ENGINEERING MATHEMATICS

ALL BRANCHES

Probability

Introduction to Random Variable

DPP-04 Solution

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Q1. The probability that a k-digit code does NOT contain the digits 0,5 or 9 is

A. 0.3^k

B. 0.6^k

C. 0.7^k

D. 0.9^k

_ _ _ .... K digits

7 ways 7 ways

$\cancel{0}$, 1, 2, 3, 4, $\cancel{5}$, 6, 7, 8, $\cancel{9}$

10 digits

→ Favourable no. of outcomes $= 7^K$

→ Total no. of ways $= 10^K$

$$P(E) = \frac{n(E)}{n(S)} = \frac{7^K}{10^K} = 0.7^K$$

Q2. **In a pathology class , the professor decided to conduct a test of swine flu on all the students. Test result says that one student in every ten is having swine flu. What is the probability that out of 5 students expected to attend the class, at least 4 will not have swine flu?**

Soln :- p = prob. that student will not have swine flue $= \frac{9}{10}$

q = " " " " have " $= \frac{1}{10}$

$$P(X=4)+P(X=5) = {}^5C_4\, p^4 q^1 + {}^5C_5\, p^5 q^0 = 5\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right) + 1\left(\frac{9}{4}\right)^{10}$$

$$= \left(\frac{9}{10}\right)^4 \left\{5\left(\frac{1}{10}\right) + 1\right\}$$

$$\frac{3}{2}\left(\frac{9}{10}\right)^4 = 0.98415$$

$$= 98.415\%$$

Q3. A fair dice is tossed eight times. The probability that in first three throws three sixes is observed in a total of eighth throws is

Soln:-

$$\underset{6}{\underset{\downarrow}{\_}}\ \underset{6}{\underset{\downarrow}{\_}}\ \underset{6}{\underset{\downarrow}{\_}}\ \_\ \_\ \_\ \_\ \_$$

$$\{\underbrace{1,2,3,4,5}_{\frac{5}{6}},\underset{\frac{1}{6}}{6}\}$$

$$\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}$$

$$=\frac{3125}{6^8}$$

Q4. **A fair dice is tossed eight times. The probability that exactly three sixes is observed in a total of eighth throws is**

Soln:- { _ _ _ _ _ _ _ _ }

Total no. of favourable ways $= {}^8C_3$

Total no. of ways $= 6^8$

$$\frac{{}^8C_3}{6^8}$$

Q5 A fair dice is tossed eight times. The probability that a third six is observed on the eighth throw is

$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_}_{2\text{ six }(7\text{ throws})}\ \underset{8^{th}\text{ throw}}{\underset{\downarrow}{\underline{6}}}$$

$$\frac{{}^7C_2}{6^7} \times \frac{1}{6}$$

Q6. P(4^{th} head in 10^{th} throw when coin is tossed 10 times)

$$\underbrace{\_\;\_\;\_\;\_\;\_\;\_\;\_\;\_\;\_}_{\substack{\text{3 Heads} \\ \text{(9 throws)}}}\;\;\underset{\substack{\downarrow \\ 10^{th}\text{ throw}}}{\overset{\substack{4^{th} \\ H}}{\_}}$$

$$\frac{{}^{9}C_{3}}{2^{9}} \times \frac{1}{2}$$

Q7. The random variable X takes on the values 1, 2 or 3 with probabilities $2 + 5P/5$, $1 + 3P/5$, $1.5 + 2P/5$ respectively. The values of P and E(X) are respectively

A. 0.05, 1.87

B. 1.90,5.87

C. 0.05, 1.10

D. 0.25,1.40

X	1	2	3
P(x)	$\frac{2+5P}{5}$	$\frac{1+3P}{5}$	$\frac{1.5+2P}{5}$

$$\sum p(x) = 1; \quad \frac{2+5P}{5} + \frac{1+3P}{5} + \frac{1.5+2P}{5} = 1$$

$$4.5 + 10P = 5$$

$$10P = 0.5$$

$$\boxed{P = 0.05}$$

$$E(x) = \sum_{i=1}^{3} x\,p(x) = 1\left(\frac{2+5P}{5}\right) + 2\left(\frac{1+3P}{5}\right) + 3\left(\frac{1.5+2P}{5}\right) = 1.87$$

Q8 Let X be a random variable with probability density function

$$f(x)\begin{cases} 0.2, & \text{for } |X| \le 1 \\ 0.1, & \text{for } 1 < |X| \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$\rightarrow -1 \le X \le 1$

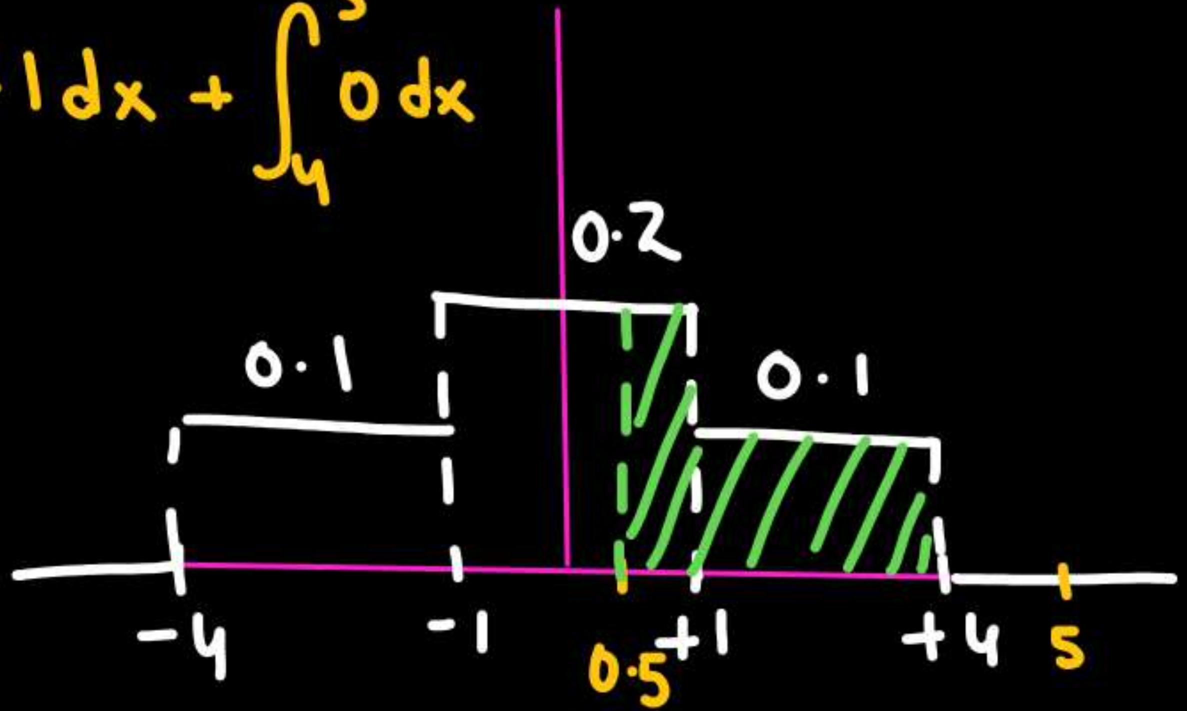
$\rightarrow -4 \le x < -1 ; 1 < x \le 4$

The probability $P(0.5 < X < 5)$ is 0.4.

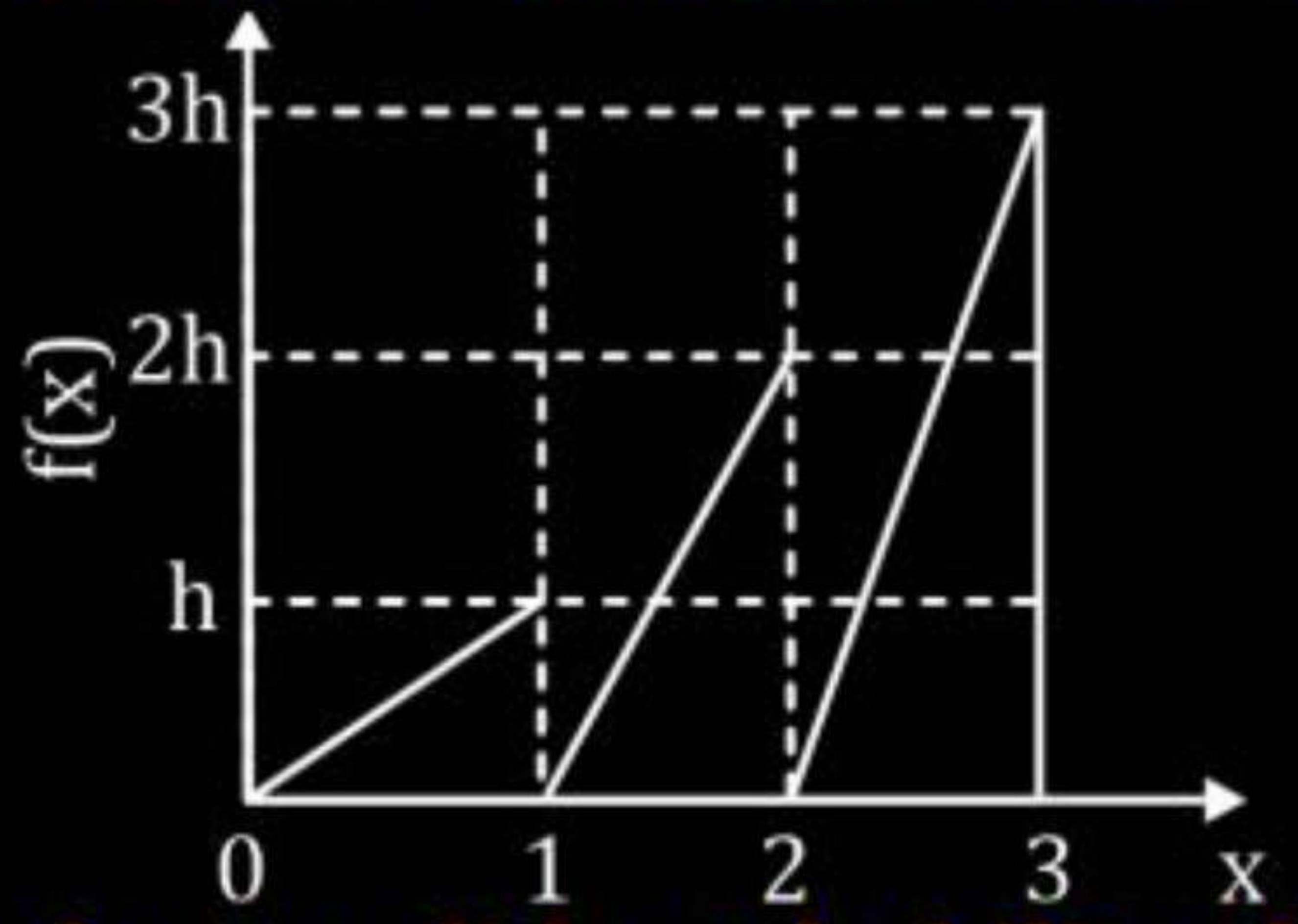
$$P(0.5 < X < 5) = \int_{0.5}^{5} p(x) = \int_{0.5}^{1} 0.2\,dx + \int_{1}^{4} 0.1\,dx + \int_{4}^{5} 0\,dx$$

$$[0.2x]_{0.5}^{1} + [0.1x]_{1}^{4} + 0$$

$$0.1 + 0.3 + 0 = 0.4$$



Q.9. The graph of a function f(x) is shown in the figure



For f(x) to be a valid probability density function the value of h is

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A 1/3

C 1

B 2/3

D 3

Soln :-

$$\int_{-\infty}^{+\infty} f(x) = 1 \quad \text{For a fn. to be valid}$$

$$\frac{1}{2} \times 1 \times h + \frac{1}{2} \times 1 \times 2h + \frac{1}{2} \times 1 \times 3h = 1$$

$$\frac{h}{2} + h + \frac{3h}{2} = 1 \quad \Rightarrow 3h = 1 \Rightarrow \boxed{h = 1/3}$$

Q10. A manufacturing company supplies condensers with 1% defective pieces. Condensers are packed in boxes of 100. Find the probability that a box picked at random will have four or more faulty condensers.

Soln:-

$$\lambda = np = 100 \times \frac{1}{100} = 1$$

$$P(X \geq 4) = 1 - \{P(X=0) + P(X=1) + P(X=2) + P(X=3)\}$$

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= 1 - \left\{\frac{e^{-1}.(1)^0}{0!} + \frac{e^{-1}(1)^1}{1!} + \frac{e^{-1}(1)^2}{2!} + \frac{e^{-1}(1)^3}{3!}\right\}$$

$$= 1 - e^{-1}\left\{1 + 1 + \frac{1}{2} + \frac{1}{6}\right\} = 0.019$$

Thank you

GW Soldiers !