ENGINEERING MATHEMATICS

ALL BRANCHES

Probability
Probability Distribution
Concepts

DPP-07 Solution

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Question 1

For a random variable $x(-\infty < x < \infty)$ following normal distribution, the mean is $\mu = 100$. If the probability is $P = \alpha$ for $x \geq 110$. Then the probability of x lying between 90 and 110 i.e $P(90 \leq x \leq 110)$ and equal to

A $1 - 2\alpha$

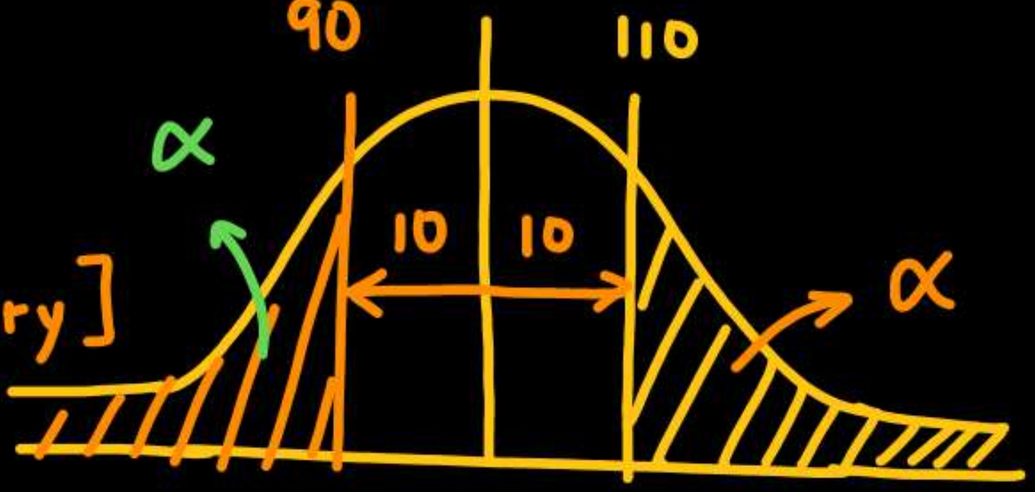
B $1 - \alpha$

C $1 - \alpha/2$

D 2α

$P(x \geq 110) = \alpha$

$\Rightarrow P(x \leq 90) = \alpha$ [Using symmetry]



$P(90 \leq x \leq 110) = 1 - P(x \leq 90) - P(x \geq 110)$ $\mu = 100$

$1 - \alpha - \alpha$

$= 1 - 2\alpha$

Question 2

Let X be a random variable following Normal **distribution** with mean + 1 and variance 4. Let Y be another normal variable with mean –1 and variance unknown.

If $P(X \le -1) = P(Y \ge 2)$. The S.D. of Y is __3__.

$X \rightarrow$ Normal C.R.V $\boxed{\mu_x = +1}$, $Var(X) = \sigma_x^2 = 4 \Rightarrow \boxed{\sigma_x = 2}$

$Y \rightarrow$ Normal C.R.V. $\boxed{\mu_y = -1}$, $Var(Y) = \sigma_y^2$; $\sigma_y = ?$

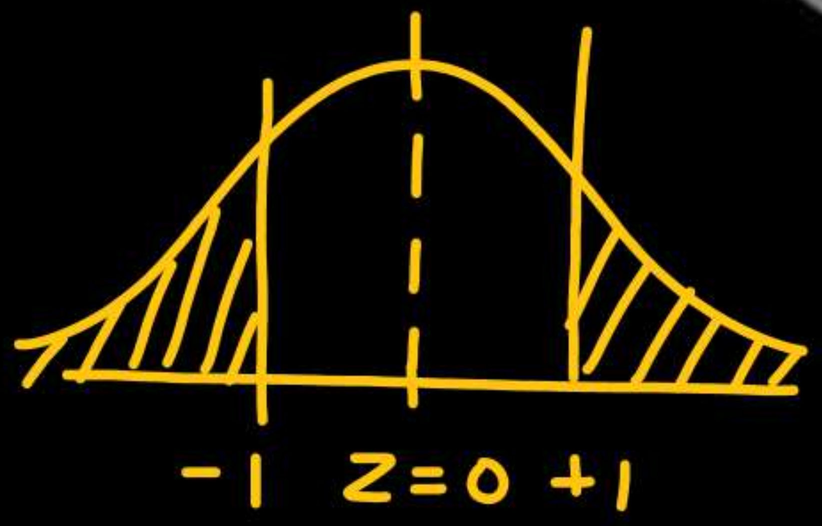
$$Z_x = \frac{X - \mu_x}{\sigma_x} \qquad P(X \le -1) = P(Y \ge 2)$$

$$Z_y = \frac{Y - \mu_y}{\sigma_y} \qquad P\left(Z_x \le \frac{-1-1}{2}\right) = P\left(Z_y \ge \frac{2-(-1)}{\sigma_y}\right)$$

$$P\left(Z_x \le -1\right) = P\left(Z_y \ge \frac{3}{\sigma_y}\right)$$

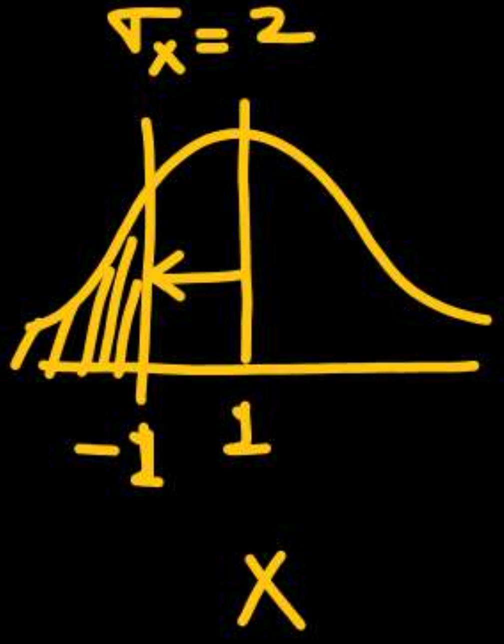
$$\therefore \frac{3}{\sigma_y} = 1$$

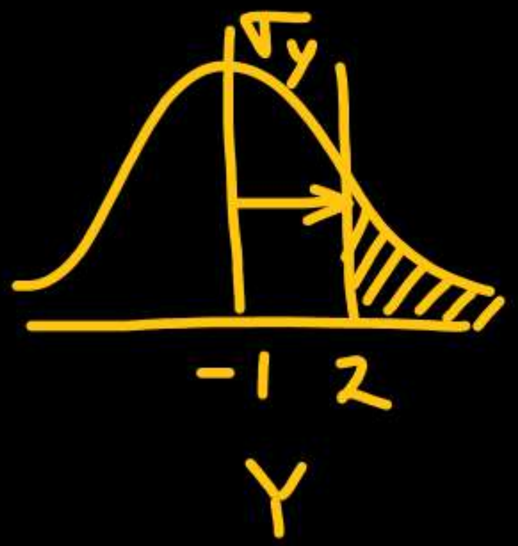
$$\sigma_y = 3$$



$$P(X \le -1) = P(Y \ge 2)$$

$$P(Z \le -a) = P(Z \ge a)$$





$$\therefore \sigma_y = 3$$

Question 3

A continuous random variable X has a probability density function $f(x) = e^{-x}, (0 < x < \infty)$ Then $P\{X > 1\}$ is

$$f(x) = e^{-x} \; ; \; 0 < x < \infty$$

A 0.368

$$P(X > 1) = \int_1^\infty f(x)\,dx = \int_1^\infty e^{-x}\,dx$$

B 0.5

C 0.632

$$= -\left[e^{-x}\right]_1^\infty = -\left[0 - e^{-1}\right]$$

$$= e^{-1}$$

$$= \frac{1}{e} = 0.368$$

D 1.0

Question 4

Which one of the following statements is not true?

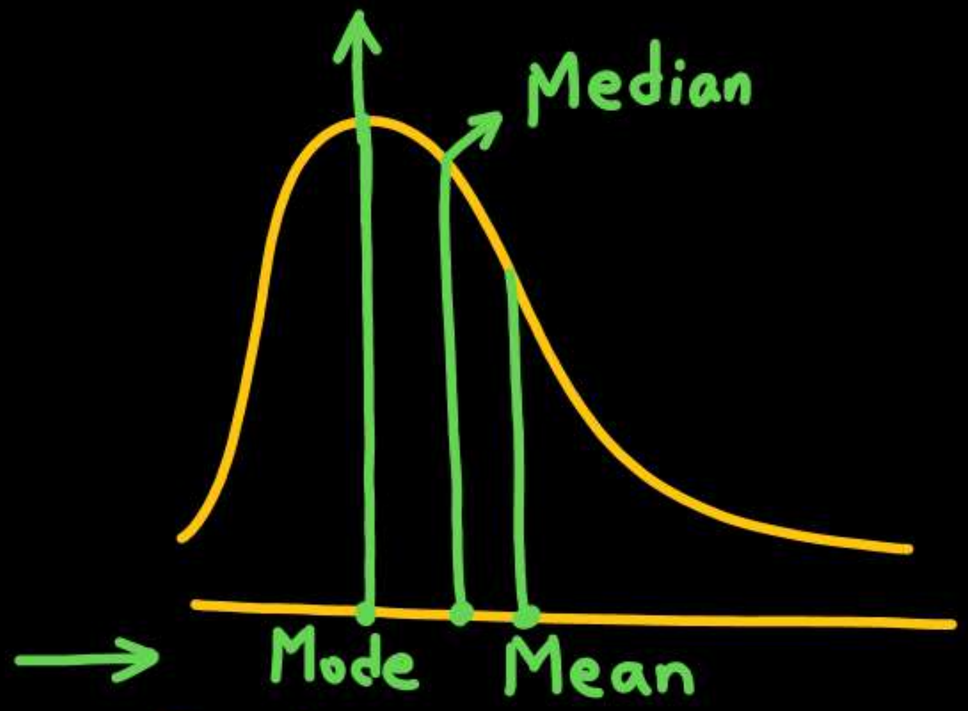
A. The measure of skewness is dependent upon the amount of dispersion

B. In a symmetric distribution, the values of mean, mode and median are the same

C. In a positively skewed distribution, mean > median > mode

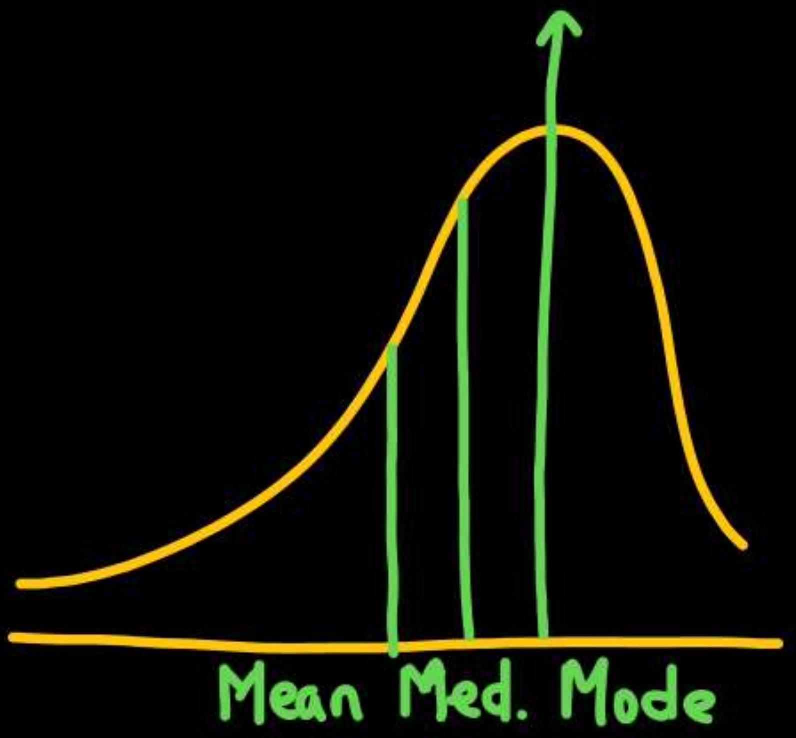
D. In a negatively skewed distribution, mode > mean > median

Mode > Median > Mean



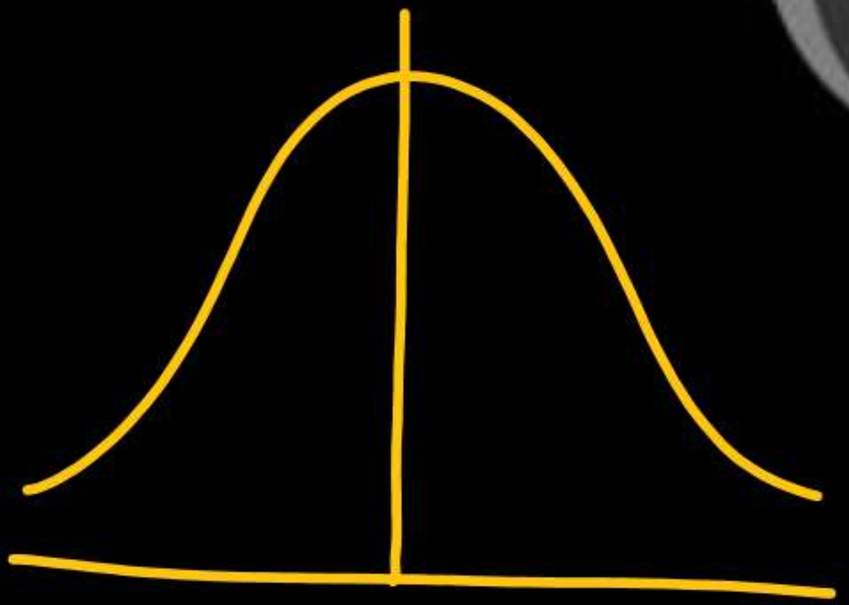
Positively skewed curve
(Left - β distribution)

(Mode < Median < Mean)



Negatively skewed
(Right - β distribution)

(Mean < Median < Mode)



[Symmetrical]

(Mean = Median = Mode)

Question 5

A random variable X has the density function $f(x) = K\frac{1}{1+x^2}$, where $-\infty < x < \infty$. Then the value of K is

A π

B $1/\pi$

C 2π

D $1/2\pi$

$$f(x) = K\frac{1}{1+x^2} \; ; (-\infty < x < \infty)$$

$$\boxed{\int_{-\infty}^{+\infty} f(x)\,dx = 1} \Rightarrow \int_{-\infty}^{+\infty} K\left(\frac{1}{1+x^2}\right)dx$$

$$2\int_0^{\infty} K\left(\frac{1}{1+x^2}\right)dx$$

$$K\left[\tan^{-1}x\right]_{-\infty}^{+\infty} = 1$$

$$K\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = 1$$

$$\boxed{K = \frac{1}{\pi}}$$

Question 6

A random variable X has a probability density function

$$f(x) = \begin{cases} kx^n e^{-x}; & x \geq 0 \\ 0; & \text{otherwise} \end{cases} \text{(nis an interger)}$$

with mean 3. The values of $\{k, n\}$ are

A $\left\{\frac{1}{2}, 1\right\}$ ×

B $\left\{\frac{1}{4}, 2\right\}$

C $\left\{\frac{1}{2}, 2\right\}$ ✓

D $\{1, 2\}$

Let $n = 1$

$f(x) = Kxe^{-x}\ ; x \geq 0$

$\int_0^\infty Kxe^{-x}dx = 1$

$K\left[(x)(-e^{-x}) - (1)(e^{-x})\right]_0^\infty = 1$

$K[0-(-1)] = 1$

$K = 1$

For $n = 1, K = 1$

Let $n = 2$;

$$f(x) = Kx^2e^{-x} \quad ; x \geq 0$$

$$\int_0^\infty f(x)\,dx = 1 = \int_0^\infty Kx^2e^{-x}\,dx$$

$$K\left[(x^2)(-e^{-x}) - (2x)(+e^{-x}) + (2)(-e^{-x})\right]_0^\infty = 1$$

$$K[0 - (-2)] = 1$$

$$K = 1/2$$

For $n = 2$; $K = 1/2$

$$E(x) = 3$$

$$\int_0^\infty x\,f(x)\,dx = 3$$

$$\int_0^\infty x\;Kx^ne^{-x}\,dx = 3$$

Question 7

What is the probability that at most 5 defective fuses will be found in a box of 200 fuses, if 2% of such fuses are defective?

A 0.82

B 0.79

C 0.59

D 0.85

$n = 200 \quad p = 2\% = \frac{2}{100}$ are defective.

$\lambda = np = 200 \times \frac{2}{100} = 4$ defective.

$P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$$P(x \le 5) = \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^3}{3!} + \frac{e^{-\lambda}\lambda^4}{4!} + \frac{e^{-\lambda}\lambda^5}{5!}$$

$$= e^{-4}\left[1 + 4 + \frac{4^2}{2} + \frac{4^3}{6} + \frac{4^4}{24} + \frac{4^5}{120}\right] = 0.7845 \simeq 0.79$$

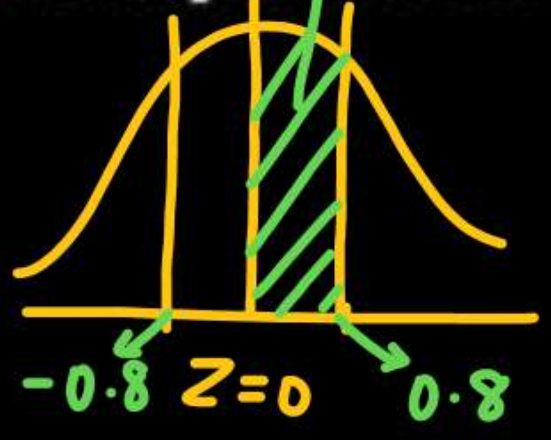
Question 8

If X is a normal variate with mean 30 and standard deviation 5, what is Probability $(26 \leq X \leq 34)$, given $A(z = 0.8) = \underbrace{0.2881}$ where A represents area.

$A(0<z<0.8)$

$\mu_x = 30, \ \sigma_x = 5$

$$P(26 \leq x \leq 34) = P\left(\frac{26-30}{5} \leq z \leq \frac{34-30}{5}\right)$$



$$P(-0.8 \leq z \leq 0.8) = 2P(0<z<0.8)$$

$$= 2 \times 0.2881$$

$$= 0.5762$$

$$z = \frac{x-\mu}{\sigma}$$

A 0.2881

B 0.5762

C 0.8181

D 0.1616

Question 9

In a sample of 100 students, the mean of the marks (only integers) obtained by them in a test is 14 with its standard deviation of 2.5 (marks obtained can be fitted with a normal distribution). The percentage of students scoring (less than) 16 marks is (Area under standard normal Curve between $z = 0$ and $z = 0.6$ is 0.2257; and between $z = 0$ and $z = 1.0$ is 3413)

A 36

B 23

C 12

D 10

$P(14 < x < 16)$

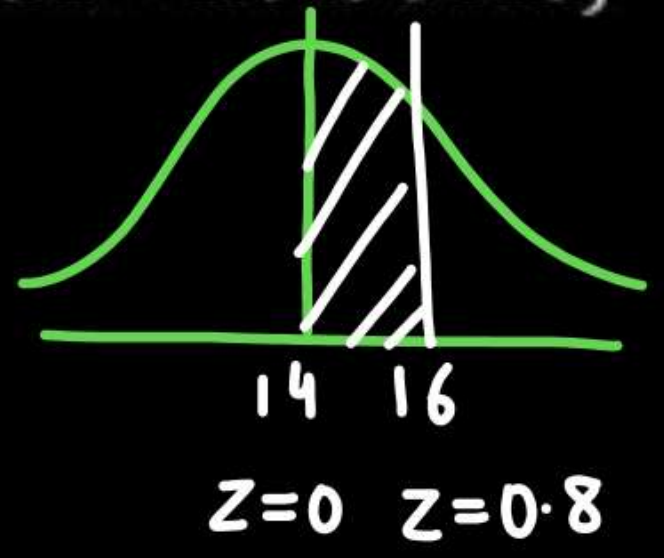
$P(0 < z < 0.8)$

$\mu = 14 \quad \sigma = 2.5$

$z = \frac{x-\mu}{\sigma} = \frac{16-14}{2.5} = \frac{2}{2.5} = 0.8$

$P(0 < z < 0.6) = 0.2257$

$P(0 < z < 1) = 0.3413 \Rightarrow P(0 < z < 0.8) \approx 23\%$



Question 10

Consider a random variable to which a Poisson distribution is best fitted. It happens that $P_{(x=1)} = \frac{2}{3} P_{(x=2)}$ on this distribution plot. The variance of this distribution will be

$$P(x=1) = \frac{2}{3} P(x=2)$$

$$\frac{\cancel{e^{-\lambda}}\lambda^1}{1!} = \frac{\cancel{2}}{3}\,\frac{\cancel{e^{-\lambda}}\lambda^2}{\cancel{2}!}$$

$$3\lambda - \lambda^2 = 0$$

$$\lambda(3-\lambda) = 0$$

$$\lambda = \cancel{0}, 3$$

Mean = Variance = λ

= 3

A 3

B 2

C 1

D 2/3

Thank you

GW Soldiers !