CS & IT ENGINEERING

DISCRETE MATHS GRAPH THEORY

Lecture No. 1

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TOPICS TO BE COVERED

01 Definition of Graph

02 Handshaking Lemma

03 Types of Graphs

04 No of Graphs

05 Simple Graphs theorem

Basics of Graph

10

Graph Theory. (4 - 6)

logic (2 - 4)

Set theory (2 - 4)

Combinatorics. (2 - 4)

Basics of Graph

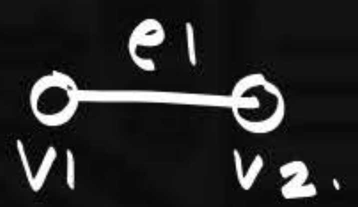
1) A

2) all bridges.

Basics of Graph

point/joint/node → vertex/vertices.

line/arc/branch → edge/edges.



Graph $G = (V, E)$

V → set of vertices.

E → set of edges.

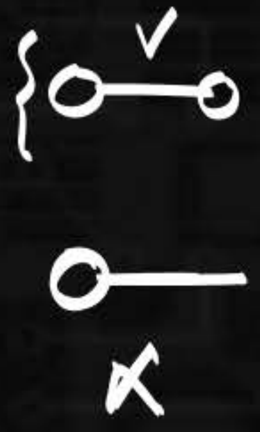
$e_1 \rightarrow (v_1, v_2)$

each edge must be associated with unordered pair of vertices.

Basics of Graph

$G = (V, E)$

$V = \{v1, v2, v3, v4, v5\}$.



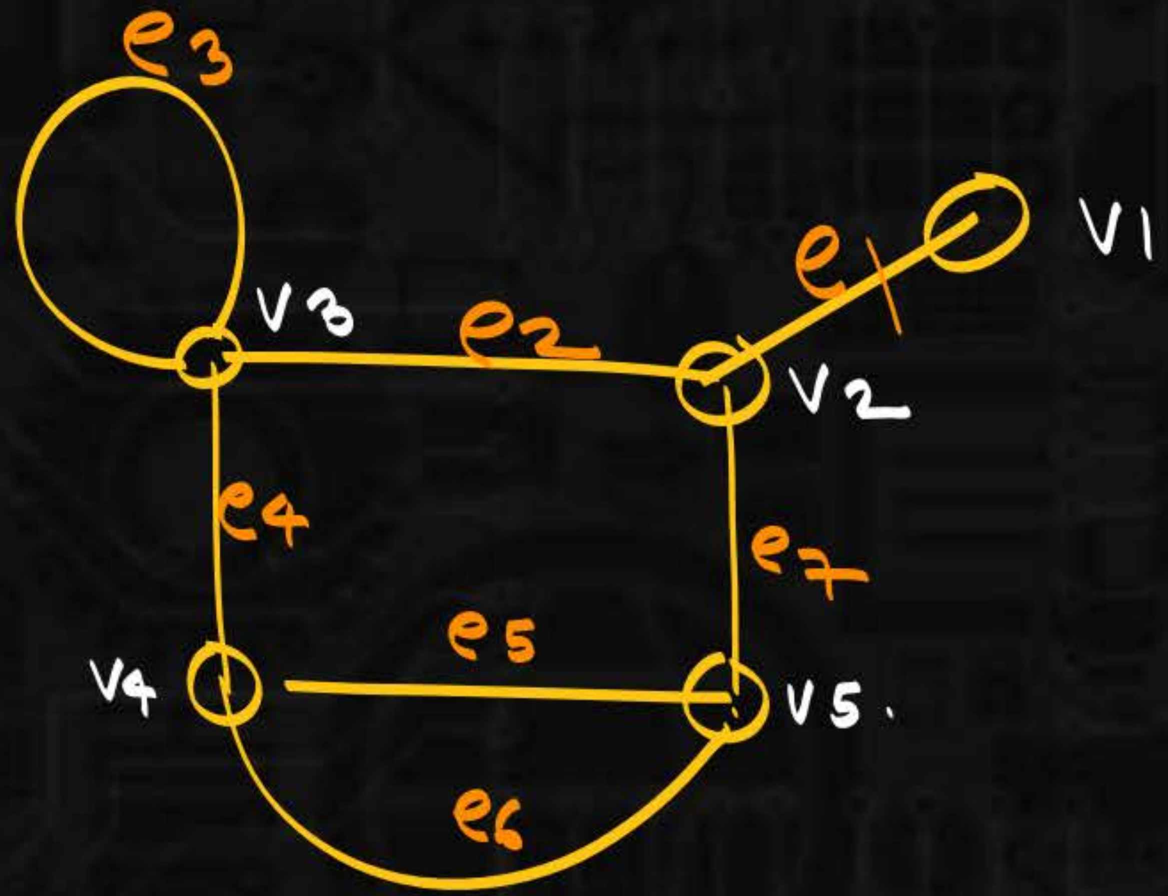
$E = \{e1, e2, e3 \ldots e7\}$.

$e1 \to (v1, v2) \mid (v2, v1)$

$e2 \to (v2, v3)$

$e3 \to (v3, v3)$

$\vdots$



Basics of Graph

$$G = (V, E, \psi)$$

$V = \{ \ldots\ldots \}$

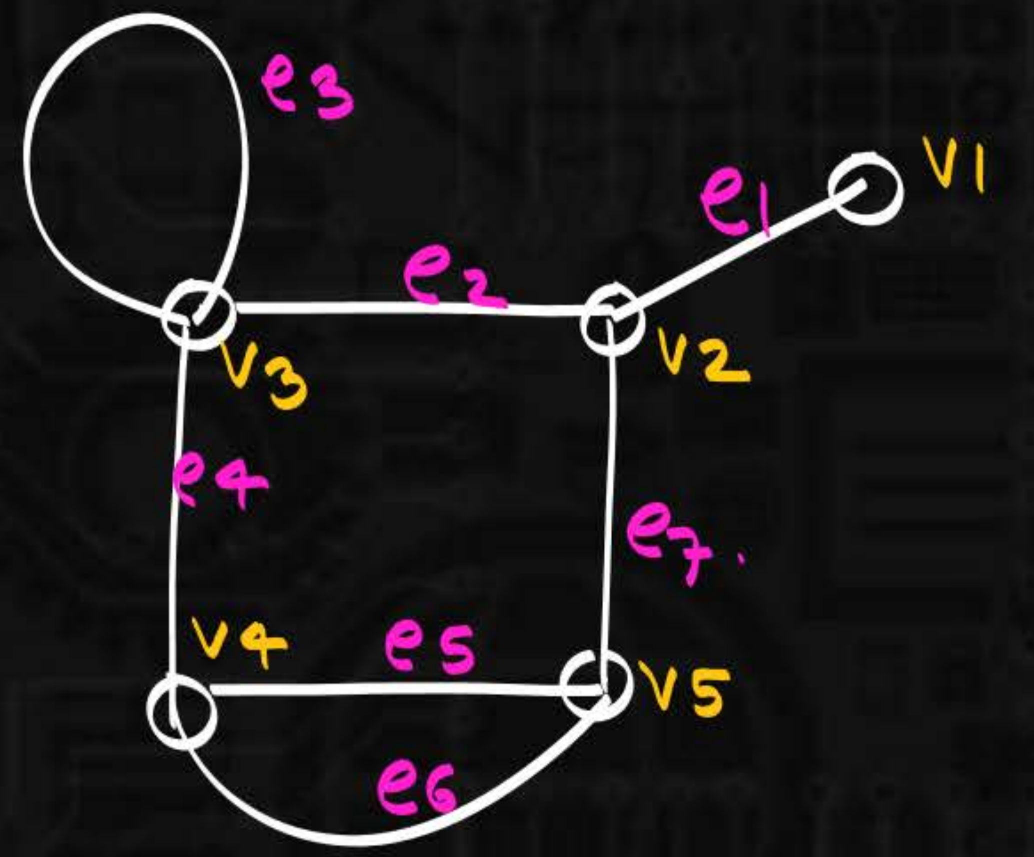
$E = \{ \ldots\ldots \}$

$\psi : E \rightarrow V \times V$

$e_1 \rightarrow (\underline{v_1}, \underline{v_2})$

each edge must be associated order pair of vertices.

Basics of Graph



incident point :

meeting point of vertex & edge.

end vertices :

each edge is associated with unorder pair of vertices called as endvertices.

$e_3 \rightarrow (v_3, v_3)$

loop/self-loop : if endvertices are same, edge is called loop.

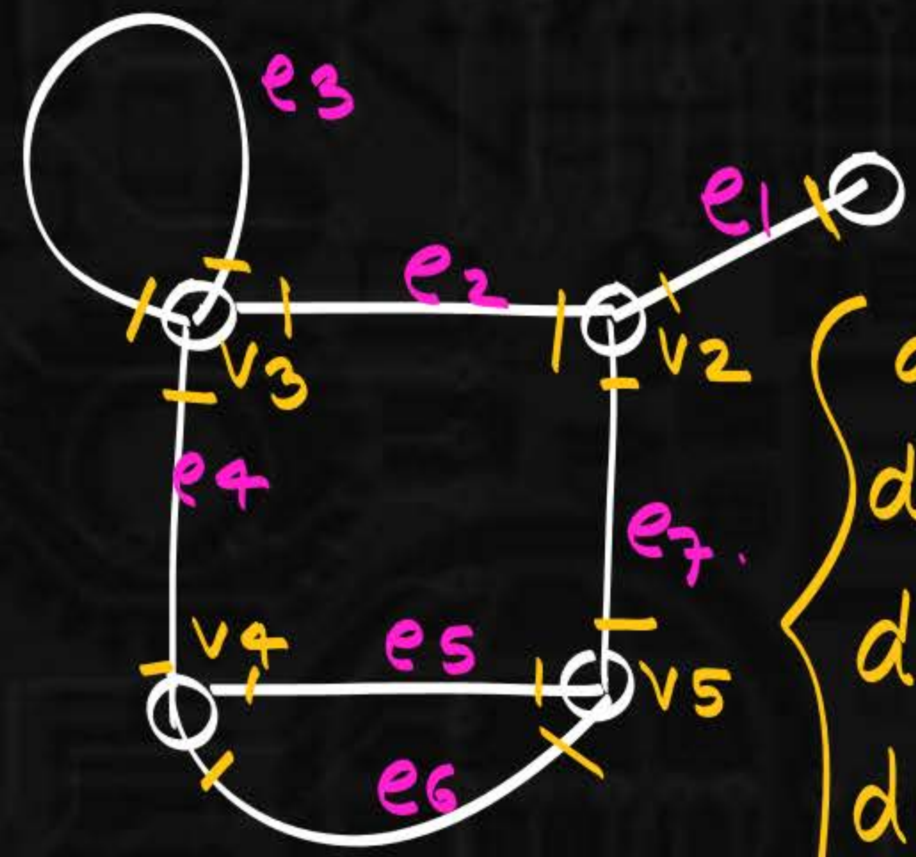
Basics of Graph

|| edges:

2 or more edges associated with sam end vertices

$e5 \rightarrow (v4, v5)$

$e6 \rightarrow (v4, v5)$



$d(v_1) = 1$

$d(v_2) = 3$

$d(v_3) = 4$

$d(v_4) = 3$

$d(v_5) = 3$

Degree/valency $(d(v_i))$

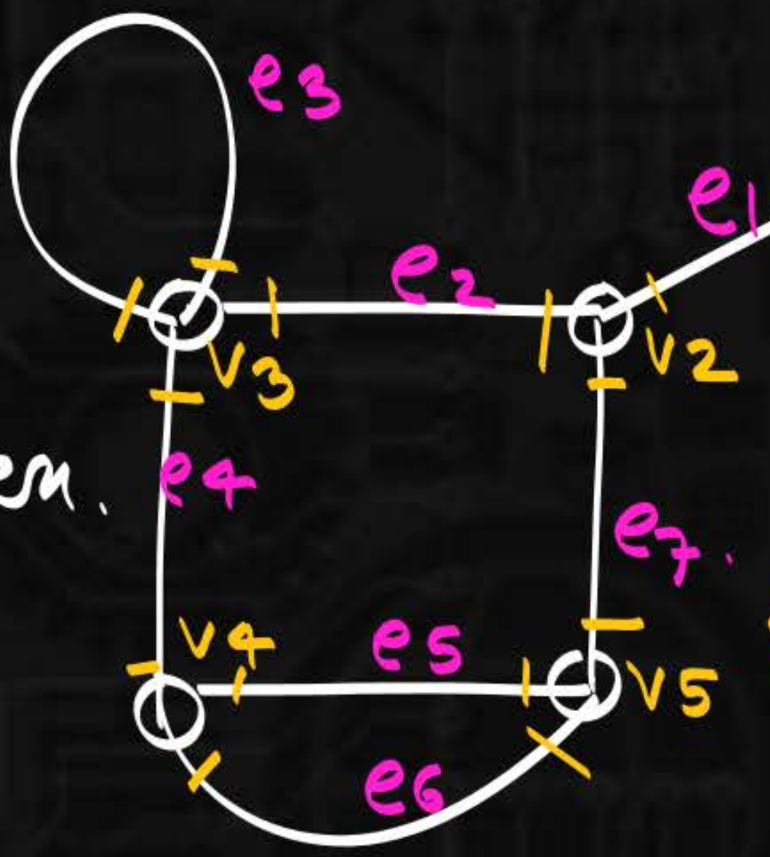
no. of edges incident with vertex.

Basics of Graph

Pendant vertex: $deg(v_1) = 1$.

Degree 1 vertex is called pendant vertex.

eg: v_1.



$$
\begin{cases}
d(v_1) = 1 \\
d(v_2) = 3 \\
d(v_3) = 4 \\
d(v_4) = 3 \\
d(v_5) = 3
\end{cases}
$$

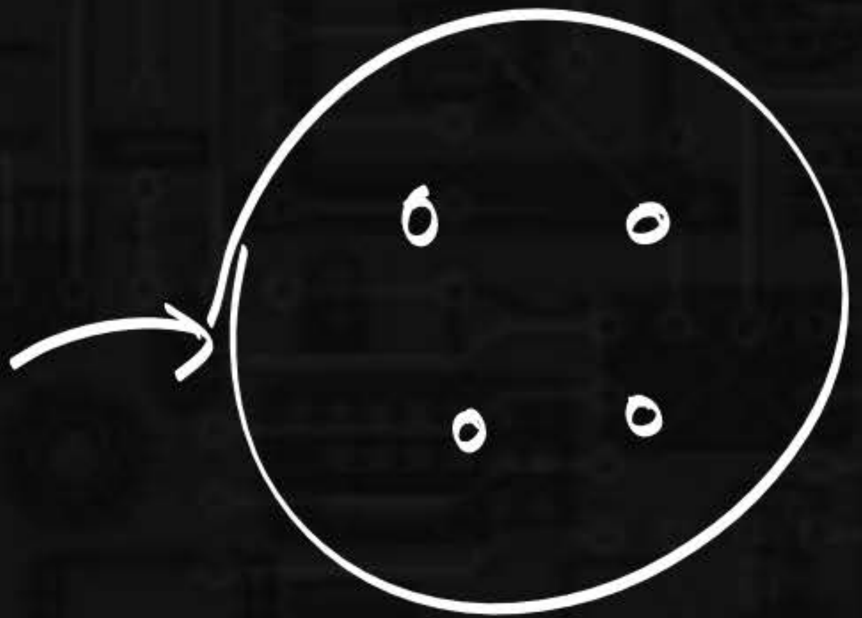
Isolated vertex:

Degree 0 vertex is called isolated vertex.

$deg(v_5) = 0$

Basics of Graph

null graph: set of isolated vertices

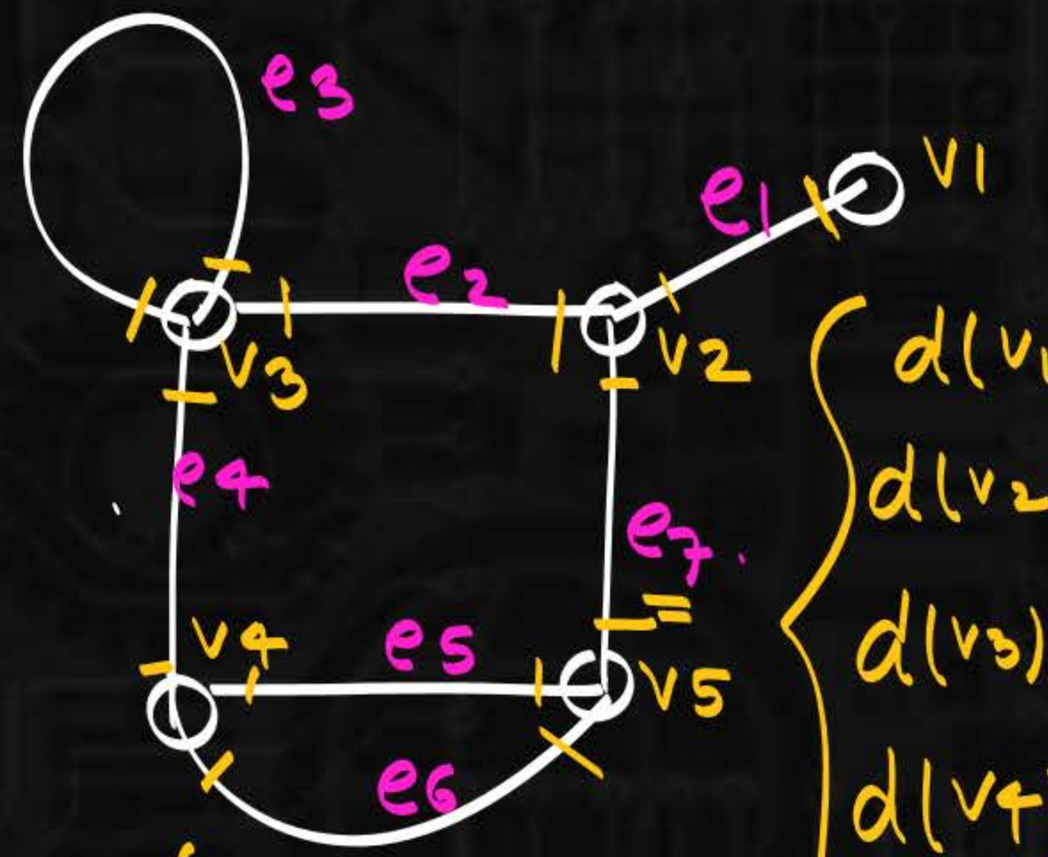


Basics of Graph

$\deg(v_1)+\deg(v_2)+\deg(v_3)+\deg(v_4)+\deg(v_5)$

$= 1+3+4+3+3$

$= 14 = 2\times 7 \rightarrow$ no. of edges.

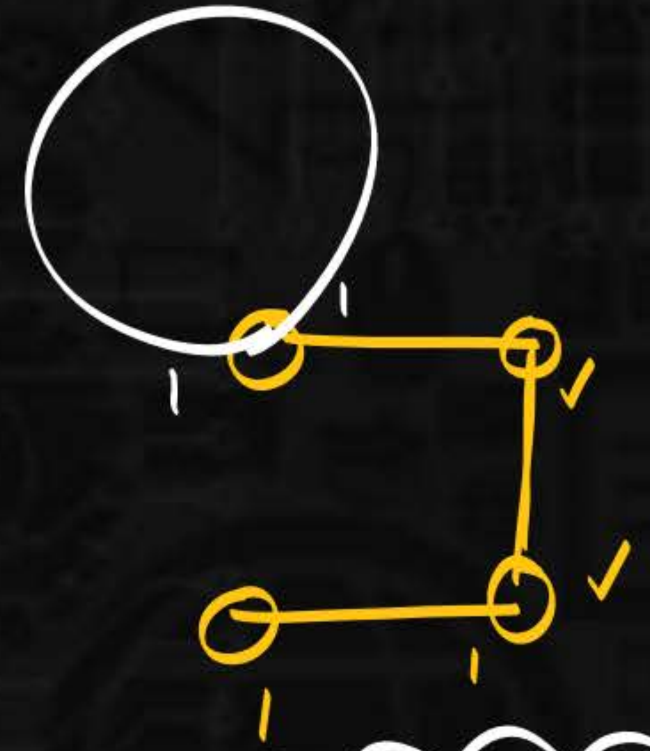


$d(v_1) = 1$
$d(v_2) = 3$
$d(v_3) = 4$
$d(v_4) = 3$
$d(v_5) = 3$

Thm: Sum of degrees of all vertices is equals to twice the no. of edges.

Basics of Graph

$$\boxed{L.H.S = R.H.S.}$$



L.H.S = R.H.S

Degrees = 2 × no of edges.

$$2 = 2(1)$$

$$2+2 = 2(1+1)$$

$$2+2+2 = 2(1+1+1)$$

$$\boxed{\sum d(v_i) = 2e}$$

Basics of Graph

1+3+5

$d(v_1) + d(v_2) + d(v_3) \ldots\ldots = 2E$

$\boxed{o, o, o \mid e + e + e}$ = Even

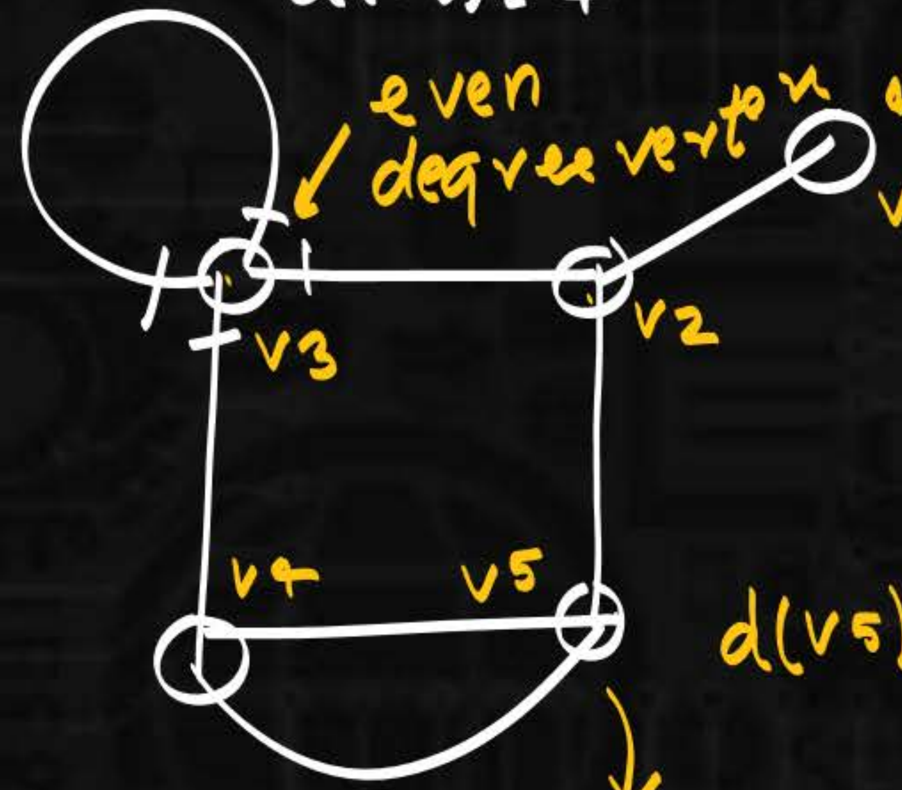
odd + Even = Even

odd = Even

→ sum of degrees of all vertices will be even.

$d(v_3) = 4$

even degree vertex



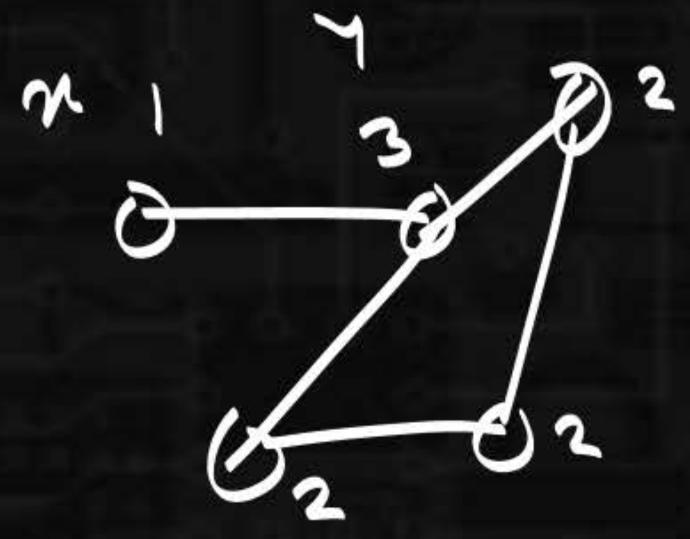
$d(v_5) = 3$

odd degree vertex

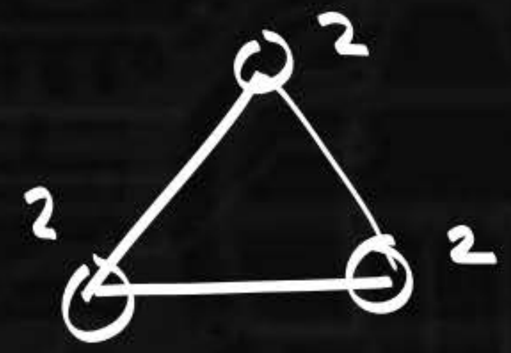
Basics of Graph

Thm2: no. of odd degree vertices in a graph will always be even.

no. of odd degrees vertices = 2.



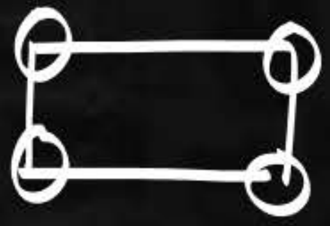
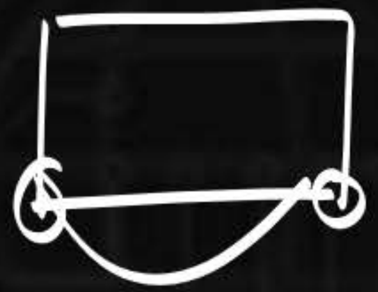
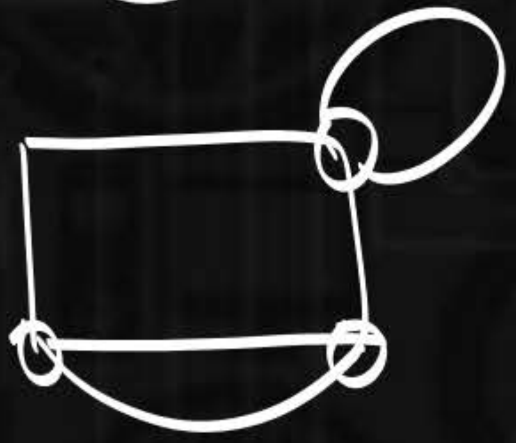
—"— = 2



—"— = 0

↓ even.

Basics of Graph

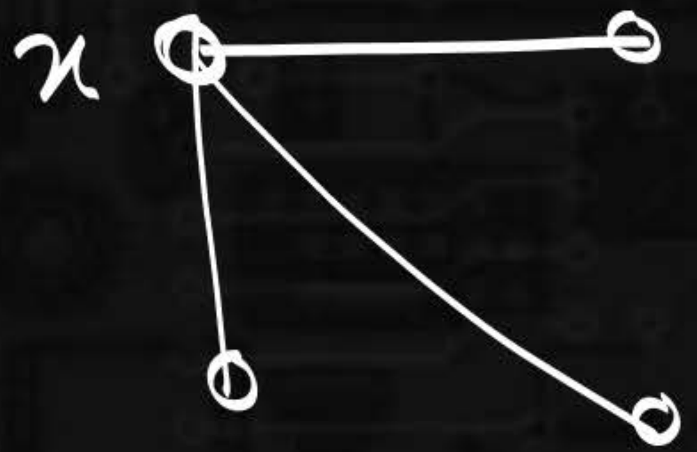
	loop	∥edges	
Simple Graph.	✗	✗	
multigraph.	✗	✓	
Pseudograph.	✓	✓	

Basics of Graph

Thm3 : maximum degree in simple graph $\leq n-1$.

$n =$ Total no. of vertices.

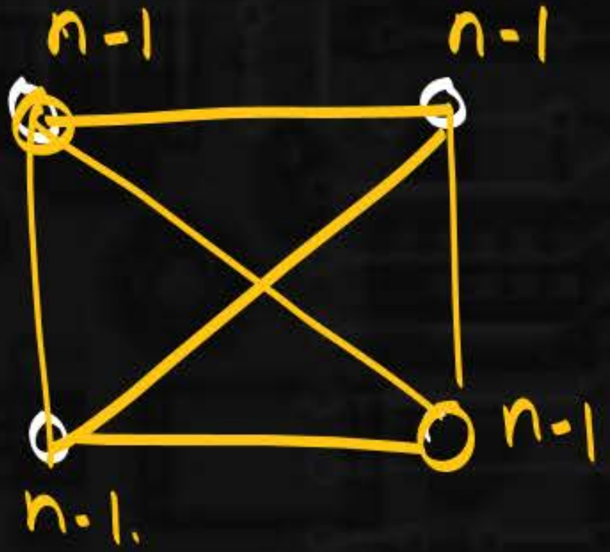
$n = 4$.



Basics of Graph

Thm4: maximum no. of edges in simple Graph $\leq \frac{n(n-1)}{2}$

$n=4$



$\Sigma d(v_i) = 2e$

$n \times n-1 = 2e$

$$\boxed{e = \frac{n(n-1)}{2}}$$

$n = 4$

$e = 0$

$n = 4$

$e = 1$

$n = 4$

$e = 2$

$n = 4$

$$e = \frac{n(n-1)}{2}$$

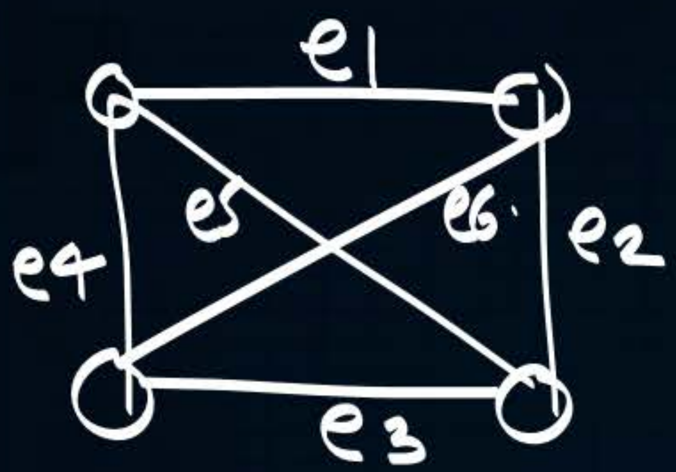
$n = 4$

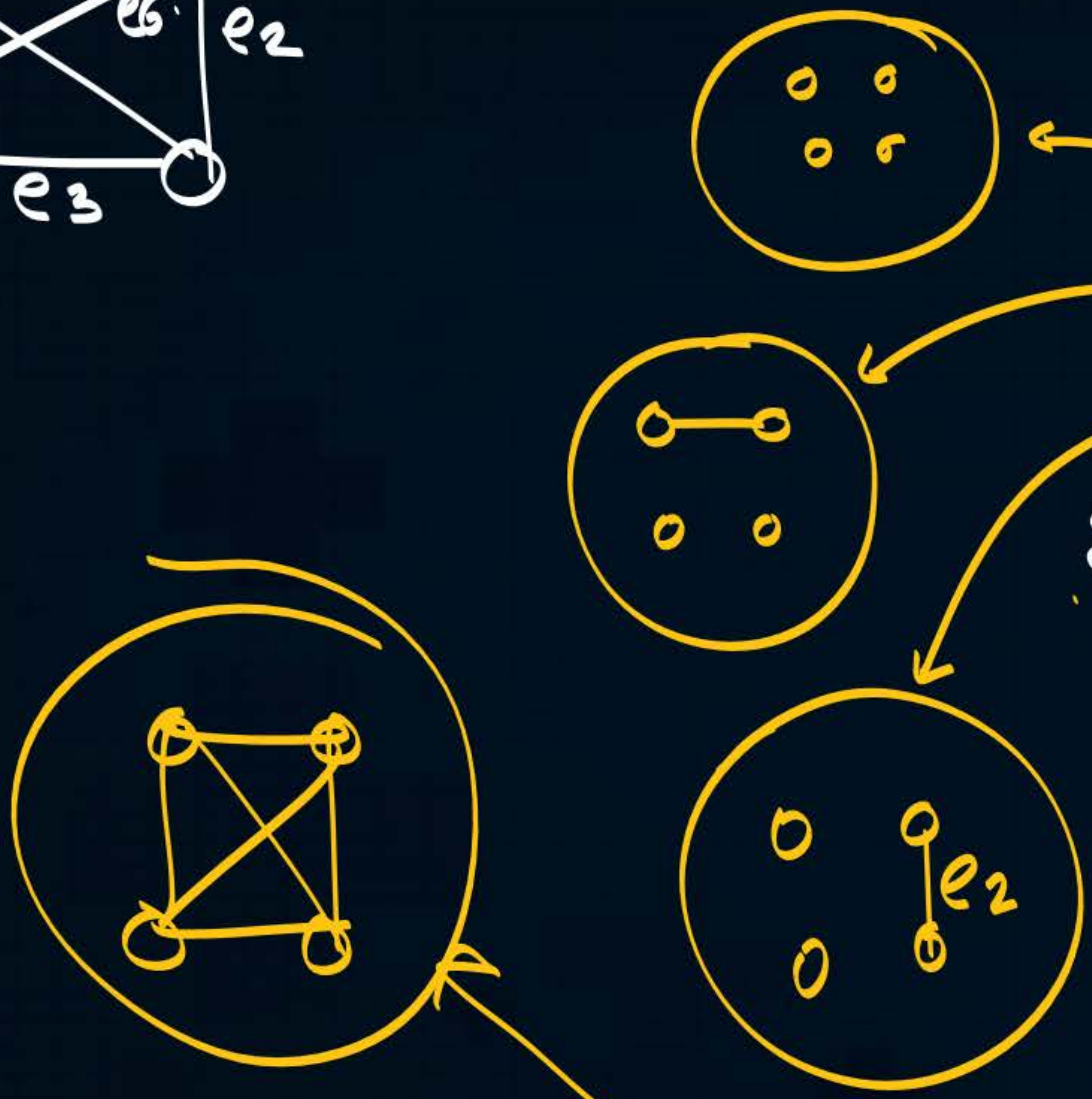
$$e = \frac{4 \cdot 3}{2}$$

$e = 6$

$\rightarrow n-1$

$$\rightarrow \frac{n(n-1)}{2}$$





$$
2^6 \left\{
\begin{array}{cccccc}
e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
 & \vdots & {}^6C_1 & & & \\
1 & 1 & 0 & 0 & 0 & 0 \\
 & & & & & \\
1 & 1 & 1 & 1 & 1 & 1
\end{array}
\right.
$$

$n = 4$.

maximum edges $= \frac{4 \cdot 3}{2} = 6$.

Total no. of Graphs $= 2^6 = 2^{\frac{4 \cdot 3}{2}}$.

n = Total vertices.

Total no. of Graphs $= 2^{\frac{n(n-1)}{2}}$.

Q.2: How many graphs are possible with 4 vertices & 1 edge.

$^{6}C_{1}$.

* How many graphs are possible with n vertices & e edges

$^{\frac{n(n-1)}{2}}C_{e}$.

4 vertices → 1 edge. → $^6C_1$

4 vertices → 2 edges → $^6C_2$.

$^6C_0 + {^6C_1} + \boxed{^6C_2 + {^6C_3} \cdots {^6C_6}} = 2^6$.

↑ Total Graphs.

→ how many graphs are possible with 4 vertices & atleast 2 edges.

m1. → $^6C_2 + {^6C_3} + {^6C_4} + {^6C_5} + {^6C_6}$.

m2 → $2^6 - {^6C_0} - {^6C_1}$.

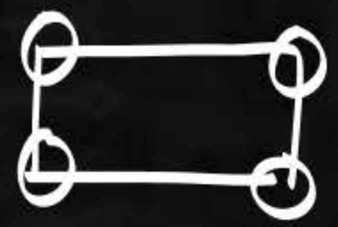
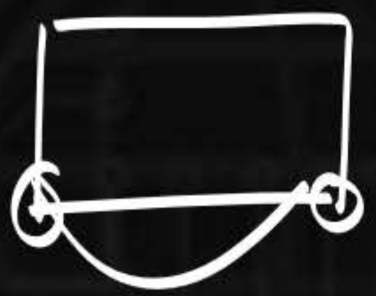
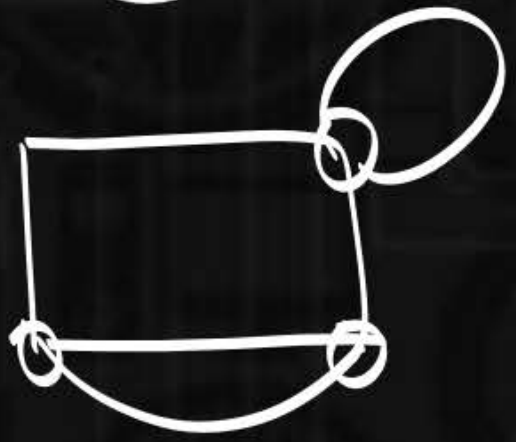
note: if degrees of all vertices are n-1, then it have exactly $\frac{n(n-1)}{2}$ edges.

$G = (V, E)$ $V = 10$

degree of each vertex is $(n-1)$ OR 9.

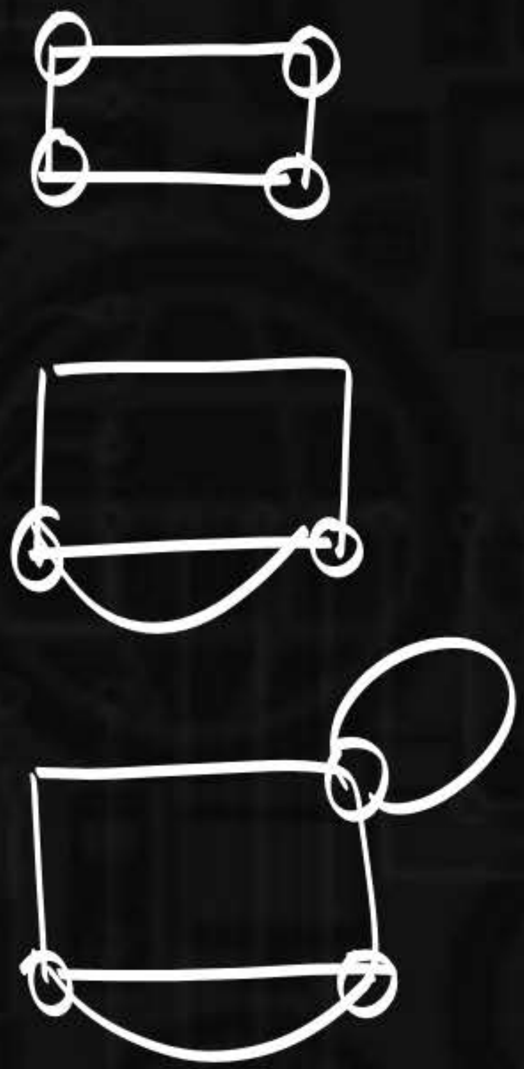
$E = 45 = \frac{n(n-1)}{2} = \frac{10 \cdot 9}{2} = 45.$

Basics of Graph

	loop	∥edges	
Simple Graph.	✗	✗	
multigraph.	✗	✓	
Pseudograph.	✓	✓	

Basics of Graph

	loop	‖edges
Simple Graph.	✗	✗
multigraph.	✗	✓
Pseudograph.	✓	✓

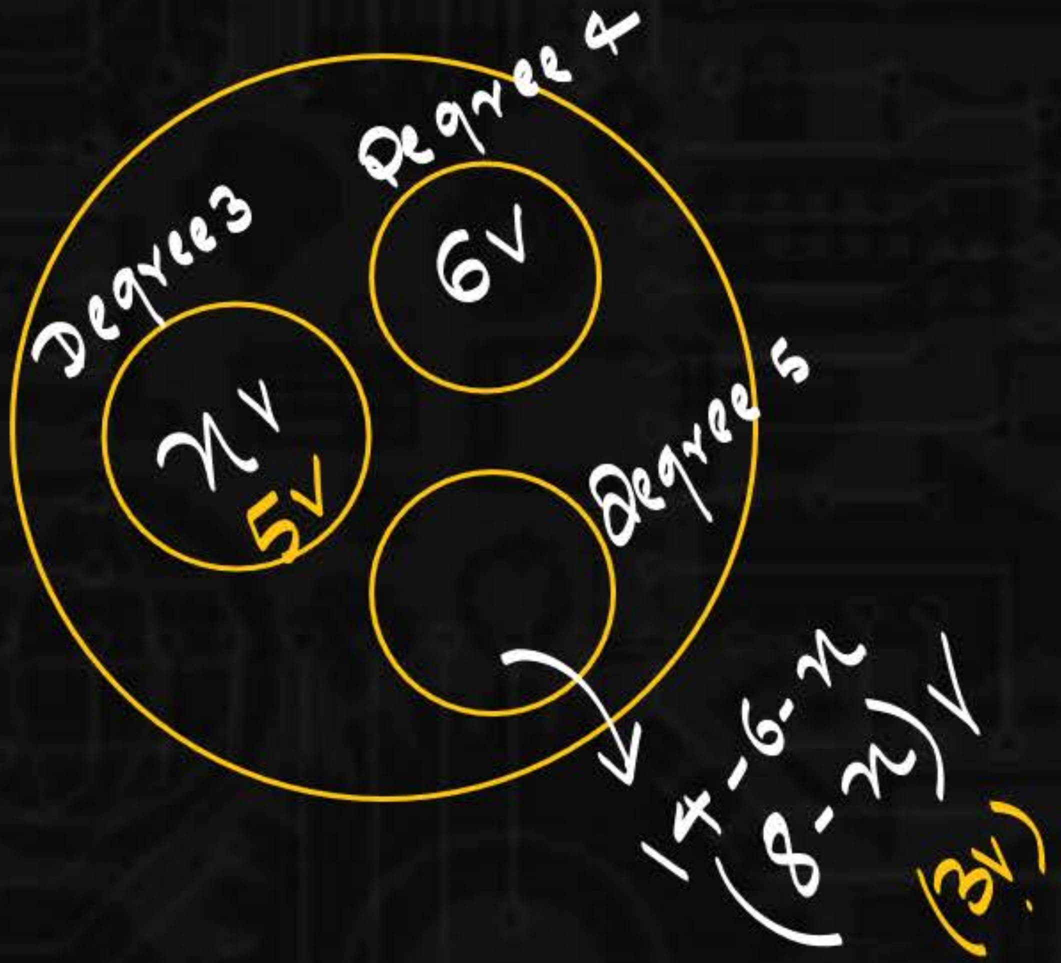


Q.1 A certain graph G has order 14 and size 27. The degree of each vertex of G is 3. 4 or 5. There are six vertices of degree 4. How many vertices of G have degree 3 and how many have degree 5?

$n = 14 \quad E = 27.$

$\Sigma d(v_i) = 2e.$

$6 \times 4 + x \times 3 + (8 - x) \times 5 = 2 \cdot 27.$



$x = 5$


THANK YOU GW SOLDIERS !