Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics
Graph Theory

DPP-04

[MCQ]

1. If a hypercube (Q_n) is given with edges 193, then the number of vertices will be

(a) 6
(b) 5
(c) 7
(d) None of these

[MCQ]

2. consider the following statements:

S_1: Every hypercube graph is a bipartite graph.

S_2: Every bipartite graph is also a hypercube.

Which of the following options is True?

(a) S_1 only (b) S_2 only
(c) Both S_1 and S_2 (d) Neither S_1 nor S_2

[NAT]

3. A certain graph G has order 16 and size 29. The degree of each vertex of G is 3, 4 or 5. There are six vertices of degree 4. How many vertices of G having degree 5?

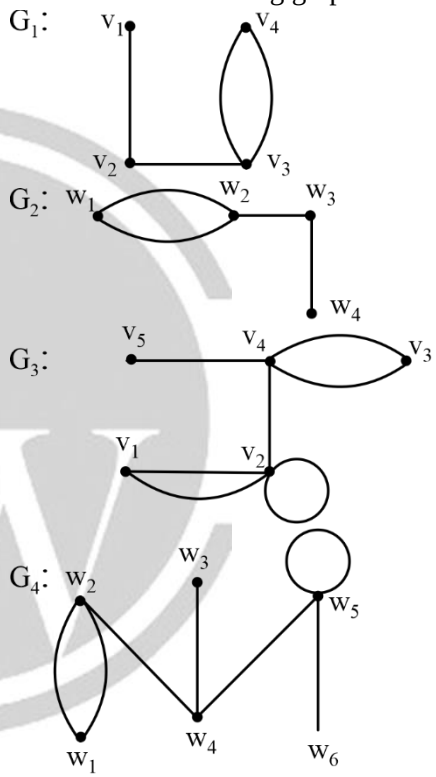
[MCQ]

4. If the sequence x, 7, 7, 5, 5, 4, 3, 2 is graphical then what are the possible value of x ($0 \le x \le 4$)?

(a) 0 (b) 2
(c) 3 (d) 1

[MSQ]

5. Which of the following graphs are isomorphic graph?



(a) G_1 and G_2 are isomorphic
(b) G_3 and G_4 are isomorphic
(c) G_1 and G_2 are not isomorphic
(d) G_3 and G_4 are not isomorphic

Answer Key

1. **(d)**
2. **(a)**
3. **(2)**
4. **(c)**
5. **(a, d)**

Hints and solutions

1. (d)

As we know that the number of edges in a hypercube (Q_n) is given as:

Number of edges = $n.2^{n-1}$

$\therefore \quad 193 = n.\ 2^{n-1}$

Here for any integer value of 'n', the hypercube would not contain 193 edges.

Hence, the correct option is d.

2. (a)

Statement S_1 : True

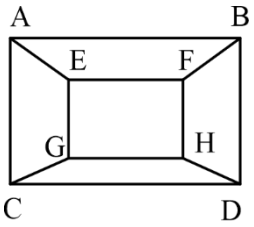
Every hypercube have cycle of length even hence it is possible to divide the vertices into two sets.

So, Every hypercube is a bipartite graph.

Statement S_2 : False

Every bipartite graph is not hypercube graph.

Example:



The above graph is bipartite graph as V_1 = {A, G, F, D} and V_2 ={B, E, H, C} with number of edge = 12 and vertices = 8.

Still the above graph is not hypercube.

3. (2)

I. In the problem number of vertices is given 16 and number of edges given is 29.

Now, we have 6 vertices of degree 4.

Assume x is total number of vertices with degree 5.

So, number of vertices with degree 3 will be :

(16 – 6 –x) = (10 – x)

II. Now, by using Handshaking lemma:

Sum of degree = 2 * |E|

$\therefore \quad (6 * 4) + (x * 5) + (10 - x) * 3 = 2 * 29$

$\Rightarrow \quad 24 + 5x + 30 - 3x = 58$

$\Rightarrow \quad 5x - 3x = 58 - 54$

$\Rightarrow \quad 2x = 4$

$\therefore \quad x = 2$

Hence, we have 2 vertices with degree 5 and 8 vertices with degree 3.

4. (c)

I. In any graph the number of odd degree vertices must be even. Now, in the given degree sequence, we have 5 vertices with odd degree

{7, 7, 5, 5, 3}

Thus, the value of x must be odd number between 0 to 4 that is either 1 or 3.

II. Now, case I assume x = 1:

Degree sequence: 1, 7, 7, 5, 5, 4, 3 ,2

⇓ order

7, 7, 5, 5, 4, 3, 2, 1

as we know that if we have 2 vertices with maximum degree (n - 1) then the degree of each vertex must be ≥ 2.

Thus, x will be 3 only.

Hence, right answer is option c.

5. (a, d)

I. G_1 and G_2 are isomorphic as it has equal number of vertices, edges and same degree sequence.

II. G_3 and G_4 are not isomorphic because incident property not satisfied.

Degree of V_2 is 5 in G_3 but there is not any vertex in G_4 with same degree.