CS & IT ENGINEERING

Digital Logic

MINIMIZATION

Lecture No. 1

By- CHANDAN SIR

01 THEOREM

02 D-MORGAN'S Law

03 QUESTION PRACTICE

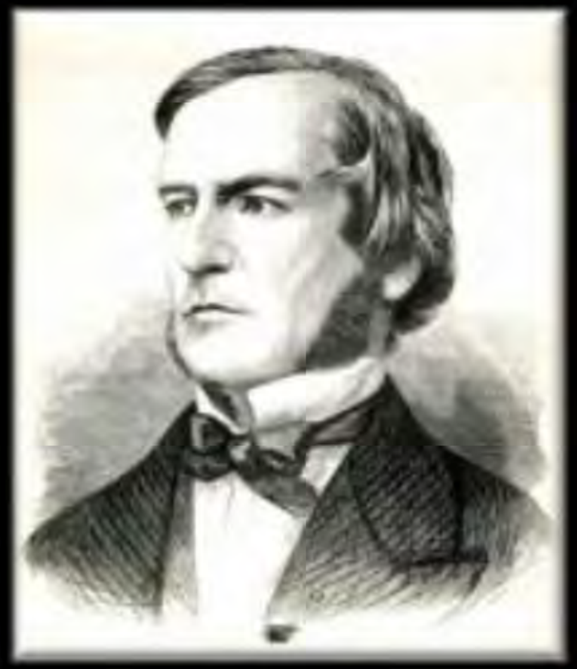
04 DUAL & SELF DUAL

05 DISCUSSION

Laws of Boolean Algebra

- 1854- George Boole

"An Investigation of Law of Thoughts"

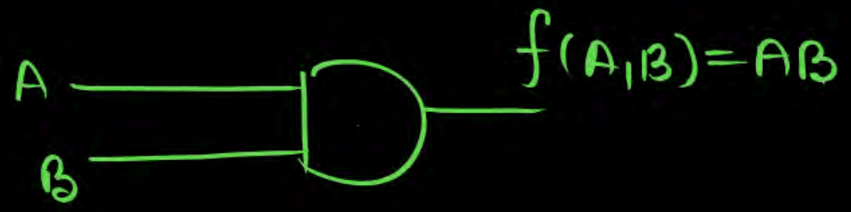


Laws of Boolean Algebra

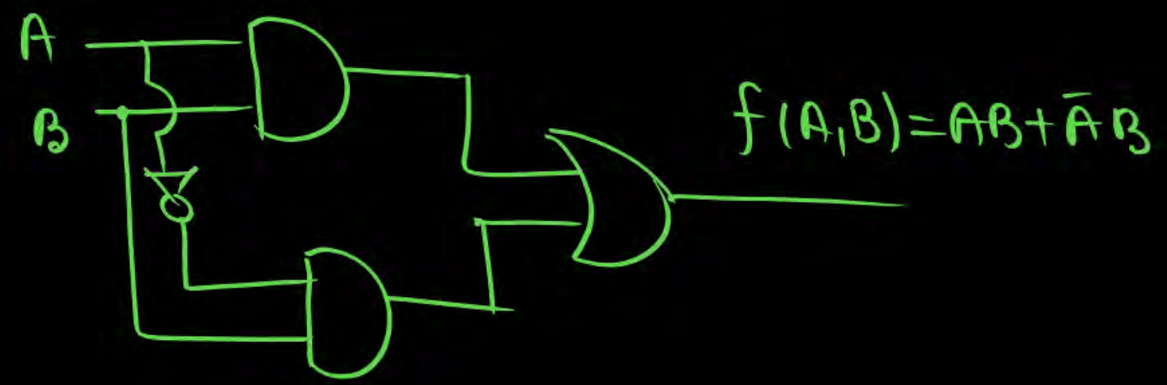
- **BOOLEAN ALGEBRA**

Ex 1.

$f(A, B) = AB$



Ex.2. $f(A,B) = AB + \bar{A}B$



Laws of Boolean Algebra

Boolean Function- It is the combination of inputs on which output is depends.

Ex $f(A,B) = AB + \bar{A}\bar{B}$

number of Variables = 2

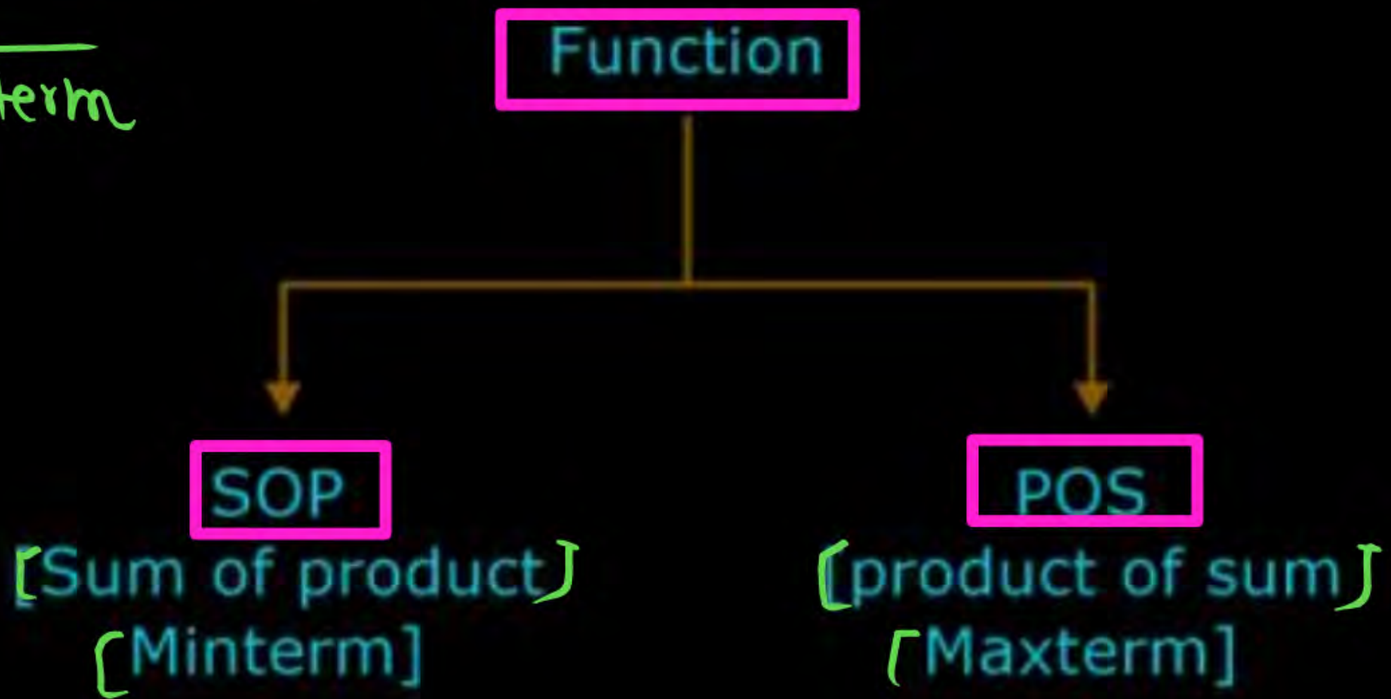
Number of terms = 2

Literals = 4

Laws of Boolean Algebra

- **BOOLEAN ALGEBRA**

$\text{Minterm} = \overline{\text{Maxterm}}$



Standard canonical form

→ Each term should contain all the variables.

Ex. $f(A,B) = \bar{A}B + A\bar{B}$ ✓

$f(A,B) = A + \bar{A}B$

→ Canonical form ✗

Laws of Boolean Algebra

aise hi likha hai.

Decimal	ABC	Min Term	Max Term	Function
0	000	$\bar{A}\bar{B}\bar{C}$	$A+B+C$	1 ✓
1	001	$\bar{A}\bar{B}C$	$A+B+\bar{C}$	0
2	010	$\bar{A}B\bar{C}$	$A+\bar{B}+C$	1 ✓
3	011	$\bar{A}BC$	$A+\bar{B}+\bar{C}$	1 ✓
4	100	$A\bar{B}\bar{C}$	$\bar{A}+B+C$	0
5	101	$A\bar{B}C$	$\bar{A}+B+\bar{C}$	0
6	110	$AB\bar{C}$	$\bar{A}+\bar{B}+C$	0
7	111	ABC	$\bar{A}+\bar{B}+\bar{C}$	1

Standard canonical Sop form :→

$$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + ABC$$

$$= m_0 + m_2 + m_3 + m_7$$

$$= \Sigma m(0,2,3,7)$$

$$= \Sigma(0,2,3,7)$$

Standard canonical Pos form

$$F(A,B,C) = (A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+B+\bar{C}) \cdot (\bar{A}+\bar{B}+C)$$

$$= M_1 \cdot M_4 \cdot M_5 \cdot M_6$$

$$= \Pi M(1,4,5,6)$$

$$= \Pi(1,4,5,6)$$

Q Number of terms present in standard canonical sop form?

$f(A,B,C) = A + BC$

$= A(\bar{B}+B)(\bar{C}+C) + (\bar{A}+A)BC$

$= A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}BC.$

(A) 4

(B) 5

(C) 6

(D) 7

5

① DISTRIBUTION THEOREM :→

$(A+B)(A+C)$

$\Rightarrow A\cdot A + AC + AB + BC$

$\Rightarrow A + AC + AB + BC$

$\Rightarrow A[1+C+B] + BC$

$\Rightarrow A\cdot 1 + BC$

$\Rightarrow A + BC$

$A + BC = (A+B)\cdot(A+C)$

$A + BCD = (A+B)(A+C)(A+D)$

Ex $A + \bar{A}B$

$(A+\bar{A})(A+B)$

$1\cdot(A+B)$

$= A+B$

Q $f(A,B) = \bar{A} + A\bar{B}$

$= (\bar{A} + A)(\bar{A} + \bar{B})$

$= \bar{A} + \bar{B}$

② Concensus Theorem

$$f(A,B,C) = AB + \bar{A}C + BC$$

$$= AB + \bar{A}C + (\bar{A}+A)BC$$

$$= AB + \bar{A}C + \bar{A}BC + ABC$$

$$= AB[1+C] + \bar{A}C[1+B]$$

$$= AB + \bar{A}C$$

$BC \rightarrow$ Redundant term

- ✓ 3 Variable function
- ✓ each term consist of 2 Variable
- ✓ Each Variable Repeated Twice Except one.
- ✓ One Variable repeated in form of complement

Ex $AB + \bar{B}C + AC$

$AB + \bar{B}C$

AUP

$AC \rightarrow$ Redundant

Ex $\bar{A}\bar{B} + \bar{A}C + \bar{B}\bar{C}$

$\bar{A}C + \bar{B}\bar{C}$

$\bar{A}\bar{B}$ → Redundant term

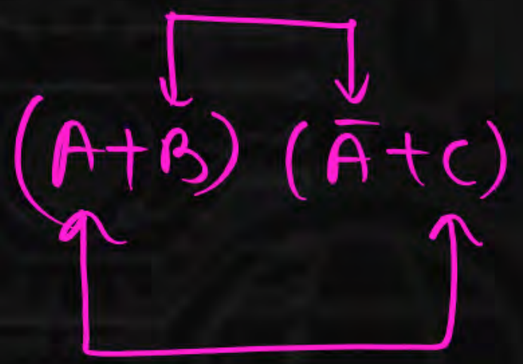
③ TRANPOSE THEOREM

$$(A+B)(\bar{A}+C)$$

$$\Rightarrow A\cdot\bar{A} + AC + \bar{A}B + BC$$

$$\Rightarrow AC + \bar{A}B + BC$$

$$= AC + \bar{A}B$$



$$= AC + \bar{A}B$$

④ D-Morgans Law.

$$\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$$

$$\overline{A+B+C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$$

Laws of Boolean Algebra

- **Theorem**

1) Distribution theorem

$(A + B)(A + C) = A + BC$ $\qquad$ $A \cdot (B + C) = AB + AC$

2) Consensus theorem

$AB + \bar{A}C + BC = AB + \bar{A}C$

3) Transpose theorem

$(A + B) \cdot (\bar{A} + C) = AC + \bar{A}B$

4) D-Morgan's Law

$\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$

$\overline{A + B + C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$

Laws of Boolean Algebra

- **Theorem**

5) Annulment Law

$A \cdot 0 = 0$, $A + 1 = 1$

6) Identity Law

$A + 0 = A$, $A \cdot 1 = A$

7) Idempotent Law

$A + A = A$, $A \cdot A = A$

8) Absorptive Law

$A + AB = A$

$A \cdot (A + B) = A$

Q.1

Find the minimum number of the NAND gate required to implement the Boolean function given below:

$f(A, B, C) = A + ABC + AB\bar{C}$

(A) 0

(B) 1

(C) 2

(D) 7

$A[1 + BC + B\bar{C}]$

$= A$

Q.2

Minimize the expression:

$f(A, B) = A + A\overline{B}$

Q.3

Minimize the expression.

$f(A, B) = \overline{A}\,\overline{B} + \overline{A}B + AB$

Q.4

Minimize the expression.

$f(A, B) = \overline{A}\,\overline{B} + \overline{A}B + A\overline{B} + AB$

Q.5

Minimize the expression.

$f(A, B) = \overline{A}B + A\overline{B}$

Q.6

Minimize the expression.

$f(A, B) = AB + \overline{A}C + BC$

Q.7

Minimize the expression.

$f(A, B, C) = \overline{A}\,\overline{B} + \overline{A}C + \overline{B}\,\overline{C}$

Q.8

Minimize the expression.

$f(A, B, C) = (A + B)(A + C)(\overline{B} + C)$

Thank you

GW

Soldiers !