

# CS & IT ENGINEERING

Discrete Maths  
Graph Theory



Lecture No. 13



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# TOPICS TO BE COVERED

01 Planarity

02 Eulers Formula

03 planarity inequality thm

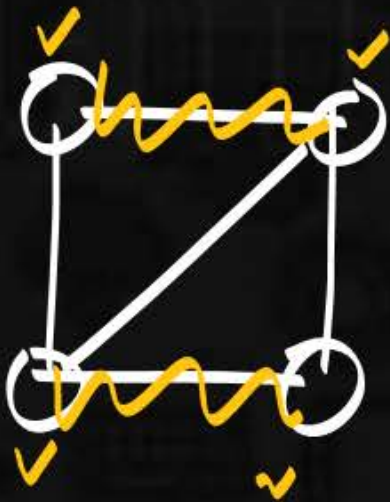
04 region

05 Practice

# Graph Theory

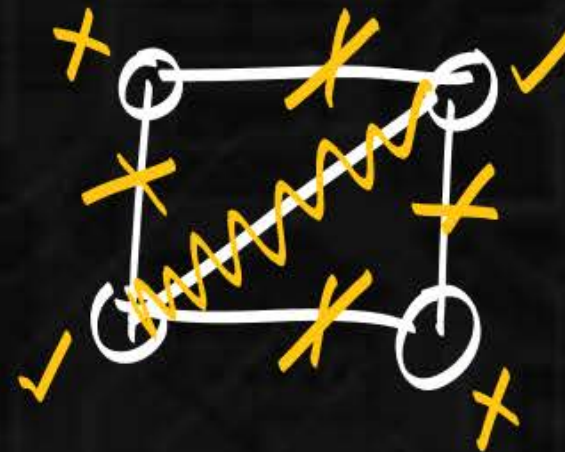


Perfect match  $\rightarrow$  maximal matching set.



maximal matching set

$\nrightarrow$  perfect matching.

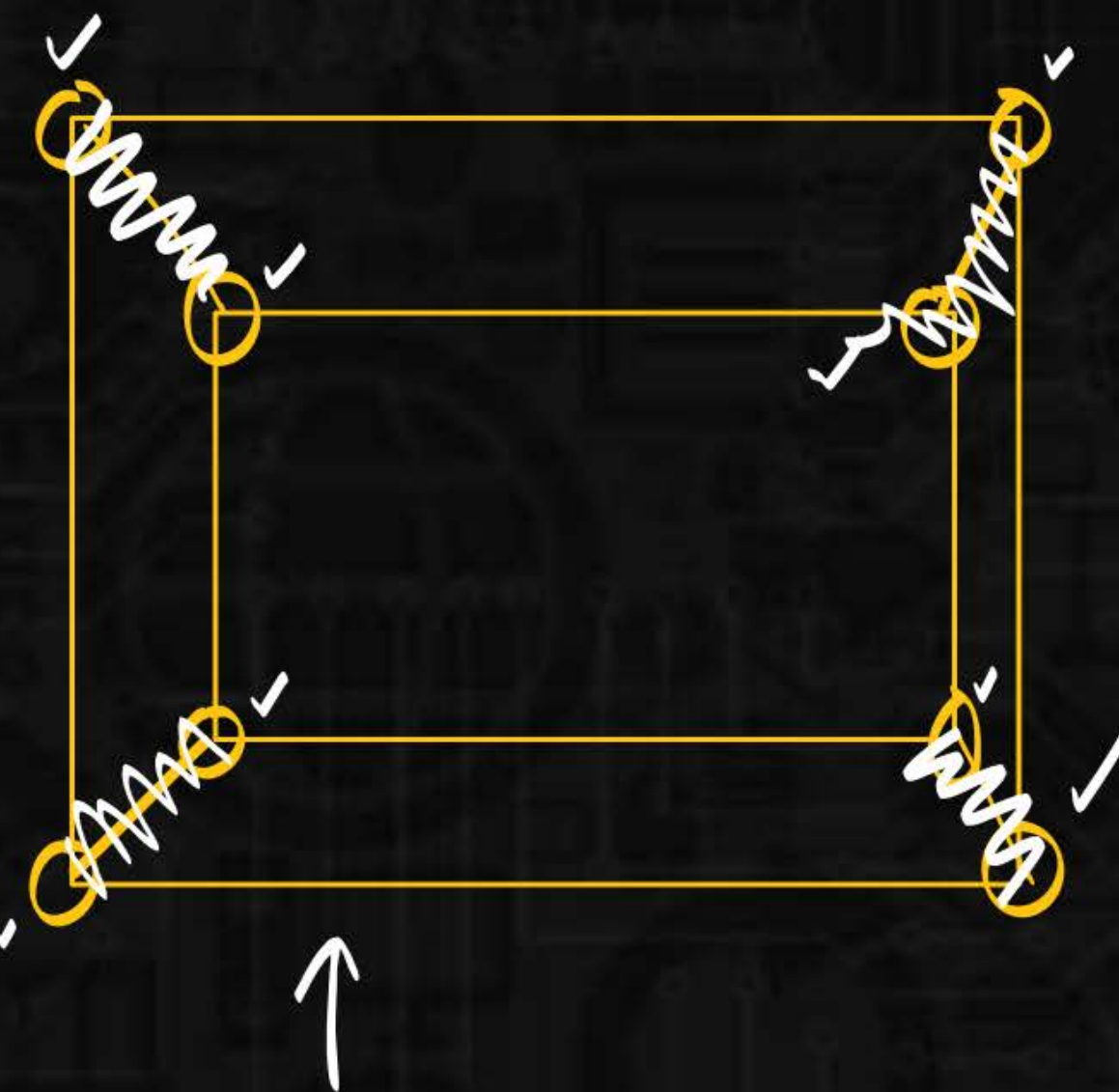
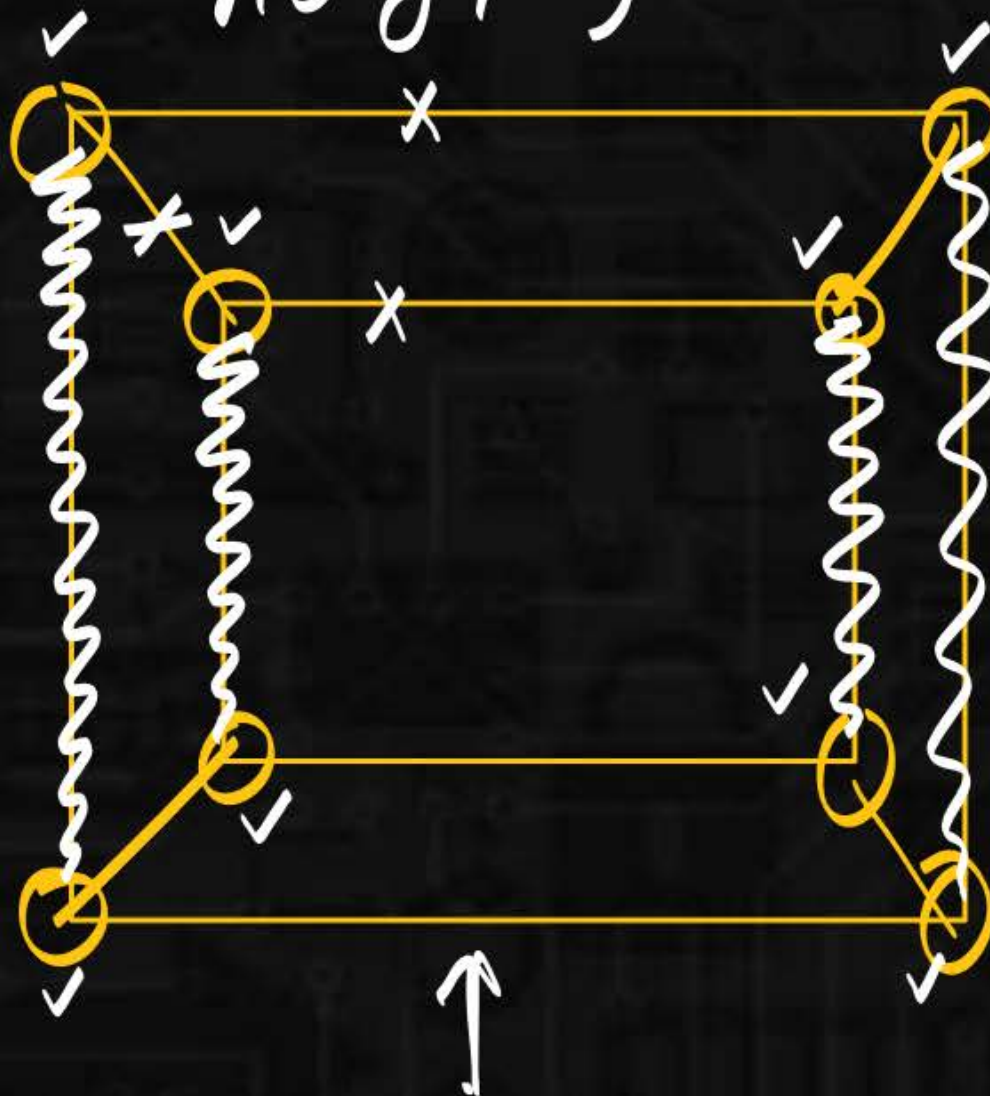
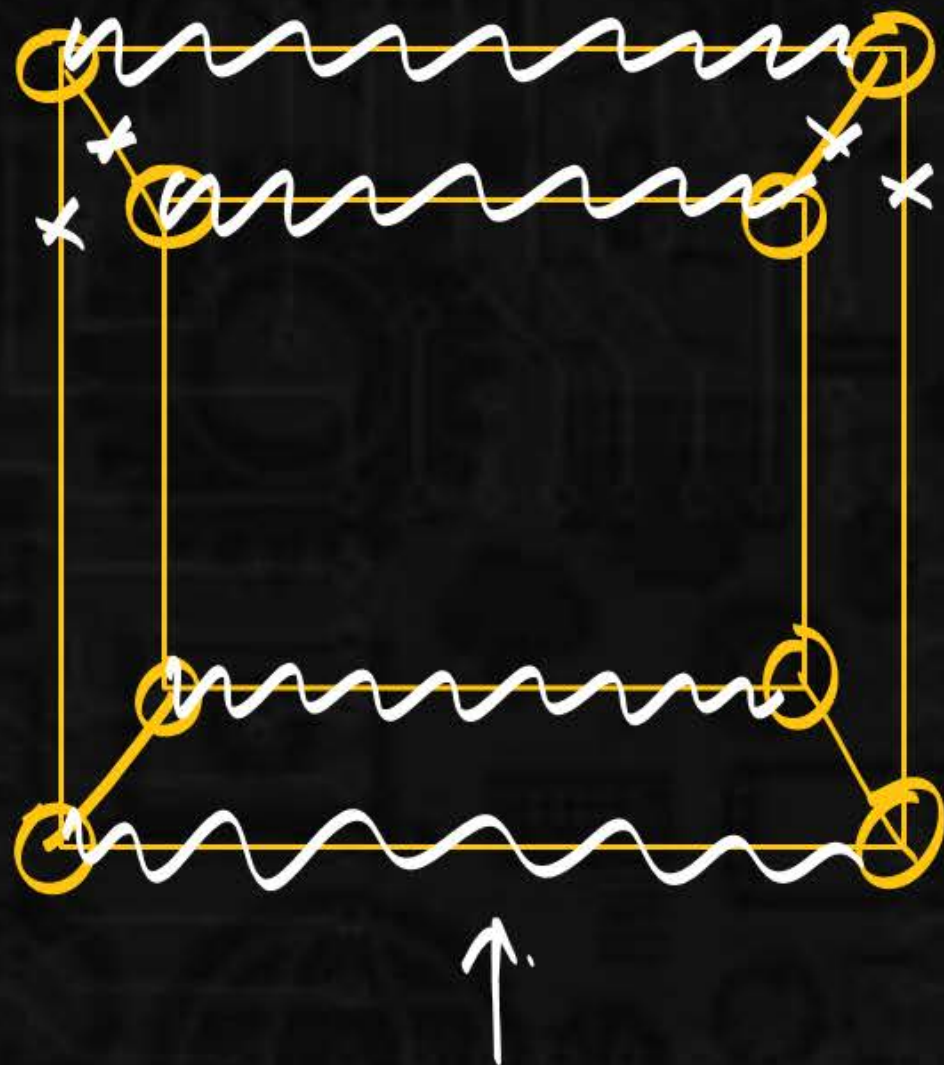




# Graph Theory



no. of perfect matching = 3.

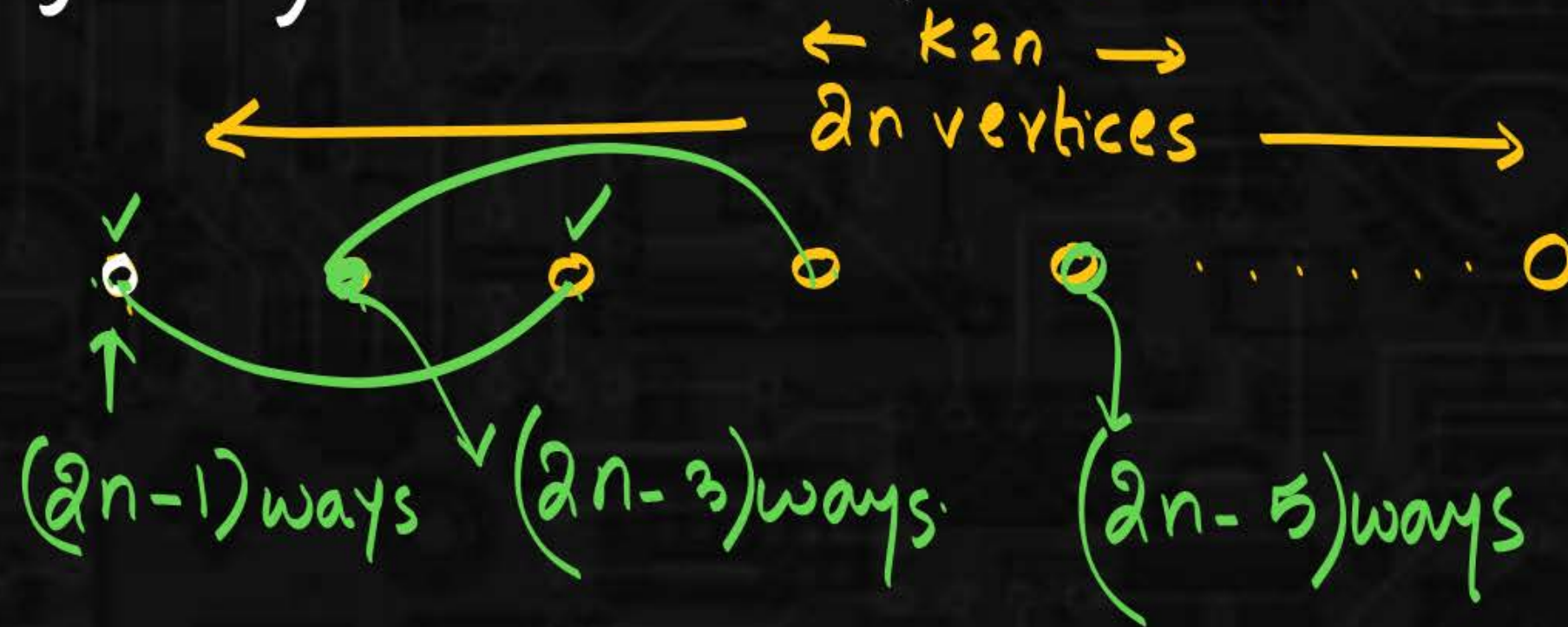


# Graph Theory



no. of perfect matching in  $K_{2n}$ .

Total vertices =  $2n$ .



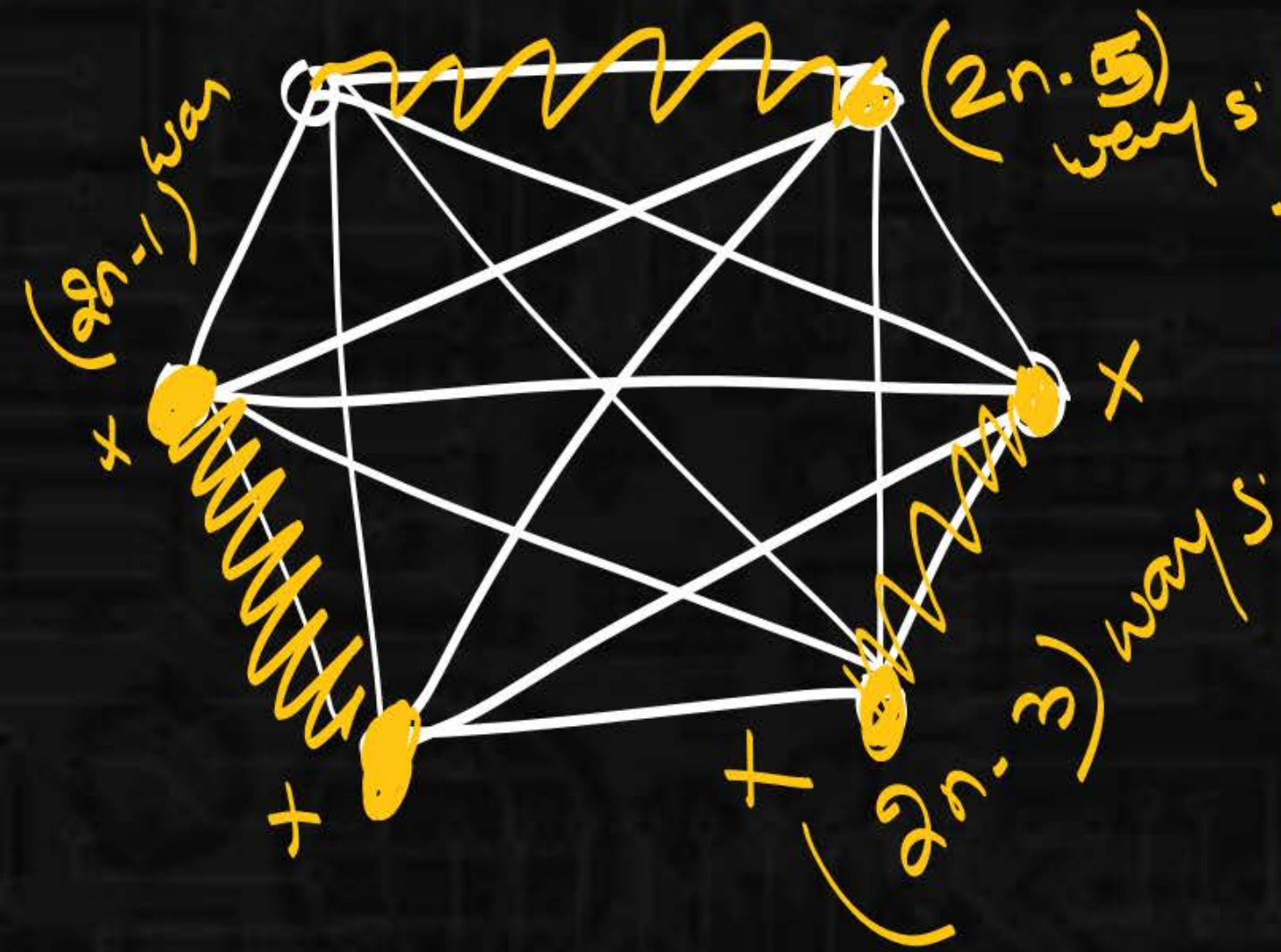


# Graph Theory

$K_6$

$2n$  vertices

$2n$  vertices.



# Graph Theory



Total perfect matching

$$= (2n-1) \times (2n-3) \times (2n-5) \dots$$

$$= \frac{2n}{2n} (2n-1) \frac{2n-2}{2n-2} (2n-3) \frac{2n-4}{2n-4} (2n-5) \dots$$

$$= \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5) \dots}{2n (2n-2) (2n-4) \dots} = \frac{(2n)!}{2^n \cdot n!}$$



# Graph Theory



$$(2n) \times (2n-2) \times (2n-4) \dots$$

take 2 common.

$$2 \times n \times 2(n-1) \times 2 \times (n-2) \dots$$

$$2 \cdot 2 \cdot 2 \dots (n)(n-1) \times (n-2) \dots$$

$$2^n \cdot n!$$

$$\frac{(2n)!}{2n \cdot (2n-2) \cdot (2n-4) \dots}$$

$$= \frac{(2n)!}{2^n \cdot n!}$$



# Graph Theory



GATE: what will be P.M in complete Graph of 6 vertices.

$$2n = 6$$

$$n = 3$$

$$\frac{(2n)!}{2^n \cdot n!}$$

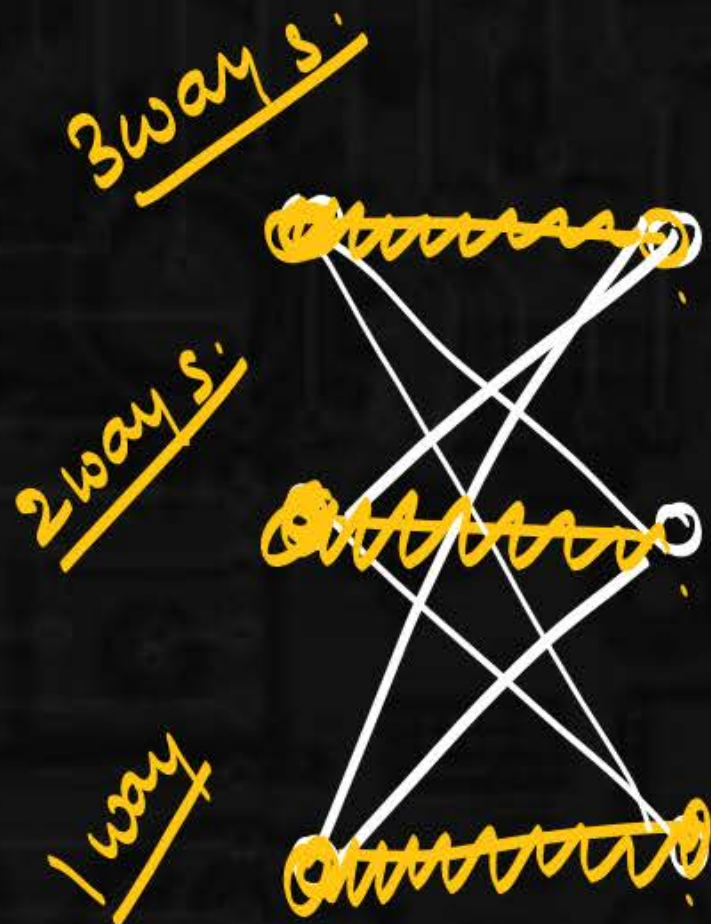
(ANS: 15)

$$K_{n,n} \quad (n \geq 1)$$

# Graph Theory



$K_{3,3}$



$$n \cdot (n-1) \cdot (n-2) \cdot \dots$$

$$= n!$$

$K_{n,n}$   $\rightarrow$   $(n!)$



# Graph Theory



Tree:



{ if p.m exist in Tree, it will be unique.

→ { Total p.m in Tree at most 1.

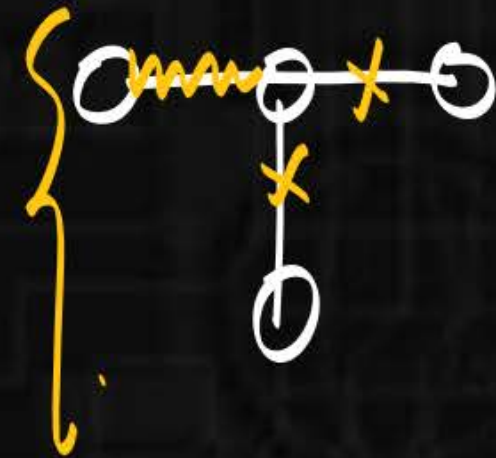
# Graph Theory



if p.m exist in Tree it will be mique.

p.m doesnot exist in all Tree eg:  $0-0-0$

but somehow if it exist then it will have only  
representation there is no other representation  
So that is why it is unique.

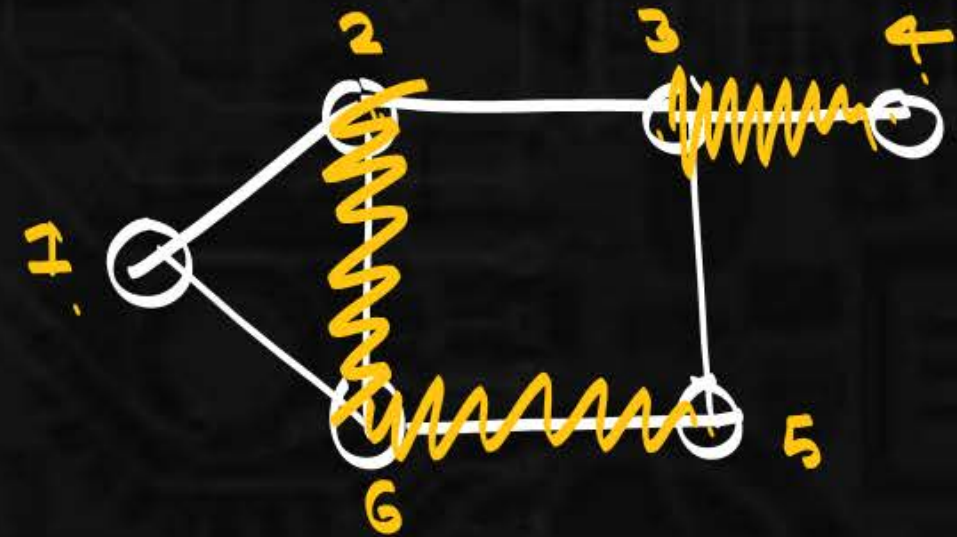




# Graph Theory



Covering set :: (at least 1. marriage proposal)  
vertices.  
edge.



→ set of edges, such that all vertices must incident on at least 1 edge.

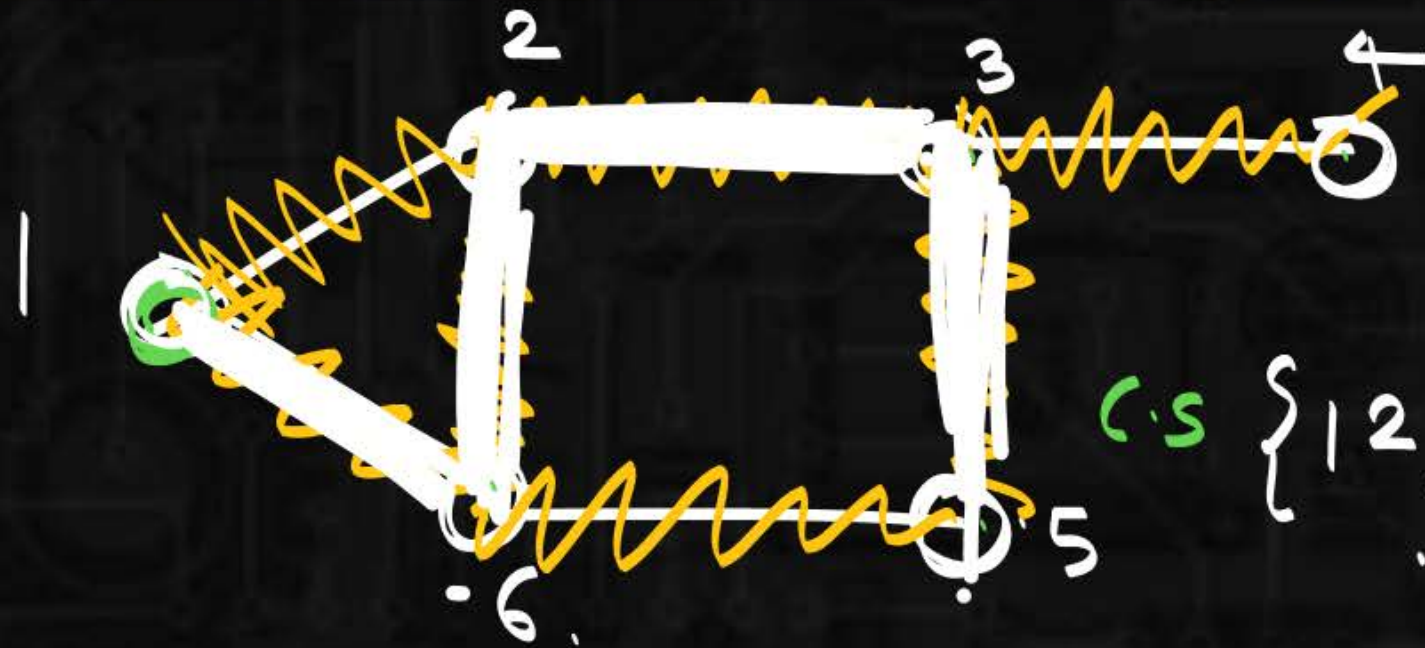
\* covering vertex set :: Set of vertices, such that all edges, must incident on at least 1 vertex.



# Graph Theory



→ all edges → covering set

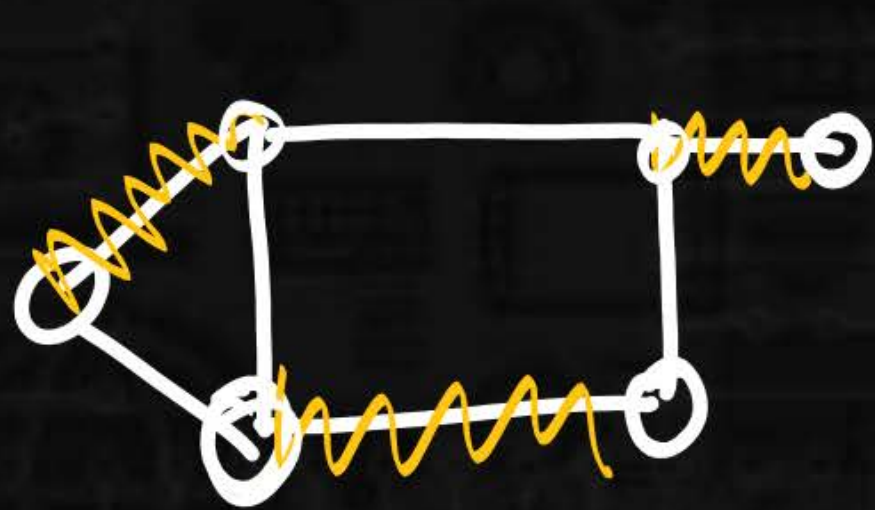


C.S.  $\{12, 16, 65, 23, \cancel{35}, \cancel{34}, \cancel{26}\}$

Set of edges

↓  
all vertices

↓ incident  
at least 1 edge.



C.S.  $\{12, \cancel{16}, \cancel{65}, \cancel{35}, \cancel{34}, \cancel{26}\}$

C.S.  $\{12, 65, 34\}$

→ minimal covering set



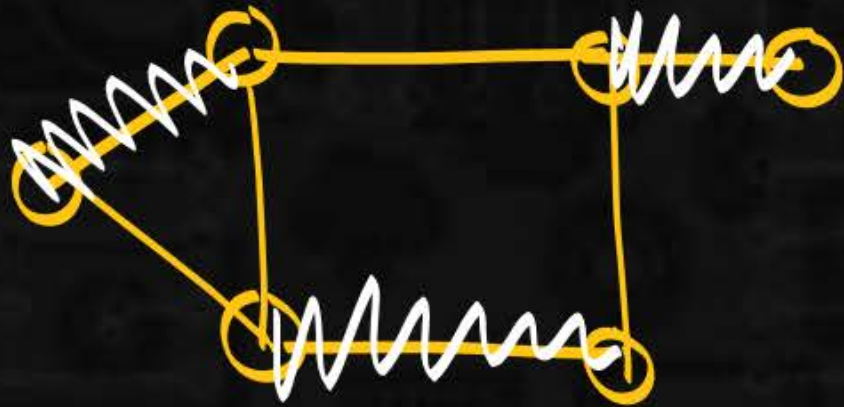
minimal covering set: covering set such that we can not remove new edge from this.

covering no( $c(G)$ ): no. of edges present in smallest minimal covering set

# Graph Theory



Every p.m will be minimal covering set.



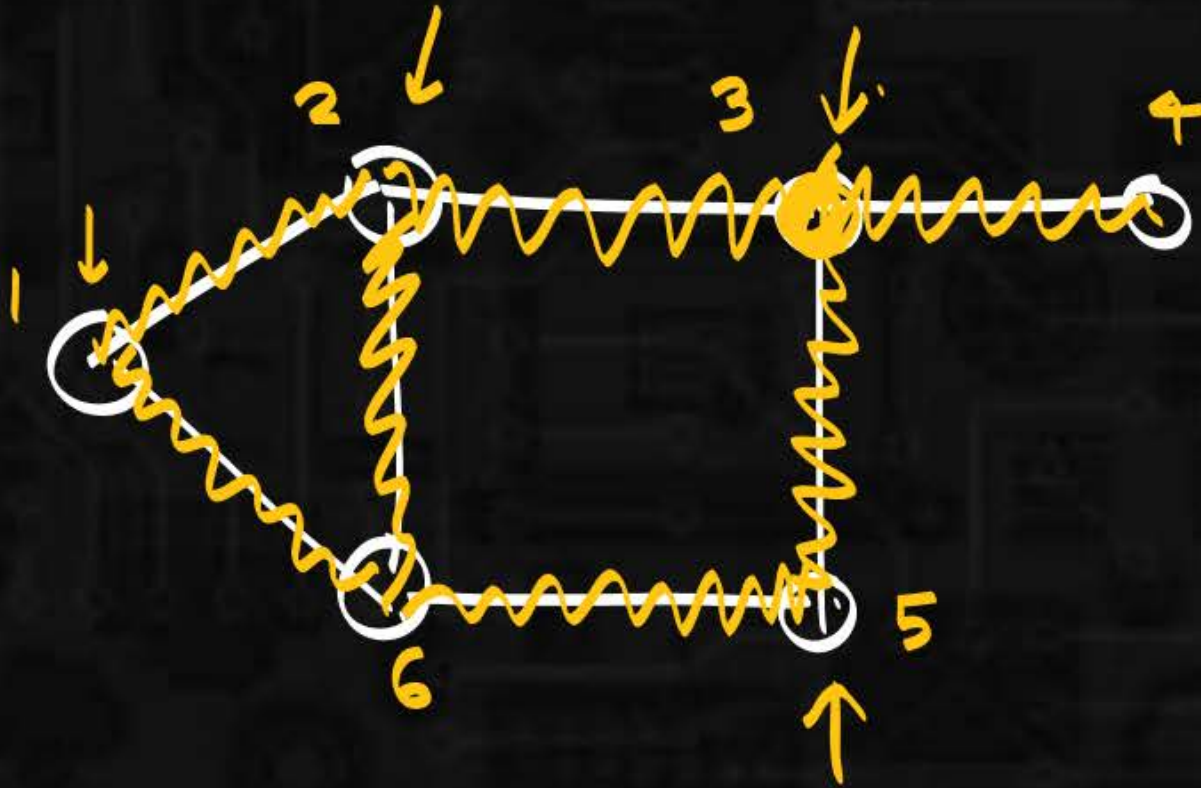
Every minimal covering set would be perfect matching.  
(False).



{ pendant will always be included into covering.



# Graph Theory



set of vertices.



all edges.



at least 1 vertex.

# Graph Theory



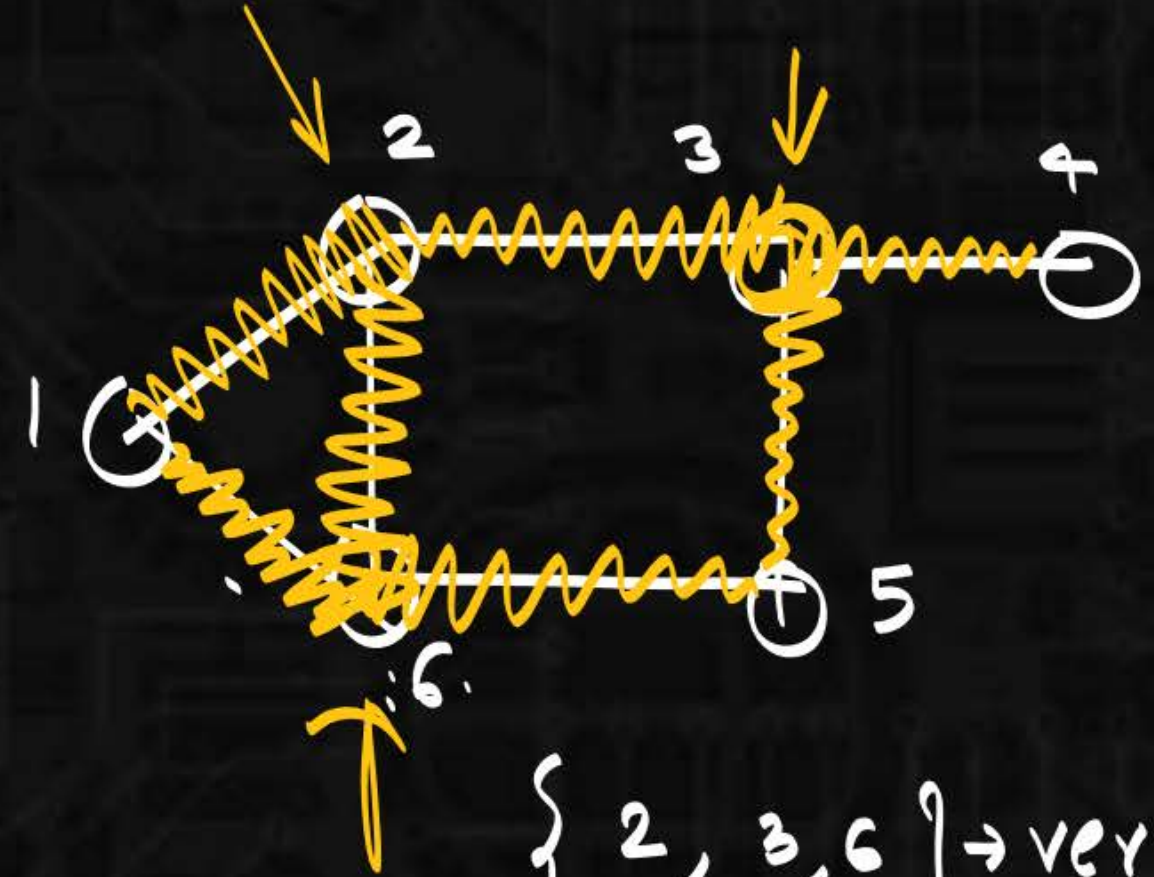
(policement → watching → gully)

covering vertex set

set of vertices

all edges

at least 1 vertex



{ 2, 3, 6 } → vertex covering set.



