ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-08

Probability

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Topics to be Covered

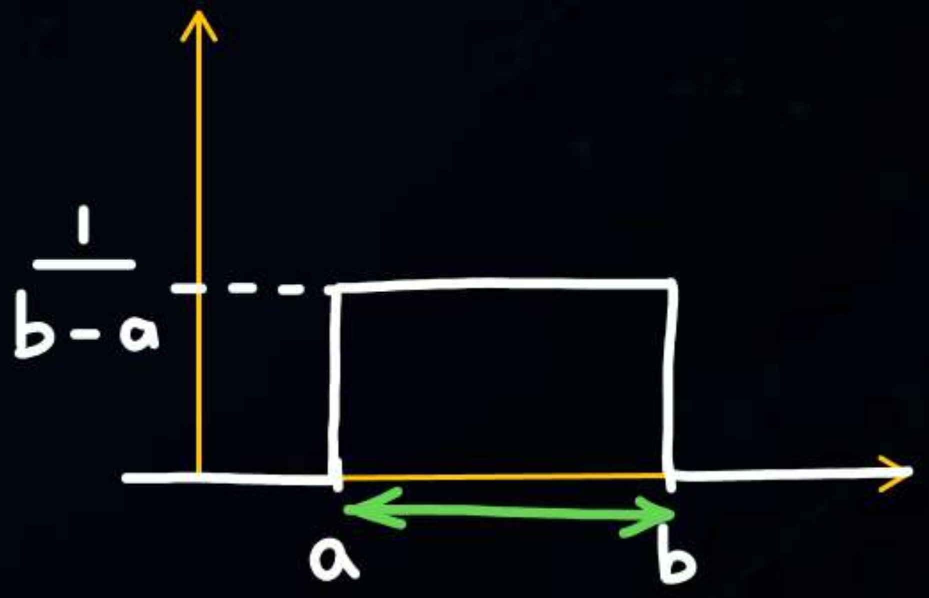
- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM
- STATISTICS – I (PROBABILITY DISTRIBUTIONS)
- STATISTICS – II (CORRELATION AND REGRESSION)

Types of Continuous Random Variable

1) ** Uniform Random Variable

A Random variable is uniformly distributed b/w a and b, then its p.d. function is

$$f(x) = \begin{cases} 0 ; & x < a \\ \frac{1}{b-a} ; & a < x < b \\ 0 ; & x > b \end{cases}$$



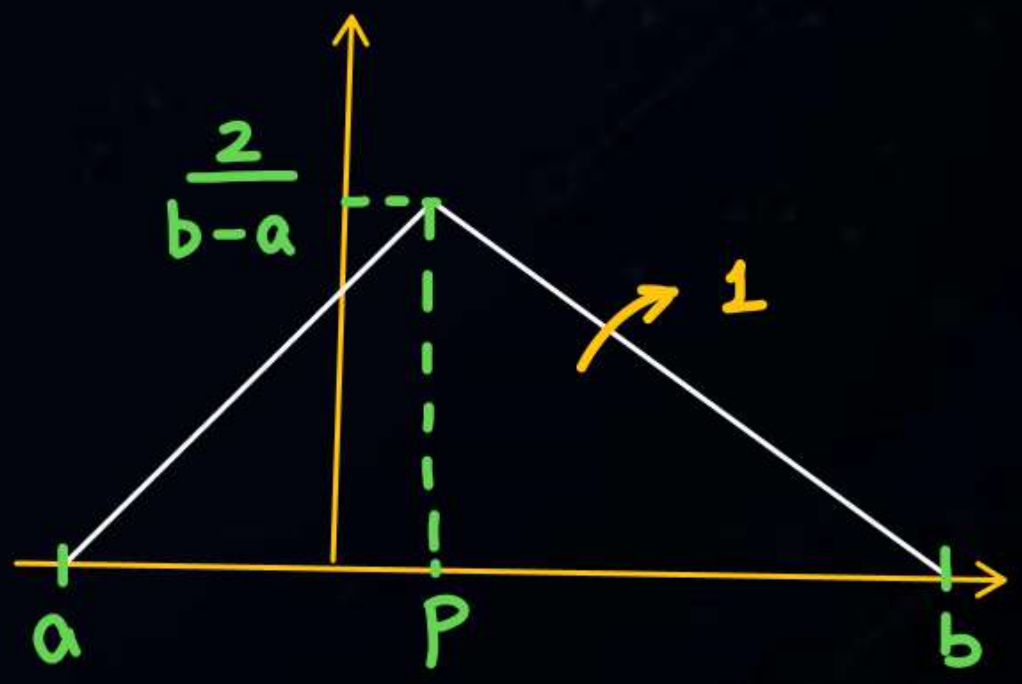
Parameters → 2 (a,b)

- Mean = $E(x) = \mu_x = \frac{a+b}{2}$
- $E(x^2) = \frac{a^2+b^2+ab}{3}$
- Variance = $\frac{(b-a)^2}{12}$

Types of Continuous Random Variable

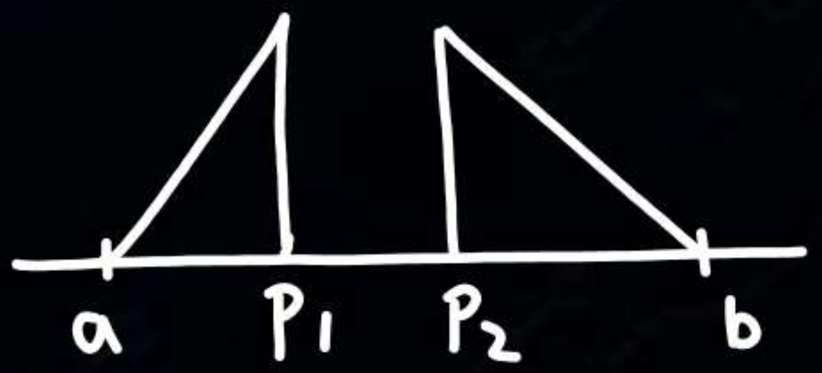
2) Triangular Random Variable.

$$f(x) = \begin{cases} m_1 x + c_1 & ; \ a < x < p \\ m_2 x + c_2 & ; \ p < x < b \end{cases}$$



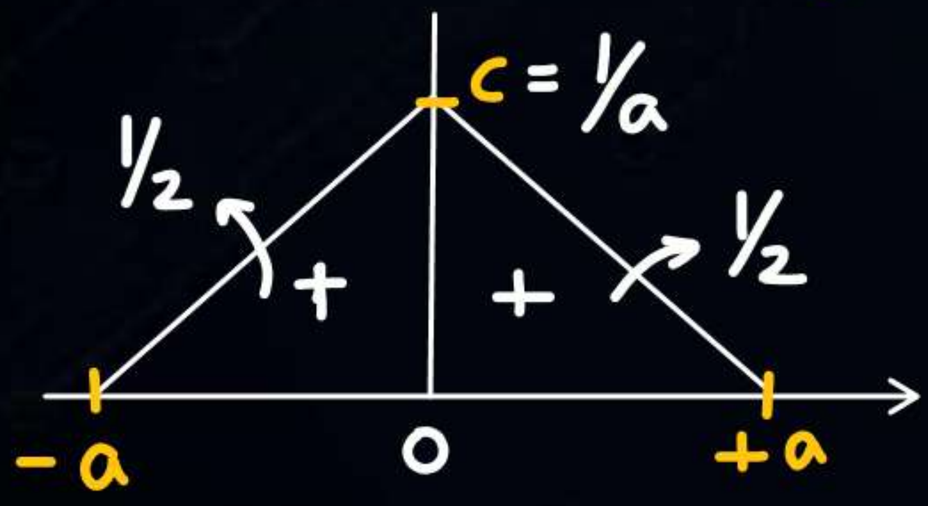
$$E(x) = \text{Mean} = \mu_x = \frac{a+p+b}{3}$$

Inclined straight lines combination



Parameters $\rightarrow$ 3 (a, p, b)

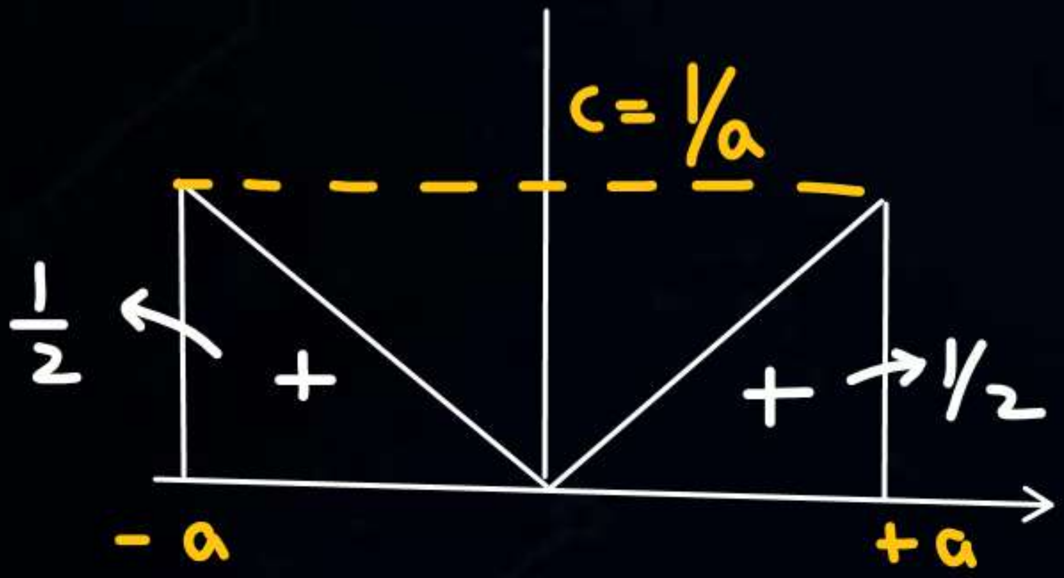
Symmetrical triangular pdf.



$f(x) \to \text{Even}$

$$E(x) = \int_{-a}^{+a} \overbrace{\underbrace{x}_{\text{Odd}}\ \underbrace{f(x)}_{\text{Even}}}^{\text{Odd}}\, dx = \boxed{0}$$

$$Var(x) = E(x^2) = \boxed{a^3/6}$$



$f(x) \to \text{Even}$

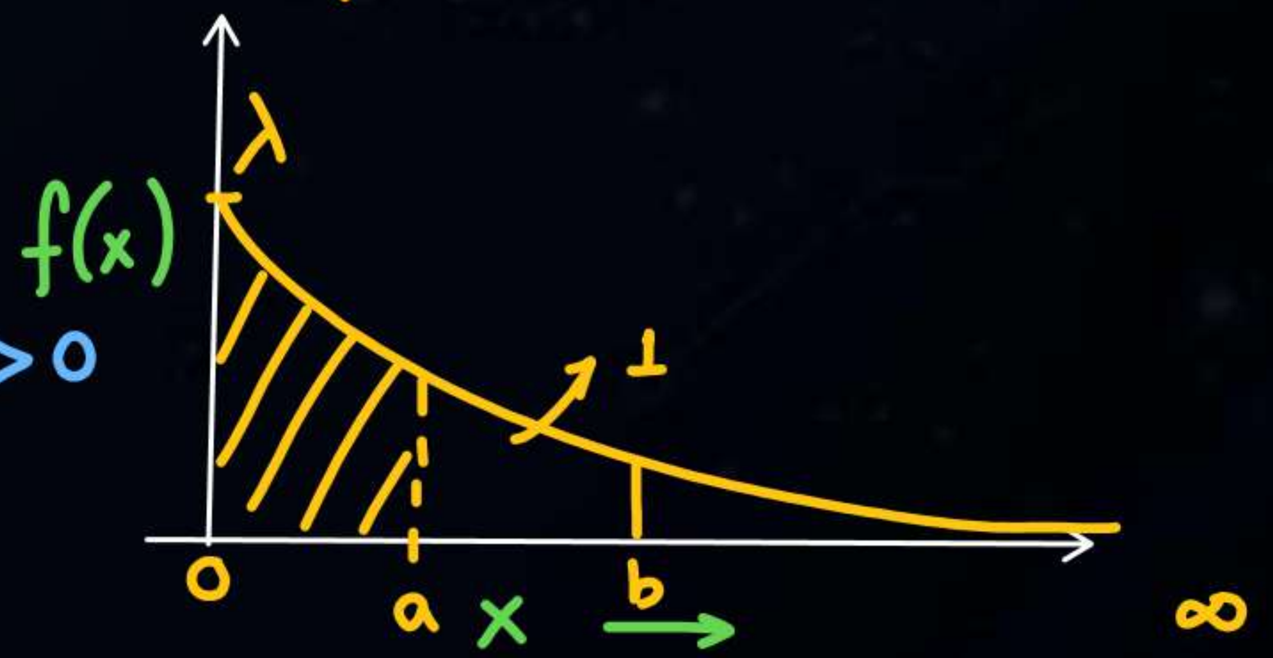
$$E(x) = \int_{-a}^{a} x\, f(x)\, dx = \boxed{0}$$

$$Var(x) = E(x^2) = \boxed{a^3/2}$$

$(1-e^{-\lambda b})-(1-e^{-\lambda a})$

3. Exponential Random Variable

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & ; x\geq 0\\ 0 & ; x<0\end{cases}\qquad \lambda>0$$



- $P(0<x<a)=\int_0^a \lambda e^{-\lambda x}\,dx=\lambda\left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^a$

$$=-(e^{-\lambda a}-1)$$

Parameters → $\lambda>0$

$\boxed{P(x<a)=1-e^{-\lambda a}}$ $\boxed{P(x>a)=e^{-\lambda a}}$

- $E(x)=\text{Mean}=\int_0^\infty x\,f(x)=\int_0^\infty x\,\lambda e^{-\lambda x}=\frac{1}{\lambda}$

- $Var(x)=\frac{1}{\lambda^2}$

$\boxed{P(a<x<b)=e^{-\lambda a}-e^{-\lambda b}}$

Ex:- Duration of phone call is exponentially distributed. On an average, duration is 15 min. Find the probability that

i) $P(0 < x < 30)$

ii) $P(15 < x < 60)$

iii) $P(x > 60)$

$$f(x) = \frac{1}{15} e^{-\frac{x}{15}}$$

$$\text{Mean} = 15 = \frac{1}{\lambda}$$

$$\boxed{\lambda = 1/15}$$

i) $$P(0<x<30) = \int_0^{30} \frac{1}{15} e^{-\frac{x}{15}} dx = 1 - e^{-\frac{1}{15}\times 30} = 1 - e^{-2} = 0.864$$

ii) $$P(15<x<60) = \int_{15}^{60} \frac{1}{15} e^{-\frac{x}{15}} dx = e^{-\frac{1}{15}\times 15} - e^{-\frac{1}{15}\times 60} = e^{-1} - e^{-4} = 0.349$$

$a < x < b$

iii) $$P(x>60) = \int_{60}^{\infty} \frac{1}{15} e^{-x/15} dx = 1 - P(x<60) = e^{-\frac{1}{15}\times 60} = e^{-4} = 0.018$$

4) Gamma Random Variable

$$f(x) = \begin{cases} \dfrac{\lambda e^{-\lambda x} (\lambda x)^{\alpha}}{\lfloor\underline{\alpha}} & ; \ x \geq 0 \\ 0 & ; \ x < 0 \end{cases}$$

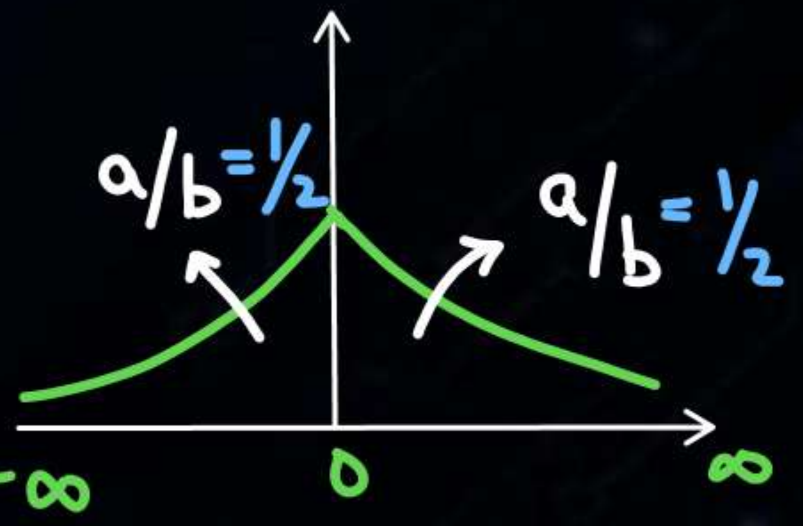
Parameters
$\lambda > 0, \ \alpha > 0$

5) Laplacian Random variable

$$f(x) = a e^{-b|x|} \qquad (-\infty, \infty)$$

Parameters $\begin{cases} \to a > 0 \\ \to b > 0 \end{cases}$

$$\boxed{\int_{-\infty}^{+\infty} a e^{-b|x|} \, dx = 2 \frac{a}{b}} \to 1$$



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6 .Normal Random Variable/Normal Distribution/ Gaussian Distribution

It is the most prominent probability distribution in statistics.

Gaussian function
$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

Parameters → Mean (μ) = $\int_{-\infty}^{+\infty} x\, f(x)\, dx$

→ S.D. (σ) = $\sqrt{Var(x)}$

Abritrary normal distribution (x) ⟶ Standard normal distribution (z)

$X \to \mu, \sigma$

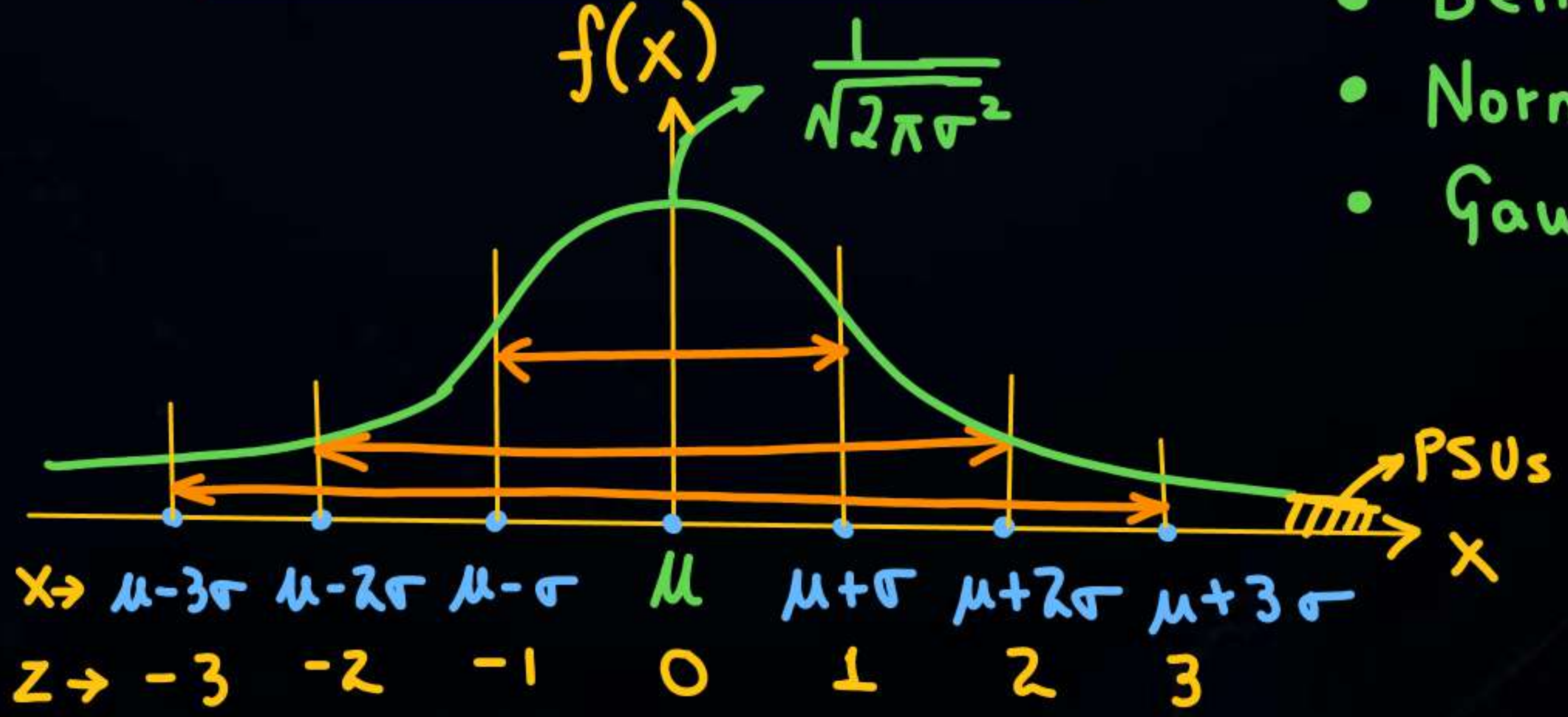
Mean = 0
S.D. = 1

$Z \to \mu = 0$
$\sigma = 1$

Standard Normal Distribution

Standard normal deviate ; $z = \frac{x - \mu}{\sigma}$

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2}}$$



- Bell shaped curve
- Normal dist. curve
- Gaussian dist. curve

Note:

i) $P(-1 < Z < 1) = 68\%$ $= P(\mu - \sigma < x < \mu + \sigma)$

ii) $P(-2 < Z < 2) = 95.5\%$ $= P(\mu - 2\sigma < x < \mu + 2\sigma)$

iii) $P(-3 < Z < 3) = 99.7\%$ $= P(\mu - 3\sigma < x < \mu + 3\sigma)$

Ex:- In a GATE Paper, Mean = 15 Marks & Standard deviation = 20 Marks. Find the probability candidate

i) Crosses cutoff $P(x > \mu + \sigma) = P(x > 35) \Rightarrow 16\% = 0.16$

ii) $P(x > 55) = P(x > 55) = P(x > \mu + 2\sigma) \Rightarrow 2.25\% = 0.0225$

iii) $P(x > 75) = P(x > 75) = P(x > \mu + 3\sigma) \Rightarrow 0.15\% = 0.0015$

iv) $P(\mu < x < \mu + \sigma) = 34\% = 0.34$

Thank you

GW Soldiers !