CS & IT ENGINEERING

Algorithm

Dynamic Programming

Lecture No.- 02

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Recap of Previous Lecture

Topic Introduction to Dynamic Programming

Topic The General Method

DP Vs DandC

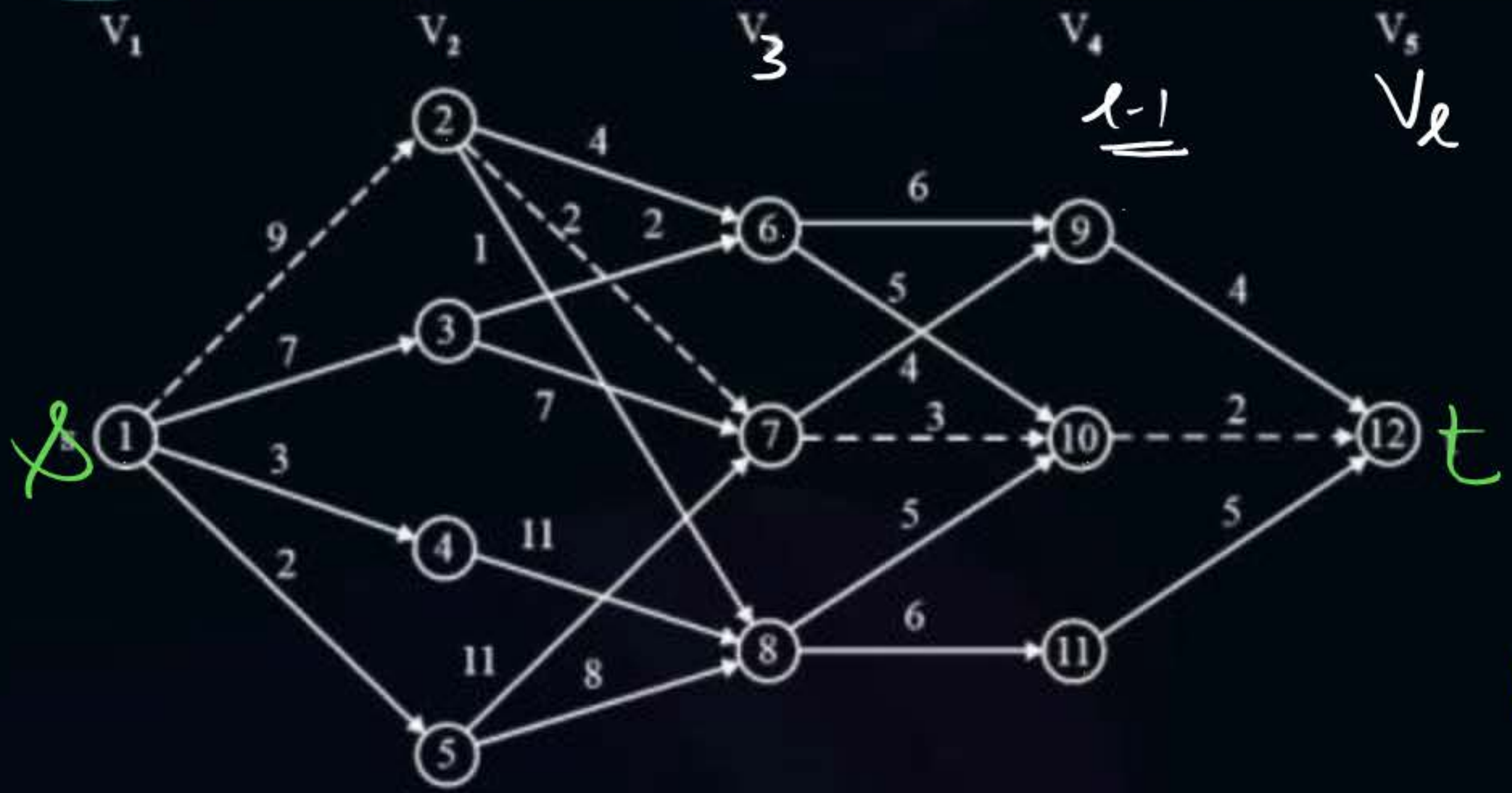
Fibonacci Implementation

Topics to be Covered

Topic — Multistage Graphs

Topic — Travelling Salesperson Problem

Topic : Dynamic Programming: (DP)



$COST(i, j)$ = Cost of Path from vertex 'j' in Stage 'i' to reach dest. vertex 't';

$$COST(1,1) = \min_{\substack{K\in V_2}} \{ c(1,K) + COST(2,K) \} \quad \text{— (1)}$$

$(1 - (K \sim\sim\sim t))$: $COST(1,1)$

$K \in V_2$ $\quad COST(2,K)$

$\langle 1,K\rangle \in E$

$$COST(i,j) = \min_{\substack{K\in V_{i+1} \\ \langle j,K\rangle \in E}} \{ c(j,K) + COST(i+1,K) \} \quad \text{— (2)}$$

$$COST(l-1, j) = c(j,t)$$

I/P

C	1	2	3	. .	12
1					
2					
3					
⋮					
12					

$c(i,j)$ = edge cost

$$\text{COST}(i,j) = \min_{\substack{k \in V_{i+1} \\ \langle j,k\rangle \in E}} \{ c(j,k) + \text{COST}(i+1,k) \} \quad \text{— (1)}$$

$$\text{COST}(k-1,j) = c(j,t) \quad \text{— (2)}$$

$D(i,j) = $ 'k' that minimizes Eq (1)

$$\text{COST}(1,1) = \min\{ \underbrace{c(1,2)+\text{COST}(2,2)}_{9+7}, \underbrace{c(1,3)+\text{COST}(2,3)}_{7+9}, \underbrace{c(1,4)+\text{COST}(2,4)}_{3+18}, \underbrace{c(1,5)+\text{COST}(2,5)}_{2+15} \}$$

$k=2$, $k=3$, $k=4$, $k=5$

$= \boxed{16}$

$D(1,1) = 2$

Path Construction :

$$\langle 1 - 2 - 7 - 10 - 12 \rangle$$

$D(1,1)$

$D(2, D(1,1)) = D(2,2)$ $\quad D(3,7)$

$$COST(4,9) = c(9,t) = 4$$

$$COST(4,10) = c(10,t) = 2$$

$$COST(4,11) = c(11,t) = 5$$

$$COST(3,6) = \min\left\{ \begin{array}{l} k=9 \quad 6+4 \\ c(6,9) + COST(4,9), \\ k=10 \\ c(6,10) + COST(4,10) \\ \quad 5+2 \end{array} \right\} = 7$$

$$D(3,6) = 10$$

$$COST(3,7) = 5$$

$$D(3,7) = 10$$

$$COST(3,8) = 7$$

$$D(3,8) = 10$$

$$COST(2,2) = \min\{4+7;\ 2+5;\ 1+7\} = 7$$

$$D(2,2) = 7$$

$$COST(2,3) = \min\{2+7;\ 7+5\} = 9$$

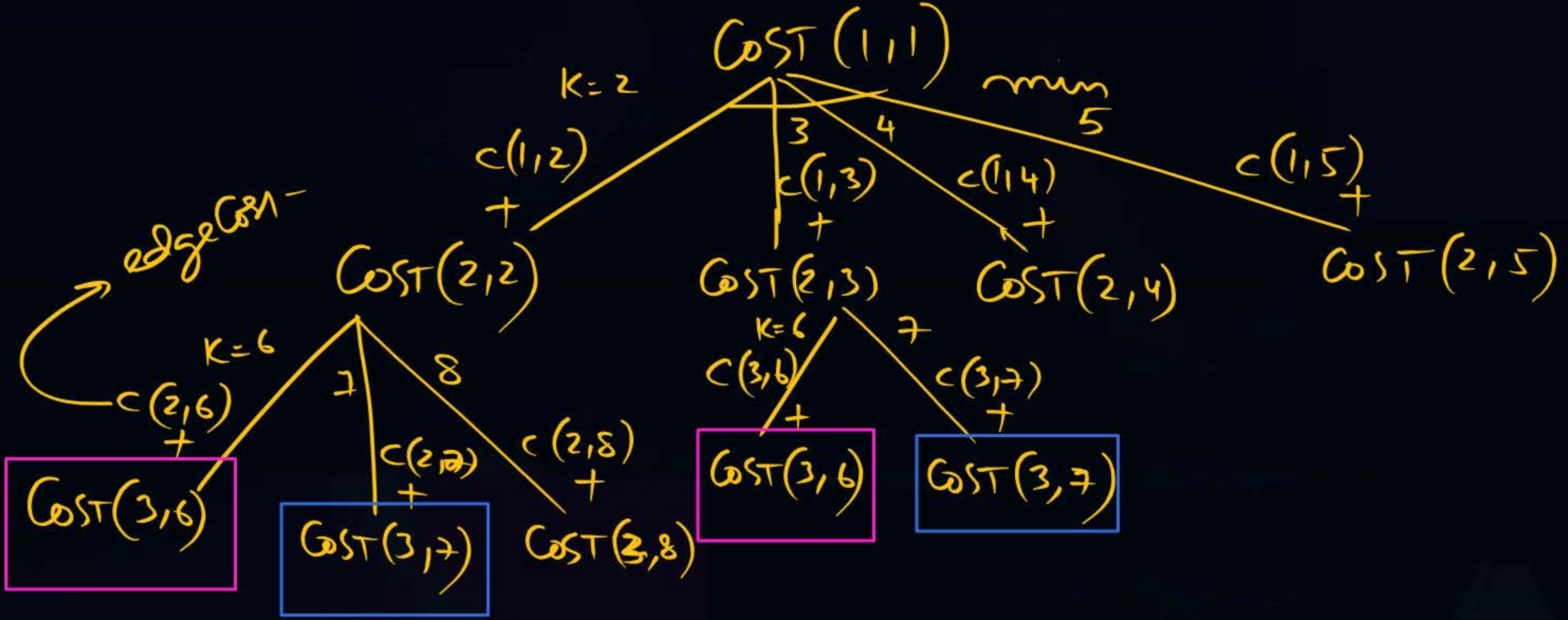
$$D(2,3) = 6$$

$$COST(2,4) = 18$$

$$D(2,4) = 8$$

$$COST(2,5) = 15$$

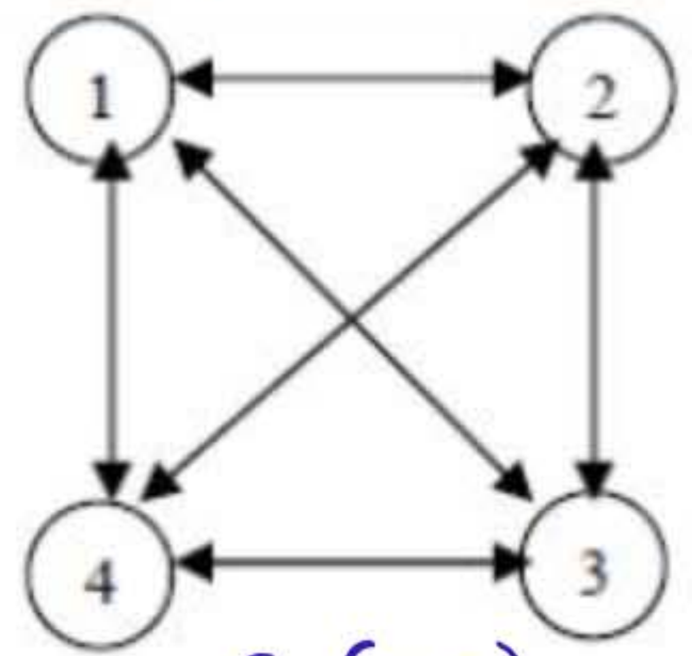
$$D(2,5) = 8$$


COST(1,1)
K=2
min
3
4
5
c(1,2) +
c(1,3) +
c(1,4) +
c(1,5) +
COST(2,2)
COST(2,3)
COST(2,4)
COST(2,5)
edgeCost
K=6
7
8
c(2,6) +
c(2,7) +
c(2,8) +
COST(3,6)
COST(3,7)
COST(3,8)
K=6
7
c(3,6) +
c(3,7) +
COST(3,6)
COST(3,7)

2) Travelling Salesperson Problem (TSP)

"Tour" v_0 = home city

⇒ The tour of TSP should start from home city v_0 & visit remaining $(n-1)$ cities exactly once & come back to home city (v_0), S.T, the cost of the Tour is minimum



$G=(V,E)$

$$
\begin{array}{c|cccc}
C & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 10 & 15 & 20 \\
2 & 5 & 0 & 9 & 10 \\
3 & 6 & 13 & 0 & 12 \\
4 & 8 & 8 & 9 & 0
\end{array}
$$

Greedy method $(v_0 = 1)$: $(1-2-3-4-1)$: 39

$(1-2-4-3-1)$: 35

$10+10+9+6$

Brute-force : $(n-1)!$

Let $g(i, S)$ repr. Cost of the Tour of T.S.P from vertex 'i', & visiting all vertices in the set 'S' exactly once and terminating the tour at V_0;

$V_0 = 1$

$$g(1, \underbrace{\{2,3,4\}}_{S}) = \min_{\substack{k \in S \\ \langle 1,k \rangle \in E}} \left\{ c(1,k) + g(k, S - \{k\}) \right\}$$

$$\left\langle \overset{c(1,k)}{1 - k} \sim\sim\sim V_0 \right\rangle$$

$k \in S$

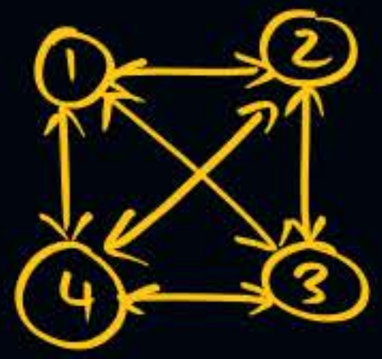
$\langle 1,k \rangle \in E$

$$g(i, S) = \min_{\substack{k \in S \\ \langle i,k \rangle \in E}} \left\{ c(i,k) + g(k, S - \{k\}) \right\} \quad -(1)$$

$$g(i, \phi) = c(i, V_0)$$

$i - V_0$

$J(i, S) \equiv$ Value of 'k' that minimizes Eq (1)



c	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

$v_0 = 1$

$$g(1,\{2,3,4\}) = \min\Big\{ \overset{k=2}{\underbrace{c(1,2)}_{10} + \underbrace{g(2,\{3,4\})}_{25}},\ \overset{k=3}{\underbrace{c(1,3)}_{15} + \underbrace{g(3,\{2,4\})}_{25}},\ \overset{k=4}{\underbrace{c(1,4)}_{20} + \underbrace{g(4,\{2,3\})}_{23}} \Big\} = 35$$

$|S| = 3$

$J(1,\{2,3,4\}) = 2$

$|S| = 0$

$g(2,\emptyset) = c(2,1) = 5$

$g(3,\emptyset) = c(3,1) = 6$

$g(4,\emptyset) = c(4,1) = 8$

$|S| = 1$

$g(2,\{3\}) = c(2,3) + g(3,\emptyset) = 9 + 6 = 15$

$g(2,\{4\}) = 10 + 8 = 18$

$g(3,\{2\}) = 13 + 5 = 18$

$g(3,\{4\}) = 12 + 8 = 20$

$g(4,\{2\}) = 8 + 5 = 13$

$g(4,\{3\}) = 9 + 6 = 15$

$|S| = 2$

$g(2,\{3,4\}) = 25$

$J(2,\{3,4\}) = 4$

$g(3,\{2,4\}) = 25$

$J(3,\{2,4\}) = 4$

$g(4,\{2,3\}) = 23$

$J(4,\{2,3\}) = 2$

$$g(2,\{3,4\}) = \min\Big\{ \overset{k=3}{\underbrace{c(2,3) + g(3,\{4\})}_{9+20}},\ \overset{k=4}{\underbrace{c(2,4) + g(4,\{3\})}_{10+15}} \Big\} = 25$$

$J(2,\{3,4\}) = 4$

Tour-Construction :

$v_0 = 1$

$$\boxed{1 - 2 - 4 - 3 - 1} = 35$$

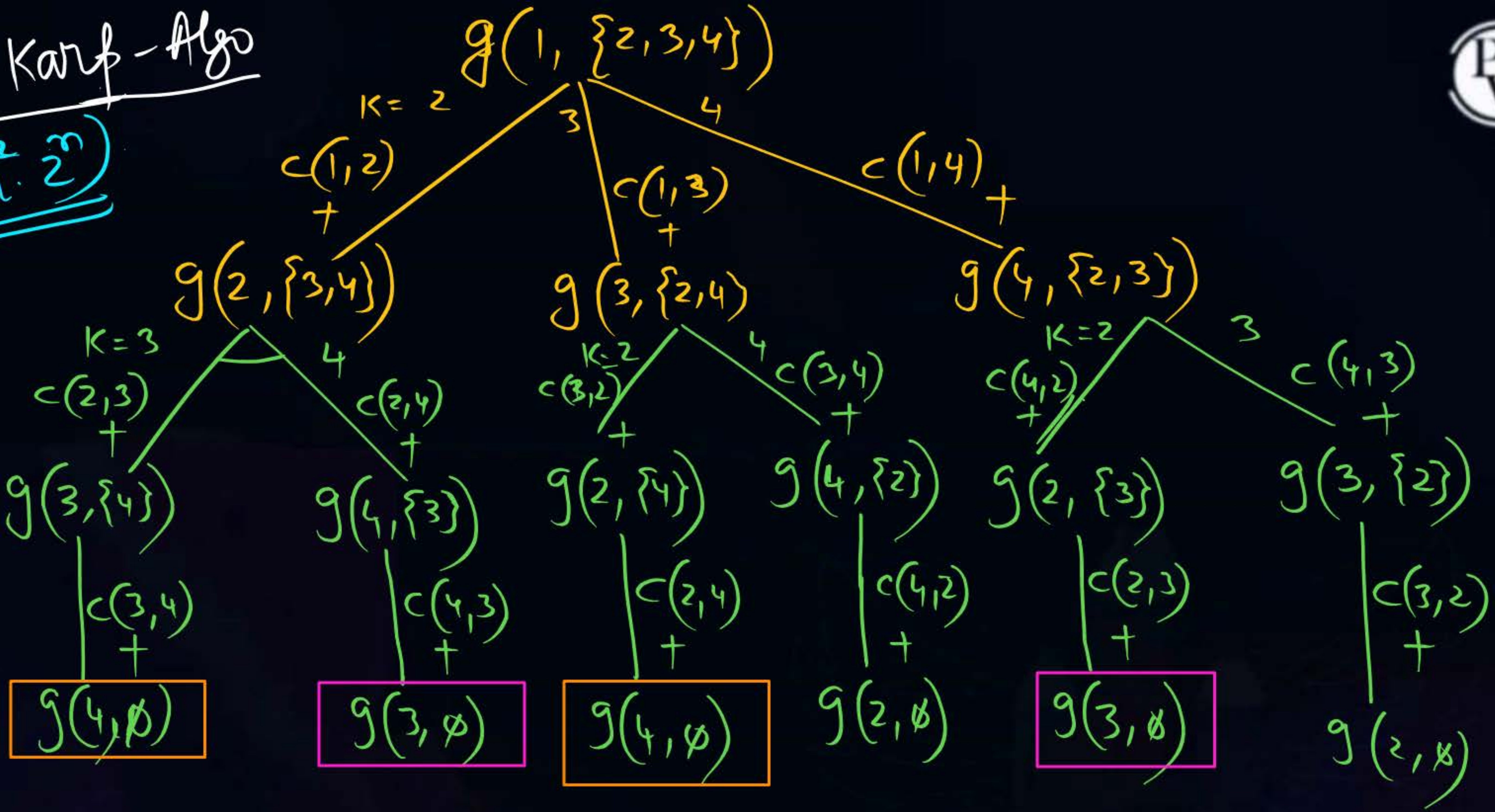
$J(1, \{2,3,4\}) = 2$

$J(2, \{3,4\}) = 4$

$\underline{\underline{v_0 = 4}}$ ✓ H/W

Held-Karp-Algo

$O(n^2 \cdot 2^n)$



(T.S.P is one of the Problem, for which there is Polynomial Time Algo. in the Literature)

3) All-Pairs Shortest Paths (Flyod-warshall's Algorithms)

(Solvable by Greedy Method also)

1

Single Src. Sht. Paths (Dijkstra's Algo)

(Apply it n-times, changing Src vertex each Time)

2

Flyod-warshall's Algo (D.P)

i K j

THANK - YOU