Branch : CSE & IT Batch : Hinglish

DISCRETE MATHEMATICS

Mathematical Logic

DPP-06

[MCQ]

1. Consider a function, P(x, y, z) = x + y + z = 15 and domain = z, then which of the following is correct?

(a) $\forall x \exists y \exists z$ P(x, y, z) (b) $\exists z \forall x \forall y$ P(x, y, z)

(c) $\forall x \exists z \forall y$ P(x, y, z) (d) $\exists z \exists y \forall x$ P(x, y, z)

[MCQ]

2. Consider an asymmetric function P(x, y) = $x^2 + y^2$ = 10.0 on domain integer, then which of the following is correct?

(a) $\exists x \exists y$ P(x, y) (b) $\forall x \exists y$ P(x, y)

(c) $\forall y \exists x$ P(x, y) (d) None of these

[MSQ]

3. Which of the following is/ are negation of $[\forall x \exists y \forall z(P(x, y, z) \oplus Q(x, y, z))]$

(a) $\exists x \forall y \exists z$ (~P(x, y, z) $\oplus$ ~ Q(x, y, z))

(b) $\exists x \forall y \exists z$ (P(x, y, z) $\Rightarrow$ ~ Q(x, y, z))

(c) $\exists x \forall y \exists z$ (P(x, y, z) $\Leftrightarrow$ Q(x, y, z))

(d) $\exists x \forall y \exists z$ (~ P(x, y, z) $\Leftrightarrow$ ~ Q(x, y, z))

[NAT]

4. Consider the following logical expressions

(a) $\forall x \forall y$ P(x, y) $\leftrightarrow \exists y \forall x$ P(x, y)

(b) $[\forall x$ P(x)] $\vee$ Q $\leftrightarrow \forall x[P(x) \vee Q]$

(c) $\forall x[P(x) \wedge Q] \leftrightarrow [\forall x$ P(x)] $\wedge$ Q

(d) $\exists x[P(x) \vee Q] \leftrightarrow [\exists x$ P(x)] $\wedge$ Q

Total invalid expressions are ___?

[MCQ]

5. Consider the following statements

S_1: There is someone who is loved by everyone.

S_2: Every real number has its corresponding negative.

Here L(x, y) denotes "x loves y"

P(x, y) denotes "x + y = 0"

Which of the following represent the correct predicate logic of the given statement?

(a) S_1:$\exists x \forall y$ L(x, y), S_2: $\exists y \forall x$ p(x, y)

(b) S_1:$\forall x \exists y$ L(x, y), S_2: $\forall x \forall y$ p(x, y)

(c) S_1: $\exists y \forall x$ L(x, y), S_2: $\forall x \exists y$ p(x, y)

(d) None of these.

Answer Key

1. **(a)**
2. **(d)**
3. **(c, d)**
4. **(2)**
5. **(c)**

Hints and Solutions

1. (a)

(a) $\forall x \exists y \exists z\ P(x, y, z)$

$z + y = 15 - x$

15 – integer = integer **True**

(b) $z = 15 - x - y$ **False**

z must be independent, here z depends on x and y.

(c) $z = 15 - x - y$ **False**

z should not depend on y.

(d) $y + z = 15 - x$ **False**

the value of (y + z) is depending on x, (y + z) must be independent, so this expression is also **False.**

2. (d)

(a) $\forall x \exists y\ P(x, y)$ **False**

$x^2 + y^2 = 10.0$

F(1, 3) = 1 + 9 = 10

Here, 10 is integer but output must be 10.0, it will never come because 10.0 is not an integer.

(b) $\forall x \exists y\ P(x, y)$ **False**

10.0 will never come.

(c) $\forall y \exists x\ P(x, y)$ **False**

Hence, option (d) is correct

3. (c, d)

Negation of XOR operator is biconditional.

p	q	$p \oplus q$	$p \leftrightarrow q$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

(c) $\sim [\forall x \exists y \forall z (P(x, y, z) \oplus Q(x, y, z))]$

$[\exists x \forall y \exists z \sim (P(x, y, z) \oplus Q(x, y, z))]$

$[\exists x \forall y \exists z\ (P(x, y, z) \Leftrightarrow Q(x, y, z))]$ True

(d) Property:

$P \leftrightarrow Q \equiv \sim P \leftrightarrow \sim Q$

$P'Q' + PQ \equiv P'Q' + PQ$

$\sim [\forall x \exists y \forall z (P(x, y, z) \oplus Q(x, y, z))]$

$[\exists x \forall y \exists z\ (\sim P(x, y, z) \Leftrightarrow \sim Q(x, y, z))]$ True

Hence, option (c, d) are correct.

4. (2)

(a): Invalid

$\forall x \forall y P(x, y) \rightarrow \exists y \forall x\ P(x, y)$ (One way true)

$\forall y \forall x P(x, y) \rightarrow \exists y \forall x\ P(x, y)$

(b): $[\forall x P(x)] \vee Q \leftrightarrow [\forall x P(x) \vee Q]$

$(P_1 \wedge P_2) + Q \equiv (P_1 \vee Q) \wedge (P_2 \vee Q)$

$P_1P_2 + Q \equiv P_1P_2 + P_1Q + P_2Q + Q$

$P_1P_2 + Q \equiv P_1P_2 + Q$ (valid)

(c): $\forall x[P(x) \wedge Q] \leftrightarrow [\forall x\ P(x)] \wedge Q$

$(P_1 \wedge Q) \wedge (P_2 \wedge Q) \equiv (P_1 \wedge P_2) \wedge Q$

$P_1QP_2\ Q \equiv P_1\ P_2\ Q$

$P_1\ P_2Q \equiv P_1\ P_2Q$

Valid

(d): $\exists x(P(x) \vee Q) \leftrightarrow [\exists x\ P(x)] \wedge Q$

$(P_1 \vee Q) \vee (P_2 \vee Q) \equiv (P_1 \vee P_2) \wedge Q$

$P_1 + Q + P_2 + Q \equiv (P_1 + P_2)\ Q$

$P_1 + P_2 + Q \not\equiv (P_1 + P_2)\ Q$ Invalid

Total 2 expressions are invalid

5. (c)

Statement S_1: There is someone who is loved by everyone.

- Assume, variables x and y denote people
- A predicate L(x, y): denotes "x loves y"

$\therefore \exists y \forall x\ L(x, y)$ there is someone who is loved by everyone.

Statement S_2: Every real number has its corresponding negative.

- Assume, a real number is denoted as x and its negative as y.
- A predicate p(x, y) denotes "x + y = 0"

$\therefore\ \forall x \exists y\ p(x, y)$

Hence, option c is correct answer.