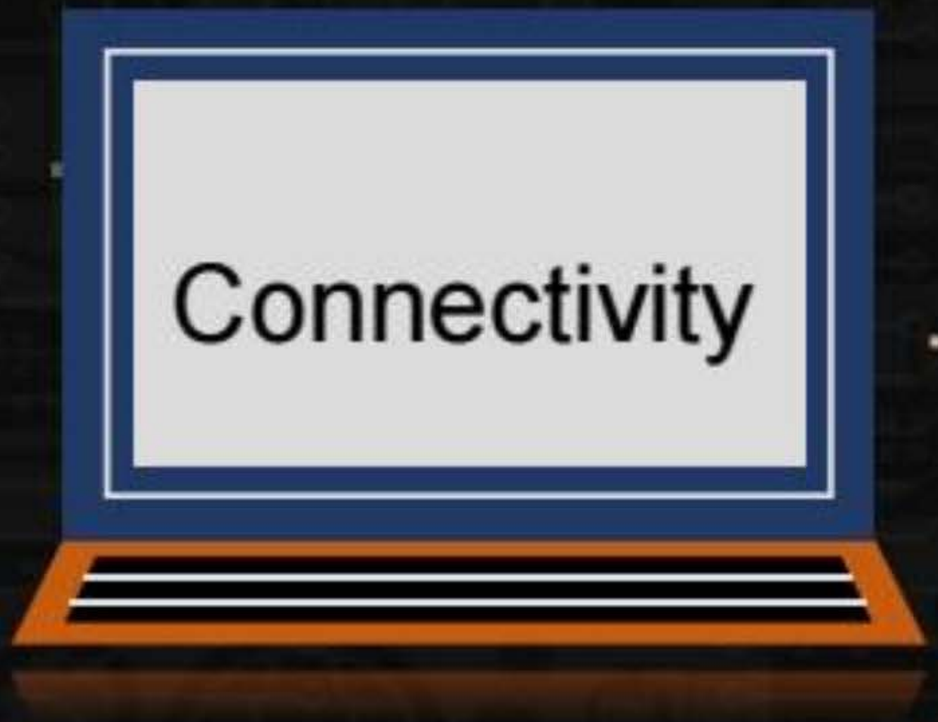
CS & IT ENGINEERING

DISCRETE MATHS GRAPH THEORY

Lecture No. 3

By- SATISH YADAV SIR



01 Degree Sequence

02 Types Of graphs

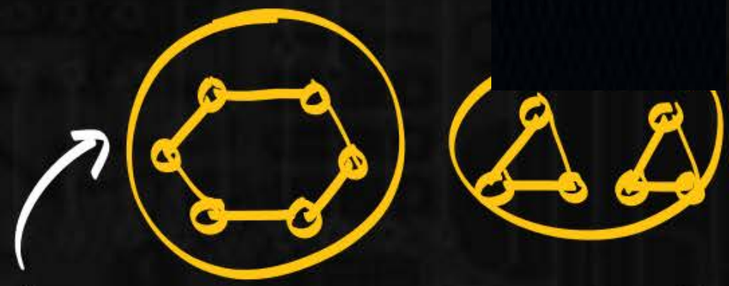
03 Practice

Graphical?

A) 5, 4, 3, 2, 1.

B) 4, 4, 3, 2, 1

C) 2, 2, 2, 2, 2, 2



not Graphical:

Reason 1: 5, 4, 3, 2, 1.

Thm 3: Total vertices = 5

$n = 5$

$\Delta(G) \leq n-1$.

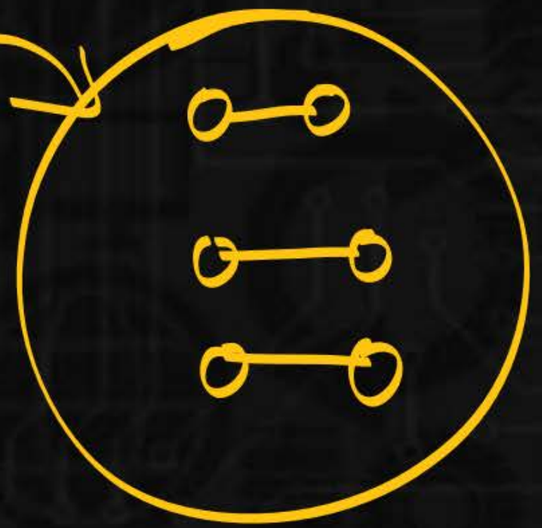
$\Delta(G) \leq 4$.

Reason 2.: Thm 2:

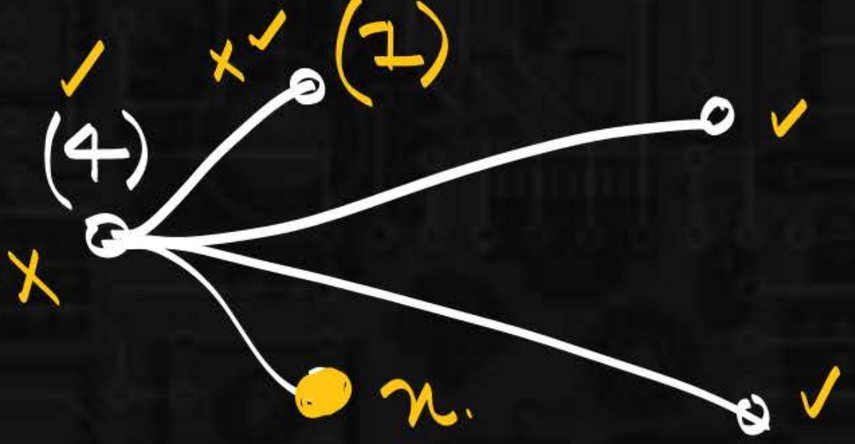
(5), 4, (3), 2, (1).

Thm 2.

D) 1, 1, 1, 1, 1, 1.



B) 4, 4, 3, 2, 1.



Demand: 3.

eg: 3, 3, 3, 1.
4, 4, 3, 2, 1.

Total vertices = n = 5.

n-1, n-1

Total vertices = n.

→ { n-1, n-1 ..... 1.

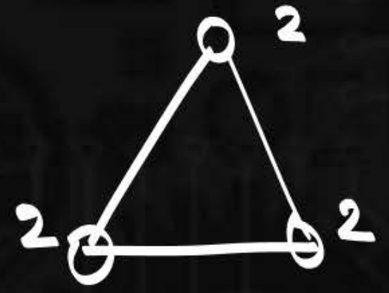
$\{n-1, n-1 \ldots\ldots 1.$

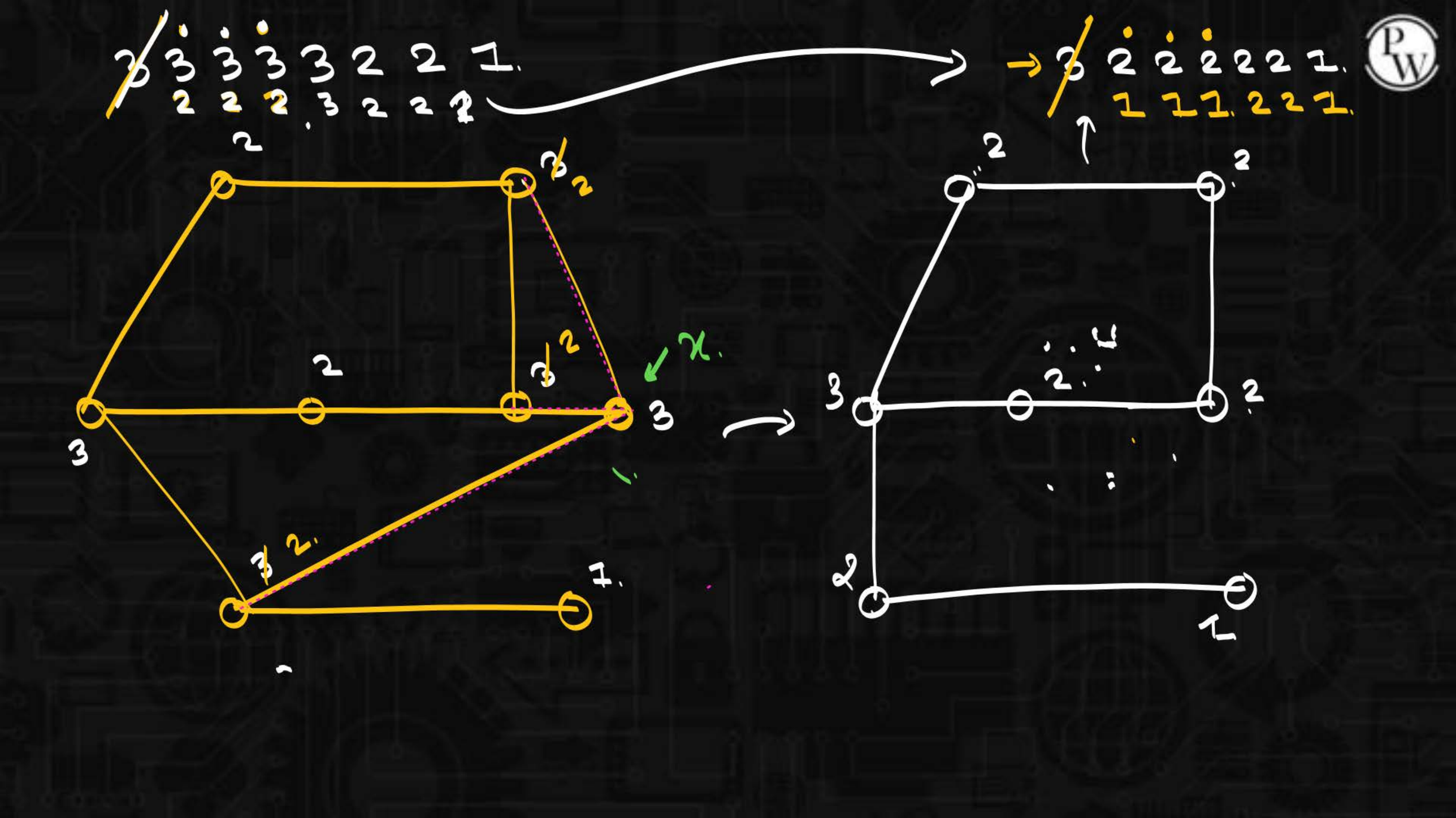
↳ not Graphical.

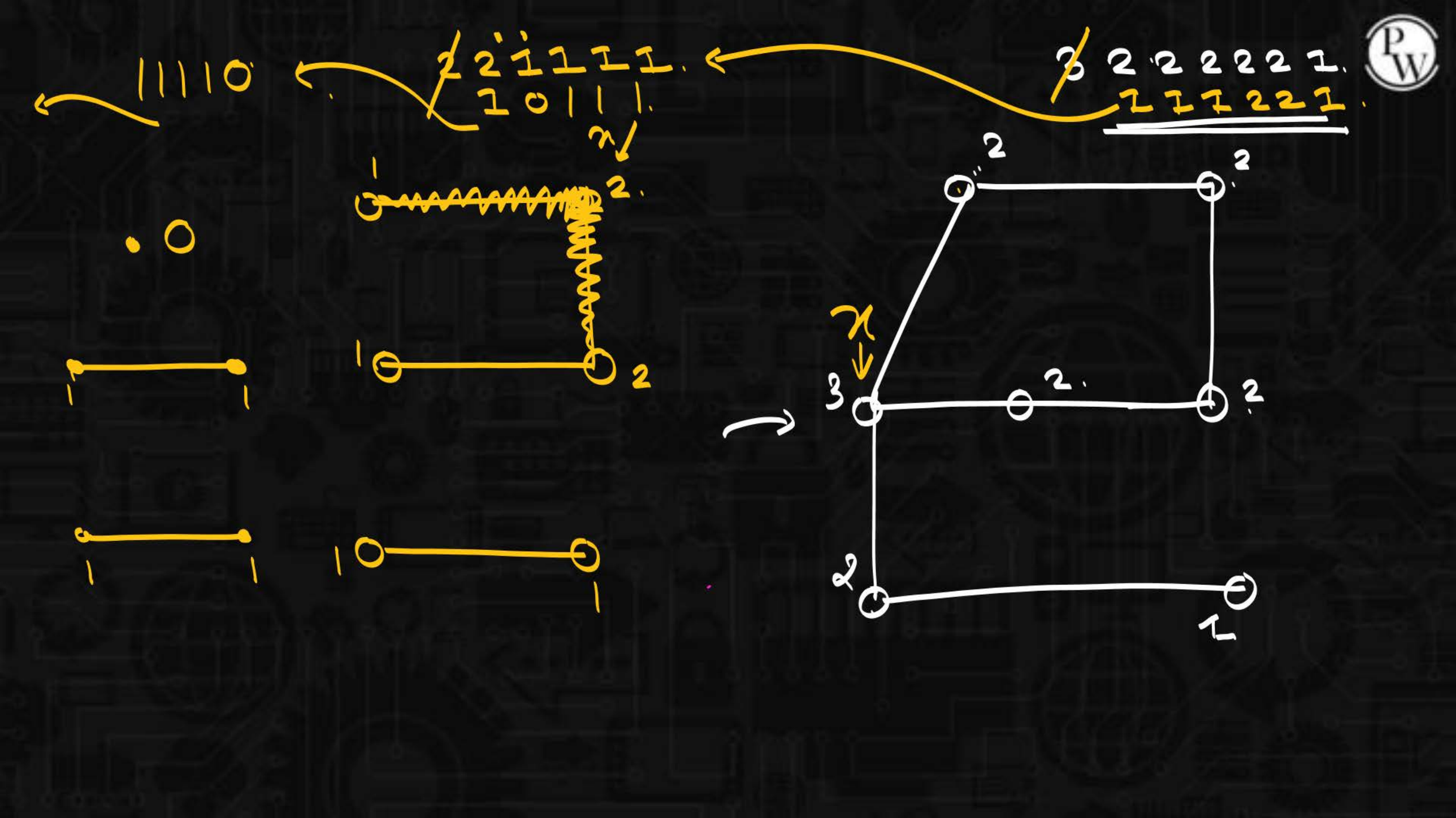
$\{n-1, n-1 \ldots\ldots\}$

2, 2, 2. $(n=3)$

$n-1, n-1, n-1.$




3 3 3 3 2 2 1
2 2 2 3 2 2 2
3 2 2 2 2 2 1
1 1 1 2 2 1
x


11110
2 2 1 1 1 1
1 0 1 1 1
3 2 2 2 2 2 1
1 1 1 2 2 1
2
1
2
1
2
3
2
2
2
1

→ 3 3 3 3 3 2 2 1 Graphical ?

2 2 2 3 2 2 1

count

→ mark

dlt (-1)

→ 3 2 2 2 2 2 1 (ordering)

1 1 1 2 2 1

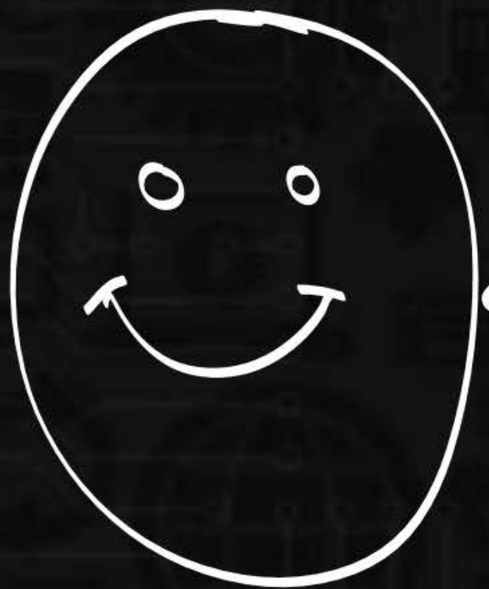
→ 2 2 1 1 1 1 (ordering)

1 0 1 1 1

→ 1 1 1 1 0 (ordering)

0 1 1 1 0

→ 1 1 0 0 (ordering)



(GATE)

→ 7 6 5 4 4 3 2 1 ✓

→ 6 6 6 6 3 3 2 2 ✗

→ 7 6 6 4 4 3 2 2 ✓

→ 8 7 7 6 4 2 1 1 ✗

$n = 8$

$\Delta(G) \leq 7$

→ 7 6 5 4 4 3 2 1

5 4 3 3 2 1 0 (ordering)

3 2 2 1 0 0 (ordering)

1 1 0 0 0

6 6 6 3 3 2 2

5 5 5 2 2 1 2.

5 5 5 2 2 2 1.

4 4 1 1 1 1.

3 0 0 0 1.

3 1 0 0 0 → no simple graph.

n-1, n-1 . . . . 1.

4 4 3 2 1.

3 2 1 0

1 0 -1.

4 3 2.

↳ Thm 2. X.

Steps:

→ check Thm 2.

→ check Thm 3.

→ n-1, n-1 ..... 1 (not possible)

⇒ all degrees are distinct
graph is not possible.

→ Havell-Hakimi

Thm6: In Simple Graph atleast 2 vertices will have same degree ($n \geq 2$)

→ all degrees are distinct → no simple Graph.

→ $5, 4, 3, 2, 1$.
n

$0 \leq x \leq 5$

$x, 1, 2, 3, 5, 5.$

for which value of x it is graphical?

not graphical.

$n-1, n-1 \ldots \ldots 1$

$5, 5, 3, 2, 1, x$

Total vertices = 6.

$0 \le x \le 7$.

7 6 5 4 3 2 1 x.

x can not be odd.

0 ~~1~~ 2 ~~3~~ 4 ~~5~~ 6 ~~7~~.

0 2 4 6.

for which value it is graphical

all degrees are distinct

$x = 0$ 7 6 5 4 3 2 1 0 ✗

$x = 2$ 7 6 5 4 3 2 2 1. ✓

$x = 4$

$x = 6$.

7 6 5 4 3 2 1 x

x = 4.

mistake :

7 6 5 4 4 3 2 1 4.

no ordering.

graphical ✓

7 6 5 4 4 3 2 1.

HW

$x, 7, 7, 5, 5, 4, 3, 2.$

$0 \leq x \leq 7.$

value of x. for which it is graphical ?

6.

null graph

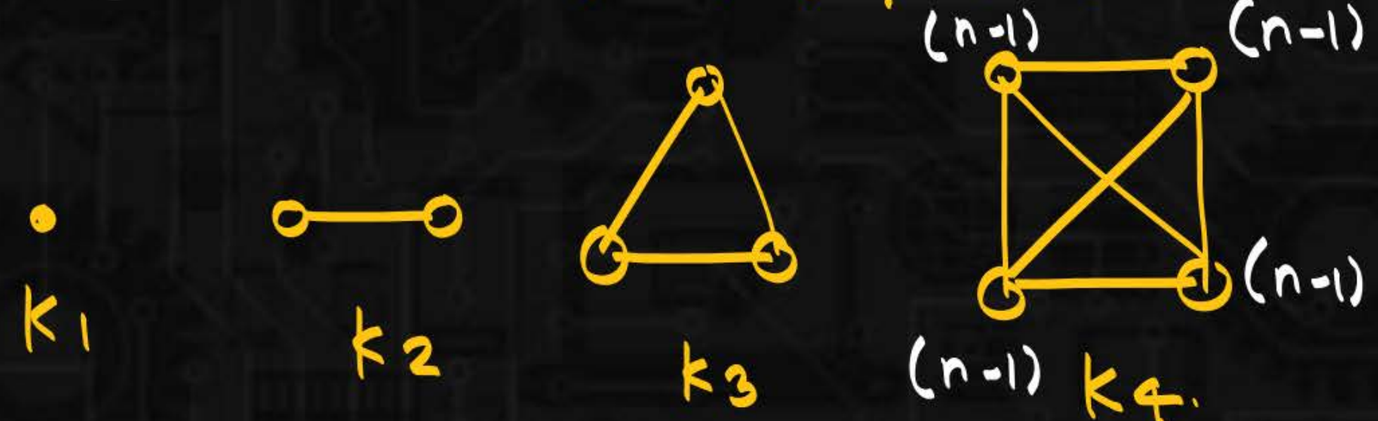
Thm 1 ✓ Type-1 ($\sum d(v) = 2e$) ✓

Thm 5 ✓ Type-2 (atleast atmost) ✓

Degree sequence (Type-3) ✓

Types of Graphs :

Complete Graph: (K_n) $(n \geq 1)$ $\left(\delta(G) = \frac{2e}{n} = \Delta(G) = n-1.\right)$



Degree of all vertices are n-1.

$\Sigma d(v) = 2e.$

$n(n-1) = 2e$

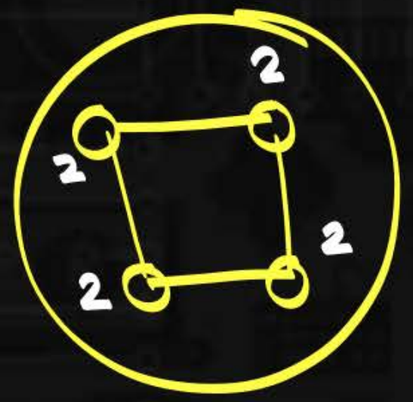
$$e = \frac{n(n-1)}{2}.$$

* n-1 Regular Graph.
* all K_n are Regular Graph (T)
* all Regular Graphs are K_n (False)

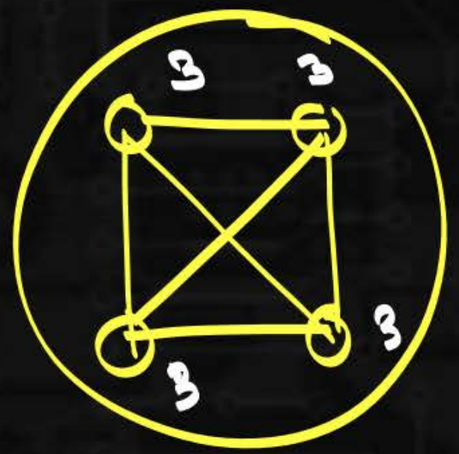
eg:

Regular Graph: $\left(\delta(G) = \frac{2e}{n} = \Delta(G)\right)$

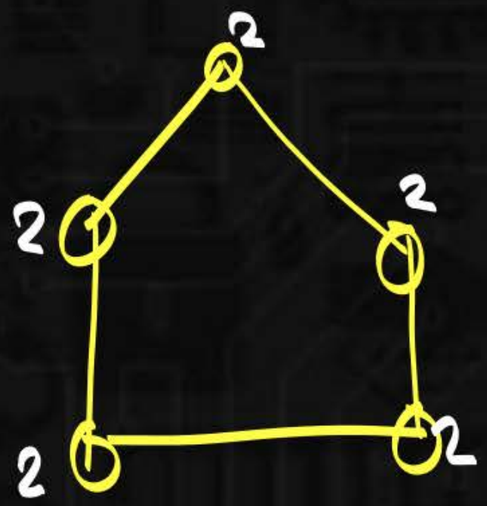
→ if degrees of all vertices are same then it is called Regular Graph.



2-Regular



3-Regular

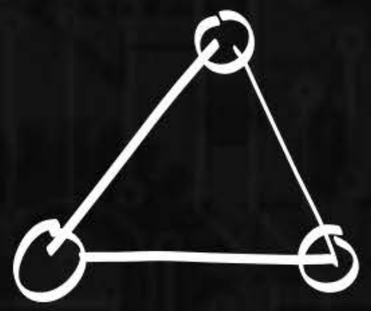


2-Regular

$\delta(G) = \frac{2e}{n} = \Delta(G) = k$

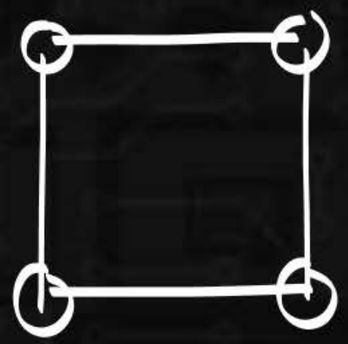
k-Regular Graph.

Cycle Graph. $(C_n)(n \geq 3)$ $\left(\delta(G) = \frac{2e}{n} = \Delta(G) = 2\right)$



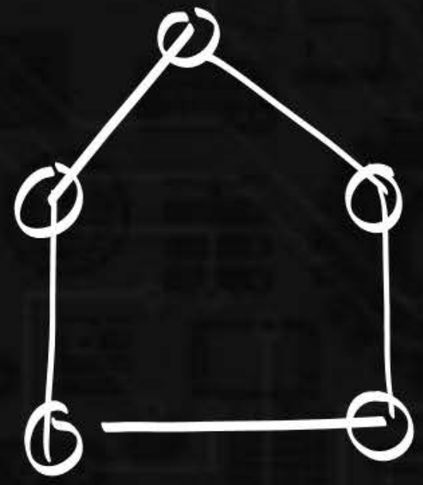
C_3

$n = e = 3$



C_4.

$n = 4 = e$



C_5

$n = e = 5$.

Degrees of all vertices are 2.

$\Sigma d(v_i) = 2e$.

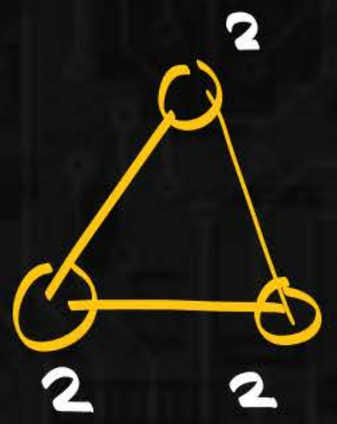
$n \cdot 2 = 2 \cdot e$.

$\star$ $n = e$

→ all c_n are Regular Graph.

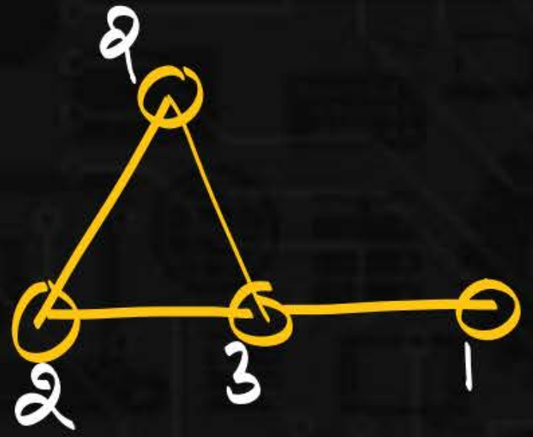
→ all Regular Graph are C_n.

C_3



2 2 2

Graph containing cycle.

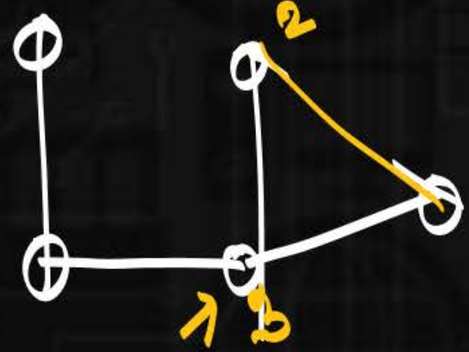


3 2 2 1.

if Graph is $C_n \rightarrow n = e$. (T)

* if $n = e \longrightarrow$ then it is cycle Graph. (False)

$n = 5 = e$




THANK YOU GW
SOLDIERS !