ENGINEERING MATHEMATICS

ALL BRANCHES

Rank of Matrix-II
Linear Algebra

DPP-05 Solution

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Question 1

If $A = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$, then adj. A is equal to

$$A^T = \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

A A

B c^T

C $3A^T$

D 3 A

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = -3$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -6$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = 6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = 3$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -6$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = 6$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -6$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = 3$$

$$\operatorname{Adj} A = [\text{Cofactor matrix}]^T$$

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = 3A^T$$

Question 2

If the rank of the matrix, $A = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 7 & \lambda \\ 1 & 4 & 5 \end{vmatrix}$ is 2, then the value of λ is

A – 13

B 13

C 3

D None of these

Since $\rho(A) = 2$ it's 3×3 minor will be 0.

$$\therefore \begin{vmatrix} 2 & -1 & 3 \\ 4 & 7 & \lambda \\ 1 & 4 & 5 \end{vmatrix} = 0$$

$$2(35 - 4\lambda) + 1(20 - \lambda) + 3(16 - 7) = 0$$

$$70 - 8\lambda + 20 - \lambda + 27 = 0$$

$$9\lambda = 117$$

$$\boxed{\lambda = 13}$$

Question 3

Let A and B be non-singular square matrices of the same order. Consider the following statements.

(I) $(AB)^T = A^T B^T$ ✗

(II) $(AB)^{-1} = B^{-1} A^{-1}$ ✓

(III) ✗ $\operatorname{adj}(AB) = (\operatorname{adj}.A)(\operatorname{adj}.B)$

(IV) $\rho(AB) = \rho(A)\rho(B)$ ✗

(V) ✓ $|AB| = |A| \cdot |B|$

Which of the following statements are false?

$(AB)^T = B^T A^T$

$(AB)^{-1} = B^{-1} A^{-1}$

$\operatorname{adj}(AB) = \operatorname{adj}(B).\operatorname{adj}(A)$

A. I, III & IV ✓

B. IV & V

C. I & II

D. All the above

Question 4

The rank of the matrix $A = \begin{vmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{vmatrix}$ is

A 3

B 2

C 1

D None of these

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_1} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

No. of non-zero rows $= 2$

$\therefore \rho(A) = 2$

Question 5

If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then the value of x is

A 1

B 2

C 1/2

D None of these

We Know, $AA^{-1} = I$

$$\begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore 2x = 1$$

$$\boxed{x = 1/2}$$

Question 6

The rank of 3×3 matrix C (= AB), found by multiplying a non-zero column matrix A of size 3×1 and a non-zero row matrix B of size 1×3, is

A 0

B 1

C 2

D 3

Matrix $C = AB$

$$A \rightarrow \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}_{3\times 1}$$

Max $\rho(A), \rho(B) \rightarrow \min(m, n)$

$\therefore \rho(A) = \rho(B) = 1$

$$B \rightarrow \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}_{1\times 3}$$

$\Rightarrow \rho(AB) \leq \min.(\rho(A), \rho(B))$

$C \rightarrow A_{3\times 1}\, B_{1\times 3}$

$\rho(AB) \leq 1 \quad (0, 1)$

$C \rightarrow AB_{3\times 3}$

$\therefore \rho(AB) = 1 \quad \because AB \neq 0$

Question 7

Given matrix $[A]=\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}_{3\times 4}$, the rank of the matrix is

A 4

B 3

C 2

D 1

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 4 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row echelon form

No. of non-zero rows = 2

$\therefore \rho(A) = 2$

Question 8

The rank of the matrix is $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}_{3\times 4}$ is 2.

$R_3 \to R_3 - 2R_1 + R_2$

$$\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} -2 & 14 & 8 & 18 \\ 6 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \to R_2 + 3R_1} \begin{bmatrix} -2 & 14 & 8 & 8 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 14 & 8 & 8 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of non zero rows $= \rho(A) = 2$

Question 9

Let $A = [a_{ij}]$ $1 \le i, j \le n$ with $n \ge 3$ and $a_{ij} = i.j$ the rank of the A is

A 0

B 1

C $n-1$

D n

$$\begin{bmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & a_{22} & \cdots \\ \vdots & & \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 & 1\times2 & 1\times3 & \cdots \\ 2\times1 & 2\times2 & \cdots & \\ \vdots & & & \end{bmatrix}$$

All rows are proportional

$\rho(A) = 1$

each row is scalar multiple of first row.

Question 10

The rank of the matrix $M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$ is

Δ about 1st column;

$\Delta = 5(0-12) - 1(60-60) + 3(20-0)$

$\Delta = -60 + 0 + 60 = 0$

$\therefore \rho(M) < 3$

$M_{11} = \begin{vmatrix} 5 & 10 \\ 1 & 0 \end{vmatrix} \neq 0$ hence $\rho(M) = 2$.

A 0

B 1

C 2

D 3

Thank you

GW Soldiers !