CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 03

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TOPICS

01 Pigeonhole principle

02 Euler totient function

3 Exercise

5 P

Some hole →

Worst case →

atleast 2	H1	H2	H3	H4		atleast 4	atleast 3
✓	O O O O	O			(4,1)	✓	✓
✓		O O O O	O		(4,1)	✓	✓
✓	O O O	O O			(3,2)	✗	✓
✓			O O O	O O	(3,2)	✗	✓
✓	O O O	O	O		(3,1,1)	✗	✓
✓		O O O	O	O	(3,1,1)	✗	✓
✓		O	O		(3,1,1)	✗	✓
✓	O O	O	O	O	(2,1,1,1)	✗	✗

Thm: In Simple Graph atleast 2 vertices will have same degree. $(n \geq 2)$.

Assumption: let's take all vertices will have diff. degrees.

Degree possibilities: $\{1, 2, 3, 4 \ldots n-1\}$ Total diff degrees $= n-1$.

Total vertices $= n$.

$\rightarrow$

d_1	d_2	..	d_{n-1}
v_1 v_n	v_2	...	v_{n-1}

v_1 — d_1

v_2 — d_2

v_3 —

v_{n-1} — d_{n-1}

v_n — d_2

$\hookrightarrow$ Assumption is wrong.

So the thm.

** → if we have n+1 pigeons _ n holes, then some of the holes contains atleast 2 pigeons

1500 flights are taking off in a single day.

prove that at least 2 flights are taking off in single min.

$m_1 \quad m_2 \quad m_3 \quad \dots \quad m_{1440}$

$\circ \quad \circ \quad \quad \quad \quad \circ$

$\circ \dots \dots \circ$

60

$1D = 24$

$= 24 \times 60$

$= \underline{1440}$

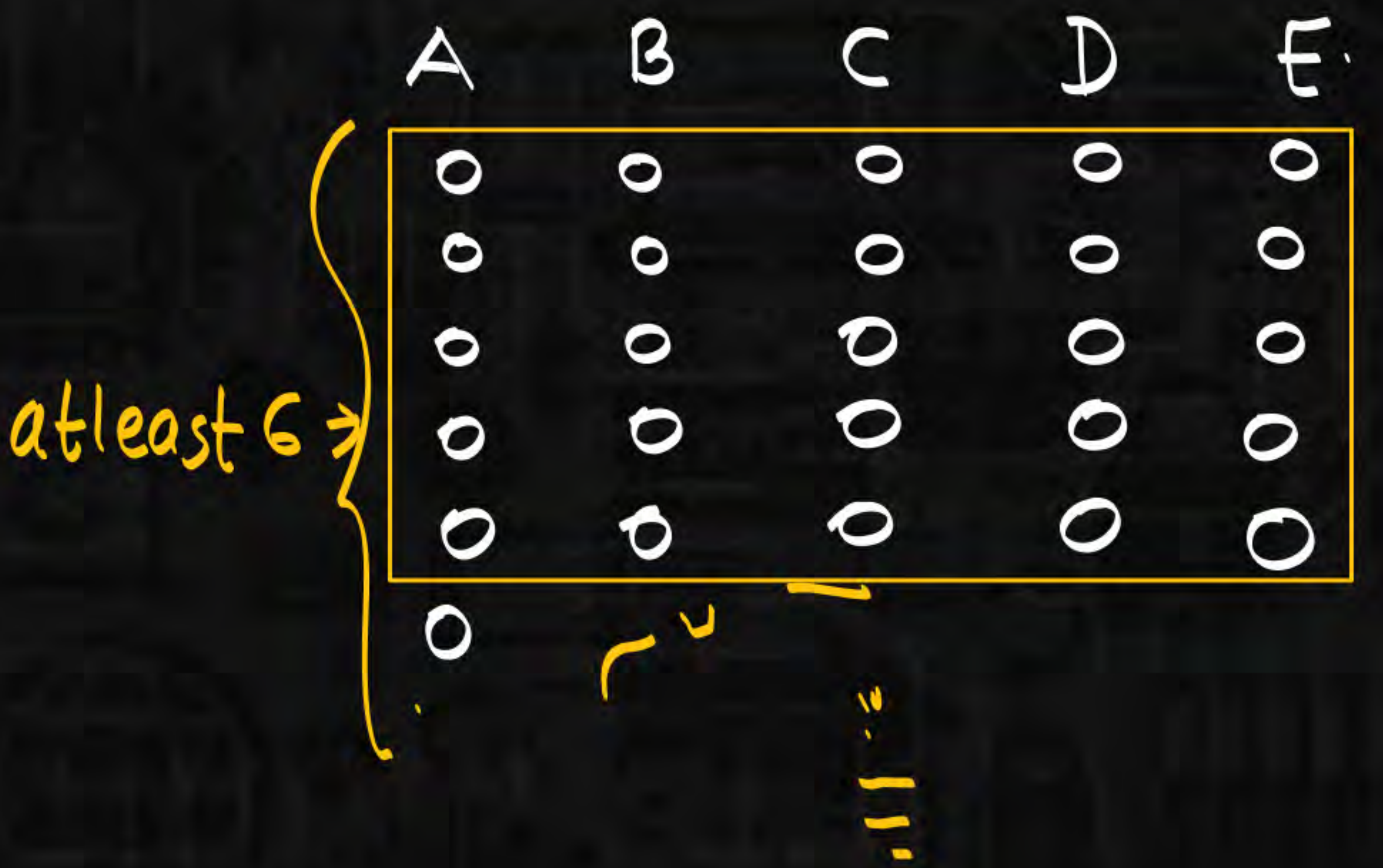
what will be min. no. of students requires such that at least 2 students will have same month bday ?.

m_1 m_2 . . . . m_{12}

o   o   o

o

What will min no. of students requires such that atleast 6 will have same grade, in school, where it contains five diff grade system.



$5 \times 5 + 1$

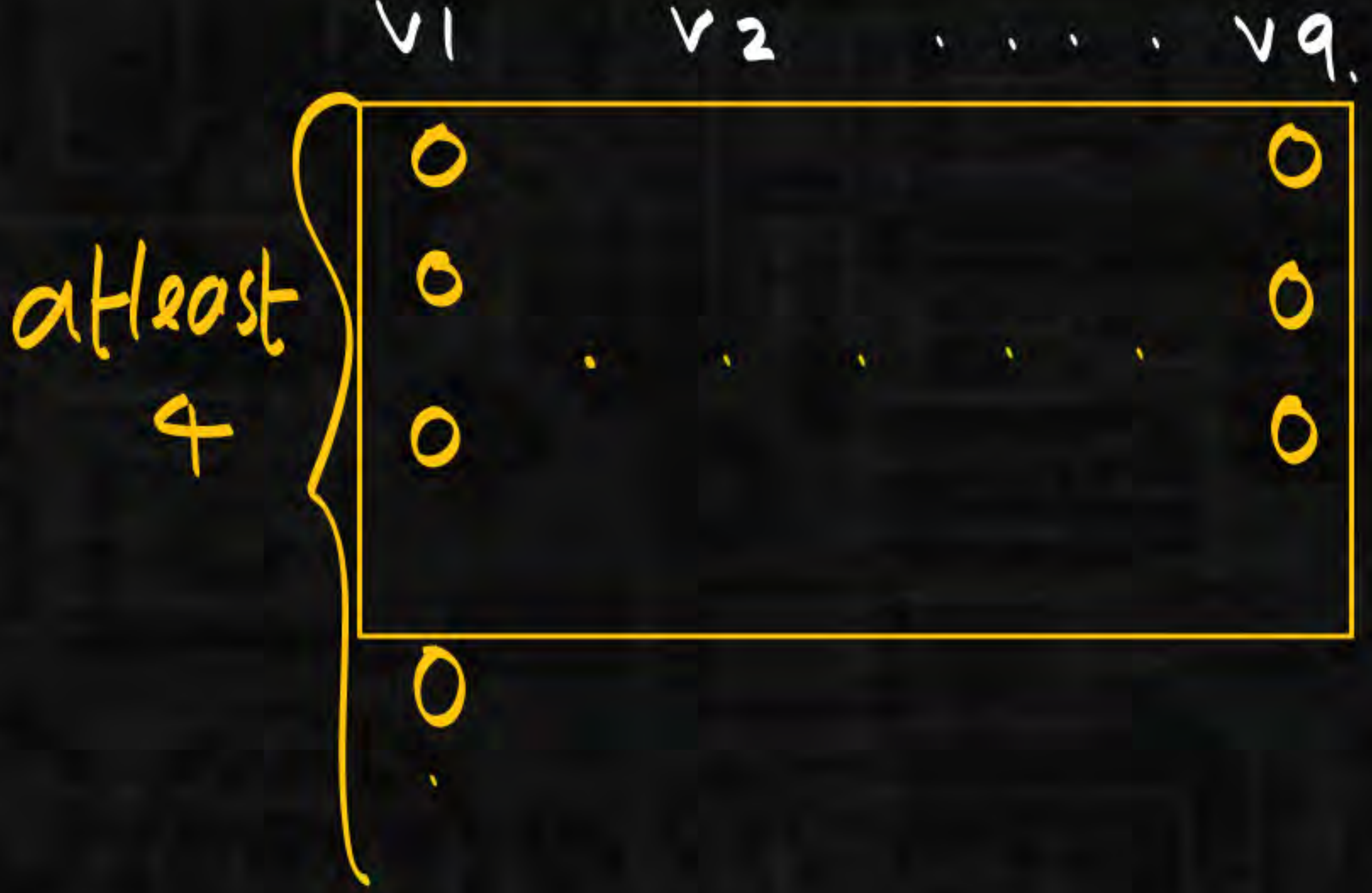
$= 26.$

GATE:

what will be min. no. of cards will be drawn such that at least 3 belongs to same suite?

	H	D	S	C
	o	o	o	o
atleast3	o	o	o	o
	o			

→ In a Graph of 9 vertices, prove that some the vertices contains atleast 4 degrees. if sum of degrees of all vertices is **atleast 27**.



$9 \times 3 = 27$.

$\sum d(v_i) =$ even

$\sum d(v_i) =$ atleast 27

28, 30, 32

min. no. of students requires atleast 9, will born in same month?

$12 \times 8 + 1 = 97.$

A bag contains 6 Red, 8 blue, 10 Green, 15 white, 20 Yellow

min. no. of balls atleast 6 balls of same clr?

6R	8bl	10G	15w	20Y

$5 \times 5 + 1$

$= 26.$

A bag contains 6 Red, 8 blue, 10 Green, 15 white, 20 Yellow

min. no. of balls atleast 9 balls of same clr?

Ans: 39

6R	8bl	10G	15w	20Y
0 0 0 0 0 0	6	6	6	6
	0 0	0 0 0	0 0	0 0

atleast 9

$6\times 5 + 8 + 1$

$S = \{1, 2, \ldots 24, 25\}$ what will no. of elements we will choose, such that one element divides another?

$13 + 1 = 14$

1 2 3 4 5 6 7 8 9 10 11 12

$\boxed{25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13}$

$\left\lceil \frac{n}{2} \right\rceil + 1$

$\{1, \ldots 200\}$ Ans: 101

$S = \{0, 1, \ldots, 9\}$ what is the smallest positive integer K such that any subset of S of size K contains two distinct subset of size two $\{\{x_1, x_2\}\ \{y_1, y_2\}\}$

$0 \ldots 9$

$\{\{x_1, x_2\}, \{y_1, y_2\}\}$ $x_1 + x_2 = y_1 + y_2 = 9$

$\{x_1 + x_2 = y_1 + y_2 = 9\}$

$\{\{0, 9\}\ \{1, 8\}\}$ $K = 4$.

$S = \{0, 1, \dots, 9\}$

subset $= K = \{9, 8, 7, \{6, \{5 \leftarrow 5$ ⟶ 6element

$3\} 4\} \leftarrow 2$

Case 2:

$0, 1, 2, \{3 | \{4$

$6 | 5$

$k = 7.$

Ans: 7.

$\{x_1, x_2\}$

$\{y_1, y_2\}$

$\{x_1, x_2\}$ $x_1 + x_2$

$=$

$\{y_1, y_2\}$ $y_1 + y_2$

$= 9.$

3. An auditorium has a seating capacity of 800. How many seats must be occupied to guarantee that at least two people seated in the auditorium have the same first and last initials?

$26^2 + 1 = 677$

4. Let $S = \{3, 7, 11, 15, 19, \ldots, 95, 99, 103\}$. How many elements must we select from S to insure that there will be at least two whose sum is 110?

Subdivide the set S into the 14 subsets: $\{3\}, \{7, 103\}, \{11, 99\}, \{15, 95\}, \ldots, \{43, 67\}, \{47, 63\}, \{51, 59\}, \{55\}$. By the Pigeonhole Principle if we select at least 15 elements of S then we must have the elements in one of the two-element subsets and these sum to 110.

20. How many times must we roll a single die in order to get the same score (a) at least twice? (b) at least three times? (c) at least n times, for $n \geq 4$?

20. (a) 7 (b) 13 (c) $6(n-1)+1$

24. Given 8 Perl books, 17 Visual BASIC[7] books, 6 Java books, 12 SQL books, and 20 C++ books, how many of these books must we select to insure that we have 10 books dealing with the same computer language?

24. 42

1. Given a group of n women and their husbands, how many people must be chosen from this group of $2n$ people to guarantee the set contains a married couple?

1. $n + 1$.

There are 20 small towns in a region of west Texas. We want to get three people from one of these towns to help us with a survey of their town. If we go to any particular town and advertise for helpers, we know from past experience that the chances of getting three respondents are poor. Instead, we advertise in a regional newspaper that reaches all 20 towns. How many responses to our ad do we need to assure that the set of respondents will contain three people from the same town?

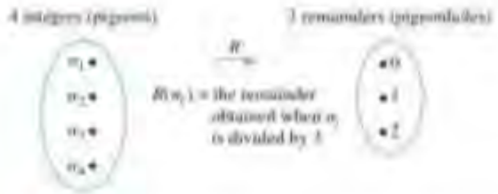
we need more than $2 \times 20 = 40$ responses.

which one is true ?

a.Given any set of four integers, must there be two that have the same remainder when divided by 3? Why?

b. Given any set of three integers, must there be two that have the same remainder when divided by 3?

Ans : a



21. Compute $\phi(n)$ for n equal to (a) 51; (b) 420; (c) 12300.

22. Compute $\phi(n)$ for n equal to (a) 5186; (b) 5187; (c) 5188.

21. (a) 32 (b) 96 (c) 3200

22. (a) $5186 = (2)(2593)$, and $\phi(5186) = (5186)(1/2)(2592/2593) = 2592$.
(b) $5187 = (3)(7)(13)(19)$, so $\phi(5187) = (5187)(2/3)(6/7)(12/13)(18/19) = (2)(6)(12)(18) = 2592$.
(c) $5188 = (2^2)(1297)$, and $\phi(5188) = (5188)(1/2)(1296/1297) = 2592$.
Hence $\phi(5186) = \phi(5187) = \phi(5188)$.

23. Let $n \in \mathbf{Z}^+$. (a) Determine $\phi(2^n)$. (b) Determine $\phi(2^n p)$, where p is an odd prime.

23. (a) 2^{n-1} (b) $2^{n-1}(p-1)$

25. How many positive integers n less than 6000 (a) satisfy $\gcd(n, 6000) = 1$? (b) share a common prime divisor with 6000?

25. (a) $\phi(6000) = \phi(2^4 \cdot 3 \cdot 5^3) = 6000(1-(1/2))(1-(1/3))(1-(1/5)) = 1600$.
(b) $6000 - 1600 - 1$ (for 6000) $= 4399$.


THANK YOU GW
SOLDIERS !