CS & IT ENGINEERING

Algorithms

Greedy Method

Lecture No.- 05

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Recap of Previous Lecture

Topic Minimum Cost Spanning Trees

Topics to be Covered

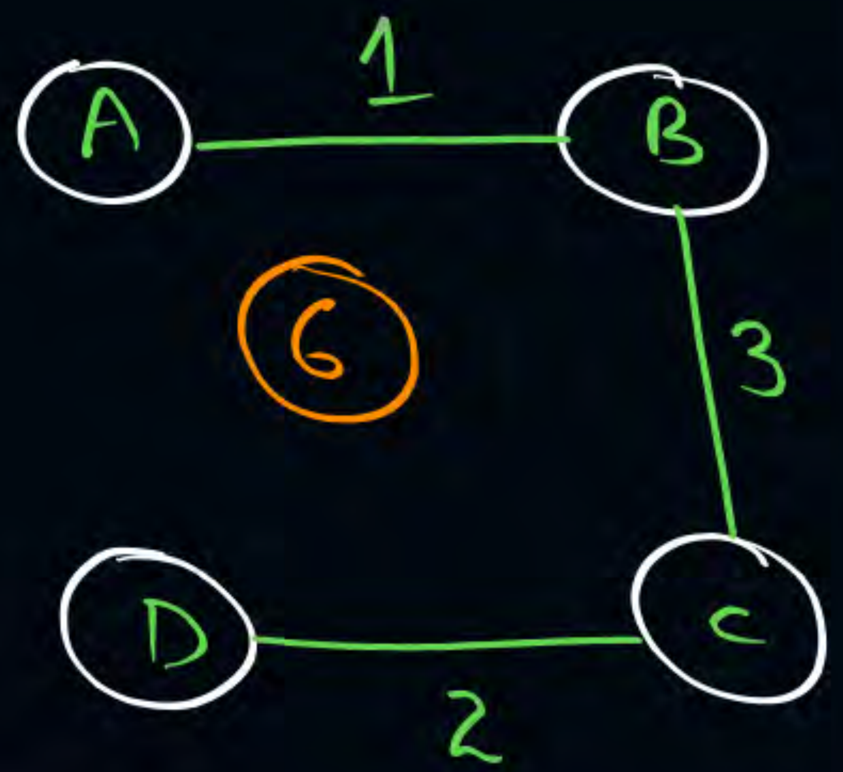
Topic Minimum Cost Spanning Trees

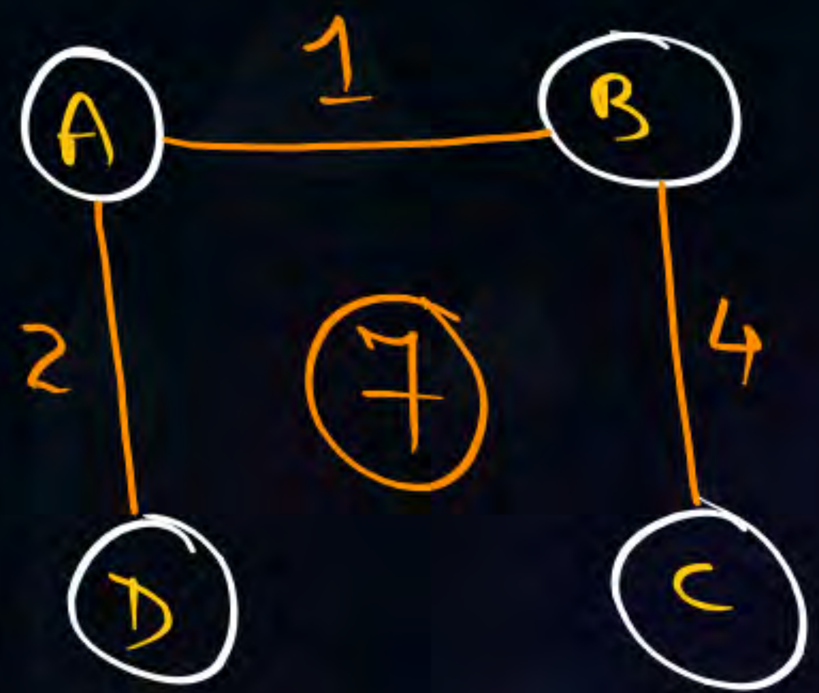
Shortest Paths

Topic : III. Greedy Method

Q. Let G be a complete undirected graph with 4 vertices and edge weights are {1, 2, 3, 4, 5, 6}. The maximum possible weight that a minimum weight Spanning Tree can have is 7.

MCST





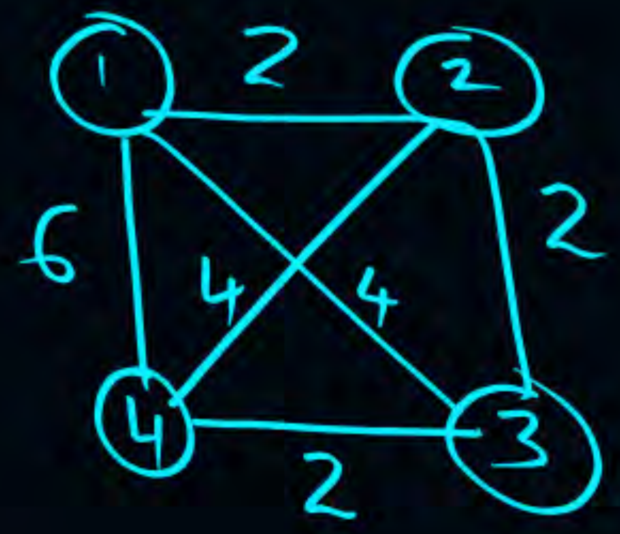
Topic : III. Greedy Method

Q. Consider a complete weighted Graph with n vertices numbered V_1 to V_n. Two vertices V_i & V_j having edge between them has a cost value of $2|i-j|$. The weight of minimum cost Spanning Tree of such a graph is ______.

$n = 3$

$(2+2)$

$n = 4$



$V_i \rightarrow V_{i+1} : 2$

'n'

$\downarrow$

$: 2(n-1)$

$\Rightarrow (2n-2)$ ✓

99

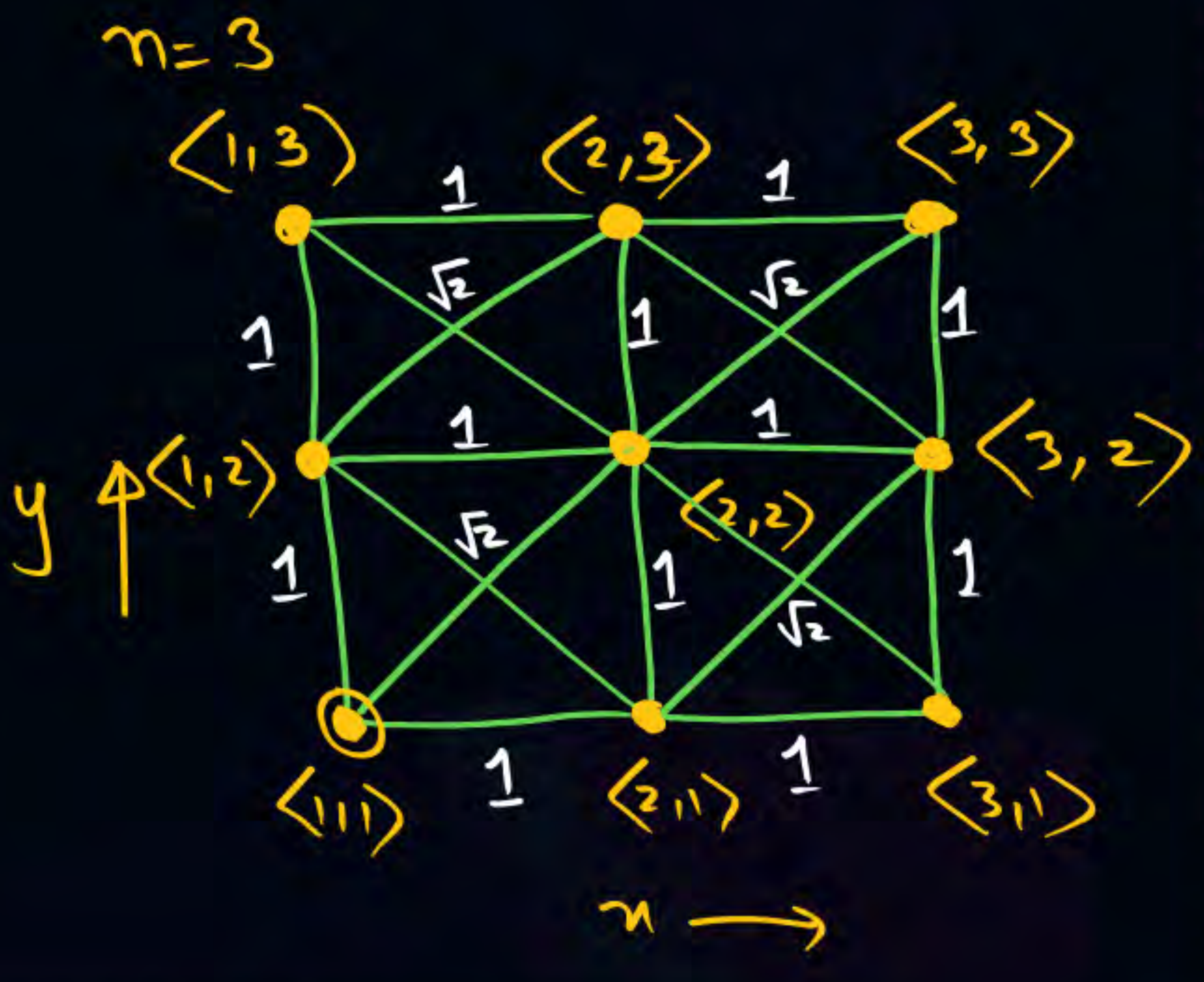
Q. Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes 995 .

+5

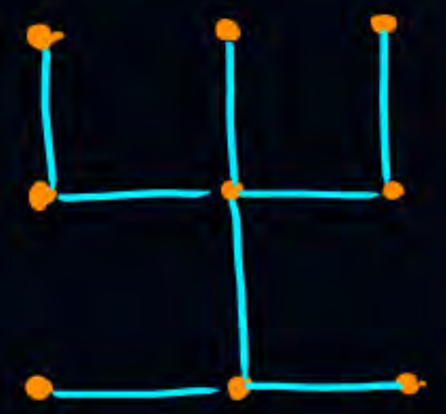
$\Longrightarrow 500 + 99 \times 5 = 995$

Topic : III. Greedy Method

Q. Consider a Graph whose vertices are points in a plane with integer co-ordinates (x, y) where $1 \le x \le n$, $1 \le y \le n$, $n > 2$ is an integer. 2 vertices $<x_1, y_1>$ & $<x_2, y_2>$ are adjacent iff $|x_1 - x_2| \le 1$ & $|y_1 - y_2| \le 1$. The cost of such an edge is given by the distance between them. Compute the weight of min cost Spanning Tree of such graph for a given value of n.



$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$



$n = 3$

Cost = 8

$(3^2-1) \times 1$

For a value of 'n'

No. of vertices of 'G' : (n^2)

No. of edges of MCST : $(n^2-1) \cdot 1$

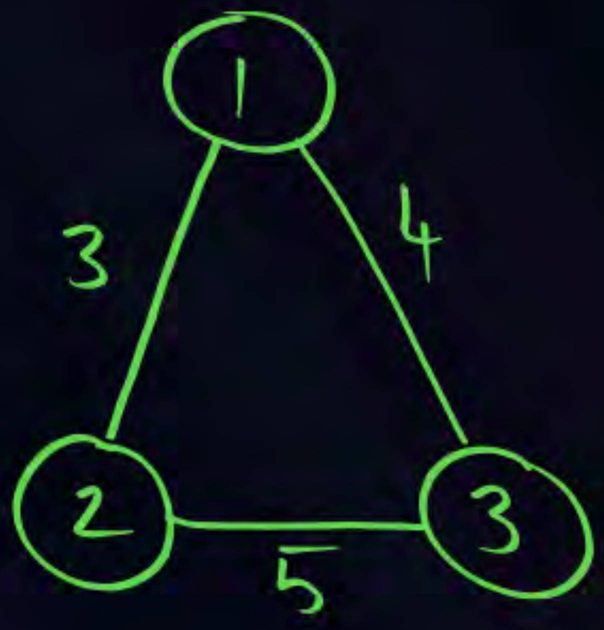
$: (n^2-2)\sqrt{2}+1$

Q2) What is the wt. of Max. Cost Sp. Tree,

Topic : III. Greedy Method

Q. Consider a graph with 'n' vertices $n > 2$. The vertices are numbered V_1 to V_n. Two vertices V_i & V_j are adjacent iff $0 < |i - j| \leq 2$. The weight of such an edge is $i + j$. The weight of minimum cost Spanning Tree of such a graph for a value of n is ________.

$n = 3$



$\therefore (3+4)$

$(3 + 2 * 2)$

$n = 4$

$3 + 2(2+3)$

$\therefore (3+4+6)$

'n' :

$\therefore 3 + 2(2+3+4+\cdots+n-1)$

$\therefore 3 + 2\left[\frac{n(n-1)}{2} - 1\right]$

$\therefore 3 + \cancel{2}\left[\frac{n^2-n-2}{\cancel{2}}\right]$

$\boxed{n^2 - n + 1}$

$\left({}^{n}C_{r}\right)$

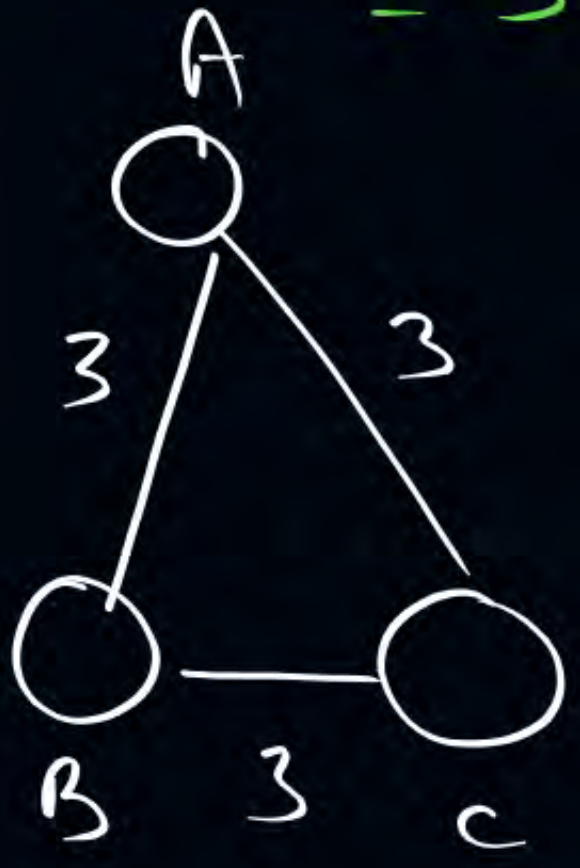
(1)

No. of MCST : $\left(2C_1\right)$

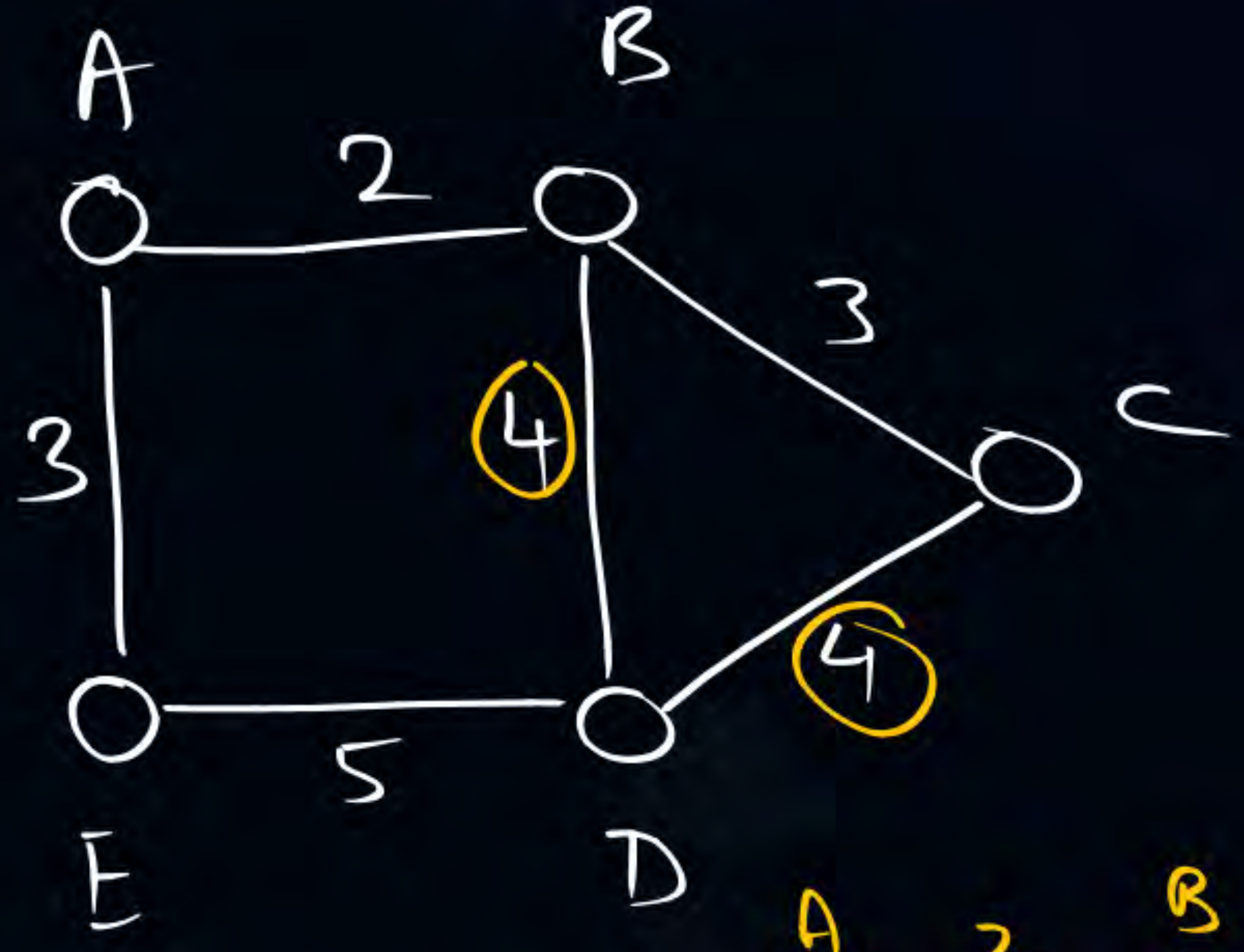
0

$\left(3C_2\right)$

$= 3$ ✓



$\left(2C_1\right)$



No. of MCST's : (2)

Topic : III. Greedy Method

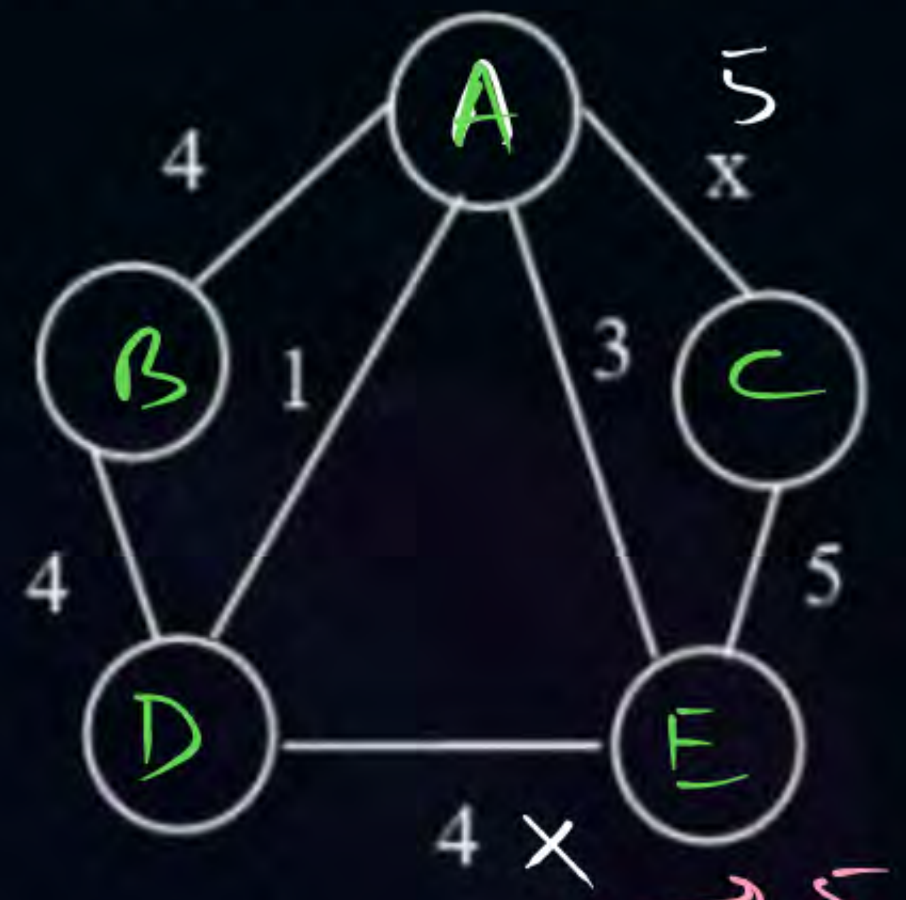
(i) If $x > 5$ then $N = 2$

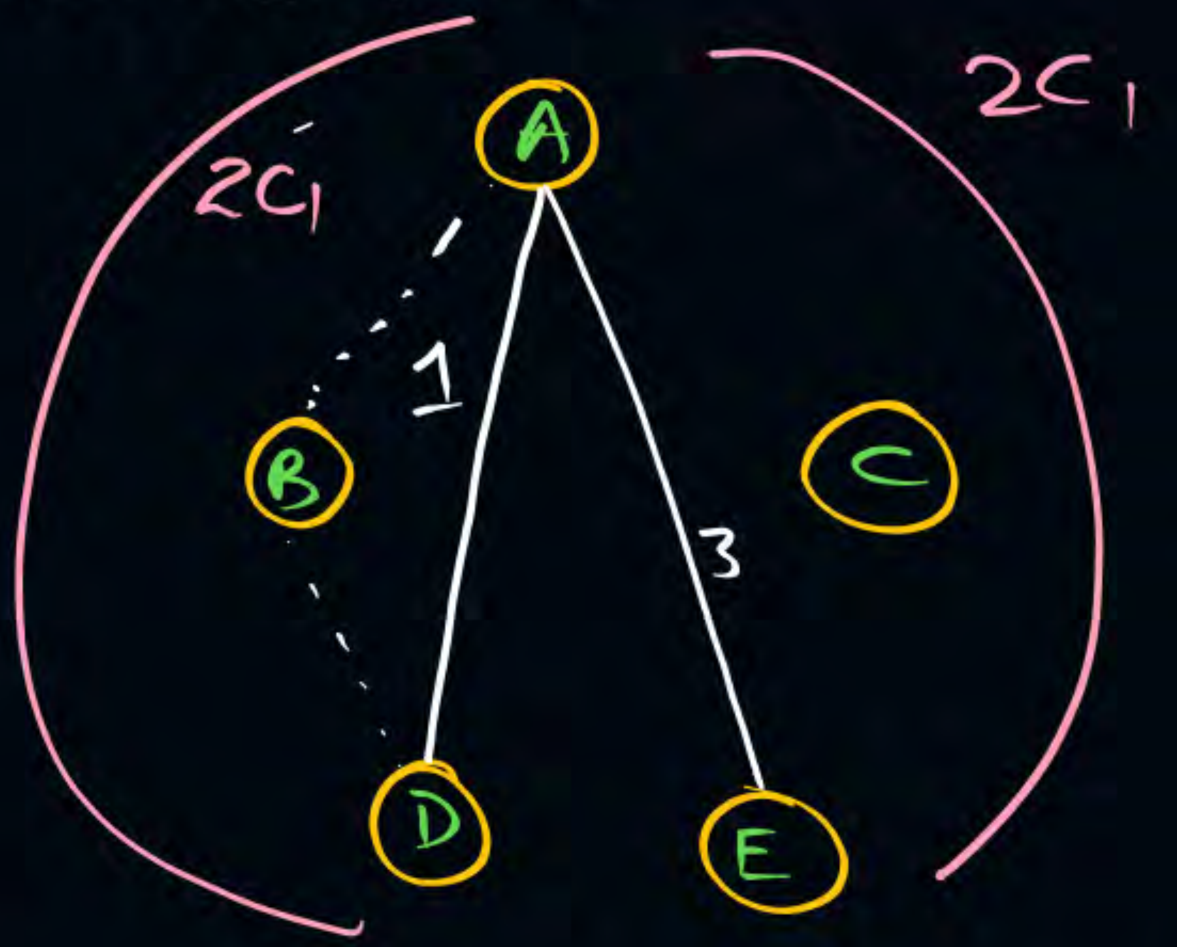
(ii) If $x < 5$ $N = 2$

(iii) If $x = 5$

Q. Consider the following undirected graph G:

(*)





Total No. of MCSTs $= {}^2C_1 * {}^2C_1$

$= 4$

Choose a value for x that will maximize the number of minimum weight spanning trees (MWSTs) of G. The number of MWSTs of G for this value of x is ____4____.

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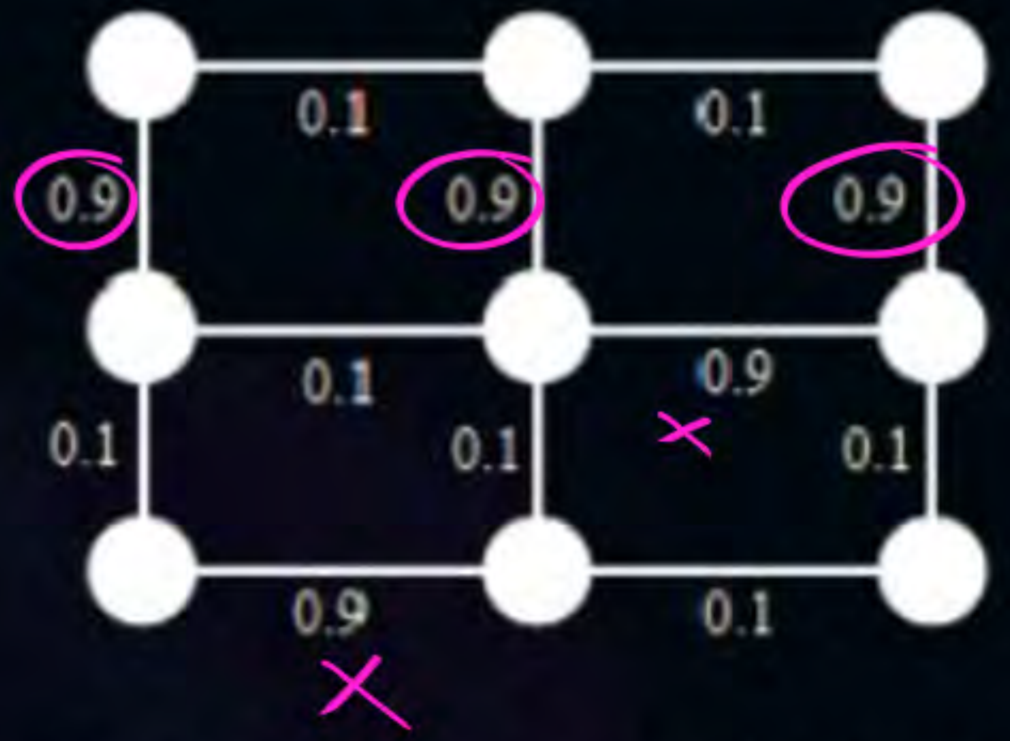
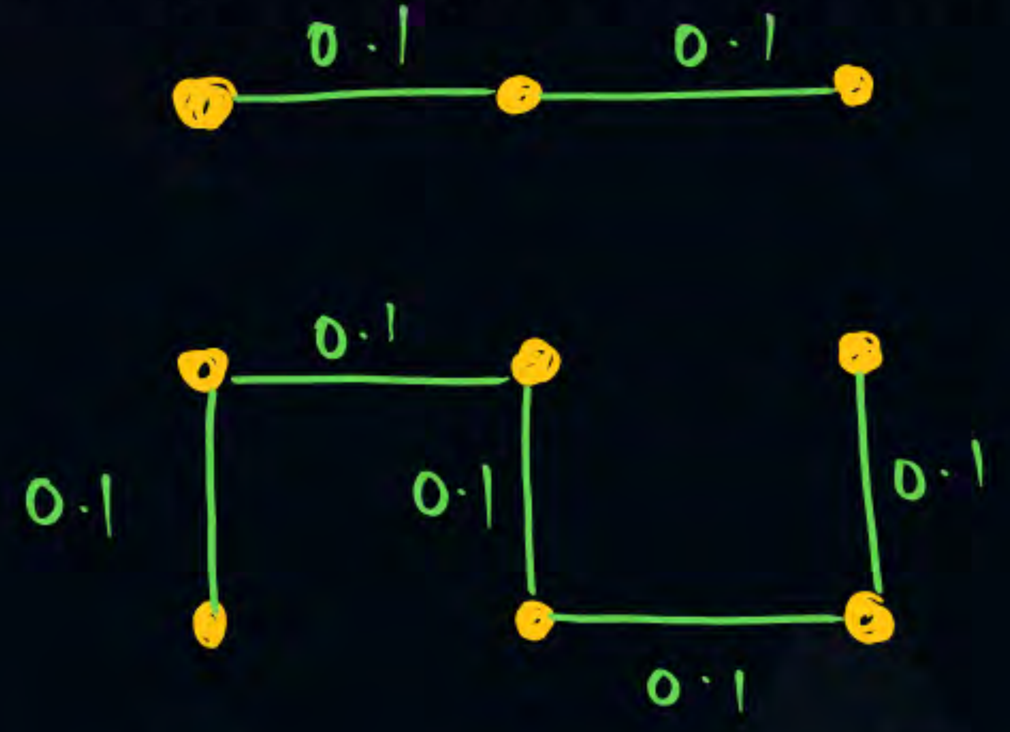
Topic : III. Greedy Method

Q. Consider the following undirected graph with edge weights as shown





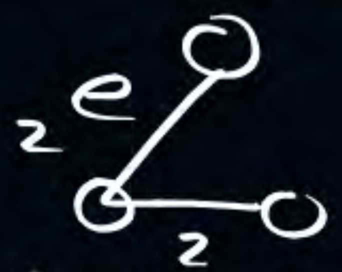
$^3C_1$

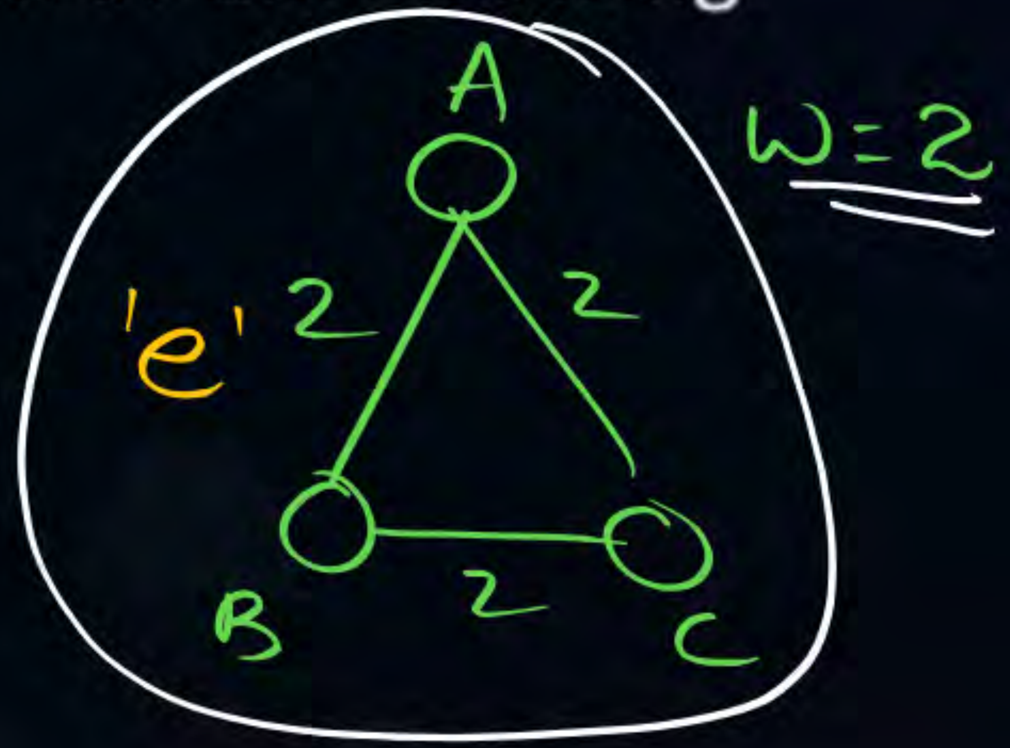
The number of minimum-weight spanning trees of the graph is 3

Topic : III. Greedy Method

Q. Let w be the minimum weight among all edge weights in an undirected connected graph. Let 'e' be a specific edge of weight 'w'. Which of the following is False?

one



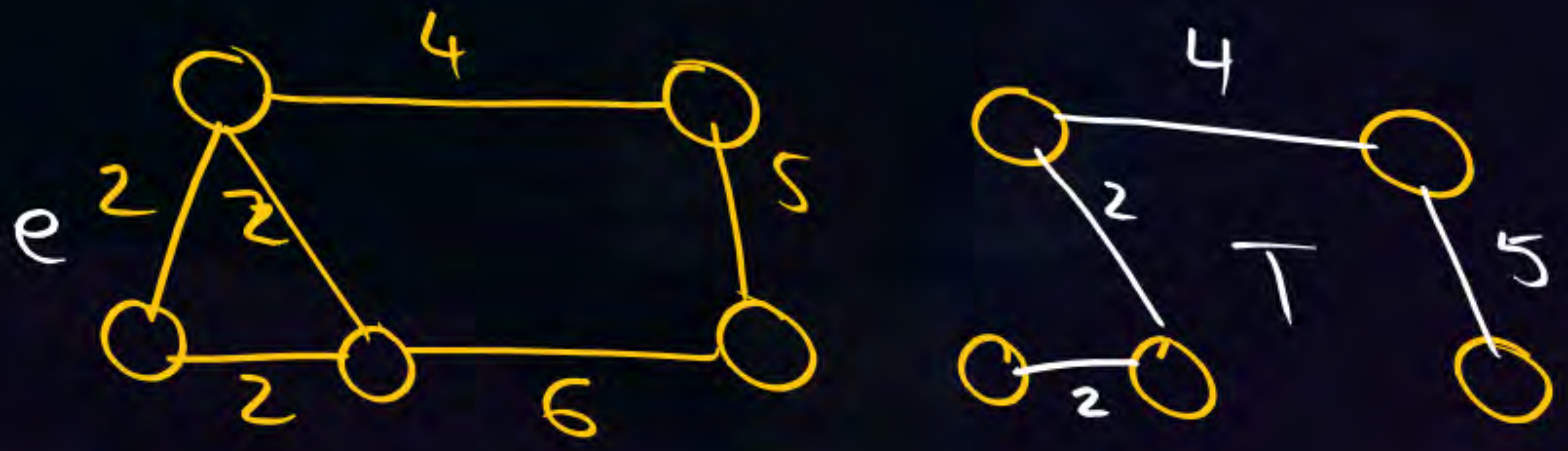


T i. There is a minimum Spanning Tree containing 'e' always.

T ii. Every minimum Spanning Tree has an edge of weight 'w'.

F iii. 'e' is present in every minimum Spanning Tree.

T iv. If 'e' is not present in a minimum Spanning Tree named 'T' then there will be a cycle formed by adding 'e' to T.



Q. G = (V, E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?

I. If e is the lightest edge of some cycle in G, then every MST of G includes e

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e

(a) I only

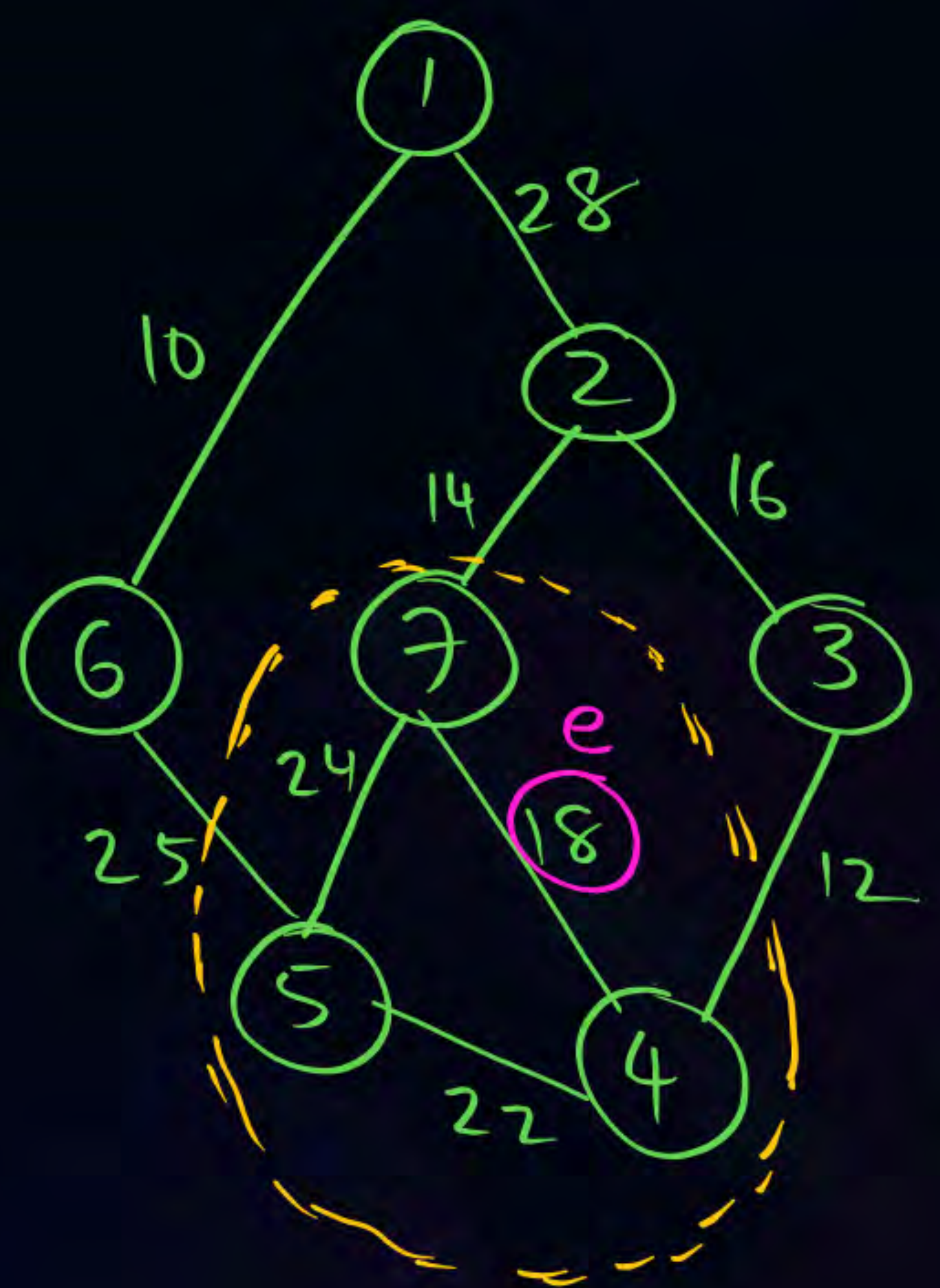
(b) II only

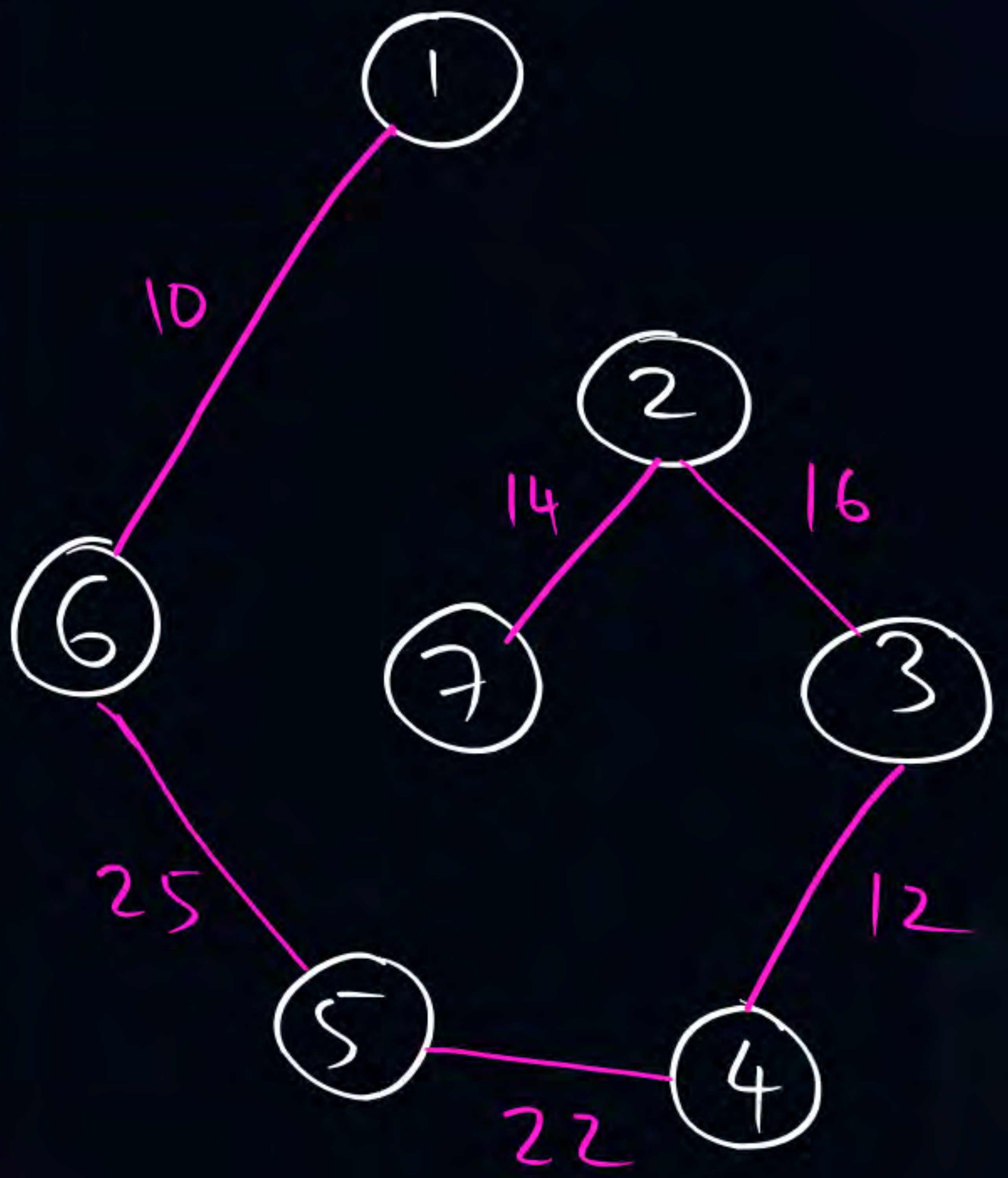
(c) both I and II

(d) neither I nor II

T

⟨Dijkstra's Algo. property⟩


1
2
3
4
5
6
7
28
10
14
16
24
18
e
25
12
22


1
2
3
4
5
6
7
10
14
16
25
12
22

Topic : III. Greedy Method

Q. Let G be a connected undirected weighted graph. Consider the following two statements.

F S1: There exists a minimum weight edge in G which is present in every minimum spanning tree of G

S2: If every edge in G has distinct weight, then G has a unique minimum spanning tree.
T

Which one of the following options is correct?

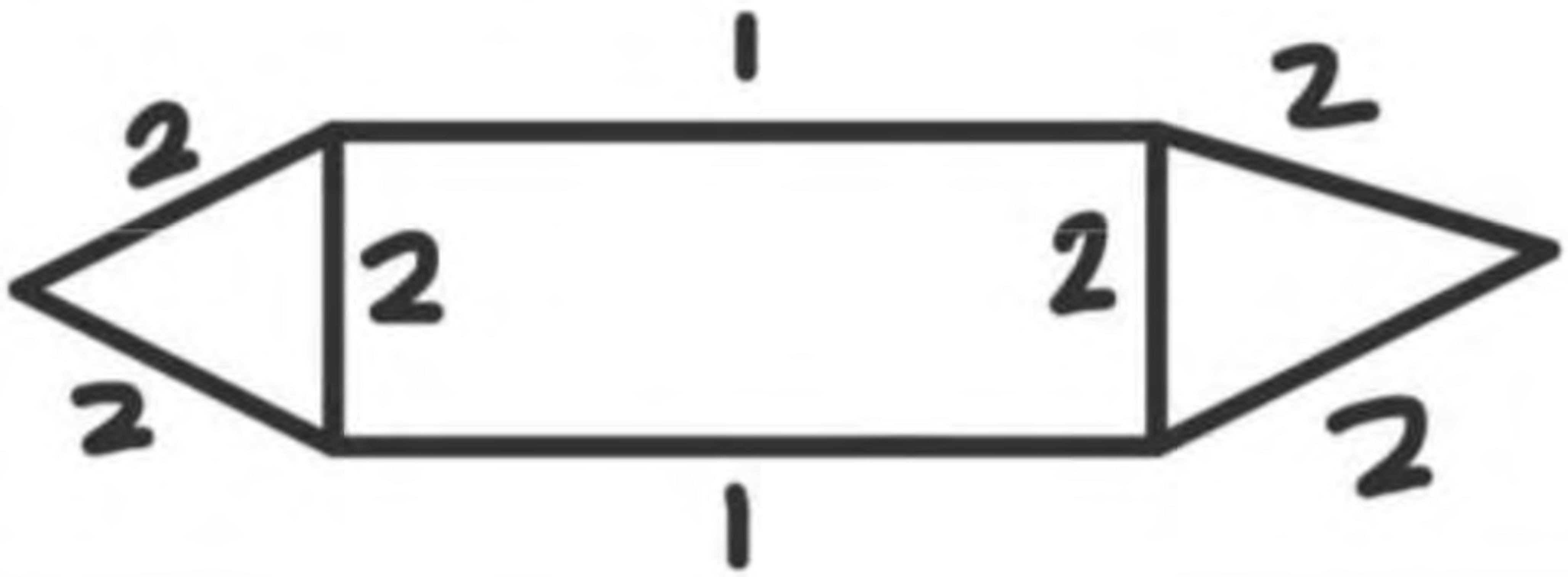
(a) Both S1 and S2 are true

(b) S1 is true and S2 is false

(c) S1 is false and S2 is true

(d) Both S1 and S2 are false.

No of minimal spanning trees



H/W

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6) Single Source Shortest Paths (SSSP)

< Dijkstra's Algorithm >

(i) Single pair Shortest Path

trivial

unicast

s ——— d

(ii) Single Source Shortest Paths

(Broadcasting)

[s → d_1, d_2, …, d_{n-1} } (n-1) dest's]

Dijkstra's Algo (Greedy)

May NOT work with -ve wt. edges

Bellman-Ford (D.P)

(Can work with -ve wt. edges)

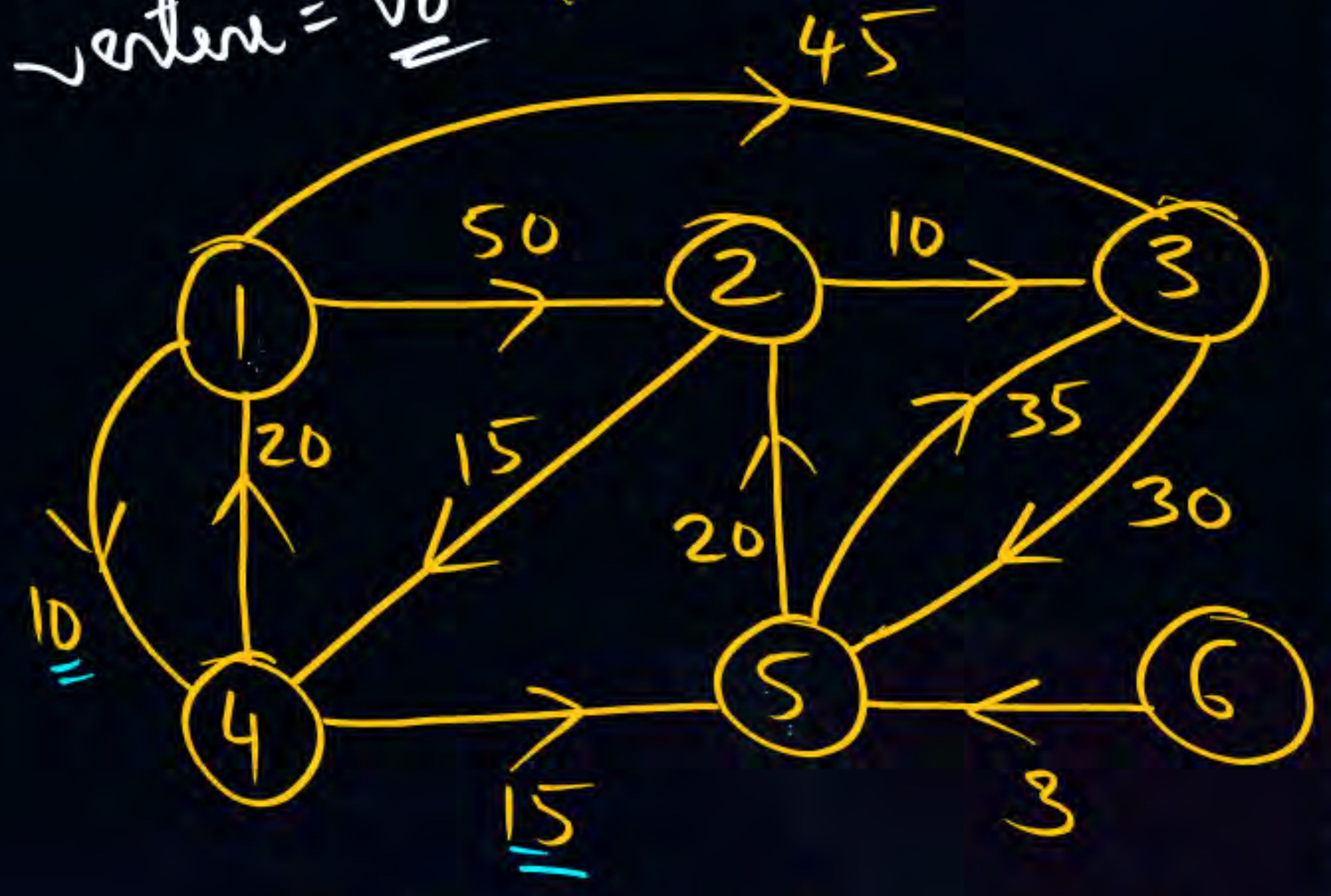
(iii) All-Pairs Shortest Paths

< Flyod-warshall's Algo. > [D.P]

(<i, j> every pair Sht. Path)

(Dijkstra's Greedy Algo)

Source vertex = V_0



C	1	2	3	. .	6
1	0	50	45		
2					
3					
.					
.					
6					

$c\langle i,j\rangle$ = edge Cost

a) Matrix-Method : $V_0 = 1$

$V_0 = 6$

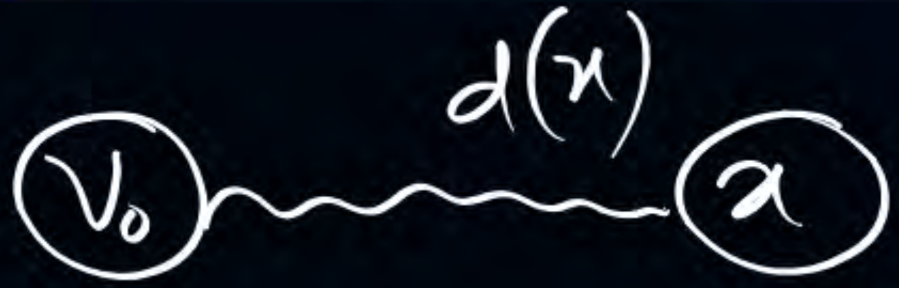
	Vertex Selected	d-values 1	2	3	4	5	6
Step 1	1	–	50	45	10	∞	∞
Step 2	{1,4}	–	50	45	10	25	∞
Step 3	{1,4,5}	–	45	45	10	25	∞
Step 4	{1,2,4,5}	–	45	45	10	25	∞
Step 5	{1,2,3,4,5}	–	45	45	10	25	∞

$v_0 = 6$

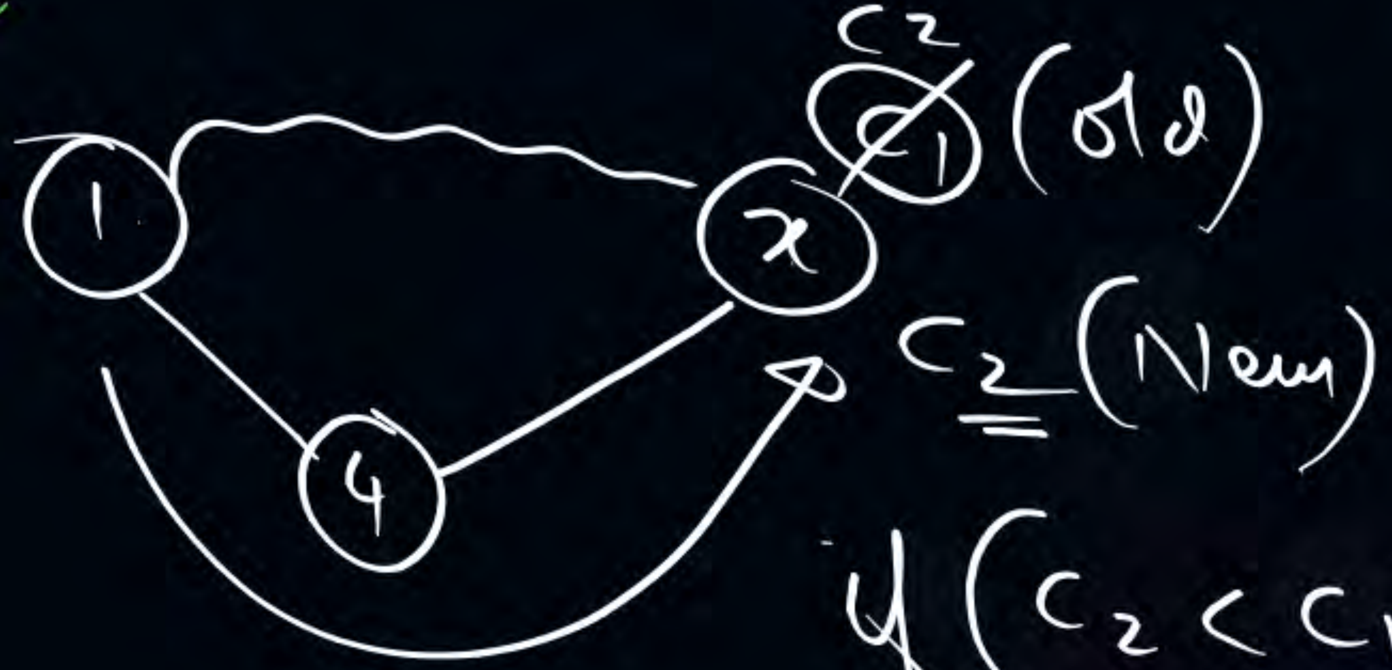
Vertex Selected		1	2	3	4	5	6
Step 1	{6}	∞	∞	∞	∞	3	—
Step 2	{6, 5}	∞	23	38	∞	3	—
Step 3	{6, 2, 5}	∞	23	33	38	3	—
Step 4	{6, 2, 3, 5}	∞	23	33	38	3	—
Step 5	{6, 2, 3, 5, 4}	58	23	33	38	3	—

d-values

$d(x)$ = distance from source (V_0) to vertex 'x' known so far



(Relaxation Process)



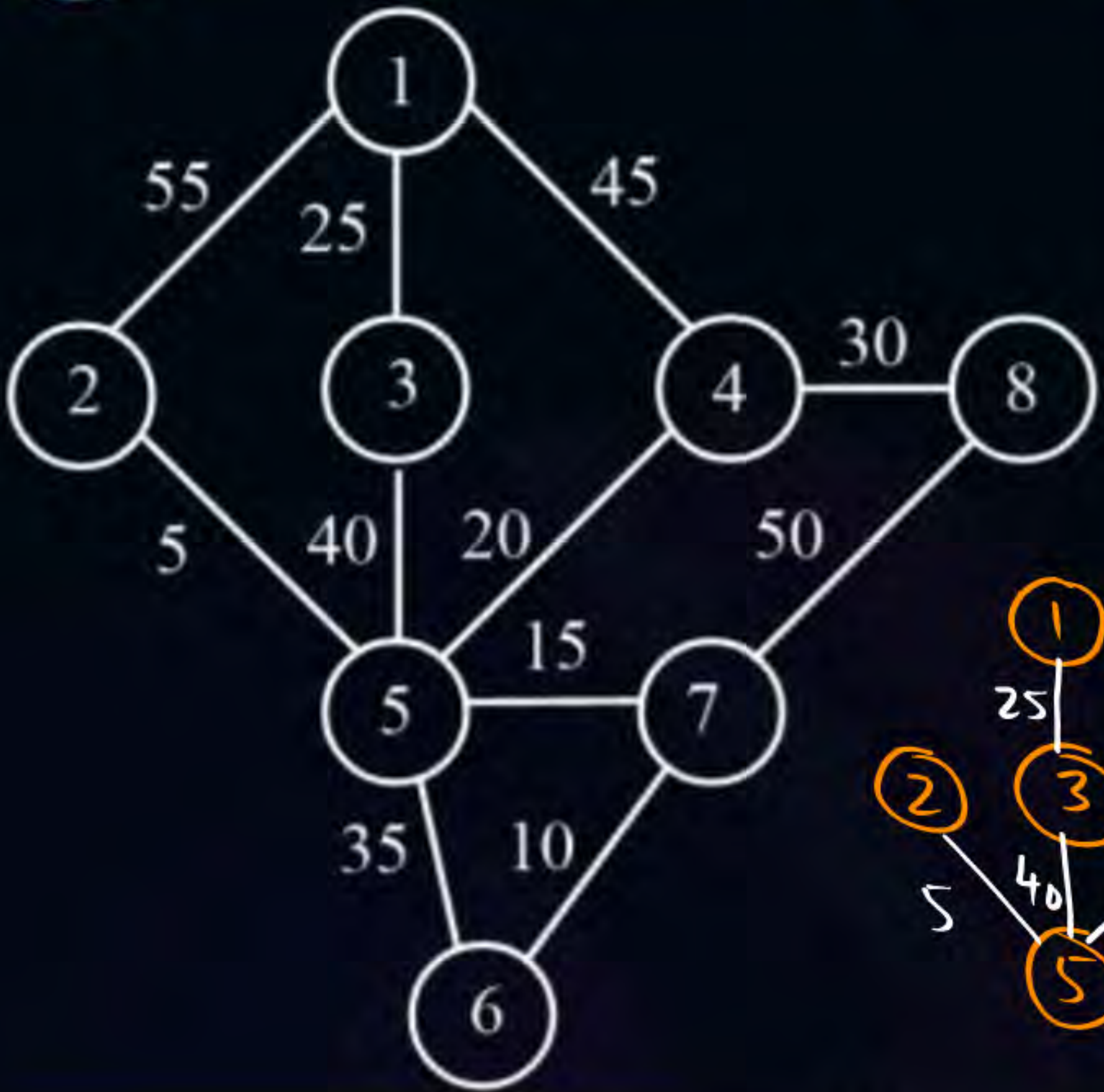
if ($c_2 < c_1$)
update (d-value of x)

Topic : Greedy Method

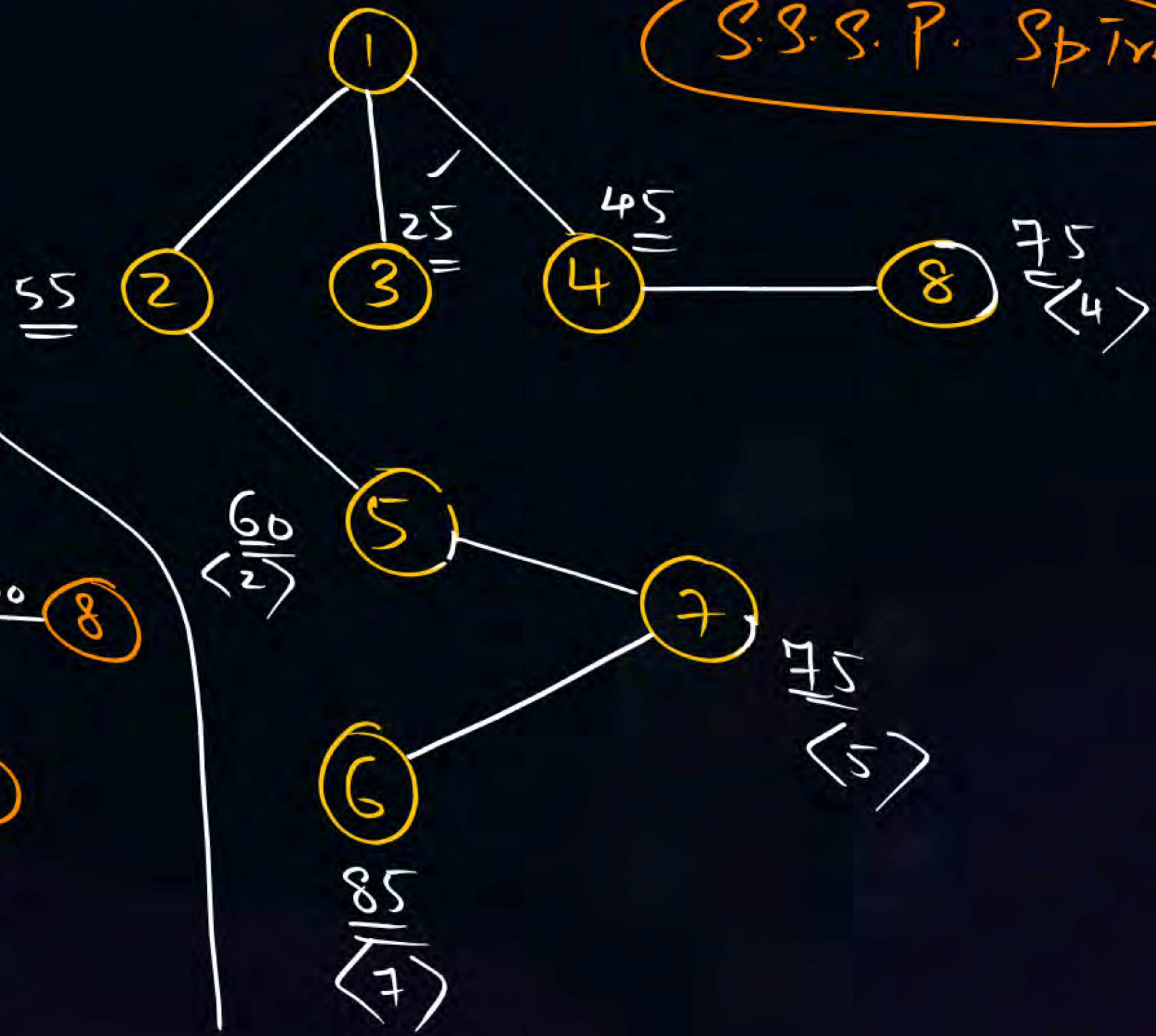
* Dijkstra's Algo [Spanning Tree]

$v_0 = 1$

S.S.S.P. Sp Tree



M.C.S.T



THANK - YOU