ENGINEERING MATHEMATICS

ALL BRANCHES

Numerical Methods

Numerical Integration & Solution of Differential equation

DPP - 03 Solution

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Question 1

Consider an ordinary differential equation $\boxed{\frac{dx}{dt} = 4t + 4}$ If $x = x_0$, at $t = 0$, the increment in x calculated using Runge-Kutta fourth order multi-step method with a step size of $\underbrace{\Delta t = 0.2}$ is

A 0.22

B 0.44

C 0.66

D 0.88

$h = 0.2$

$\frac{dx}{dt} = 4t + 4 \qquad t_0 = 0 \qquad x = x_0$

$(0, x_0)$

$\downarrow \quad \downarrow$

$0.2,$

$x_1 = x_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = h\,f(t_0, x_0) = 0.2\,f(0, x_0) = 0.2(4\times 0 + 4) = \boxed{0.8}$

$K_2 = h\,f\left(t_0 + \frac{h}{2}, x_0 + \frac{K_1}{2}\right) = 0.2\,f(0.1, x_0 + 0.4) = 0.2(4\times 0.1 + 4)$

$= \boxed{0.88}$

$$K_3 = h f\left(t_0 + \frac{h}{2}, x_0 + K_2/2\right) = 0.2 f(0.1, x_0 + 0.44)$$

$$= 0.2(4 \times 0.1 + 4) = \boxed{0.88}$$

$$K_4 = h f(t_0 + h, x_0 + K_3) = 0.2 f(0.2, x_0 + 0.88)$$

$$= 0.2(4 \times 0.2 + 4) = \boxed{0.96}$$

$$x_1 = x_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$\text{Increment} = x_1 - x_0 = \frac{1}{6}(0.8 + 2 \times 0.88 + 2 \times 0.88 + 0.96)$$

$$\Delta x = \boxed{0.88}$$

Question 2

The ordinary differential equation $\frac{dx}{dt} = -3x + 2$, with $x(0) = 1$ is to be solved using the forward Euler method. The largest time step that can be used to solve the equation without making the numerical solution unstable is $\underline{2/3}$.

$$x_{n+1} = x_n + h f(t_n, x_n)$$

$$= x_n + h(-3x_n + 2)$$

$$x_{n+1} = \underbrace{(1-3h)}\, x_n + 2h$$

For method to be stable

$$|1-3h| < 1$$

$$-1 < 1-3h < 1$$

$$-2 < -3h < 0$$

$$\boxed{\frac{2}{3} > h > 0}$$

$\therefore$ Max. value $= 2/3$

Question 3

Consider the aquation $\frac{du}{dt} = 3t^2 + 1$with $u = 0$ at $t = 0$. This is numerically solved by using the forward Euler method with a step size, $\Delta t = 2$.

The absolute error in the solution at the end of the first-time step is ____

Euler method; $u_1 = u_0 + hf(t_0, u_0)$

$= 0 + 2(3t_0^2 + 1) = 0 + 2(3 \times 0^2 + 1) = 2$

Now for exact value, $\int \frac{du}{dt} = \int 3t^2 + 1$

$u = [t^3 + t]_0^2 = 2^3 + 2 = 10$

Absolute error $= 10 - 2 = 8$

Question 4

Match the problem type Group-I with the numerical method in Group-2

Group-I		Group-II	
(P)	System of linear algebraic equations	(i)	Newton-Raphson
(Q)	Non-linear algebraic equations	(ii)	Gauss-seidel
(R)	Ordinary differential equations	(iii)	Simphson's Rule
(S)	Numerical integrations	(iv)	Runge-Kutta

Choose the correct set of combinations.

A P – II, Q – I, R – III, S – IV

B P – IV, Q – III, R – II, S – I

C P – I, Q – II, R – IV, S – III

D P – II, Q – I, R – IV, S – III

Question 5

Consider a differential equation $\frac{dy(x)}{dx} - y(x) = x$ with the initial condition $y(0) = 0$. Using Euler's first order method with a step size of 0.1, the value of $y(0.3)$ is

A 0.01

B 0.031

C 0.0631

D 0.1

$y_1 = y_0 + h f(x_0, y_0)$ $\quad f(x,y) = x + y$

$= 0 + 0.1(0+0) = 0$

$y_2 = y_1 + h f(x_1, y_1)$

$= 0 + 0.1(x_1 + y_1) = 0 + 0.1(0.1 + 0)$

$= 0.01$

$y_3 = y_2 + h f(x_2, y_2)$

$= 0.01 + 0.1(x_2 + y_2) = 0.01 + 0.1(0.2 + 0.01) = \boxed{0.031}$

x	y
0	0
0.1	0
0.2	0.01
0.3	0.031

Question 6

Match the CORRECT pairs

Numerical integration Scheme		Order of Fitting Polynomial	
P.	Simpson's 3/8 Rule	1.	First
Q.	Trapezoidal Rule	2.	Second
R.	Simpson's 1/3 Rule	3.	Third

A P – 2, Q – 1, R – 3

B P – 3, Q – 2, R – 1

C P – 1, Q – 2, R – 3

D P – 3, Q – 1, R – 2

Question 7

The values of function $f(x)$ at 5 discrete points are given below.
Using Trapezoidal rule with step size of 0.1 the value of $\int_0^4 f(x)dx$ is ____.

x	0	0.1	0.2	0.3	0.4
$f(x)$	0 y_0	10 y_1	40 y_2	90 y_3	160 y_4

$$\int_0^4 f(x)\,dx = \frac{h}{2}\left\{(y_0+y_4)+2(y_1+y_2+y_3)\right\}$$

$$= \frac{0.1}{2}\left\{(0+160)+2(10+40+90)\right\}$$

$$= \boxed{22}$$

Question 8

The estimate of $\int_{0.5}^{1.5} \frac{dx}{x}$ obtained using Simpson's rule with three-point function evaluation exceeds the exact value by

$$\int_{0.5}^{1.5} \frac{1}{x}\, dx$$

$h = 0.5$

x	0.5	1	1.5
f(x)	2	1	2/3
	y_0	y_1	y_2

A 0.235

B 0.068

C 0.024

D 0.012

$$\int_{0.5}^{1.5} f(x)\,dx = \frac{h}{3}\left\{y_0 + 4y_1 + y_2\right\}$$

$$= \frac{0.5}{3}\left\{2 + 4\times 1 + \frac{2}{3}\right\}$$

$$= 1.1116$$

$$\int_{0.5}^{1.5} \frac{1}{x}\,dx = [\log x]_{0.5}^{1.5} = 1.0986$$

It exceeds by 0.012

Question 9

A calculator has an accuracy up to 8 digits after decimal. The value of $\int_0^{2\pi} \underline{\sin x}\, dx$ when evaluated using this calculator by trapezoidal method with 8 equal intervals, to 5 significant digits is

A 0.00000

B 1.0000

C 0.00500

D 0.00025

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
f(x)	0	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}$	0

$$\int_0^{2\pi} \sin x\, dx = \frac{\pi/4}{2}\left\{(0+0)+2\left(\frac{1}{\sqrt{2}}+1+\frac{1}{\sqrt{2}}+0-\frac{1}{\sqrt{2}}-1-\frac{1}{\sqrt{2}}\right)\right\}$$

$$= 0.00000$$

Question 10

Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ rule and

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.2	0.1	1/17	1/26	1/37
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^6 \frac{1}{1+x^2}\,dx = \frac{3\times1}{8}\left\{\left(1+\frac{1}{37}\right)+2\times0.1+3\left(0.5+0.2+\frac{1}{17}+\frac{1}{26}\right)\right\}$$

$$= 1.357080836$$

Thank you

GW

Soldiers !