ENGINEERING MATHEMATICS

ALL BRANCHES

Probability

Continuous Random Variable

DPP-06 Solution

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Question 1

X is a uniformly distributed random variable that takes values between 0 and 1. the value of $E(X^3)$ will be

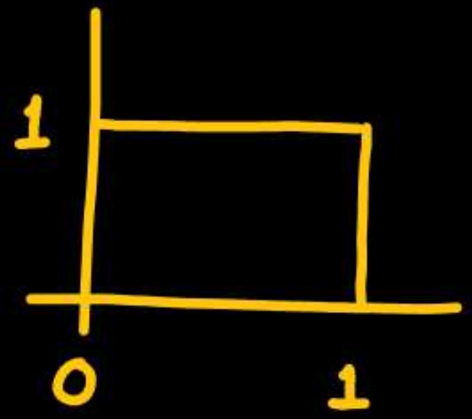
A 0

B $\frac{1}{8}$

C $\frac{1}{4}$

D $\frac{1}{2}$

$$f(x) = \begin{cases} \dfrac{1}{1-0} = 1 & 0 < x < 1 \\ = 0 & \text{otherwise} \end{cases}$$



$$E(x^3) = \int_0^1 x^3 f(x)\,dx$$

$$= \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4}$$

Question 2

A random variable is uniformly distributed over the interval 2 to 10. Its variance will be

A 16/3

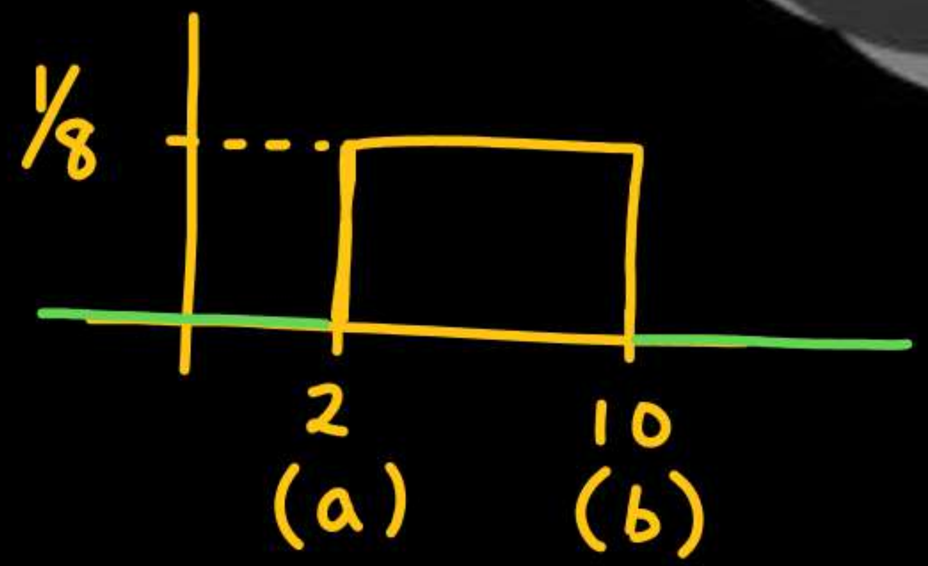
B 6

C 256/9

D 36

$$f(x) = \frac{1}{10-2} = \frac{1}{8} \quad ; \quad 2 < x < 10$$

$$= 0 \quad ; \text{ otherwise}$$



$$Var(x) = E(x^2) - [E(x)]^2 = \frac{(b-a)^2}{12}$$

$$E(x) = \int_a^b x\, f(x)\, dx = \int_2^{10} x \, \frac{1}{8} = \frac{1}{8}\left[\frac{x^2}{2}\right]_2^{10} = 6$$

$$E(x^2) = \int_a^b x^2 f(x)\, dx = \int_2^{10} x^2 \frac{1}{8} dx = \frac{1}{8}\left[\frac{x^3}{3}\right]_2^{10} = \frac{\overset{124}{\cancel{992}}}{\cancel{8} \times 3} = \frac{124}{3}$$

$$Var(x) = \frac{124}{3} - (6)^2 = \frac{16}{3}$$

$$\frac{(10-2)^2}{12} = \frac{64}{12} = \frac{16}{3}$$

Question 3

Consider a Gaussian distributed random variable with zero mean and standard deviation σ. The value of its cumulative distribution function at the origin will be ________.

A 0

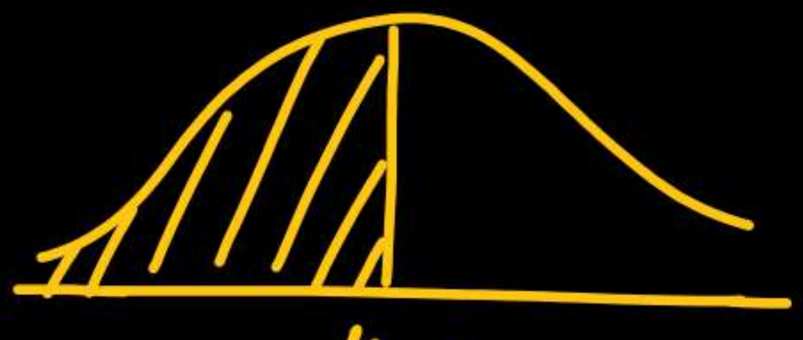
B 0.5

C 1

D $10\,\sigma$

$\text{Mean}(\mu) = 0$

$\text{S.D.} = \sigma$



Value of C.D.F. at origin.

$$F_x(0) = \int_{-\infty}^{0} f(x)\,dx = \frac{1}{2} = 0.5$$

Question 4

The independent random variables X and Y are uniformly distributed in the interval [–1, 1]. The probability that max [X,Y] is less than 1/2 is

$X \rightarrow -1$ to $+1$

$Y \rightarrow -1$ to $+1$

A 3/4

B 9/16

C 1/4

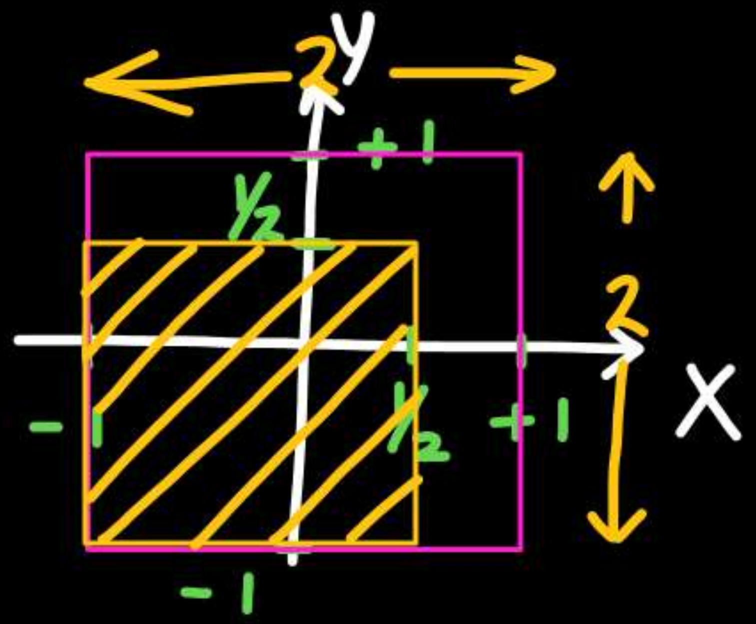
D 2/3

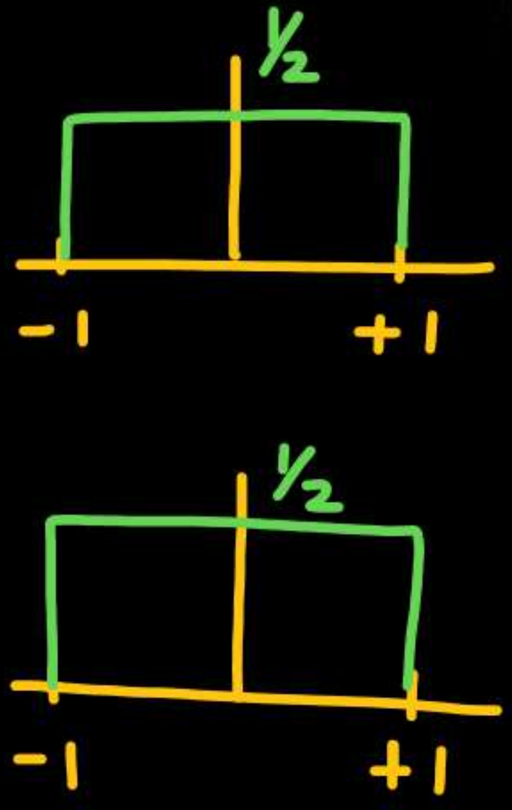
$$f(x) = f(y) = \begin{cases} \dfrac{1}{1-(-1)} = \dfrac{1}{2} & -1 < x, y < 1 \end{cases}$$

$$P\left(\max.\{x,y\} < \frac{1}{2}\right) = \frac{\text{Shaded area}}{\text{Total area}}$$

$$= \frac{\frac{3}{2} \times \frac{3}{2}}{2 \times 2}$$

$$= \frac{9}{16}$$





$-1 < y < 1/2$

$-1 < x < 1/2$

$-1 < \max(x,y) < 1/2$

Question 5

Let X be a random variable which is uniformly chosen from the set of positive odd numbers less than 100. The expectation, E[X] is

50

$S = \{1, 3, 5, 7 \ldots\ldots\ldots 99\}$ (50 numbers)

X	1	3	5	...	99
$p(x)$	$1/50$	$1/50$	$1/50$		$1/50$

D.R.V.

$$E(x) = \sum_{i=1}^{50} x_i p(x_i) = \frac{1}{50}\underbrace{[1+3+5+7 \ldots\ldots 99]}_{n^2 = 50^2 = 2500} = 50$$

Question 6

A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is 0.265

Soln :-

$\lambda = 5/\text{day}$

$\dfrac{e^{-\lambda}\lambda^{r}}{r!}$

$P(X<4) \Rightarrow P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$P(X\le 3) \Rightarrow \dfrac{e^{-\lambda}\lambda^{0}}{0!}+\dfrac{e^{-\lambda}\lambda^{1}}{1!}+\dfrac{e^{-\lambda}\lambda^{2}}{2!}+\dfrac{e^{-\lambda}\lambda^{3}}{3!}$

$\Rightarrow e^{-5}\left[1+5+\dfrac{25}{2}+\dfrac{125}{6}\right] = 0.265$

Question 7

An observer counts 240 veh/h at a specific highway location. Assume that the vehicles arrival at the location is Poisson distributed, the probability of Having one vehicle arriving over a 30-second time interval is 0.27.

$$\text{Average no. of vehicles passing in 30 sec } (\lambda) = \frac{24\cancel{0} \text{ veh}}{36\cancel{00} \text{ sec}} \times 3\cancel{0} \text{ sec}$$

$$\lambda = 2 \text{ veh}/30 \text{ sec}$$

$$\text{Count} = 240 \frac{\text{veh}}{\text{hr}}$$

$$\text{Reqd. probability } P(x=1) = \frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-2}\,2^1}{1!} = 0.27$$

Question 8

A simple random sample of 100 observations was taken from a large population. The sample mean & the standard deviation were determined to be 80 and 12 respectively. The standards error of mean is 1.2.

Size of sample $(n) = 100$

μ (Mean) $= 80$

σ (S.D.) $= 12$

* $\boxed{\text{Std. error of mean} = \frac{\sigma}{\sqrt{n}}} = \frac{12}{\sqrt{100}} = \frac{12}{10} = 1.2$

Question 9

The standard deviation of a uniformly distributed random variable between 0 and 1 is

A $1/\sqrt{12}$

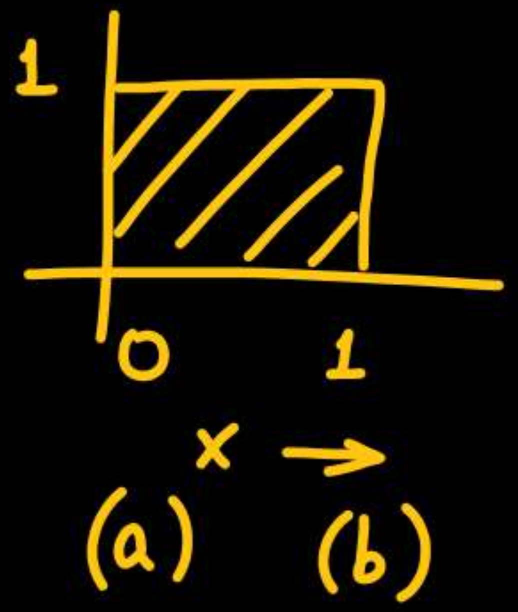
B $1/\sqrt{3}$

C $5/\sqrt{12}$

D $7/12$

$$f(x) = \begin{cases} \dfrac{1}{1-0} & ; \ 0 < x < 1 \end{cases}$$

$$Var(x) = E(x^2) - [E(x)]^2 = \frac{(b-a)^2}{12}$$



$$Var(x) = (S.D.)^2 = \sigma^2$$

$$S.D.(\sigma) = \sqrt{Var(x)}$$

$b = 1$

$a = 0$

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(1-0)^2}{12}} = \frac{1}{\sqrt{12}}$$

Question 10

Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

A $8/(2e^3)$

B $9/(2e^3)$

C $17/(2e^3)$

D $26/(2e^3)$

$\lambda = 3 \text{ cars/min}$

$$P(X<3) \Rightarrow P(X \le 2) \Rightarrow P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!}$$

$$= e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2}\right]$$

$$= e^{-3}\left[1 + 3 + \frac{3^2}{2}\right] = e^{-3}\left(\frac{2+6+9}{2}\right)$$

$$= \frac{17}{2e^3}$$

5 – 6pm

No. of cars $\rightarrow$ P

Thank you

GW

Soldiers !