac{1}{2}} \times a \times c$

$f(x) \rightarrow$ Even

$$E(x) = \int_{-a}^{a} \underbrace{x}_{\text{odd}} \, \underbrace{f(x)}_{\text{Even}} \, dx = \boxed{0}$$

$$Var(x) = E(x^2) = \boxed{a^3/6}$$




$f(x) \rightarrow$ Even

$$E(x) = \int_{-a}^{a} x \, f(x) \, dx = \boxed{0}$$

$$Var(x) = E(x^2) = \boxed{a^3/2}$$

# Thank you

# GW Soldiers !