CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 2

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TOPICS

01 combination with reptn

02 Examples

3 Problems

Combination: Selecting 2 elements is same as Shifting 2 lines.

Places C line

$^3C_2$.

$\{a, b, c\}$

110 $\leftarrow \{a, b\} \rightarrow \{|, |, -\}$

011 $\{b, c\} \rightarrow \{-, |, |\}$

101 $\{a, c\} \rightarrow \{|, -, |\}$

1 2 3

$\{|\,|\,-\}$

$\{|\,-\,|\}$

$\{-\,|\,|\}$

$\{a, b, c\}$

Select 1 element = Shifting 1 line.

$$\begin{array}{ll} & \quad\ 1 \quad\ 2 \quad\ 3 \\ \{a\} & \{a,\ \_\ \_\} \\ \{b\} & \{\_\ \ ,\ \_\} \\ \{c\} & \{\_\ \_\ \ \} \end{array}$$

Places = 3

line = 1.

$$^{\text{Places}}C_{\text{line}} = {}^{3}C_{1}$$

A mall which contains 3 container A, P, O how many ways we can choose 4 fruits ?

Ans: 15

4A	3A 1O	2A 2O	2A 1O 1P
4O	3A 1P	2A 2P	2O 1A 1P
4P	3O 1A	2P 2O	2P 1A 1O
	3O 1P		
	3P 1A		
	3P 1O		

$3 + 6 + 3 + 3 = 15$

A _ 0 P

$$15 \begin{cases} 3A\,1\,0 & \{ \underset{1}{A}\ \underset{2}{A}\ \underset{3}{A} \mid \underset{4}{} \ \underset{5}{0} \mid \underset{6}{} \} \\ 2A\,2\,0 & \{ AA \mid 00 \mid \} \\ 3\,0 \mid A & \{ A \mid 000 \mid \} \\ 2\,0 \mid A \mid P & \{ A \mid 00 \mid P \} \\ \vdots & \end{cases}$$

places $= 6$

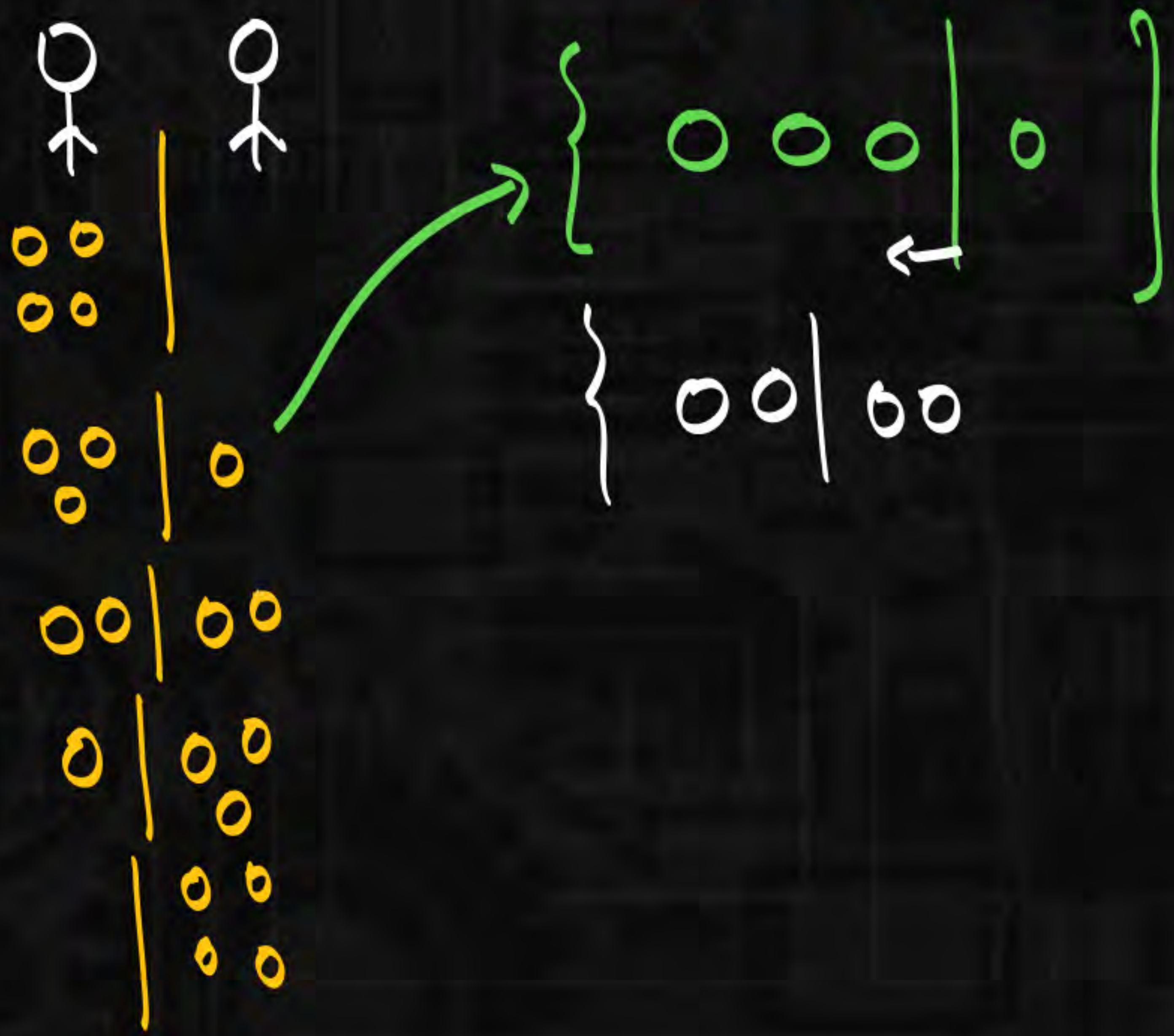
line $= 2$

Places C line.

$$6C_2 = \frac{6 \cdot 5}{2 \cdot 1} = \underline{\underline{15}}.$$

$$\text{Places}\ C_{\text{line}} = {}^6C_2$$

How many ways we can distribute 4 chocolates among 2 girls ?



Places C line.

$^5C_1 = \underline{\underline{5}}$.

How many ways we can distribute 10 chocolates among 3 students?

$^{12}C_2 = 66.$

0000 | 00 | 0000

1 2 3 4 5 6 7 8 9 10 11 12

0000000000 | X | X

how many non negative soln are possible.

$x_1 + x_2 + x_3 = 10 \quad \boxed{x_i \geq 0}$

$x_1 = 4 \quad x_2 = 2 \quad x_3 = 4.$

$x_1 = 3 \mid x_2 = 3 \mid x_3 = 4.$

000 | 000 | 0000

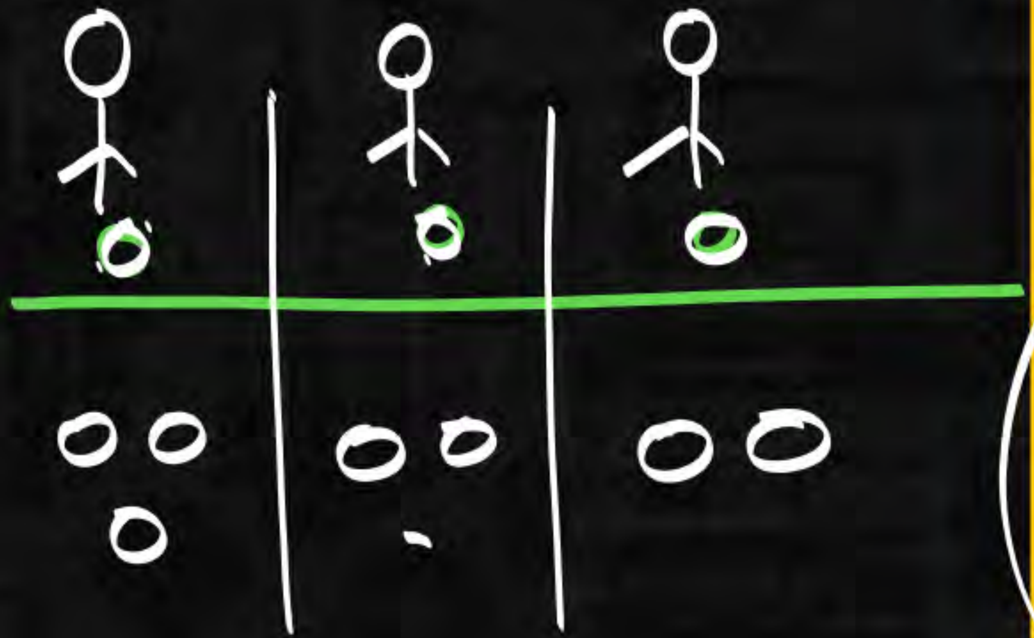
$x_1 = 10 \quad x_2 = 0 \quad x_3 = 0$

$x_1 = 0 \quad x_2 = 10 \quad x_3 = 0$

Ans: $^{12}C_2.$

How many ways we can distribute 10 chocolates among 3 students, each child gets atleast 1 chocolate?

$$10-3 = 7$$



$${}^{7+2}C_2$$

how many non negative soln are?

$$x_1 + x_2 + x_3 = 10 \quad \boxed{x_i \geq 1}$$

$$y_i = x_i - 1 \qquad \underbrace{x_i - 1}_{y_i} \geq 0$$

$$y_i + 1 = x_i$$

$$y_1 + 1 + y_2 + 1 + y_3 + 1 = 10 \quad y_i \geq 0$$

$$y_1 + y_2 + y_3 = 10 - 3 = 7 \quad y_i \geq 0$$

$$x_1 + x_2 + x_3 = 20$$

$$x_1 \geqslant 2 \qquad x_2 \geqslant 3 \qquad x_3 \geqslant 4.$$

$$\underbrace{x_1 - 2}_{y_1} \geqslant 0 \qquad \underbrace{x_2 - 3}_{y_2} \geqslant 0 \qquad \underbrace{x_3 - 4}_{y_3} \geqslant 0$$

$$y_1 = x_1 - 2$$

$$y_1 + 2 = x_1 \qquad y_2 + 3 = x_2 \qquad y_3 + 4 = x_3.$$

$$y_1 + 2 + y_2 + 3 + y_3 + 4 = 20 \qquad y_i \geqslant 0$$

$$y_1 + y_2 + y_3 = 20 - 9 = 11 \qquad {}^{13}C_2$$

$x_1 + x_2 + x_3 = 20 \qquad x_i > 1.$

$\downarrow$

$x_i \geq 2$

$x_i - 2 \geq 0$

$x_1 + x_2 + x_3 + x_4 \ldots\ldots x_{10} = 40 \quad x_i \geq 0$

$x_1 + x_2 + x_3 = 10$

Ans: $^{12}C_2 \times {}^{36}C_6$.

$$\underbrace{\boxed{x_1 + x_2 + x_3}}_{10+2_{C_2}}^{=10} + \underbrace{\boxed{x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}}}_{30+6_{C_6}} = 40$$

$\cancel{10, 0, 0} \Leftrightarrow 36, 0 \ldots\ldots 0$

$35, 1, \ldots\ldots$

$34, 1, 1 \ldots\ldots$

$\vdots$

$^{36}C_6$.

GATE:

How many ways we can distribute m identical balls among n boxes, such that each box contains $\boxed{\text{at least } K}$ balls $(m \geq kn)$?

Lines = n-1.

$K + K + K \ldots = nK.$

after distbⁿ Remaining balls = m-nk.

$^{m-nk+n-1}C_{n-1}.$

How many ways we can distribute 10 Roses, 15 daffodils among 2 girls ?

(GATE)

Ans: ${}^{11}C_1 \times {}^{16}C_1$.

mistake :

$10 + 15 = 25$

${}^{26}C_1 = 26$

wrong

10 Roses

${}^{11}C_1$

5R 5R

10R 0R

15 D

${}^{16}C_1$

7D | 8D

8D | 7D

$x_1 + x_2 + x_3 \leq 10 \qquad x_i \geq 0$

$x_1 + x_2 + x_3 = 0$

$x_1 + x_2 + x_3 = 1$

$= 2$

$= 3$

$\vdots$

$= 10 \quad (a = 0)$

$x_1 + x_2 + x_3 + a = 10$

$\boxed{x_1 + x_2 + x_3 = 10} \quad a = 0$

$\boxed{x_1 + x_2 + x_3 = 9} \quad a = 1$

$a = 2$

$a = 3$

$\vdots$

$a = 10$

${}^{10+3}C_3$

$= {}^{13}C_3$.

$$x_1+x_2+x_3 \le 5$$

$$x_1+x_2+x_3+a = 5$$

$a=0$

$a=1$

$a=2$

$$x_1+x_2+x_3=0$$
$$=1$$
$$=2$$
$$=3$$
$$=4$$
$$x_1+x_2+x_3=5$$

x_1	x_2	x_3	a
			$\underline{0}$

$= 5$

1. In how many ways can 10 (identical) dimes be distributed among five children if (a) there are no restrictions? (b) each child gets at least one dime? (c) the oldest child gets at least two dimes?

(a) The number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 10$, $0 \le x_i$, $1 \le i \le 5$, is $\binom{5+10-1}{10} = \binom{14}{10}$. Here $n = 5$, $r = 10$.
(b) Giving each child one dime results in the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 5$, $0 \le x_i$, $1 \le i \le 5$. There are $\binom{5+5-1}{5} = \binom{9}{5}$ ways to distribute the remaining five dimes.
(c) Let x_5 denote the amount for the oldest child. The number of solutions to $x_1 + x_2 + x_3 + x_4 + x_5 = 10$, $0 \le x_i$, $1 \le i \le 4$, $2 \le x_5$ is the number of solutions to $y_1 + y_2 + y_3 + y_4 + y_5 = 8$, $0 \le y_i$, $1 \le i \le 5$, which is $\binom{5+8-1}{8} = \binom{12}{8}$.

2. In how many ways can 15 (identical) candy bars be distributed among five children so that the youngest gets only one or two of them?

3. Determine how many ways 20 coins can be selected from four large containers filled with pennies, nickels, dimes, and quarters. (Each container is filled with only one type of coin.)

2. Let x_i, $1 \le i \le 5$, denote the number of candy bars for the five children with x_1 the number for the youngest. $(x_1 = 1)$: $x_2 + x_3 + x_4 + x_5 = 14$. Here there are $\binom{4+14-1}{14} = \binom{17}{14}$ distributions. $(x_1 = 2)$: $x_2 + x_3 + x_4 + x_5 = 13$. Here the number of distributions is $\binom{4+13-1}{13} = \binom{16}{13}$. The answer is $\binom{17}{14} + \binom{16}{13}$ by the rule of sum.

3. $\binom{4+20-1}{20} = \binom{23}{20}$

7. Determine the number of integer solutions of

$$x_1 + x_2 + x_3 + x_4 = 32,$$

where

a) $x_i \ge 0, \quad 1 \le i \le 4$ **b)** $x_i > 0, \quad 1 \le i \le 4$

c) $x_1, x_2 \ge 5, \quad x_3, x_4 \ge 7$

d) $x_i \ge 8, \quad 1 \le i \le 4$ **e)** $x_i \ge -2, \quad 1 \le i \le 4$

f) $x_1, x_2, x_3 > 0, \quad 0 < x_4 \le 25$

7. (a) $\binom{4+32-1}{32} = \binom{35}{32}$ (b) $\binom{4+28-1}{28} = \binom{31}{28}$
(c) $\binom{4+8-1}{8} = \binom{11}{8}$ (d) 1
(e) $x_1 + x_2 + x_3 + x_4 = 32$, $x_i \ge -2$, $1 \le i \le 4$. Let $y_i = x_i + 2$, $1 \le i \le 4$. The number of solutions to the given problem is then the same as the number of solutions to $y_1 + y_2 + y_3 + y_4 = 40$, $y_i \ge 0$, $1 \le i \le 4$. This is $\binom{4+40-1}{40} = \binom{43}{40}$.
(f) $\binom{4+28-1}{28} - \binom{4+3-1}{3} = \binom{31}{28} - \binom{6}{3}$, where the term $\binom{6}{3}$ accounts for the solutions where $x_4 \ge 26$.

8. In how many ways can a teacher distribute eight chocolate donuts and seven jelly donuts among three student helpers if each helper wants at least one donut of each kind?

9. Columba has two dozen each of n different colored beads. If she can select 20 beads (with repetitions of colors allowed) in 230,230 ways, what is the value of n?

8. For the chocolate donuts there are $\binom{3+5-1}{5} = \binom{7}{5}$ distributions. There are $\binom{3+4-1}{4} = \binom{6}{4}$ ways to distribute the jelly donuts. By the rule of product there are $\binom{7}{5}\binom{6}{4}$ ways to distribute the donuts as specified.

9. $230,230 = \binom{n+20-1}{20} = \binom{n+19}{20} \Longrightarrow n = 7$

12. Determine the number of integer solutions for

$$x_1 + x_2 + x_3 + x_4 + x_5 < 40,$$

where

a) $x_i \geq 0, \quad 1 \leq i \leq 5$

b) $x_i \geq -3, \quad 1 \leq i \leq 5$

(a) The number of solutions for $x_1 + x_2 + \ldots + x_5 < 40, x_i \geq 0, 1 \leq i \leq 5$, is the same as the number for $x_1 + x_2 + \ldots + x_5 \leq 39, x_i \geq 0, 1 \leq i \leq 5$, and this equals the number of solutions for $x_1 + x_2 + \ldots + x_5 + x_6 = 39, x_i \geq 0, 1 \leq i \leq 6$. There are $\binom{6+39-1}{39} = \binom{44}{39}$ such solutions.

(b) Let $y_i = x_i + 3, 1 \leq i \leq 5$, and consider the inequality $y_1 + y_2 + \ldots + y_5 \leq 54, y_i \geq 0$. There are [as in part (a)] $\binom{6+54-1}{54} = \binom{59}{54}$ solutions.

18. a) How many nonnegative integer solutions are there to the pair of equations $x_1 + x_2 + x_3 + \cdots + x_7 = 37$, $x_1 + x_2 + x_3 = 6$?

b) How many solutions in part (a) have $x_1, x_2, x_3 > 0$?

18. (a) There are $\binom{3+6-1}{6} = \binom{8}{6}$ solutions for $x_1 + x_2 + x_3 = 6$ and $\binom{4+31-1}{31} = \binom{34}{31}$ solutions for $x_4 + x_5 + x_6 + x_7 = 31$, where $x_i \geq 0, 1 \leq i \leq 7$. By the rule of product the pair of equations has $\binom{8}{6}\binom{34}{31}$ solutions.

(b) $\binom{5}{3}\binom{34}{31}$


THANK YOU GW
SOLDIERS !