ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-09

Probability

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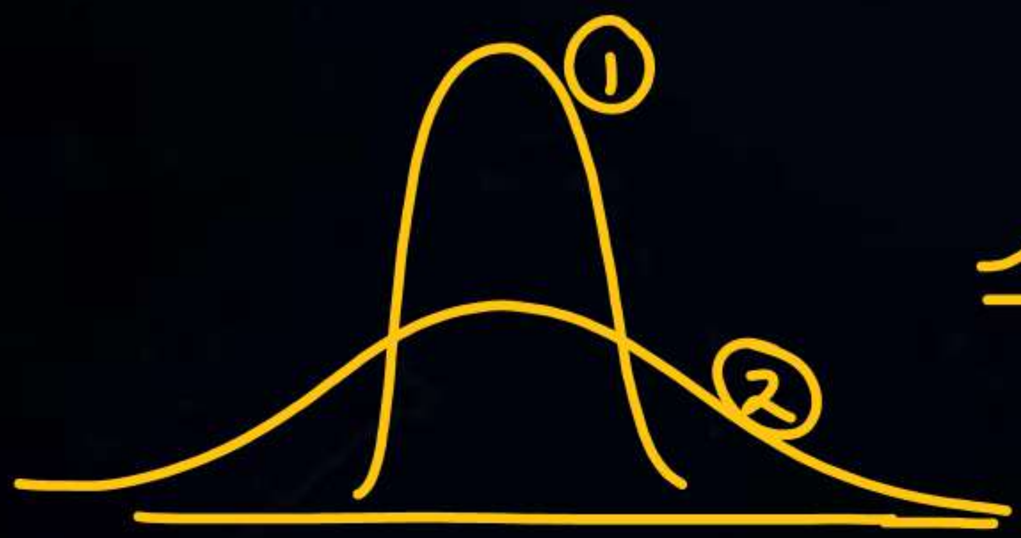
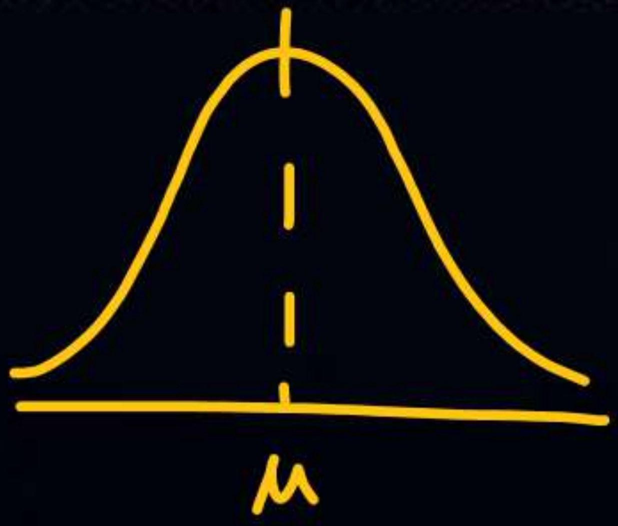
Topics To Be Covered

- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM
- STATISTICS – I (PROBABILITY DISTRIBUTIONS)
- STATISTICS – II (CORRELATION AND REGRESSION)

- Function f(x) is unimodal curve & attains it max. value at $x = \mu$
- Function f(x) is symmetric around the point $x = \mu$

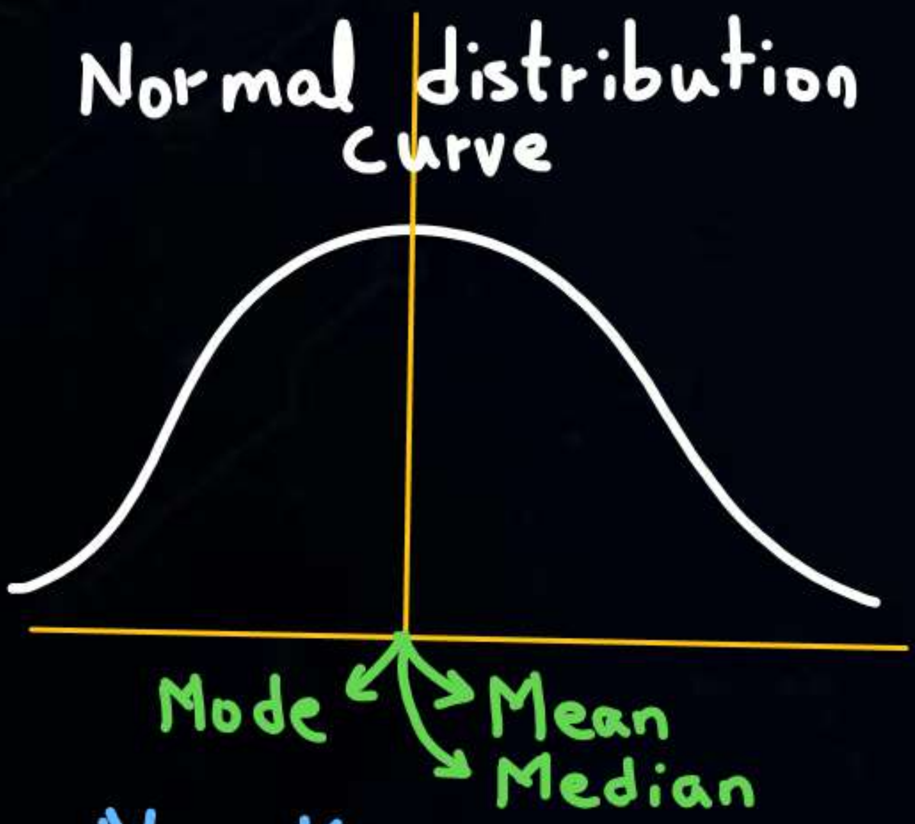




$Var \Rightarrow 2 > 1$

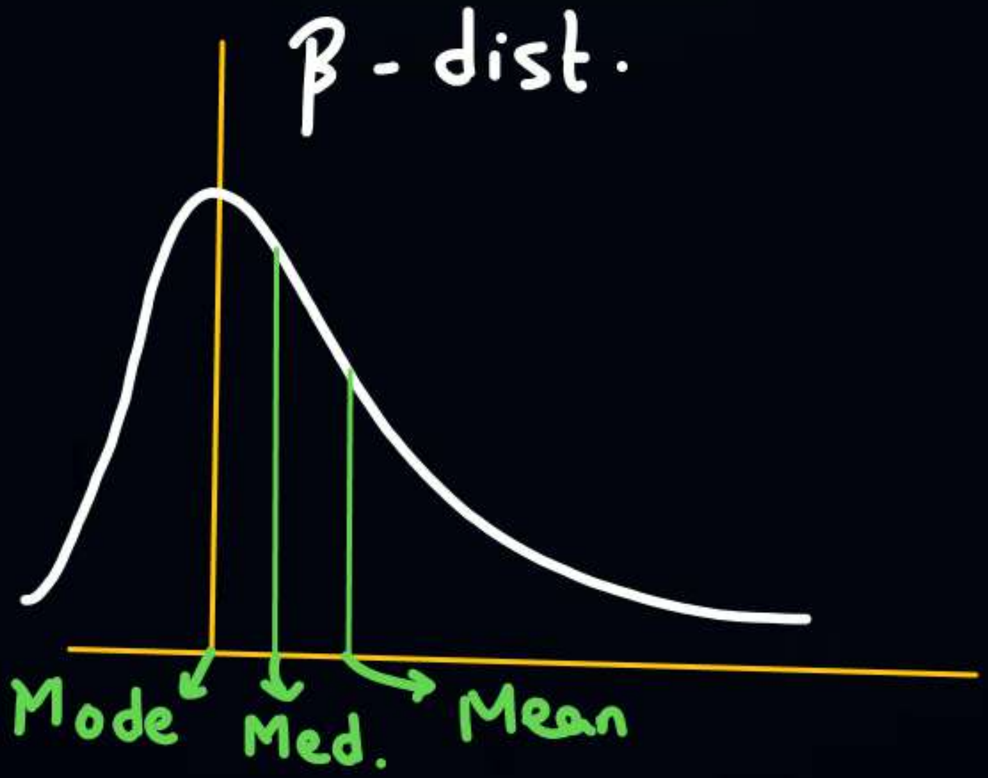
$\sigma \Rightarrow 2 > 1$

Skewness in Normal Curve



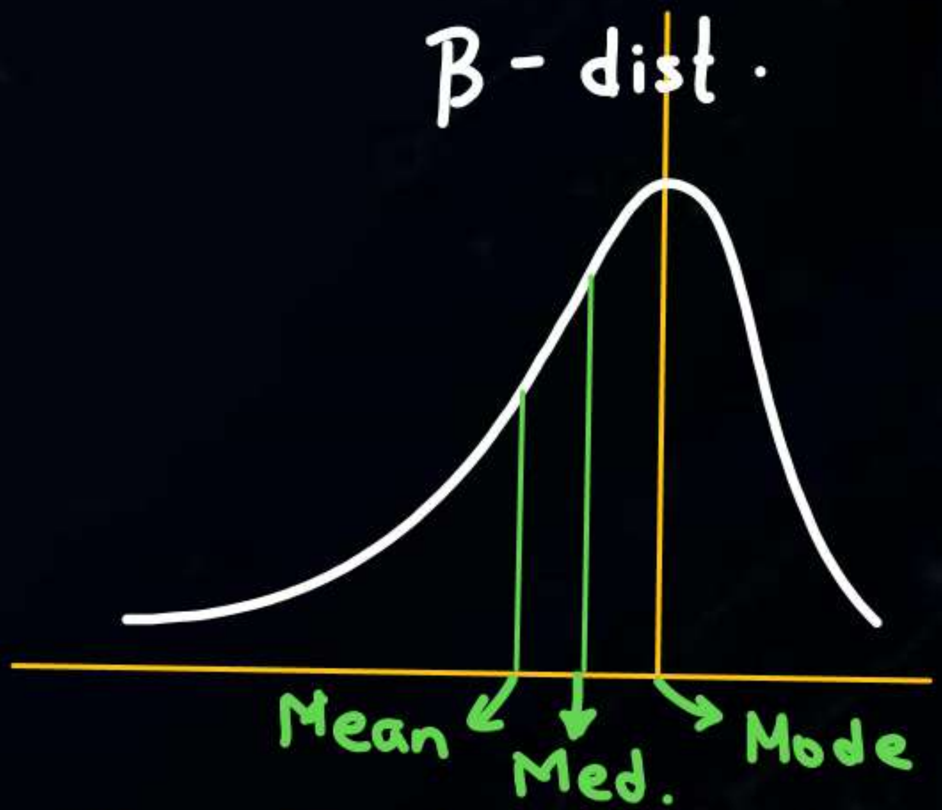
No skewness
(Symmetrical)

$\boxed{\text{Mean} = \text{Median} = \text{Mode}}$



Positively skewed
(Right - skewed dist.)

$\boxed{\text{Mode} < \text{Median} < \text{Mean}}$



Negatively skewed
(left skewed dist.)

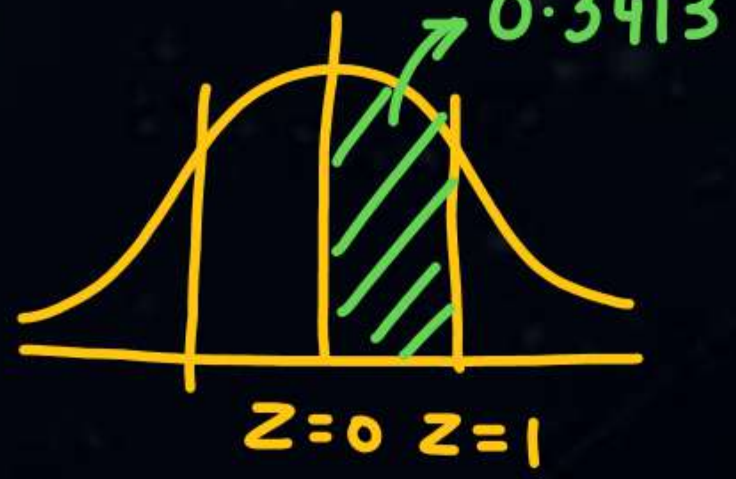
$\boxed{\text{Mode} > \text{Median} > \text{Mean}}$

$$\int_{-\infty}^{\infty} p(x)dx = 1$$

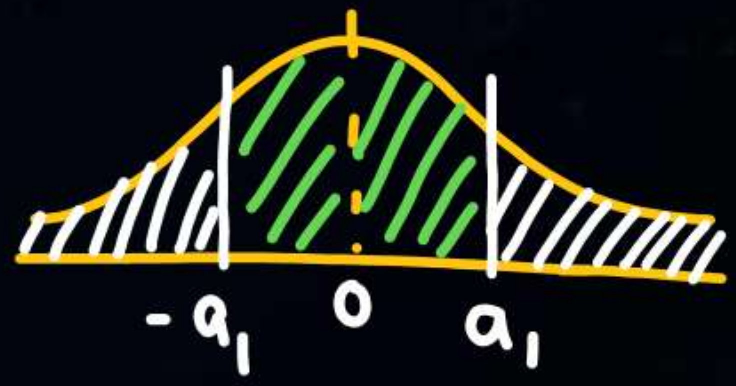
- Max. point of the curve occurs at $x = \mu$ & $P(x)_{max} = \frac{1}{\sqrt{2\pi\sigma^2}}$
- $z(-\infty \to 0) \to$ Exponentially↑
- $z(0 \to \infty) \to$ Exponentially↓

STATISTICS –I (PROBABILITY DISTRIBUTIONS

- $P(0 < Z < 1) = 0.3413$
  $P(0 < z < 2) = 0.4775$

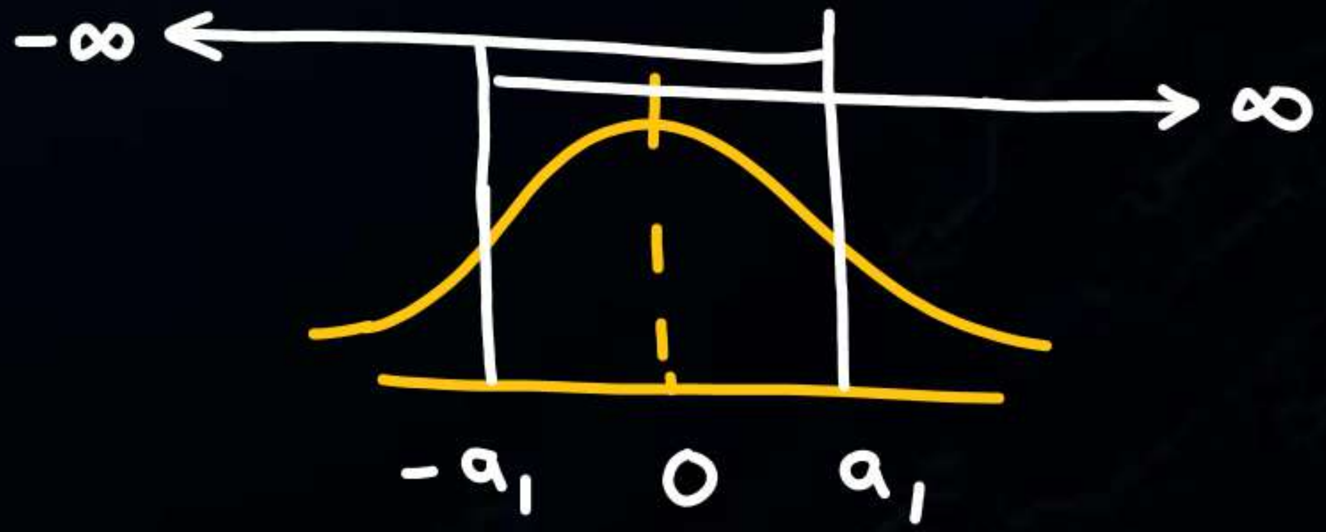


- $P(z < -a_1) = P(z > a)$



- $P(-a_1 < z < 0) = P(0 < z < a_1)$

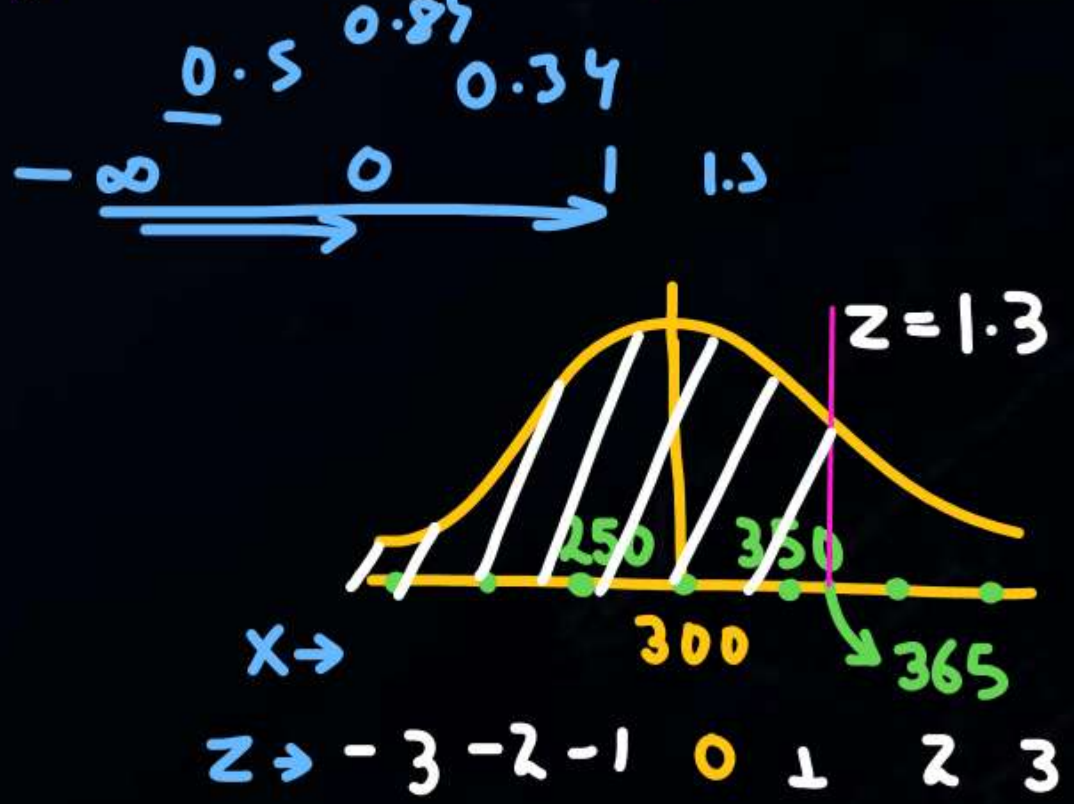
- $P(z > -a_1) = P(z < a_1)$



Ex:- An avg. light bulb lasts around 300 days with standard deviation equal to 50 days. Assuming that bulb life is normally distributed, what is the probability that light bulb will last at most 365 days?

Normal C.R.V. $X \rightarrow$ Life of bulb $\qquad \mu = 300$ days $\qquad \sigma = 50$ days

$$z = \frac{x-\mu}{\sigma} = \frac{365-300}{50} = 1.3$$



$P(x<365)$
$P(z<1.3)$ $= \displaystyle\int_{-\infty}^{1.3} \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2}\, dz = 0.9$

$$= P(z<0) + P(0<z<1.3)$$
$$0.5 + 0.4 \quad = 0.9$$

STATISTICS –I (PROBABILITY DISTRIBUTIONS)

Ex:- Suppose X → normal random variable with mean = 0 & variance = 4. Then the mean of absolute value of X is

A. $\frac{1}{\sqrt{2\pi}}$

B. $\frac{2\sqrt{2}}{\sqrt{\pi}}$ ✓

C. $\frac{2\sqrt{2}}{\pi}$

D. $\frac{2}{\sqrt{\pi}}$

✓ $\mu = 0$

$Var = \sigma^2 = 4$

✓ $\sigma = 2$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$f(x) = \frac{1}{2\sqrt{2\pi}} e^{-x^2/8}$$

$$E(|X|) = \int_{-\infty}^{+\infty} |x|\, f(x)\, dx = \cancel{2}\int_0^{\infty} x \cdot \frac{1}{\cancel{2}\sqrt{2\pi}} \cdot e^{-x^2/8}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{t=0}^{t=\infty} 4\, e^{-t}\, dt = \frac{4}{\sqrt{2\pi}} \left[\frac{e^{-t}}{-1}\right]_0^{\infty}$$

$$= \frac{4}{\sqrt{2\pi}} \cdot -[0-1]$$

$$= \frac{4}{\sqrt{2\pi}} = \frac{2\sqrt{2}}{\sqrt{\pi}}$$

$$\frac{x^2}{8} = t$$

$$\frac{2x}{8}\, dx = dt$$

$$x\, dx = 4\, dt$$

Properties of C.d.f :- (Cumulative distribution function)

$F_x(x)$

i) $F(a) = P(x \le a) = 1 - P(x > a)$

ii) $\lim_{x \to -\infty} F(x) = 0$

iii) $\lim_{x \to \infty} F(x) = 1$

iv) $F(-\infty) + F(\infty) = 1$

v) $F(x) \rightarrow$ non-decreasing $\begin{cases} \nearrow \text{increasing} \\ \searrow \text{constant} \end{cases}$

vi) $\boxed{0 \le F(x) \le 1}$

vii) $F(x)$ is not continuous always.

but it is always continuous from right.

$\{1, 2, \ldots 6\}$	$(0, 100)$
D.R.V.	C.R.V
$\rightarrow$ p.m.f (Prob. at single pt.)	$\rightarrow$ p.d.f. (Prob. in a range)
$\rightarrow P(x = a) \neq 0$	$\rightarrow P(a < x < b) \neq 0$
	$\rightarrow P(x = a) = 0$
(impulse)	(continuous) piecewise

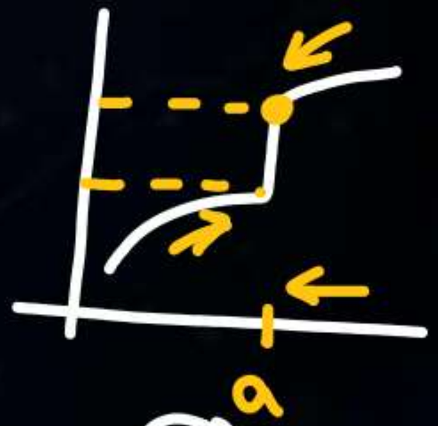
$F(a) = F(a^+)$

- → Continuous $F(a^-) = F(a) = F(a^+)$
- → Discontinuous $F(a^-) \neq F(a) = F(a^+)$

viii) Prob. at a single point

$$P(X=a) = F(a^+) - F(a^-)$$

①



②

If $F(x)$ is continuous at $x=a$ (C.R.V.)

$$P(X=a) = 0$$

If $F(x)$ is discontinuous at $x=a$ (D.R.V.)

$$P(X=a) = \text{Height of jump}$$

ix) Prob. in a range

$$P(a < X \le b) = F(b^+) - F(a^+)$$

Properties of p.d.f :- (Probability distribution function)

i) $$f(x) = \frac{d}{dx}[F(x)]$$ p.d.f $= \frac{d}{dx}$ (c.d.f).

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$ p.d.f. $\rightarrow f_x(x)$

ii) $F(x) \rightarrow$ non-decreasing
- $\rightarrow$ increasing
- $\rightarrow$ constant

$$\boxed{f(x) \geq 0}$$ $0 \leq f(x) < \infty$

iii) p.d.f. can never be odd fn.
- $\rightarrow$ Even fn. $\int_{-\infty}^{+\infty} f(x)dx = 2\int_{0}^{\infty} f(x)dx$
- $\rightarrow$ Neither even nor odd. $= 1$

$$\left[\because \int_{-\infty}^{+\infty} f(x)dx \neq 0\right]$$

iv) Prob. at a single point

$$P(X=a) = \int_{-a}^{+a} f(x)\,dx \begin{cases} \to 0 & \text{for all std. C.R.V.} \\ \to \neq 0 & \text{for impulse (Ex:- D.R.V., Mixed) at } x=a \end{cases}$$

v) Prob. in a range

$$P(a<x\le b) = \int_{a^+}^{b^+} f(x)\,dx = \text{Area under p.d.f.}$$

DRV	CRV
→ p.d.f ⇒ only impulse	→ p.d.f. ⇒ no impulse
→ c.d.f. ⇒ stepped (stairs)	→ c.d.f ⇒ no jump / steps.

Mean

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}}$$

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum f_i}}$$

1, 5, 9, 10

$1 \rightarrow 20$
$2 \rightarrow 10$
$3 \rightarrow 5$

$\underline{1, 1, 1, 1, 1, \ldots 1}, \underline{2, 2, 2.}$

$$\frac{1 \times 20 + 2 \times 10 + 3 \times 5}{35}$$

Median

Arrange the data in ascending or descending order.

n is odd ; Median = $\left(\frac{n+1}{2}\right)^{th}$ observation

n is even ; Median = $\dfrac{\left(\frac{n}{2}\right)^{th} \text{ ob.} + \left(\frac{n}{2}+1\right)^{th} \text{ ob.}}{2}$

→ middle most value of data.

Ex :- 1, 3, $\boxed{5}$, 9, 15 $(n=5)$ $\quad \left(\frac{5+1}{2}\right)^{th}$ ob. $= 3^{rd}$ ob. $= 5$

Ex :- 1, 3, $\boxed{5, 9}$, 15, 17 $(n=6)$ $\quad \dfrac{\left(\frac{6}{2}\right)^{th} + \left(\frac{6}{2}+1\right)^{th}}{2} = \dfrac{5+9}{2} = 7$

Mode – Value in a data which occurs most frequently.

Ex:- 60, 60, 60, 70, 40, 60, 50, 50

Mode = 60

Thank you

GW Soldiers !