CS & IT ENGINEERING

DIGITAL LOGIC

Number System

Lecture No. 01

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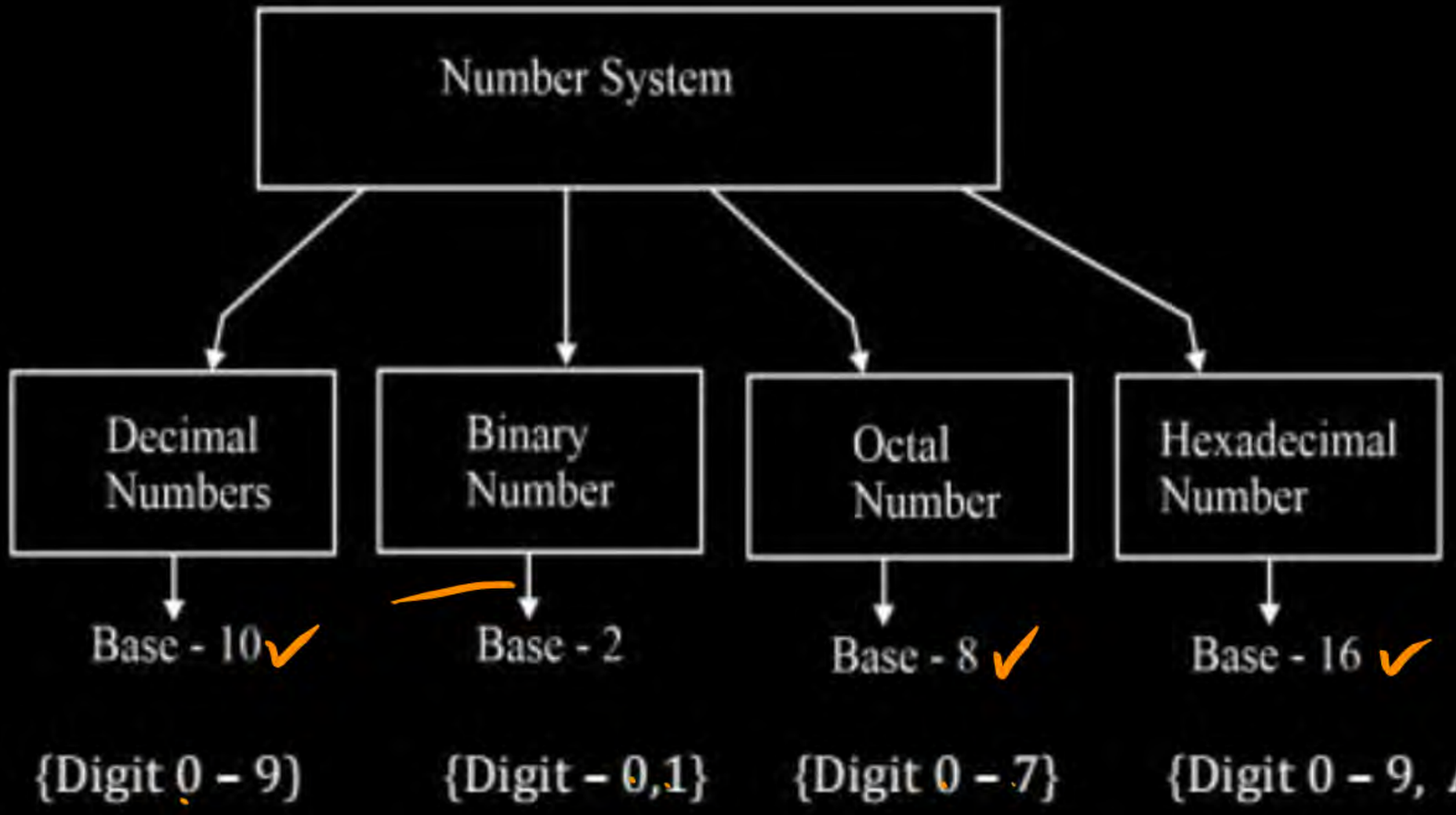
01 Conversions

02 Magnitude Comparator

03 Practice & DISCUSSION

Base (Radix)

Total number of digit used in the system



Base (Radix)

Decimal :→

...	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3} ...
...	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3} ...

a_i → Coefficient of decimal number system

10^i → Weight of decimal number system

Example : $(501.23)_{10}$

10^2	10^1	10^0	10^{-1}	10^{-2}
5	0	1	2	3

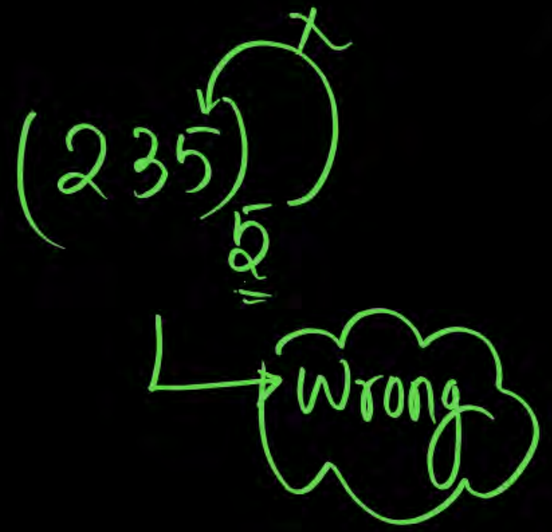
weight → 10^2 10^1 10^0 10^{-1} 10^{-2}

cofficient → $(125.75)_{10}$

Base	Digit
2	0, 1
3	0, 1, 2
4	0, 1, 2, 3
5	0, 1, 2, 3, 4
6	0, 1, 2, 3,4,5
7	0, 1, 2, 3, 4, 5, 6
8	0, 1, 2, 3, 4, 5, 6, 7
9	0, 1, 2, 3, 4, 5, 6, 7, 8
10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
11	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A
12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B
13	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C
14	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D
15	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E
16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

$(12C)_{13, 14, 15, 16}$

$(101)_{2,3,4,5,6 \ldots\ldots}$



Binary Number System (Base (Radix) = 2)

...	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3} ...
...	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3} ...

$2^i \rightarrow$ Weight of Binary number system

$a_i \rightarrow$ Coefficient of Binary number system {0, 1}

Example:- $(101.11)_2$

2^2	2^1	2^0	2^{-1}	2^{-2}
1	0	1	1	1

Octal Number System (Base (Radix) = 8)

...	8^3	8^2	8^1	8^0	8^{-1}	8^{-2}	8^{-3}...
...	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3}...

$8^i \rightarrow$ Weight of Octal number system

$a_i \rightarrow$ Coefficient of Octal number system {0 -7}

Example:- $(720.64)_8$

8^2	8^1	8^0	8^{-1}	8^{-2}
7	2	0	6	4

Hexadecimal Number System (Base (Radix) = 16)

...	16^3	16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3}...
...	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3}...

$16^i \rightarrow$ Weight of Hexadecimal number system

$a_i \rightarrow$ Coefficient of Hexadecimal number system {0 – 9, A–F}

10 ⇒ A
11 ⇒ B
12 ⇒ C
13 ⇒ D
14 ⇒ E
15 ⇒ F

Example: $(A2C.F)_{16}$

$16^2\ 16^1\ 16^0\ 16^{-1}$

16^2	16^1	16^0	16^{-1}
A	2	C	F

In base conversion 2 key points are there:

(A) Any base to Decimal conversion $(\ )_{?} \longrightarrow (\ )_{10}$

(B) Decimal to any other base conversion $(\ )_{10} \longrightarrow (?)_{?}$

(A) Any base to Decimal conversion :

$$(a_3\ a_2\ a_1\ a_0\ .\ a_{-1}\ a_{-2})_{r} = (\quad)_{10}$$

$$(a_3 \times r^3 + a_2 \times r^2 + a_1 \times r^1 + a_0 \times r^0 + a_{-1} \times r^{-1} + a_{-2} \times r^{-2})_{10}$$

Case (1) : Binary to Decimal conversion

$2^3\ 2^2\ 2^1\ 2^0\ 2^{-1}\ 2^{-2}$

Ex. $(1011.11)_2 = (\ ?\ )_{10}$

$$\Rightarrow \left[(1\times 2^3)+(0\times 2^2)+(1\times 2^1)+(1\times 2^0)+(1\times 2^{-1})+(1\times 2^{-2})\right]_{10}$$

$$\left[8+0+2+1+0.5+0.25\right]_{10}$$

$$\Rightarrow (11.75)_{10}$$

Case (2) : Octal to Decimal conversion

$8^2\ 8^1\ 8^0\ 8^{-1}$

Ex. $(721.4)_8 = (\ ?\ )_{10}$

$$\Rightarrow \left[(7\times8^2)+(2\times8^1)+(1\times8^0)+(4\times8^{-1})\right]_{10}$$

$$\Rightarrow \left[448+16+1+0.5\right]_{10}$$

$$\Rightarrow (465.5)_{10}$$

Case (3) : Hexadecimal to Decimal conversion

$16^2\ 16^1\ 16^0\ 16^{-1}$

Ex. $(A2B.C)_{16} = (\ ?\ )_{10}$

$$\Rightarrow \left[(A\times16^2)+(2\times16^1)+(B\times16^0)+(C\times16^{-1})\right]_{10}$$

$$\Rightarrow \left[(10\times256)+(2\times16)+(11\times1)+(12\times16^{-1})\right]_{10}$$

$$\Rightarrow \left[2560+32+11+0.75\right]_{10}$$

$$\Rightarrow (2603.75)_{10}$$

Case (4) : Base 5 to Decimal conversion

Ex. $(432.22)_5 = (\quad)_{10}$

$\Rightarrow \left[(4\times5^2)+(3\times5^1)+(2\times5^0)+(2\times5^{-1})+(2\times5^{-2})\right]_{10}$

$\Rightarrow \left[100+15+2+0.4+0.08\right]_{10}$

$\Rightarrow (117.48)_{10}$

Q $(4365.23)_{7^3\,7^2\,7^1\,7^0\,7^{-1}\,7^{-2}} = (\,?\,)_{10}$

$\Rightarrow [(4\times 7^3)+(3\times 7^2)+(6\times 7^1)+(5\times 7^0)+(2\times 7^{-1})+(3\times 7^{-2})]_{10}$

$\Rightarrow (1566.346)_{10}$

Ans

Q $(62C) = (?)_{10}$

By minimum base convert into Decimal ?

A=10
B=11
C=12
D=13
E=14
F=15

$(62C)_{13,14,15,\cdots}$

$\hookrightarrow$ minimum = 13

$(62C)_{13} = (?)_{10}$

$(6\times13^{2}+2\times13^{1}+12\times13^{0})_{10}$

$(1052)_{10}$ Ans

(B) Decimal to any other Base conversion

$(\ )_{10} \longrightarrow (?)_r$

r	a_3	a_2	a_1	a_0	Remainder
r					b_0
r					b_1
					b_2
					b_3

$(a_3\ a_2\ a_1\ a_0 \cdot a_{-1}\ a_{-2}\ a_{-3})_{10} = (\ \{\ )_r$

Before Decimal | After Decimal

$(a_3 a_2 a_1 a_0 \cdot a_{-1} a_{-2} a_{-3})_{10} = (b_3 b_2 b_1 b_0 \cdot x_0 x_1 x_2)_r$

$0 \cdot a_{-1} a_{-2} a_{-3} \times r = x_0 \cdot x_{-1} x_{-2}$ → x_0

$0 \cdot x_{-1} x_{-2} \times r = x_1 \cdot x_{-3} x_{-4}$ → x_1

$0 \cdot x_{-3} x_{-4} \times r = x_2 \cdot x_{-5} x_{-6}$ → x_2

$$(a_3 a_2 a_1 a_0 \cdot a_{-1} a_{-2} a_{-3})_{10} = (b_3 b_2 b_1 b_0 \cdot x_0 x_1 x_2)_r$$

Case (1) : Decimal to Binary Base conversion.

Ex. $(19.75)_{10} = (\ ?\ )_2$

Before Decimal R After Decimal

2	19	1
2	9	1
2	4	0
2	2	0
	1	1

$0.75 \times 2 = 1.5$ 1

$0.5 \times 2 = 1.0$ 1

$(19.75)_{10} = (10011.11)_2$

Case (2) : Decimal to Octal Base conversion.

Ex. $(210.23)_{10} = (\ ?\ )_8$

Before Decimal Remainder — After Decimal

8	210	2
8	26	2
	3	3

$0.23 \times 8 = 1.84$ → 1
$0.84 \times 8 = 6.72$ → 6
$0.72 \times 8 = 5.76$ → 5

$(210.23)_{10} = (322.165)_8$

Case (3): Decimal to Hexadecimal Base conversion.

Ex.

$(1228.55)_{10} = (\ ?\ )_{16}$

Before Decimal Remainder | After Decimal

16	1228	12 (C)
16	76	12 (C)
	4	4

$0.55 \times 16 = 8.8 \quad 8$

$0.8 \times 16 = 12.8 \quad 12(C)$

$(1228.56)_{10} = (4CC.8C)_{16}$

Q $(93.25)_{10} = (?)_5$

		R
5	93	3
5	18	3
	3	3

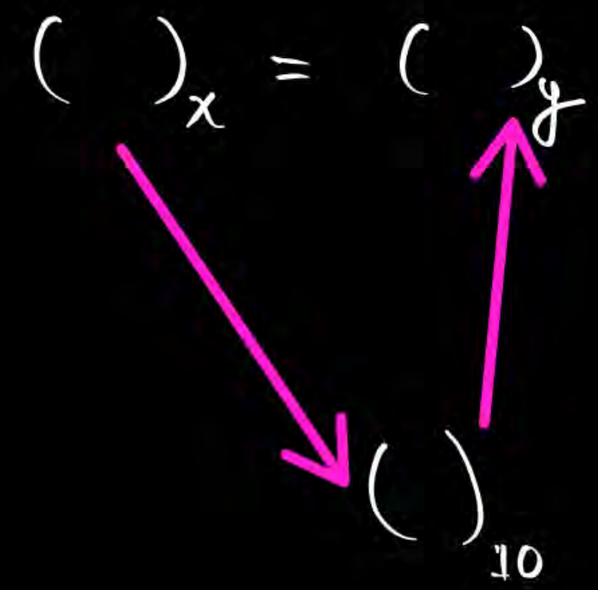
$0.25 \times 5 = 1.25$ 1

$0.25 \times 5 = 1.25$ 1

$(333.11\cdots)_5$

Ans

NOTE :



Some Special Case

Case (1): Binary to Octal base conversion

Ex. $(10110111)_2 = (\quad)_8$

Octal → means base 8

$8 = 2^3$

$8 = 2^{3}$

Every three digits of binary represent one digit of octal

010	110	111
2	6	7

Hence $(10110111)_2 = (267)_8$ Ans

Method ① $(10110111)_2 \rightarrow (\quad)_{10} \rightarrow (\quad)_8$

Some Special Case

Case (2) : Binary to Hexadecimal base conversion

$(\quad)_2 \Rightarrow (\quad)_{10} \rightarrow (\quad)_{16}$

Ex. $(1011011)_2 = (\quad)_{16}$

Hexadecimal → means base 16

$16 = 2^4$

Every four digits of binary represent one digit of Hexadecimal.

0101 1011

5 11(B)

Hence $(1011011)_2 = (5B)_{16}$

Q $(3221)_4 = (\quad)_2$

$4 = 2^2$ → Write in 2 bits of binary

$(11\ 10\ 10\ 01)_2$

Ans

Q $(330123)_4 = (?)_8$

Method ① $(330123)_4 = (\ )_{10} \longrightarrow (\ )_8$

Method ② $(\quad)_4 \Rightarrow (\ )_2 \longrightarrow (\ )_8$

$4 = 2^2$

$(11\ 11\ 00\ 01\ 10\ 11)_2 = (\quad)_8$

2^3

$(7433)_8$ Ans

Q $(7634)_8 = (\ ?\ )_{16}$

$(7634)_8 = (\ )_2$

$\hookrightarrow 2^3$

111	110	011	100

$(F9C)_{16}$ Ans

BCD (Binary Coded Decimal)

In this each digit of the decimal number is represented by its four-bit binary equivalent. It is also called natural BCD or 8421 code. It is weighted code.

Excess – 3 Code: This is an non weighted binary code used for decimal digits. Its code assignment is obtained from the corresponding value of BCD after the addition of 3.

BCO (Binary Coded Octal): In this each digit of the Octal number is represented by its three-bit binary equivalent.

BCH (Binary Coded Hexadecimal): In this each digit of the hexadecimal number is represented by its four bit binary equivalent.

Decimal Digits	BCD 8421	Excess – 3	Octal digits	BCO	Hexadecimal Digits	BCH
0	0000	0011	0	000	0	0000
1	0001	0100	1	001	1	0001
2	0010	0101	2	010	2	0010
3	0011	0110	3	011	3	0011
4	0100	0111	4	100	4	0100
5	0101	1000	5	101	5	0101
6	0110	1001	6	110	6	0110
7	0111	1010	7	111	7	0111
8	1000	1011			8	1000
9	1001	1100			9	1001
					A	1010
					B	1011
					C	1100
					D	1101
					E	1110
					F	1111

Q. Find the base values which can satisfy the following equation.

$$\left(\frac{39}{3}=13\right)_x$$

A. Only 10

B. Any base

C. Any base > 9

D. Any base ≥ 9

$$\left(\frac{39}{3}=13\right)_x$$

$$\frac{(39)_x}{(3)_x}=(13)_x$$

$$\frac{3x+9}{3}=x+3$$

$3x+9=3x+9 \rightarrow$ This equation satisfy for all value of x.

But x we have to choose $x>9$

$$\frac{3\times x^1+9x^0}{(3\times x^0)}=[1\times x^1+3\times x^0]$$

Q. Find the base values which can satisfy the following two equations simultaneously.

(i) $2 + 3 = 5$

(ii) $2 \times 4 = 10$

(i) $(2)_x + (3)_x = (5)_x$

$(2y\,x^0) + (3x\,x^0) \pm (5x\,x^0)$

$2 + 3 = 5$

$x > 5$

$\boxed{x \geq 6}$ ✓

(ii) $2 \times 4 = 10$

$(2)_x \times (4)_x = (10)_x$

$(2x\,x^0) \times (4x\,x^0) = (1x\,x^1 + 0x\,x^0)$

$2 \times 4 = x + 0$

$\boxed{x = 8}$

$x = 8$ will satisfy both.

A. Any base ≥ 6

B. 8

C. ✗ Any base ≥ 8

D. 6

Q. Find number of solutions of x and y which can satisfy the equation

$(43)_8 = (x\ 0)_y$

$y > x$

$(43)_8 = (x\ 0)_y$

$(4\times8^1 + 3\times8^0)_{10} = (x\cdot y + 0\times y^0)_{10}$

$35 = x\cdot y$

$y > x$

x	y
35	1
1	35 ✓
5	7 ✓
7	5

A 1

B 2

C 3

D 4

Q. Find number of solutions of x and y which can satisfy the equation

$(123)_5 = (x\,8)_y$

$y > 8$
$y > x$

$(123)_5 = (x\,8)_y$

$(1\times5^2)+(2\times5^1)+(3\times5^0) = xy^1 + 8xy^0$ ✓ $y>8$ $y>x$

$25+10+3 = xy+8$

$38 = xy+8$

$xy = 30$

x	y
30	1
1	30
15	2
2	15
10	3
3	10
6	5
5	6

$y > x$ $(y>8)$

A. 1

B. 2

C. 3

D. 4

Q. The result of addition operation 34 + 43 performed on minimum base is stored in an 8-bit register. The content of register will be

A. 01000011

B. 00101010

C. 01010101

D. 01010100

$(34)_5 + (43)_5$

$[(3\times5^1+4\times5^0)+(4\times5^1+3\times5^0)]_{10}$

$[19+23]_{10}$

$(42)_{10}$

128	64	32	16	8	4	2	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	1	0	1	0	1	0

Q. $7\times512 + 3\times256 + 6\times128 + 5\times32 + 3\times16 + 3$

If this number will be written in binary then number 1's present will be ___?

$2^0 = 1 \Rightarrow 1$

$2^1 \Rightarrow 10$

$2^2 \Rightarrow 100$

$2^4 \Rightarrow 1000$

$2^5 \Rightarrow 10000$

$(4+2+1)2^9 + (2+1)2^8 + (4+2)2^7 + (4+1)2^5 + (2+1)2^4 + 2+1$

$2^{11} + 2^{10} + 2^9 + 2^9 + 2^8 + 2^9 + 2^8 + 2^7 + 2^5 + 2^5 + 2^4 + 2^1 + 2^0$

$2^{11} + 2^{10} + 2^9(3) + 2^8(2) + 2^7 + 2^6 + 2^4 + 2^1 + 2^0$

$2^{11} + 2^{10} + 2^{10} + 2^9 + 2^9 + 2^7 + 2^6 + 2^4 + 2^1 + 2^0$

$2^{11} + 2^{11} + 2^{10} + 2^7 + 2^6 + 2^4 + 2^1 + 2^0$

$2^{12} + 2^{10} + 2^7 + 2^6 + 2^4 + 2^1 + 2^0$

(7)

Thank you

GW

Soldiers !