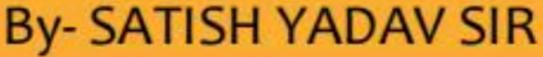
# CS & IT





Lecture No. 6







01 Basics of relations

02 Types of relations

**03 Number Of relations** 



demand diagonal > 2 Reflexive YaEA (a,a) ER.

Symmetric  $\forall a \forall b [(a,b) \in R \rightarrow (b,a) \in R] \rightarrow 2^{n \cdot 2 \cdot 2 \cdot 2}$ 

TRR YaEA (a.a) &R.

(hates

same clement) 2

sallows same element

4 demands flipping.



Antisymmetric. Dallows same element.

Yayb [ (a,b) \in R \ (b,a) \in R \ \a=b ]

$$R_{2} = \left\{ (1, 1) \right\} \underbrace{Anti}_{Anti}$$



$$RA = \{ (12) \} \text{ Antiv}$$

$$(a,b) \in R \land (b,a) \in R \rightarrow a = b$$

$$(1,2) \in R \land F \rightarrow F$$

$$True$$

$$R5 = \{ \} \$$

$$R6 = \{ (22)(11) \} \$$

$$R4 = \{ (23)(11) \} \$$

$$R7 = \{ (23)(11) (22)(33)(13) \} \$$

$$R8 = \{(23)\}$$

$$R9 = \{(13)\}$$

$$\{(13)\}$$

$$\{(13)\}$$

$$\{(13)\}$$

$$\{(13)\}$$

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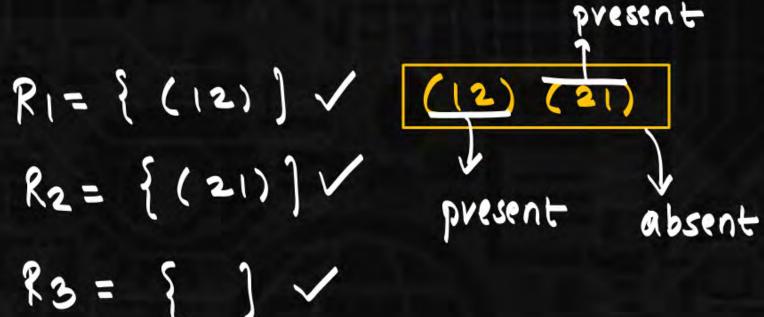
$$\{(13)\}$$

$$\{(13)\}$$

elations

Antipallows same element.

Antipallows flipping.



Pw.

boxes = 
$$\frac{n^2 - n}{2}$$

$$A \times A = \begin{cases} (11)(22)(33)(12)(21)(13)(31)(23)(32) \\ \frac{n^2 - n}{2} \\ \frac{n^2 - n}{2} \end{cases}$$



$$(2,1)$$
  $(2,2)$   $(2,3)$   $(3,3)$   $(3,3)$ 

Total =  $n^2$ . Quagon = nnon diagonal =  $n^2 - n$ . boxes =  $\frac{n^2 - n}{2}$ 3 choices

3,3)) achoices

2<sup>n.</sup>

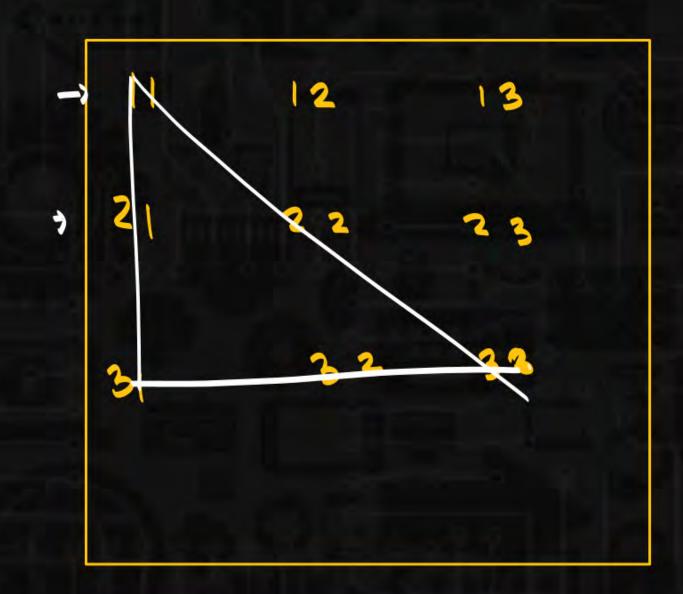




Totalno.of
Anti
212

smakest size. (11) (22) (33) (12)(11) argest size.  $\left\{ \frac{n^{2}-n}{(11)(22)(33)(12)(13)(23)} \right\}$  $n + \frac{n^2 - n}{2}$ 





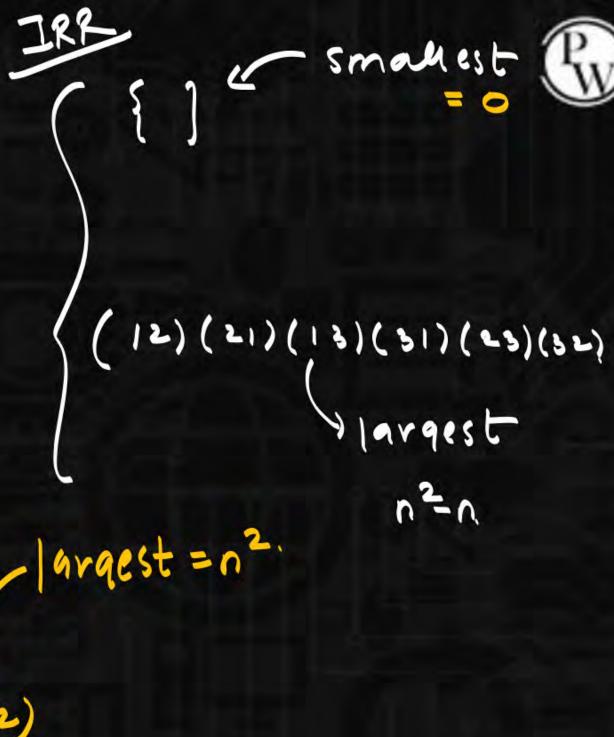


```
smallest size = 0
Symmetric.
       (11)(22)(33)(12)(21)(13)(31)(23)(32)
```

```
Reflexive:

[ smallest = n.

(11) (22) (53)
```



(11) (22) (33) (12) (21) (13) (31) (23) (32)



A symmetric: -) does not allows same element.

Hayb((a,b) \in R -) (b,a) \neq R)

$$R_{1} = \{ \}$$

$$(a,b) \in R \rightarrow (b,a) \notin R.$$

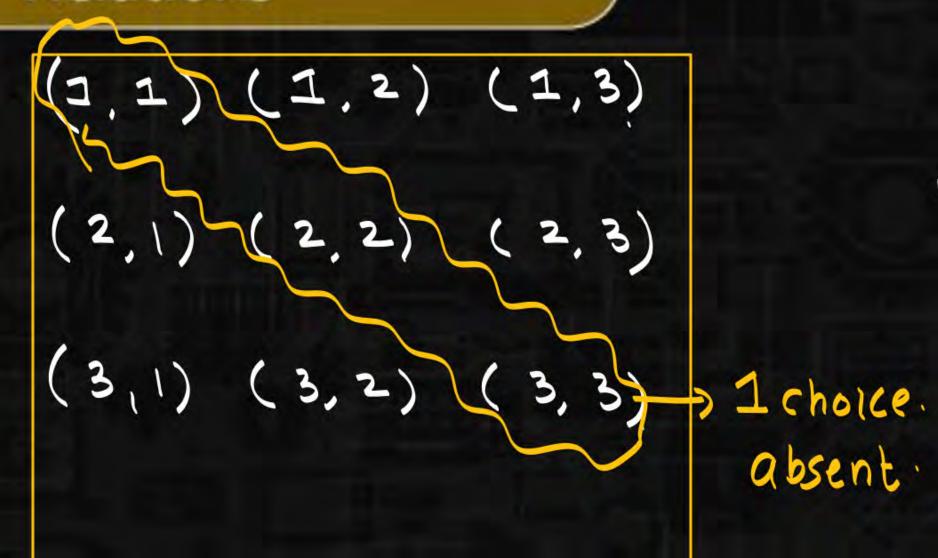
$$+ \rightarrow$$

$$T_{TVUE}$$



$$R_{3} = \{ (12) \} \text{ Asy } / \\ (a,b) \in R \rightarrow (b,a) \notin R. \\ (1,2) \in R \rightarrow (2,1) \notin R. \\ T.$$





N2-0 present absent. Present absent.

## Symmetric

- -, allows same element
- -> demands flipping
  - $\frac{n^2-n}{2}$

 $(a,b)\in R\rightarrow (b,a)\in R$ 

## Antisymmetric

→ allows same.
element:

→ no flipping.

 $(a,b) \in R \land (b,a) \in R \rightarrow a = b$ 

## Asymmetric.

no same elements

-> no flipping.

3

(a,b) ∈ R → (b.a) & R.



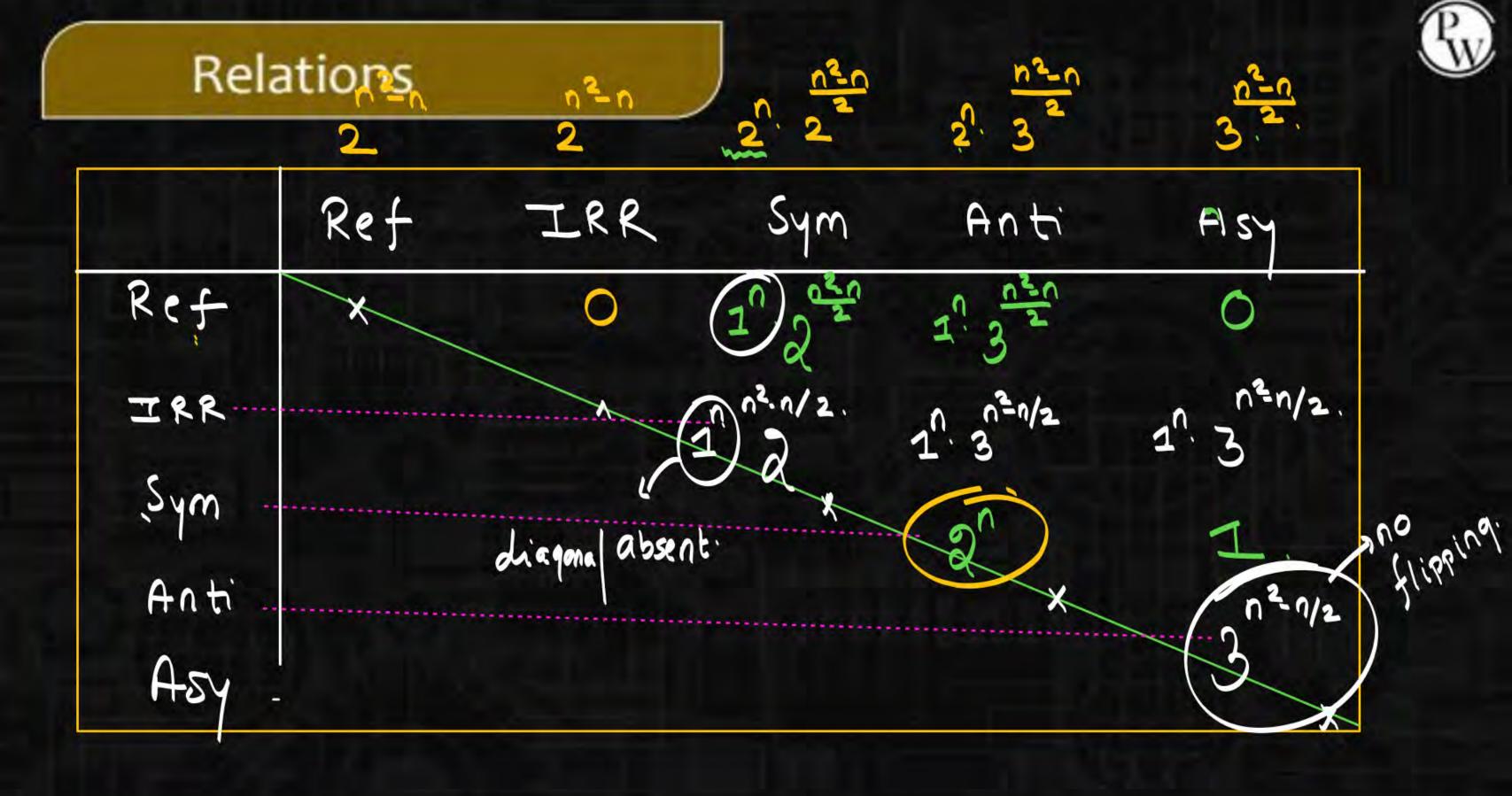
$$\{=\}(11)(21)(22)(23)(24)(31)(32)(33)(34)\}$$





(GATE-09)

Sym &
Anti &





Reflexive 1 I rreflexive. {(11)(22)(33).....







$$R_1 = \{ 11 22 33 () () ... \}$$
 $R_2 = \{ 11 22 33 () () ... \}$ 



	Union (U)	Inter— (n)
RI, RZ reflexive		
RI.RZ ZRR		
RI RZ Symm		
Anti	X	
ASY	X	

$$R_1 = \{(12)\}$$

$$R_2 = \{(21)\}$$
Antiv

RIUR2 = 
$$\{(12)(2i)\}$$
 not Anti  
RIUR2 =  $\{\}$ 



#### check for

reflexive, symmetric, antisymmetric, or transitive.

 $\Re \subseteq \mathbf{Z}^+ \times \mathbf{Z}^+$  where  $a \Re b$  if  $a \mid b$  (read "a divides b,"

- **b)**  $\Re$  is the relation on **Z** where  $a \Re b$  if a | b.
- c) For a given universe  $\mathcal U$  and a fixed subset C of  $\mathcal U$ , define  $\mathcal R$  on  $\mathcal P(\mathcal U)$  as follows: For A,  $B \subseteq \mathcal U$  we have  $A \mathcal R B$  if  $A \cap C = B \cap C$ .
- **d)** On the set A of all lines in  $\mathbb{R}^2$ , define the relation  $\mathcal{R}$  for two lines  $\ell_1$ ,  $\ell_2$  by  $\ell_1$   $\mathcal{R}$   $\ell_2$  if  $\ell_1$  is perpendicular to  $\ell_2$ .
- e)  $\Re$  is the relation on **Z** where  $x \Re y$  if x + y is odd.
- f)  $\Re$  is the relation on **Z** where  $x \Re y$  if x y is even.
- g) Let T be the set of all triangles in  $\mathbb{R}^2$ . Define  $\mathcal{R}$  on T by  $t_1 \mathcal{R}$   $t_2$  if  $t_1$  and  $t_2$  have an angle of the same measure.
- h)  $\Re$  is the relation on  $\mathbb{Z} \times \mathbb{Z}$  where  $(a, b)\Re(c, d)$  if  $a \le c$ . [Note:  $\Re \subseteq (\mathbb{Z} \times \mathbb{Z}) \times (\mathbb{Z} \times \mathbb{Z})$ .]



- (a) reflexive, antisymmetric, transitive
- (b) transitive
- (c) reflexive, symmetric, transitive
- (d) symmetric
- (e) (odd): symmetric
- (f) (even): reflexive, symmetric, transitive
- (g) reflexive, symmetric
- (h) reflexive, transitive
- (a) If R is a relation on A and |R| ≥ n, then R is reflexive.
- b) If  $\Re_1$ ,  $\Re_2$  are relations on A and  $\Re_2 \supseteq \Re_1$ , then  $\Re_1$  reflexive (symmetric, antisymmetric, transitive)  $\Rightarrow \Re_2$  reflexive (symmetric, antisymmetric, transitive).
- c) If  $\mathcal{R}_1$ ,  $\mathcal{R}_2$  are relations on A and  $\mathcal{R}_2 \supseteq \mathcal{R}_1$ , then  $\mathcal{R}_2$  reflexive (symmetric, antisymmetric, transitive)  $\Rightarrow \mathcal{R}_1$  reflexive (symmetric, antisymmetric, transitive).
- (a) False: Let A = (1,2) and R = ((1,2),(2,1)).
- i) (i) Reflexive: True
  - (ii) Symmetric: False. Let  $A = \{1,2\}, \mathcal{R}_1 = \{(1,1)\}, \mathcal{R}_2 = \{(1,1),(1,2)\}.$
  - (iii) Antisymmetric & Transitive: False. Let  $A = \{1, 2\}, \mathcal{R}_1 = \{(1, 2)\}, \mathcal{R}_2 = \{(1, 2), (2, 1)\}.$
- (c) (i) Reflexive: False. Let  $A = \{1, 2\}, R_1 = \{(1, 1)\}, R_2 = \{(1, 1), (2, 2)\}.$ 
  - (ii) Symmetric: False. Let  $A = \{1,2\}, \mathcal{R}_1 = \{(1,2)\}, \mathcal{R}_2 = \{(1,2),(2,1)\}$ .
  - (ili) Antisymmetric: True
  - (iv) Transitive: False. Let  $A = \{1, 2\}, \mathcal{R}_1 = \{(1, 2), (2, 1)\}, \mathcal{R}_2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}.$

10. If A = {w, x, y, z}, determine the number of relations on A that are (a) reflexive; (b) symmetric; (c) reflexive and symmetric; (d) reflexive and contain (x, y); (e) symmetric and contain (x, y); (f) antisymmetric; (g) antisymmetric and contain (x, y); (h) symmetric and antisymmetric; and (i) reflexive, symmetric, and antisymmetric.



- (b)  $(2^4)(2^6) = 2^{10}$ (e)  $(2^4)(2^5) = 2^6$ (h)  $(2^4)$
- (c) 2<sup>6</sup> (f) 2<sup>4</sup> · 3<sup>6</sup> (i) 1

- (a) 2<sup>12</sup> (d) 2<sup>11</sup> (g) 2<sup>4</sup> · 3<sup>5</sup>

- 17. Let  $A = \{1, 2, 3, 4, 5, 6, 7\}$ . How many symmetric relations on A contain exactly (a) four ordered pairs? (b) five or-



