ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-01

Numerical Methods

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Topics to be Covered

- GAUSS JACOBI METHOD
- GAUSS SEIDEL METHOD OF ITERATION

Topicsto be Covered

NUMERICAL SOLUTION OF LINEAR EQUATION

Direct method (Exact method)

- Gauss elimination method
- Do little's method
- Crout's method

NUMERICAL SOLUTION OF LINEAR EQUATION

Iterative methods (Numerical Methods)

- Gauss – Jacobi method
- Gauss - seidel method

Note: Iterative methods are applicable only on diagonally dominant system

NUMERICAL SOLUTION OF LINEAR EQUATION

$$a_{11}x + a_{12}y + a_{13}z = d_1$$

$$b_{11}x + b_{12}y + b_{13}z = d_2$$

$$c_{11}x + c_{12}y + c_{13}z = d_3$$

$$|a_{11}| > |a_{12}| + |a_{13}| \; ; \; |b_{12}| > |b_{11}| + |b_{13}| \; ; \; |c_{13}| > |c_{11}| + |c_{12}|$$

$\Rightarrow$ Diagonally dominant system

$$\left.\begin{aligned} 9x + 4y + z &= -17 \\ x - 2y - 6z &= 14 \\ x + 6y \quad &= 4 \end{aligned}\right\} \Rightarrow \begin{aligned} \boxed{9}x + 4y + z &= -17 \\ x + \boxed{6}y \quad &= 4 \\ x - 2y - \boxed{6}z &= 14 \end{aligned}$$

1) Gauss-Jacobi Method

Solve for x, y, z (whose coefficients are larger) in terms of other variables.

$$x = \frac{1}{a_{11}}(d_1 - a_{12}y - a_{13}z)$$

$$y = \frac{1}{b_{13}}(d_2 - b_{11}x - b_{13}z)$$

$$z = \frac{1}{c_{13}}(d_3 - c_{11}x - c_{12}y)$$

→ In absence of initial values of x, y, z we take (0,0,0)
→ Assume $x_0\ y_0\ z_0$ → Put initial values, find $x_1\ y_1\ z_1$ → Put $x_1\ y_1\ z_1$ find $x_2\ y_2\ z_2$
→ Keep on iterating, until convergence.

Q.

$$4x + y + 3z = 17$$
$$x + 5y + z = 14$$
$$2x - y + 8z = 12$$

$x = \frac{1}{4}(17 - y - 3z)$ — i)

$y = \frac{1}{5}(14 - x - z)$ — ii)

$z = \frac{1}{8}(12 - 2x + y)$ — iii)

$x_0 \quad y_0 \quad z_0$

$x_1 \quad y_1 \quad z_1$ ✓

$x_2 \quad y_2 \quad z_2$ ✓

$x_3 \quad y_3 \quad z_3$ ✓

$x_4 \quad y_4 \quad z_4$ ✓

$$\boxed{x = 3,\ y = 2,\ z = 1}$$

Assume		Trial I	Trial II	Trial III	Trial IV	Trial V	Trial VI	Trial VII
x	0	17/4	97/40	3.25	2.885	3.05	3	3
y	0	14/5	33/20	2.16	1.93	2.03	2	2
z	0	3/2	63/80	1.1	0.96	1.02	1	1

[GOUSS SEIDEL METHOD]

2) Gauss Seidel Method

Note: Convergence of gauss seidel method is greater than gauss Jacobi method

$$x = \frac{1}{a_{11}}\left(d_1 - a_{12}y - a_{13}z\right)$$

$$y = \frac{1}{b_{12}}\left(d_2 - b_{11}x - b_{13}z\right)$$

$$z = \frac{1}{c_{13}}\left(d_3 - c_{11}x - c_{12}y\right)$$

$$x_0, y_0, z_0 \longrightarrow x_1, y_1, z_1 \longrightarrow x_2, y_2, z_2 \longrightarrow x_3, y_3, z_3 \longrightarrow$$

x_1	y_1	z_1	x_2	y_2	z_2	x_3	y_3	z_3
↓	↓	↓	↓	↓	↓	↓	↓	↓
y_0	x_1	x_1	y_1	x_2	x_2	y_2	x_3	x_3
z_0	z_0	y_1	z_1	z_1	y_2	z_2	z_2	y_3

Q.

$$54x + y + z = 110$$
$$2x + 15y + 6z = 72$$
$$-x + 6y + 27z = 85$$

$$x = \frac{1}{54}[110 - y - z]$$

$$y = \frac{1}{15}[72 - 2x - 6z]$$

$$z = \frac{1}{27}[85 + x - 6y]$$

$x = 1.926$
$y = 3.574$
$z = 2.425$

Assume		Trial I	Trial II	Trial III	Trial IV	Trial V
x	0	2.037	1.912	1.925	1.926	1.926
y	0	4.528	3.658	3.581	3.574	3.574
z	0	2.217	2.406	2.424	2.425	2.425

NUMERICAL SOLUTION OF ALGEBRAIC & TRANSCENDENTAL EQUATION

Algebraic equation example :-

$$x^2 + 5x - 7 = 0 \quad , \quad x^4 - 2x^3 = 0$$

Transcendental equation example :-

I, L, T, E

$$3\sin x - \cos x + e^x = 0 \qquad e^x \cos x + 1 = 0$$

* Angle is in radians.

Direct methods

Iterative methods

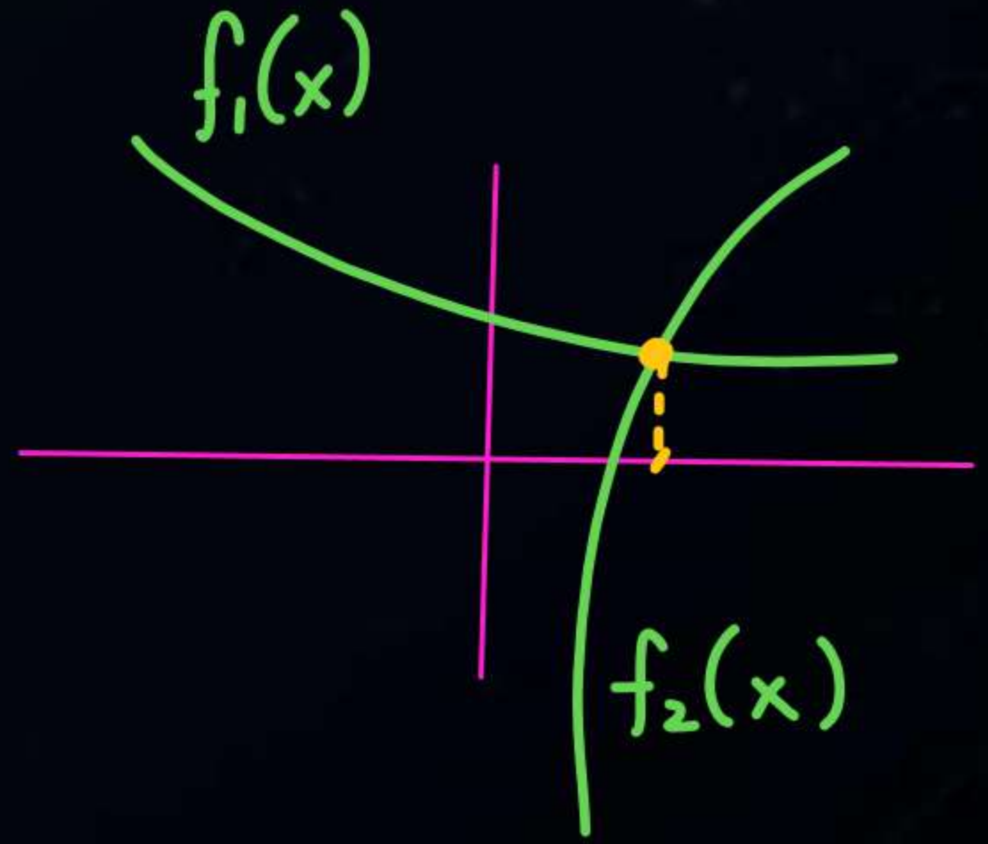
- Assume initial roots & take trials.

Graphical Method

$$f(x) = 0$$

$\hookrightarrow f_1(x) - f_2(x) = 0$

$\hookrightarrow f_1(x) = f_2(x)$



Q. Find approx. root of $\underline{x - \sin x - 1} = 0$

$f(x) = 0$

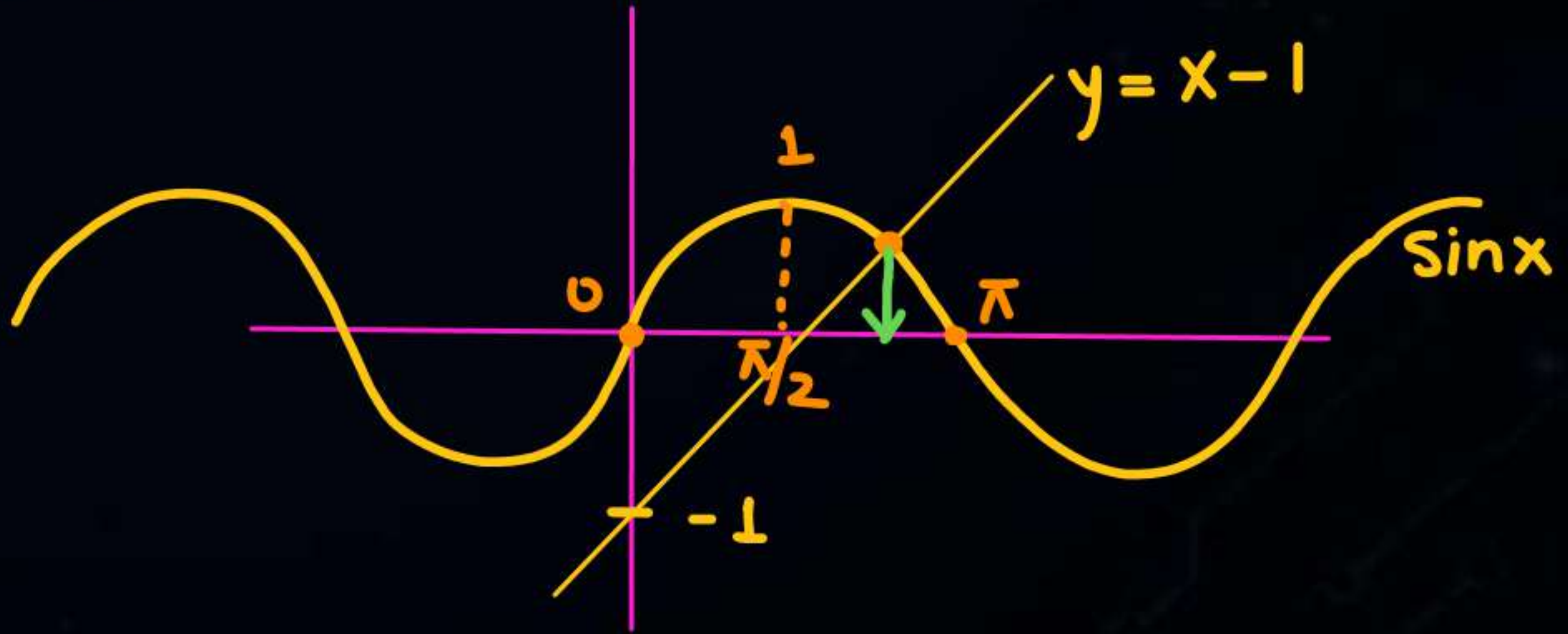
$x - 1 = \sin x$

$f_1(x) = f_2(x)$

$f_1(x) = x - 1$

$f_2(x) = \sin x$

Approx. root $= \underline{1.9}$



1.57 3.14

A) 0.7

B) 1.0

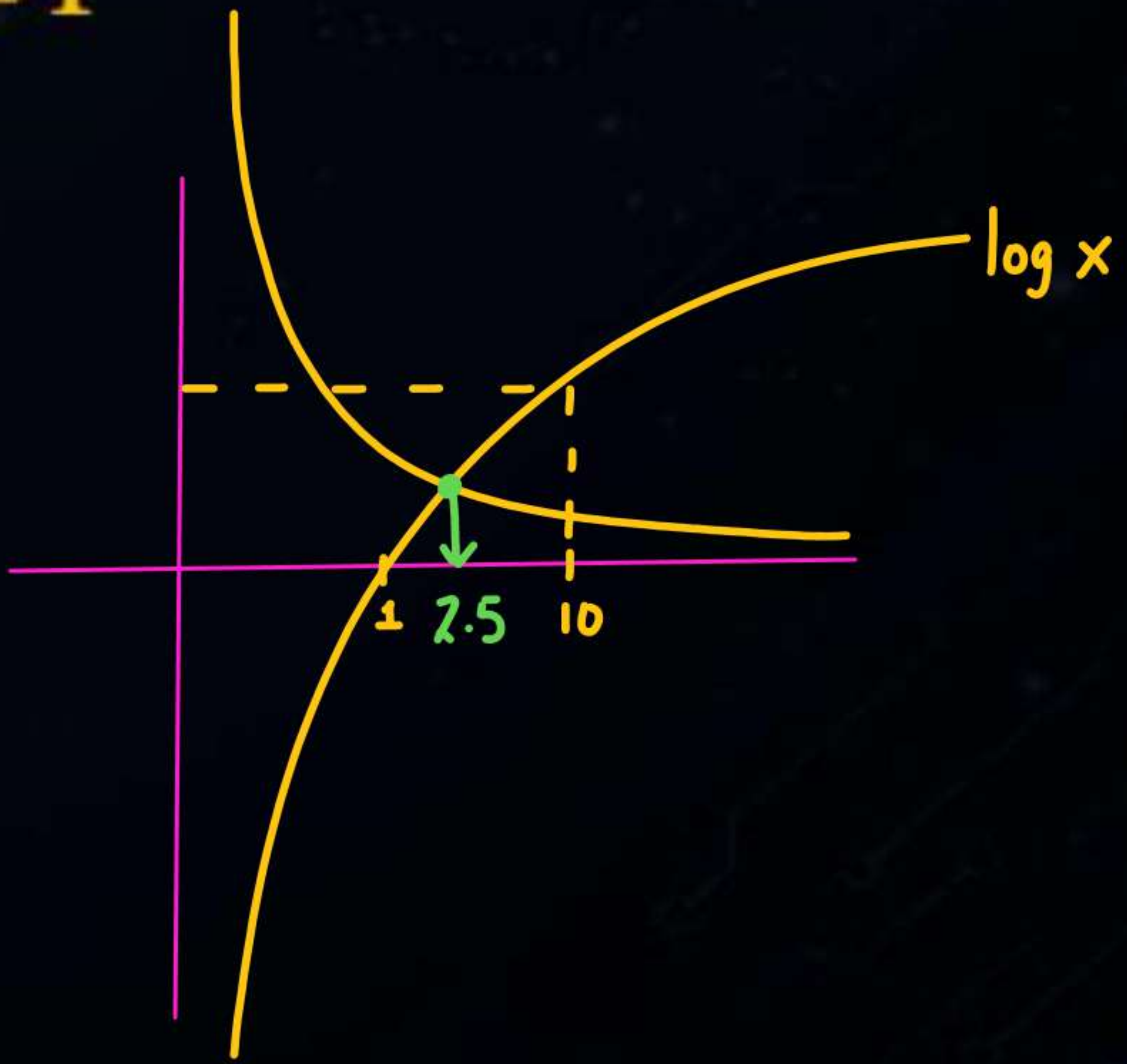
C) 1.9

Q. Solve graphically $x \log_{10} x = 1$

$$\log_{10} x = \frac{1}{x}$$

$$f_1(x) = f_2(x)$$

Approx. root = 2.5



Thank you

GW Soldiers !