CS & IT ENGINEERING

Algorithm

Backtracking and Branch & Bound

One Shot

Topics to be Covered

Topic Backtracking and Branch & Bound

Backtracking

Represents one of the most general techniques. Many problems which deal with searching for a set of solutions or which ask for an optimal solution satisfying some constraints can be solved using the backtracking formulation.

The name backtrack was first coined by D. H. Lehmer in the 1950's. Early workers who studied the process were R. J. Walker who gave an algorithmic account of it in 1960 and Golomb and Baumert who presented a very general description of backtracking coupled with a variety of applications. (See the references for further details).

In order to apply the backtrack method, the desired solution must be expressible as an n-tuple (x_1, x) where the x_i are chosen from some finite set S_i. Often the problem to be solved calls for finding one vector which maximizes (or minimizes or satisfies) a criterion function $P(x_1, \ldots\ldots\ldots x_n)$. Sometimes it seeks all such vectors which satisfy P.

① n-Queens : n-Queens $(q_1 \cdots q_n)$; $n \times n$ CHBRD

$n = 4$ $\langle q_1 \cdots q_4 \rangle$

$X[1 \cdots n] = \langle x_1\ x_2\ x_3\ x_4 \rangle$

x_i = Position of q_i i.e placed in row 'i';

$\boxed{1 \le x_i \le n}$

1 2 3 4

q_1

q_2

q_3

q_4

$X : \langle 2, 4, 1, 3 \rangle$ $\langle 3, 1, 4, 2 \rangle$

Solution Space

Backtking

(State-Space)

4!

$\langle 1, 2, 3, 4 \rangle$

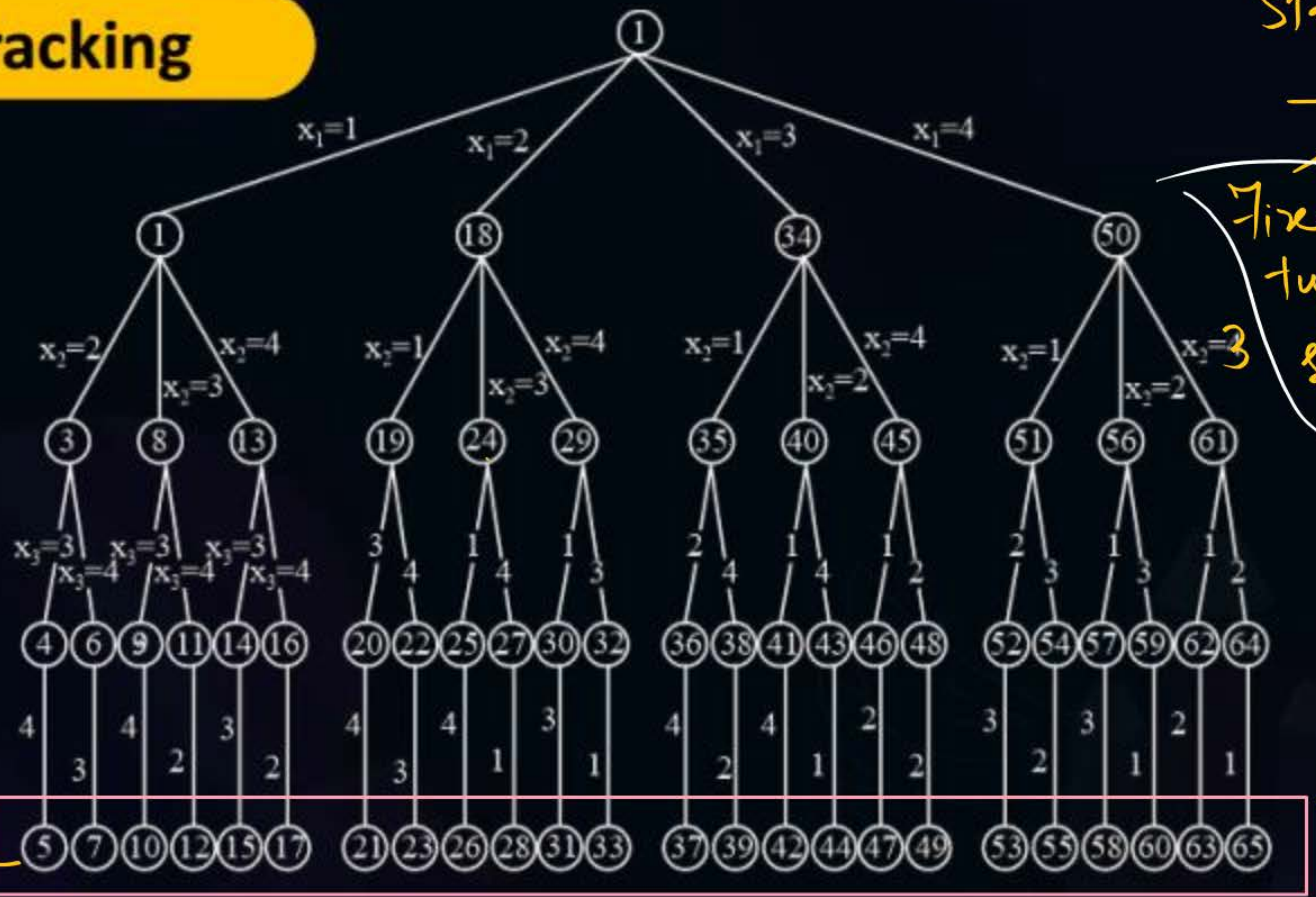
$\langle 1, 2, 4, 3 \rangle$

$\langle 1, 3, 2, 4 \rangle$

$\langle 1, 3, 4, 2 \rangle$

⋮

Backtracking



Tree organization of the 4-queens solution space. Nodes are numbered as in depth first search

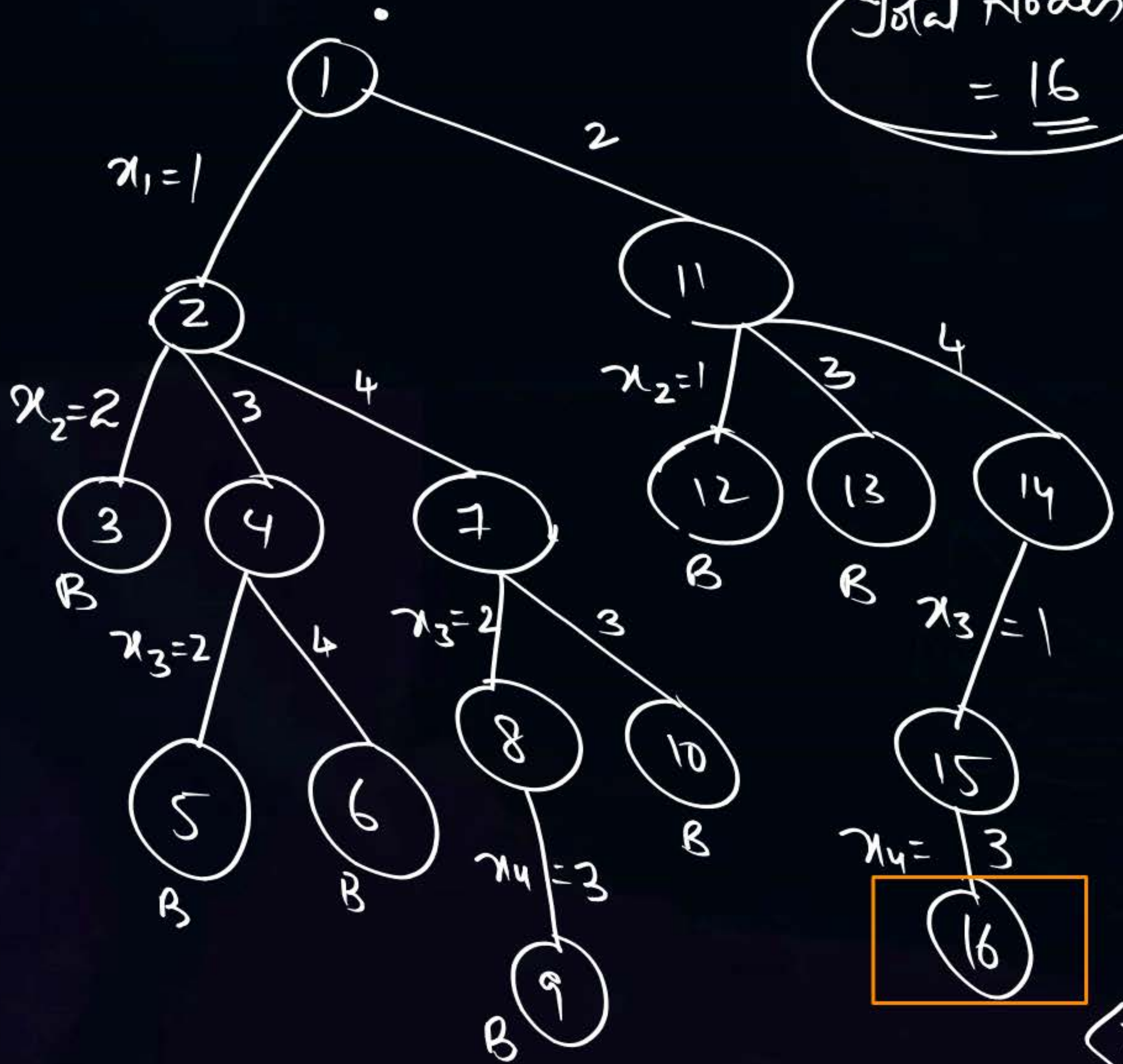
STATE Space Tree

$\Longrightarrow$ Backtracking uses D.F.S to search for $\frac{\text{Feasible}}{\text{Optimal}}$ Soln's in the Solution Space, & it also applies $\frac{\text{Bounding Fn's}}{\text{Criterian Fn's}}$ to kill/Bound those Nodes, that are Not feasible/optimal

$$\boxed{\text{Backtracking} = \text{D.F.S} + \text{Bounding fns}}$$

Portion of State-Space Tree Generated due to Backtracking Process for 4-Q's

Total Nodes = 16



$\langle 2,4,1,3 \rangle$

STACK

Variable tuple Size Formulation of State-Space Tree

Sum-of-Subsets: $\langle S.S \rangle$

n-elements; M

$A : \langle 1 .. n \rangle$

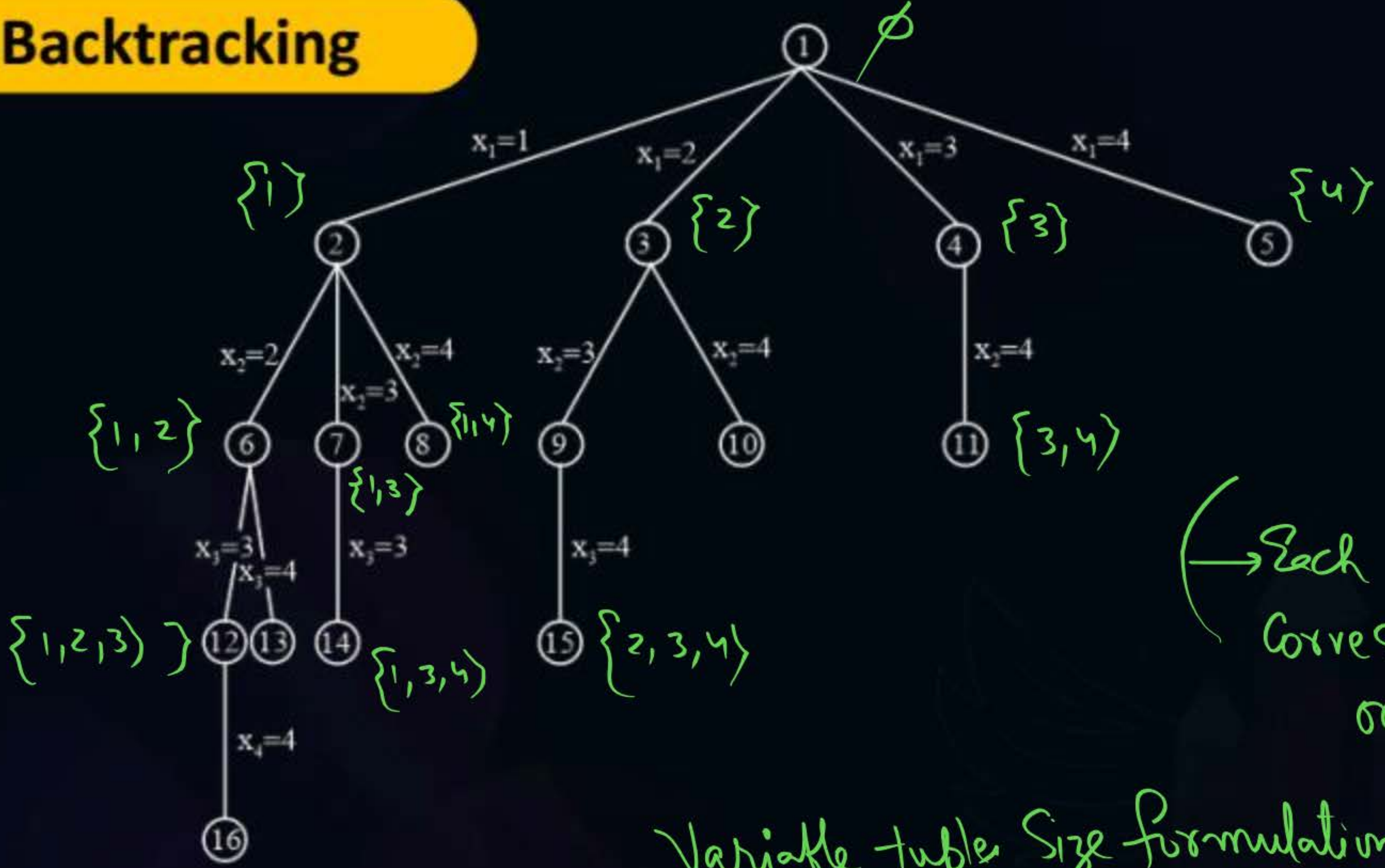
$n = 5$; $M = 50$

$A : \langle 10, 20, 30, 40, 50 \rangle$ $\Big\}\{2^n\}$

$\{20, 30\}$ $\{50\}$

$\{10, 40\}$

Backtracking



$n=4$

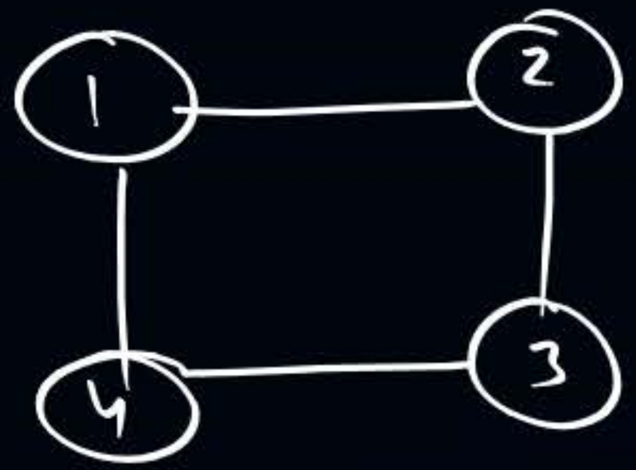
$: 2^4 = 16$

Each Node Corresponds to one Subset

Variable tuple Size formulation for Sum of Subsets

3) Graph coloring: It is reqd to paint the graph in such a way that no two adjacent vertices should have the same color;

$\langle x_1, x_2 x_3 x_4 \rangle$

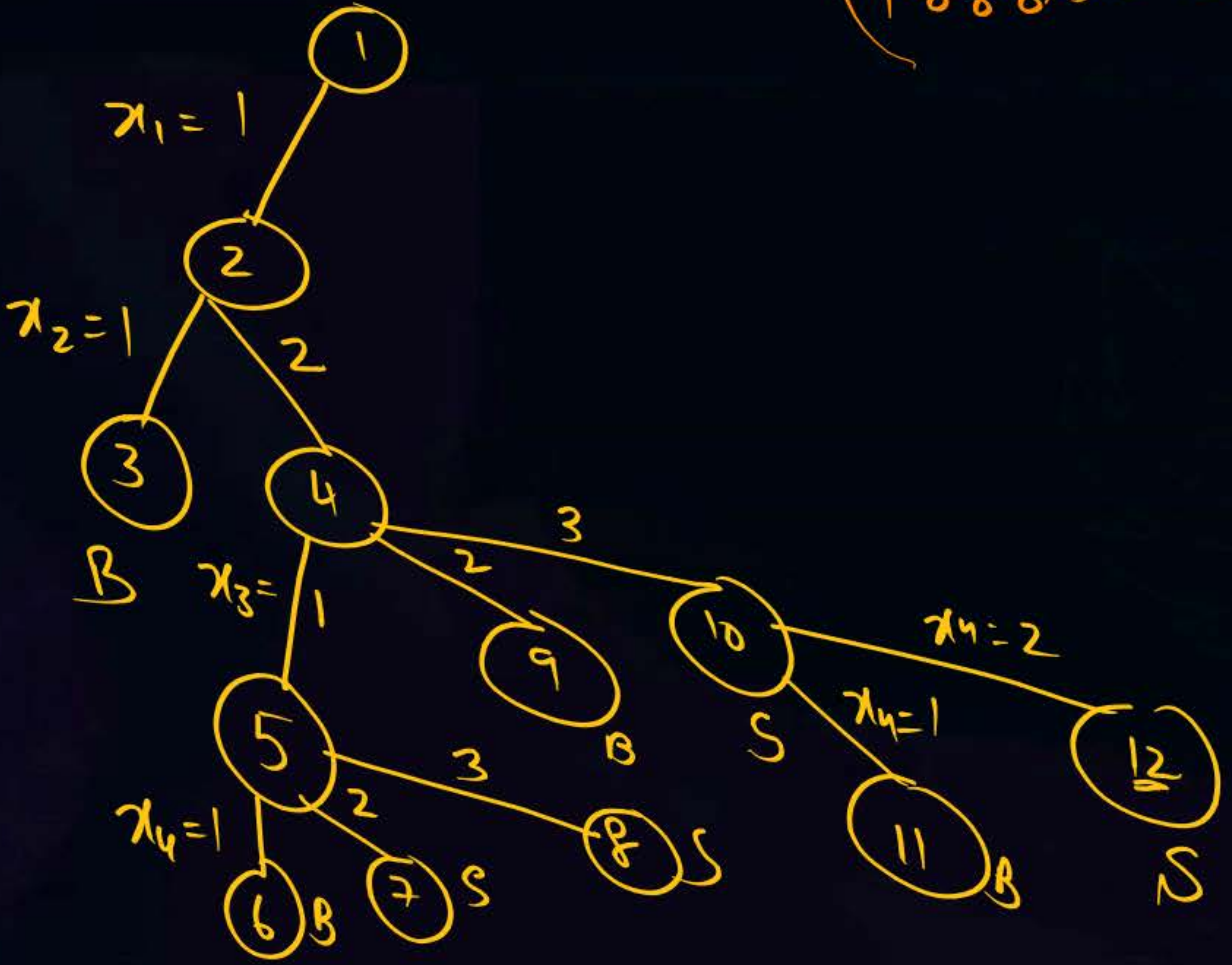


no. of colors $m = 3$

(Possible orderings)

$\langle$chromatic No.$\rangle$

(Painting of Atlas)



Popular Problems Solved by Backtracking

(i) N- Queens

(ii) Sum of Subsets

(iii) Graph Coloring

(iv) Hamiltonian Cycle

(vi) 0/1 KNAPSACK

Branch & Bound:

1) Backtracking = DFS + Bounding fn's

Stack

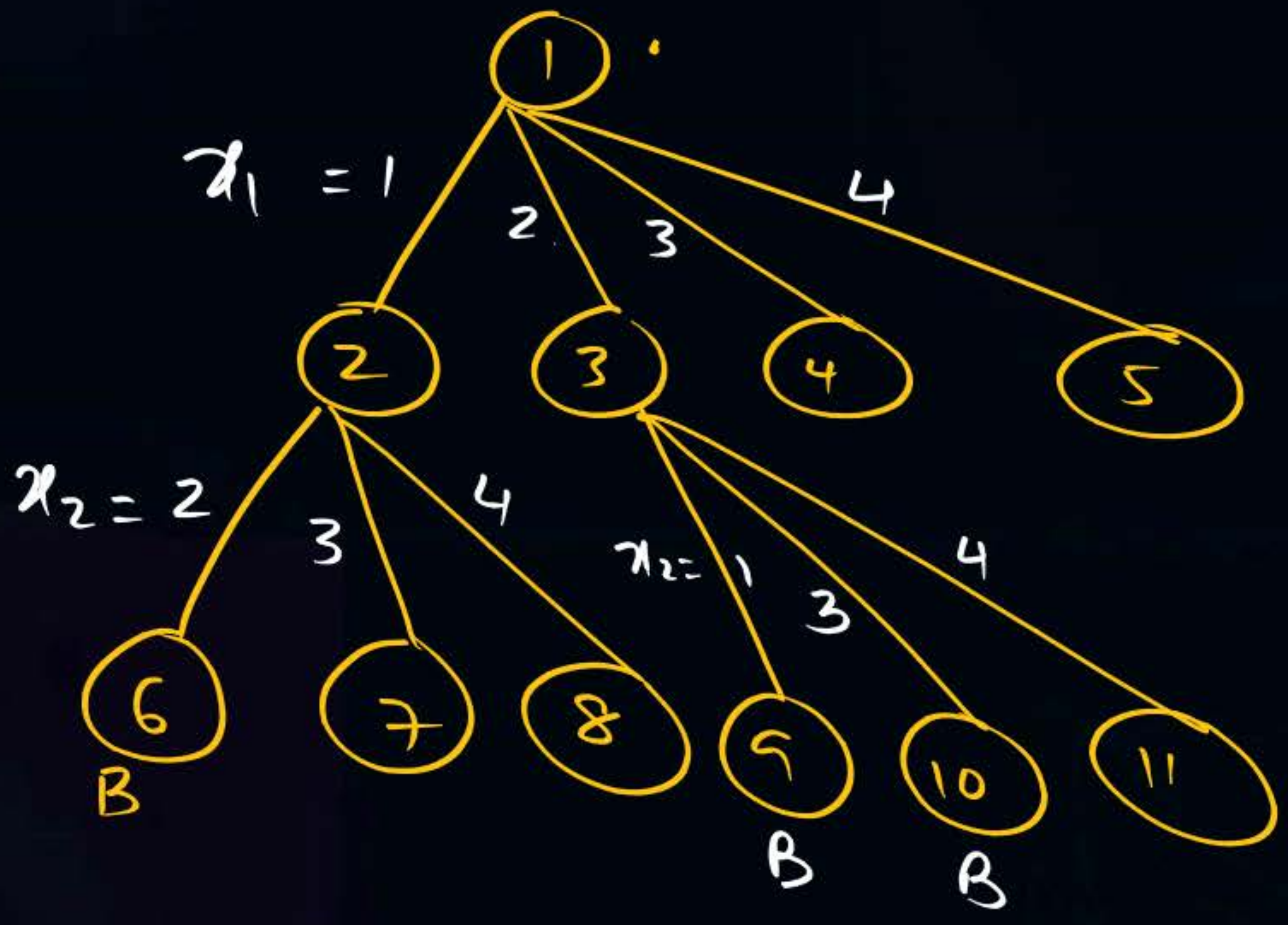
2) Branch & Bound = BFS + "

Queue:
- FIFO ✓
- LIFO ✓

FIFO-BB

LIFO-BB

Solving 4-Queens Problem using FIFO-BB



Q(FIFO)

~~2~~	~~3~~	~~4~~	5	7	8	11

No. of Nodes Generated = 31

Branch & Bound

1) n - Queens
2) 15 - Puzzle Problem
3) J. S. D
4) T. S. P
5) 0/1 KNAP

8	3	12	15
4	9	5	
6	2	1	14
7	10	11	13

1) Complete the Syllabus

↳ (Notes)

2) Practice Problems (Class)

3) P.Y.Q's ⟨ 25-30 yrs ⟩

4) Test-Series

- Time Mgmnt
- Coverage of Syllabus

5) No Peer Comparison

THANK - YOU