ENGINEERING MATHEMATICS

ALL BRANCHES

Rank of Matrix (Part-01)
Linear Algebra

DPP-04 Solution

By- CHETAN SIR

Question 1

The rank of $(m \times n)$ matrix (where $m < n$) cannot be more than

A m

B n

C mn

D None

$\text{Rank}(A) \leq \min.(m, n)$

Ex:-

i) $A_{3\times 4}$.

Possible ranks $\rightarrow 0, 1, 2, 3$

ii) $A_{2\times 3}$

Possible ranks $\rightarrow 0, 1, 2$

Question 2

The rank of the following $(n+1) \times (n+1)$ matrix, where 'a' is a real number is

$$\begin{bmatrix} 1 & a & a^2 & \dots & a^n \\ 1 & a & a^2 & \dots & a^n \\ \vdots & & & & \\ 1 & a & a^2 & \dots & a^n \end{bmatrix}_{(n+1)\times(n\times 1)} \xrightarrow[\substack{\vdots \\ R_{n+1} \to R_{n+1} - R_1}]{\substack{R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1}} \begin{bmatrix} 1 & a & a^2 & \dots & a_n \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

No. of non-zero rows $= 1$

$\rho(A) = 1$

$\therefore$ When all rows/columns are proportional then $\rho(A) = 1$

A $\checkmark$ 1

B 2

C n

D Depends on value of a

Question 3

The rank of the matrix $\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$

A 3

B 1

C 2

D 4

$$\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 4 & 8 & 7 \\ 4 & 2 & 3 & 1 \\ 0 & 0 & 3 & 0 \\ 3 & 12 & 24 & 2 \end{bmatrix} \xrightarrow[R_4 \to R_4 - 3R_1]{R_2 \to R_2 - 4R_1} \begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & -14 & -29 & -27 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -19 \end{bmatrix}$$

Row echelon form

No. of non-zero rows in Echelon form $= 4$

$$\therefore \rho(A) = 4$$

Question 4

Two matrices A and B are given below:

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix} B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$$

If the rank of matrix A is N, then the rank of matrix B is

A $\frac{N}{2}$

B N–1

C N

D 2N

$$\begin{array}{cc} A & A^T \\ \begin{bmatrix} p & q \\ r & s \end{bmatrix} & \begin{bmatrix} p & r \\ q & s \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$$

$\therefore AA^T = B$

$\rho(A) = \rho(A^T) = \rho(A^*) = \rho(A^{-1}) = \rho(AA^T)$

$\therefore \rho(A) = \rho(AA^T) = N$

Question 5

Let A be a 4 × 3 real matrix with rank 2. Which one of the following statement is TRUE?

$A_{4\times 3}$

Given $\rho(A) = 2$

A. Rank of $A^T A$ is less than 2.

B. Rank of $A^T A$ is equal to 2.

$\rho(A) = \rho(A^T) = \rho(A^{-1}) = \rho(AA^T) = \rho(A^T A)$

$\therefore \rho(A) = \rho(A^T A) = 2$

C. Rank of $A^T A$ is greater than 2.

D. Rank of $A^T A$ can be any number between 1 and 3.

Question 6

If v is a non-zero vector of dimension 3 × 1, then the matrix A = vv^T has a rank ___1___ .

Let $V = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1}$ $\longrightarrow$ Max Rank of V $= \min(1, 3)$

$\therefore \rho(V) = 1$

$A = VV^T = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}_{1\times 3}$

$\therefore \rho(V) = \rho(VV^T)$

$\rho(V) = \rho(A) = 1$

$A = \begin{bmatrix} x_1^2 & x_1x_2 & x_1x_3 \\ x_2x_1 & x_2^2 & x_2x_3 \\ x_3x_1 & x_3x_2 & x_3^2 \end{bmatrix}_{3\times 3}$

Question 7

If for a matrix, rank equals both the number of rows and number of columns, then the matrix is called

A Non-singular

B singular

C transpose

D minor

$A \rightarrow$ Matrix $(n \times n)$

Rank = No. of rows = No. of columns

Rank $\rightarrow$ Order of largest non-zero minor

Rank $\rightarrow n$

$|A| \neq 0$

$\therefore$ A is non-singular

Question 8

The rank of matrix $\begin{vmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{vmatrix}$

A 0

B 1

C 2

D 3

Since 3×3 minor $|A| = 0$ $\therefore \rho(A) < 3$

Now any 2×2 minor $|A| \neq 0$ $\therefore \rho(A) = 2$

Alternatively,

$$A = \begin{bmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix} \xleftrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 3 & 1 & 1 \\ 9 & 3 & 5 \\ 0 & 0 & -3 \end{bmatrix} \xleftrightarrow{R_2 \to R_2 - 3R_1} \begin{bmatrix} 3 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & -3 \end{bmatrix} \xleftrightarrow{R_3 \to 2R_3 + 3R_2}$$

Row Echelon form $\begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \rho(A) =$ No. of non-zero rows $= 2$

Thank you

GW Soldiers !