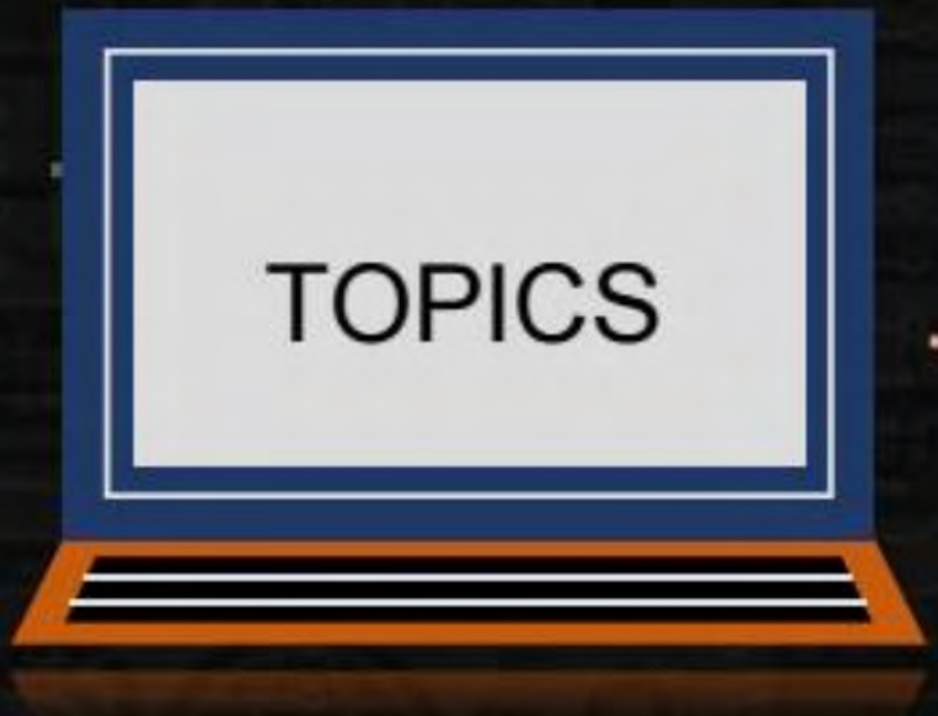
CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 12

By- SATISH YADAV SIR



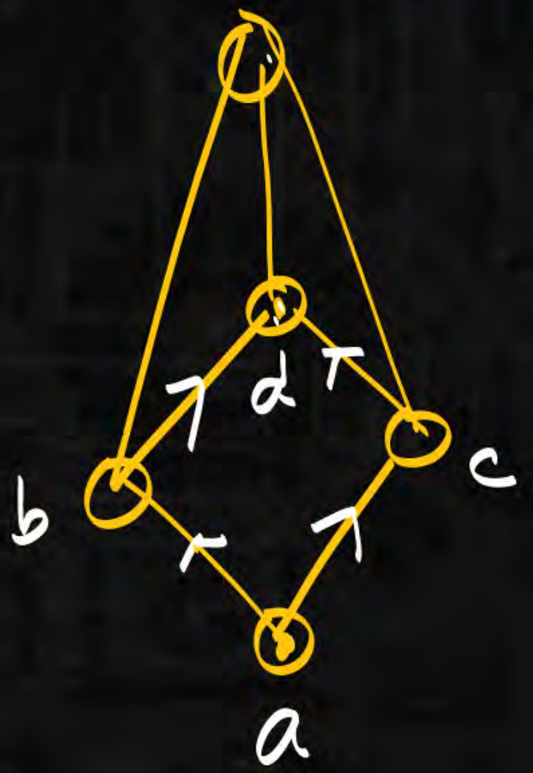
01 GLB / LUB

02 LATTICE

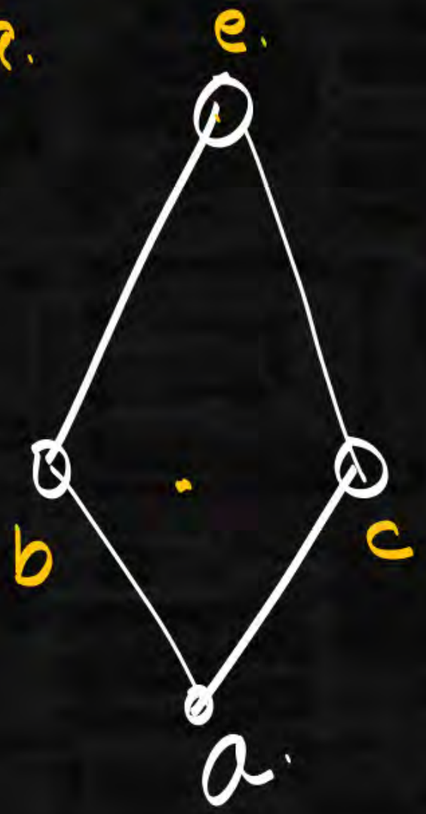
3 TYPES OF LATTICE

lub(b,c) = d.

glb(b,c) = a.



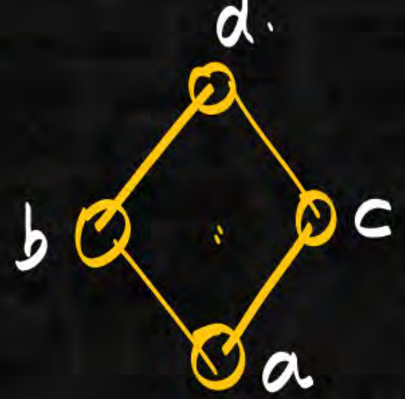
1.



2.

lub(b,c) = e.

glb(b,c) = a.



3.

B is sublattice of A

1) $B \subseteq A$.

2) glb & lub of each pair of B must be same as in A.

Distributive lattice:

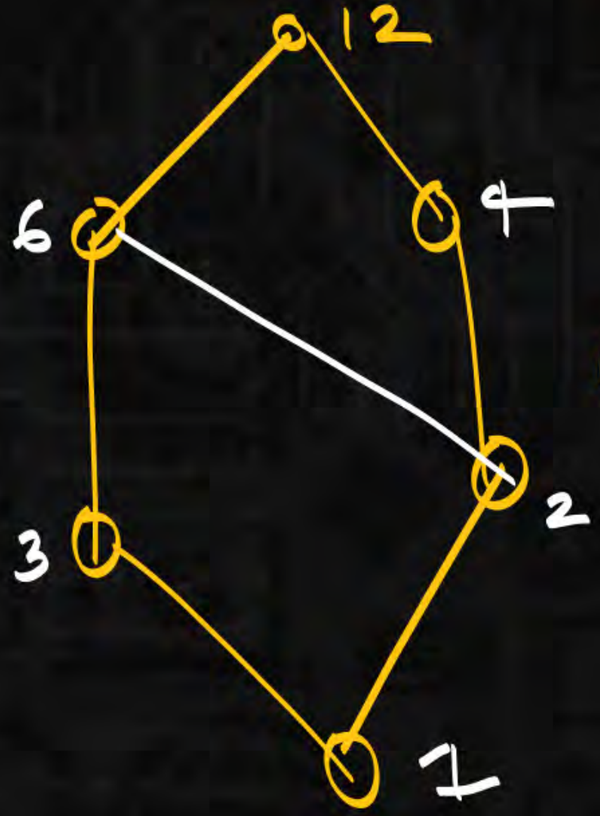
$\wedge \rightarrow$ glb $\vee \rightarrow$ lub.

$\forall a, b, c$

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \checkmark$$

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \checkmark$$

$(D_{12}, |)$ Distributive lattice / not complement lattice



$6 \vee (4 \wedge 2) = (6 \vee 4) \wedge (6 \vee 2)$

$6' \neq$ n.A

$4 \wedge 2$ → $\gcd(4,2)$ → 2

$6 \vee 2$ → $\text{lub}(6,2)$ → $\text{lcm}(6,2)$ → 6

$6 \vee 4$ → $\text{lcm}(6,4)$ → 12

$6 \vee 2$ → $\text{lcm}(6,2)$ → 6

$12 \wedge 6$ → $\gcd(6,12)$ → 6

$6 \wedge (4 \vee 2) = (6 \wedge 4) \vee (6 \wedge 2)$

2, 3, 12

$(D_n, |)$ → lattice.

↳ Distributive lattice.

$(D_{60}, |)$

glb → gcd.
lub → lcm.

$(P(A), \subseteq)$ → Distributive lattice.

lub → ∪ → ∨
glb → ∩ → ∧.

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$

$A \cup \phi = A \cap A$

$A = A$

$(D_{60}, 1)$

$(\{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}, 1)$ → lcm/gcd.

$10 \vee (12 \wedge 15) = (10 \vee 12) \wedge (10 \vee 15)$

$12 \wedge 15$

↓

$\gcd(12, 15)$

↓

$10 \vee -$

$\text{lcm}(10, -)$

$= \cdot$

$(\{1,2,3,4,5\}, \leq)$ $\vee \to lub \to \max\{\ ,\ \}$
$\wedge \to glb \to \min\{\ ,\ \}$

Every TOSET is always Distributive lattice

$lub(1,2) = \max\{1,2\} = 2.$

$glb(1,2) = \min\{1,2\} = 1$

$1 \vee (2 \wedge 3) = (1 \vee 2) \wedge (1 \vee 3)$

$\max(1, \min\{2,3\}) = (\max\{1,2\}, \max\{1,3\})\ \min.$

$\max(1,2) = 2 \quad \min(2,3)$

$= 2$

$\vee \rightarrow lub \rightarrow max.$
$\wedge \rightarrow glb \rightarrow min$

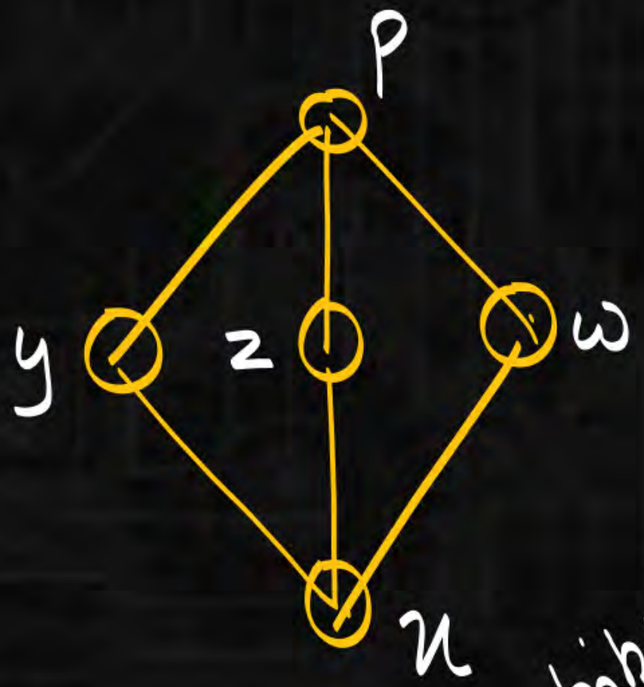
$a \vee (b \wedge c)$ $\qquad$ $(a \vee b) \wedge (a \vee c)$

$max(a, min\{b, c\})$ $\qquad$ $min(max\{a, b\}, max\{a, c\})$

GE/LE

bounded.

complement



not distributive lattice

$y \wedge (z \vee w) = (y \wedge z) \vee (y \wedge w)$

$y \wedge (z \vee w)$

$\downarrow$ lub(z, w)

$y \wedge p$

$\downarrow$

glb(y, p)

$\downarrow$

y

$(y \wedge z) \vee (y \wedge w)$

glb(y, z) $\vee$ glb(y, w)

$\downarrow$ $\downarrow$

x $\vee$ x

$\downarrow$

x

$\neq$

x

complement ✓

Distributive X.

Boolean algebra = lattice ∧ complement ∧ distributive

lattice:

$a + a = a$

$a \cdot a = a$

$a + b = b + a$

$a \cdot b = b \cdot a$

$a + (b + c) = (a + b) + c$

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

$a + (a \cdot b) = a$

$a \cdot (a + b) = a$

complement:

$a + b = 1$

$a \cdot b = 0$

distributive:

$a + (b \cdot c) = (a + b)(a + c)$

$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

$(P(A), \subseteq)$


THANK YOU GW
SOLDIERS !