CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 08

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TOPICS TO BE COVERED

- 01 Basics of relations
- 02 Types of relations
- 03 Number Of relations

Relations

Equivalence Relation :

$A = \{1, 2, 3\}$

(RST) Reflexive
Symmetric
Transitive.

$R_1 = \{ \underbrace{(1\,1)\ (2\,2)\ (3\,3)}_{}\ \boxed{(1\,2)\ (2\,1)} \}$

(RST)

Relations

Set: z.

(EQR)

$R_1 = \{ (a, b) \mid a+b = \text{even} \}$

Re: aRa

$a+a = \text{even}$.

$2a = \text{even}$.

Sy:

$aRb \to bRa$.

$\downarrow$

$a+b = \text{even} \to b+a = \text{even}$.

if $a+b = \text{even}$ then $b+a = \text{even}$.

$(0,4) \in R \to (4,0) \in R$.

$0+4 = \text{even} \to 4+0 = \text{even}$.

Tran ✓

$aRb \wedge bRc \to aRc$.

$\underset{=\text{even}}{a+b} \wedge \underset{\text{even}}{b+c=} \to \underset{=\text{even}}{a+c}$.

$(0,4) \in R \wedge (4,6) \in R$.

$\underset{=\text{even}}{0+4} \wedge \underset{=\text{even}}{4+6} \longrightarrow \underset{=\text{even}}{0+6}$.

$1+3 = \text{even} \wedge$

$3+5 = \text{even} \to 1+5 = \text{even}$

Relations

$Z \times Z$

$(1,1)\ (1,2)\ (1,3)$

$(0,3)\ (0,0)$

$\cancel{\not\exists}\ 1+2 = \text{even}$

$a+b = \text{even}.$

$\rightarrow$ Result: $\{(1,1)\ \underline{(1,3)}\ \underline{(3,5)\ (1,5)}$

T.

Relations

(EQR)

$R_2 = \{(a,b) \mid a \equiv b \pmod 4\}$

a, b are having same remainder w.r.t 4.

R: aRa ✓

$a \equiv a \pmod 4$

Sy: $aRb \to bRa$.

$a \equiv b \pmod 4 \to b \equiv a \pmod 4$

$(0,4) \in R \to (4,0) \in R$.

$aRb \wedge bRc \to aRc$.

$a \equiv b \pmod 4$

$\wedge\, b \equiv c \pmod 4$

$\to a \equiv c \pmod 4$

✓ $(0,4) \in R$.

$0 \equiv 4 \pmod 4$

✓ $(1,5) \in R$.

$1 \equiv 5 \pmod 4$

$(0,3) \notin R$.

$0 \not\equiv 3 \pmod 4$

Relations

$$\overset{0}{(0,4)} \in R \wedge \overset{0}{(4,8)} \in R \to \overset{0}{(0,8)} \in R.$$

$$\underbrace{0 \equiv 4 \pmod 4}_{0} \wedge \underbrace{4 \equiv 8 \pmod 4}_{0} \to \underbrace{0 \equiv 8 \pmod 4}_{0}$$

Relations

→ equivalence Relation creates partition on set.

Set : A

partition

{ . . . . }

A × A

(RST)

Relation

Check (E.Q.R) ✓

Result. { . . . . . }

Relations

$A = \{1, 2, 3, 4\}$

$R = \{ (1,2)\ (3,4) \}$

$(1,3) \notin R$

$(1,2) \in R$

1,2 will go in one partion

how to create a partion.

$\{(a,b) \in R$ OR aRb

a, b will go in one partition.

Set: A

$1,2 \rightarrow P_1$

$3,4 \rightarrow P_2$.

Relations

Set: $\mathbb{Z}$

partition

$\mathbb{Z} \times \mathbb{Z}$

$R = \{(a,b) \mid a+b = \text{even}\}$

(RST)

EQR ✓

$(1,3) \in R \quad 1+3 = \text{even}$

$(0,0) \in R \quad 0+0 = \text{even}$

$(0,1) \notin R \quad 0+1 \neq \text{even}$

$(0,2) \in R \quad 0+2 = \text{even}$

even no

0, 2, 4, 6, 8 ....

odd no

1, 3, 5, 7, 9 ....

Relations

$R_2 = \{(a,b) \mid a \equiv b \pmod 4\}$

(RST) ✓

EQR ✓

$(0,4) \in R \quad 0 \equiv 4 \pmod 4$

$(1,5) \in R \quad 1 \equiv 5 \pmod 4$

0, 4
1, 5
2, 6
3, 7

Relations

end with 1

$R : \{ (s_1, s_2) \mid s_1, s_2 \text{ will go in same state} \}$

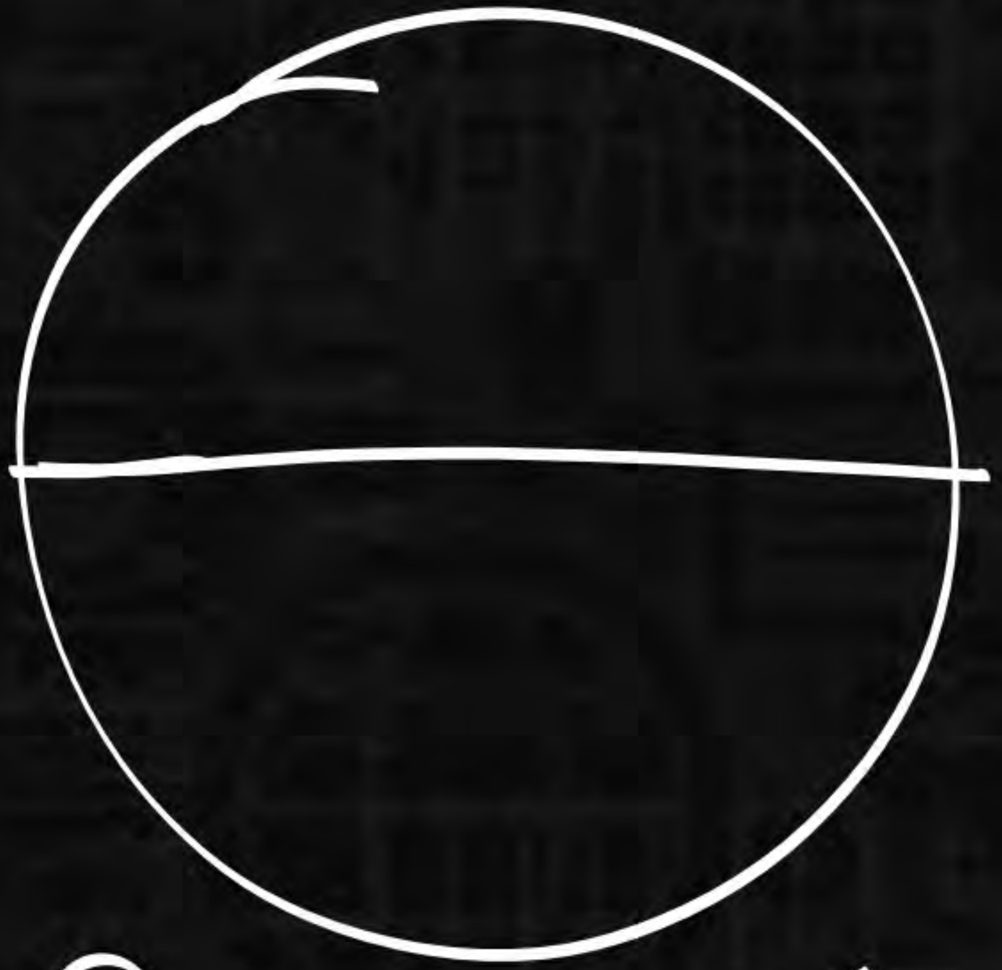
$R : sRs$

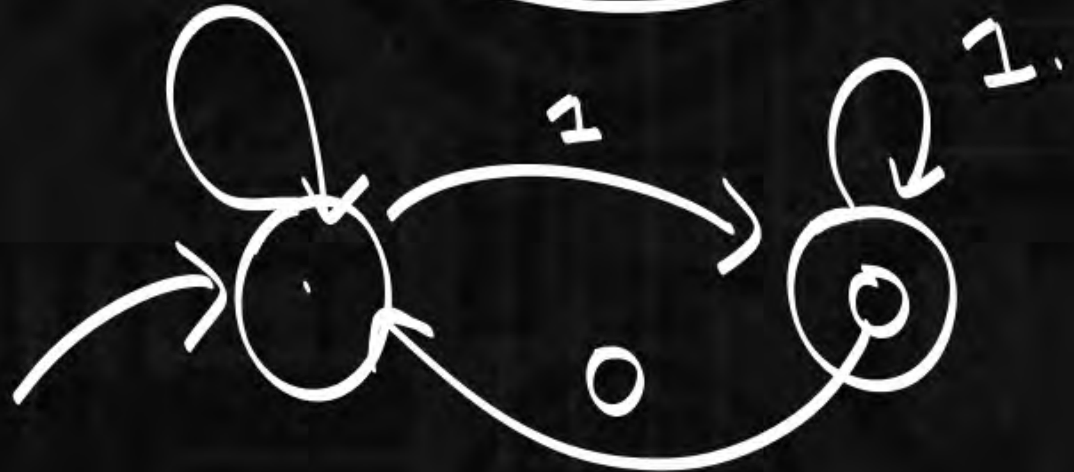
EQR

Sy : $s_1 R s_2 \rightarrow s_2 R s_1$

T : $s_1 R s_2 \wedge s_2 R s_3$

$011 R 01 \wedge 01 R 0111 \rightarrow 011 R 0111$





Relations

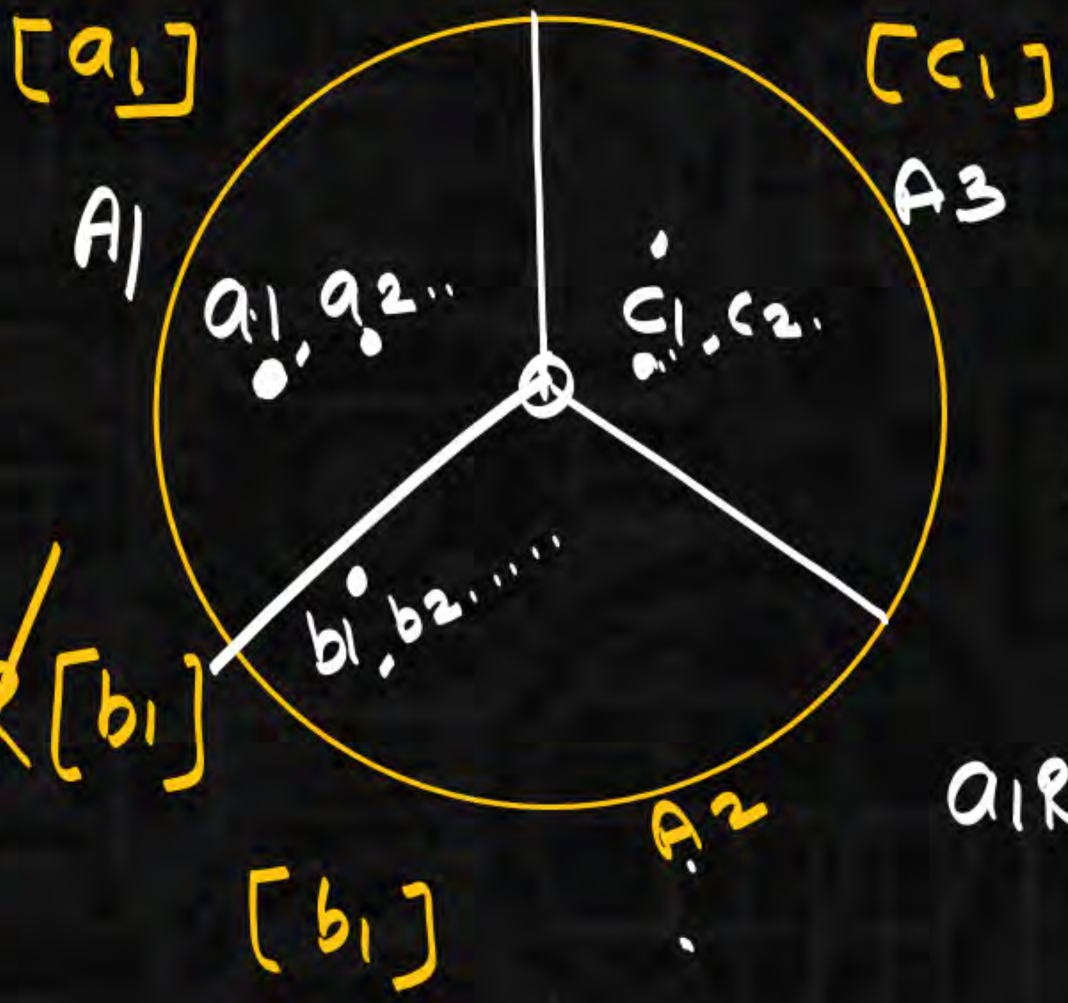
Set : A

EQR creates partition on a set A.

$\underbrace{\text{partition}}$ ↳ equivalence classes.

(A_1, A_2, A_3)

$[a_1] \not R [b_1]$



$a_2 R c_1$

$\underline{a_1 R c_1}$

$a_1 R a_2 \rightarrow a_2 R a_1$

1) $A_1 \cup A_2 \cup A_3 = A$

2) $A_1 \cap A_2 \cap A_3 = \phi$

$[\ ]_R$: Representative of a class.

Relations

Total equivalence
= Total no. of partition.

Set : A = {1, 2, 3}.

R|S/T

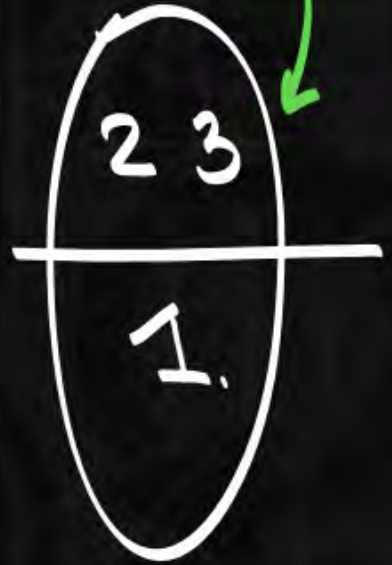
R_1 : { (11) (22) (33) }

R/S/T.

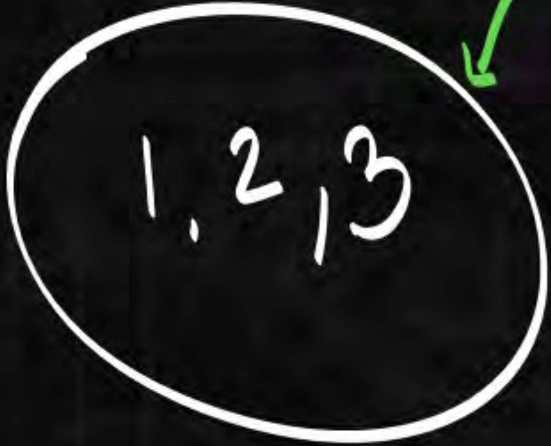
R_2 : { (11) (22) (33) (12) (21) }

R_3 : { (11) (22) (33) (13) (31) }

R4 : { 11 22 33 23 32 }



R 5 : { 11 22 33 12 21 23 32 31 13 }



Relations

G//

Total no. of partitions = 5. = Total no. of equivalence

$A = \{1, 2, 3\}$

$S(3,1)$ $S(3,2)$ $S(3,3)$

$R_5 = \{11\ 22\ 33,\ 12\ 21,\ 13\ 31,\ 23\ 32\}$

$\{11\ 22\ 33,\ 12,\ 21\}$ RST.

$\{11\ 22\ 33,\ 23\ 32\}$

$\{11\ 22\ 33,\ 13,\ 31\}$

$\{11,\ 22,\ 33\}$ RST.

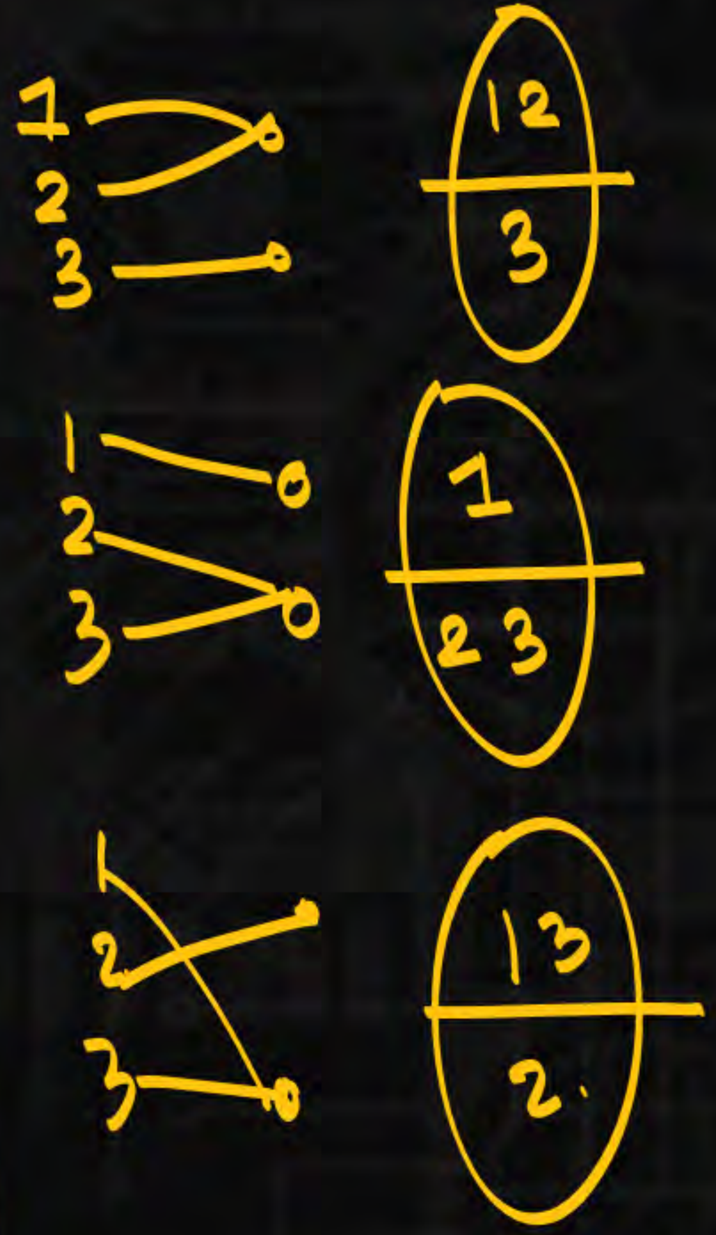
Relations → 6 onto

Onto 3 diff quest → 2 diff room.

1	R_1	1	R_1	1	R_1
2		2		2	
3	R_2	3	R_2	3	R_2

1	R_1	1	R_1	1	R_1
2		2		2	
3	R_2.	3	R_2	3	R_2.

$S(\underset{\text{diff}}{3}, \underset{\text{identical.}}{2})$



$S(3, 2)$

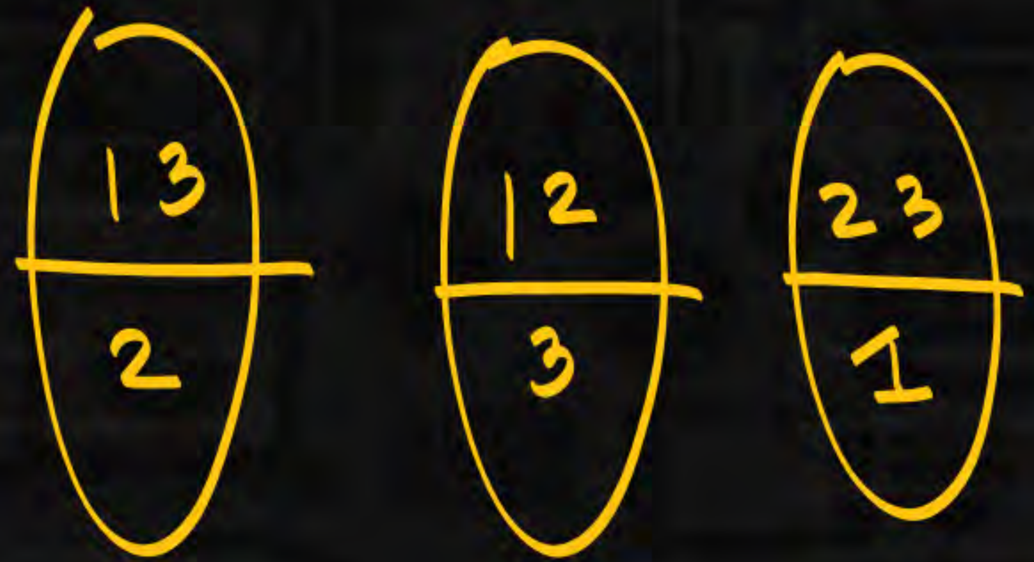
Relations

Total Equivalence relation = Total partions.

$$= S(3,1) + S(3,2) + S(3,3)$$

Relations

how many ways we can adjust 3 quest 2 identical rooms.



$S(\underset{\text{diff}}{3}, \overset{\text{identical}}{2})$

how many ways we can adjust 3 quest to 1 room.

$S(3, 1)$

3 quest → 3 identical room.

$S(3, 3)$

Relations

$$S(3,1)+S(3,2)+S(3,3)$$

$$\sum_{i=1}^{3} S(3,i)$$

$$B(n)=\sum_{i=1}^{n} S(n,i)$$

Relations

Triangle :

B_0	1				
B_1	1	2			
B_2	2	3	5		
$B_3 \rightarrow$	5	7	10	15	
$B_4 \rightarrow$	15	20	27	37	52

$B_5 \rightarrow 52$.

	b		c	
a		a+b		a+b+c

a+b+c.

B_3: how many partitions we can create if set is having 3 elements.

3. If $A = \{1, 2, 3, 4, 5\}$ and $\mathcal{R}$ is the equivalence relation on A that induces the partition $A = \{1, 2\} \cup \{3, 4\} \cup \{5\}$, what is $\mathcal{R}$?

4. For $A = \{1, 2, 3, 4, 5, 6\}$, $\mathcal{R} = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4), (4, 5), (5, 4), (5, 5), (6, 6)\}$ is an equivalence relation on A. (a) What are $[1]$, $[2]$, and $[3]$ under this equivalence relation? (b) What partition of A does $\mathcal{R}$ induce?

7. Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$, and define $\mathcal{R}$ on A by $(x_1, y_1)\ \mathcal{R}\ (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.

a) Verify that $\mathcal{R}$ is an equivalence relation on A.

b) Determine the equivalence classes $[(1, 3)]$, $[(2, 4)]$, and $[(1, 1)]$.

c) Determine the partition of A induced by $\mathcal{R}$.

8. If $A = \{1, 2, 3, 4, 5, 6, 7\}$, define $\mathcal{R}$ on A by $(x, y) \in \mathcal{R}$ if $x - y$ is a multiple of 3.

a) Show that $\mathcal{R}$ is an equivalence relation on A.

b) Determine the equivalence classes and partition of A induced by $\mathcal{R}$.

9. For $A = \{(-4, -20), (-3, -9), (-2, -4), (-1, -11), (-1, -3), (1, 2), (1, 5), (2, 10), (2, 14), (3, 6), (4, 8), (4, 12)$ define the relation $\mathcal{R}$ on A by $(a, b)\ \mathcal{R}\ (c, d)$ if $ad = bc$.

a) Verify that $\mathcal{R}$ is an equivalence relation on A.

b) Find the equivalence classes $[(2, 14)]$, $[(-3, -9)]$, and $[(4, 8)]$.

11. How many of the equivalence relations on $A = \{a, b, c, d, e, f\}$ have (a) exactly two equivalence classes of size 3? (b) exactly one equivalence class of size 3? (c) one equivalence class of size 4? (d) at least one equivalence class with three or more elements?

12. Let $A = \{v, w, x, y, z\}$. Determine the number of relations on A that are (a) reflexive and symmetric; (b) equivalence relations; (c) reflexive and symmetric but not transitive; (d) equivalence relations that determine exactly two equivalence classes; (e) equivalence relations where $w \in [x]$; (f) equivalence relations where $v, w \in [x]$; (g) equivalence relations where $w \in [x]$ and $y \in [z]$; and (h) equivalence relations where $w \in [x]$, $y \in [z]$, and $[x] \neq [z]$.

13. If $|A| = 30$ and the equivalence relation $\mathcal{R}$ on A partitions A into (disjoint) equivalence classes A_1, A_2, and A_3, where $|A_1| = |A_2| = |A_3|$, what is $|\mathcal{R}|$?


THANK YOU GW
SOLDIERS !