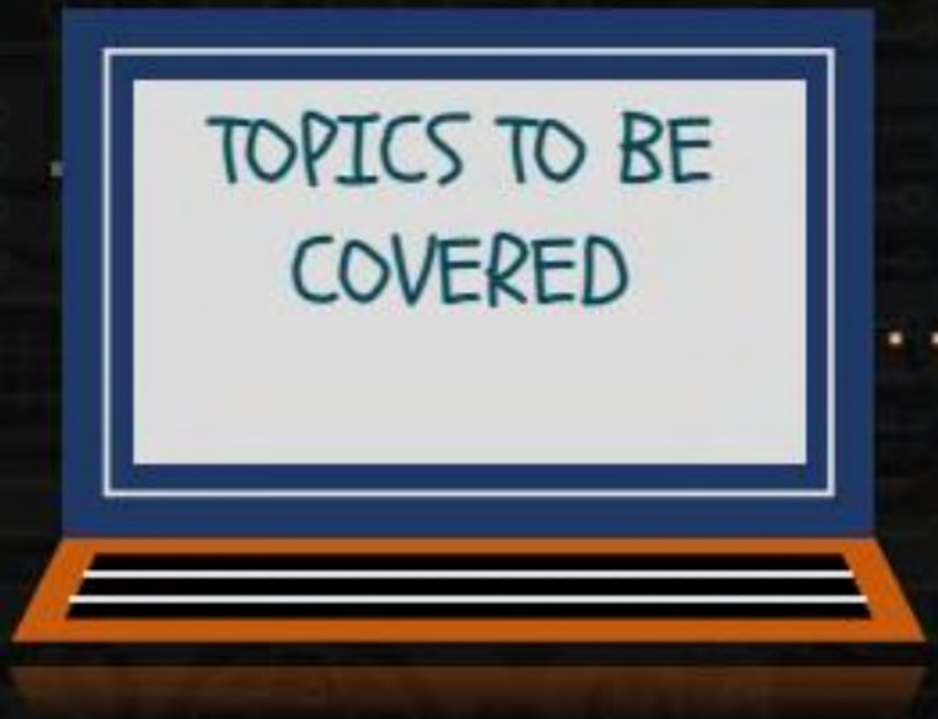
CS & IT ENGINEERING

Discrete Mathematics

Set Theory

DPP 10 Discussion

SATISH YADAV SIR



01 Question

02 Discussion

Q.1 The set of all positive rational numbers forms an abelian group under the composition * defined by a * b = (ab)/2.

Which of the following is/are TRUE? $(Q, *)$ $a*b=\frac{ab}{2}$ [MSQ]

(a, b, c)

A. The identity element is 2 ✓

B. The inverse of a is 4/a. ✓

C. The inverse of 4 is 1 ✓

D. The identity element is 1 ✗

$a * e = a$

$\frac{ae}{2} = a \Rightarrow e = 2.$

$a * a^{-1} = 2$

$\frac{a \cdot a^{-1}}{2} = 2$

$a^{-1} = \frac{4}{a}$

$a = 4$

$a^{-1} = \frac{4}{a}$

$= \frac{4}{4} = 1.$

Q.2 Let r be the set of all real numbers and * is a binary operation defined by

$a * b = a + b + ab$.

Which of the following is TRUE? [MSQ]

A. Identity element is 0. ✓

B. The inverse of –1 is 1. ✗

C. The inverse of a is $-a/(a+1)$. ✓

D. R is not a group. ✗

$a * e = a$

$a + e + ae = a$

$e + ae = 0$

$e(1+a) = 0 \quad e = 0$

$a * a^{-1} = e$

$a + a^{-1} + a \cdot a^{-1} = 0$

$a^{-1} + a \cdot a^{-1} = -a$

$a^{-1}(1+a) = -a$

$a^{-1} = \frac{-a}{1+a} \quad (a \neq -1)$

Q.3 The set G = {0, 1, 2, 3, 4, 5} is a group with respect to addition modulo 6.

[MCQ]

Which of the following is false?

A. The inverse of 2 is 4 ✓ $2^{-1} = 4$

B. The inverse of 3 is 3 ✓

C. The inverse of 5 is 2 ✗

D. The inverse of 1 is 5 ✓

Q.4 G = {1, –1, i, –i}is a group w.r.t multiplication. The order –i is

[NAT]

$(-i)^1 = -i$

$(-i)^2 = -1$

$(-i)^3 = i^2 \times -i$

$= -1 \times -i = i$

$(-i^4) = -i^2 \times -i^2 = -1 \times -1 = 1$

$\langle -i \rangle = \{-i, -1, i, 1\}$

Ans: 4

Q.5 If G is a group of order p, where p is a prime number. Then the number of sub groups of G is____.

[MCQ]

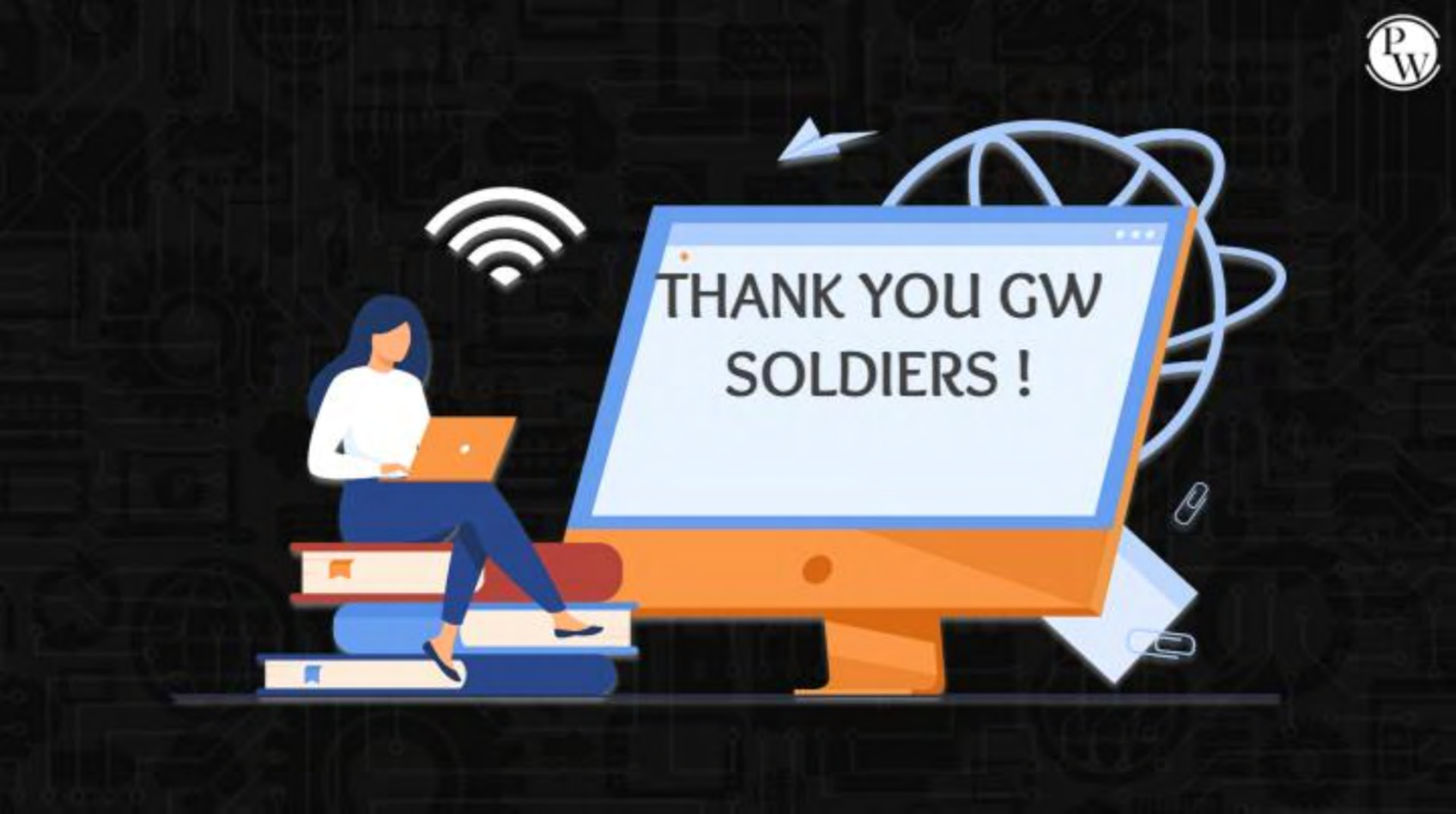
A. 1

B. 2 ✓

C. p – 1

D. p

1
p


THANK YOU GW
SOLDIERS !