ENGINEERING MATHEMATICS

ALL BRANCHES

Differential Equation
Introduction & formation of DE
DPP-01 Solution

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Question - 01

The differential $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\sin y=0$ is

A linear ✓

B non-linear

C homogenous

D of degree two ✗

Order = 2

- Degree = 1 ✓
- y' ✓
- $y \times$ D.C. ✓
- D.C.$'$ ✓

$\}$ linear

Question - 02

The necessary and sufficient condition for the differential equation of the form $M(x, y)\,dx + N(x, y)\,dy = 0$ to be exact is

A $M = N$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

B $\dfrac{\partial M}{\partial x} = \dfrac{\partial N}{\partial y}$

C $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

D $\dfrac{\partial^2 M}{\partial x^2} = \dfrac{\partial^2 N}{\partial y^2}$

Question - 03

Match each of the items A, B, C with an appropriate item from 1, 2, 3, 4 and 5

a $a_1\frac{d^2y}{dx^2}+a_2y\frac{dy}{dx}+a_3y=a_4$ — 1

b $a_1\frac{d^3y}{dx^3}+a_2y=a_3$ — 2

c $a_1\frac{d^3y}{dx^3}+a_2x\frac{dy}{dx}+a_3x^2y=0$ — 3

1 non-linear differential equation

2 linear differential equation with constant coefficients

3 linear differential equation

4 non-linear homogeneous differential equation

5 non-linear first order differential equation

A a-1, b-2, c-3

B a-3, b-4, c-2

C a-2, b-4, c-3

D a-3, b-1, c-2

Question - 04

The differential equation $y'' + (y^3 \sin x)^5 y' + y = \cos x^3$ is

Order = 2

Degree = 1

A homogeneous

B non-linear (y^{power} ; power > 1)

C second order linear

D non-homogeneous with constant coefficients

Question - 05

Biotransformation of an organic compound having concentration (x) can be modeled using an ordinary differential equation $\frac{dx}{dt} + kx^2 = 0$, where k is the reaction rate constant. If $x = a$ at $t = 0$, the solution of the equation is

A $x = ae^{-kt}$

B $\frac{1}{x} = \frac{1}{a} + kt$

C $x = a\left(1 - e^{-kt}\right)$

D $x = a + kt$

$$\frac{dx}{dt} + Kx^2 = 0$$

$$\int \frac{dx}{x^2} = -\int K\,dt$$

$$-\frac{1}{x} = -Kt + c$$

At $t = 0$, $x = a$

$$-\frac{1}{a} = -K(0) + c$$

$$\boxed{c = -\frac{1}{a}}$$

$$-\frac{1}{x} = -Kt - \frac{1}{a}$$

$$\frac{1}{x} = \frac{1}{a} + Kt$$

Question - 06

The following differential equation has $3\left(\frac{d^2y}{dt^2}\right)+4\left(\frac{dy}{dt}\right)^3+y^2+2=x$

Order = 2

Degree = 1

A degree = 2, order = 1

B degree = 1, order = 2

C degree = 4, order = 3

D degree = 2, order = 3

Question - 07

The equation of the curve, for which the angle between the tangent and the radius vector is twice the vectorial angle is $r^2 = A\sin 2\theta$. This satisfies the differential equation

A $r\dfrac{dr}{d\theta} = \tan 2\theta$

B $r\dfrac{d\theta}{dr} = \tan 2\theta$

C $r\dfrac{dr}{d\theta} = \cos 2\theta$

D $r\dfrac{d\theta}{dr} = \cos 2\theta$

$r^2 = A\sin 2\theta \quad -1)$

$\cancel{2}r\dfrac{dr}{d\theta} = \cancel{2}A\cos 2\theta \quad -2)$

$r\dfrac{d\theta}{dr} = \tan 2\theta \quad 1/2$

Question - 08

The differential equation of the family of circles of radius r whose center lies on the x-axis is

A $y\frac{dy}{dx}+y^2+r^2$

B $y\left(\frac{dy}{dx}+1\right)=r^2$

C $y^2\left[\left(\frac{dy}{dx}\right)+1\right]=r^2$

D $y^2\left[\left(\frac{dy}{dx}\right)^2+1\right]=r^2$

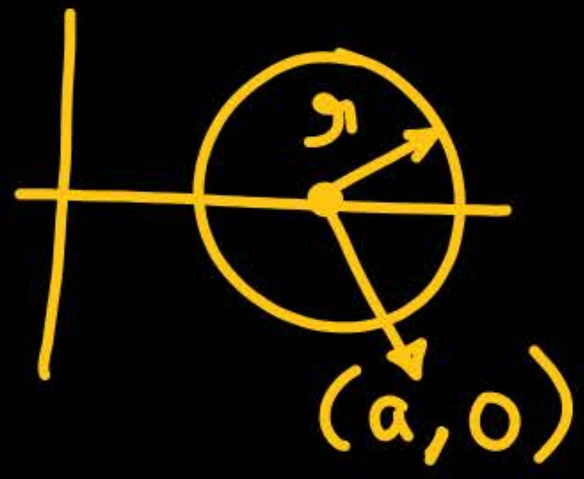
$$(x-a)^2+y^2=r^2$$

$$2(x-a)+2y\frac{dy}{dx}=0$$

$$(x-a)=-y\frac{dy}{dx}$$

$$\left(-y\frac{dy}{dx}\right)^2+y^2=r^2$$

$$y^2\left[\left(\frac{dy}{dx}\right)^2+1\right]=r^2$$



Question - 09

If $x = A\cos(mt - \alpha)$, then the differential equation satisfying this relation is

A $\frac{dx}{dt} = 1 - x^2$

B $\frac{d^2x}{dt^2} = -\alpha^2 x$

C $\frac{d^2x}{dt^2} = -m^2 x$

D $\frac{dx}{dy} = -m^2 x$

$$x = A\cos(mt - \alpha)$$

$$\frac{dx}{dt} = -mA\sin(mt - \alpha)$$

$$\frac{d^2x}{dt^2} = -m^2 \underbrace{A\cos(mt - \alpha)}$$

$$\frac{d^2x}{dt^2} = -m^2 x$$

Question - 10

The solution of the differential equation $2x\frac{dy}{dx}=2-y$

A $y=2-\sqrt{\frac{c}{x}}$

B $y=2+\sqrt{\frac{c}{x}}$

C $y=2-c\sqrt{x}$

D $y=2+c\sqrt{x}$

$$2\frac{dy}{2-y}=\frac{dx}{x}$$

$$-2\ln(2-y)=\ln x-\ln c$$

$$\ln(2-y)^{-2}+\ln c=\ln x$$

$$c(2-y)^{-2}=x$$

$$\frac{c}{x}=(2-y)^2$$

$$y=2-\sqrt{\frac{c}{x}}$$

Thank you

GW Soldiers !