

Discrete Mathematics

Set Theory

DPP-07

[MCQ]

1. Which of the following pairs of elements are comparable in the poset $(N, |)$ where N is set of all integers.

- (a) 6, 7 (b) 3, 5
(c) 5, 15 (d) 4, 6

[MCQ]

2. Which of the following pairs of sets are not comparable in the poset $[P(A), \subseteq]$ where

$$A = \{0, 1, 2\}$$

- (a) $\{0\}, \{0, 1\}$ (b) $\phi, \{0, 1, 2\}$
(c) $\{1, 2\}, \{0, 1, 2\}$ (d) $\{0\}, \{1\}$

[NAT]

3. Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, with $A = P(U)$, and let R

be the subset relation on A . For

$$B = \{\{1\}, \{2\}, \{2, 3\}\} \subseteq A, \text{ then the number of upper}$$

bounds that exist for B is X and the number of lower bounds that exist for B is Y , then value of $X + Y$ is?

[MCQ]

4. Let $(A, R_1), (B, R_2)$ be two posets. On $A \times B$, define relation R by $(a, b)R(x, y)$ if $a R_1 x$ and $b R_2 y$.

Consider the following statement:

I. R is a partial order.

II. R is not a partial order.

- (a) Only **I** is true.
(b) Only **II** is true.
(c) Both **I** and **II** are true.
(d) Neither **I** nor **II** is true.

[NAT]

5. Define the relation R on the set Z by $a R b$ if $a - b$ is a nonnegative even integer. Then consider the following statement regarding relation R :

I. R is a partial order.

II. R is a total order.

The number of correct statements for R is/are ?

Answer Key

1. (c)
2. (d)
3. (17)

4. (a)
5. (1)



Hints and Solutions

1. (c)

Two elements a and b are said to be comparable with respect to R , if (aRb) or (bRa) .

\therefore Only 5 and 15 are comparable

2. (d)

The sets $\{0\}$ and $\{1\}$ are not comparable because neither of the two is a subset of the other.

3. (17)

The number of upper bounds that exist for B

$$\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 2^4 = 16$$

$X = 16$

The number of lower bounds that exist for B

One – namely ϕ

$Y = 1$, therefore $X + Y = 16 + 1 = 17$.

4. (a)

For all $a \in A, b \in B, aR_1a$ and bR_2b so

$(a, b)R(a, b)$, and R is reflexive. Next

$(a, b)R(c, d), (c, d)R(a, b) \Rightarrow aR_1c, cR_1a$ and

$bR_2d, dR_2b \Rightarrow a = c, b = d \Rightarrow (a, b) = (c, d)$, so

R is antisymmetric. Finally,

$(a, b)R(c, d), (c, d)R(e, f) \Rightarrow aR_1c, cR_1e$ and

$bR_2d, dR_2f \Rightarrow aR_1e, bR_2f \Rightarrow (a, b)R(e, f)$, and R is transitive. Consequently, R is a partial order.

5. (1)

For each $a \in \mathbb{Z}$ it follows that aRa because $a - a = 0$, an even nonnegative integer. Hence R is reflexive. If $a, b, c \in \mathbb{Z}$ with aRb and bRc then

$$a - b = 2m, \text{ for some } m \in \mathbb{N}$$

$$b - c = 2n, \text{ for some } n \in \mathbb{N},$$

and $a - c = (a - b) + (b - c) = 2(m + n)$, where

$m + n \in \mathbb{N}$. Therefore, aRc and R is transitive.

Finally, suppose that aRb and bRa for some

$a, b \in \mathbb{Z}$. Then $a - b$ and $b - a$ are both

nonnegative integers. Since this can only occur for

$a - b = b - a$, we find that $[aRb \wedge bRa] \Rightarrow a = b$,

so R is antisymmetric.

Consequently, the relation R is a partial order for \mathbb{Z} .

But it is not a total order. For example, $2, 3 \in \mathbb{Z}$ and

we have neither $2R3$ nor $3R2$, because neither -1

nor 1 , respectively, is a non-negative even integer.



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