Branch : CSE/IT Batch : Hinglish

Discrete Mathematics
Graph Theory

DPP-06

[NAT]

1. Let G be a simple graph with 15 edges and $\overline{G}$ be a complement graph of G has 21 edges, then the number of vertices in graph G is ___.

[MSQ]

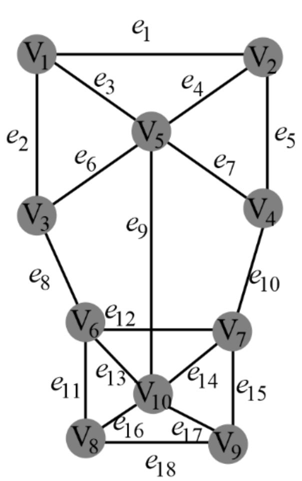
2. Which of the following is true for a graph with vertex connectivity is 3 and edge connectivity is 4?
(a) Removal of any 3 vertes can disconnect the graph.
(b) Removal of any 4 edges can disconnect the graph.
(c) Removal of some 3 vertices will increase the number of connected components.
(d) Removal of some 4 edges will increase the number of connected componets.

[MCQ]

3. What is the maximum value of vertex connectivity and edge connectivity possible with a graph of order 10 and size 16?
(a) $1 \leq$ VC, EC ≤ 3
(b) $1 \leq$ VC, EC ≤ 4
(c) $0 \leq$ VC, EC ≤ 3
(d) $0 \leq$ VC, EC ≤ 4

[MSQ]

4. Consider the given connected graph G



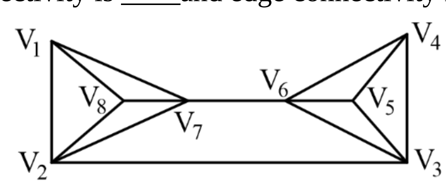
Which of the following is not the cut set?
(a) $\{e_6, e_7, e_9\}$
(b) $\{e_8, e_9, e_{10}, e_{12}\}$
(c) $\{e_8, e_9, e_{10}\}$
(d) $\{e_1, e_2, e_3\}$

[MCQ]

5. Consider the following statements:
S_1: The vertex connectivity of the graph is 1 if and only if graph has cut vertex.
S_2: The edge connectivity of the graph is 1 if and only if graph has cut edge.
Which of the following statements is true?
(a) S_1 only
(b) S_2 only
(c) Both S_1 and S_2
(d) Neither S_1 nor S_2

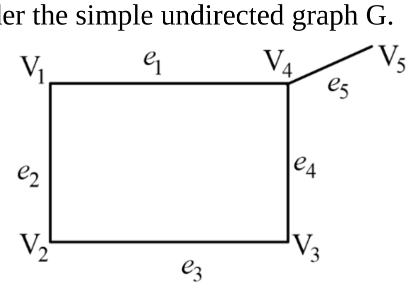
[NAT]

6. For the graph below, vertex.
Connectivity is ____and edge connectivity is ____.



[NAT]

7. Consider the simple undirected graph G.



Find the number of cut set for the above graph G?

Answer Key

1. **(9)**
2. **(c, d)**
3. **(a)**
4. **(a, b)**
5. **(c)**
6. **(2, 2)**
7. **(7)**

Hints and Solutions

1. (9)

As we know that the maximum number of edges in graph with n vertices is $n_{c_2} = \frac{n(n-1)}{2}$.

Now, the graph G and its complement graph together form a complete graph

So, Complete graph with 9 vertices = 15 + 21

$\Rightarrow n_{c_2} = 36$

$\Rightarrow n_{c_2} = \frac{n(n-1)}{2}$

$\Rightarrow n^2 - n = 72$

$\Rightarrow n^2 - n - 72 = 0$

$\Rightarrow n(n-9) + 8(n-9) = 0$

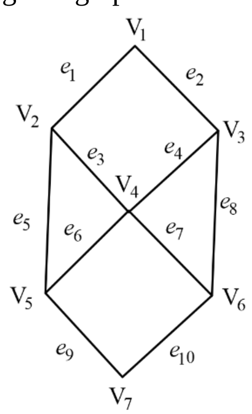
$\Rightarrow (n+8)(n-9) = 0$

$\therefore$ n = 9

Hence, the graph G have 9 vertices.

2. (c, d)

Consider the given graph below:



Now, the vertex connectivity and edge connectivity for the above graph is 2.

Option a: False

Removal of vertices $\{V_1, V_2\}$ would not disconnected the graph. Hence, removal of any 2 vertices for the above graph will not disconnected the graph. Thus, option a is False.

Option b: False

Removal of edges $\{e_6, e_7\}$ would not disconnect the graph.

Option c: True

As removal of some vertices $\{V_5, V_6\}$ will disconnect the graph. Thus, it increases the number of components.

Option d: True

As removal of some edges $\{e_9, e_{10}\}$ will disconnect the graph.

3. (a)

Here the number of vertices is given 10 and edges are 16.

Now, first find the minimum degree for the above graph:

$\delta(G) \le \left\lfloor \frac{2|E|}{n} \right\rfloor$

$\delta(G) \le \left\lfloor \frac{2*16}{10} \right\rfloor$

$\delta(G) \le \lfloor 3.2 \rfloor$

$\therefore$ Minimum degree $\delta(G)$ will be 3.

Now, the relation between the VC, EC and minimum degree

$VC \le EC \le \delta(G)$

$\therefore$ $VC \le EC \le 3$.

Hence, the possible value of VC and EC will be

$1 \le VC, EC \le 3$.

4. (a, b)

I. A cut set is a minimal set of edges whose removal disconnect the graph.

II. No proper subset of cut set should be able to disconnect the graph.

Now, let's understand the options:

Option A: Correct, as removal of edges $\{e_6, e_7, e_9\}$ will not disconnect the graph. It is not a cut set.

Option B: Correct, as the proper subset of edges $\{e_8, e_9, e_{10}\}$ will disconnect the graph. Hence, it is also not a cut set. So option c and d are the minimal set of edges whose removal disconnect the graph.

5. (c)

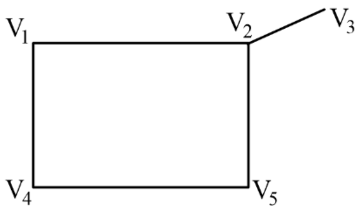
Statement S_1: True

The vertex connectivity of the graph is 1 mean we have a vertex whose removal disconnect the graph.

Statement S_2: True

The edge connectivity of the graph is 1 mean a pendant vertex (degree 1) is connect to graph with 1 edge.

Example:



The VC is 1: Removal of V_2 will disconnect the graph.

The EC is also 1: Removal of edge ($V_2 - V_3$) will disconnect.

6. (2, 2)

I. First find the minimum degree of the given graph $\delta(G)$.

So, $\delta(G) = 3$.

II. Now, the relation between VC, EC and S(G) is as follows:

$$VC \le EC \le S(G)$$

$$\therefore \quad VC \le EC \le 3$$

From the above relation, we can conclude that the VC and EC would be at most 3.

III. Now, if we delete the vertices either $\{V_6, V_3\}$ or $\{V_7, V_2\}$. It will disconnect the graph as well as if we delete the edges $\{(V_7 - V_6), (V_2 - V_3)\}$ it will also disconnect the graph .

Hence, the VC and EC for the given graph is 2.

7. (7)

I. The vertex V_5 is pendant vertex. Hence, e_5 will be one of the cut set.

II. Now, in the above given we have a cycle of length '4': $\{ V_1 - V_2 - V_3 - V_4 - V_1\}$

So, if we select any 2 edges from the cycle, it will disconnect the graph.

So, the number of cut set from the cycle

$$= 4C_2 = \frac{4 \times 3}{2} = 6$$

Hence, the total number of cut set is 6 + 1 = 7.

(I)	{e5}	(II)	{e1, e2}
(III)	{e2, e3}	(IV)	{e3, e4}
(V)	{e4, e1}	(VI)	{e2, e4}
(VII)	{e1, e3}		