ENGINEERING MATHEMATICS

ALL BRANCHES

Numerical Methods
Numerical Solution of Algebraic
& Transcendental equations

DPP -02 Solution

By- CHETAN SIR

Question 1

In the interval $[0, \pi]$ the equation $x = \cos x$ has

$f_1(x) = f_2(x)$

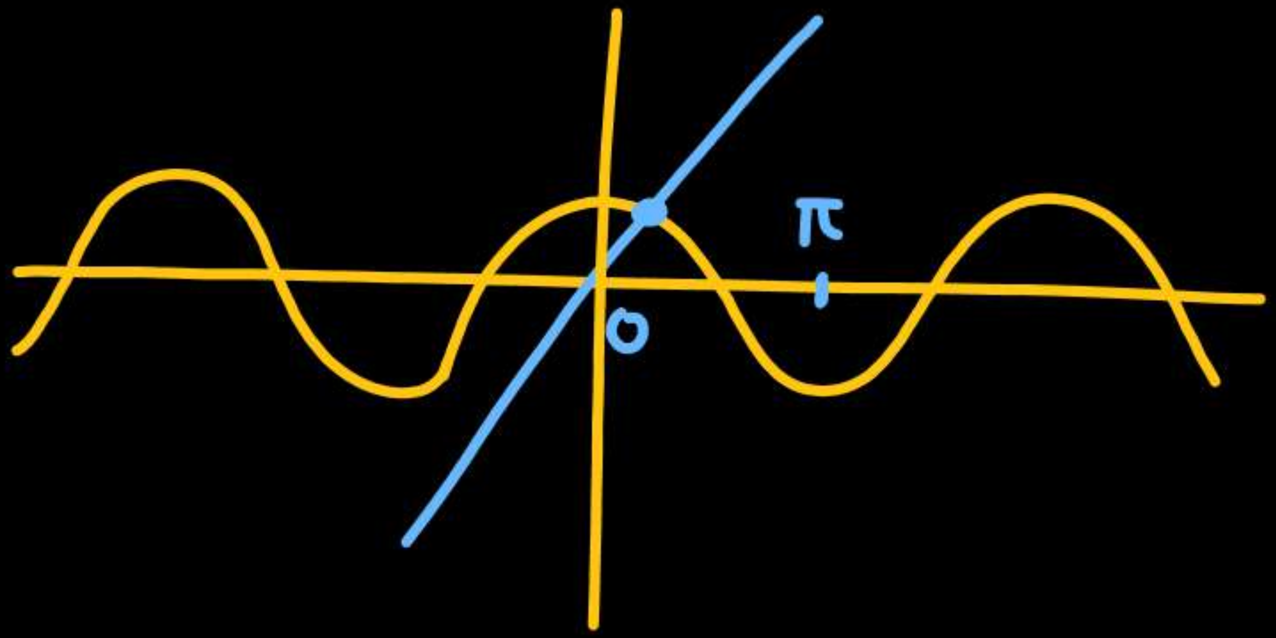
$x = \cos x$

A No solution

B Exactly one solution

C Exactly Two solution

D An infinite number of solution



Question 2

The Newton-Raphson method is used to find the root of the equation $x^2 - 2$. If the iterations are started from -1, then the iteration will-

A converge to -1

B converge to $\sqrt{2}$

C converge to $-\sqrt{2}$

D not converge

$f(x) = x^2 - 2 = 0$

$f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 2}{2x_n} = x_n - \frac{x_n}{2} + \frac{1}{x_n} = \frac{x_n}{2} + \frac{1}{x_n}$$

$$x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}$$

$$x_0 = -1$$

$$x_1 = -\frac{1}{2} + \frac{1}{-1} = -1.5$$

$$x_2 = \frac{-1.5}{2} + \frac{1}{-1.5} = -1.4166$$

$$x_3 = -\frac{1.4166}{2} + \frac{1}{-1.4166} = -1.4141$$

$$= -\sqrt{2}$$

Question 3

The equation $x^3 - x^2 + 4x - 4 = 0$ is to be solved using the Newton-Raphson method. If $x = 2$ is taken as the initial approximation of the solution, the next approximation using this method will be-

A $\frac{2}{3}$

B $\frac{4}{3}$

C 1

D $\frac{3}{2}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{x_0^3 - x_0^2 + 4x_0 - 4}{3x_0^2 - 2x_0 + 4}$$

$$= 2 - \frac{2^3 - 2^2 + 4 \times 2 - 4}{3(2)^2 - 2(2) + 4}$$

$$= 4/3$$

Question 4

Consider the series $X_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}, x_0 = 0.5$ obtained from the Newton-Raphson method. The series converges to-

$$x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$$

A 1.5

B $\sqrt{2}$

$$\alpha = \frac{\alpha}{2} + \frac{9}{8\alpha}$$

C 1.6

D 1.4

$$\alpha = \frac{4\alpha^2 + 9}{8\alpha}$$

$$8\alpha^2 = 4\alpha^2 + 9$$

$$4\alpha^2 = 9$$

$$\alpha = \pm 1.5$$

Question 5

Equation $e^x - 1 = 0$ is required to be solved using Newton's method with an initial guess $x_0 = -1$. Then, after one step of Newton's method, estimate x_1 of the solution will be given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{e^x - 1}{e^x}$$

$$= -1 - \frac{e^{-1} - 1}{e^{-1}} = -1 - 1 + \frac{1}{e^{-1}}$$

$$= -2 + e$$

$$= 0.71828$$

A 0.71828

B 0.36784

C 0.20587

D 0.00000

Question 6

The real root of the equation $xe^x = 2$ is evaluated using Newton-Raphson's method. If the first approximation of the value of x is 0.8676, the 2nd approximation of the value of x correct to three decimal places is-

A 0.865

B 0.853

C 0.849

D 0.838

$f(x) = xe^x - 2 \qquad x_1 = 0.8676$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = x_1 - \frac{x_1 e^{x_1} - 2}{x_1 e^{x_1} + e^{x_1}} = 0.8676 - \frac{0.8676\, e^{0.8676} - 2}{0.8676\, e^{0.8676} - e^{0.8676}}$$

$\hookrightarrow 0.853$

Question 7

The square root of a number N is to be obtained by applying the Newton-Raphson iterations to the equation $X^2 - N = 0$. If i denotes the iteration index the correct iterative scheme will be-

A $X_{i+1} = \frac{1}{2}\left(X_i + \frac{N}{X_i}\right)$

B $X_{i+1} = \frac{1}{2}\left(X_i^2 + \frac{N}{X_i^2}\right)$

C $X_{i+1} = \frac{1}{2}\left(X_i^2 + \frac{N^2}{X_i}\right)$

D $X_{i+1} = \frac{1}{2}\left(X_i - \frac{N}{X_i}\right)$

$$f(x) = x^2 - N$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = x_i - \frac{x_i^2 - N}{2x_i}$$

$$= x_i - \frac{x_i}{2} + \frac{N}{2x_i} = \frac{x_i}{2} + \frac{N}{2x_i}$$

$$\boxed{x_{i+1} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)}$$

Question 8

How many distinct values of x satisfy the equation sin(x) = x/2, where x is in radians?

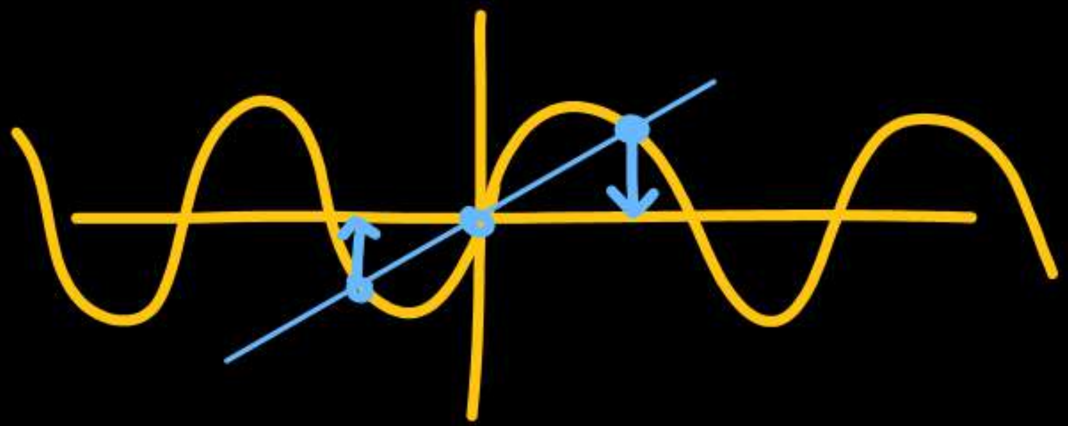
$f_1(x) = f_2(x)$

A 1

B 2

C 3

D 4 or more



Question 9

Only one of the real roots of $f(x) = x^6 - x - 1$ lies in the interval $1 \leq x \leq 2$ and bisection method is used to find its value. For achieving an accuracy of 0.001, the required minimum number of iterations is 10.

Min. no. of iterations $\frac{|b-a|}{2^n} < \varepsilon$

$$\frac{2-1}{2^n} < 0.001$$

$$1000 < 2^n$$

$$\boxed{n \geq 10}$$

Question 10

What is value of $(1525)^{0.2}$ to 2 decimal places?

A 4.33

B 4.36

C 4.38

D 4.30

$$x = (1525)^{0.2}$$

$$f(x) = x^5 - 1525 = 0$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \Rightarrow x_{i+1} = x_i - \frac{x_i^5 - 1525}{5x_i^4}$$

Let initial root = 4

$$x_1 = 4 - \frac{4^5 - 1525}{5 \times 4^4} = 4.39$$

$$x_2 = 4.39 - \frac{4.39^5 - 1525}{5(4.39)^4} = 4.33$$

$$x_3 = 4.33 - \frac{4.33^5 - 1525}{5(4.33)^4} = 4.33$$

Hence root is 4.33

Thank you

GW

Soldiers !