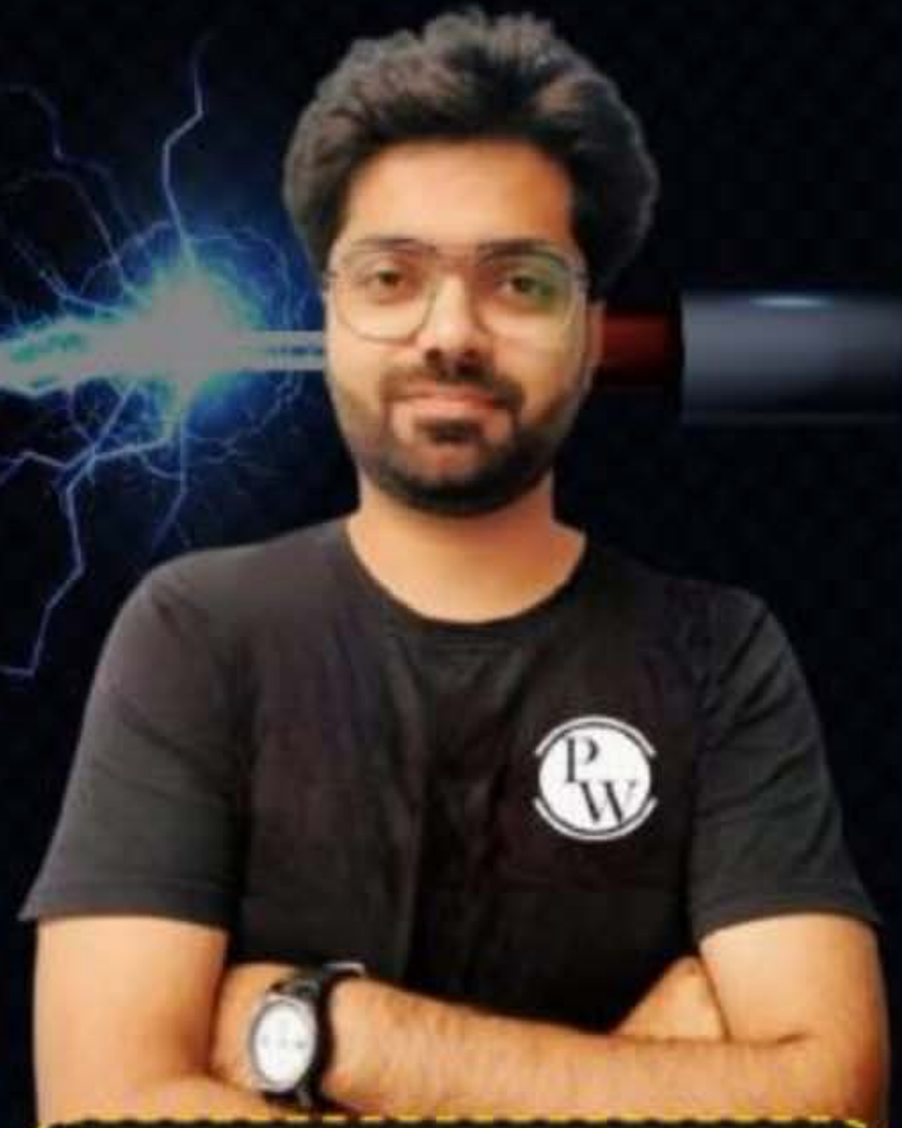
ENGINEERING MATHEMATICS

ALL BRANCHES



Types of Matrices & Operations on Matrices

DPP-01 Solution

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Q.1 Let A is a matrix of order 3 defined as, $A = [a_{ij}]3 \times 3$ where

$$a_{ij} = \lim_{x \to 0} \frac{\sin(ix)}{\tan(jx)}, \forall\ 1 \leq i, j \leq 3$$ Then A^2 is

$$a_{ij} = \lim_{x \to 0} \frac{\sin ix}{\tan jx} = \frac{ix}{jx} = \frac{i}{j}$$

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \end{bmatrix}$$

(A) 4A

(B) 3A ✓

(C) 2A

(D) A

$$A^2 = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 2/3 \\ 3 & 3/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 2/3 \\ 3 & 3/2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 3/2 & 1 \\ 6 & 3 & 2 \\ 9 & 9/2 & 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 2/3 \\ 3 & 3/2 & 1 \end{bmatrix}$$

$$A^2 = 3A$$

Q.2

For α, β, γ, let $A = \begin{bmatrix} \alpha^2 & 6 & 8 \\ 3 & \beta^2 & 9 \\ 4 & 5 & \gamma^2 \end{bmatrix}$ $B = \begin{bmatrix} 2\alpha & 3 & 5 \\ 2 & 2\beta & 6 \\ 1 & 4 & 2\gamma - 3 \end{bmatrix}$ If $T_r(A) = T_r(B)$

then the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$ $= \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$

(A) 1

(B) 2

(C) 3

(D) 4

Given $Tr(A) = Tr(B)$

$$\alpha^2 + \beta^2 + \gamma^2 = 2\alpha + 2\beta + 2\gamma - 3$$

$$(\alpha^2 - 2\alpha + 1) + (\beta^2 - 2\beta + 1) + (\gamma^2 - 2\gamma + 1) = 0$$

$$(\alpha - 1)^2 + (\beta - 1)^2 + (\gamma - 1)^2 = 0$$

iff $\Rightarrow \alpha - 1 = \beta - 1 = \gamma - 1 = 0$

$$\alpha = \beta = \gamma = 1$$

Q.3 If the product of n matrices $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\dots\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ equal to the matrix $\begin{bmatrix}1 & 378\\0 & 1\end{bmatrix}$ then the value of n is equal to

(A) 26

(B) 27 ✓

(C) 377

(D) 378

We have, $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}$

Now, $\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 6\\0 & 1\end{bmatrix}$

$\therefore \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\dots\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}1 & \frac{n(n+1)}{2}\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 378\\0 & 1\end{bmatrix}$

$\therefore \frac{n(n+1)}{2} = 378 \Rightarrow n = 27$

Q.4

If $A=\begin{bmatrix}1 & 2 & x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2 & y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ and $AB = I_3$, then x + y equal

(A) 0

(B) –1

(C) 2

(D) -2

$$AB = I_3$$

$$\begin{bmatrix}1 & 2 & x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & x+y\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\therefore x+y=0$$

Q.5 If $A = \begin{bmatrix} 3 & 4 \\ 1 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 6 & 1 \end{bmatrix}$ then X such that $A + 2X = B$ equals

(A) $\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 5 \\ -1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 2 \\ -1 & 0 \end{bmatrix}$

(D) None of these

$$A + 2X = B$$

$$X = \frac{B - A}{2}$$

$$= \frac{\begin{bmatrix} -2 & 5 \\ 6 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ 1 & -6 \end{bmatrix}}{2}$$

$$X = \frac{1}{2}\begin{bmatrix} -5 & 1 \\ 5 & 7 \end{bmatrix}$$

Q.6

If $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$, the x equals

(A) $\pm 2\sqrt{3}$

(B) $\pm 4\sqrt{3}$

(C) $\pm 3\sqrt{2}$

(D) $\pm 4\sqrt{2}$

$$\begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}_{1\times 3} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}_{3\times 1} = 0$$

$$x(x-2) - 40 + 2x - 8 = 0$$

$$x^2 - \cancel{2x} - 40 + \cancel{2x} - 8 = 0$$

$$x^2 = 48$$

$$x = \sqrt{48}$$

$$x = \pm 4\sqrt{3}$$

Q.7

Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ then Tr(A) – Tr(B) has the value equal to

(A) 0

(B) 1

(C) 2

(D) 3

$Tr(A+2B) = -1 \Rightarrow Tr(A) + 2Tr(B) = -1$ ....(1)

$Tr(2A-B) = 3 \Rightarrow 2Tr(A) - Tr(B) = 3$ ...(2)

On solving (1) and (2)

$Tr(A) = 1$ and $Tr(B) = -1$

$\therefore Tr(A) - Tr(B) = 1-(-1) = 2$

Q.8

A is an involutary matrix given by $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ then the inverse of $\frac{A}{2}$ will be

(A) 2A

(B) $\frac{A^{-1}}{2}$

(C) $\frac{A}{2}$

(D) A^2

$A^2 = I$ [Involutary matrix]

$\Rightarrow A = A^{-1}$

$\left(\frac{A}{2}\right) \cdot (2A) = I$

$\left(\frac{A}{2}\right)^{-1} = 2A$

Q.9 Let A= $\begin{bmatrix} \beta & -1 \\ 1 & 2\beta \end{bmatrix}$ and det.$(A^4) = 16$, then the product of all possible real value of β equals

(A) $\frac{1}{2}$

(B) $\frac{-1}{2}$

(C) 0

(D) 2

$$|A| = 2\beta^2 - (-1) = 2\beta^2 + 1$$

Given, $|A^4| = 16$

$$\Rightarrow |A|^4 = 16$$

$$(2\beta^2+1)^4 = 16$$

$$(2\beta^2+1)^4 = (\pm 2)^4$$

$$2\beta^2 + 1 = \pm 2$$

$$\beta^2 = \frac{1}{2} \text{ and } \beta^2 = -\frac{3}{2}$$

$$\beta = +\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, +\sqrt{\frac{3}{2}}\,i, -\sqrt{\frac{3}{2}}\,i$$

$\therefore$ Product of real values of β $= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$

Q.10

Let a = 2; b = -4; c = 1 and d = – 2, then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

Let $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$

(A) Idempotent $(A^2 = A)$

(B) Involutary $(A^2 = I)$

(C) Non-singular $|A| \neq 0$

(D) Nilpotent

$$A^2 = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = 0$$

A is nilpotent matrix of order 2.

$$\begin{bmatrix} a & -a^2 \\ 1 & -a \end{bmatrix}$$

Thank you

GW Soldiers !