ENGINEERING MATHEMATICS

ALL BRANCHES

Probability

Discrete Random Variable

DPP-05 Solution

By- CHETAN SIR

Q.1 The number of parameters in the univariate exponential and Gaussian distributions, respectively are

A 2 and 2

B 1 and 2 ✓

C 2 and 1

D 1 and 1

Exponential $\rightarrow \lambda\ (1)$

Gaussian $\rightarrow \mu, \sigma\ (2)$

Q2 For the function $f(x) = a + bx, 0 \leq x \leq 1$, to be a valid probability density function, which one of the following statements is correct?

A $a = 1, b = 4$

B $a = 0.5, b = 1$

C $a = 0, b = 1$

D $a = 1, b = -1$

$$\int_{-\infty}^{+\infty} f(x) = 1$$

$0 \leq x \leq 1$

$$\int_0^1 a + bx \; dx = 1$$

$$\left[ax + b\frac{x^2}{2}\right]_0^1 = 1$$

$$\boxed{a + \frac{b}{2} = 1}$$

If $a = 0.5$ & $b = 1$ then

Q3 If f(x) and g(x) are two probability density functions,

$$f(x)=\begin{cases}\frac{x}{a}+1: & -a\le x<0\\ -\frac{x}{a}+1: & 0\le x\le a\end{cases}$$

$-\left|\frac{x}{a}\right|+1$

↳ Even

Ex:- $|x|$; $-a<x<a$

$-x$; $-a<x<0$

x ; $0<x<a$

$$g(x)=\begin{cases}-\frac{x}{a}: & -a\le x<0\\ \frac{x}{a}: & 0\le x\le a\\ 0: & \text{otherwise}\end{cases}$$

$\left|\frac{x}{a}\right|$

↳ Even

Continue To Next Slide

Which of the following statement is true?

A Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are same

B Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are different

C Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are same

D Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are different

We Know $f(x)$ and $g(x)$ are even functions

Mean

$$E[f(x)] = \int_{-a}^{+a} \underset{O}{x} \cdot \overset{E}{f(x)}\, dx = \boxed{0}$$

$$E[g(x)] = \int_{-a}^{+a} x\, g(x)\, dx = \boxed{0}$$

$$Var[f(x)] = E(X^2) - [E(X)]^2$$

$$\rightarrow Var[f(x)] = \int_{-a}^{+a} x^2 \cdot f(x)\, dx - 0 = 2\int_0^a x^2 \cdot f(x)\, dx = 2\int_0^a x^2 \cdot \left(-\frac{x}{a} + 1\right)$$

$$= 2\int_0^a -\frac{x^3}{a} + x^2\, dx = 2\left[-\frac{x^4}{4a} + \frac{x^3}{3}\right]_0^a = \boxed{\frac{a^3}{6}}$$

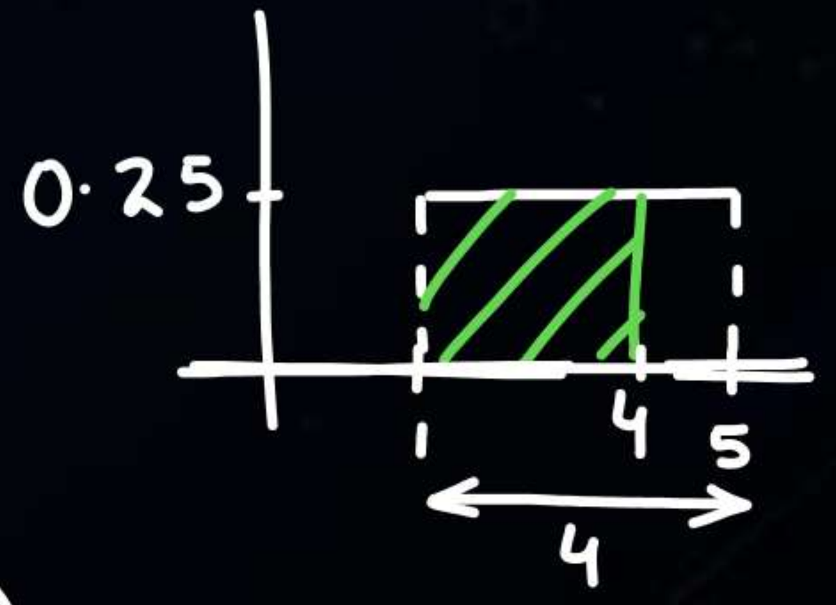
$$\rightarrow Var[g(x)] = \int_{-a}^{+a} x^2 \cdot g(x)\, dx - 0 = 2\int_0^a x^2 \cdot \frac{x}{a}\, dx = \frac{2}{a}\left[\frac{x^4}{4}\right]_0^a = \boxed{\frac{a^3}{2}}$$

Q4. **Probability density function of a random variable X is given below**

$$f(x) = \begin{cases} 0.25 & \text{if } 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

$P(X \le 4)$ is

$$P(x \le 4) = \int_1^4 0.25\,dx$$



$$= 0.25\,[x]_1^4 = 0.25(4-1)$$

$$= 0.75$$

Q5 Given that x is a random variable in range $[0, \infty]$ with a probability density function $e^{-x/2}/K$, the value of the constant K is_____.

$$X \rightarrow [0, \infty]$$

$$p(x) \rightarrow \frac{e^{-x/2}}{K}$$

$$\int_{-\infty}^{+\infty} p(x)\,dx = 1 \quad \text{For a valid p.d.f.}$$

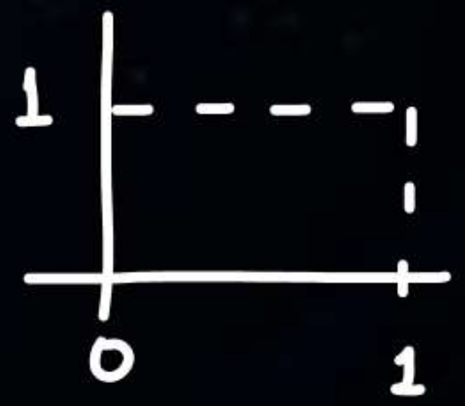
$$\int_0^{\infty} \frac{e^{-x/2}}{K} = \left[\frac{e^{-x/2}}{-\frac{1}{2}K}\right]_0^{\infty} = -\frac{2}{K}(0-1) = \frac{2}{K} = 1$$

$$\boxed{K=2}$$

Q6. Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is __0.25__.

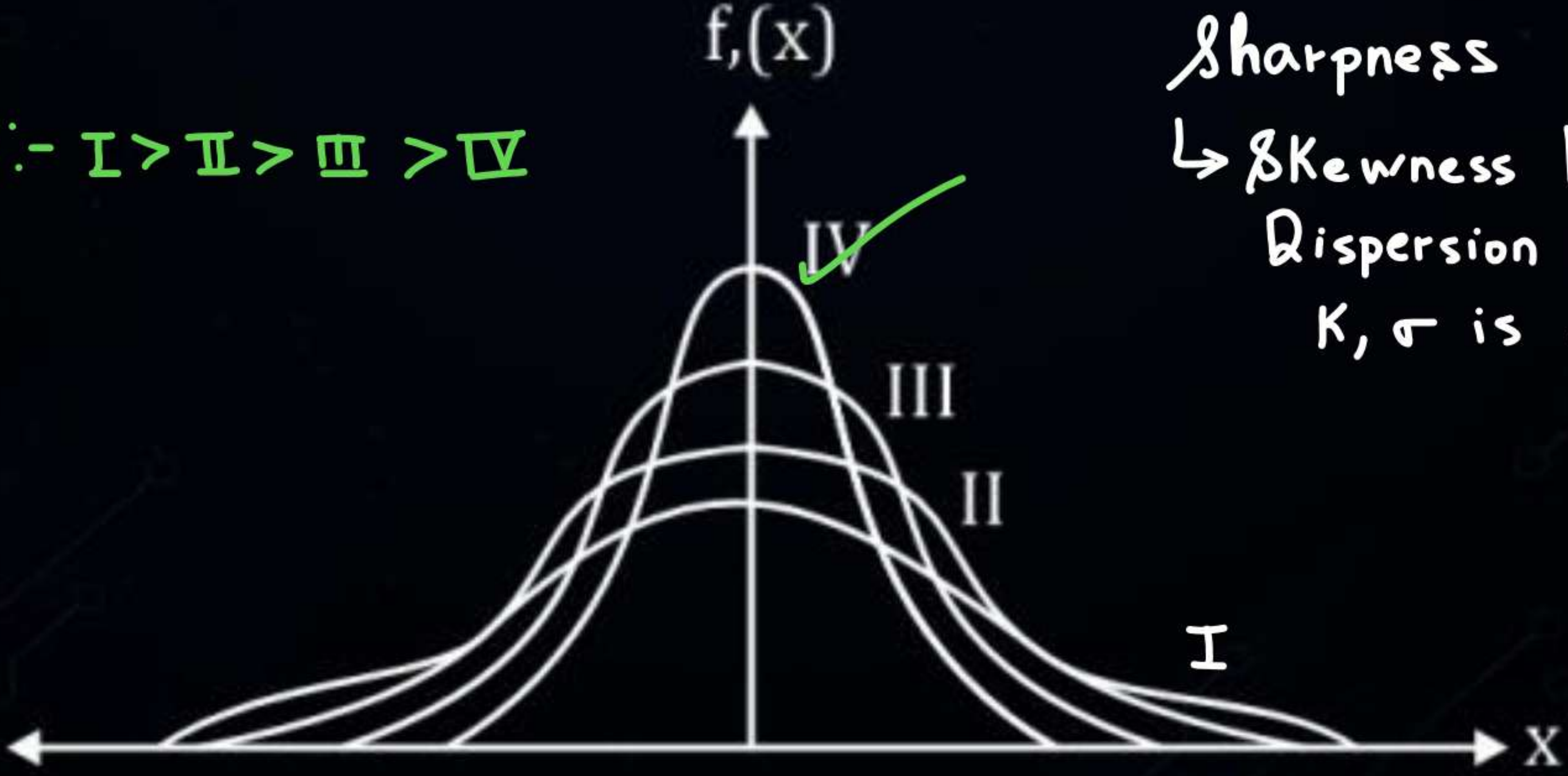
$$f(x) = \begin{cases} 1 ; & 0 \le x \le 1 \\ 0 ; & \text{otherwise} \end{cases}$$

$X \to (0,1)$ uniformly



$$E(X) = \int_0^1 x f(x) = \int_0^1 x \cdot 1 = \left[\frac{x^2}{2}\right]_0^1 = 0.25$$

Q7 **Among the four normal distributions with probability density function as shown below, which one has the least density and the lowest variance?**



Variance:- I > II > III > IV

Sharpness
↳ Skewness least
Dispersion is least
K, σ is least

Q8 A simple random sample of 100 observations was taken from a large population. The sample mean and the standard deviation were determined to be 80 and 12 respectively. The standards error of mean is 1.2.

Soln:-

$\mu = 80$; $n = 100$

$\sigma = 12$

$$\text{Standard error of mean} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{100}} = 1.2$$

Q9. The standard normal probability function can be approximated as

$$F(X_n) = \frac{1}{1+\exp(-1.7255X_N|X_N|^{0.12})}$$. where X_N = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be between 90 cm and 102 cm is

A. 66.7%

B. 50.0%

C. 33.3%

D. 16.7%

$\mu = 102\text{ cm}\ ;\ \sigma = 27\text{ cm}$

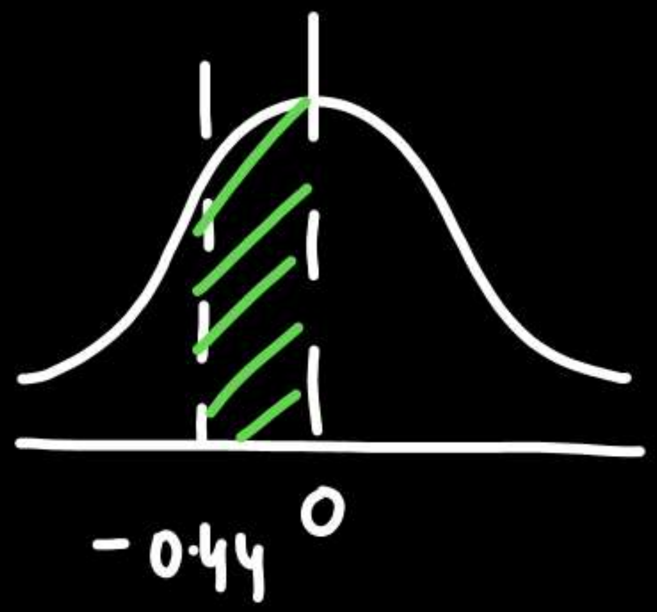
$$P(90 < X < 102) = P\left(\frac{90-102}{27} < X_N < \frac{102-102}{27}\right) \qquad Z = \frac{x-\mu}{\sigma}$$

$$\Rightarrow P(-0.44 < X_N < 0) = F(0) - F(-0.44)$$

$$= \frac{1}{1+1} - \frac{1}{1+e^{-1.7255 \times -0.44\,|-0.44|^{0.12}}}$$

$$= 0.166$$

$$\approx 16.66\%$$



Q10 $P_X(X) = Me^{(-2|x|)} + Ne^{(-3|x|)}$ is the probability density function for the real random variable X, over the entire x-axis, M and N are both positive real numbers. The equation relating M and N is

A $M + \frac{2}{3}N = 1$

C $M + N = 1$

B $2M + \frac{1}{3}N = 1$

D $M + N = 3$

$$\int_{-\infty}^{+\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{+\infty} \left(M e^{-2|x|} + N e^{-3|x|}\right) dx = 1$$

$$2\int_{0}^{\infty} M\, e^{-2x} + N\, e^{-3x}\, dx = 1$$

$$2\left[M\left[\frac{e^{-2x}}{-2}\right]_0^{\infty} + N\left[\frac{e^{-3x}}{-3}\right]_0^{\infty}\right] = 1$$

$$2\left[\frac{M}{2} + \frac{N}{3}\right] = 1$$

$$\boxed{M + 2\frac{N}{3} = 1}$$

Thank you

GW Soldiers !