

CS & IT ENGINEERING

Discrete Mathematics

Combinatorics

DPP 01 Discussion Notes



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TOPICS TO BE COVERED

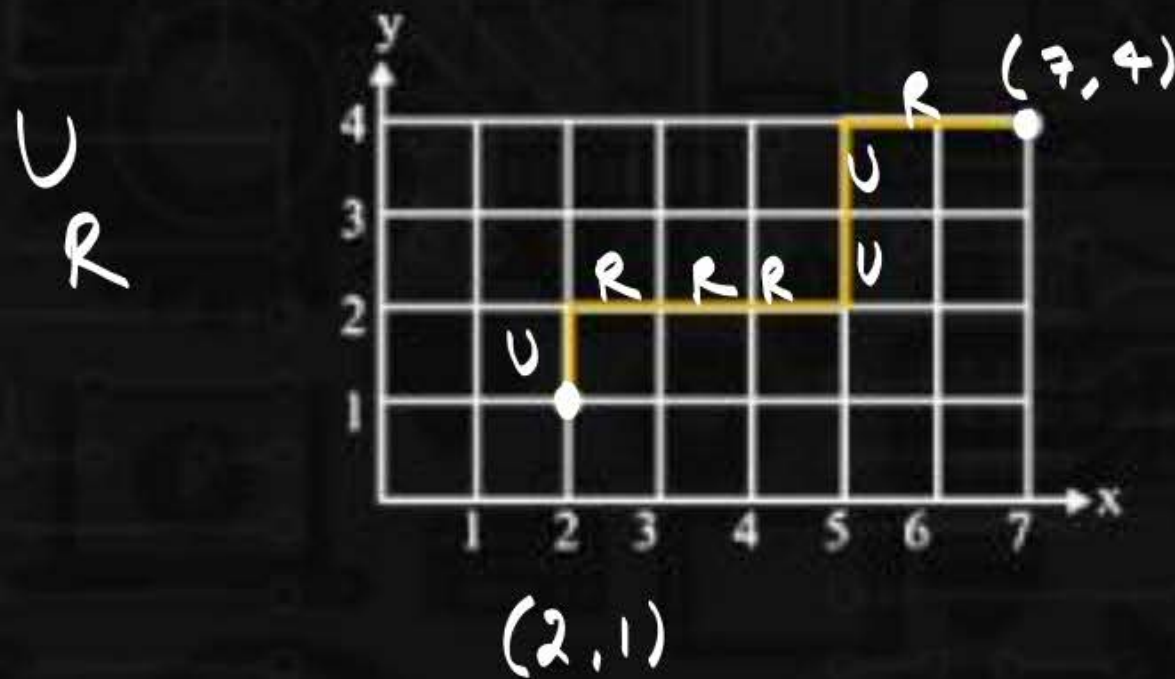
01 Question

02 Discussion

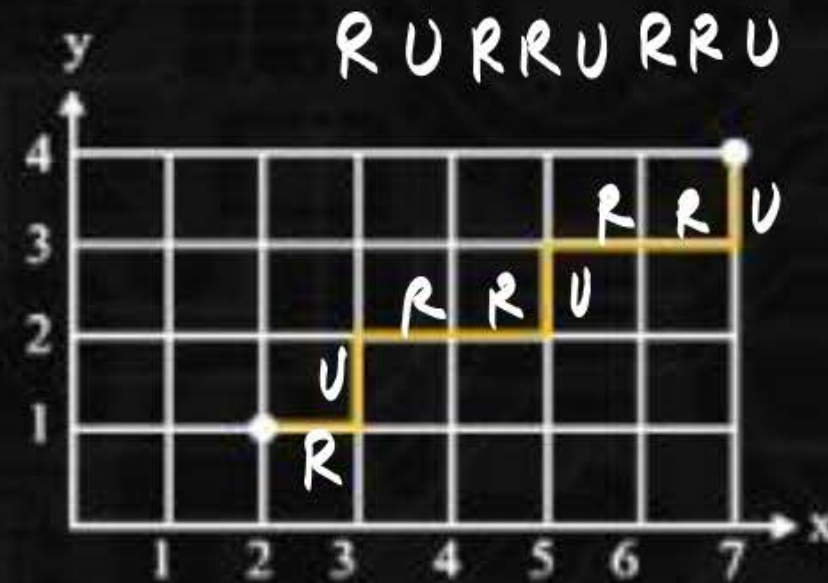
Q.1



Determine the number of (staircase) paths in the xy - plane from $(2, 1)$ to $(7, 4)$, where each such path is made up of individual steps going one unit to the right (R) or one unit upward (U). The bold lines in Figure. show two of these paths. **[NAT]**



U R R R U U R



→ R U R R U R R U

Ans:
$$\frac{8!}{3! \cdot 5!}$$

= 56 ways
$$\frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5}}{\cancel{3} \cdot \cancel{5}}$$

Q.2

How many nonnegative integer solutions are there to the pair of equations $x_1 + x_2 + x_3 + \dots + x_7 = 37$, $x_1 + x_2 + x_3 = 6$? [MCQ]



A. $\binom{42}{12} \binom{34}{31}$

B. $\binom{5}{3} \binom{34}{31}$

✓ C. $\binom{8}{6} \binom{34}{31}$

D. None of these

$8C_2 \times 34C_3$

$8C_6 \times 34C_{31} = 6$

$31 + 3C_3$

$34C_3$

$37 - 6 = 31$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 37$

$6 + 2C_2 = 8C_2$

Q.3

Determine the number of integer solution of $x_1 + x_2 + x_3 + x_4 + x_5 < 40$, where $x_i \geq 0, 1 \leq i \leq 5$.



[MCQ]

☒ A. $\binom{44}{39}$

$$\frac{x_1 + x_2 + x_3 + x_4}{10} \leq 10$$

☐ B. $\binom{39}{38}$

$$x_1 + x_2 + x_3 + x_4 + x_5 < 40$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 39$$

$$44 \leq 5$$

☐ C. $\binom{41}{11}$

$$x_1 + x_2 + x_3 \leq 10$$

☐ D. None of these

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ = 1 \\ = 2 \\ = 9 \checkmark \\ = 10 \checkmark \end{cases}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 39$$

$$39 + 5 \leq 5$$

$$44 \leq 5 = 44 \leq 39$$

$${}^n C_r = {}^n C_{n-r}$$

Q.4

Determine the number of integer solution of

$$x_1 + x_2 + x_3 + x_4 + x_5 < 40, \text{ where}$$

$$x_i \geq -3, \quad 1 \leq i \leq 5$$

[NAT]



$$x_i \geq -3$$

$$\frac{x_i + 3}{y_i} \geq 0$$

$$y_i = x_i + 3$$

$$y_i - 3 = x_i$$

$$x_1 + x_2 + x_3 + x_4 + x_5 < 40$$

$$y_1 - \underline{3} + y_2 - \underline{3} + y_3 - \underline{3} + y_4 - \underline{3} + y_5 - \underline{3} < 40$$

$$\leq 39$$

$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 = 39 + 15$$

$$54 + 5C_5$$

$$59C_5$$

$$\begin{array}{r} 39 \\ 15 \\ \hline 54 \end{array}$$

Q.5

How many ways are there to place 12 marbles of the same size in five distinct jars if the marbles are all black?

[NAT]

$$12 + 4 \text{ } {}^nC_4$$

$$= 16 \text{ } {}^nC_4$$

Q.6

Determine the number of integer solutions of

[MCQ]



$$x_1 + x_2 + x_3 + x_4 = \underline{32},$$

where

(I) $x_i \geq 0, 1 \leq i \leq 4$

(II) $x_i > 0, 1 \leq i \leq 4$

niz!

A. $\binom{11}{8}, \binom{35}{32}$

C. $\binom{35}{32}, \binom{11}{8}$

~~B. $\binom{35}{32}, \binom{31}{28}$~~

D. None of these

$$32 + 3C_3 = 35C_3 = 35C_{32}$$

$$31C_3 = 31C_{28}$$

Q.7

A certain ice cream store has 31 flavors of ice cream available. In how many ways can we order a dozen ice cream cones if a flavor may be ordered as many as 12 times? **[MCQ]**



A. $\binom{42}{12}$

B. $\binom{31}{12}$

☒ C. $\binom{42}{11}$

D. None of these

$31 + 11$
 C_{11}
 $42 C_{11}$

Q.8

In how many ways can 10 (identical) dimes be distributed among five children if there are no restrictions?



[MCQ]

A. $\binom{12}{8}$

☒ B. $\binom{14}{10}$

C. $\binom{23}{11}$

D. $\binom{14}{9}$

$10 + 4 = 14$

$14C4 = 14C10$

Q.9

Find the coefficient of x^{16} in the expansion of $\left(2x^2 - \frac{x}{2}\right)^{12}$.



[NAT]

$$\left(2x^2 - \frac{x}{2}\right)^{12}$$

$${}^{12}C_k (2x^2)^{12-k} \left(-\frac{x}{2}\right)^k$$

$${}^{12}C_k (2)^{12-k} x^{24-2k} \left(-\frac{1}{2}\right)^k (x)^k$$

$${}^{12}C_k x^{12-k} (2)^{12-k} x^{24-2k+k}$$

$$\left(-\frac{1}{2}\right)^k {}^{12}C_k (2)^{12-k} x^{24-k}$$

$$x^{24-k} = x^{16}$$

$$24-k = 16$$

$$k = 8$$

$$\left(-\frac{1}{2}\right)^8 \times {}^{12}C_8 \times 2^{12-8}$$

$$= \frac{495}{16}$$

Q.10

How many ways are there to place 12 pebbles of the same size in five distinct jars if each pebble is a different color?

[MCQ]



A. $\binom{16}{12}$

☒ B. 5^{12}

C. 12^5

D. $\binom{14}{12}$

