CS & IT ENGINEERING

Discrete Mathematics

Set Theory

DPP 01

Discussion notes

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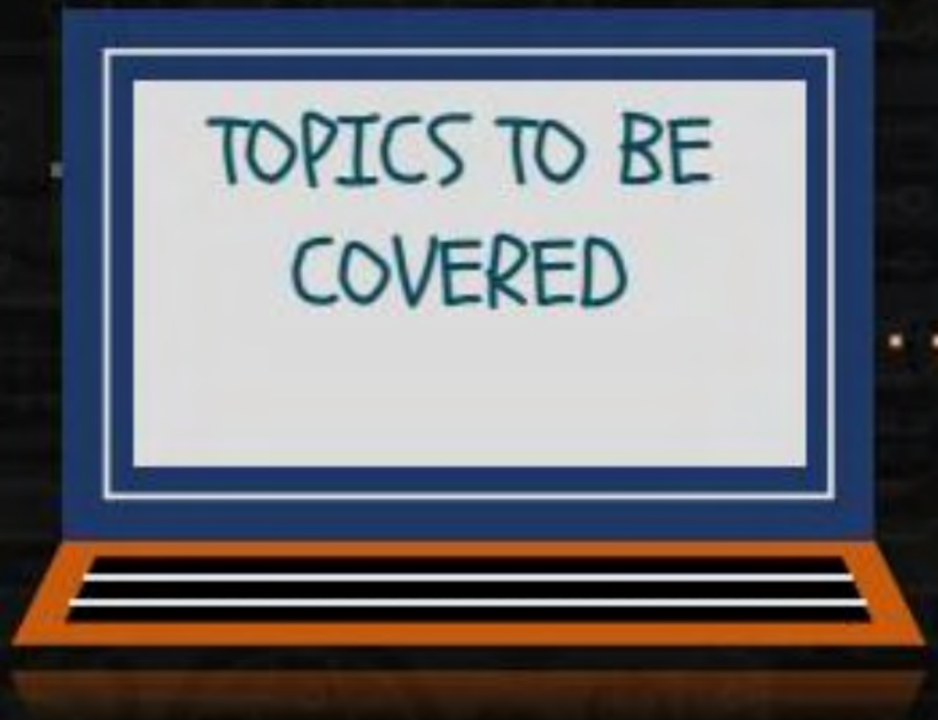

01 Question

02 Discussion

Q.1 Which of the following statements are true? [NAT]

I. $\phi \in \phi$

II. $\phi \subset \phi$

III. $\phi \subseteq \phi$

IV. $\phi \in \{\phi\}$

V. $\phi \subset \{\phi\}$

VI. $\phi \subseteq \{\phi\}$

$\phi, \{\}, \{\phi\}$

Empty set

ϕ : No. of subset : 2^n

$2^0 = 1$

$\{\phi\}$

$A = \{1, 2\}$

Subset - $\{\phi, \{1\}, \{2\}, \{1,2\}\}$

I: $\phi \in \phi$: False

Empty

II: $\phi \subset \phi$: False

III: $\phi \subseteq \phi$: True

Empty Empty

True, True, True, False, True, True

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Q.2 If a set A has 63 proper subsets, then what is the cardinality of A? [NAT]

$A = \{1, 2, 3\}$: 2^n : $2^3 = 8$

Subset: $\{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}$ ~~$\{1,2,3\}$~~ $\}$

proper subset of A: 7 element

: $2^n - 1$

$2^3 - 1 = 7$

A: 63 proper subset

$63 = 2^n - 1 \Rightarrow 2^n = 64 \Rightarrow 2^n = 2^6$

$n = 6$

$\{1, 2, 3 \ldots 6\}$

Q.3 If a set A has 64 subsets of odd cardinality, then what is |A|? [MCQ]

A. 6

B. 63

C. 7 ✓

D. 128

no. of elements in th subset must be odd.

$A = \{1, 2, 3, 4, 5\}$

Subset(1): $\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$

Subset(2): $\{\{1,2\}, \{1,3\}\}$: even

Subset(3): $\{\{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \ldots\}$

$A = \{n \text{ element}\}$

$2^{n-1} = \{{}^nC_1 + {}^nC_3 + {}^nC_5 \ldots\ldots {}^nC_n\}$

$\therefore \quad 2^{n-1} = 64$ odd card. subsets

$\Rightarrow 2^{n-1} = 2^6$

$\Rightarrow n-1 = 6$

$\Rightarrow n = 6+1$

$\Rightarrow \boxed{n = 7}$ ~~$\boxed{|A| = 7}$~~

Q.4

How many subset of {1, 2, 3, ..., 11} contain at-least one even integer?

[NAT]

1984

A : {1, 2, 3 . . . . 11}

Subset : { 1, 2, 3, 5 }

Method 1 : Complement method

Total subsets : 2^{11}

{ 1, 3, 5 }

odd values { 1, 3, 5, 7, 9, 11 } : 6 elements

Subsets : 2^6 : 64 Subsets

Subset with atleast one even : $2^{11} - 64$

$2^{10} \times 2 - 64$

$1024 \times 2 - 64$

$2048 - 64$

$= 1984$

Method 2:

Set: $\{2, 4, 6, 8, 10\}$

No. of subsets: 2^5 : $\{\phi\}$

$2^5 - 1 = 31 * 2^6 = 1984$

$A : \{1, 2, 3, 4\}$

$Set = \{2, 4\}$

$2^2 = 4 - 1 = 3$

$\{\cancel{\phi}, \{2\}, \{4\}, \{2,4\}\}$

$\{3\}, \{4\}\ \{2,4\}$,

$\{2,3\}, \{4,3\}, \{2,4,3\}$

$Set = \{1, 3\}$

$2^2 = 4$

$\{\phi, \{1\}, \{3\}, \{1,3\}\}$

Q.5 [NAT]

Let A = {1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18}

How many subsets of A contain six elements?

$A = \{1, 2, 3\}$

Subsets: $\{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

2 elements : 3 Subsets

$\{1,2,3\}$ $^3C_2 = \frac{3!}{2!\,1!} = \frac{3*2*1}{2*1*1} = 3$

$(1,2)$ $\{1,3\}, \{2,3\}$

Total number of elements: 12 elements

Select 6 elements for subsets.

$\therefore$ Total no. of subsets (6): $^{12}C_6 = \frac{12!}{6!\,6!}$

$$\frac{12 * 11 * 10 * 9 * 8 * 7 * 6!}{6! * 6 * 5 * 4 * 3 * 2}$$

$$= \boxed{924}$$

Q.6

Let A = {2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15}

[NAT]

How many six-elements subsets of A contain four even integers and two odd integers?

#225

Subsets with 6 elements : $^{12}C_6 = 924$

Even numbers in sets

{2, 4, 6, 8, 10, 12}

ODD numbers in sets

{5, 7, 9, 11, 13, 15}

46 + 20

$^6C_4 * {}^6C_2$

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Q.7 [NAT]

Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} → 12 elements

How many subsets of A contain only odd integers?

63

{1, 3, 5, 7, 9, 11}

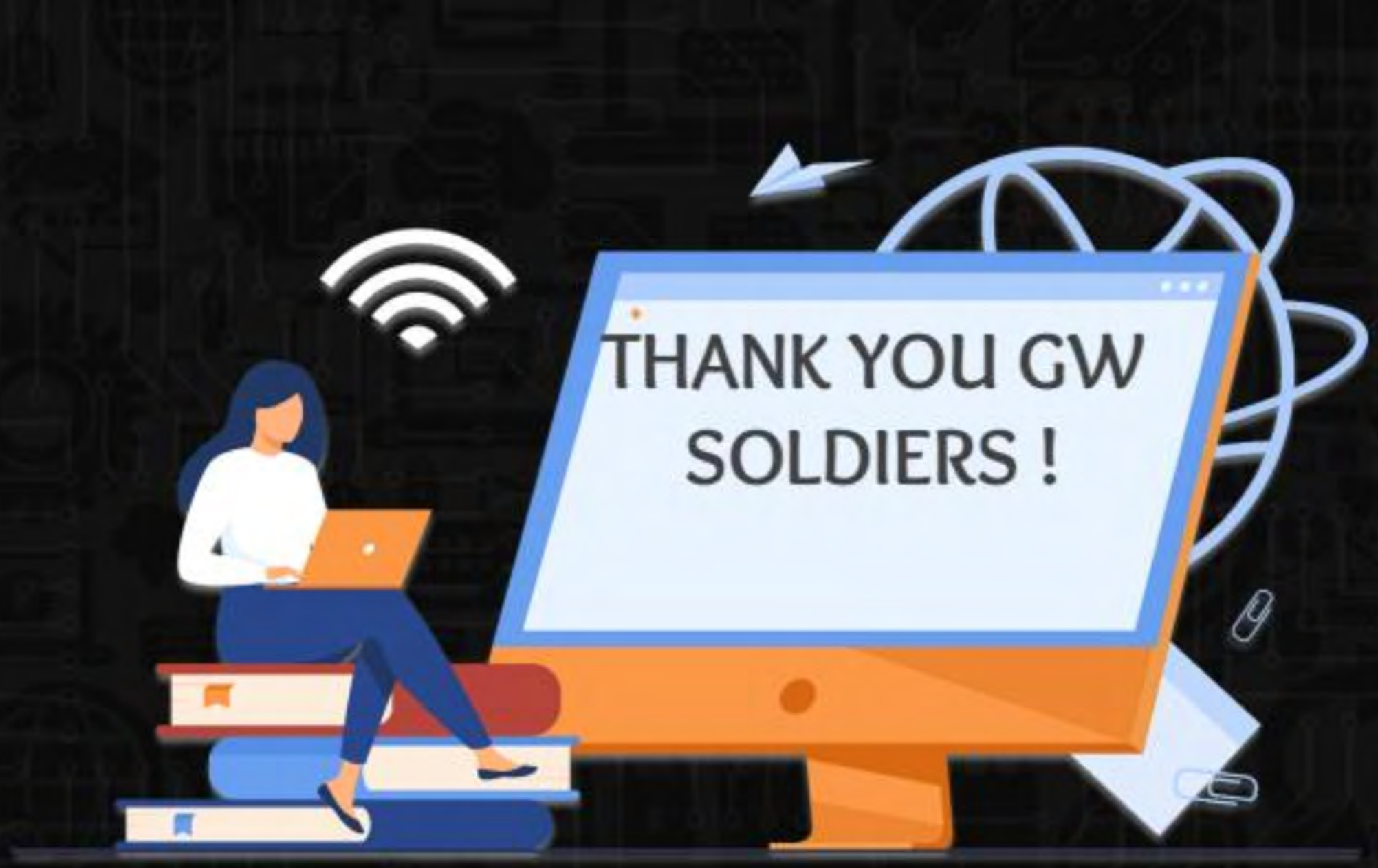
6 elements

No. of subsets: $2^6 - 1$

$64 - 1 = 63$

{ {1}, {3}, {5}, {7}, {9}, {11}

− {1, 3}, {1, 5} − − − }


THANK YOU GW
SOLDIERS !