CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 5

By- SATISH YADAV SIR

TOPICS

01 Binomial coefficient

02 Extended Binomial coefficient

3 Exercise

$$(a+b)^2 = a^2 + 2ab + b^2.$$

$$= \underbrace{1}\cdot a^2 + \underbrace{2}ab + \underbrace{1}\cdot b^2$$

$$(a+b)\times(a+b)$$

$a \times a$

$$(a+b)\times(a+b)$$

$\underbrace{a\times b} + \underbrace{b \times a}.$

$2ab.$

$(a+b)$

$\downarrow$ OR $\downarrow$

$a.$ $b.$

$1.a^2 = 1$ way

we can take.

a^2 out.

$(a+b)\times(a+b)$

$2(ab)$

2 ways we can take $\underline{ab}$ out

$\underline{b\times a}$

$(a+b)\times(a+b)$

$\underline{a\times b}$

2 boxes $(a+b)$

how many ways we can take 1 b out from 2 boxes

$(\ \ +b)\times(\ \ +b)$

${}^2C_1\, ab$

$(a+b)^2 = a^2 + 2ab + b^2$

$= 1 \cdot a^2 + 2ab + 1 \cdot b^2$

out of 2 boxes

$= {}^2C_0\, a^2 \cdot b^0 + {}^2C_1\, ab + {}^2C_2\, a^0 b^2$ how many ways we take 2 b

at a same time.

2 boxes

1 b out.

2 boxes

2 b's out.

$(a+b) \times (a+b)$

$b \times b$

${}^2C_2$

$^nC_x = {^nC_{n-x}}$.

$\{a, b, c\} \to$ no. of ways to take 1 element out $=$ no. of ways we left 2 element inside.

$\{a\}$ $\{bc\}$ $= {^3C_1}$.

$\{$

$$(a+b)^2 = 1a^2 + 2ab + 1.b^2.$$

$$= {}^2C_0\, a^2 b^0 + {}^2C_1\, a^1 b + {}^2C_2\, a^0 b^2.$$

* power of a is decreasing.
* power of b is increasing.

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

$$= {}^3C_0\, a^3 b^0 + {}^3C_1\, a^2 b^1 + {}^3C_2\, a^1 b^2 + {}^3C_3\, a^0 b^3.$$

* power of a and b will always be n.

$(a+b)^n$.

$\downarrow$ $b\uparrow$ $(a+b)\times(a+b)\times(a+b)$

$$(a+b)^4 = {}^4C_0\, a^4 b^0 + {}^4C_1\, a^3 b^1 + {}^4C_2\, a^2 b^2 + {}^4C_3\, a b^3 + {}^4C_4\, a^0 b^4.$$

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 \ldots\ldots {}^nC_n\, a^0 b^n$$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \times a^{n-i} \times b^i$$

${}^nC_i \rightarrow$ binomial coefficient. $\rightarrow$ no. of ways, we can take b^i outside in (n) boxes.

→ How many ways we can select 6 students in a class of 10 ?

S1

select OR. not selected.

$(b \quad \overset{OR}{+} \quad a)$

S2

Select not (select)

$^nC_i\, a^{n-i}\, b^i$

$^{10}C_6\, a^4\, b^6$

S10

Ans : $^{10}C_6$.

Selecting 6 students.
it means 6'b out of 10 boxes

$(a+b) \times (a+b) \times (a+b) \cdots (a+b)$

$b \times b \times b$

→ finding coefficient of b^6.

→ How many ways we can select 6 students in a class of 10 ?

→ Ans : ${}^{10}C_6$.

S_1 → Select OR not selected

S_2 → Select, not (select)

S_{10}

${}^{n}C_i \, a^{n-i} b^i$

${}^{10}C_6 \, a^4 b^6$

$$(x^1 + x^0) \times (x^1 + x^0) \dots$$

$$(1+x) \times (1+x) \cdots (1+x) = (1+x)^{10}$$

x^6

Selecting 6 students it means 6'b out of 10 boxes

→ finding coefficient of — b^6

$$(a+b) \times (a+b) \times (a+b) \cdots (a+b)$$

$$(a+b)^{10}$$

Selecting 6 student.

coefficient of b^6.

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \, a^{n-i} \, b^i$$

$${}^{10}C_6 \, a^4 \cdot b^6.$$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \cdot a^{n-i} \cdot b^i$$

$a=1 \quad b=1.$

$$(1+1)^n = \sum_{i=0}^{n} {}^nC_i \,(1)^{n-i}(1)^i$$

$$2^n = \sum_{i=0}^{n} {}^nC_i = \boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 \cdots {}^nC_n = 2^n}$$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \cdot a^{n-i} \cdot b^i$$

$a = 1 \quad b = -1.$

$$(1+(-1))^n = \sum_{i=0}^{n} {}^nC_i \, (1)^{n-i} \, (-1)^i$$

$$0 = \sum_{i=0}^{n} {}^nC_i \, 1 \cdot (-1)^i = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 \cdots$$

$$\boxed{{}^nC_0 + {}^nC_2 + {}^nC_4 \cdots = {}^nC_1 + {}^nC_3 + {}^nC_5}$$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \cdot a^{n-i} \cdot b^i$$

$a = 1 \quad b = 2.$

$$(1+2)^n = \sum_{i=0}^{n} {}^nC_i \cdot (1)^{n-i} \cdot (2)^i$$

$$3^n = \sum_{i=0}^{n} {}^nC_i \cdot 2^i = 2^0\, {}^nC_0 + 2^1\, {}^nC_1 + 2^2 \cdot {}^nC_2 \cdots 2^n \cdot {}^nC_n = 3^n.$$

$^3C_1 a^2 b^1$ $\quad$ $^3C_2 a^1 b^2$ $\quad$ $^3C_3 a^0 b^3$

$(a+b)^3 =$ $\quad 1\left(^3C_0\right) \quad 3\left(^3C_1\right) \quad 3\left(^3C_2\right) \quad 1\left(^3C_3\right)$

$(a+b)^4$ $\quad 1 \; ^4C_0 \quad 4\left(^4C_1\right) \quad 6\left(^4C_2\right) \quad 4\left(^4C_3\right) \quad 1\left(^4C_4\right)$

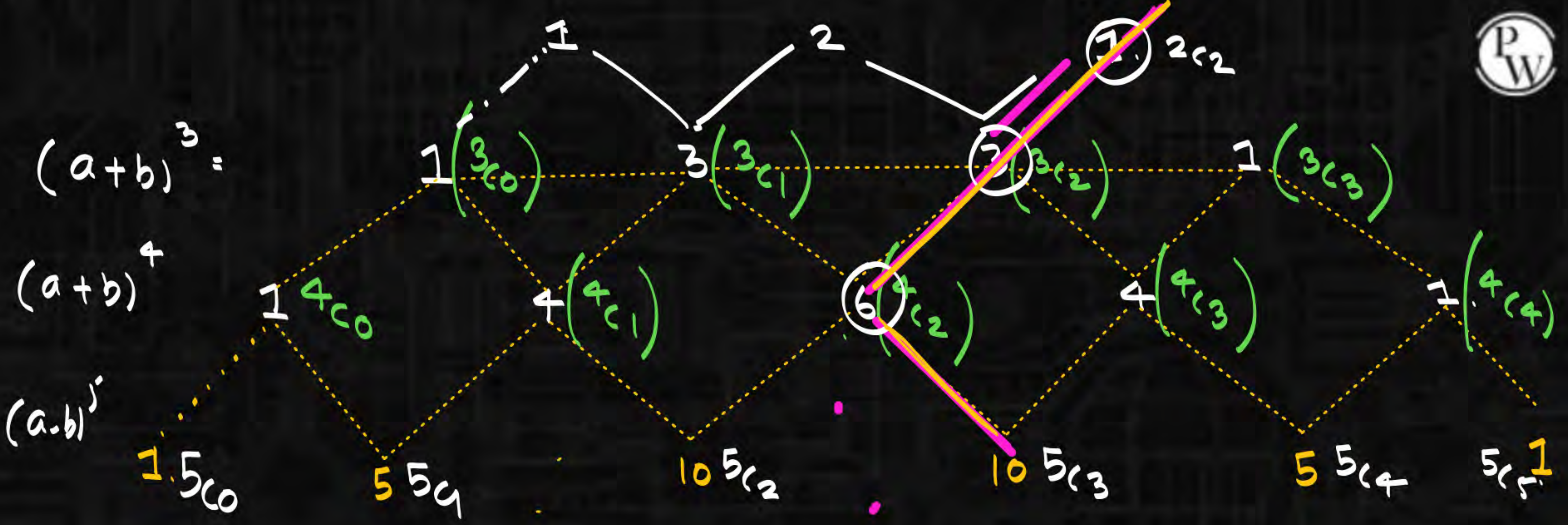
$(a-b)^5$

$+1 \quad -5 \quad +10 \quad -10 \quad +5 \quad -1$

$$^3C_1 + {}^3C_2 = {}^4C_2$$

$$^nC_x + {}^nC_{x+1} = {}^{n+1}C_{x+1}$$

for − use alterna +/−



$$1 + 3 + 6 = 10$$

$$\underline{{}^2C_2} + \underline{{}^3C_2} + \underline{{}^4C_2} = {}^5C_3$$

$(x+y)^{30}$ coefficient of $y^{10} = {}^{30}C_{10} = {}^{30}C_{20}$

${}^{30}C_{10}\, x^{20} y^{10}$

$(2x+3y)^{30}$ coefficient of $y^{10} = \underbrace{{}^{30}C_{10}\, 2^{20} \cdot 3^{10}}$

${}^{30}C_{10}\, (2x)^{20} (3y)^{10} = \underbrace{{}^{30}C_{10}}\, 2^{20} \cdot 3^{10}\, x^{20} \cdot y^{10}$

Extended binomial coefficient :

$$^{11}C_2 = \frac{(11)!}{2! \times 9!} = \frac{11 \times 10 \times \cancel{9!}}{2! \times \cancel{9!}} = \frac{11 \times 10}{2 \times 1} = {}^{11}C_2 = \frac{11 \cdot 10}{2 \cdot 1}$$

$$^{11}C_3 = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1}$$

k^{th} places.

$$^{n}C_{K} = \frac{n\cdot(n-1)\cdot(n-2)\cdots(n-k+1)}{K!}$$

$$^{11}C_{3} = \frac{11\cdot(11-1)(11-2)}{3!}$$

$$^{-n}C_{K} = \frac{(-n)(-n-1)(-n-2)\cdots(-n-k+1)}{K!}$$

$$= \frac{(-n-k+1)\cdots(-n-2)(-n-1)(-n)}{K!} = (-1)^{K}\frac{(n+k-1)\cdots(n+2)(n+1)(n)}{K!}$$

take -1 common.

$$^{-n}C_K = \frac{(-1)^K (n+K-1)\cdots(n+2)(n+1)(n)}{K!} \cdot \frac{(n-1)!}{(n-1)!}$$

$\left\{ \div \text{ by } N/D \text{ by } (n-1)! \right\}$

$$= (-1)^K \frac{(n+K-1)!}{K! \times (n-1)!}$$

**

$$\boxed{^{-n}C_K = (-1)^K \cdot {}^{n+K-1}C_K}$$

$$\rightarrow (-1)^K \frac{(n+K-1)!}{K! \times (n+\cancel{K}-1-\cancel{K})!} = (-1)^K \frac{(n+K-1)!}{K! \cdot (n-1)!}$$

$-{}^{11}C_2 = {}^{12}C_2$ | $-{}^{11}C_3 = -{}^{13}C_3$ | $-{}^{k}C_0 = 1$ | $-{}^{1}C_1 = -1$

$-{}^{11}C_2 = -{}^{n}C_k$ → k even (Total +ve), odd (Total -ve)

$n = 11 \quad k = 2$

$= (-1)^k \; {}^{n+k-1}C_k$

$= (-1)^2 \; {}^{11+2-1}C_2$

$= {}^{12}C_2$

Express each of the sums in closed form

$$\sum_{k=0}^{n}\binom{n}{k}5^k$$

$$\sum_{i=0}^{n}\binom{n}{i}x^i$$

$$\sum_{j=0}^{2n}(-1)^j\binom{2n}{j}x^j$$

$$\sum_{i=0}^{m}\binom{m}{i}p^{m-i}q^{2i}$$

$$\sum_{i=0}^{m}(-1)^i\binom{m}{i}\frac{1}{2^i}$$

$$\sum_{i=0}^{n}(-1)^i\binom{n}{i}5^{n-i}2^i$$

$$\sum_{k=0}^{n}\binom{n}{k}5^k=\sum_{k=0}^{n}\binom{n}{k}1^{n-k}5^k=(1+5)^n=6^n$$

$$\sum_{i=0}^{n}\binom{n}{i}x^i=\sum_{i=0}^{n}\binom{n}{i}1^{n-i}x^i=(1+x)^n$$

$$\sum_{j=0}^{2n}(-1)^j\binom{2n}{j}x^j=\sum_{j=0}^{2n}\binom{2n}{j}1^{2n-j}(-x)^j=(1-x)^{2n}$$

$$\sum_{i=0}^{m}(-1)^i\binom{m}{i}\frac{1}{2^i}=\sum_{i=0}^{m}\binom{m}{i}1^{m-i}\left(-\frac{1}{2}\right)^i$$

$$=\left(1-\frac{1}{2}\right)^m=\frac{1}{2^m}$$

$$\sum_{i=0}^{n}(-1)^i\binom{n}{i}5^{n-i}2^i=\sum_{i=0}^{n}\binom{n}{i}5^{n-i}(-2)^i=(5-2)^n=3^n$$

Find the coefficient of x^{16} in the expansion of $\left(2x^2 - \frac{x}{2}\right)^{12}$.

$$\binom{12}{k}(2x^2)^{12-k}\left(-\frac{x}{2}\right)^k = \binom{12}{k}2^{12-k}\left(-\frac{1}{2}\right)^k x^{24-k}.$$

We want $24 - k = 16$; thus, $k = 8$. The coefficient is $\binom{12}{8}2^4\left(-\frac{1}{2}\right)^8 = \frac{1}{16}\binom{12}{8}$
$\frac{495}{16}$.


THANK YOU GW
SOLDIERS !