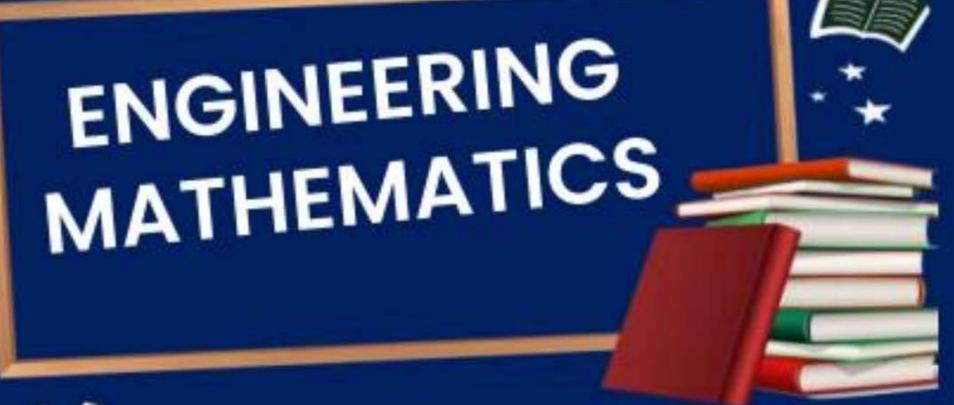


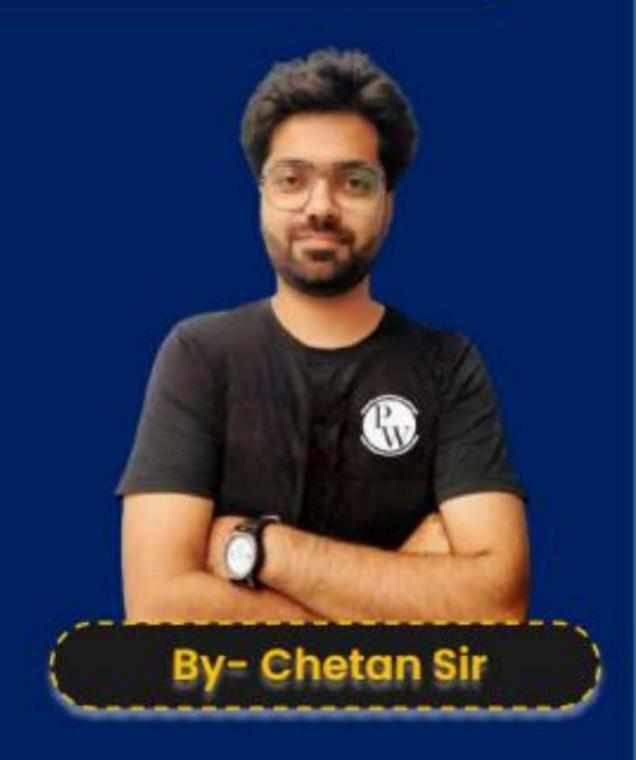
ALL BRANCHES





Lecture No.-02

Differential equations





Topics to be Covered

DEFINITION & TYPES

ORDER & DEGREE OF DE

SOLUTION OF DE

FORMATION OF DE

WRONSKIAN & LD/LI SOLUTIONS

METHODS OF SOLVING DE

PARTIAL DIFFERENTIAL EQUATIONS

WRONSKIAN & LD/LI SOLUTION



 \rightarrow In y_1, y_2 are two solns. of 2nd order D.E. $a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$ then Wronskian is defined as –

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

The y1, y2, y3 are 3 solns. of 3^{rd} DF. $a_0 \frac{d^3y}{dx^3} + a_1 \frac{d^2y}{dx^2} + a_2 \frac{dy}{dx} + a_3 y = 0$ then Wronskian is defined as—



Linearly Dependent (LD)/Linearly independent (LI) Solution

-> let y1, y2, y3 are solns. of D.E. if there exist a relation b/w y1, y2, y3

Such that C1y1+C2y2+C3y3=0 where C1, C2&C3 are non-zero

then they are linearly dependent.

-> Let y1, 42, 43 are solns. of D.E if there does not exist a relation b/w y1, y2, y3 such that zy1+zy2+z3y3=0 when <,, <2, <3=0 then they are linearly independent.

In such case; W = 0



Ex:- Prove that 1, x, x² are linearly independent & form the DE

$$y_1 = 1$$
 $y_2 = x$ $y_3 = x^2$ $\rightarrow 3^{rd} \text{ order D.E.}$
 $W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 0 & L & 2x \\ 0 & 0 & 2 \end{vmatrix} = 2 \neq 0 \quad \text{W} \neq 0$

Hence given polutions

Alence general poly: $y = C_1(1) + C_2(x) + C_3(x^2)$ are L.I.

 $y' = C_2 + 2C_3x$
 $y''' = 0$
 $y'''' = 0$
 $y''' = 0$
 $y''' = 0$
 $y''' = 0$



Ex:-LI Solutions (e^x, xe^x, x²e^x) form DE
Soln:
$$W \neq 0$$
; then general soln is $y = C_1 e^x + C_2 (x e^x) + C_3 (x^2 e^x)$
 $y' = C_1 e^x + C_2 e^x + C_3 x e^x + C_3 e^x + C_3 e^x + C_3 e^x$
 $y'' = y' + C_2 e^x + 2C_3 x e^x$
 $y'' = y' + (y' - y) + 2C_3 e^x$
 $y''' = y'' + y'' - y' + 2C_3 e^x$
 $y''' = 7y'' - y' + (y'' - 7y' + y)$
 $y''' = 3y'' - 3y' + y$
 $y''' = 3y'' - 3y' + y$

Methods of Solving D.E .: -



```
(1) Observation Method
    2) D.E. of first order & first degree
    Variable separable mtd. b) Homogenous D.E. mtd.
ODE Dinear D.E. mtd.
   3) Exact differential equations (Non exact D.E. -> Exact D.E.)
    14) L.D.E. of nth order with _ constant coefficients } (C.F. + P.I.)
    5) Methods for solving non-linear D.E.
PDE [6) Methods for solving P.D.E.
```



Observation method

•
$$d(xy) = xdy + ydx$$

•
$$d\left(\frac{x}{y}\right) = \frac{y dx - x dy}{y^2}$$

•
$$d\left(\frac{x}{\lambda}\right) = \frac{x dy - y dx}{x^2}$$

$$\begin{cases} x \frac{dy + y}{dx} - x^2 \frac{dx}{dx} = 0 \\ \int \frac{d(xy) - x^2}{3} + c = 0 \\ xy - \frac{x^3}{3} + c = 0 \end{cases}$$

$$\begin{cases} \frac{y \frac{dx - x}{dy} - \frac{y^2}{y^2} \frac{dy}{y^2} = 0}{y^2} \\ \int \frac{d(\frac{x}{y}) - 1}{y} \frac{dy}{y^2} = 0 \end{cases}$$

$$\frac{x}{y} - y + c = 0$$



DE of first order & firstdegree
 (a) Variable separable method

Ex:- $\tan y \sec^2 x dx + \tan x \sec^2 y dy = 0$

$$\int \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

$$\log (\tan x) + \log (\tan y) = \log c$$

$$\log (\tan x \tan y) = \log c$$

$$\tan x \cdot \tan y = c$$

$$\frac{dy}{dx} + xy = 0$$

$$\frac{dy}{dx} = -\int x dx$$

$$\log y = -\frac{x^2}{2} + C$$

$$tan x = t$$

 $sec^2 x dx = dt$



Ex:-
$$y - x \frac{dy}{dx} = a(y^2 + \frac{dy}{dx})$$

$$y - ay^2 = \frac{dy}{dx} (x + a)$$

$$\frac{dx}{x + a} = \frac{dy}{y(1 - ay)}$$

$$\int \frac{dx}{x + a} = \int \frac{dy}{y} + \int \frac{a}{1 - ay} dy$$

$$\log(x + a) = \log y + \cancel{x} \frac{\log(1 - ay)}{-\cancel{x}} + \log c$$

$$\log(x + a) = \log y + \log(1 - ay)^{-1} + \log c$$

$$x + a = \frac{cy}{y}$$

$$\frac{1}{y(1 - ay)} = \frac{A}{y} + \frac{B}{1 - ay}$$

$$\frac{1}{y(1 - ay)} = \frac{A(1 - ay) + By}{y(1 - ay)}$$

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DE of first order & firs degree

Let
$$y/x = \sqrt{y} = \sqrt{y}$$

Jomo genous
$$\rightarrow x, y \rightarrow (x, vx) \rightarrow (v, x)$$
function

Let
$$x/y = v$$

 $\sqrt{x} = vy \rightarrow Diff \cdot w \cdot r \cdot t \cdot y$
 $\sqrt{\frac{dx}{dy}} = v \cdot 1 + y \frac{dv}{dy}$

$$(x,y) \rightarrow (y,y) \rightarrow (y,y)$$

Variable separable then solve & put value of v.



DE of first order & first degree 2) (b) Homogenous DE method

Ex:-
$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

$$y = Y \times j \quad \frac{dy}{dx} = V \cdot 1 + \times \frac{dv}{dx}$$

$$\begin{cases} \frac{1-v}{x} & \text{d}v = \int \frac{dx}{x} \\ \frac{1-v}{x} & \text{d}v = \int \frac{dx}{x} \end{cases}$$

$$\begin{cases} \frac{1-v}{1+v^2} & \text{d}v = \int \frac{dx}{x} \\ \frac{1-v}{1+v^2} & \text{d}v = \int \frac{dx}{x} \end{cases}$$

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$$\int \frac{1-v}{1+v^{2}} dv = \int \frac{dx}{x}$$

$$\int \frac{1}{1+v^{2}} - \int \frac{2v}{2(1+v^{2})} = \int \frac{dx}{x}$$

$$\tan^{-1}v - \frac{1}{2}\log(1+v^{2}) = \log x + c$$

$$\tan^{-1}(\frac{y}{x}) - \frac{1}{2}\log(1+(\frac{y}{x})^{2}) = \log x + c$$



DE of first order & firs degree(b) Homogenous DE method

Homogenous
$$f_n \rightarrow y^{\circ} f(\frac{x}{y})$$

Ex:-
$$(1 + e^{x/y})dx + e^{x/y}(1-x/y)dy = 0$$

 $x = yy$ $\frac{dx}{dy} = y \cdot 1 + y \frac{dy}{dy}$
 $(1 + e^y) dx + e^y(1-y) dy = 0$
 $\frac{dx}{dy} = -\frac{e^y(1-y)}{1+e^y}$
 $y \frac{dy}{dy} = -\frac{e^y + ye^y}{1+e^y}$
 $y \frac{dy}{dy} = -\frac{e^y + ye^y}{1+e^y}$

$$\int \frac{(1+e^{v})}{v+e^{v}} dv = -\int \frac{dy}{y}$$

$$\log_{e}(v+e^{v}) = -\log_{e}y + \log c$$

$$\log_{e}y (v+e^{v}) = \log_{e}c$$

$$y(\frac{x}{y}+e^{x/y}) = c$$

$$x+ye^{x/y} = c$$



2) DE of first order & firs degree

$$\frac{dy}{dx} + Py = Q$$

$$P, Q \rightarrow f(x)$$
Find I.F. = e $\int Pdx$

$$\int e^{\int P dx} \frac{dy}{dx} + e^{\int P dx} \cdot Py = \int Q e^{\int P dx}$$

$$y \cdot e^{\int P dx} = \int Q (I.F.)$$

$$y \cdot (I.F.) = \int Q (I.F.)$$

$$\frac{dx}{dy} + Px = Q$$

$$P, Q \rightarrow f(y)$$

$$T.F. = e^{\int Pdy}$$

$$\int e^{\int Pdy} \frac{dx}{dy} + e^{\int Pdy} Px = Q e^{\int Pdy}$$

$$x. e^{\int Pdy} = Q(I.F.)$$

$$x(I.F.) = Q(I.F.)$$



DE of first order & firs degree
 (c) Linear D.E method

Ex:-
$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

$$\frac{dy}{dx} + \int y = Q$$

$$T.F. = e$$

$$\int \frac{dy}{dx} + \int \frac{dx}{dx} = e$$

$$\int \frac{dx}{dx} = e$$

$$\int \frac{dx}{dx} + \frac{dx}{dx} = e$$

$$\int \frac{dx}{d$$



DE of first order & firs degree
 (c) Linear D.E method

(c) Linear D.E method
$$P = \frac{2}{x}, Q = e^{x}$$

$$Ex: -\frac{dy}{dx} + \frac{2y}{x} = e^{x}$$

$$T.F. = e^{\int Pdx} = e^{x}$$

$$y \cdot x^{2} = \int e^{x}(x^{2})$$

$$y \cdot x^{2} = (x^{2})(e^{x}) - (2x)(e^{x}) + (2)(e^{x}) + C$$

$$y = e^{x}(1 - 2/x + 2/x^{2}) + C/x^{2}$$



DE of first order & firs degree
 (c) Linear D.E method

Ex:-
$$\frac{dy}{dx} + \frac{y}{x} = x$$
 with the condition that $y(1) = 1$, is

(a)
$$Y = \frac{2}{3x^2} + \frac{y}{x}$$
 (b) $Y = \frac{x}{2} + \frac{1}{2x}$

(c)
$$Y = 2/3 + x/3$$
 (d) $Y = \frac{2}{3x} + \frac{x^2}{3}$

$$\frac{dy}{dx} + \frac{y}{x} = x$$

$$T.F. = x$$

$$y \cdot x = \int x \cdot x$$

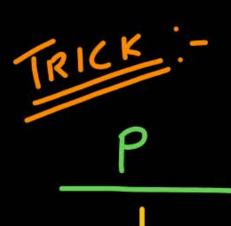
$$y \cdot x = \frac{x}{3} + c$$

$$1.1. = \frac{1^3}{3} + C$$

$$C = \frac{2}{3}$$

$$y = \frac{x^2}{3} + \frac{2}{3x}$$





$$x^{-1} \rightarrow \frac{1}{x}$$

$$x^{2} \rightarrow x^{2}$$

$$x^{-2} \rightarrow \frac{1}{x^{2}}$$

$$x^{3} \rightarrow x^{3}$$

$$\chi^{-3} \rightarrow \chi^{\chi_3}$$



DE of first order & firs degree 2)

(d) Bernoulli DE method (reducible to L DE method)

Divide by
$$y^n$$
 $\frac{dy}{dx} + Py = Q y^n$; $n > 0$

$$\frac{dy}{dx} + Py = Q y^n$$
; $n > 0$

$$\frac{dy}{dx} + Py = Q y^n$$
; $n > 0$

$$\frac{dy}{dx} + Py = Q y^n$$
; $n > 0$

$$\frac{dy}{dx} + Py = Q y^n$$
; $n > 0$

$$\frac{dy}{dx} + Q y^n$$
; $\frac{dy}{dx} + Q y^n$; $\frac{dy}{dx}$

Reducible to L.D.E.
$$\frac{dz}{dx} + P(1-n) z = Q(1-n)$$

$$z \cdot (T.F) = \int Q(1-n) T.F.$$

$$I \cdot F. = e^{\int P(1-n) dx}$$



Ex:
$$\frac{dy}{dx} + \frac{y}{x} = x^2y^3$$

$$\frac{1}{y^3} \frac{dy}{dx} + \frac{1}{x} \frac{1}{y^2} = x^2$$

$$\frac{1}{-2} \frac{dz}{dx} + \frac{1}{x} z = x^2$$

$$(z,x) \Rightarrow \frac{dz}{dx} \left(-\frac{z}{x} \right) z = -2x^2$$

$$z \cdot x^{-2} = \int -2x^2 \cdot y^2$$

$$\frac{z}{x^2} = -2x + c$$

$$\frac{1}{y^2} = z$$

$$-\frac{2}{4} \frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{1}{x^2y^2} = -2x + C$$



Ex:
$$\frac{dy}{dx} + \frac{y}{x}$$
. $(\log y) = \frac{y(\log y)^2}{x}$

Rivide $y(\log y)^2$



Thank you

Seldiers!

