CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 11

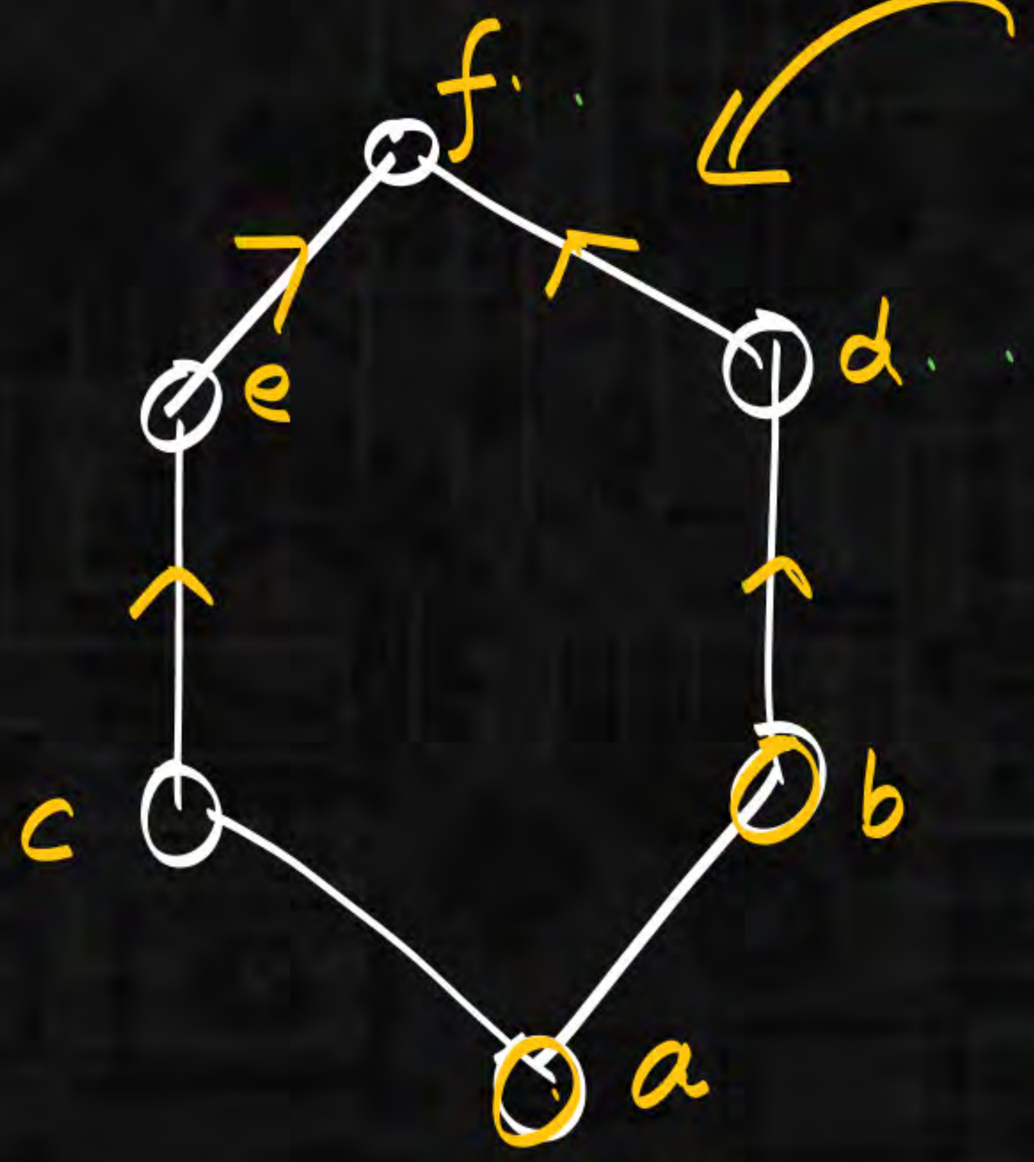
By- SATISH YADAV SIR

TOPICS

01 GLB / LUB

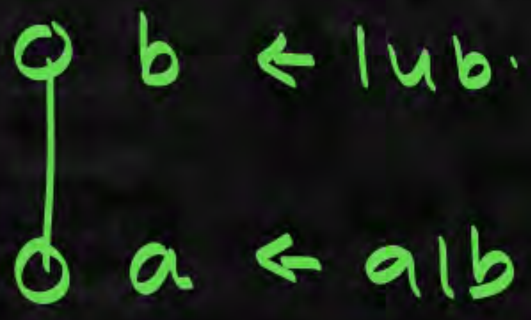
02 LATTICE

3 TYPES OF LATTICE



(A, R) lattice

$$\begin{cases} \text{glb}(a, b) = a \\ \text{lub}(a, b) = \end{cases}$$



$\text{lub}(b, c) = f$

$\text{glb}(b, c) = a$

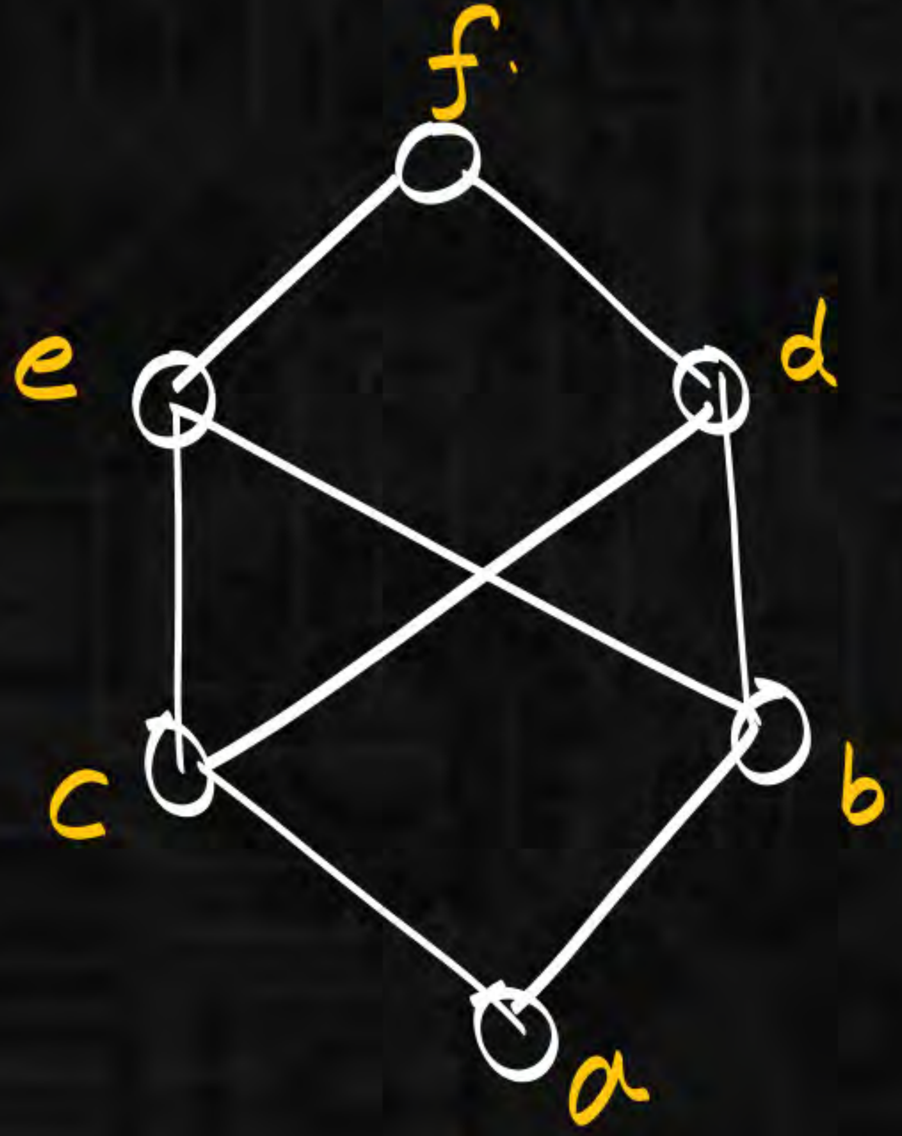
$\text{lub}(e, b) = f$

$\text{glb}(e, b) = a$

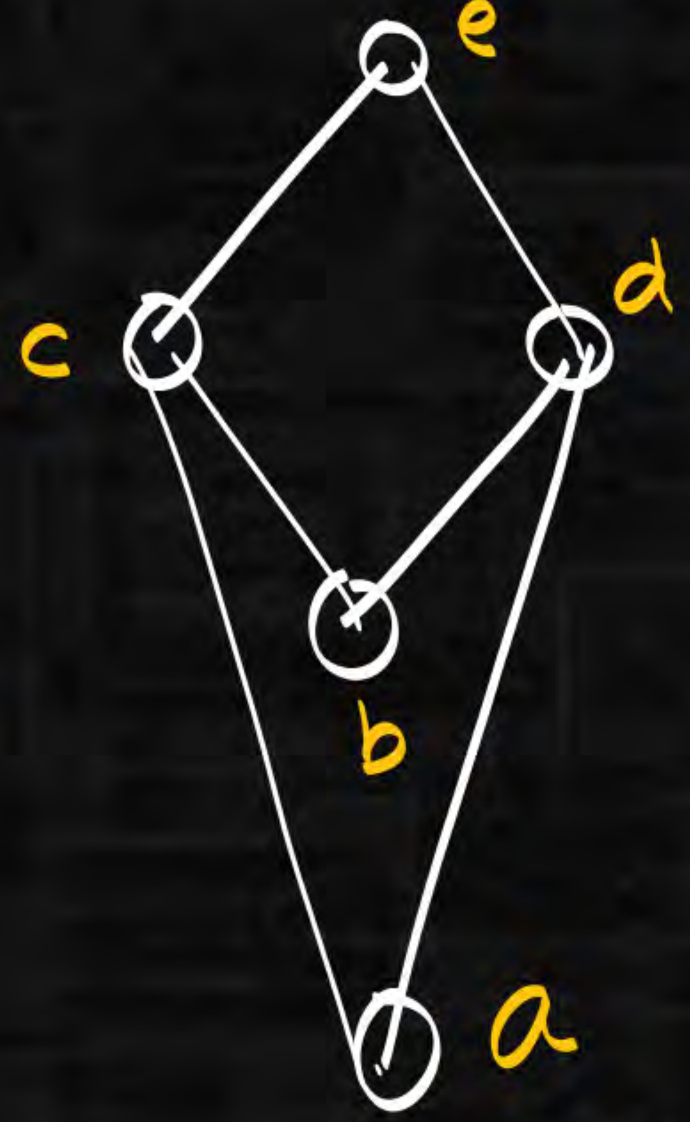
(A, R) poset

↳ when glb & lub exist for all pairs.

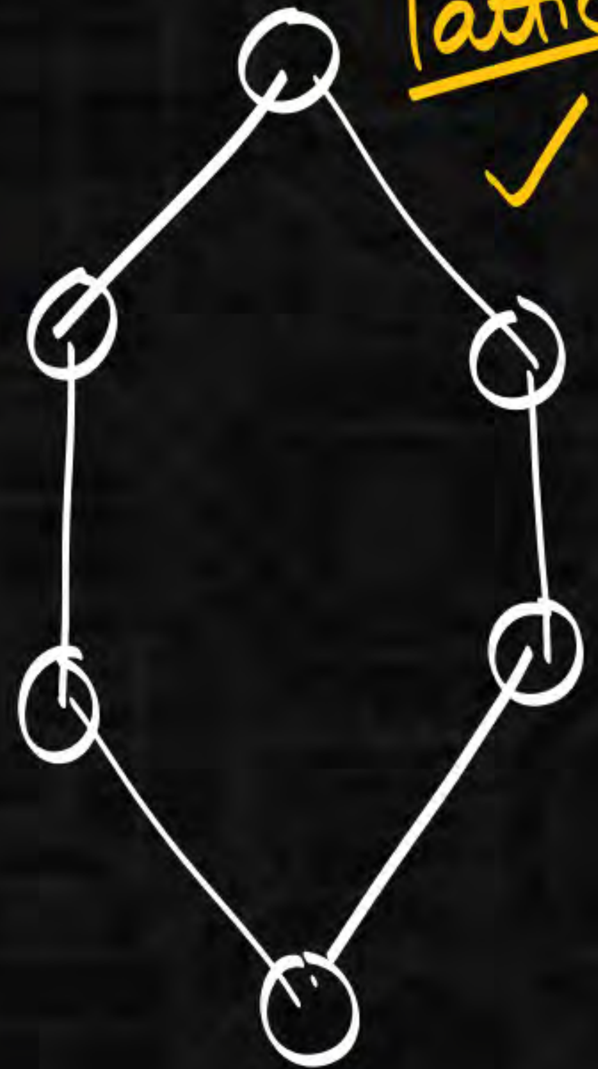
→ (A, R) lattice



i)

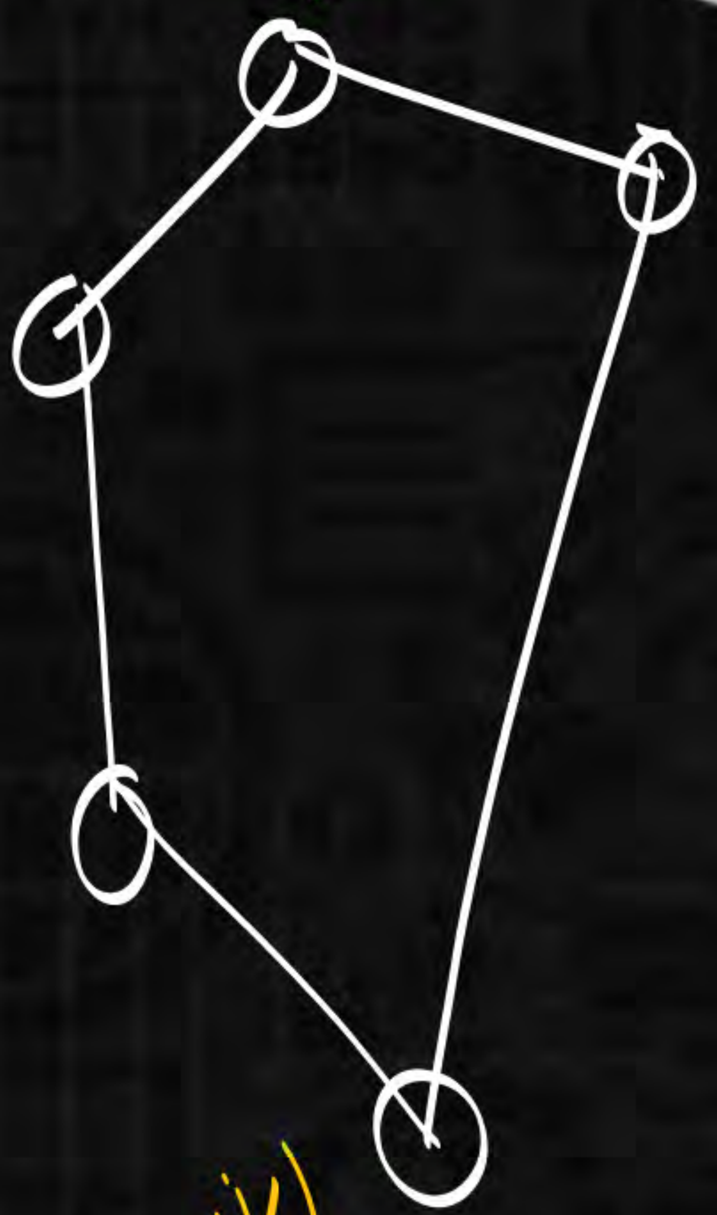


ii)

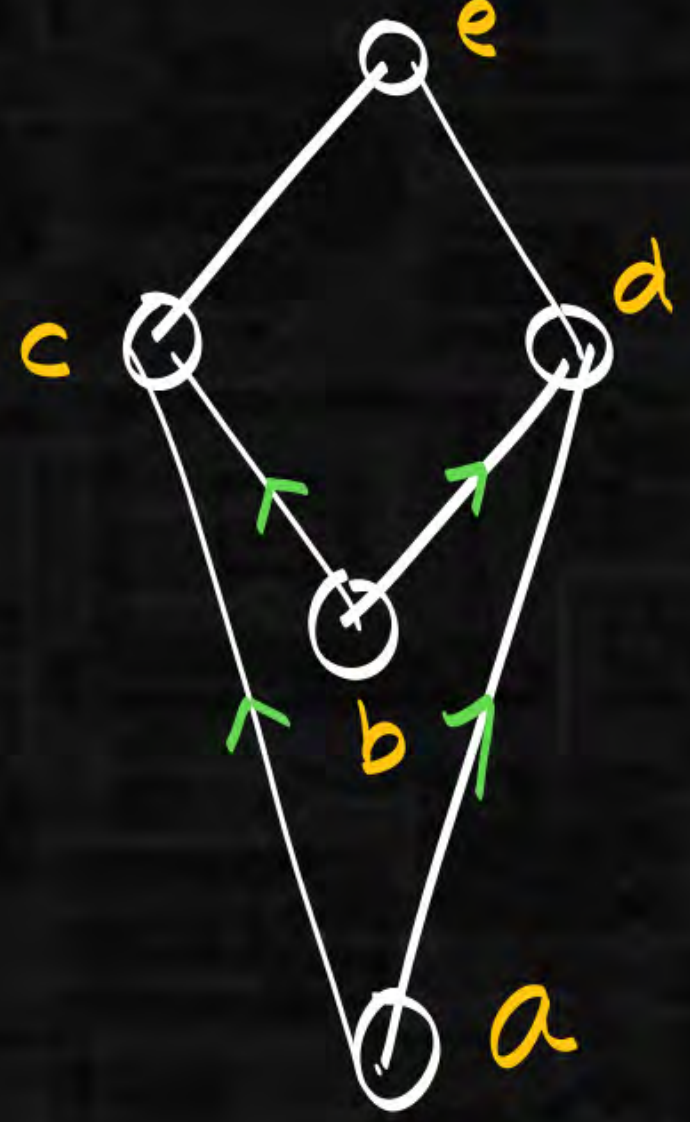


lattice

iii)



iv)

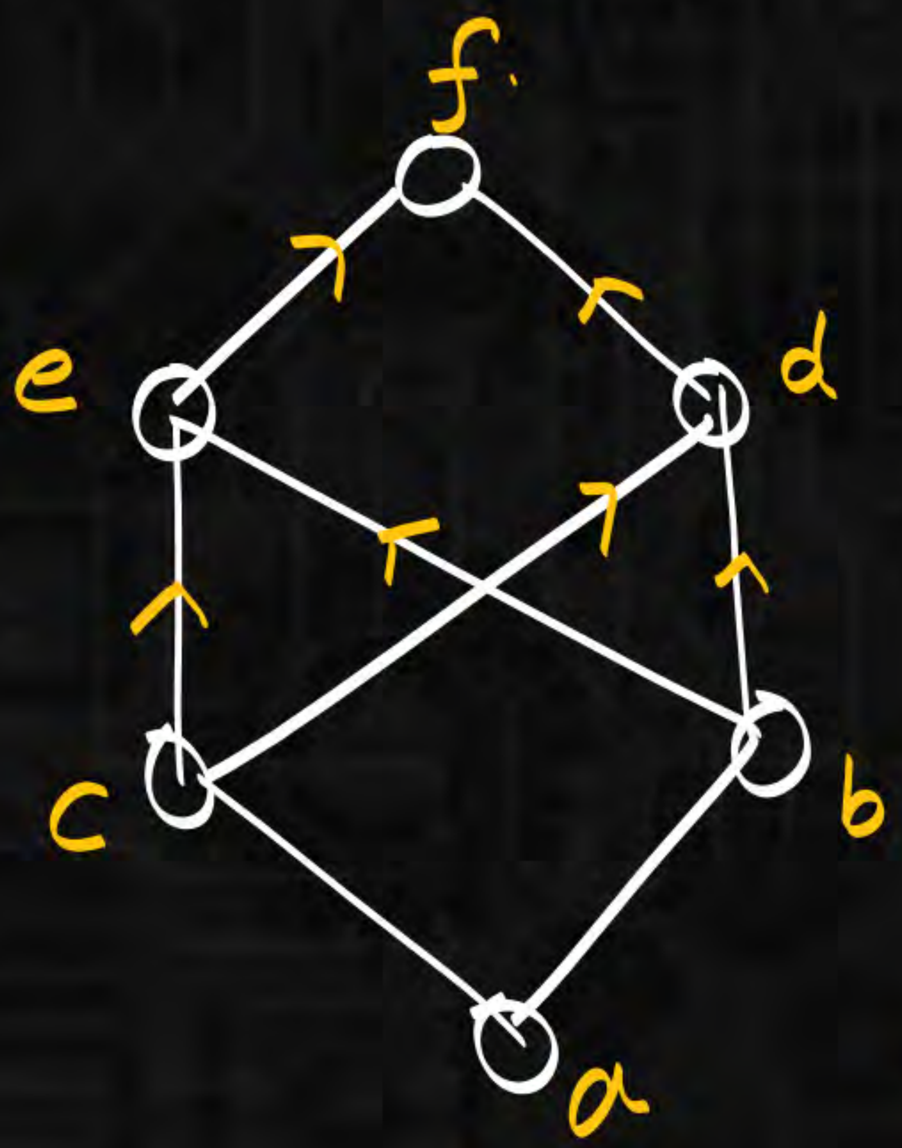


ii) X

$glb(c,d) =$

$\hookrightarrow lb(c,d) \to \{a, b\}$

Check(a)	Check(b)
$ab \le a$	$ab \le b$
$a \le a$ (T)	$a \le b$ (F)
$b \le a$ (F)	



i) ✗

$glb\{e, d\} = N.A.$

$lub(c, b) = N.A$

$ub\{c, b\} \rightarrow \{e, d, f\}$

Check(e)

lub $e \le edf$

✗ $e \le e$ (T)

$e \le d$ (F)

check(d)

$d \le edf$

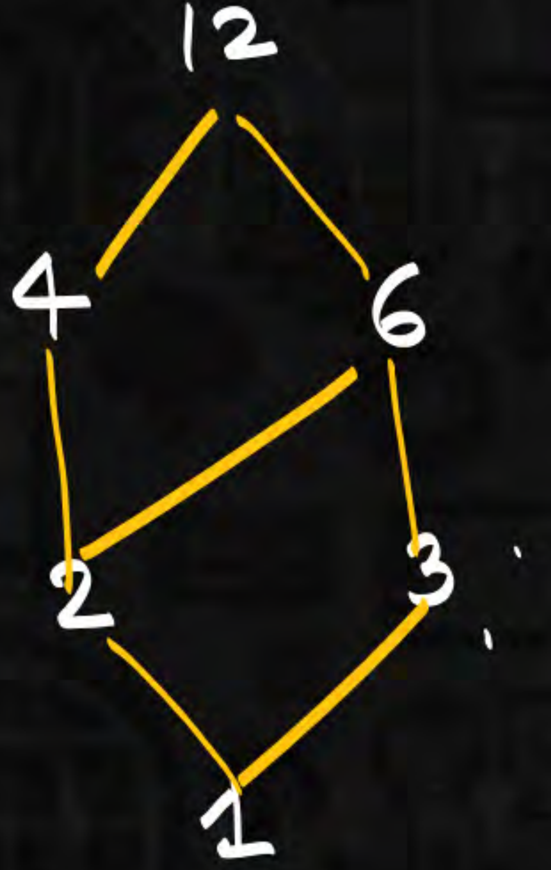
$d \le e$ (F)

check(f)

$f \le edf$

$f \le e$ (F)

$(D_{12}, |)$



$lub(4,6) = 12 \qquad lcm(4,6) = 12.$

$glb(4,6) = 2. \qquad gcd(4,6) = 2.$

$(D_n, |)$

$glb \rightarrow gcd$

$lub \rightarrow lcm.$

$A = \{1, 2, 3\}$

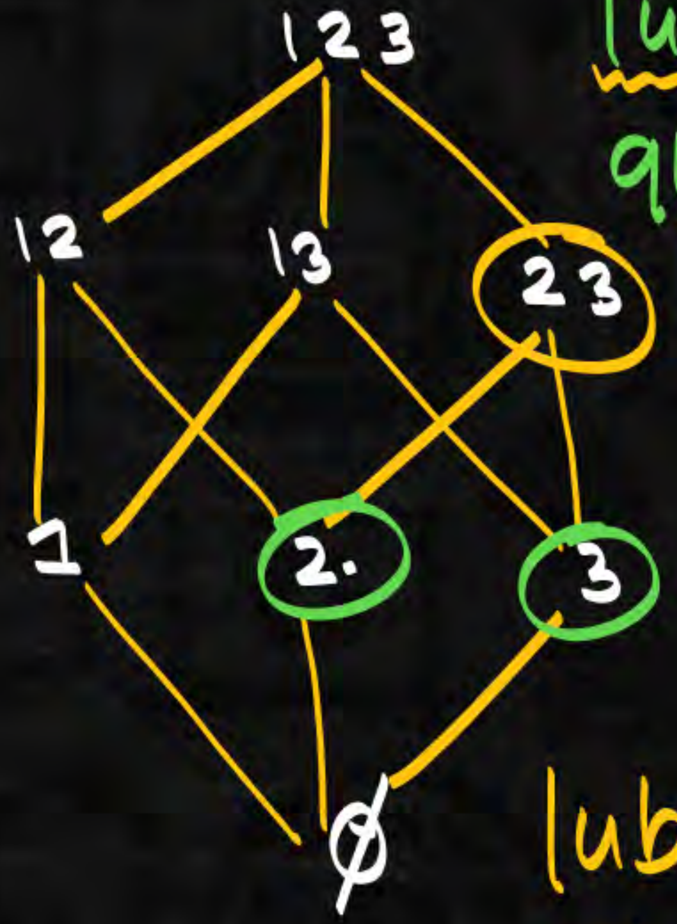
$(P(A), \subseteq)$ poset

$\{2\} \cup \{3\} = \{23\}$

lub$(2,3) = 23$

glb$(2,3) = \phi$

$\{2\} \cap \{3\} = \phi$



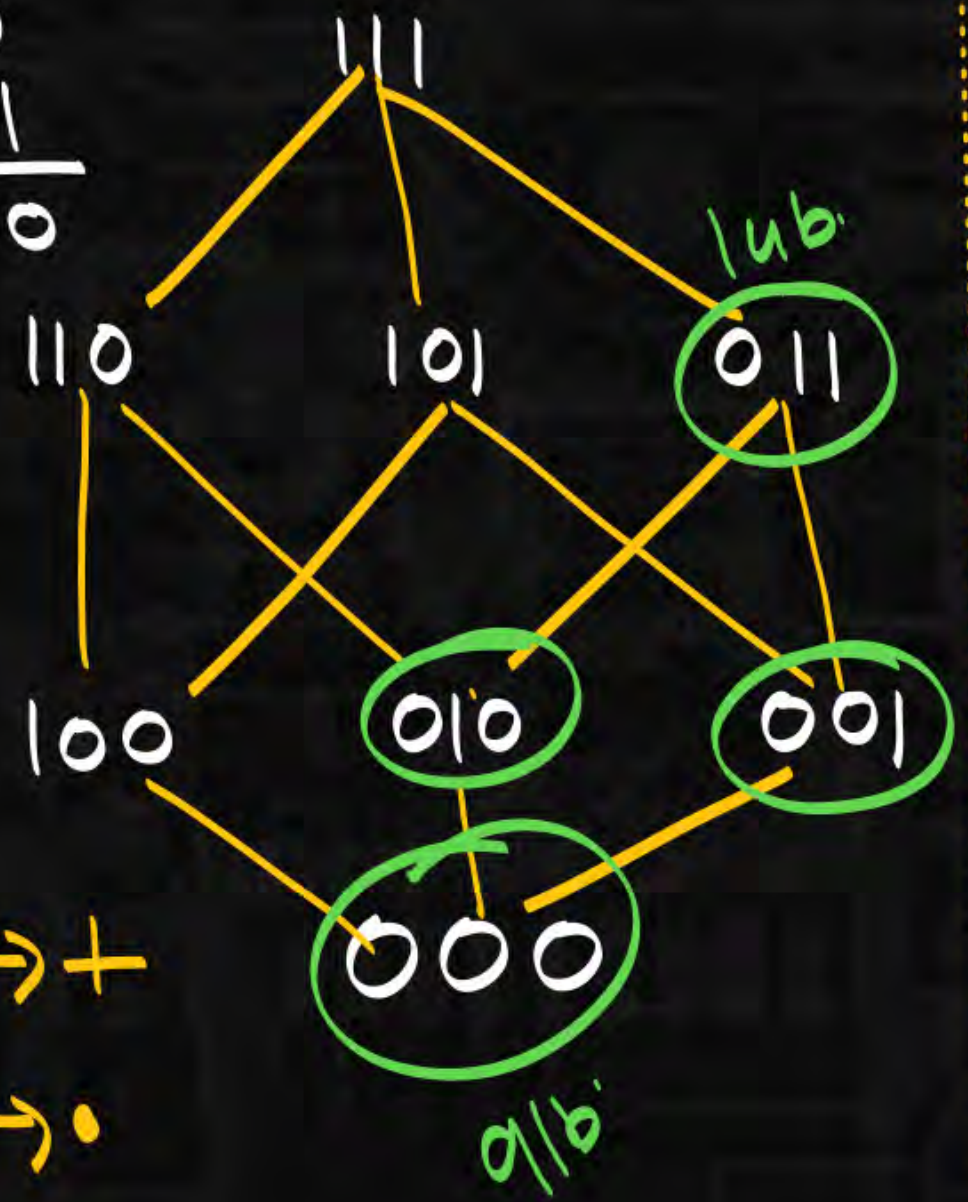
lub → ∪

glb → ∩

lub$(010, 001) = 011$

glb$(010, 001) = 000$

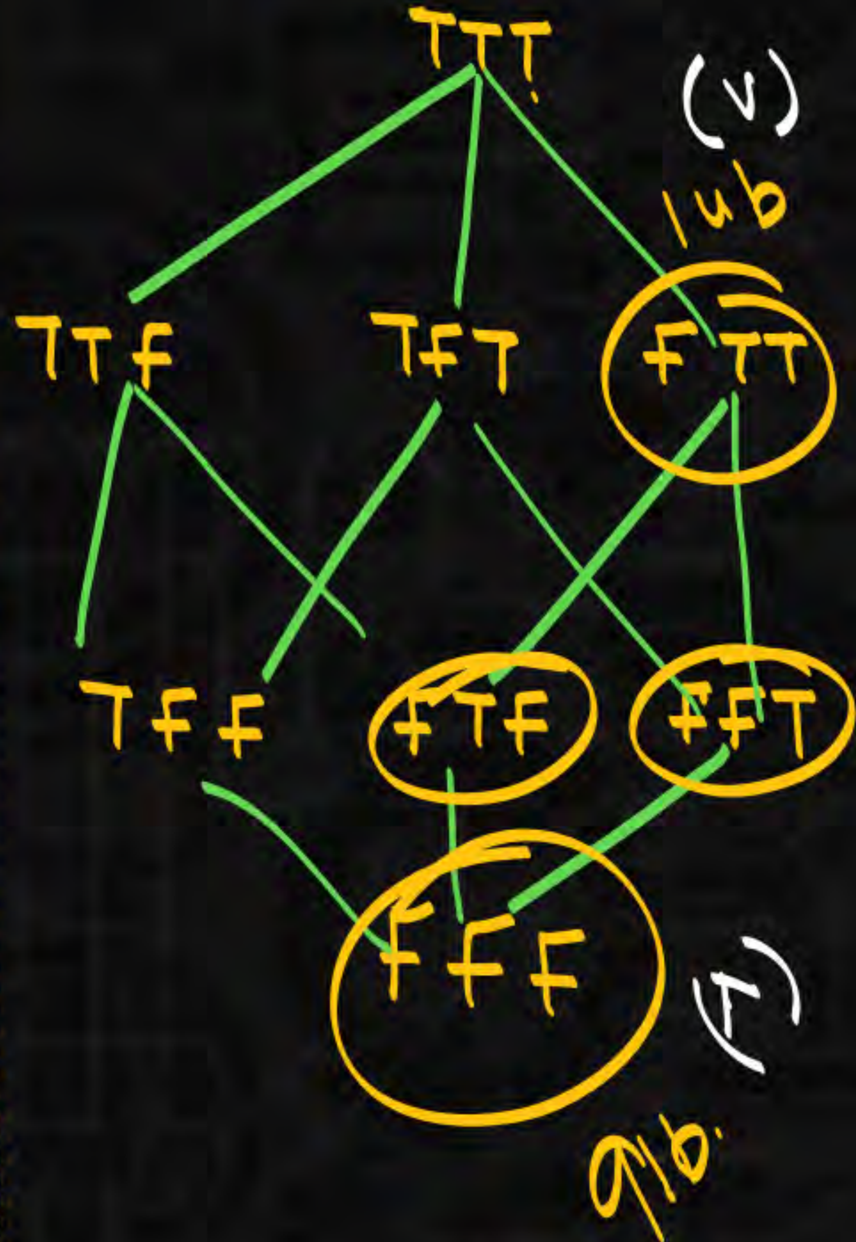
$$\begin{array}{r} 010 \\ +\ 001 \\ \hline 011 \end{array} \qquad \begin{array}{r} 010 \\ \cdot\ 001 \\ \hline 000 \end{array}$$



lub → +

glb → ·

$$\begin{array}{r} FTF \\ \vee\ FFT \\ \hline FTT \end{array} \qquad \begin{array}{r} FTF \\ \wedge\ FFT \\ \hline FFF \end{array}$$



$\vee$	$\cup$	$+$
$\wedge$	$\cap$	$\cdot$
T	U	1
F	$\emptyset$	0

* Every TOSET (chain) is lattice.

$\boxed{\begin{array}{l}(\vee)\ \text{lub} \\ (\wedge)\ \text{glb}\end{array}}$

$[L, \vee, \wedge]$ lattice

$\text{lub}(a, \text{lub}(b,c)) =$

$\text{lub}(\text{lub}(a,b), c)$

1) $a \vee a = a \qquad a \wedge a = a$

$\text{lub}(a,a) = a \qquad \text{glb}(a,a) = a.$

2) $a \vee b = b \vee a \qquad a \wedge b = b \wedge a.$

$\text{lub}(a,b) = \text{lub}(b,a) \qquad \text{glb}(a,b) = \text{glb}(b,a)$

3) $a \vee (b \vee c) = (a \vee b) \vee c \qquad a \wedge (b \wedge c) = (a \wedge b) \wedge c.$

4) $a \vee (a \wedge b) = a \qquad a \wedge (a \vee b) = a$

3) $a \vee (b \vee c) = (a \vee b) \vee c$

$a \vee (b \vee c) = (a \vee b) \vee c$

$a \vee (b \vee c) \to \text{lub}(b, c) \to a \vee b$

$\text{lub}(a, b) \to b$

$(a \vee b) \vee c \to \text{lub}(a, b) \to b \vee c$

$\text{lub}(b, c) \to b$

$a \wedge (b \wedge c) = (a \wedge b) \wedge c$

$a \vee (a \wedge b) = a$ $\qquad$ $a \wedge (a \vee b) = a.$

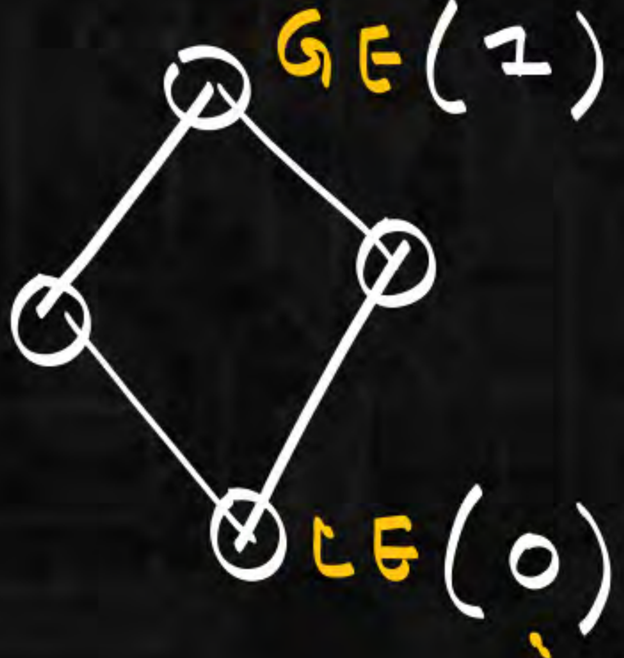
Case 1:

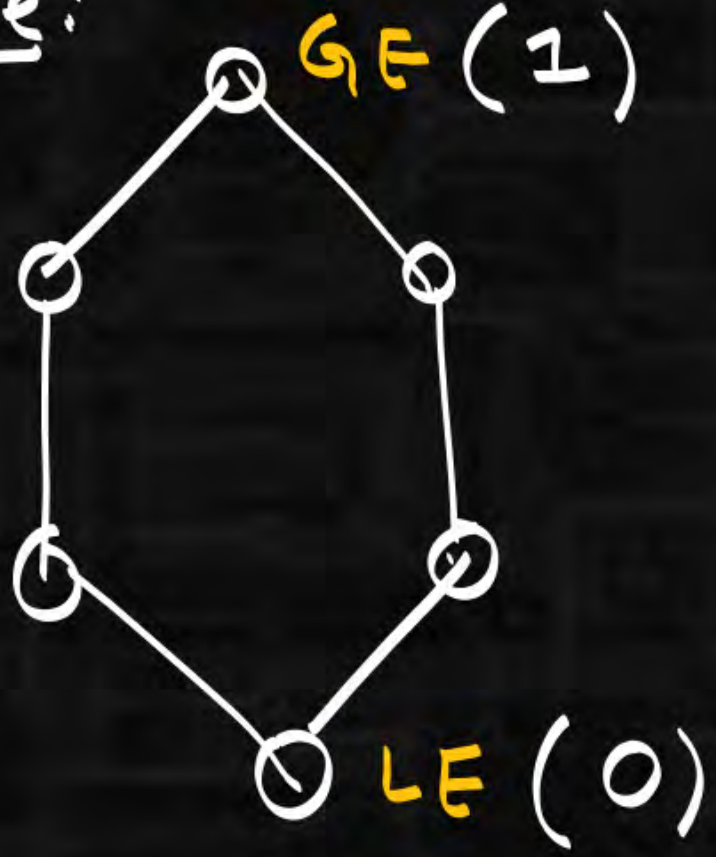
Case 2:

$a \vee \underbrace{(a \wedge b)}_{glb(a,b)} = a.$

$(a \vee a) = a.$

Bounded lattice:





Lattice $[L, \vee, \wedge]$

$\downarrow$

(G.E & L.E) are existing.

$\downarrow$

bounded lattice.

$[L, \vee, \wedge, 1, 0]$

$(Z^+, \leq)$ toset → lattice.

→ G.E → NA.

LE. → 1.

$(Z, \leq)$ → $\boxed{\text{Toset} \to \text{lattice.}}$

→ GE : N.A

→ LE → NA.

** Every finite lattice will be bounded lattice.

$1 \to GE \quad 0 \to LE$

$a + b = 1$

$a \cdot b = 0$

a, b are complement to each.

$a \vee b = GE$

$a \wedge b = LE$

Complement lattice

$a + b = 1$ | $lub(a,b) = GE(1)$

$a \cdot b = 0$ | $glb(a,b) = LE(0)$

bounded lattice, atleast 1 $\boxed{\text{Complement}}$ exist for all elements.

$lub(b,x) = d$

$glb(b,x) = a$

$lub(\quad) = GE$

$glb(\quad) = LE.$

$b' = \{x, c\}$

$a' = d$

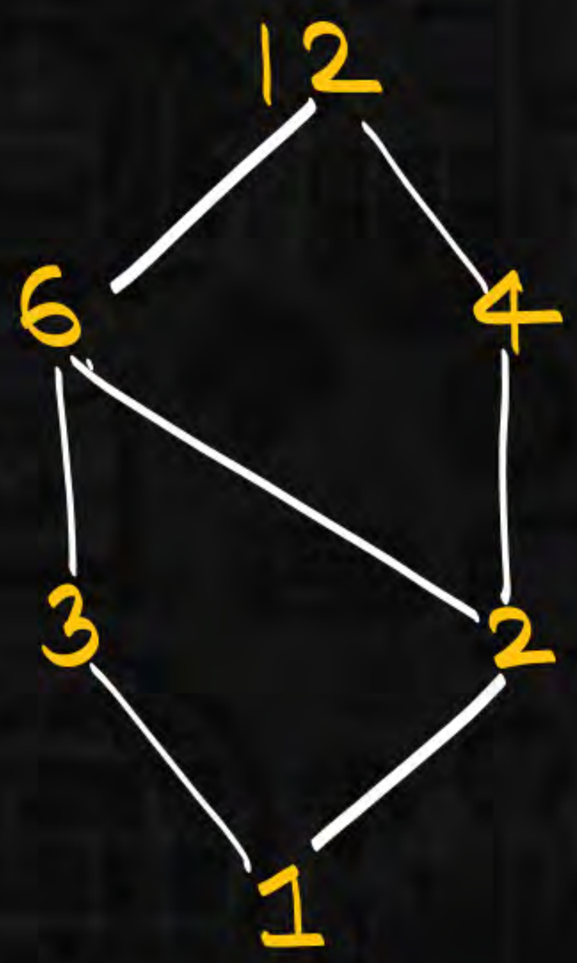
$lub(b,c) = d(1)$ $\quad b' = c.$

$glb(b,c) = a(0)$

$lub(a,d) = d(1)$ $\quad a + d = 1.$

$glb(a,d) = a(0)$ $\quad a \cdot d = 0.$

$(D_{12}, |)$ complement lattice?



$3' = ?$

$3 + \quad = 12$ $\quad$ $lcm(3, 4) = 12$

$3 \cdot \quad = 1$ $\quad$ $gcd(3, 4) = 1$

$3' = 4.$

$6' = NA.$

$6 + \quad = 12$

$lcm(6, 12) = 12$

$6 \cdot \quad = 1.$

$gcd(6, 12) = 1.$

$(D_{12}, |)$ complement lattice?

$3' = 4$
$4' = 3$

$2 + \quad = 12$

$lcm(2, \quad) = 12$

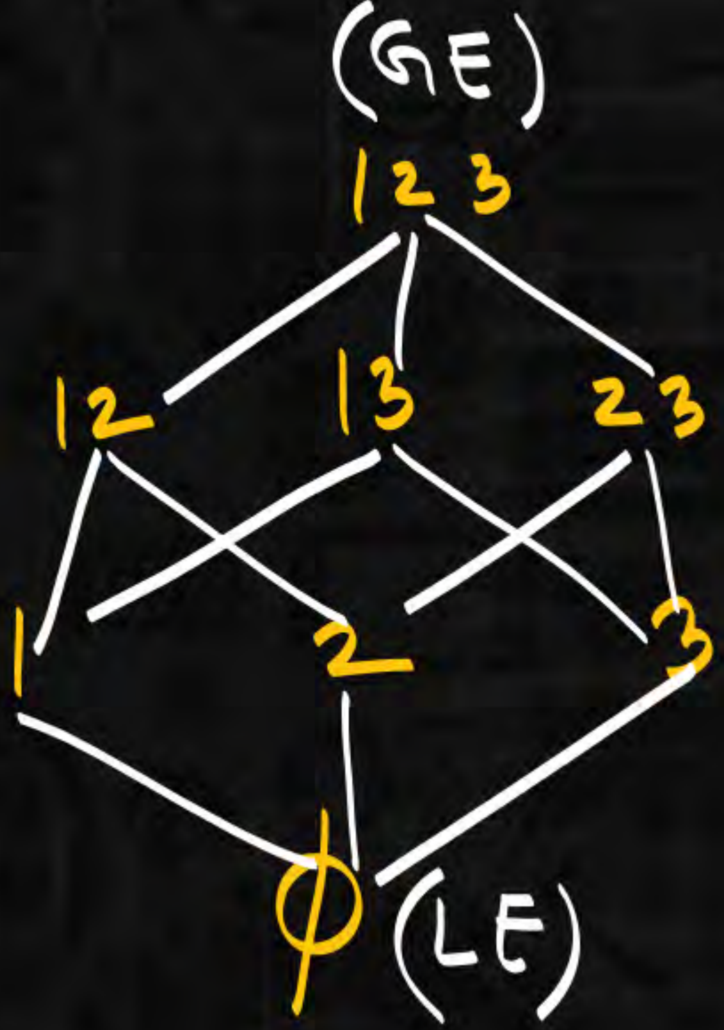
put 3
$lcm(2, 3) \neq 12$
put 4
$lcm(2, 4) \neq 12$

put 6
$lcm(2, 6) \neq 12$
put 12
$lcm(2, 12) = 12$

$gcd(2, 12) \neq 1$

$A = \{1, 2, 3\}$

$(P(A), \subseteq)$



$a + b = 1.$ $\quad a \cup b = GE.$

$$\begin{cases} a \cup b = \{1\,2\,3\} \\ a \cap b = \phi \end{cases}$$

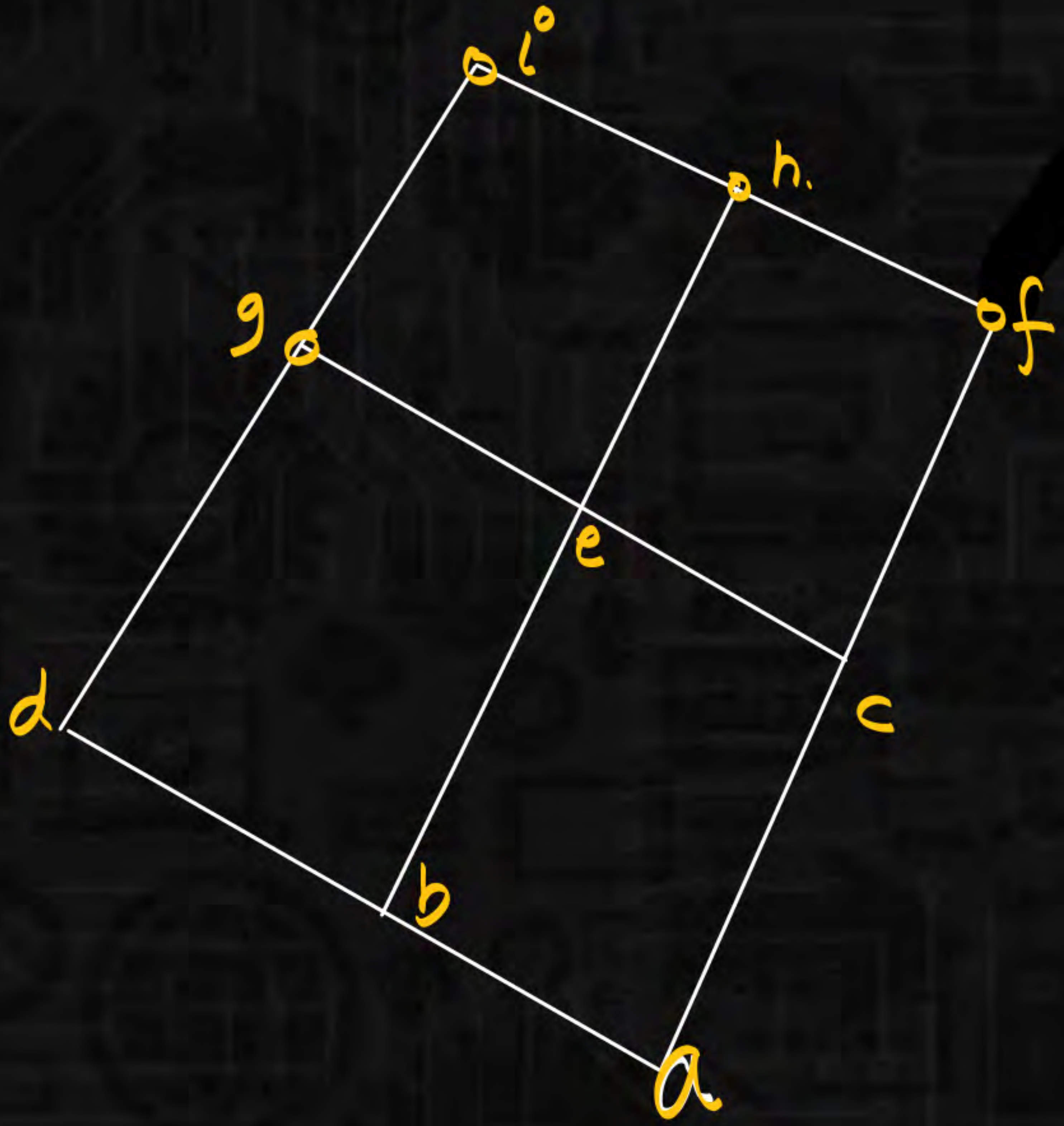
$\{1\} \cup \{23\} = \{1\,2\,3\}$

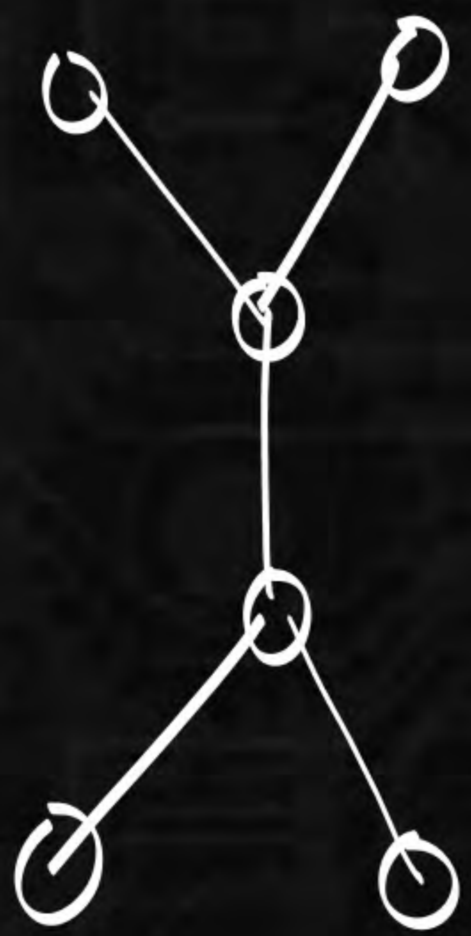
$\{1\} \cap \{23\} = \phi$

$\{1\}' = \{23\}$

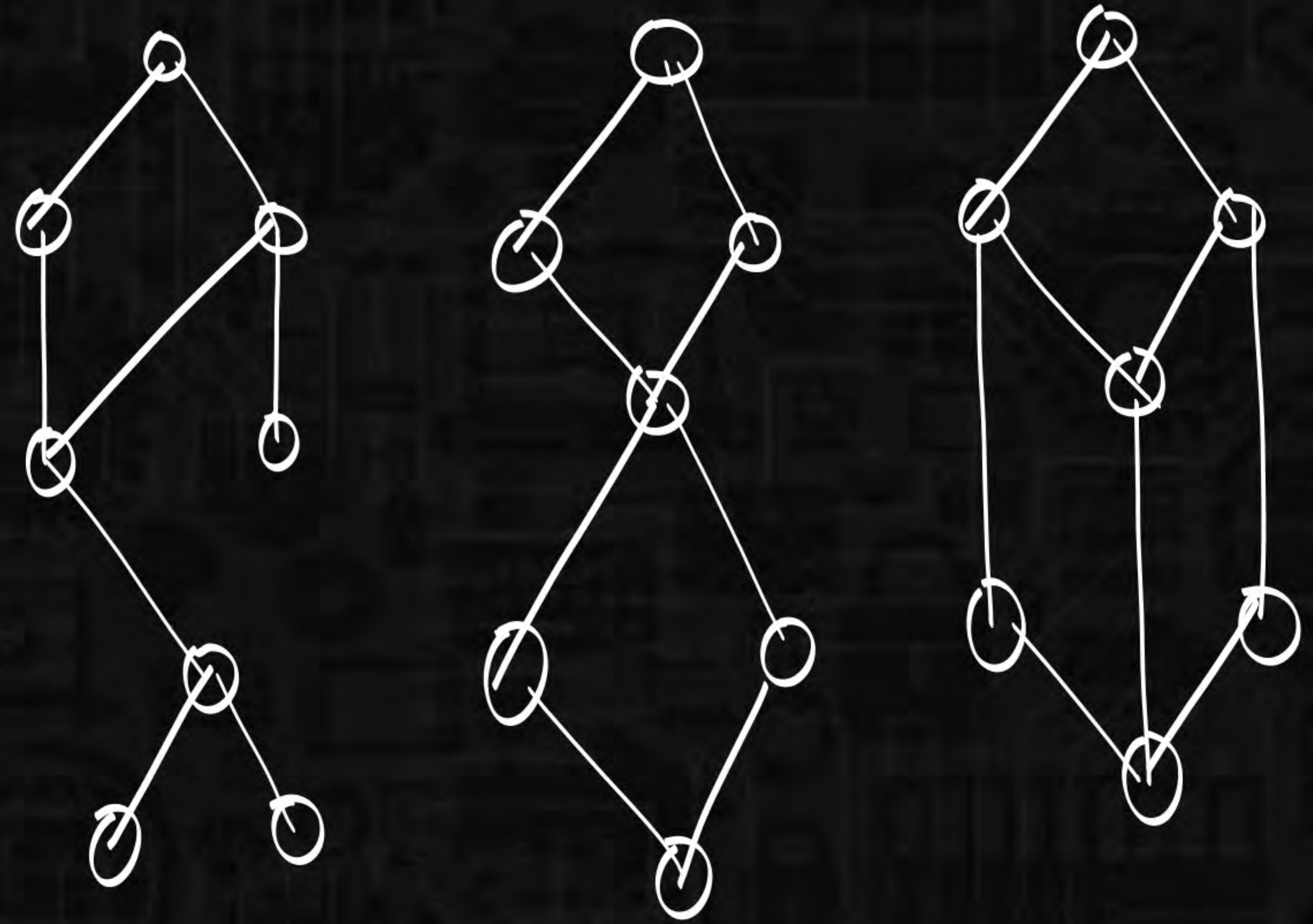
$\{2\} \cup \{13\} = \{1\,2\,3\}$

$\{2\} \cap \{13\} = \phi$





lattice?



$$\left(\{2, 3, 6, 12, 24, 36, 72\}, \mid\right)$$

lattice ?

complement ?


THANK YOU GW
SOLDIERS !