Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics

Set Theory

DPP-08

[NAT]

1. Let (A, R) be a poset, then the number of below statements that are false is?

I. If (A, R) is a lattice, then it is a total order.

II. If (A, R) is a total order, then it is a lattice.

[MCQ]

2. If (A, R) is a lattice, with A finite, then (A, R) has:

(a) Only greatest element.

(b) Only least element.

(c) A greatest element and a least element.

(d) None of these

[MCQ]

3. Let M (R) be the relation matrix for relation R on A, with |A| = n. If (A, R) is total order, then the number of 1's appear in M(R)?

(a) $n+\binom{n}{2}$ (b) $\frac{n+1}{2}$

(c) $\frac{n-1}{2}$ (d) $2+\binom{n}{2}$

[MCQ]

4. Let D_{30} = {1,2,3,5,6,10,15, 30} and let the relation '|' be a partial ordering on D_{30}. The greatest lower bound of 10 and 15 is?

(a) 3 (b) 1

(c) 5 (d) 6

[MCQ]

5. For U = {1, 2, 3}, Let A = P (U). Define the relation R on A by B R C if $B \subseteq C$. How many ordered pairs are there in the relation R?

(a) 24 (b) 25

(c) 26 (d) 27

Answer Key

1. **(1)**
2. **(c)**
3. **(a)**
4. **(c)**
5. **(d)**

Hints and Solutions

1. (1)

I. False, Let U = {1, 2} A = P(U), and R be the inclusion relation. Then (A, R) is a lattice where for all S, T $\in$ A, lab {S, T} = S U T and glb {S, T} = S $\cap$ T. However, {1} and {2} are not related, so (A, R) is not a total order.

II. If (A, R) is a total order, then for all x, y $\in$ A, xRy or yRx. For xRy, *lub*{*x* , *y*} = y and *glb* {*x*, *y*} = *x*. Consequently, (A, R) is a lattice.

2. (c)

- Since A is finite, A has a maximal element. If x, y (x $\neq$ y) are both maximal elements, since x, y R lub {x, y}, then lub (x, y)

  must equal either x or y. Assume lub {x, y} = x. Then y Rx , so y cannot be maximal element. Hence A has a unique maximal element x. Now for each a $\in$ A, a $\neq$ x, If lub {a, x} $\neq$ x, then we contradict x being a maximal element. Hence a Rx for all a $\in$ A, so *x* is the greatest element in A.

3. (a)

The correct answer is $n+\binom{n}{2}$ for example substituting value of n = 5, we get 15 one's

4. (c)

glb of 10 and 15 = gcd of 10 and 15 = 5

5. (d)

If C $\subseteq$ U, then $0 \le |C| \le 3$. For $0 \le k \le 3$ there are $\binom{3}{k}$ subsets C of U where |C| = k; each such subset C determines 2^k subsets B $\subseteq$ C. Hence the relation R contains $\binom{3}{0}2^0+\binom{3}{1}2^1+\binom{3}{2}2^2+\binom{3}{3}2^3 = (1+2)^3 = 3^3 = 27$ ordered pairs.