CS & IT ENGINEERING

GRAPH THEORY

Discrete Maths

Lecture No. 10

By- SATISH YADAV SIR

TOPICS TO BE COVERED

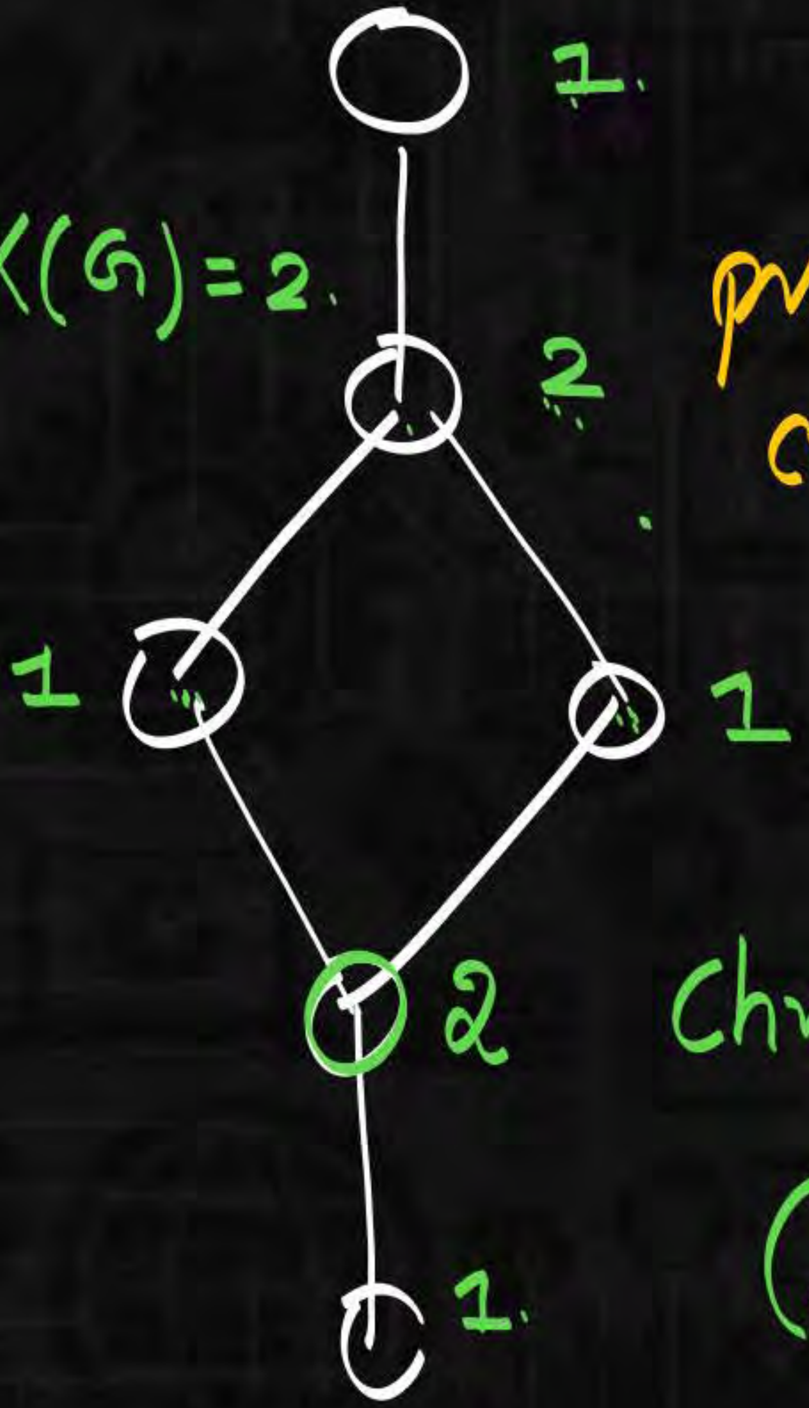
01 Properly coloring

02 Chromatic number

03 Chromatic Number in Graphs

04 Subgraphs

05 Graph operations



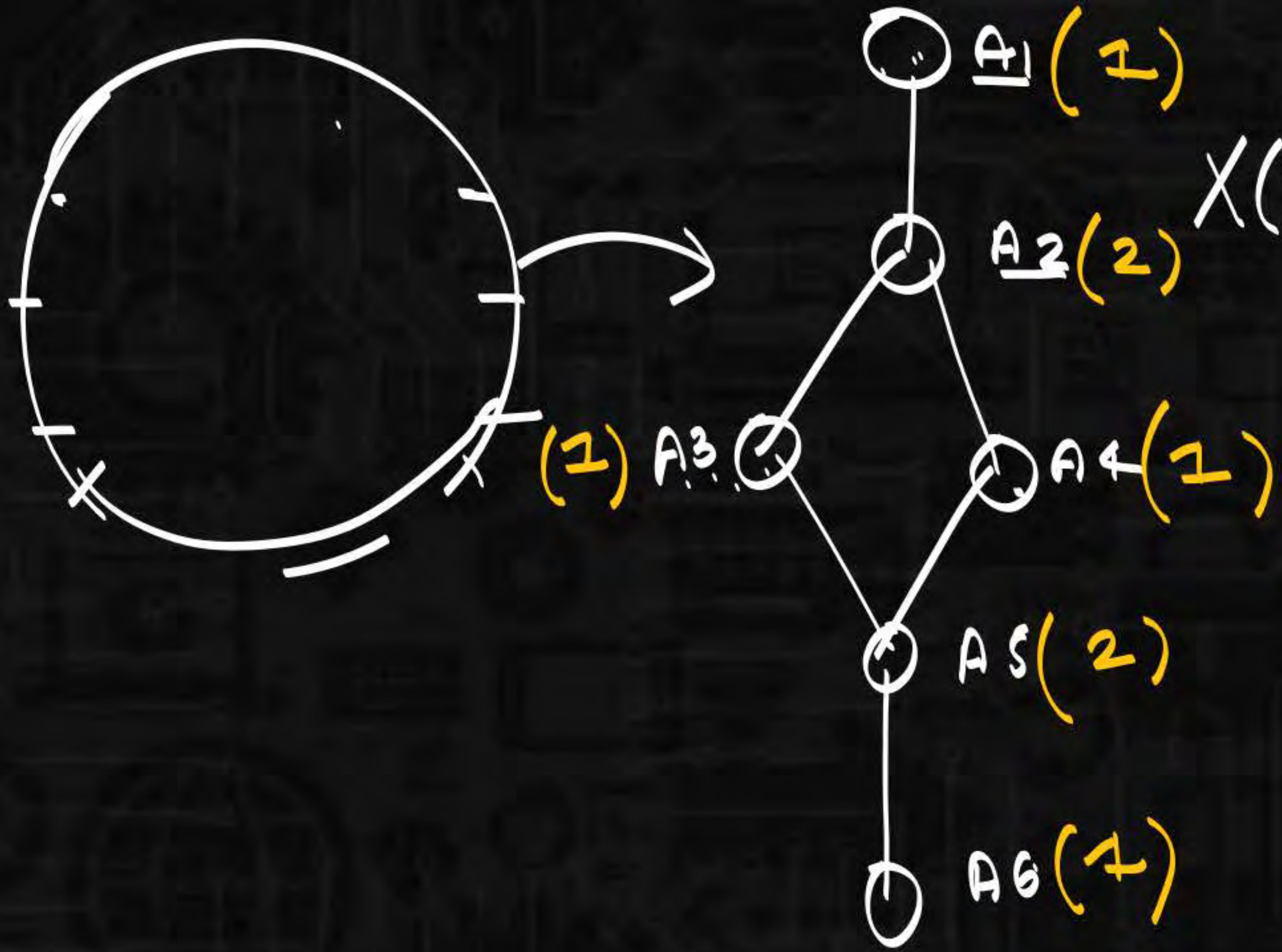
$\chi(G) = K$ k-colorable Graph.

$\chi(G) = 2.$

properly coloring.
paint all vertices with diff colors such that adjacent should not have same color.

$\boxed{\chi(G) = \text{min + properly coloring.}}$

Chromatic no. $(\chi(G))$
paint all the vertices with min no. of colors such that adjacent should not have same clr.



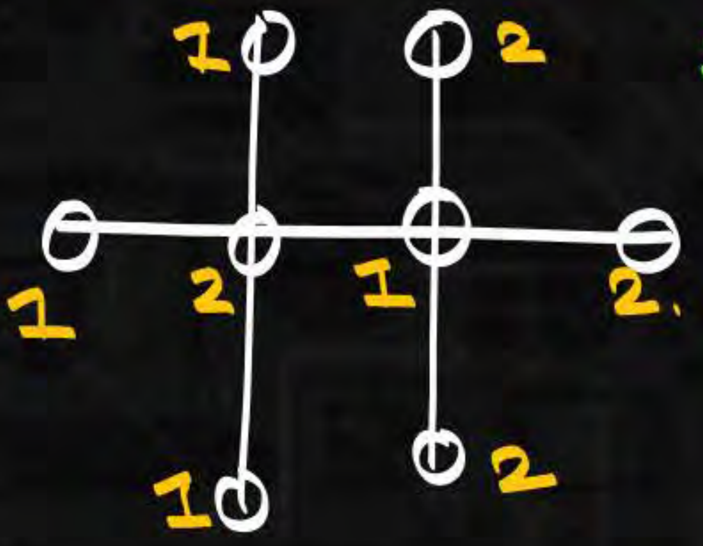
$\chi(G) = 2.$

1)  $\chi(G) = 1$.

 $\chi(G) = 1$

2) Tree:

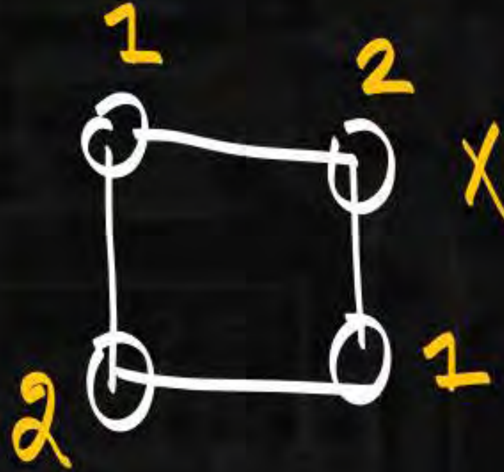
$\chi(\text{Tree}) = 2$.



* Every Tree will always be 2-colorable (True)
* Every 2-colorable will always be Tree (False)

3) Cycle Graph (C_n) ($n \geq 3$)

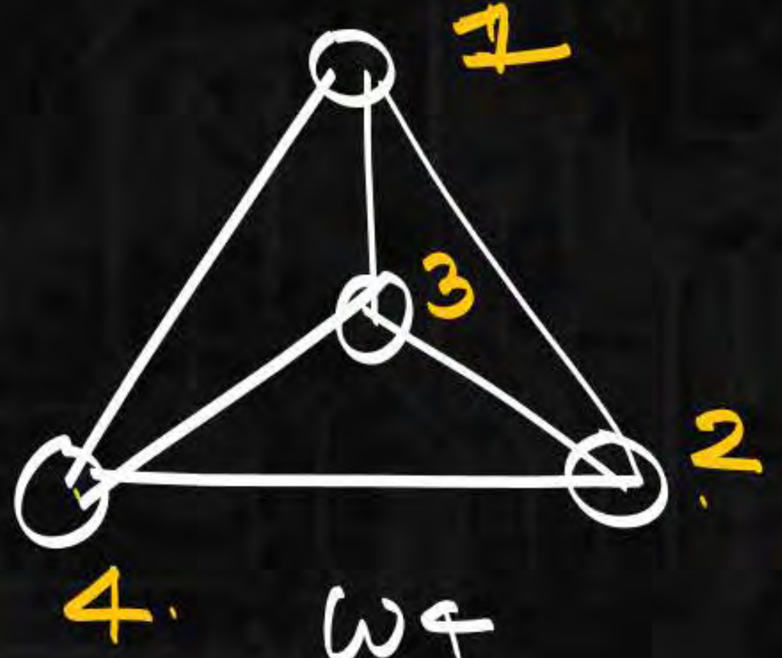
$\chi(C_3) = 3$ 



$\chi(C_4) = 2$.

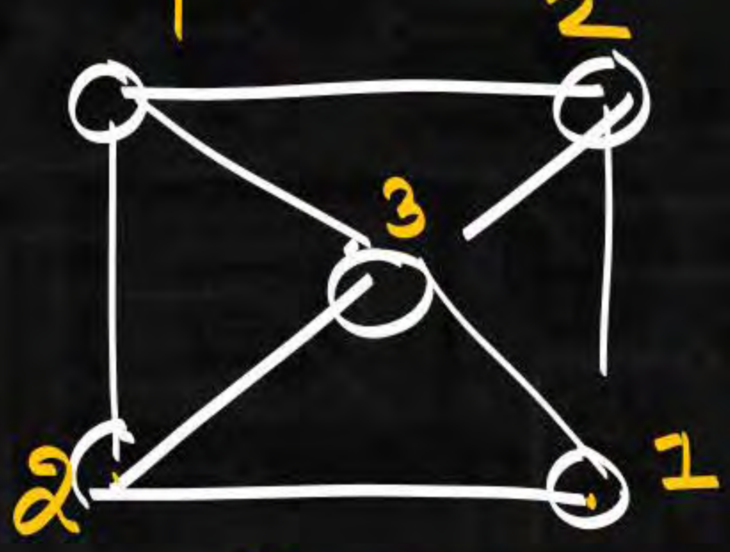
$\boxed{\begin{array}{ll} \chi(C_n) = 2 & n \text{ is even.} \\ \chi(C_n) = 3 & n \text{ is odd.} \end{array}}$

Wheel Graph (W_n) ($n \geq 4$)



W_4

$\chi(W_4) = 4$



$\chi(W_5) = 3$

$\chi(W_4) = 4$

$\chi(W_5) = 3$

$$\boxed{\begin{array}{ll} \chi(W_n) = 4 & \text{n is even} \\ \chi(W_n) = 3 & \text{n is odd} \end{array}}$$

Every Tree will always be 2-colorable. (True)

Every 2 colorable Graph will be Tree (false) eg: C4.

→ Every even length cycle will be 2-colorable (True)

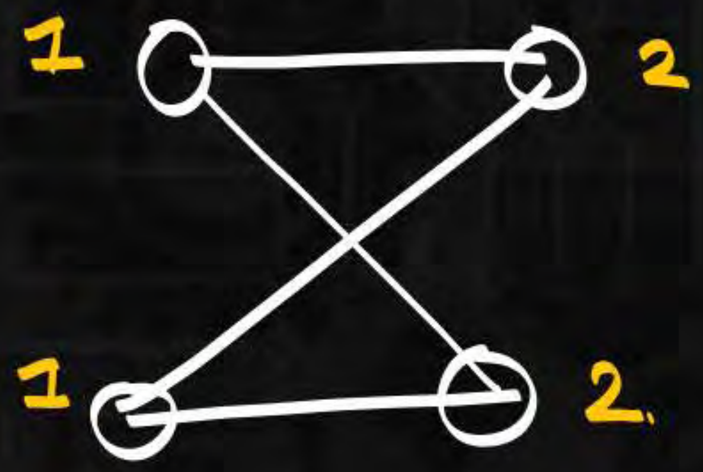
→ Every 2-colorable will always be even length cycle (false)

eg: Tree

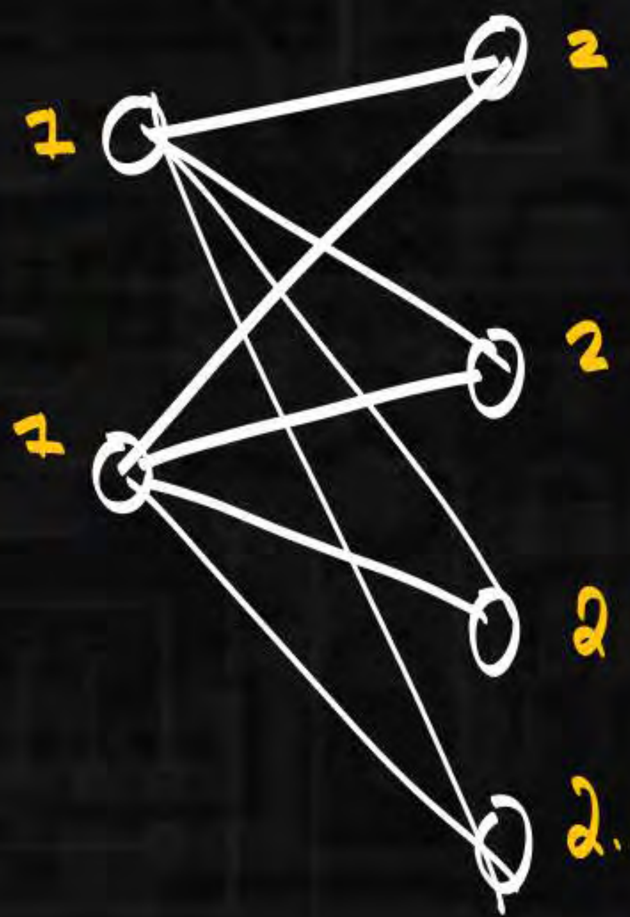
Bipartite Graph :

Every Bipartite Graph will always be 2-colorable (True)

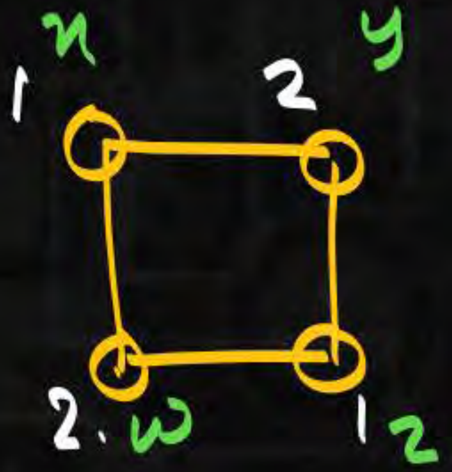
→ Every 2-colorable will always be Bipartite Graph.

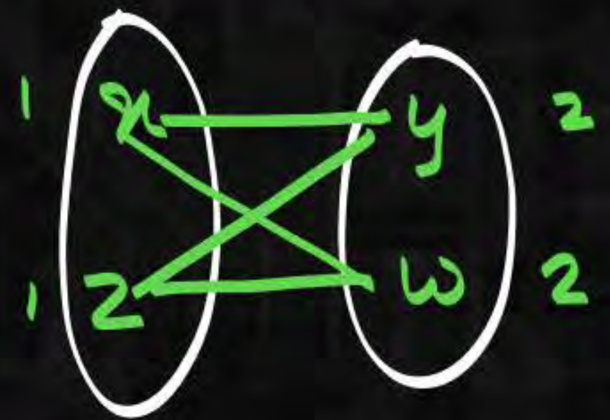


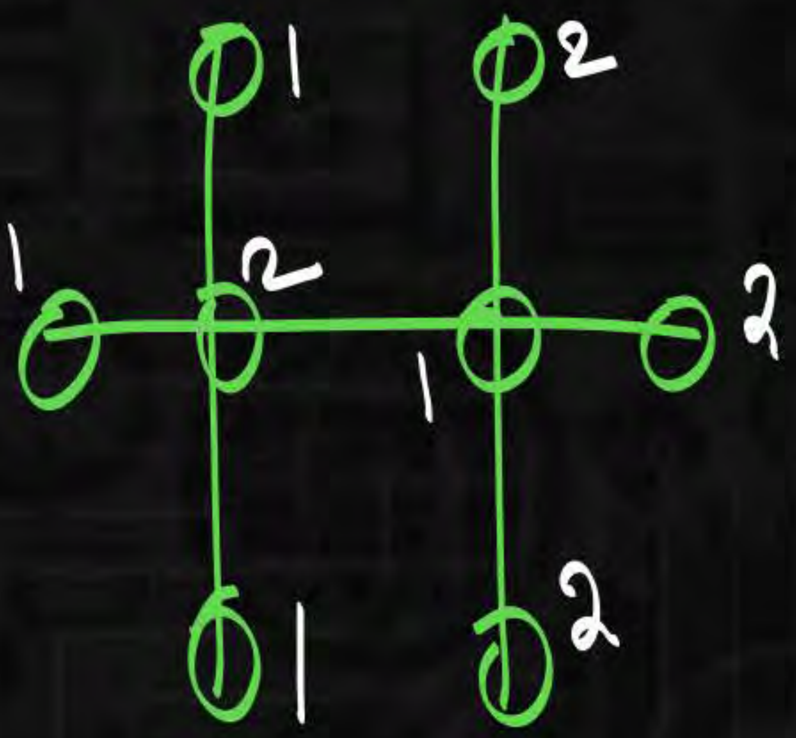
$K_{2,4}$



Every 2-colorable will always be bipartite Graph.



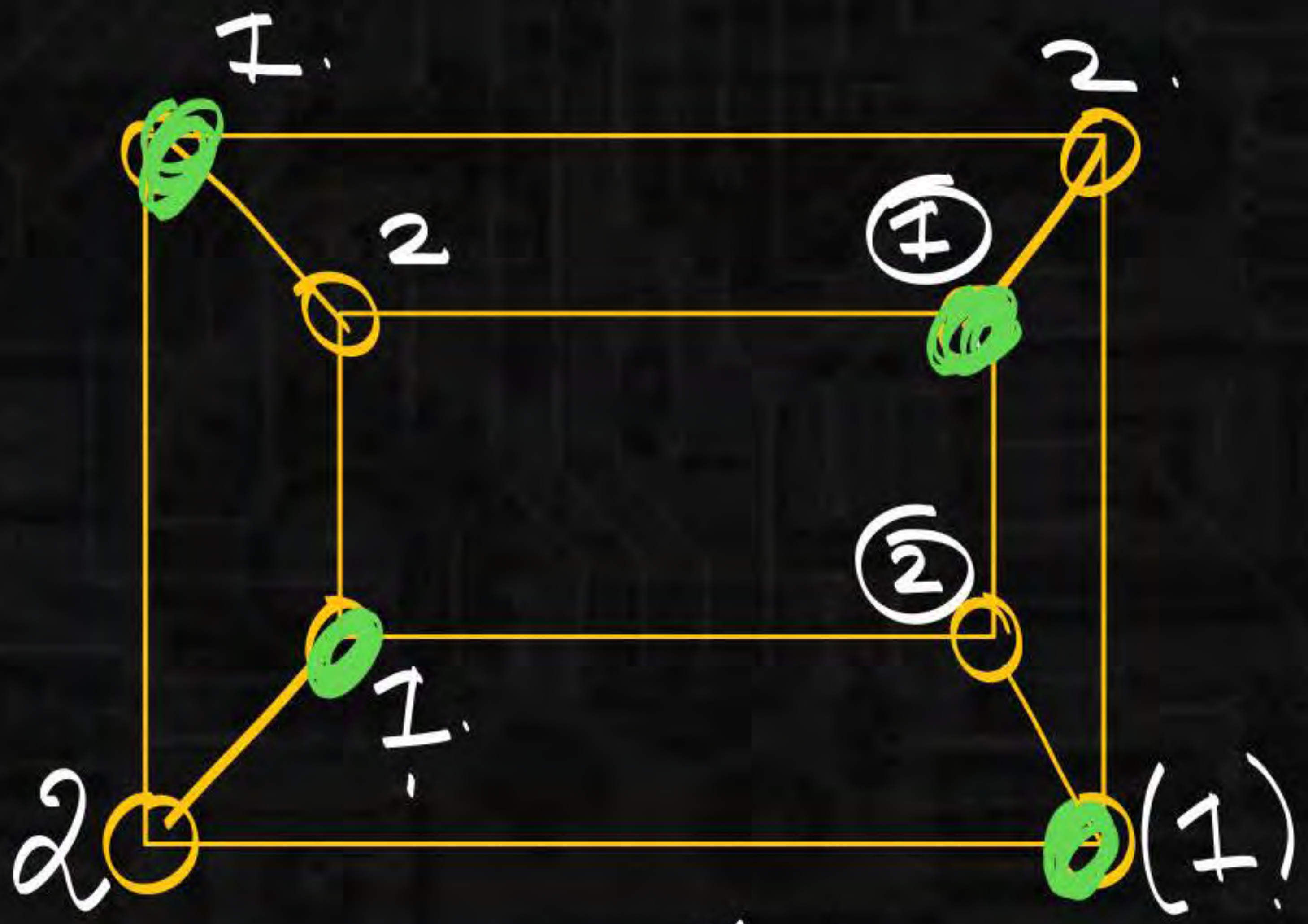




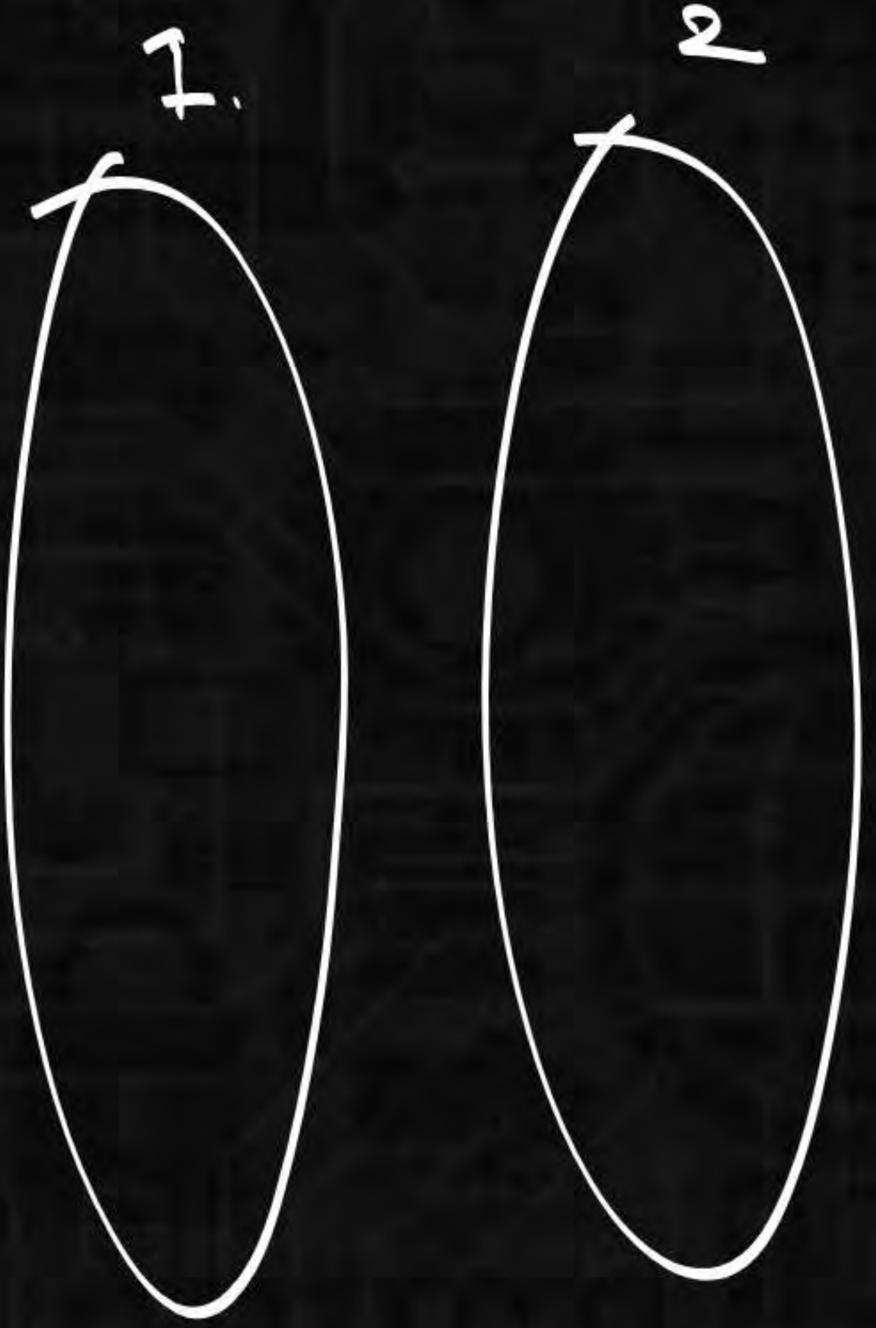
Bipartite Graph does not contains odd length cycle.

→ Tree. (2-clr)

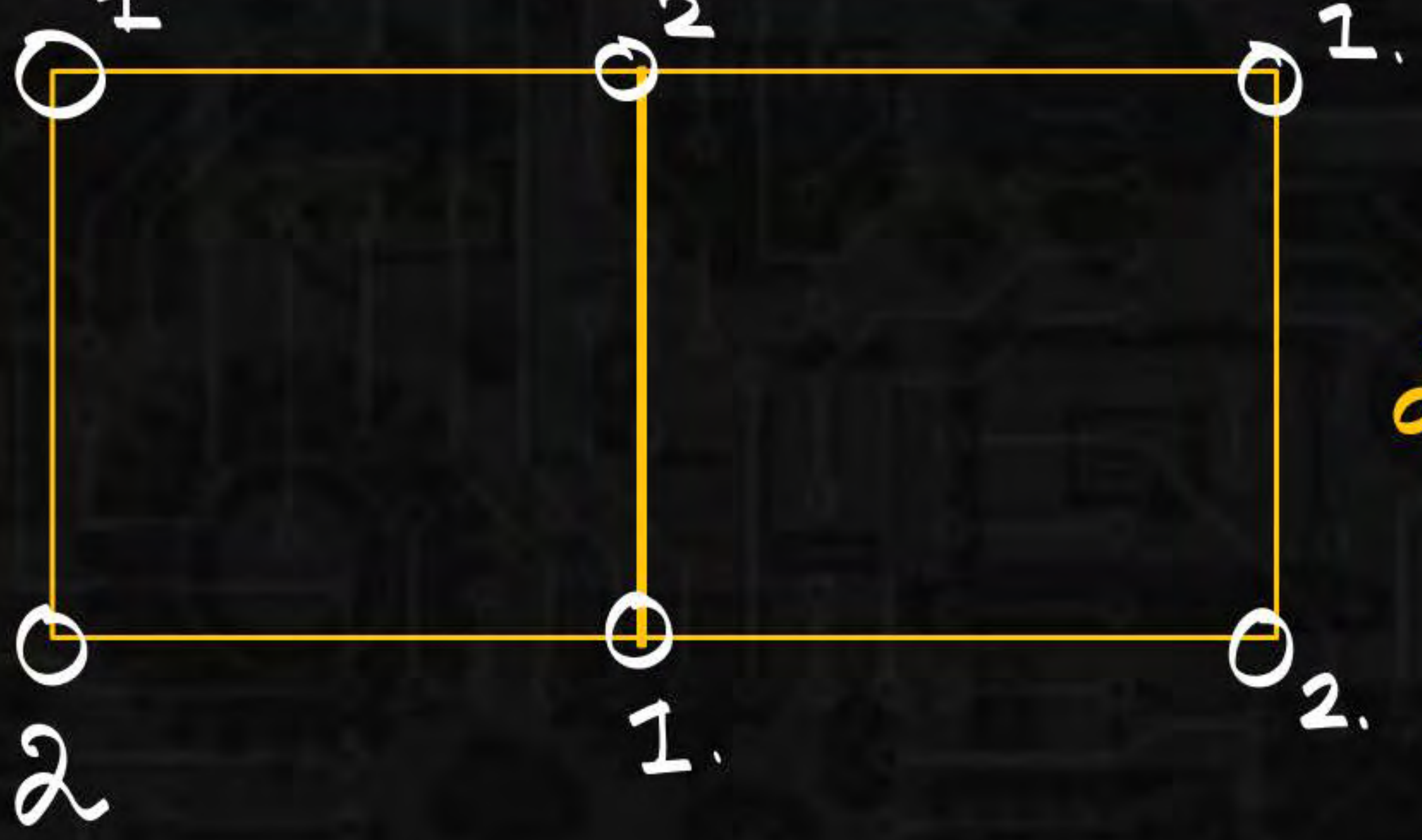
→ even leng (2-clr)



$\chi(G) = 2.$



Hypercube will be 2 colorable. So bipartite Graph.



2-colorable → B.P.G.

Every 2-colorable will always be bipartite Graph. (false)

2-or more isolated vertices will also be bipartite Graph.

with. $\chi(G) = 1$.

→ connected bipartite Graph will be 2 colorable ($n \geq 2$) (True)

→ Bipartite Graph will be 2 colorable (false)

eg:  $\chi(G) = 1$.

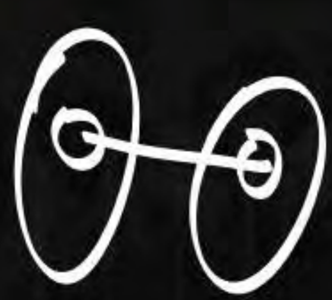
Every Bipartite Graph will be 2 colorable. (false)

$\chi(G)=2$

$\chi(G)=2$.

$\chi(G)=2$

→ Every B.P.G will be 2 colorable $\left(\begin{array}{c}\text{connected}\\ + \\ n \geq 2\end{array}\right)$ (True)



$K_n \ (n \geq 1)$

$$\boxed{\chi(G) \leq n}$$

any Graph.

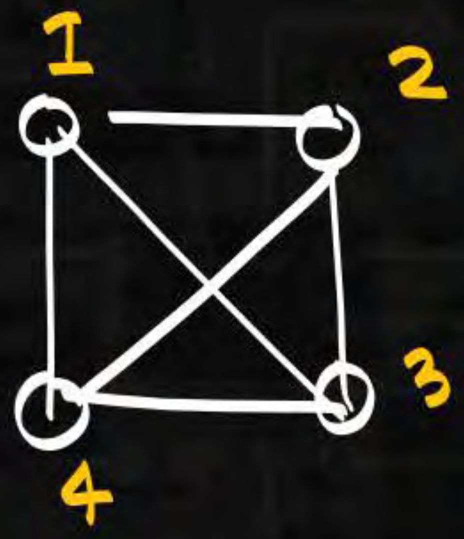
K_1

$\chi(G) = 1$

K_2

$\chi(G) = 2$

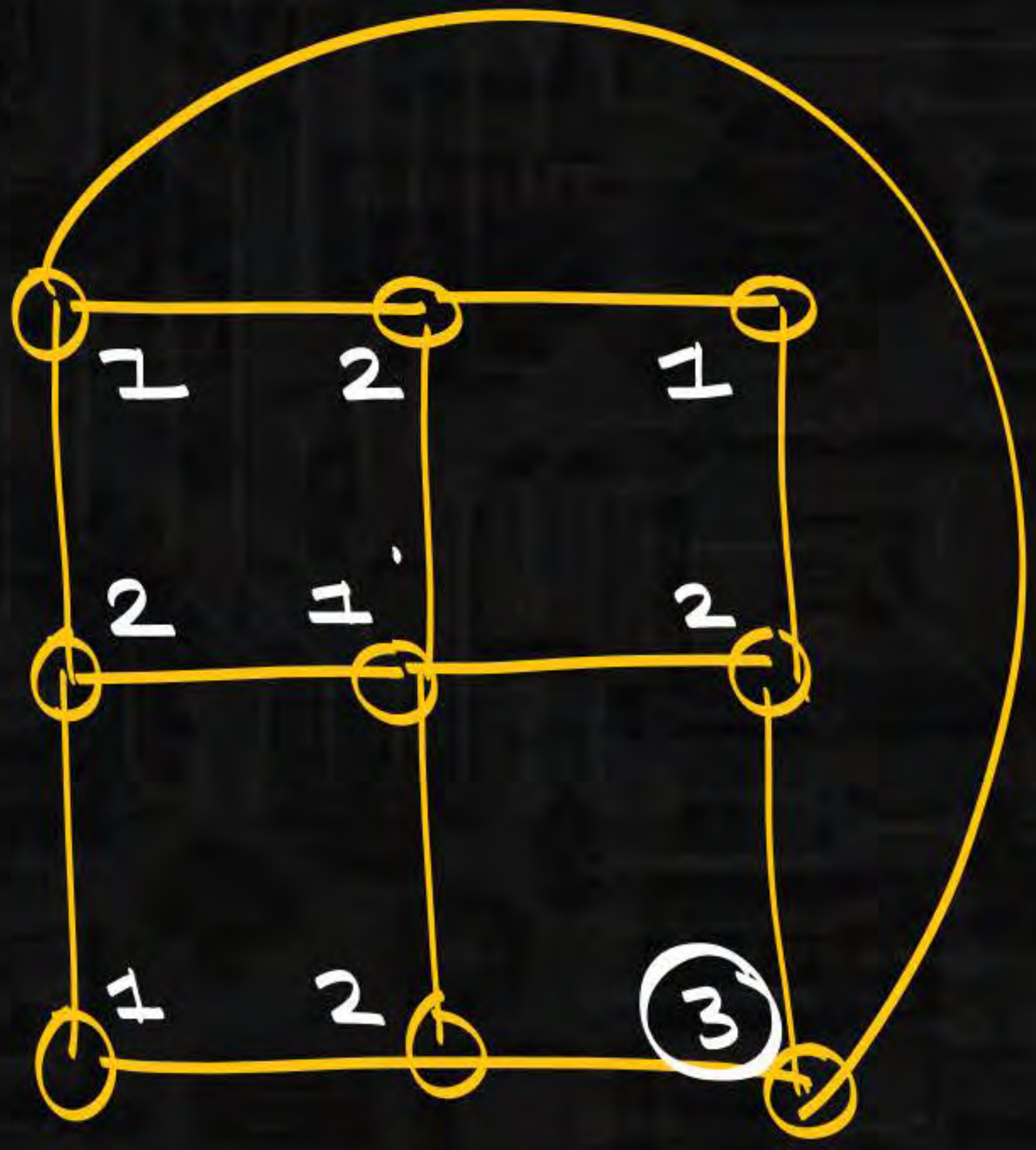
$\chi(K_3) = 3$



$\chi(K_4) = 4$

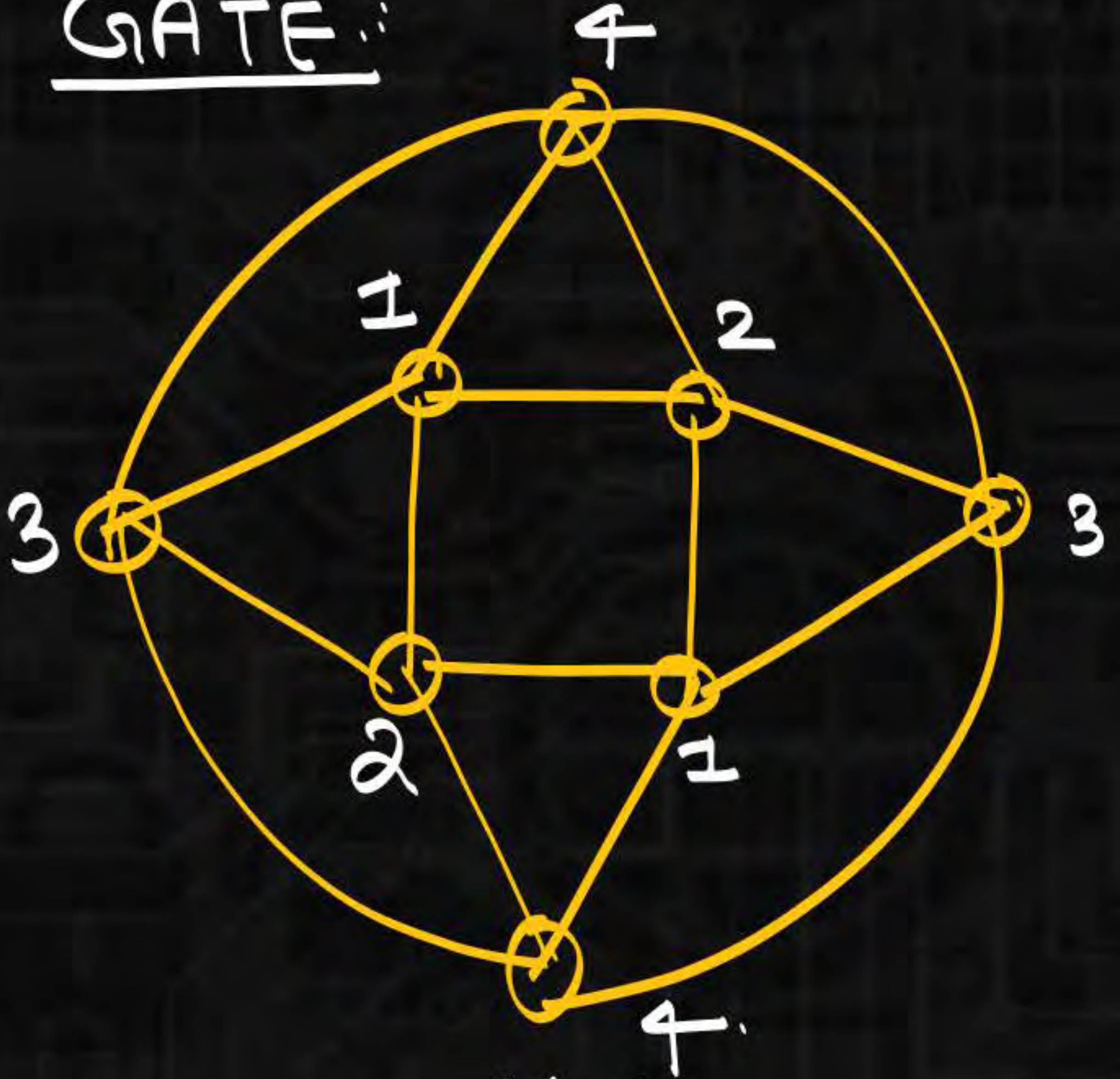
$$\boxed{\chi(K_n) = n}$$

GATE:



$\chi(G) = 3$

GATE:

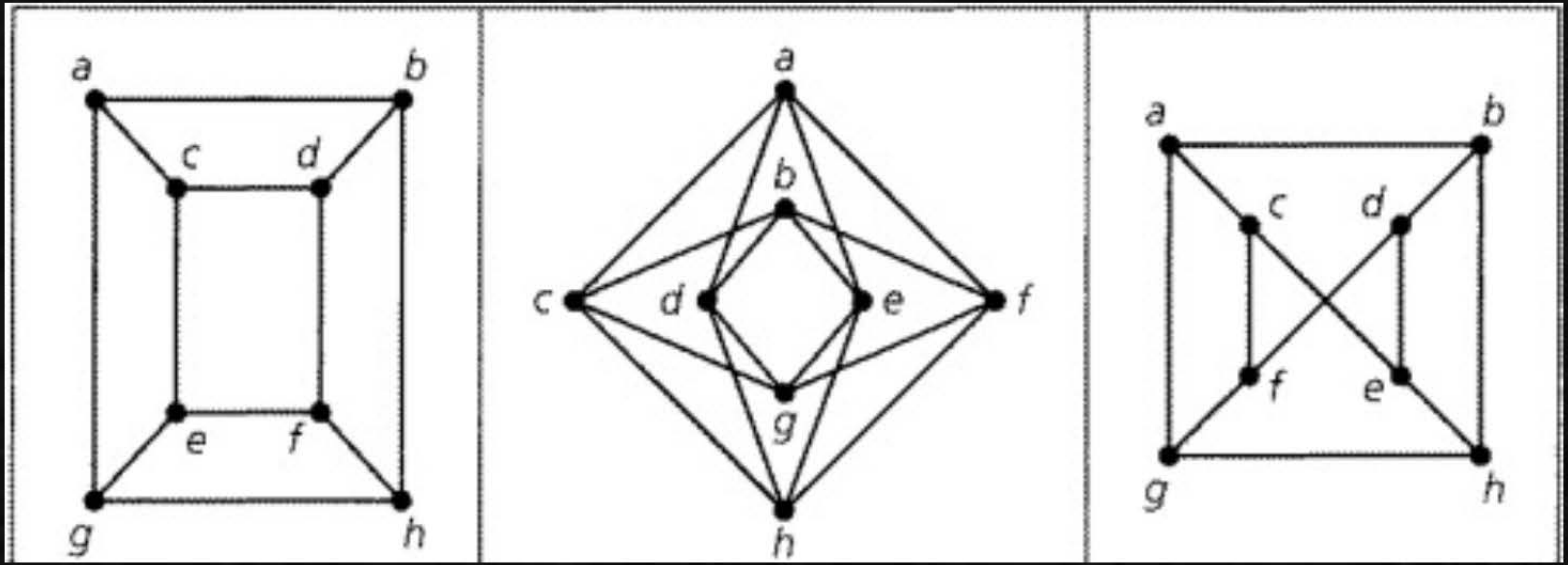


$\chi(G) = 4$

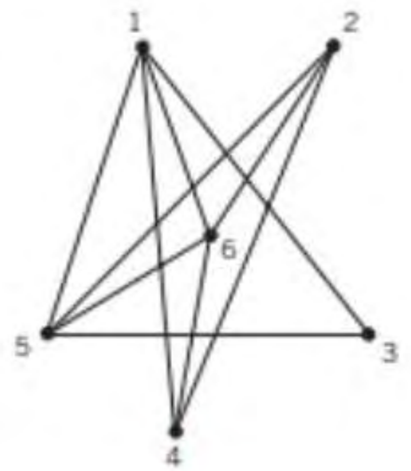
Triangle

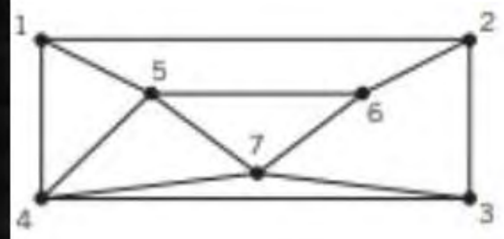
cycle.

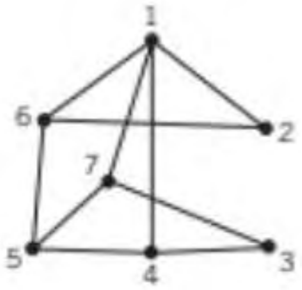
$\chi(G) = 4$

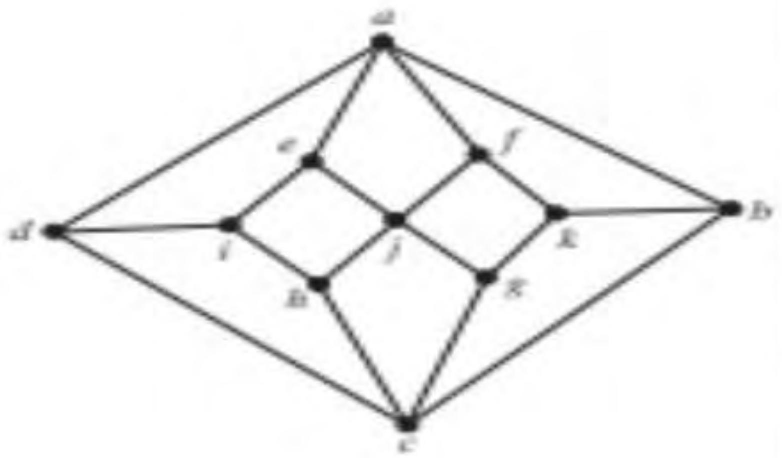


$\chi(G) = ?$



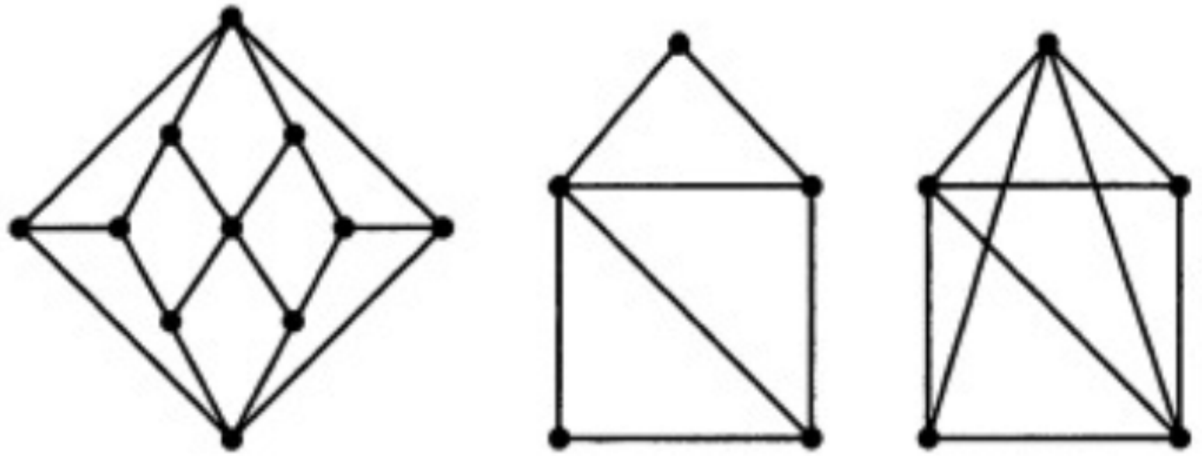






$\chi(G) = ?$

5. Determine the chromatic numbers of the following graphs:



10. What is the chromatic number of the graph obtained from K_n by removing one edge?

The *Petersen graph* $\mathcal{P}$ is the graph whose vertices are t
$\{1, 2, 3, 4, 5\}$ in which two vertices are joined by an edge
2-subsetss are disjoint.

$\chi(G) = ?$


THANK YOU GW
SOLDIERS !