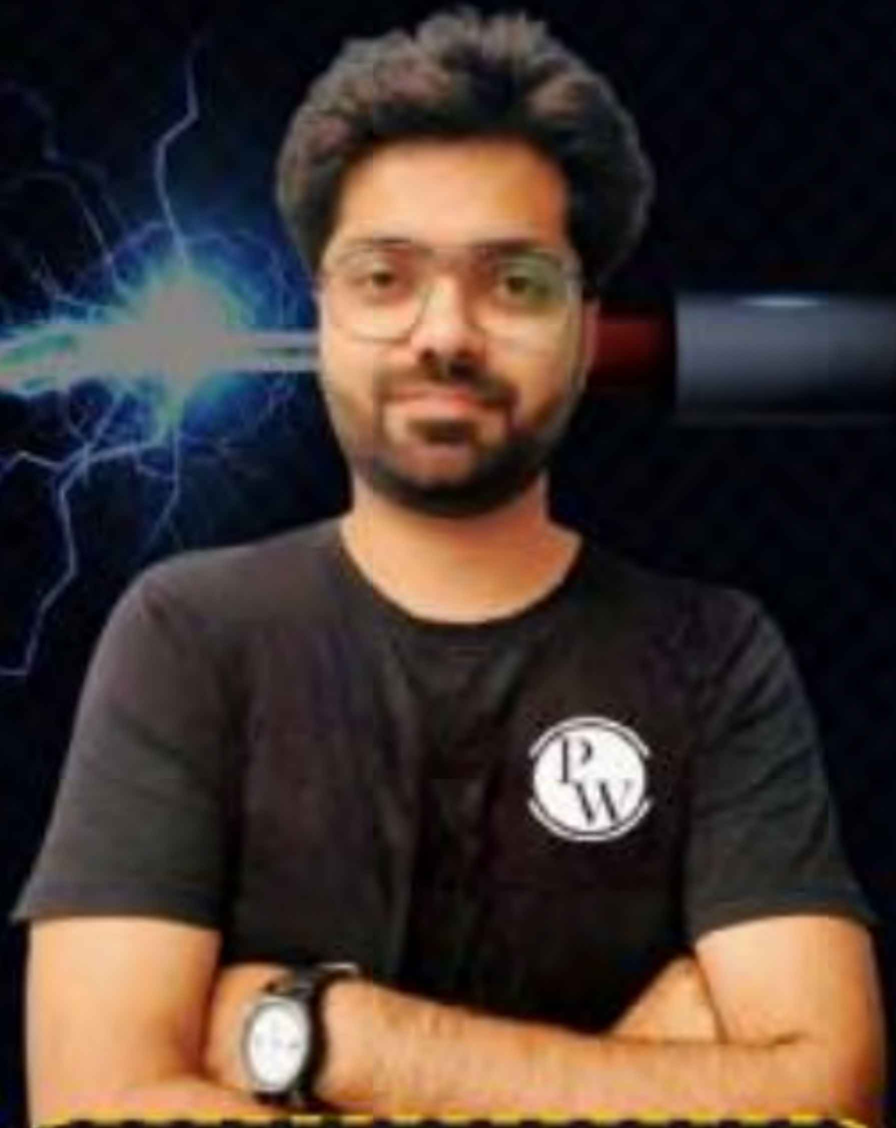
ENGINEERING MATHEMATICS

ALL BRANCHES



Determinant & its Properties

DPP-02 Solution

By- CHETAN SIR

Question 1

If A and B are square matrices of size $n \times n$, then which of the following statement is not true.

$A_{n \times n}$ $B_{n \times n}$

A $\det(AB) = \det(A)\det(B)$

$|AB| = |A||B|$

B $\det(kA) = k^n \det(A)$

$|KA| = K^n |A|$

C $\det(A + B) = \det(A) + \det(B)$

$|A^T| = |A| = \frac{1}{|A^{-1}|}$

D $\det(A^T) = 1/\det(A^{-1})$

$|A^{-1}| = \frac{1}{|A|}$

Question 2

If the determinant of matrix $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -6 \\ 2 & 7 & 8 \end{bmatrix}$ is 26, then the determinant of the matrix $\begin{bmatrix} 2 & 7 & 8 \\ 0 & 5 & -6 \\ 1 & 3 & 2 \end{bmatrix}$ is

$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -6 \\ 2 & 7 & 8 \end{bmatrix}$ $\begin{bmatrix} 2 & 7 & 8 \\ 0 & 5 & -6 \\ 1 & 3 & 2 \end{bmatrix}$

$\Delta = 26$ $\Delta = -26$

A – 26

B 26

C 0

D 52

Question 3

The determinant of the matrix $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}_{4\times4}$ is

Diag.
Scalar
unit
UTM
LTM

$\Delta = a_{11}.a_{22}.a_{33}.a_{44} = 6\times2\times4\times-1 = -48$

A 11

B -48

C 0

D -24

Question 4

If A $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ then $[AA^T]^{-1}$

A $\begin{bmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$

B $\begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$

Continue...

C $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D $\begin{bmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/4 \end{bmatrix}$

$$AA^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$ (Diagonal matrix)

NOTE:- Inverse of diagonal matrix is obtained by reciprocal of diagonal elements.

$$[AA^T]^{-1} = \begin{bmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Matrix A is not orthogonal $\quad AA^T \neq I$

Question 5

Consider the matrices $\boxed{X_{(4\times 3)}}$, $\boxed{Y_{(4\times 3)}}$ and $\boxed{P_{(2\times 3)}}$. The order of $[P(X^TY)^{-1}P^T]^T$ will be

A (2×2) ✓

B (3×3)

C (4×3)

D (3×4)

Order of $X^T \rightarrow 3\times 4$

Order of $X^TY \rightarrow 3\times 3$

Order of $(X^TY)^{-1} \rightarrow 3\times 3$

Order of $P(X^TY)^{-1} \rightarrow 2\times 3$

Order of $P(X^TY^{-1})P^T \rightarrow 2\times 2$

Order of $[P(X^TY^{-1})P^T]^T \rightarrow 2\times 2$

Question 6

For the given orthogonal matrix Q.

$$Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$$

Find Q^{-1}

A

$$Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$$

B

$$Q = \begin{bmatrix} -3/7 & -2/7 & -6/7 \\ 6/7 & -3/7 & -2/7 \\ -2/7 & -6/7 & 3/7 \end{bmatrix}$$

Continue...

C $Q=\begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$

D $Q=\begin{bmatrix} -3/7 & -6/7 & -2/7 \\ -2/7 & -3/7 & -6/7 \\ -6/7 & -2/7 & 3/7 \end{bmatrix}$

$QQ^T = I$ $\because Q$ is orthogonal

$Q^{-1}QQ^T = Q^{-1}I$ [Pre multiply by Q^{-1}]

$IQ^T = Q^{-1}$

$Q^{-1} = Q^T = \begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$

Question 7

Which one of the following does NOT equal

$$\begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}?$$

A $\begin{bmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{bmatrix}$

B $\begin{bmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{bmatrix}$

Continue...

C $\begin{bmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{bmatrix}$

D $\begin{bmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{bmatrix}$

Let us take option a)

$$\begin{vmatrix} 1 & x^2+x & x+1 \\ 1 & y^2+y & y+1 \\ 1 & z^2+z & z+1 \end{vmatrix} = \begin{vmatrix} 1 & x^2+x & x \\ 1 & y^2+y & y \\ 1 & z^2+z & z \end{vmatrix} + \begin{vmatrix} 1 & x^2+x & 1 \\ 1 & y^2+y & 1 \\ 1 & z^2+z & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} + \begin{vmatrix} 1 & x & x \\ 1 & y & y \\ 1 & z & z \end{vmatrix} + 0 = -\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + 0$$

$$\neq \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

Question 8

If any two columns of determinant $\begin{bmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{bmatrix}$ are interchanged,

$\Delta \rightarrow -\Delta$

which one of the statement is correct?

Absolute value Δ Δ

Value Δ $-\Delta$

A. Absolute value remains unchanged but sign will change.

B. Both absolute value & sign will change.

C. Absolute value will change but sign will not change.

D. Both absolute value and sign will remain unchanged.

Question 9

For a matrix $M = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by

A $-\frac{4}{5}$

B $-\frac{3}{5}$

C $\frac{3}{5}$

D $\frac{4}{5}$

$$M = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix} \qquad M^T = \begin{bmatrix} 3/5 & x \\ 4/5 & 3/5 \end{bmatrix}$$

$$M^{-1} = \frac{1}{|M|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{9/25 - 4x/5}\begin{bmatrix} 3/5 & -4/5 \\ -x & 3/5 \end{bmatrix}$$

$$M^T = M^{-1}$$

$$\begin{bmatrix} 3/5 & x \\ 4/5 & 3/5 \end{bmatrix} = \frac{1}{\frac{9-20x}{25}}\begin{bmatrix} 3/5 & -4/5 \\ -x & 3/5 \end{bmatrix}$$

On comparing

$$\therefore \frac{9-20x}{25} = 1$$

$$x = -\frac{16}{20} = -\frac{4}{5}$$

Question 10

Let $M^4 = I$, (where I denotes the identity matrix) and $M \neq I$ and $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals:

A M^{4k+1}

B M^{4k+2}

C M^{4k+3} $= M^{-1}$

D M^{4k}

$$M^4 = I$$

$$\Rightarrow M \cdot M^3 = I \quad \ldots (1)$$

We Know, $MM^{-1} = I \quad \ldots (2)$

From (1) and (2)

$$M^{-1} = M^3$$

$$M^4 M^{-1} = M^4 M^3$$

$$\therefore M^{4K+3} = [M^4]^K \cdot M^3 = I M^{-1} = M^{-1}$$

Thank you

GW Soldiers !