# ENGINEERING MATHEMATICS

**ALL BRANCHES**

Vector Calculus

## Gradient of Scalar Function & Directional Derivative

**DPP-02 Solution**

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Question 1

The directional derivative of the function $f(x, y, z) = x + y$ at the point $P(1, 1, 0)$ along the direction $\hat{\imath} + \hat{\jmath}$ is

A $\frac{1}{\sqrt{2}}$

B $\sqrt{2}$

C $-\sqrt{2}$

D $2$

$f = x + y$

$$D.D. = \nabla f \cdot \hat{a}$$

$$= \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}\right)_{(1,1,0)} \cdot \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

$$= (\hat{i} + \hat{j}) \cdot \frac{(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{1+1}{\sqrt{2}}$$

$$= \sqrt{2}$$

## Question 2

The derivative of $f(x, y)$ at point $(1, 2)$ in the direction of vector $i + j$ is $2\sqrt{2}$ and in the direction of the vector $-2j$ is $-3$. Then the derivative of $f(x, y)$ in direction $-i-2j$ is

A $2\sqrt{2}+\frac{3}{2}$

B $\frac{-7}{\sqrt{5}}$

C $-2\sqrt{2}-\frac{3}{2}$

D $\frac{1}{\sqrt{5}}$

$$\nabla f \cdot \left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = 2\sqrt{2} \quad — (1)$$

$$\nabla f \cdot \frac{(-2\hat{j})}{2} = -3 \quad — (2)$$

$$\nabla f \cdot \hat{j} = 3$$

let $\nabla f = x\hat{i} + y\hat{j}$

$$\therefore y = 3$$

$$\therefore \frac{x+3}{\sqrt{2}} = 2\sqrt{2}$$

$$x = 1$$

$$\nabla f = \hat{i} + 3\hat{j}$$

$$\nabla f \cdot \frac{(-\hat{i} - 2\hat{j})}{\sqrt{(-1)^2 + (-2)^2}}$$

$$(\hat{i} + 3\hat{j}) \frac{(-\hat{i} - 2\hat{j})}{\sqrt{5}} = \frac{-1 - 6}{\sqrt{5}} = \frac{-7}{\sqrt{5}}$$

Question 3

The directional derivative $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at point P(2, 1, 3) in the direction of the vector $\vec{a} = \vec{\imath} - \overrightarrow{2k}$ is

A $\frac{4}{\sqrt{5}}$

B $-\frac{4}{\sqrt{5}}$

C $\frac{\sqrt{5}}{4}$

D $-\frac{\sqrt{5}}{4}$

$$D.D = \nabla f \cdot \hat{a}$$

$$= (4x\hat{i} + 6y\hat{j} + 2z\hat{k}) \cdot \frac{\hat{i} - 2\hat{k}}{\sqrt{1^2 + (-2)^2}}$$

$$(8\hat{i} + 6\hat{j} + 6\hat{k}) \frac{(\hat{i} - 2\hat{k})}{\sqrt{5}}$$

$$= \frac{8 - 12}{\sqrt{5}} = -\frac{4}{\sqrt{5}}$$

## Question 4

The maximum value of the directional derivative of the function $\phi = 2x^2 + 3y^2 + 5z^2$ at a point $(1, 1, -1)$ is

A. 10

B. $-4$

C. $\sqrt{152}$

D. 152

Max value of D.D $= |\vec{\nabla}\phi|$

$= \left(4x\,\hat{i} + 6y\,\hat{j} + 10z\,\hat{k}\right)_{(1,1,-1)}$

$= 4\hat{i} + 6\hat{j} - 10\hat{k}$

$\sqrt{4^2 + 6^2 + (-10)^2} = \sqrt{152}$

Question 5

The directional derivative of the scalar function $f(x, y, z) = x^2 + 2y^2 + z$ at the point P $= (1, 1, 2)$ in the direction of the vector $\vec{a} = 3\vec{\imath} - 4\vec{\jmath}$ is

A $-4$

B $-2$

C $-1$

D $1$

$$D.D. = \vec{\nabla f} \cdot \hat{a}$$

$$= (2x\hat{i} + 4y\hat{j} + \hat{k})_{(1,1,2)} \left(\frac{3\hat{i} - 4\hat{j}}{\sqrt{3^2 + (-4)^2}}\right)$$

$$= (2\hat{i} + 4\hat{j} + \hat{k}) \frac{(3\hat{i} - 4\hat{j})}{5}$$

$$\frac{6 - 16}{5} = \frac{-10}{5} = -2$$

## Question 6

For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, the magnitude of the gradient at the point $(1, 3)$ is

$$\nabla u = \left(\frac{2x}{2}\hat{i} + \frac{2y}{3}\hat{j}\right)_{(1,3)}$$

$$\hat{i} + 2\hat{j}$$

$$|\vec{\nabla u}| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

A. $\sqrt{\frac{13}{9}}$

B. $\sqrt{\frac{9}{2}}$

C. $\sqrt{5}$

D. $9/2$

## Question 7

A scalar field is given by $f = x^{2/3} + y^{2/3}$, where $x$ and $y$ are the Cartesian coordinates. The derivative of '$f$' along the line $y = x$ directed away from the origin at the point (8, 8) is

A. $\frac{\sqrt{2}}{3}$ ✓

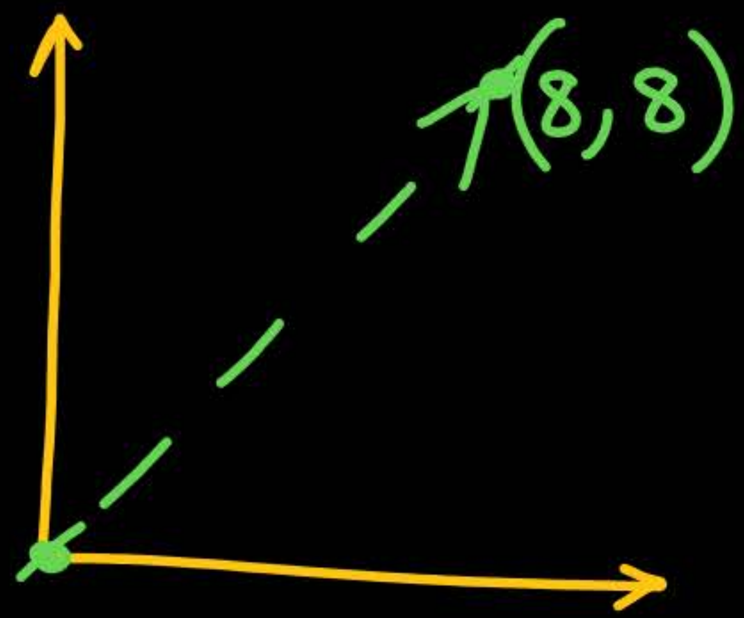
B. $\frac{\sqrt{3}}{2}$

C. $\frac{2}{\sqrt{3}}$

D. $\frac{3}{\sqrt{2}}$

$$\vec{\nabla} f = \left(\frac{2}{3} x^{-1/3}\,\hat{i} + \frac{2}{3} y^{-1/3}\,\hat{j}\right)_{(8,8)}$$

$$= \left(\frac{\hat{i}}{3} + \frac{\hat{j}}{3}\right)$$




$$\vec{\nabla} f \cdot \hat{a} = \left(\frac{\hat{i}}{3} + \frac{\hat{j}}{3}\right)\left(\frac{8\hat{i} + 8\hat{j}}{\sqrt{8^2 + 8^2}}\right) \qquad \vec{a} = 8\hat{i} + 8\hat{j}$$

$$= \left(\frac{8}{3} + \frac{8}{3}\right) \cdot \frac{1}{8\sqrt{2}} = \frac{16}{3} \cdot \frac{1}{8\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{3\cancel{\sqrt{2}\sqrt{2}}} = \sqrt{2}/3$$

Question 8

The magnitude of the gradient of the function $f = xyz^3$ at $(1, 0, 2)$ is

A 0

B 3

C 8

D $\infty$

$$\nabla f = (yz^3)\hat{i} + (xz^3)\hat{j} + (3xyz^2)\hat{k}$$

$$(0\times 2^3)\hat{i} + (1\times 2^3)\hat{j} + (3\times 1\times 0\times 2^3)\hat{k}$$

$$8\hat{j}$$

$$|\vec{\nabla} f| = 8$$

Question 9

For the function $\phi = ax^2y - y^3$ to represent the velocity potential of an ideal fluid, $\nabla^2\phi$ should be equal to zero. In that case, the value of '$a$' has to be

A $-1$

B $1$

C $-3$

D $3$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

$$\frac{\partial\phi}{\partial x} = 2xay$$

$$\frac{\partial^2\phi}{\partial x^2} = 2ay$$

$$\frac{\partial\phi}{\partial y} = ax^2 - 3y^2$$

$$\frac{\partial^2\phi}{\partial y^2} = -6y$$

$$2ay - 6y = 0$$

$$2a\cancel{y} = 6\cancel{y}$$

$$\boxed{a = 3}$$

Question 10

The gradient of field $f = y^2 x + xyz$ is

A $y(y+z)i + x(2y+z)j + xy\hat{k}$

B $y(2x+z)i + x(x+z)j + xyk$

C $y^2 i + 2yxj + xyk$

D $y(2y+z)i + x(2y+z)j + xyk$

$\vec{\nabla} f = (y^2 + yz)\hat{i} + (2yx + xz)\hat{j} + (xy)\hat{k}$

$y(y+z)\hat{i} + x(2y+z)\hat{j} + xy\hat{k}$

Question 11

The magnitude of the gradient of the function $f = xyz^3$ at $(1, 0, 2)$ is

Same as Q8.

A 0

B 3

C 8

D $\infty$

## Question 12

Let $f: R^2 \to R$ be defined by

$$\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}$$

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4+y^2} \text{ for}(x,y) \neq (0,0) \\ 0 \text{ for}(x,y) = \underbrace{(0,0)} \end{cases}$$

0

$$D.D. = \vec{\nabla} f \cdot \hat{a}$$
$$= 0 \cdot \hat{a} = 0$$

The directional derivative of $f$ at $\underbrace{(0,0)}$ in the direction of the vector $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is

$$\left[\frac{(x^4+y^2)(2xy) - (x^2y)(4x^3)}{(x^4+y^2)^2}\right]\hat{i}$$

$$\left[\frac{(x^4+y^2)(x^2) - (x^2y)(2y)}{(x^4+y^2)^2}\right]\hat{j}$$

A. 0

B. 1/2

C. $\frac{1}{2\sqrt{2}}$

D. $\frac{1}{4\sqrt{2}}$

# Thank you

# GW Soldiers !