CS & IT ENGINEERING

DISCRETE MATHS
Set theory

Lecture No. 03

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TOPICS

01 Types of functions

02 Composition of functions

03 Inverse function

Onto (Right side must be full) (Range = codomain)

Range $\subseteq$ codomain

Function: ✓

1:1 ✓

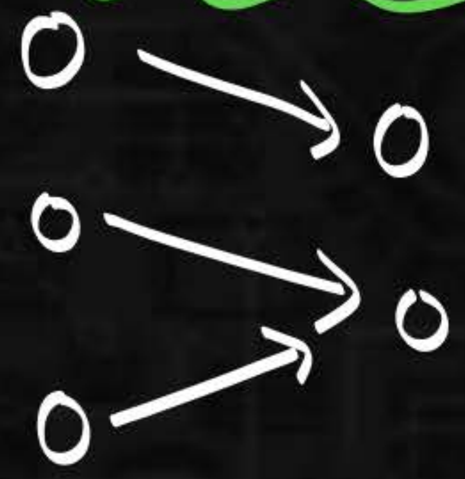
onto ✗

Range $\subseteq$ codomain

1:1 ✓

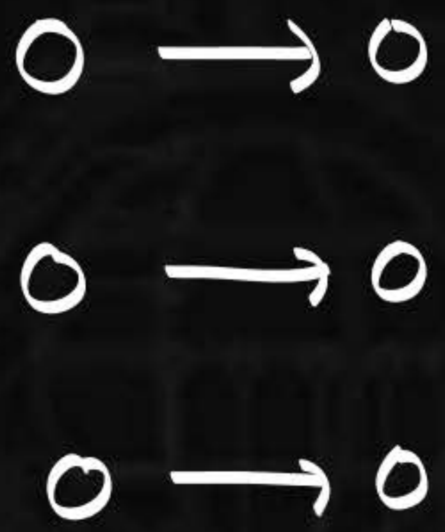
onto ✗

Range = codomain



1:1 ✗

onto ✓



$\begin{cases} \text{1:1 ✓} \\ \text{onto ✓} \end{cases}$

$f: Z \to Z$

$f(x) = x + 1$

x	a
1	2
2	3
3	4

$a \in Z$

$a - 1 \in Z$

$a - 1 \longrightarrow a$

$f(x) = x + 1$

$\downarrow$

$y = x + 1$

$y - 1 = x$

$x \qquad y$

$f : Z \to Z$

$f(x) = x^2$

$$\begin{array}{ccc} 1 & \longrightarrow & 1 \\ & & 2 \\ 2 & \searrow & 3 \\ & & 4 \end{array}$$

$y = 2$

$f(x) = x^2$

$2 = x^2$

$\sqrt{2} = x$

$\sqrt{2} \notin Z$

not onto

a) $f(x) = x + 7$ ✓

b) $f(x) = 2x - 3$ → not onto

c) $f(x) = -x + 5$ ✓

d) $f(x) = x^2$

e) $f(x) = x^2 + x$

f) $f(x) = x^3$

$f: \mathbb{Z} \to \mathbb{Z}$

$f: R \to R$ onto

$y = 2$.

$f(x) = 2x - 3$.

$y = 2x - 3$

$\frac{y+3}{2} = x$.

$f: \mathbb{Z} \to \mathbb{Z}$

$f(x) = 2x - 3$

$x = 1$.

$2(1) - 3 = 2 - 3 = -1$.

$x = 2$

$2(2) - 3 = 4 - 3 = 1$

$x = 3$

$2(3) - 3 = 3$.

$1 \to -1$

$\cdot\, 0$

$2 \to 1$

$\cdot\, 2$

$3 \to 3$

$\frac{y+3}{2} = x$.

$y = 2$

$\frac{2+3}{2} = x$

$f(x) = x + 7$

$f : Z \to Z$

$1 \longrightarrow 8$

$2 \longrightarrow 9$

$y - 7 \longrightarrow y$

$f(x) = y = x + 7$

$y - 7 = x$

$f(x) = x^2 + x$

$x = 0$

$0 \longrightarrow 0$

$1 \longrightarrow 2$

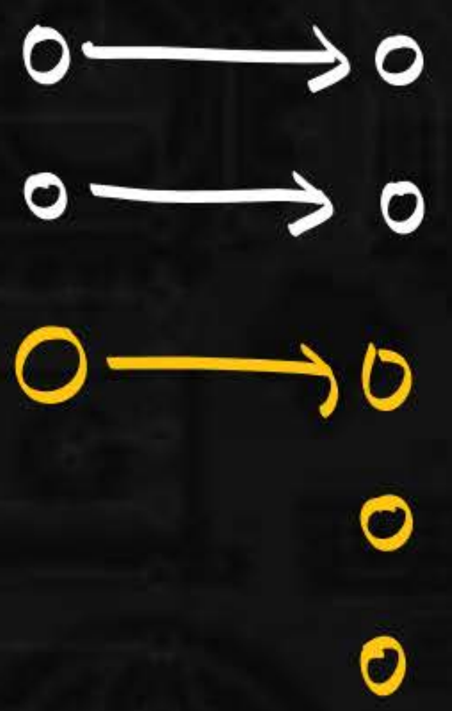
1

$f : A \to B$

$|A| = 3 \quad |B| = 5$

$|A| < |B|$

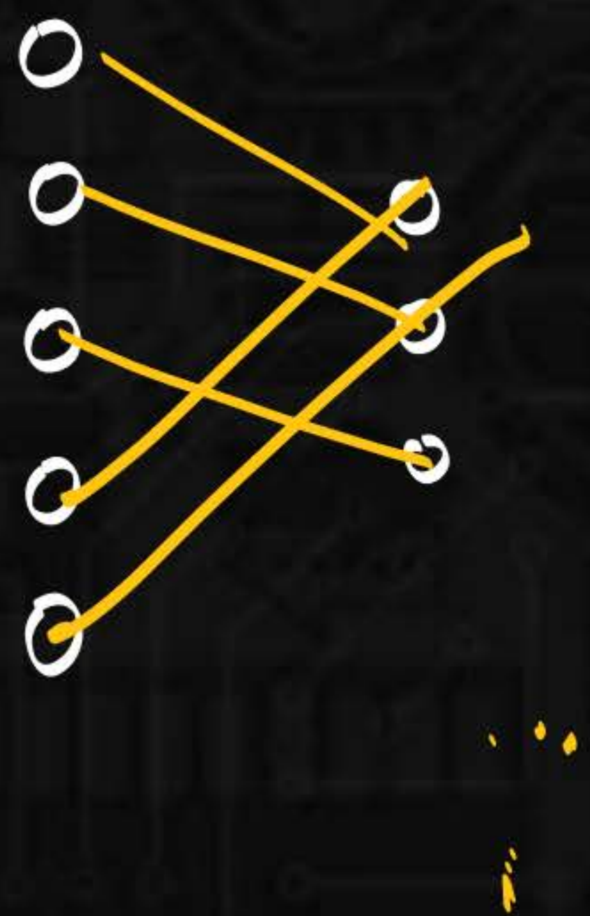
(onto is not possible)

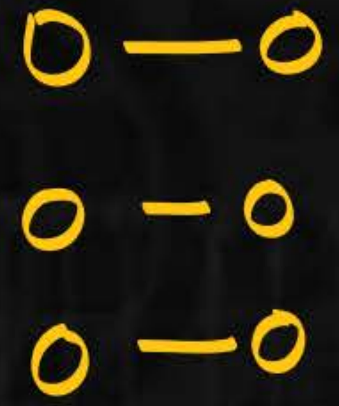


$f : A \to B$

$\boxed{|A| \geq |B|}$

$|A| = 5 \quad |B| = 3$



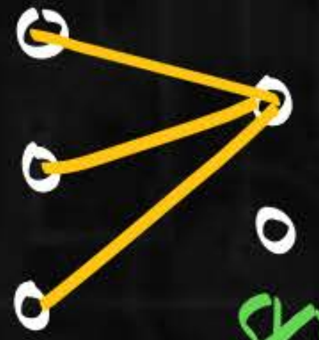


$|A| = 3$ $|B| = 2$ Total onto Functions = Total − Total non onto

non onto (R.S is not full)

$$= 2^3 - 2$$

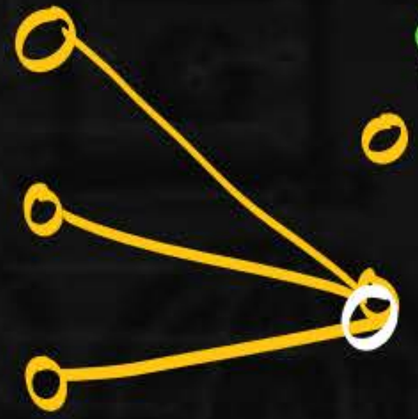
$$= 6 \text{ function (onto)}$$



(000)

Skip 1

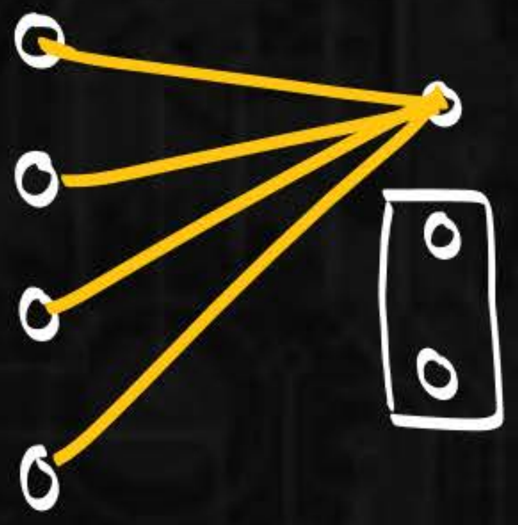
Skip



(111)

$f : A \to B \quad |A| = 4 \quad |B| = 3.$

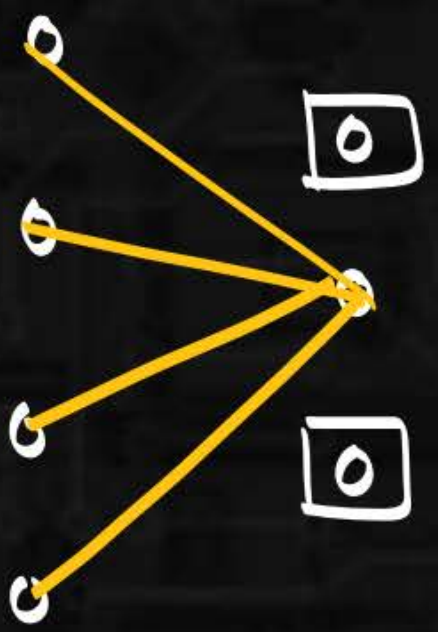
Total onto = Total − Total non onto.



1^4

non onto

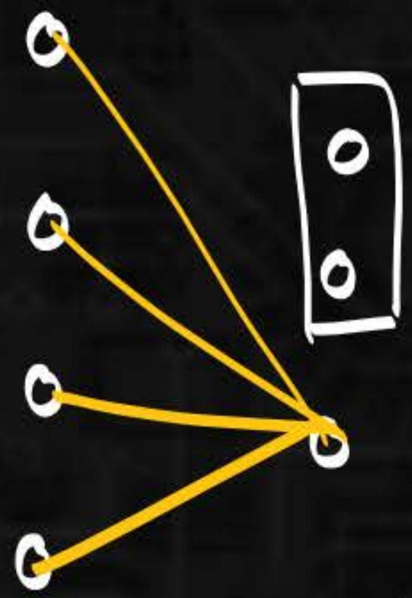
Skip 2



1^4

non onto

Skip → 2



1^4

non onto

Skip - 2.

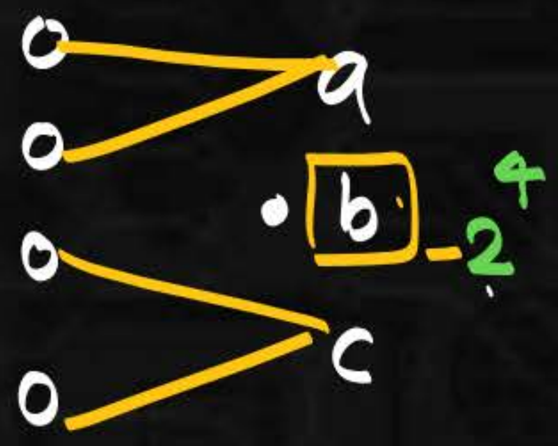
${}^3C_2 \; 1^4$

skipping 2 elements

$(RS)^{LS}$

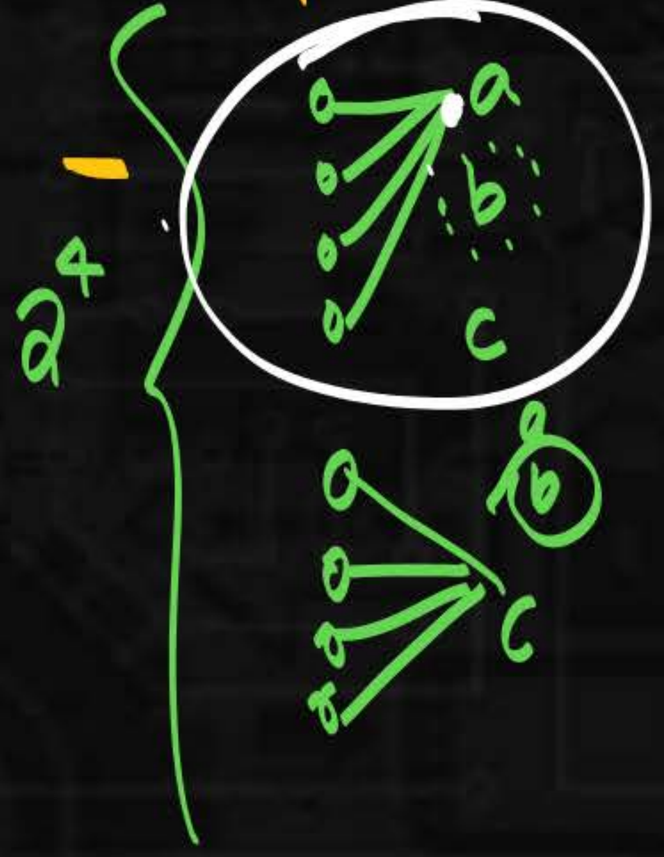
non onto

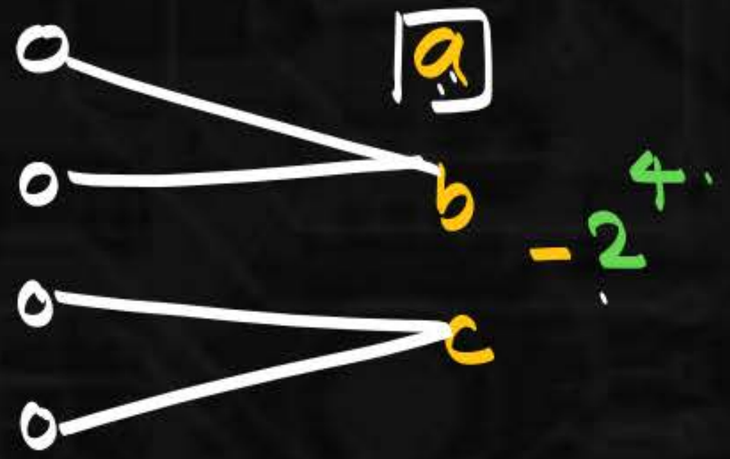
Skip c.



non onto

Skip b





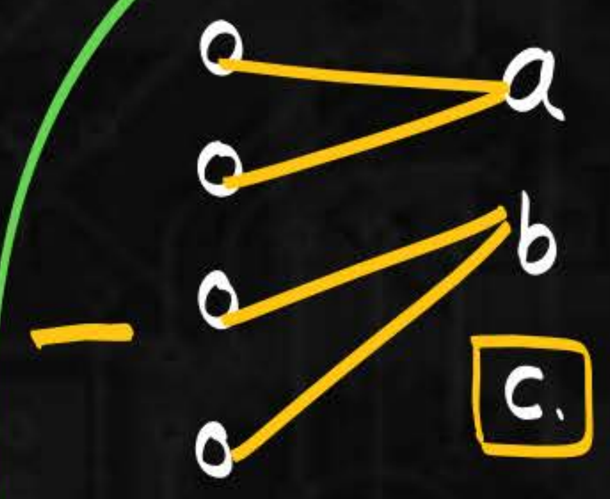
Skip 1.

$c^{\times}$ $b^{\times}$ $a^{\times}$

$2^4 + 2^4 + 2^4$

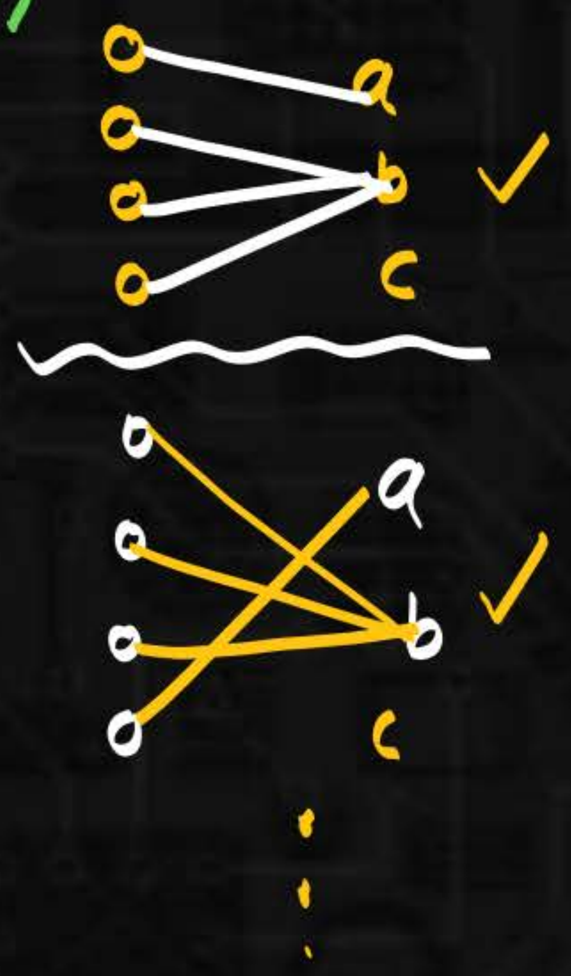
$^3C_1$

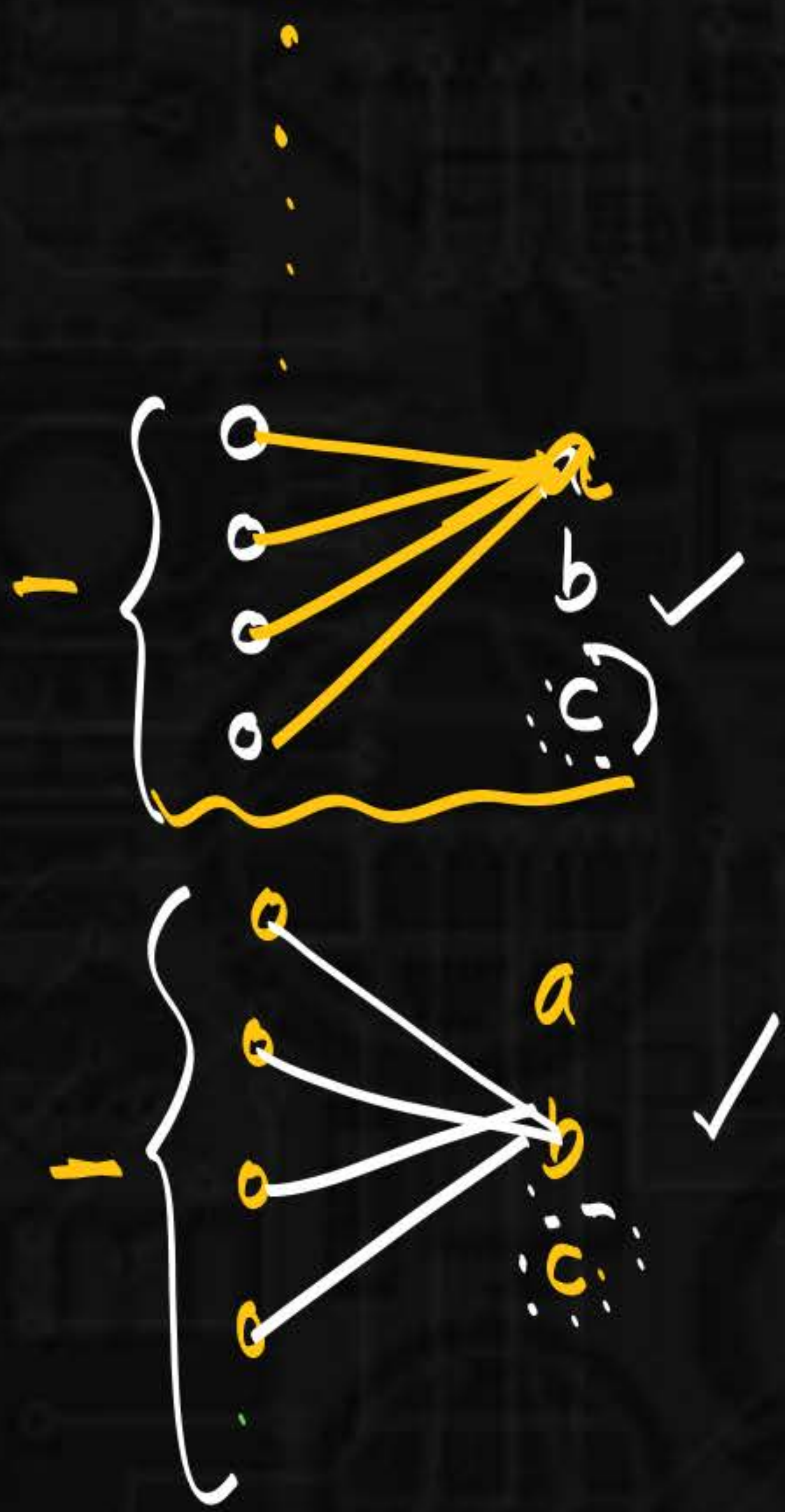
all → a, b.



non onto

Skip 1 → C.





Total Functions

if we are not pointing

on c → $2^4 = (R.S)^{L.S}$

$f: A \to B$

$|A| = 4$

$|B| = 3$

Total onto = Total Functions − Total non onto.

$$= 3^4 - {}^3C_1\, 2^4 + {}^3C_2\, 1^4$$

Skipping 1 element

Skipping 2 elements.

$f : A \to B$

$|A| = 4 = m$

$|B| = 3 = n$

Total onto = Total Functions − Total non onto.

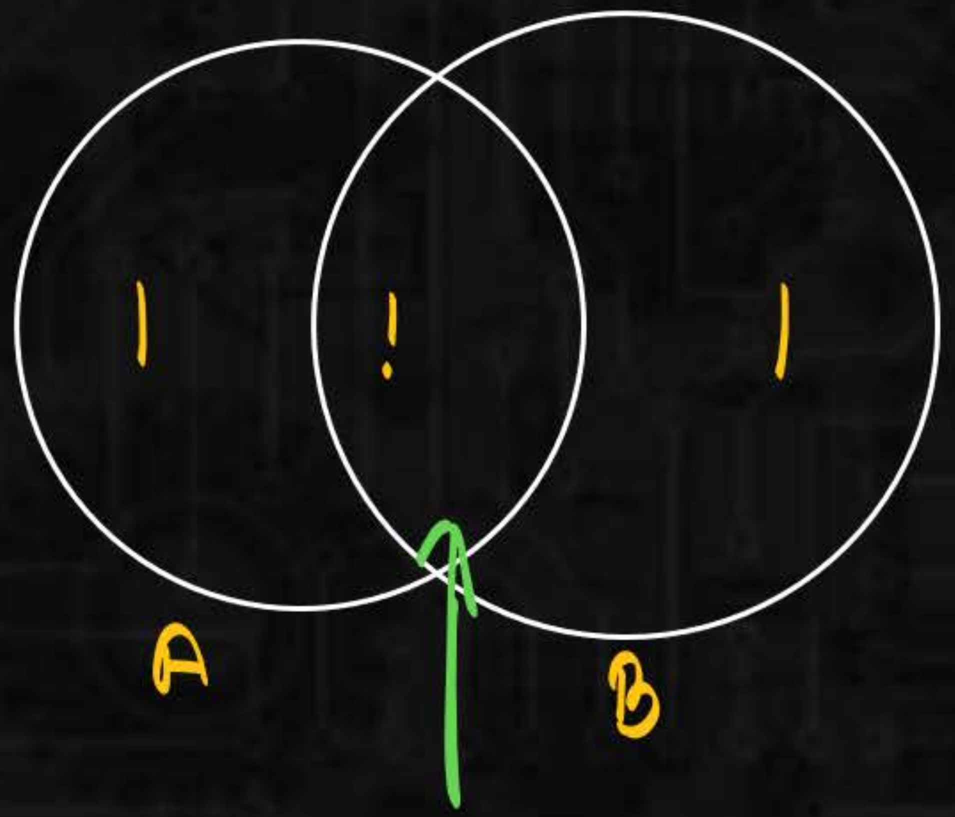
$= 3^4 - {}^3C_1\, 2^4 + {}^3C_2\, 1^4$

$= 3^4 - {}^3C_1 \cdot 2^4 + {}^3C_2 \cdot 1^4$

$$\sum_{i=0}^{n} (-1)^i \cdot {}^nC_i\, (n-i)^m$$

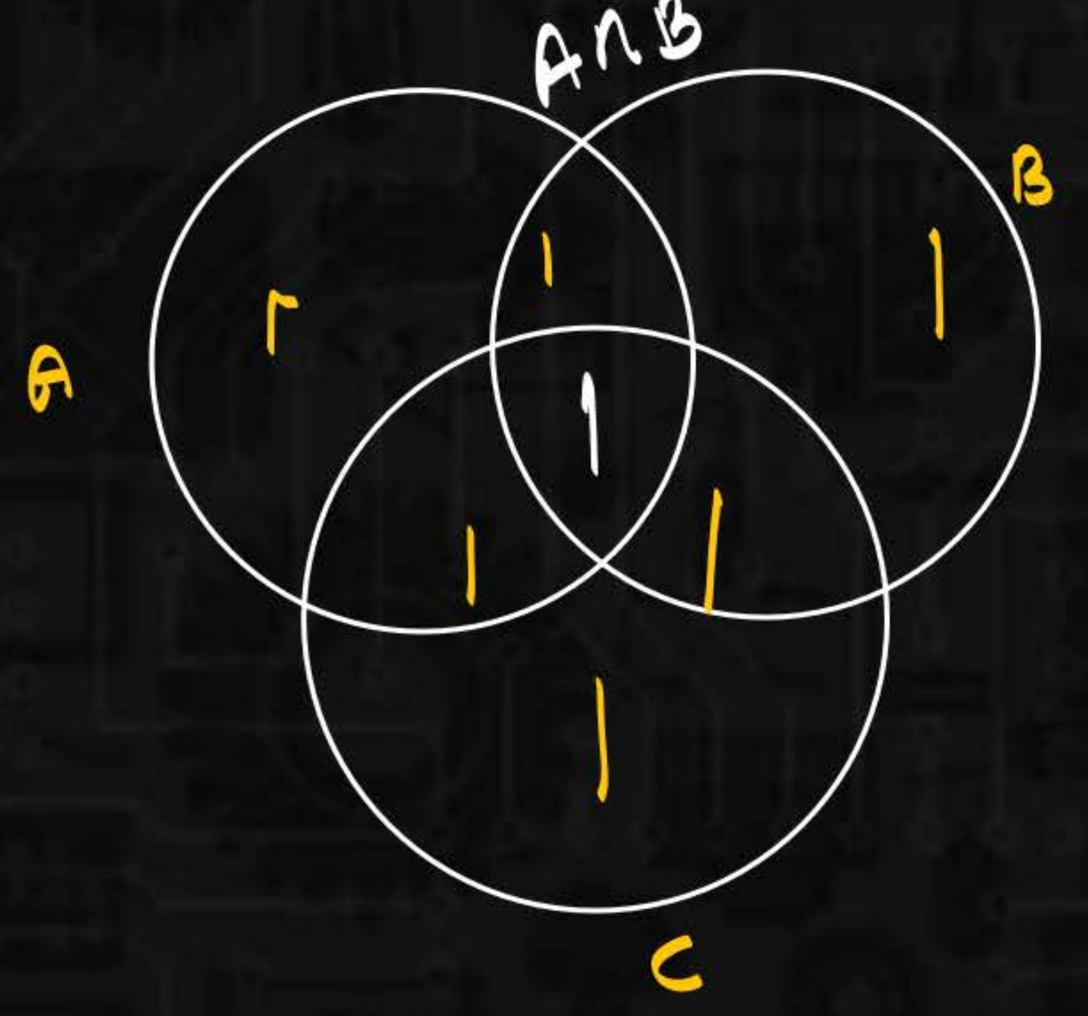
$= {}^3C_0 (3-0)^4 - {}^3C_1 (3-1)^4 + {}^3C_2 (3-2)^4$

$= {}^nC_0 (n-0)^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m - {}^nC_3 (n-3)^m$



$$A \cup B = \underline{A} + B - A \cap B.$$

$$4 - 1 = 3$$



$$= A + B + C - A\cap B - B\cap C - A\cap C$$
$$+ A\cap B\cap C.$$

$f: A \to B$

$|A| = 4$ $|B| = 3$

$m = 4$ $n = 3$

$3^4 - {}^3C_1 \cdot 2^4 + {}^3C_2 \cdot 1^4.$

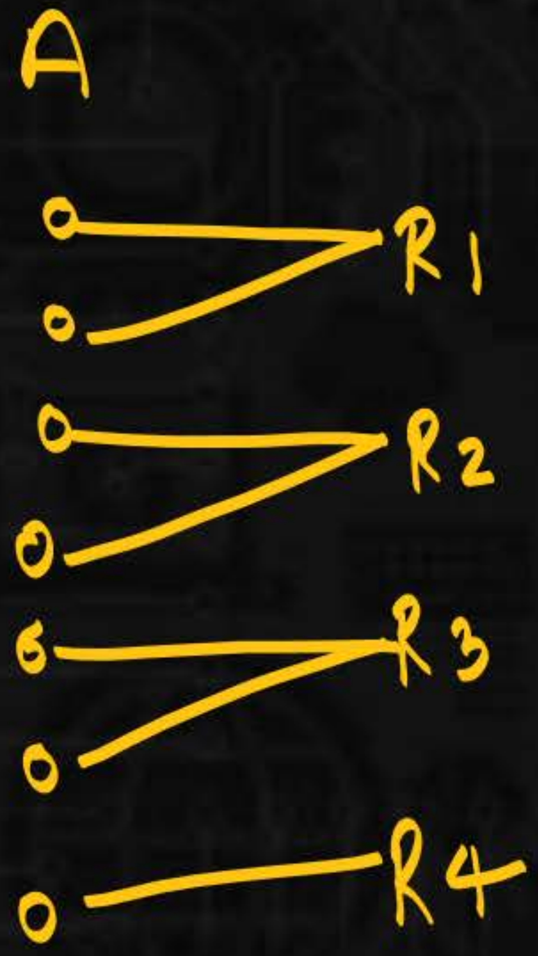
${}^3C_0(3-0)^4 - {}^3C_1(3-1)^4 + {}^3C_2(3-2)^4.$

${}^nC_0(n-0)^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - \dots$

$$\sum_{i=0}^{n} (-1)^i \cdot {}^nC_i \cdot (n-i)^m.$$

How many ways we can arrange 7 diff guest to 4 different rooms, such that none of the rooms must be empty?

Ans: 8400



$f: A \to B$

$|A| = 7 \quad |B| = 4.$

$m = 7 \quad n = 4.$

$$n_{C_0}(n-0)^m - n_{C_1}(n-1)^m + n_{C_2}(n-2)^m -$$

$$= 4_{C_0}(4-0)^7 - 4_{C_1}(4-1)^7 + 4_{C_2}(4-2)^7 - 4_{C_3}(4-3)^7 + 4_{C_4}(4-4)^7.$$

$f : A \to B$

$|A| = 3 \quad |B| = 2.$

3 diff quest $\to$ 2 same room

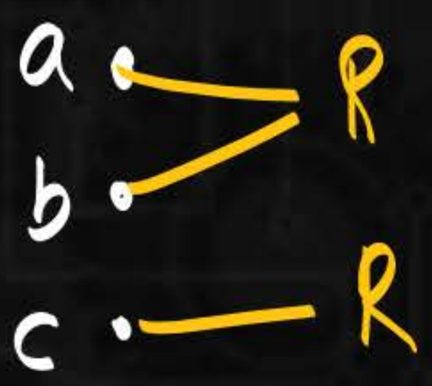
Total onto $= 2^3 - 2$

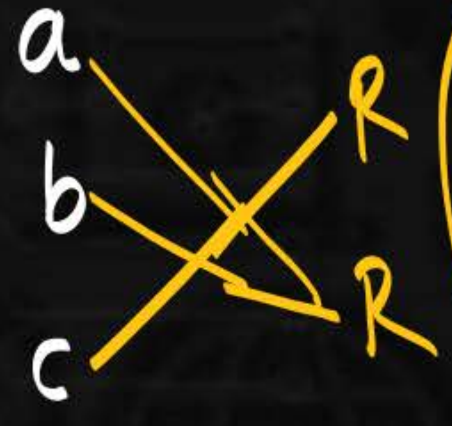
$= 8 - 2$

$= 6$

$\downarrow$

(onto)

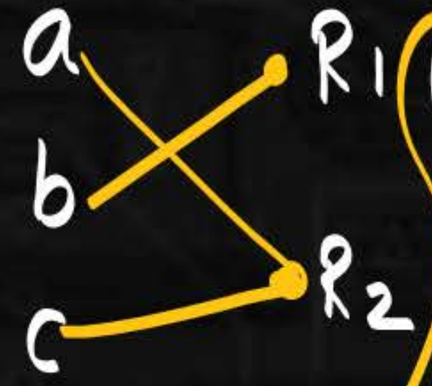


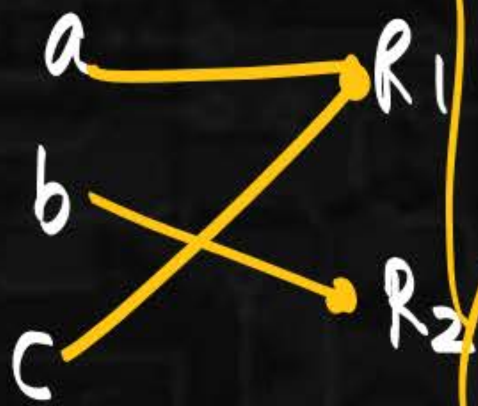


$(ab)\,(c)$

$(ab)\,(c)$

$(ab)\,(c)$

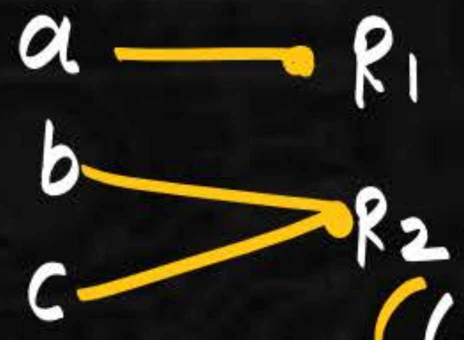


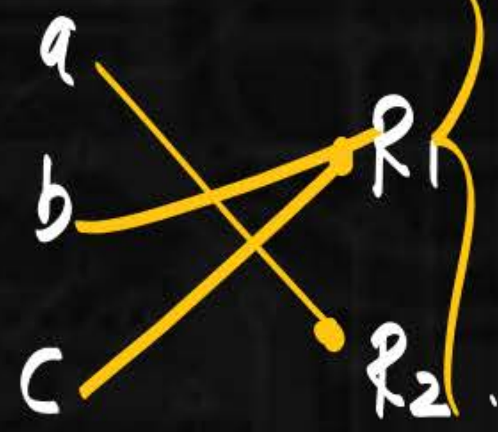


$(ac)_2\; b_1$

$(ac)_1\; b_2$

$(ac)\,(b)$





$(a)_1\,(bc)_2$

$(a)_2\,(bc)_1$

$(a)\,(bc)$

3 diff quest $\rightarrow$ 2 diff rooms onto = 6.

3 diff quest $\rightarrow$ 2 identical rooms $\frac{6}{2} = \frac{\text{onto}}{2!}$

3 diff onto

↓

2 diff

$[ab]_1 \ [c]_2$

$[ab]_2 \ [c]_1$ $\Rightarrow$ $[ab]\ [c]$

$[bc]_1 \ [a]_2$

$[bc]_2 \ [a]_1$ $\Rightarrow$ $[bc]\ [a]$

$[ac]_1 \ [b]_2$

$[ac]_2 \ [b]_1$ $\Rightarrow$ $[ac]\ [b]$

3 quest

↓

2 identical

3 diff quest → 2 identical

$$\frac{\text{onto}}{2!}$$

m diff quest → n diff room (none of the rooms should be empty)

↳ onto → $\sum_{i=0}^{n} (-1)^i \cdot {}^nC_i \cdot (n-i)^m$.

m diff quest → n identical rooms.

Sterling second kind no. $S(m, n) = \frac{\text{onto}}{n!} = \frac{1}{n!} \sum_{i=0}^{n} (-1)^i \cdot {}^nC_i \, (n-i)^m$.

m → diff
n → identical


THANK YOU GW
SOLDIERS !