CS & IT ENGINEERING

Data Structure

Arrays

Chapter -2

Lec- 03

By- Pankaj Sharma sir

TOPICS TO BE COVERED

Arrays-3

Column-Major Order

—×—

$int \; a[2][3];$

0 1 0 1 2

$$\begin{array}{c c} & \begin{array}{c c c} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ \\ 1 \end{array} & \left[\begin{array}{c c c} a_{00} & a_{01} & a_{02} \\ \\ a_{10} & a_{11} & a_{12} \end{array} \right] \end{array}$$

| a_{00} | a_{10} | a_{01} | a_{11} | a_{02} | a_{12} |
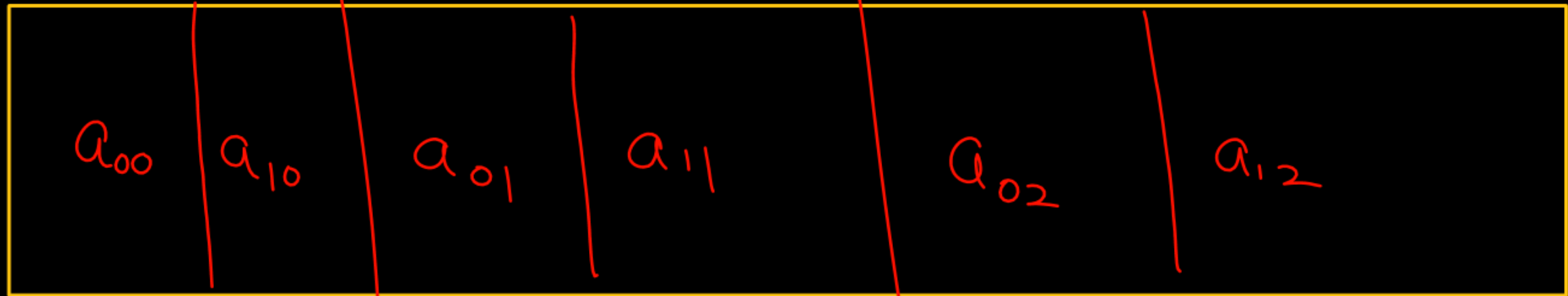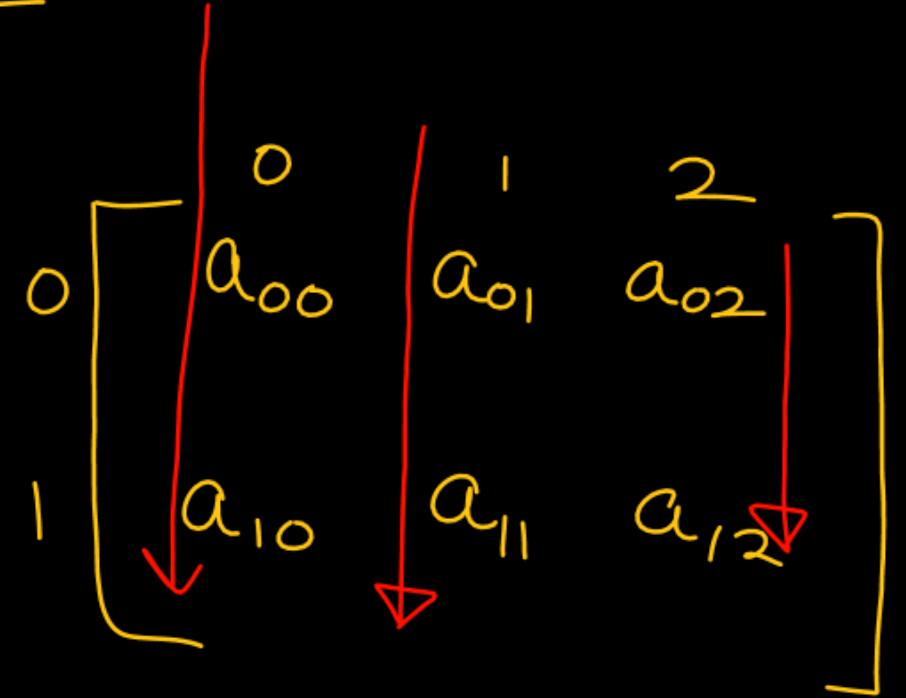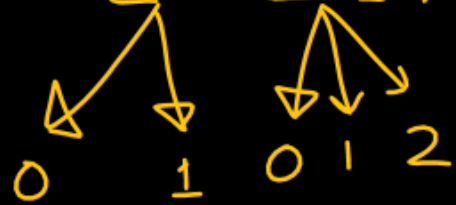
int a[3][4]

0 1 2 0 1 2 3

add(a_{23})

0 to 1
= 1-0+1 = 2

Elem. already filled before a_{23}

within col with index 3

How many col. already
filled before col. with index = 3

0 to 2 = 2-0+1 = 3col.

index

        0   1   2   3
→ 0 [a_{00} a_{01} a_{02} a_{03}]
  1 [a_{10} a_{11} a_{12} a_{13}]
  2 [a_{20} a_{21} a_{22} a_{23}]

3 col. already filled

←col index=0→ | ←col index=1→ | ←col index=2→ | ←col index=3→

| a_{00} | a_{10} | a_{20} | a_{01} | a_{11} | a_{21} | a_{02} | a_{12} | a_{22} | a_{03} | a_{13} | a_{23} |

1000   1004
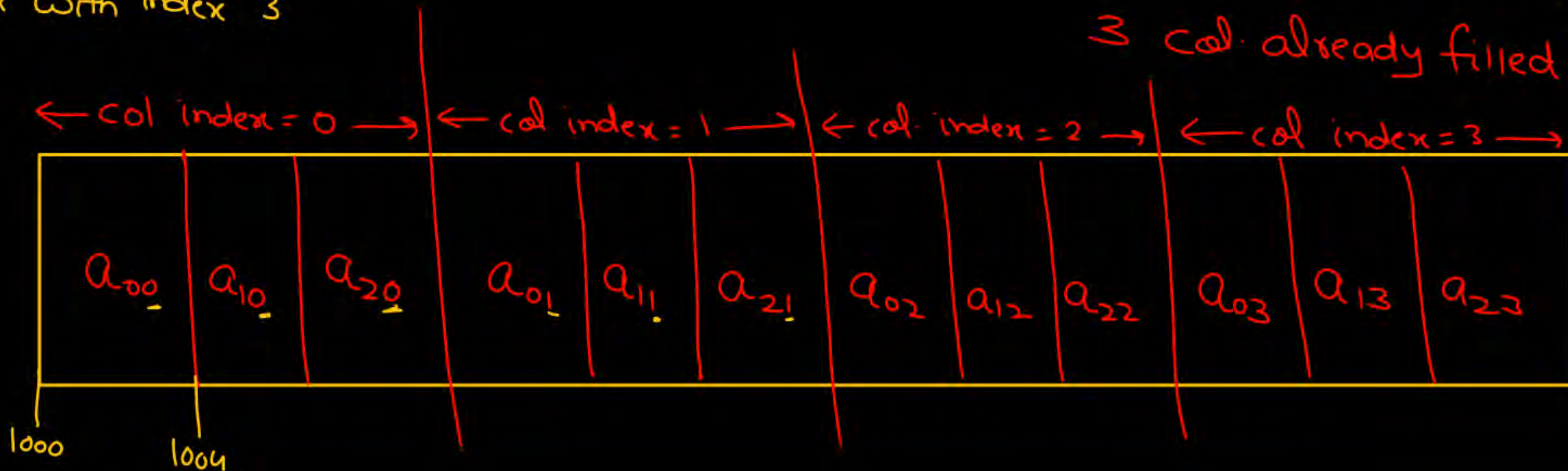
$$\text{int } a[3][4] \rightarrow \text{Every index/No rep} = 3 \text{ ele}$$

index

0 1 2 0 1 2 3

a_{23}

cal with index 3
में a_{23} से पहले
कितने ele fill

= row index 0 to 1
= 1−0+1 = 2 ele

Col जिसका index 3
है, उससे पहले कितने
Col fill = 0 to 2
= 2−0+1 = 3 col

row index → 0

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \end{bmatrix}$$

0 1 2 3

row index → 1

2

3 Col already filled

← col index = 0 → | ← col index = 1 → | ← col index = 2 → | ← col index = 3 →

| a_{00} | a_{10} | a_{20} | a_{01} | a_{11} | a_{21} | a_{02} | a_{12} | a_{22} | a_{03} | a_{13} | a_{23} |

1000   1004
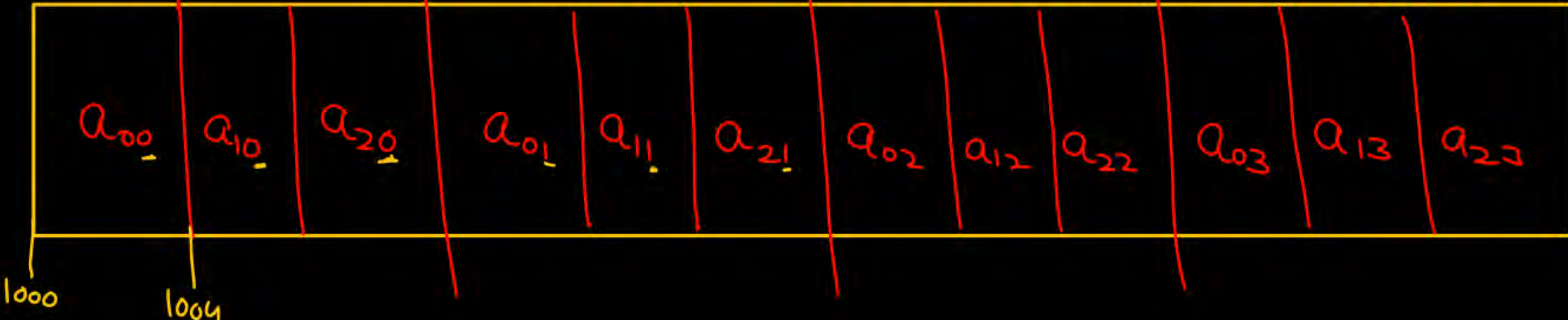
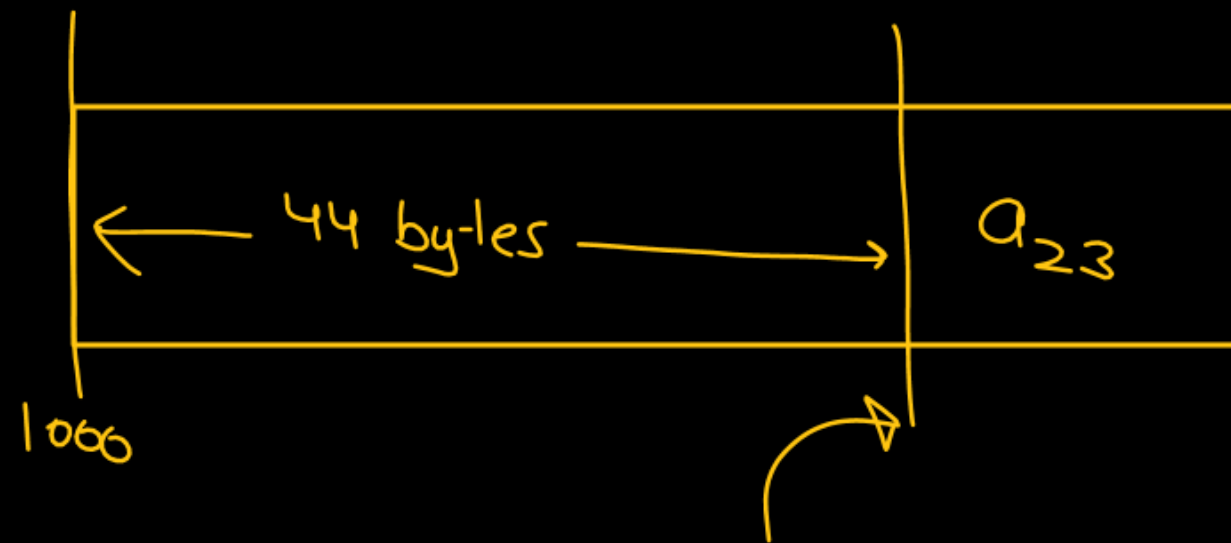$$A[3][4]$$

After 3 cols & 2 elements a_{23} is stored

$$\left(3 \times 3 + 2\right) \text{ elements}$$

Total elements already filled before $a_{23} = 11$

Memory already filled before $a_{23} = 11 \times 4 = 44$ bytes.

```
|←——— 44 bytes ————→| a₂₃ |
1000                 
```

a_{23}

1000

$$1000 + 44$$
$$= \boxed{1044}$$

Q

CMO

$$6-(-3)+1 \quad \boxed{12}$$

$$A[-5..6][-3..3]$$

Every index = 12 ele

$$\omega = 2 \text{ byte}$$

$$BA = 1000$$

$$\text{Add}(A[0][0])$$

-5 to -1
$$= -1-(-5)+1$$
$$= 5$$
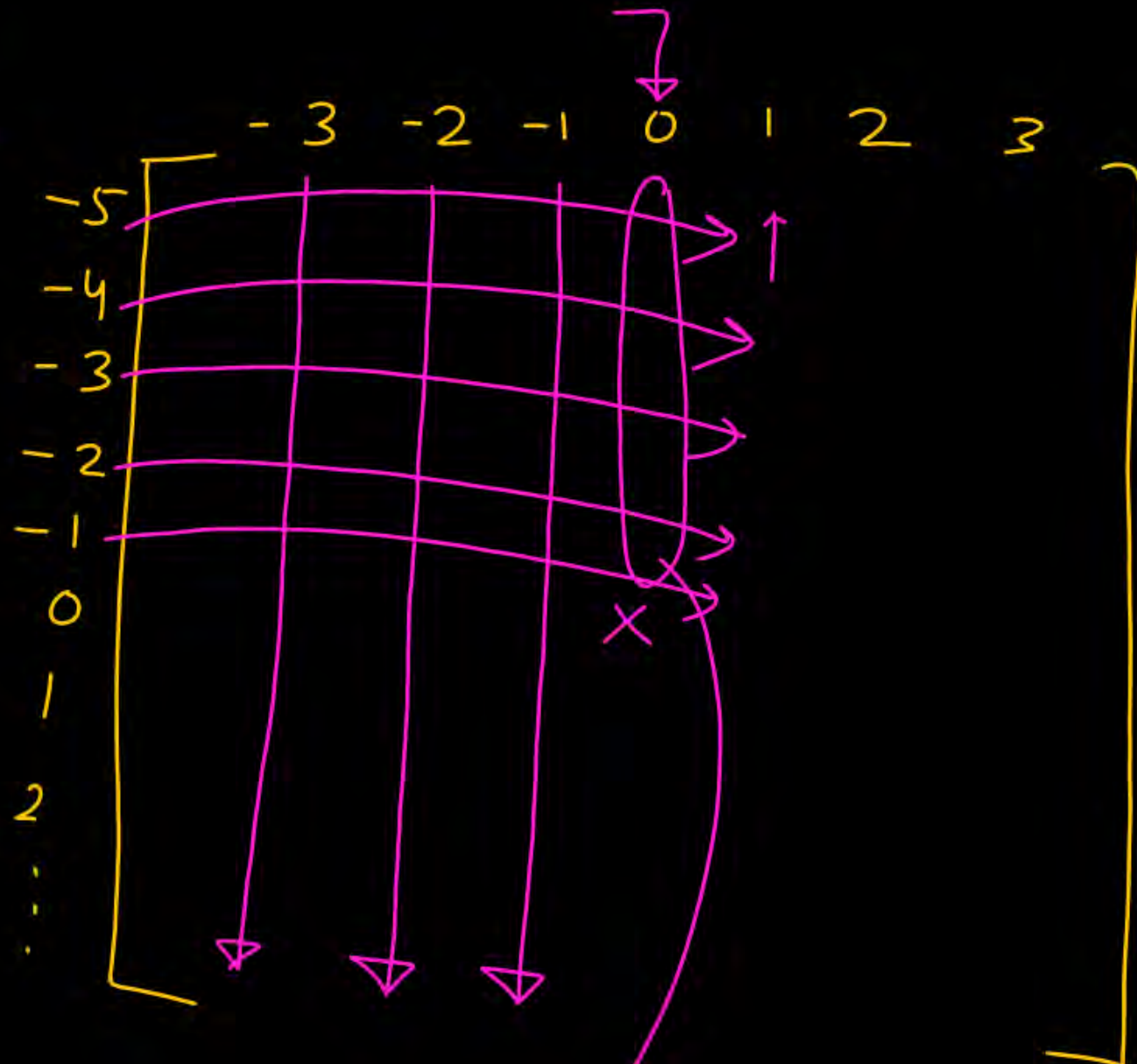
-3 to -1
$$= -1-(-3)+1$$
$$= -1+3+1 = 3 \text{ col}$$

$$3 \times 12 + 5 = 41 \text{ elem. are already filled}$$
before $A_{0,0}$

Memory filled $= 41 \times 2 = 82$ bytes

-5 to -1

$$1000 + 82$$
$$= \underline{1082}$$

$\leftarrow$ 82 byte $\rightarrow$ | A_{00}

1000

CMO

RMO

Wikibedia

(i)

$$ \overset{m}{[\quad]} \overset{n}{[\quad]} \overset{p}{[\quad]} $$

$$ \overset{m}{[\quad]} \overset{n}{[\quad]} \overset{p}{[\quad]} $$

(ii) $n \times p$ p

m $m \times n$

CMO
$w = 2$ byte
$BA = 100$
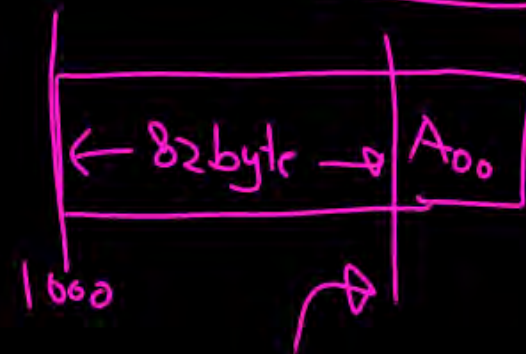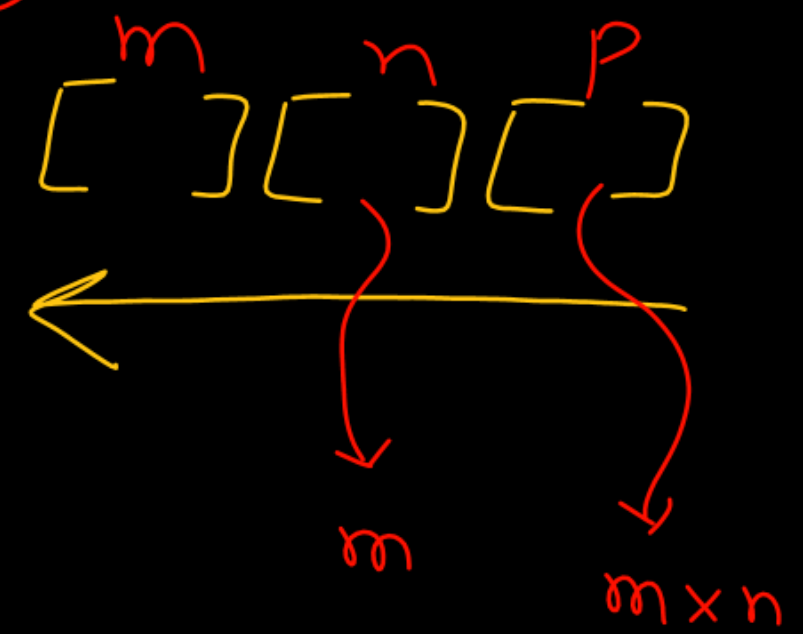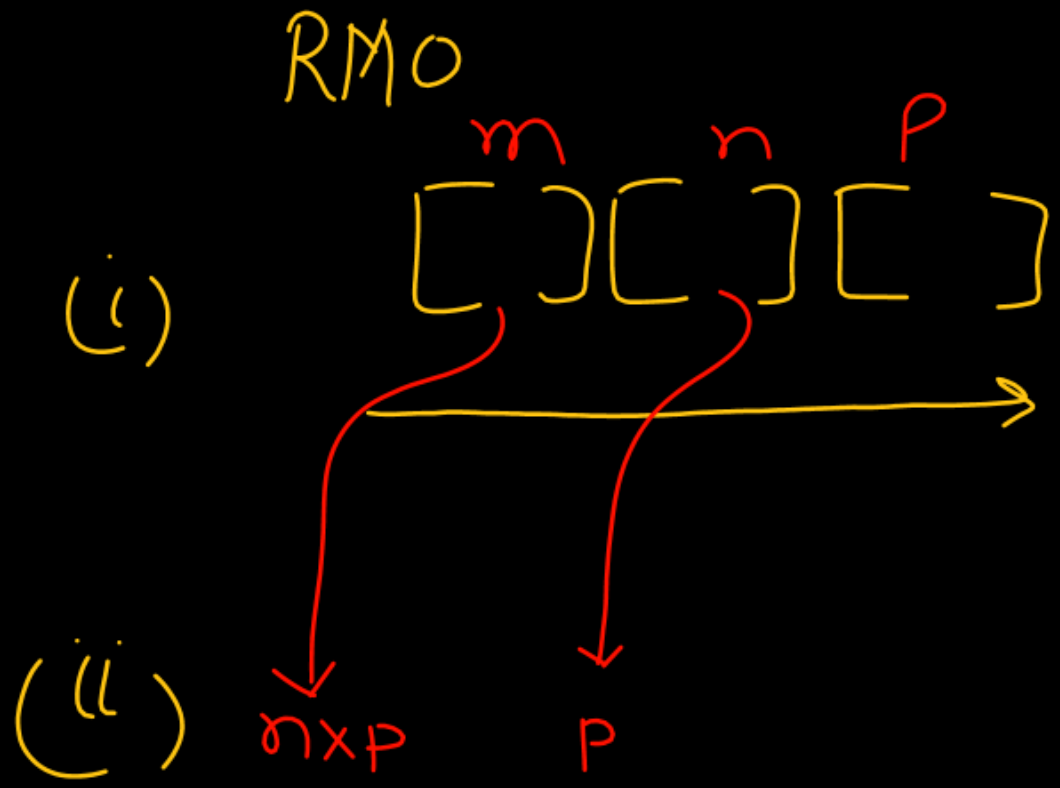
$$\underset{5-(-5)+1}{\overset{\textcircled{11}}{}} \quad \underset{3-(-3)+1}{\overset{\textcircled{7}}{}} \quad \overset{\textcircled{11}}{} $$

$$A[-5..5][-3..3][-5..5]$$

add (A_{000})

Every index = 11

Every index = 7 × 11

$\begin{cases} -5\checkmark \\ -4\checkmark \\ -3\checkmark \\ -2\checkmark \\ -1\checkmark \end{cases}$ -5 to -1
$= -1 -(-5)+1$
$= 5$

$\Rightarrow 0$

-5 to -1
$-1-(-5)+1$
5

-3 to -1
$-1-(-3)+1$
$= \textcircled{3}$

-5 to -1
$-1-(-5)+1$
5

5 3×11 $5 \times 7 \times 11$

$\left(5 \times 7 \times 11 + 3 \times 11 + 5 \right)$

423 elements

Memory already filled $= 423 \times 2$
$= 846$ bytes

100

$\longleftarrow$ 846 bytes $\longrightarrow$ | A_{000}
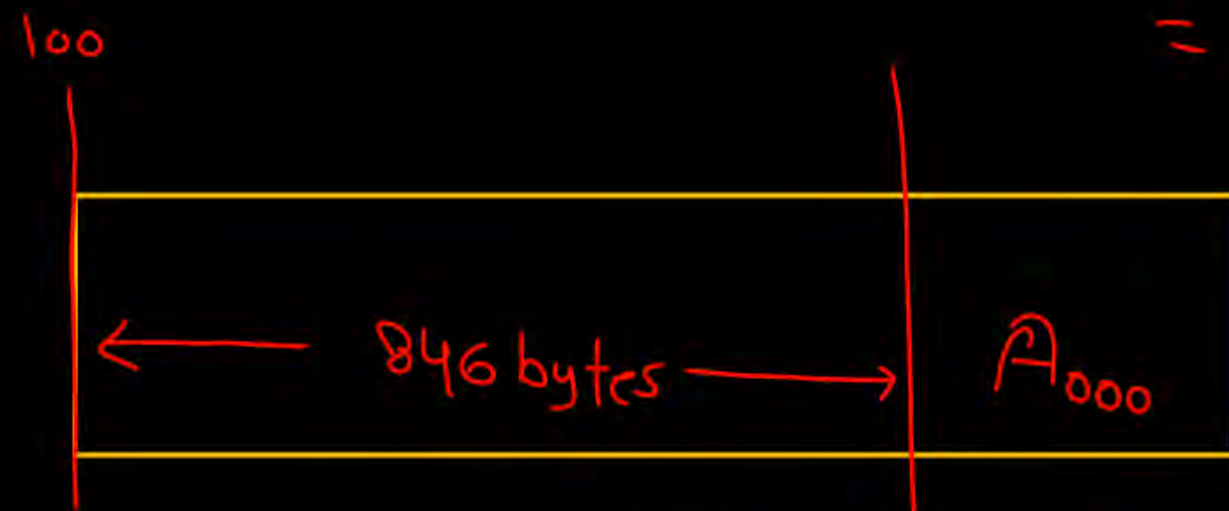
$= 100 + 846$
$= \boxed{946}$

(i)   Lower triangular matrix

(ii) Upper triangular matrix

(iii) Tri-diagonal matrix

Lower triangular matrix

A LTM is a square matrix.

$A_{12} = 0$

$A_{13} = 0$

$A_{14} = 0$

$A_{ij} = 0$ $\boxed{i < j}$

$A_{23} = 0$

$A_{24} = 0$

$A_{34} = 0$

$$\begin{array}{c c c c c}
 & 1 & 2 & 3 & 4 \\
1 & \begin{bmatrix} A_{11} & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 \\ A_{31} & A_{32} & A_{33} & 0 \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}
\end{array}$$

① Non-zero entries
RMO

Lower triangular matrix

2D-array

$$
\begin{array}{c|cccc}
 & 1 & 2 \quad X \quad 3 & & 4 \\
\hline
1 & A_{11} & 0 & 0 & 0 \\
2 & A_{21} & A_{22} & 0 & 0 \\
3 & A_{31} & A_{32} & A_{33} & 0 \\
4 & A_{41} & A_{42} & A_{43} & A_{44}
\end{array}
$$

Memory
$\{$ Non-zero $\}$ fill
element

1st row | 2nd row | 3rd row | 4th row

| A_{11} | A_{21} | A_{22} | A_{31} | A_{32} | A_{33} | A_{41} | A_{42} | A_{43} | A_{44} |

add (A_{43})

row 1,2,3

rows already filled
= 1 to 3
= 3-1+1
= 3 rows

$[1+2+3]$ elements

With row with index 4
ele already filled before
A_{43} = 1 to 2
= 2 elements

$\Rightarrow 6+2 = 8$ elements

$$\begin{array}{cccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & A_{11} & 0 & 0 & 0 & 0 \\ 2 & A_{21} & A_{22} & 0 & 0 & 0 \\ 3 & A_{31} & A_{32} & A_{33} & 0 & 0 \\ 4 & A_{41} & A_{42} & A_{43} & A_{44} & 0 \\ 5 & A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{array}$$

A_{11}	A_{21}	A_{22}	A_{31}	A_{32}	A_{33}	A_{41}	A_{42}	A_{43}	A_{44}	A_{51}	A_{52}	A_{53}	A_{54}	A_{55}

1000   1004

RMO $A_{N \times N} \Rightarrow LTM$ (A.P)

$$1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$$

add (A_{ij})

rows already filled

= index 1 to i-1

= i-1 -1+1

= (i-1) rows

row with index $\underset{\downarrow}{1}, \underset{\downarrow}{2}, \underset{\downarrow}{3}, \cdots \underset{\downarrow}{i-1}$

$1 + 2 + 3 + \cdots + (i-1) = \dfrac{(i-1)(i)}{2}$

within row whose index is i

ele = 1 to j-1

= j-1 -1+1

= (j-1)

	1	2	3	⋯	j-1	j	⋯	N
1	A_{11}	0	0	⋯	0	0	⋯	0
2	A_{21}	A_{22}	0	⋯	0	0	⋯	0
3	A_{31}	A_{32}	A_{33}	⋯	0	⋯	0	⋯ 0

(i-1)

i → Col-1 2 3 ⋯ j-1 A_{ij}

col index 1 to j-1

N
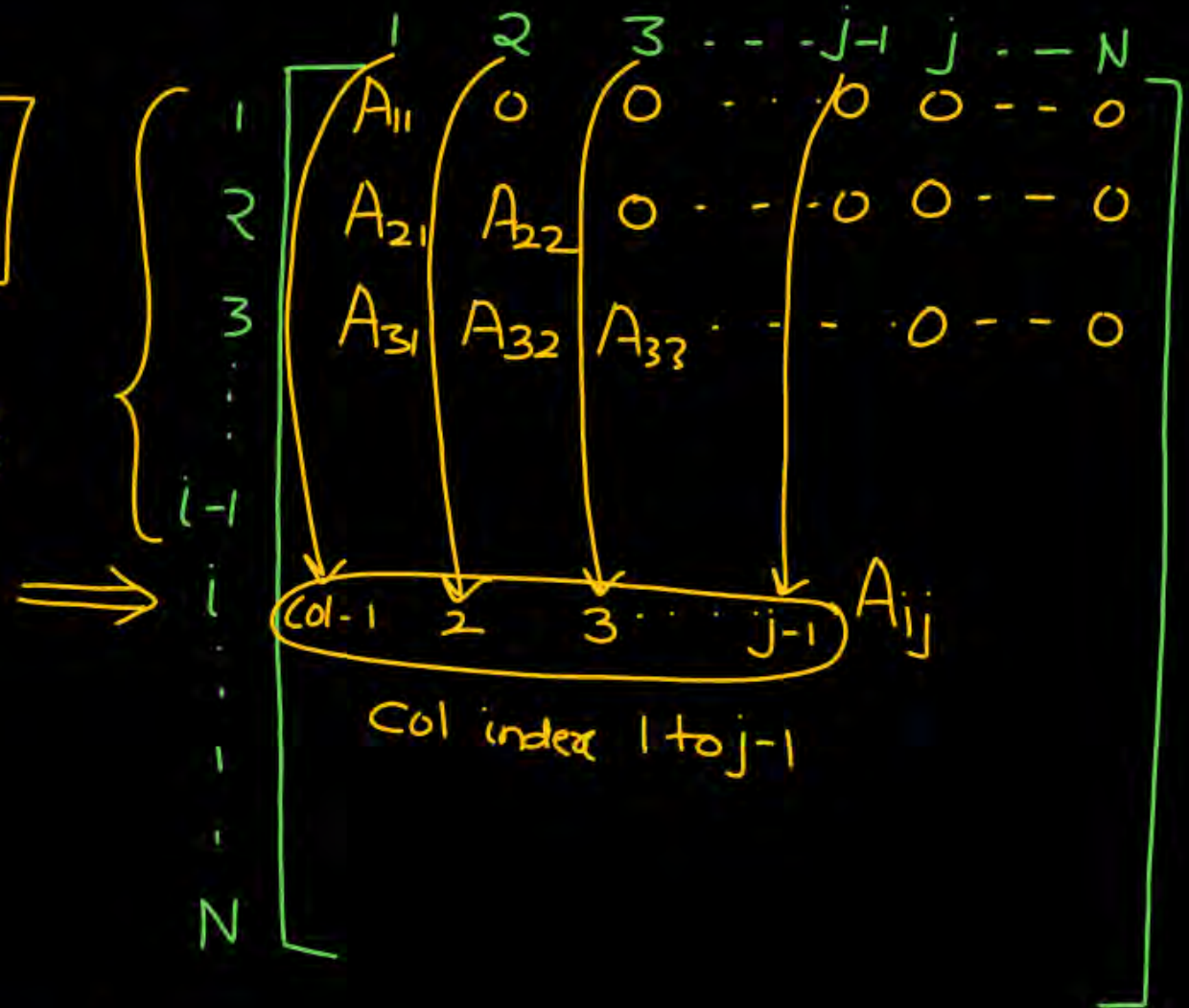
A_{11}	A_{21}	A_{22}	⋯ ⋯ ⋯ ⋯	A_{ij}	

Total elements already filled before A_{ij} = $\dfrac{(i-1)(i)}{2} + (j-1)$

Total elements already filled before A_{ij} = $\dfrac{(i-1)(i)}{2} + (j-1)$

Memory alread filled = $\left[\dfrac{(i-1)(i)}{2} + (j-1) \right] \omega$ bytes

Add (A_{ij}) = $BA + \left(\dfrac{(i-1)(i)}{2} + (j-1) \right) \omega$

२ एना
नहीं है

Q   LTM A[-5..5][-7..3]   जरूरी है

$w = 2$ bytes

RMO   BA $= 1000$

Add (A_{12})

-5 to 0 -7 to 1

$= 0 - (-5) + 1$ $1 - (-7) + 1$

$= 6$ rows 9 elements
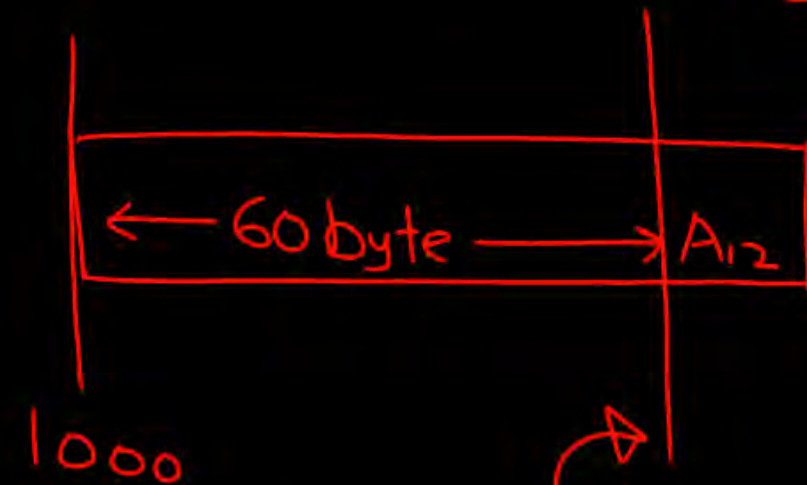
$1 + 2 + 3 + 4 + 5 + 6$

$= \dfrac{6 \times 7}{2}$

$= 21$ elements

Total elements already filled before

$A_{12} = 30$

Memory already filled $= 30 \times 2$

$= 60$ bytes

$\longleftarrow 60$ byte $\longrightarrow$ A_{12}

1000

$\Rightarrow 1000 + 60$?

$= \underline{1060}$

add $(A_{1,-2})$

Q

6 rows $\boxed{-7 \text{ to } -3}$

$\dfrac{6 \times 7}{2}$

$= 21$

$-3 - (-7) + 1$
$-3 + 7 + 1$
$= 5$

$= 26$ elem.

$\boxed{1052}$ $26 \times 2 = 52$ bytes

		-7	-6	-5	-4	-3	-2	-1	0	1	2	3
-5		X										
-4		X	X									
-3		X	X	X								
-2		X	X	X	X							
-1		X	X	X	X	X						
0		X	X	X	X	X	X					
1		X	X	X	X	X	X	X	0	0	0	
2												
3												
4												
5												

1.2	

1000

LTM — CMO

UTM — RMO, CMO

Tri-diagonal Matrix — RMO