

# CS & IT ENGINEERING

Data Structure



**Chapter-**  
**Hashing**  
**Lec-02**

07



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TOPICS TO BE  
COVERED



Hashing-II

Quad. Probing is free from primary clustering

## Quadratic Probing

$h(k) = k \bmod m \Rightarrow$  this leads to a collision

$$h(k) = k \bmod m = L_1 \quad \text{Collision}$$

$$H(k, i) = (h(k) + i^2) \bmod m$$

$$i=1$$

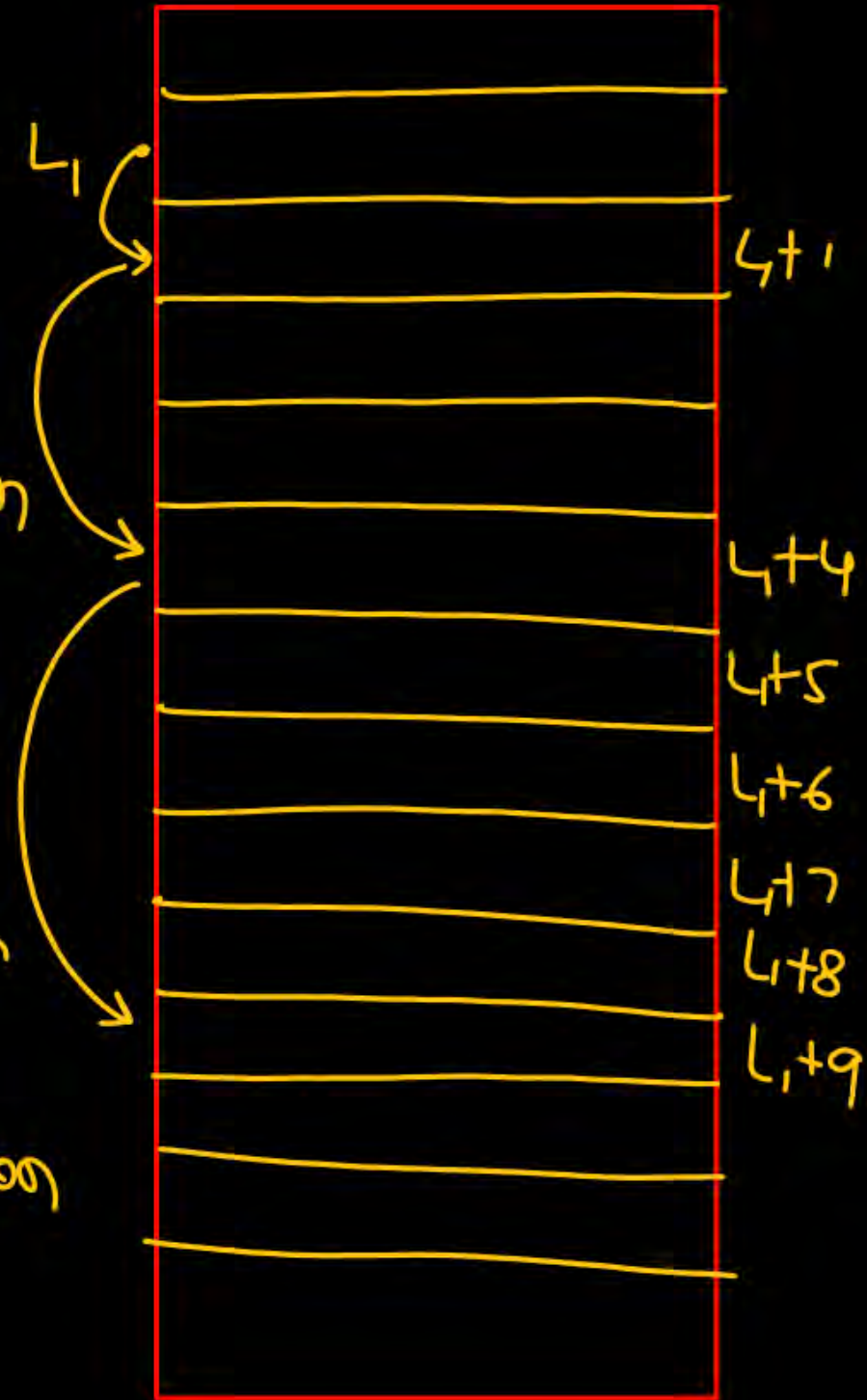
$$i=2$$

$$i=3$$

$$H(k, 1) = (h(k) + 1^2) = L_1 + 1^2 \quad \text{Collision}$$

$$H(k, 2) = (h(k) + 2^2) = L_1 + 4$$

$$H(k, 3) = (h(k) + 3^2) = L_1 + 9 \quad \text{Collision}$$





# Quadratic Probing

Keys: 24, 17, 32, 2, 13, 50, 30, 61

$$m = 11$$

$$h(24) = 24 \bmod 11 = 2$$

$$h(17) = 17 \bmod 11 = 6$$

$$h(32) = 32 \bmod 11 = 10$$

$$h(2) = 2 \bmod 11 = 2 \quad \times \text{Collision}$$

$$i = 1$$

$$H(2, 1) = (h(2) + i^2) \bmod 11 = 3$$

$$h(13) = 13 \bmod 11 = 2 \quad \times \text{collision}$$

$$i = 1$$

$$H(13, 1) = (h(13) + i^2) \bmod 11 = 3 \quad \times \text{collision}$$

$$i = 2$$

$$H(13, 2) = (h(13) + i^2) \bmod 11 = 6 \quad \times \text{collision}$$

$$i = 3$$

$$H(13, 3) = (h(13) + 3^2) \bmod 11 = 0 \quad \checkmark$$

$$h(50) = 50 \bmod 11 = 6 \quad \times$$

$$H(50, 1) = (h(50) + 1^2) \bmod 11 = 7$$

$$h(30) = 30 \bmod 11 = 8$$

$$h(61) = 61 \bmod 11 = 6 \quad \times$$

$$H(61, 1) = (h(61) + 1^2) \bmod 11 = 7 \quad \times$$

$$H(61, 2) = (h(61) + 2^2) \bmod 11 = 10 \quad \times$$

$$H(61, 3) = (h(61) + 3^2) \bmod 11 = 4 \quad \checkmark$$

0	13
1	
2	24
3	2
4	61
5	
6	17
7	50
8	30
9	
10	32



# Secondary Clustering Problem

$$\begin{cases} h(24) = 24 \bmod 11 = 2 \\ h(2) = 2 \bmod 11 = 2 \\ h(13) = 13 \bmod 11 = 2 \end{cases}$$

$i=1$

$$\begin{cases} H(24,1) = (h(24)+1) \bmod 11 = 3 \\ H(2,1) = (h(2)+1) \bmod 11 = 3 \\ H(13,1) = (h(13)+1) \bmod 11 = 3 \end{cases}$$

$i=2$

$$\begin{cases} H(24,2) = (h(24)+2^2) \bmod 11 = 6 \\ H(2,2) = (h(2)+2^2) \bmod 11 = 6 \\ H(13,2) = (h(13)+2^2) \bmod 11 = 6 \end{cases}$$

$i=3$

$$\begin{cases} H(24,3) = (h(24)+3^2) \bmod 11 = 0 \\ H(2,3) = (h(2)+3^2) \bmod 11 = 0 \\ H(13,3) = (h(13)+3^2) \bmod 11 = 0 \end{cases}$$

$$\begin{cases} i=4 \\ H(24,4) = (h(24)+4^2) \bmod 11 = 7 \\ H(2,4) = (h(2)+4^2) \bmod 11 = 7 \\ H(13,4) = (h(13)+4^2) \bmod 11 = 7 \end{cases}$$

$$\begin{cases} i=5 \\ H(24,5) = (h(24)+5^2) \bmod 11 = 5 \\ H(2,5) = 5 \\ H(13,5) = 5 \end{cases}$$

$i=6$

$$\begin{cases} H(24,6) = (h(24)+6^2) \bmod 11 = 5 \\ H(2,6) = 5 \\ H(13,6) = 5 \end{cases}$$

2, 3, 6, 0, 7, 5, 5, 7, 0, 6

$i=7$

$$\begin{cases} H(24,7) = (h(24)+7^2) \bmod 11 = 7 \\ H(2,7) = 7 \\ H(13,7) = 7 \end{cases}$$

$i=8$

$$H(24,8) = H(2,8) = H(13,8) = (2+8^2) \bmod 11 = 0$$

0	/ / / / / / / /
1	
2	/ / / / / / / /
3	/ / / / / / / /
4	
5	/ / / / / / / /
6	/ / / / / / / /
7	/ / / / / / / /
8	
9	
10	

$i=9$

$$\begin{aligned} H(24,9) &= (h(24)+9^2) \bmod 11 = 6 \\ H(2,9) &= 6 \\ H(13,9) &= 6 \end{aligned}$$



2, 3, 6, 0, 7, 5, 5, 7, 0, 6

Key = 24

2, 3, 6, 0, 7, 5, 5, 7, 0, 6, 3, 2,

2, 3, 6, 0, 7, 5

Keys that are hashed to some location follow the same resolution path bcz of which we are not able to utilize the table size efficiently.

0	/ / / / / / / / / /
1	Free slot
2	/ / / / / / / / / /
3	/ / / / / / / / / /
4	Free slot
5	/ / / / / / / / / /
6	/ / / / / / / / / /
7	/ / / / / / / / / /
8	Free slot
9	Free slot
10	Free slot

i = 9

$$H(24, 9) = (h(24) + 9^2) \bmod 11 = 6$$

$$H(2, 9) = 6$$

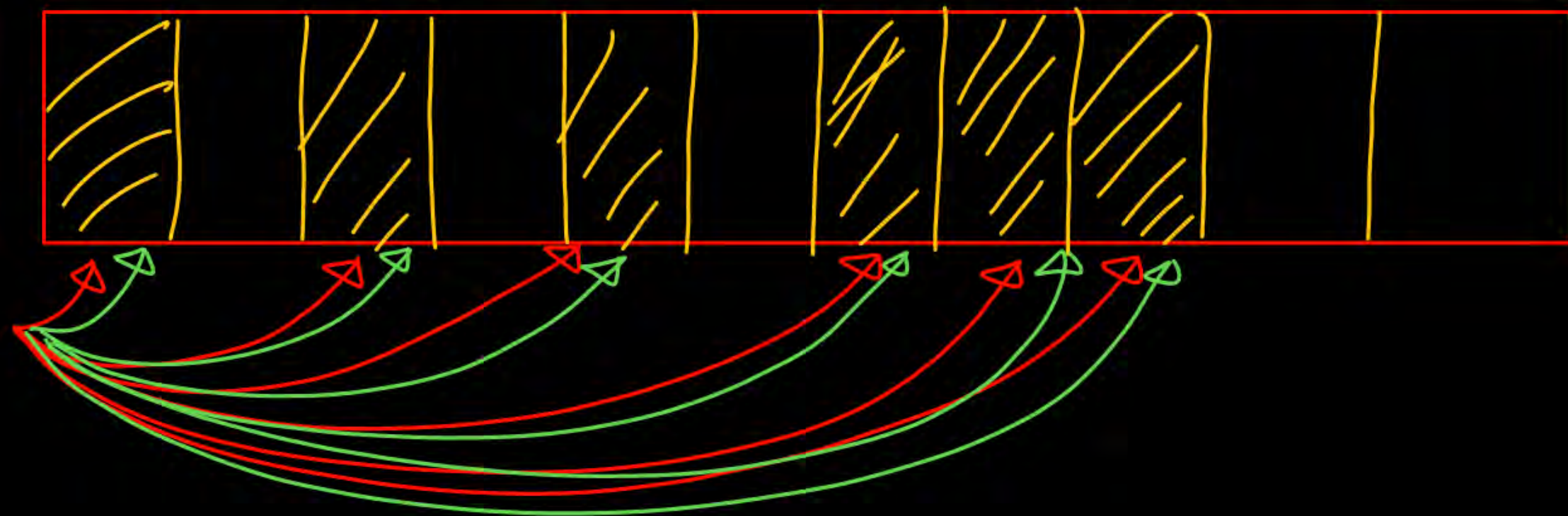
$$H(13, 9) = 6$$

m = 11

$\left(\frac{m+1}{2}\right) \rightarrow$  occupied

Inspite of almost 50% free slots we are not able to insert a new element.

OK





## Double Hashing

Let  $h(k)$  is the hash function  
 $h(k) = k \bmod m \Rightarrow$  collision

$$\begin{aligned} H(k, i) &= (h(k) + i) \bmod m && \text{L.P} \\ &= (h(k) + i^2) \bmod m && \text{Q.P} \end{aligned}$$

$$H(k, i) = (h(k) + i h'(k)) \bmod m$$

Primary  
hash function

Secondary  
hash function

Q What if  
the value generated  
by  $h'(k)$  is 0?



$h'(k)$  never generate  
0



# Double Hashing

2 6 10 7 8 9 1 5  
Key: 13, 17, 21, 2, 57, 28, 30, 27

$$h(x) = x \bmod 11$$

$$h'(x) = 7 - (x \bmod 7)$$

$$h(13) = 13 \bmod 11 = 2$$

$$h(17) = 17 \bmod 11 = 6$$

$$h(21) = 21 \bmod 11 = 10$$

$$h(2) = 2 \bmod 11 = 2^x$$

$$H(2,1) = (h(2) + 1 \cdot h'(2)) \bmod 11$$

$$h'(2) = 7 - 2 \bmod 7$$

$$= 7 - 2 = 5$$

$$H(2,1) = (2 + 1 \cdot 5) \bmod 11 = 7$$

$$h(57) = 57 \bmod 11 = 2^x$$

$$H(57,1) = (h(57) + 1 \cdot h'(57)) \bmod 11$$

$$h'(57) = 7 - 57 \bmod 7$$

$$= 7 - 1 = 6$$

$$H(57,1) = (2 + 1 \cdot 6) \bmod 11 = 8$$

$$h(28) = 28 \bmod 11 = 6^x$$

$$H(28,1) = (h(28) + 1 \cdot h'(28)) \bmod 11$$

$$h'(28) = 7 - 28 \bmod 7 = 7$$

$$H(28,1) = (6 + 1 \cdot 7) \bmod 11 = 2^x$$

$$H(28,2) = (h(28) + 2 \cdot h'(28)) \bmod 11$$

$$= (6 + 2 \cdot 7) \bmod 11 = 9^x$$

$$h(30) = 30 \bmod 11 = 8^x$$

$$H(30,1) = (h(30) + 1 \cdot h'(30)) \bmod 11$$

$$h'(30) = 7 - 30 \bmod 7$$

$$= 7 - 2 = 5$$


$$H(30,1) = (8 + 1 \cdot 5) \bmod 11 = 2^x$$

$$H(30,2) = (8 + 2 \cdot 5) \bmod 11 = 7^x$$

$$H(30,3) = (8 + 3 \cdot 5) \bmod 11 = 1$$

$$h(27) = 27 \bmod 11 = 5$$



$$h(2) = 2 \bmod 11 = 2, 7$$

$$h(57) = 57 \bmod 11 = 2, 8$$

Problem  
+ Overhead

$\Rightarrow$  2 hash function  
Computation time.

Time complexity



## Separate Chaining

Load factor ( $\lambda$ )

$$\lambda = \frac{n}{m}$$

no. of keys

Table size

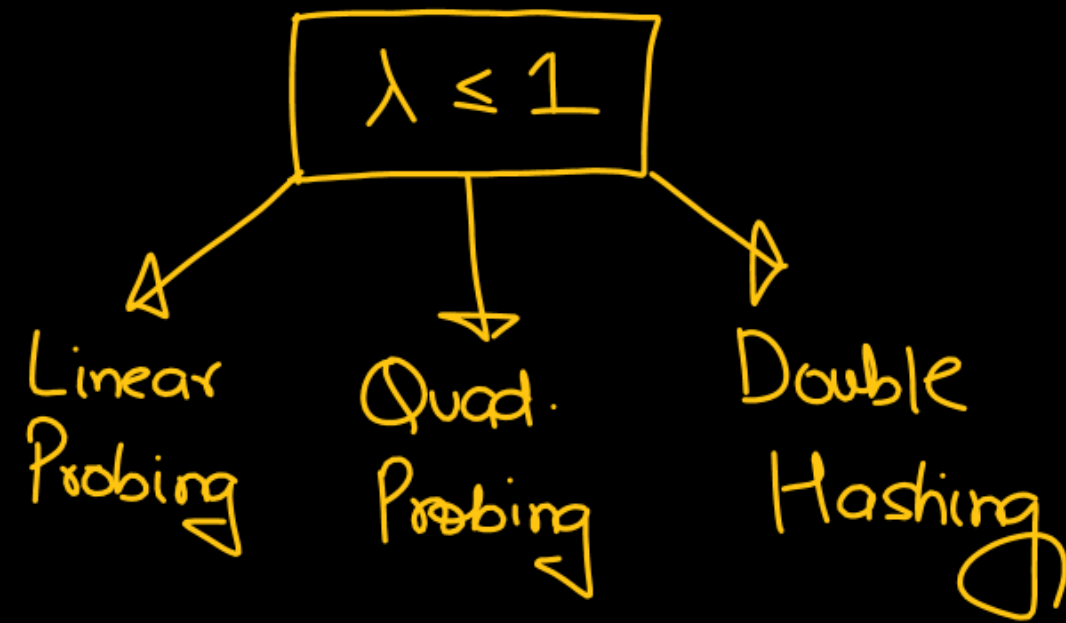
$$n = 20 \text{ keys}$$

$$m = 40$$

$$\lambda = \frac{20}{40} = \frac{1}{2} \checkmark$$

$$\begin{array}{l} n = 40 \\ m = 30 \end{array}$$





$\lambda > 1$

Separate chaining

: Collision resolve

↓  
List



Keys: 400, 500, 635, 425, 36, 86, 126, 16

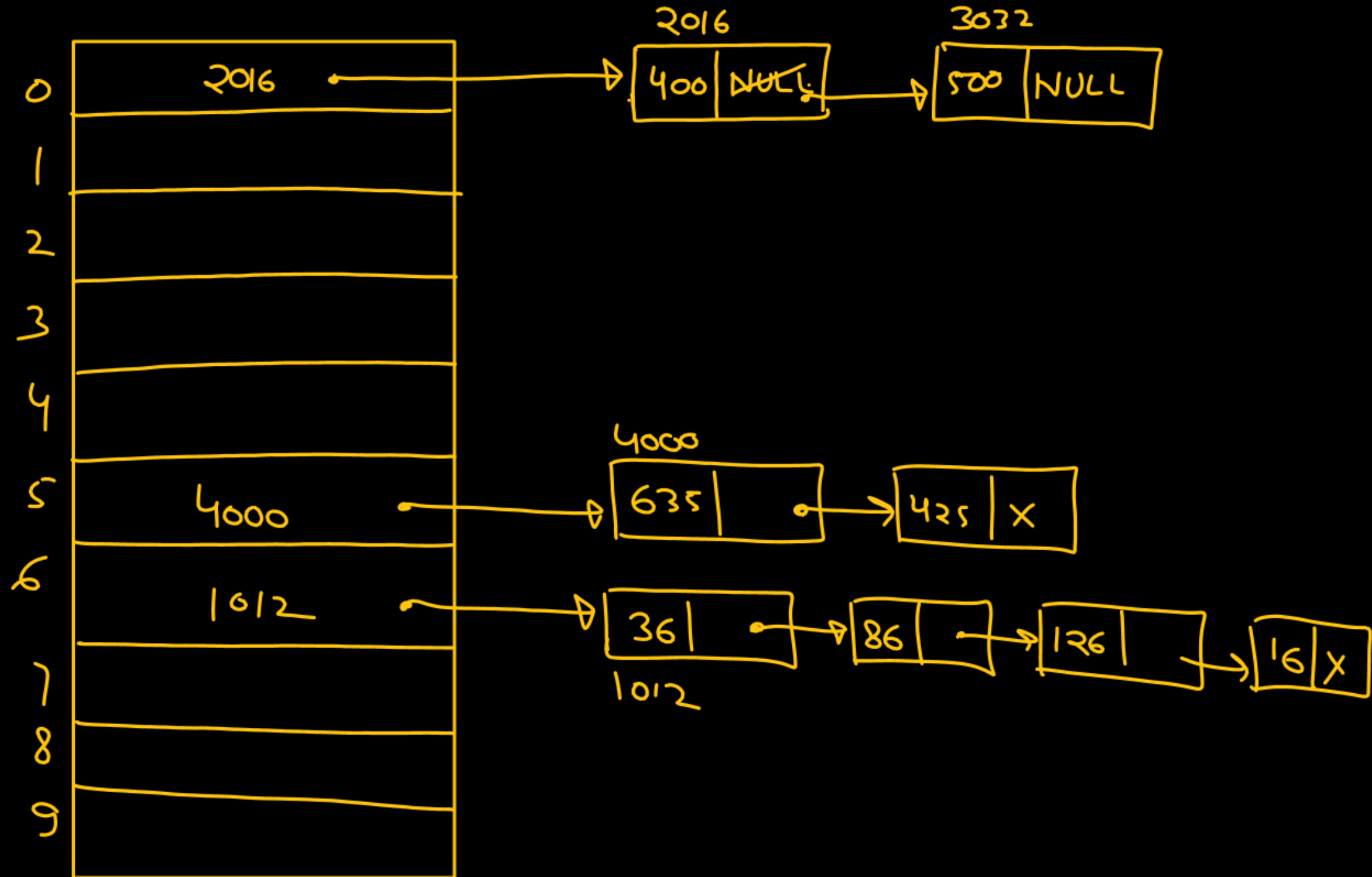
$m = 10$

$$h(400) = 400 \bmod 10 = 0$$

$$h(500) = 500 \bmod 10 = 0$$

$$h(635) = 635 \bmod 10 = 5$$

$(n+m)$   
Extra pointer



Linear  
Probing

Keys: 31, 26, 43, 27, 34, 12, 46, 14, 58

$m=12$

delete 26

Search 14

$14 \bmod 12$

$= 2$

after deletion

$\Rightarrow$  Rem. Keys

Re-hash

0	12
1	58
2	<del>26</del>
3	27
4	14
5	
6	
7	31
8	43
9	
10	34
11	46



