ENGINEERING MATHEMATICS

ALL BRANCHES

Probability

Mean, Median & Mode

DPP-08 Solution

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Question 1

The standard deviation for the data 7, 9, 11, 13, 15 is

$$\sigma = \sqrt{\frac{\Sigma(x_i - x_m)^2}{n}}$$

$$x_m = \frac{7+9+11+13+15}{5} = 11$$

A 2.4

B 2.5

$$\sqrt{\frac{(7-11)^2+(9-11)^2+(11-11)^2+(13-11)^2+(15-11)^2}{5}}$$

C 2.7

$$= \sqrt{\frac{40}{5}} = \sqrt{8} = 2\sqrt{2}$$

$$= 2.8$$

D 2.8

Question 2

Consider the continuous random variable with probability density function

$f(t) = 1 + t$ for $-1 \le t \le 0$

$= 1 - t$ for $0 \le t < 1$

$f(t) \rightarrow$ Even

The standard deviation of the random variable is

$$E(t) = \int_{-1}^{+1} \overbrace{\underset{\to O}{t}\ \underset{\to E}{f(t)}}\, dt = 0$$

A $\frac{1}{\sqrt{3}}$

B $\frac{1}{\sqrt{6}}$

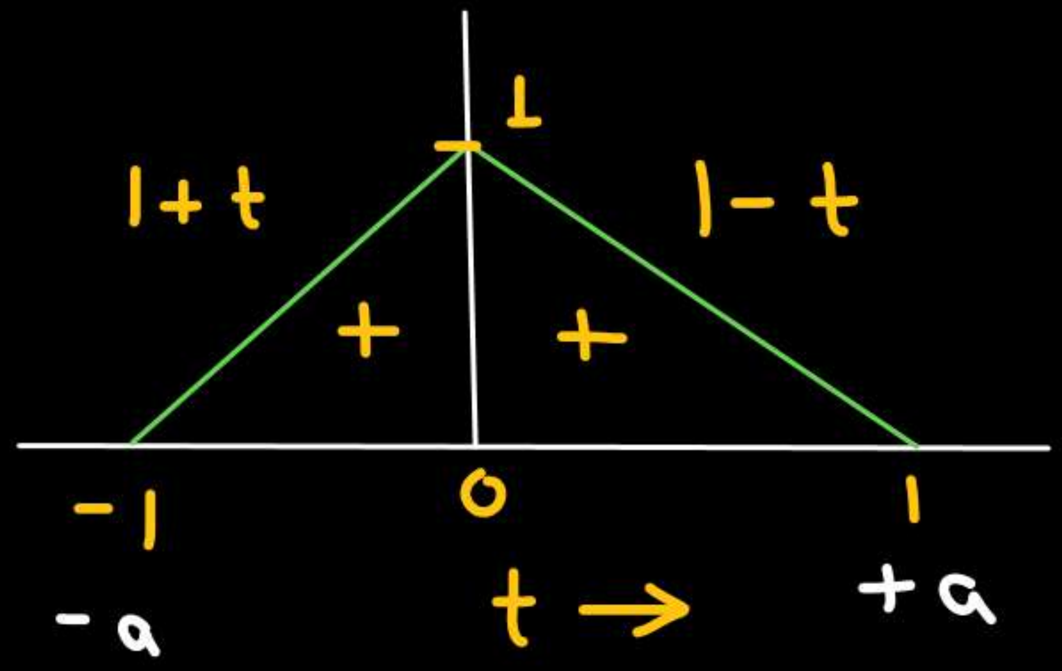
C $\frac{1}{3}$

D $\frac{1}{6}$

$$\sigma^2 = Var(t) = E(t^2) - [\underbrace{E(t)}_{0}]^2 = a^3/6 = 1^3/6 = 1/6$$

$$\sigma^2 = \int_{-1}^{+1} t^2 f(t)\, dt = 1/6$$

$$\sigma = 1/\sqrt{6}$$



Question 3

Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE?

A. $E(XY) = E(X)\,E(Y)$ ✓

B. $\text{Cov}(X, Y) = 0$ ✓

C. $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ ✓

D. $E(X^2Y^2) = (E(X))^2\,(E(Y))^2$ ✓

$Cov(x,y) = E(XY) - E(x).E(Y) = 0$

$E(X^n Y^n) = E(X^n).E(Y^n)$

if independent.

$Var(x+Y) = Var(x) + Var(Y) + 2\,Cov(x,Y)$

Question 4

In the following table, x is a discrete random variable and $p(x)$ is the probability density. The standard deviation of x is

x	1	2	3
p(x)	0.3	0.6	0.1

A 0.18

B 0.36

C 0.54

D 0.6

$E(x) = \sum_{i=1}^{3} x_i p(x_i) = 1\times0.3 + 2\times0.6 + 3\times0.1 = 1.8$

$E(x^2) = 1^2\times0.3 + 2^2\times0.6 + 3^2\times0.1 = 3.6$

$\sigma^2 = Var(x) = E(x^2) - [E(x)]^2 = 3.6 - (1.8)^2$

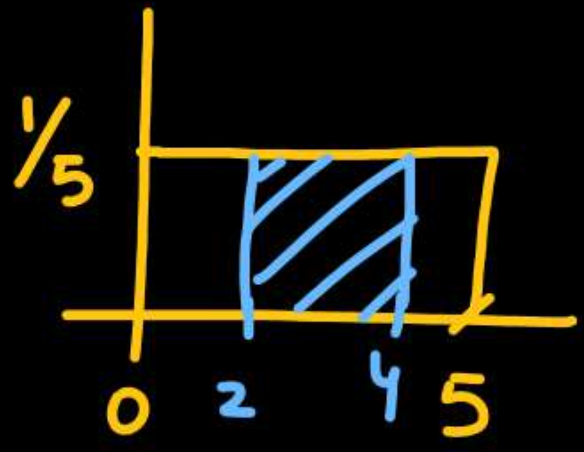
$\sigma^2 = 0.36$

$\sigma = 0.6$

Question 5

The probability density function of evaporation E on any day during a year in watershed is given by

$$f(E)=\begin{cases}\frac{1}{5} & 0\leq E\leq 5\text{mm/day}\\ 0 & \text{otherwise}\end{cases}$$



The probability that E lies in between 2 and 4 mm/day in a day in watershed is (in decimal) 0.4

$$P(2<E<4) = \int_2^4 f(E)\,dE = \int_2^4 \frac{1}{5}\,dE = \frac{2}{5} = 0.4$$

Question 6

Let X be a random variable with probability density function

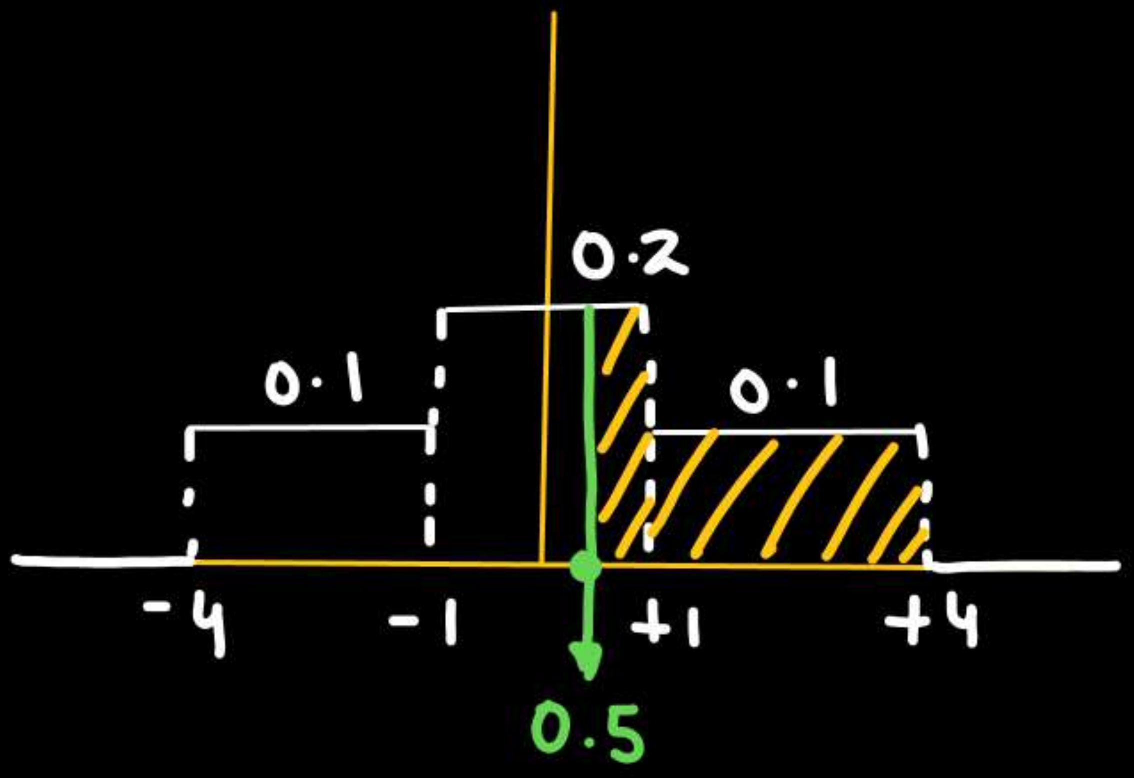
$$f(x)=\begin{cases}0.2, & \text{for } |x|\le 1 \quad -1 \le x \le +1 \\ 0.1, & \text{for } 1<|x|\le 4 \quad (-4 \le x < -1) \cup (1 < x \le 4) \\ 0, & \text{otherwise}\end{cases}$$

The probability $P(0.5 < X < 5)$ is $\underline{0.4}$.

$$P(0.5<X<5) = \int_{0.5}^{1} 0.2\,dx + \int_{1}^{4} 0.1\,dx + \int_{4}^{5} 0\,dx$$

$$= 0.2(1-0.5) + 0.1(4-1) + 0$$

$$= 0.1 + 0.3 = 0.4$$



Question 7

Mark obtained by 100 students in an examination are given in the table. What would be the mean, median and mode of the marks obtained by the students?

Sl. No.	Marks obtained	Number of students
1	25	20
2	30	20
3	35	40
4	40	20

A. Mean 33; Median 35; Mode 40

B. Mean 35; Median 32; Mode 40

C. Mean 33; Median 35; Mode 35

D. Mean 35; Median 32; Mode 35

$$\text{Mean} = \frac{\text{Total marks}}{\text{No. of students}} = \frac{25\times 20 + 30\times 20 + 35\times 40 + 40\times 20}{20+20+40+20}$$

$$N = 100 \quad \boxed{\text{Mean} = 33}$$

25, 25 . . . . , 30, 30 . . . . . . , 35, 35, ... 40, 40, 40.

$$\begin{array}{cc} 50^{th} & 51^{st} \\ \downarrow & \downarrow \\ 35 & 35 \end{array}$$

$$\text{Median} = \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2} = \frac{35+35}{2} = 35$$

$$\boxed{\text{Median} = 35}$$

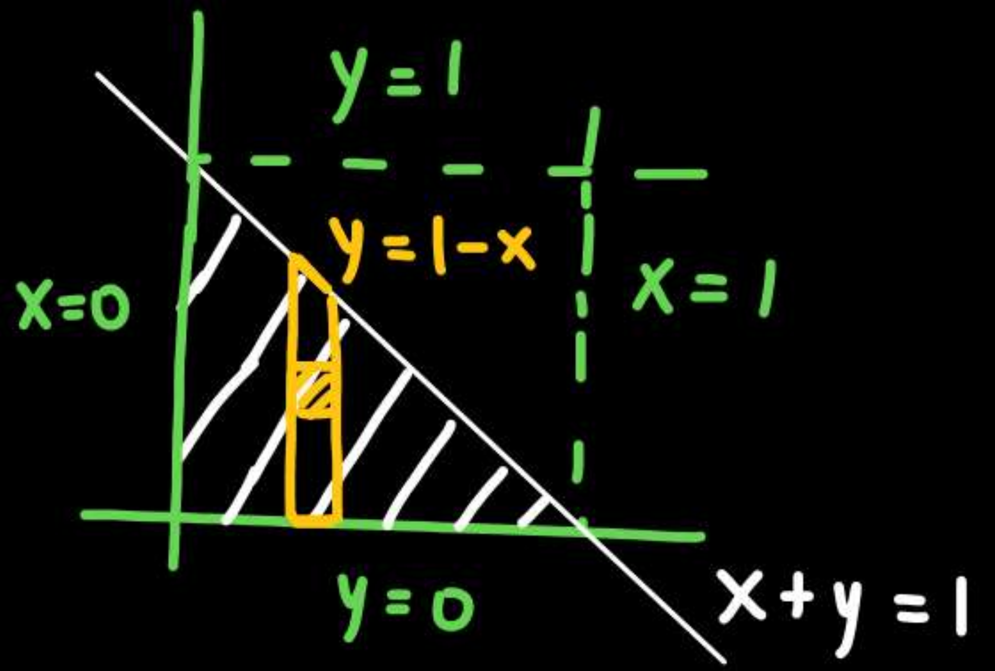
$\boxed{\text{Mode} = 35}$ (Marks having highest frequency)

Question 8

Two random variable x and y are distributed according to

$$f_{x,y}(x,y)=\begin{cases}(x+y), & 0\le x\le 1, 0\le y\le 1\\ 0, & \text{otherwise}\end{cases}$$

The probability P (x + y ≤ 1) is $\underline{1/3}$.



$$P(x+y\le 1)=\iint_R f(x,y)\,dx\,dy$$

$$=\int_{x=0}^{x=1}\int_0^{1-x}(x+y)\,dy\,dx = \int_0^1\left[xy+\frac{y^2}{2}\right]_0^{1-x}dx$$

$$\int_0^1\left(x(1-x)+\frac{(1-x)^2}{2}\right)dx = \left[\frac{x^2}{2}-\frac{x^3}{3}-\frac{(1-x)^3}{6}\right]_0^1 = \frac{1}{2}-\frac{1}{3}+\frac{1}{6}=\frac{1}{3}$$

Question 9

Suppose the probability density function of a continuous random variable x is $f(x) = 3x^2$, $0 < x < 1$. Find 'a' satisfying the following condition

(A) $P[x \leq a] = P[x \geq a]$

$$\int_0^a 3x^2 dx = \int_a^1 3x^2 dx$$

$$[x^3]_0^a = [x^3]_a^1$$

$$a^3 = 1 - a^3$$

$$2a^3 = 1$$

$$a = \sqrt[3]{\frac{1}{2}}$$

Question 10

Suppose the probability density function of a continuous random variable x is $f(x) = 3x^2, 0 < x < 1$. Find 'b' satisfying the following condition

(A) $P[x > b] = 0.05$

$$P(x > b) = \int_b^1 3x^2\,dx = 0.05$$

$$= \left[x^3\right]_b^1 = 0.05$$

$$1 - b^3 = 0.05$$

$$b^3 = 1 - 0.05$$

$$b = \sqrt[3]{0.95}$$

Thank you

GW Soldiers !