CS & IT ENGINEERING

Discrete maths

Mathematical Logic

Lecture No. 05

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TOPICS TO BE COVERED

01 Theorems on Quantifier

02 English statement to Logic Conversion

03 Problems on Quantifier

04 Type 4

05 Nested Quantifeir

1) $\forall x\,[P(x) \wedge Q(x)] \equiv \forall x P(x) \wedge \forall x Q(x)$ $\quad D: \{1, 2\}$.

$$\boxed{P(1) \wedge Q(1)} \wedge \boxed{P(2) \wedge Q(2)} \qquad \begin{pmatrix} P_1 \\ \wedge \\ P_2 \end{pmatrix} \wedge \begin{pmatrix} P_2 \\ \wedge \\ Q_2 \end{pmatrix}$$

$P(1) \wedge Q(1) \wedge P(2) \wedge Q(2) \qquad P(1) \wedge Q(1) \wedge P(2) \wedge Q(2)$

$$\forall x\,[p(x) \wedge Q(x)] \equiv \forall x\,p(x) \wedge \forall x\,Q(x).$$

$$* \quad \forall x\,[p(x) \vee Q(x)] \leftarrow \forall x\,p(x) \vee \forall x\,Q(x)$$

$$\exists x\,[p(x) \vee Q(x)] \equiv \exists x\,p(x) \vee \exists x\,Q(x)$$

$$\exists x\,[p(x) \wedge Q(x)] \rightarrow \exists x\,p(x) \wedge \exists x\,Q(x)$$

$$\forall x\,[p(x) \rightarrow Q(x)] \rightarrow \left(\forall x\,p(x) \rightarrow \forall x\,Q(x)\right)$$

$$\forall x\,[p(x) \Leftrightarrow Q(x)] \rightarrow \left(\forall x\,p(x) \Leftrightarrow \forall x\,Q(x)\right)$$

Case 1 : $\overbrace{\forall x\,[P(x) \vee Q(x)]}^{T} \not\Rightarrow \overbrace{\forall x P(x) \vee \forall x Q(x)}^{\text{false}}$ valid?

$$T \left\{ \begin{array}{c} \boxed{T \;\vee\; F} \; T \\ \wedge \\ \boxed{F \;\vee\; T} \; T \end{array} \right.$$

$$\boxed{\begin{array}{c} T \\ \wedge \\ F \end{array}} \;\vee\; \boxed{\begin{array}{c} F \\ \wedge \\ T \end{array}}$$

$$\underbrace{F \;\vee\; F}_{F}$$

$$\overbrace{\forall x P(x) \lor \forall x Q(x)}^{T} \rightarrow \overbrace{\forall x [P(x) \lor Q(x)]}^{T}$$

$$\boxed{\begin{matrix} T \\ \land \\ T \end{matrix}} \quad \lor \quad \boxed{\begin{matrix} \\ \land \\ \\ \end{matrix}}$$

$$\frac{T \lor F}{T}$$

$$\boxed{T \quad \lor \qquad\qquad} \quad T$$

$$\land$$

$$\boxed{T \quad \lor \qquad\qquad} \quad T$$

1. Let $p(x)$, $q(x)$ denote the following open statements.

$$p(x): \quad x \leq 3 \qquad\qquad q(x): \quad x + 1 \text{ is odd}$$

If the universe consists of all integers, what are the truth values of the following statements? Except all are false.

a) $q(1)$ **b)** $\neg p(3)$ **c)** $p(7) \vee q(7)$

(True) **d)** $p(3) \wedge q(4)$ **e)** $\neg(p(-4) \vee q(-3))$

f) $\neg p(-4) \wedge \neg q(-3)$

H.W

2. Let $p(x)$, $q(x)$ be defined as in Exercise 1. Let $r(x)$ be the open statement "$x > 0$." Once again the universe comprises all integers.

a) Determine the truth values of the following statements.

i) $p(3) \vee [q(3) \vee \neg r(3)]$

ii) $p(2) \rightarrow [q(2) \rightarrow r(2)]$

iii) $[p(2) \wedge q(2)] \rightarrow r(2)$

iv) $p(0) \rightarrow [\neg q(-1) \leftrightarrow r(1)]$

1. a $q(x)$: $x+1$ is odd.

$q(1)$: $1+1$ is odd.

2 is odd (F)

b) $\neg p(3)$ $\neg(T) \equiv F$

$p(x): x \leq 3$

$p(3): 3 \leq 3$ (T)

Negate and simplify each of the following.

a) $\exists x\,[p(x) \vee q(x)]$

b) $\forall x\,[p(x) \wedge \neg q(x)]$

c) $\forall x\,[p(x) \rightarrow q(x)]$

d) $\exists x\,[(p(x) \vee q(x)) \rightarrow r(x)]$

d) $\neg \exists x\,[p(x) \vee q(x) \rightarrow R(x)]$

$\neg \exists x\,[\neg (p(x) \vee q(x)) \vee R(x)]$

$\forall x\,[(p(x) \vee q(x)) \wedge \neg R(x)]$

A) $\neg \exists x\,[p(x) \vee q(x)]$

$\forall x\, \neg [p(x) \vee q(x)]$

$\forall x\,[\neg p(x) \wedge \neg Q(x)]$

b) $\exists x\,[\neg p(x) \vee q(x)]$

OR

$\exists x\,[p(x) \rightarrow q(x)]$

$p(x)$: $x^2 - 7x + 10 = 0$ $(x = 2, 5)$ D: Z

$q(x)$: $x^2 - 2x - 3 = 0$ $(x = 3, -1)$

$r(x)$: $x < 0$ (-ve) $\neg R(x): x \geq 0$

$R(x): x < 0$

$\underbrace{R(2)}_{F}: 2 < 0$

$\neg R(2): \neg(F) = T$

i) $\forall x\, [p(x) \rightarrow \neg r(x)]$ ii) $\forall x\, [q(x) \rightarrow r(x)]$

iii) $\exists x\, [q(x) \rightarrow r(x)]$ iv) $\exists x\, [p(x) \rightarrow r(x)]$

1) $\forall x\, [P(x) \rightarrow \underbrace{\neg R(x)}_{T}]$

$x = 1$ $\boxed{\underbrace{P(1)}_{F} \rightarrow \neg R(1)}$ T

$\wedge$

$\boxed{\underbrace{P(2)}_{(T)} \rightarrow \neg R(2)\ (T)}$ T

$\wedge$

$\boxed{P(3) \rightarrow \neg R(3)}$ T

$p(x)$: $x^2 - 7x + 10 = 0$ $(x = 2, 5)$

$q(x)$: $x^2 - 2x - 3 = 0$ $(x = 3, -1)$

$r(x)$: $x < 0$ (-ve) $\neg R(x): x \geq 0$

$\neg R(2): 2 \geq 0$

T → ∀ (T)

i) $\forall x\,[p(x) \rightarrow \neg r(x)]$ ii) $\forall x\,[q(x) \rightarrow r(x)]$

iii) $\exists x\,[q(x) \rightarrow r(x)]$ iv) $\exists x\,[p(x) \rightarrow r(x)]$

$\forall x\,(p(x) \rightarrow \neg R(x)]$

$x=1$: $\boxed{P(1) \rightarrow \neg R(1)}$ T

F → ∧

$x=2$: $\boxed{P(2) \rightarrow \neg R(2)}$ T

T → T ∧

$x=3$: $\boxed{P(3) \rightarrow \neg R(3)}$ T

F → ∧

$x=4$: $\boxed{P(4) \rightarrow \neg R(4)}$ T

F ∧

$x=5$: $\boxed{P(5) \rightarrow \neg R(5)}$ T

T → T

$p(x)$: $x^2 - 7x + 10 = 0$ ($x = 2, 5$)

$q(x)$: $x^2 - 2x - 3 = 0$ ($x = 3, -1$)

$r(x)$: $x < 0$ (-ve) $\neg R(x): x \geq 0$

$\neg p$

$Q: Z$

T

i) $\forall x\,[p(x) \to \neg r(x)]$ ii) $\forall x\,[q(x) \to r(x)]$

iii) $\exists x\,[q(x) \to r(x)]$ iv) $\exists x\,[p(x) \to r(x)]$

$\exists x\,(q(x) \to R(x))$

$x=1$ $\boxed{q(1) \to R(1)}$ T

F $\to$ $\vee$

$\boxed{\quad \to \quad}$

$\vee$

$\boxed{\quad \to \quad}$

$\vee$

$\boxed{\quad \to \quad}$

$\vee$

$\boxed{\quad \to \quad}$

HW:

$p(x)$: $x^2 - 8x + 15 = 0$

$q(x)$: x is odd

$r(x)$: $x > 0$

a) $\forall x\,[p(x) \rightarrow q(x)]$

b) $\forall x\,[q(x) \rightarrow p(x)]$

c) $\exists x\,[p(x) \rightarrow q(x)]$

d) $\exists x\,[q(x) \rightarrow p(x)]$

e) $\exists x\,[r(x) \rightarrow p(x)]$

f) $\forall x\,[\neg q(x) \rightarrow \neg p(x)]$

g) $\exists x\,[p(x) \rightarrow (q(x) \wedge r(x))]$

h) $\forall x\,[(p(x) \vee q(x)) \rightarrow r(x)]$

7. For the universe of all integers, let $p(x)$, $q(x)$, $r(x)$, $s(x)$, and $t(x)$ be the following open statements.

$p(x)$: $x > 0$

$q(x)$: x is even

$r(x)$: x is a perfect square

$s(x)$: x is (exactly) divisible by 4

$t(x)$: x is (exactly) divisible by 5

a) Write the following statements in symbolic form.

i) At least one integer is even.
ii) There exists a positive integer that is even.
iii) If x is even, then x is not divisible by 5.
iv) No even integer is divisible by 5.
v) There exists an even integer divisible by 5.
vi) If x is even and x is a perfect square, then x is divisible by 4.

negation of quantifier:

$$\neg \forall x\, P(x) \equiv \exists x\, \neg P(x)$$

$$\neg \exists x\, P(x) \equiv \forall x\, \neg P(x)$$

Exprssn: $\forall x\,[P(x) \rightarrow Q(x)]$

negate it

$$\neg \forall x\,[P(x) \rightarrow Q(x)]$$

$$\neg \forall x\,[\neg P(x) \vee Q(x)]$$

$$\exists x\, \neg[\neg P(x) \vee Q(x)]$$

$$\exists x\,[\neg(\neg P(x)) \wedge \neg Q(x)]$$

$$\exists x\,[P(x) \wedge \neg Q(x)]$$

English stmt → logical stmt :

All mothers are female.

All of x, x is mother x is female.

$\forall x$ $MT(x)$ $F(x)$

$$\forall x\,[\,MT(x) \quad F(x)\,]$$

$\rightarrow$

$\wedge$

All mothers are female. (True)

Case 1:

$$\forall x\,[MT(x) \wedge F(x)]$$

False

False $\wedge$	F
$\wedge$	
F $\wedge$ T	F
$\wedge$	
T $\wedge$ T	T

Case 2:

$$\forall x\,[MT(x) \rightarrow F(x)]$$

T.

F $\rightarrow$	T
$\wedge$	
F $\rightarrow$	T
$\wedge$	
T $\rightarrow$ T	T.

- BABA
- Single female.
- mother

ALL $\rightarrow$

Some cats are black.

Some of x, x is cat x is black.

$\underbrace{\text{Some of } x}_{\exists x}$, $\underbrace{x \text{ is cat}}_{c(x)}$ $\underbrace{x \text{ is black}}_{bl(x)}$

True

- kat / w
- maggie / bl.
- Dog.

(Some $\wedge$)

1) $\exists x[c(x) \rightarrow bl(x)]$

$\exists x[c(x) \wedge bl(x)]$

$\exists x[c(x) \Leftrightarrow bl(x)]$

T. $\begin{cases} \boxed{T \rightarrow F} & F \\ \vee & \\ \boxed{T \rightarrow T} & T. \\ \vee & \\ \boxed{F \rightarrow } & T \end{cases}$

T. $\begin{cases} \boxed{T \wedge F} & F \\ \vee & \\ \boxed{T \wedge T} & T \\ \vee & \\ \boxed{\wedge} & \end{cases}$

T. $\begin{cases} \boxed{T \Leftrightarrow F} & F \\ \vee & \\ \boxed{T \Leftrightarrow T} & T \\ \vee & \\ \boxed{} & \end{cases}$

Some cats are black.

$$\underbrace{\text{Some of } x}_{\exists x},\ \underbrace{x \text{ is cat}}_{c(x)}\ \underbrace{x \text{ is black}}_{bl(x)}$$

false

kat/w

maggie/w

Dog

$\exists x\,[c(x) \rightarrow bl(x)]$

$T \rightarrow F$	F
$\vee$	
$T \rightarrow F$	F
$\vee$	
$F \rightarrow$	T

T

false.

$\exists x\,[c(x) \wedge bl(x)]$

$T \wedge F$	F
$\vee$	
$T \wedge F$	F
$F \wedge$	F

All of my friends are perfect.

$$\forall x\,[F(x) \rightarrow P(x)]$$

2. not (all of my friends are perf.)

$$\neg \forall x\,[F(x) \rightarrow P(x)]$$

$$\neg \forall x\,[\neg F(x) \vee P(x)]$$

$$\exists x\,[F(x) \wedge \neg P(x)]$$

3. none of my friends are perfect $\equiv$ 4. All of my friends are not perfect.

$$\forall x\,[F(x) \rightarrow \neg P(x)] \qquad \forall x\,[F(x) \rightarrow \neg P(x)]$$

All graphs are connected.

$$\forall x\,[G(x) \to C(x)]$$

not (all graphs are connected)

$$\neg\,[\forall x\,[G(x) \to C(x)]]$$

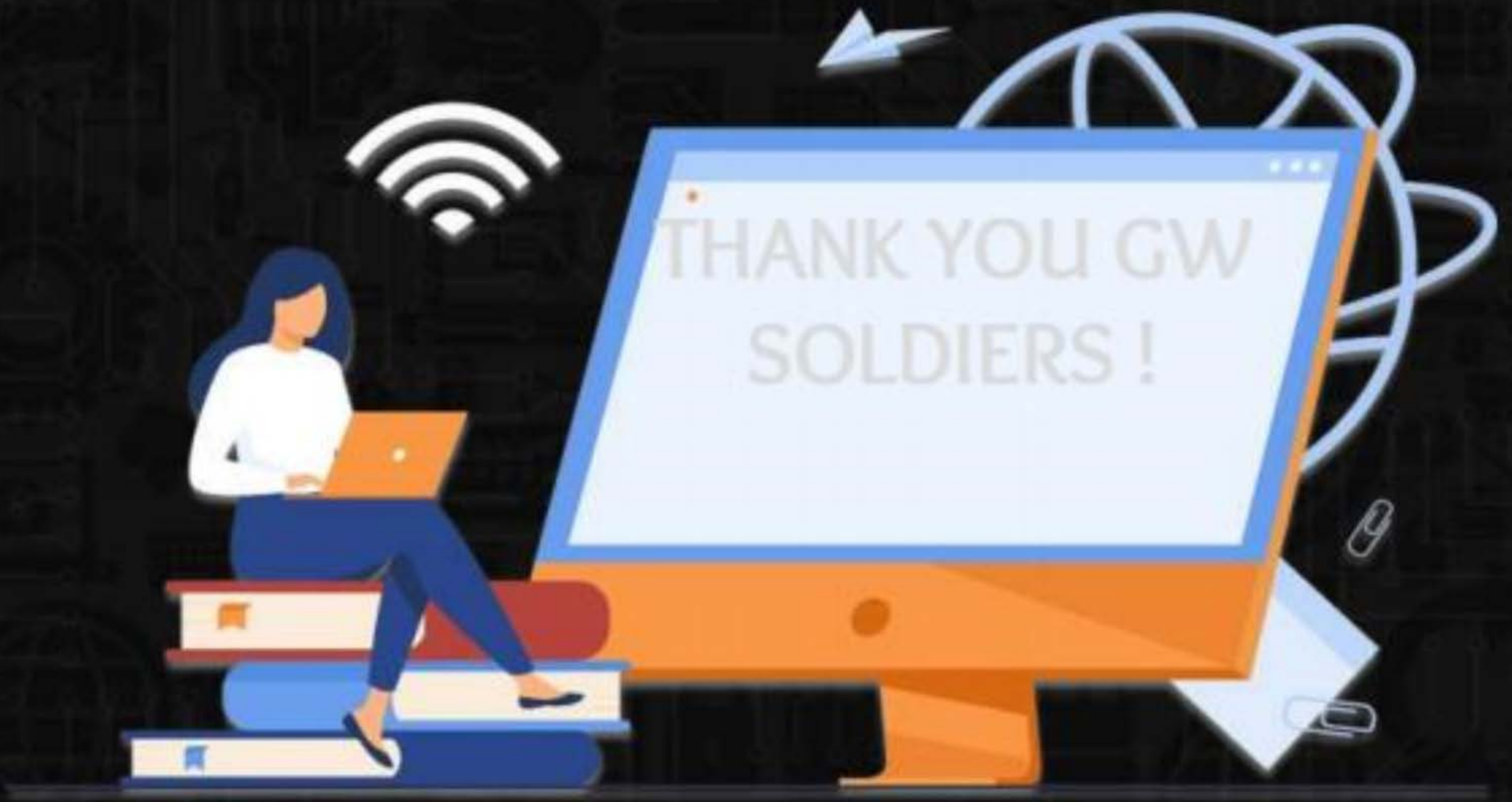
no graphs are connected $\equiv$ all graphs are not connected.

$$\forall x\,[G(x) \to \neg C(x)] \qquad \forall x\,[G(x) \to \neg C(x)]$$

not all $\neg \forall x [\quad \rightarrow \quad]$

no/none $\forall x [\quad \rightarrow \neg \quad]$

GATE


THANK YOU GW
SOLDIERS !