ENGINEERING MATHEMATICS

ALL BRANCHES

Probability
Correlation & Regression

DPP-09 Solution

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Question 1

If $\Sigma x_i = 15$, $\Sigma y_i = 36$, $\Sigma x_i y_i = 110$ and $n = 5$, then cov (x, y) is equal to

A 0.6

B 0.5

C 0.4

D 0.225

$$\text{Cov}(x,y) = E(XY) - E(X).E(Y)$$

$$= \frac{\sum x_i y_i}{n} - \left(\frac{\sum x_i}{n}\right).\left(\frac{\sum y_i}{n}\right)$$

$$= \frac{110}{5} - \left(\frac{15}{5}\right)\left(\frac{36}{5}\right)$$

$$= 0.4$$

Question 2

If cov (x, y) = –16.5, var (x) = 2.89 and var (y) = 100, then the coefficient of correlation r is equal to

A 0.36

B –0.64

C 0.97

D –0.97

$$r = \frac{Cov(x,y)}{\sqrt{Var(x).Var(y)}}$$

$$= \frac{-16.5}{\sqrt{2.89 \times 100}} = -0.97$$

Question 3

If $\Sigma x_i = 24$, $\Sigma y_i = 44$, $\Sigma x_i y_i = 306$, $\Sigma x_i^2 = 164$, $\Sigma y_1^2 = 574$ and $n = 4$, then the regression coefficient b_{yx} is equal to

A 2.1

B 1.6

C 1.225

D 1.75

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$= \frac{306 - \frac{24 \times 44}{4}}{164 - \frac{(24)^2}{4}} = 2.1$$

Question 4

If $\Sigma x_i = 30$, $\Sigma y_i = 42$, $\Sigma x_i y_i = 199$, $\Sigma x_i^2 = 184$, $\Sigma y_i^2 = 318$ and $n = 6$, then the regression coefficient b_{xy} is equal to

A. –0.36

B. –0.46

C. 0.26

D. None

$$b_{xy} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}$$

$$b_{xy} = \frac{199 - \frac{30 \times 42}{6}}{318 - \frac{(42)^2}{6}} = -0.46$$

Question 5

Let r be the correlation coefficient between x and y and b_{yx}, b_{xy} be the regression coefficient of y on x and x on y respectively then

A $r = b_{xy} + b_{yx}$

B $r = b_{xy} \times b_{yx}$

C $r = \sqrt{b_{xy} \times b_{yx}}$

D $r = \frac{1}{2}(b_{xy} + b_{yx})$

$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y}$ $\quad b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$

$r = b_{xy} \cdot \frac{\sigma_y}{\sigma_x}$ $\quad r = b_{yx} \cdot \frac{\sigma_x}{\sigma_y}$

$r^2 = b_{xy} \times b_{yx}$

$r = \sqrt{b_{xy} \cdot b_{yx}}$

Question 6

Which one of the following is a true statement.

A $\frac{1}{2}(b_{xy}+b_{yx})=r$

B $\frac{1}{2}(b_{xy}+b_{yx})<r$

C $\frac{1}{2}(b_{xy}+b_{yx})>r$

D None of these

$$\frac{1}{2}(b_{xy}+b_{yx}) > r$$

$$\frac{1}{2}\left(r\cdot\frac{\sigma_x}{\sigma_y}+r\cdot\frac{\sigma_y}{\sigma_x}\right) > r$$

$$\frac{\sigma_x^2+\sigma_y^2}{2\sigma_x\sigma_y} > 1$$

$$\sigma_x^2+\sigma_y^2-2\sigma_x\sigma_y > 0$$

$(\sigma_x-\sigma_y)^2 > 0$, which is true.

Question 7

If $b_{yx} = 1.6$ and $b_{xy} = 0.4$ and θ is the angle between two regression lines, then $\tan\theta$ is equal to

A 018

B 0.24

C 0.16

D 0.3

$$\tan\theta = \pm\left(\frac{b_{yx} - b_{xy}}{1 + b_{yx} . b_{xy}}\right)$$

$$\tan\theta = \left(\frac{1.6 - 0.4}{1 + 1.6 \times 0.4}\right) = \frac{1.2}{7.4} = 0.16$$

Question 8

If cov (X, Y) = 10, var(X) = 6.25 and var(Y) = 31.36, then $\underbrace{\rho\,(X, Y)}_{r}$ is

A $\frac{5}{7}$

B $\frac{4}{5}$

C $\frac{3}{4}$

D 0.256

$$r, \rho(x,y) = \frac{Cov(x,y)}{\sqrt{Var(x).Var(Y)}}$$

$$= \frac{10}{\sqrt{6.25 \times 31.36}} = \frac{5}{7}$$

Question 9

Using given data points tabulated below, a straight line passing through the origin is fitted using least squares method. The slope of the line is

x	y	xy	x^2
1	1.5	1.5	1
2	2.2	4.4	4
3	2.7	8.1	9
		$\Sigma xy = 14$	$\Sigma x^2 = 14$

A 0.9

B 1

C 1.1

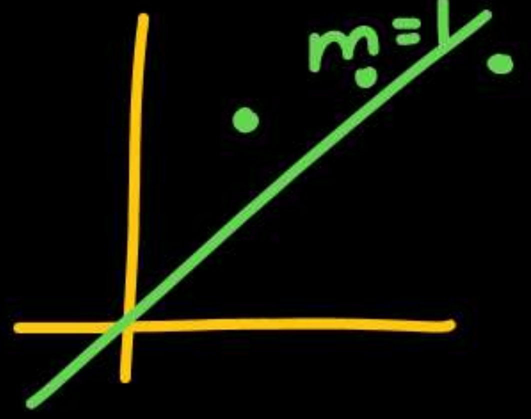
D 1.5

Let $y = mx$

$$u = \sum E^2 = \sum \{ y_i - (m x_i) \}^2$$

By least sq. mtd. $\dfrac{\partial u}{\partial m} = 0$

$$\frac{\partial u}{\partial m} = 2 \sum (y_i - m x_i)(-x_i) = 0$$



$$\sum x_i y_i - m \sum x_i^2 = 0$$

$$m = \frac{\sum xy}{\sum x^2} = \frac{14}{14} = 1$$

Question 10

Three values of x and y are to be fitted in straight line in form $y = a + bx$ by the method of least squares. Given $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$ and $\Sigma xy = 46$, values of a and b are respectively

A. 2 and 3

B. 1 and 2

C. 2 and 1

D. 3 and 2

$$y = a + bx$$

$$\sum_{1}^{3} y = an + b\sum_{1}^{3} x$$

$$\sum_{1}^{3} xy = a\sum_{1}^{3} x + b\sum_{1}^{3} x^2$$

$$21 = 3a + 6b \quad -1)$$

$$46 = 6a + 14b \quad -2)$$

On solving; $a = 3, b = 2$

$$y = a + bx$$

$$y = 3 + 2x$$

Thank you

GW Soldiers !