Branch : CSE/IT Batch : Hinglish

Discrete Mathematics

Graph Theory

DPP-02

[NAT]

1. Consider a complete graph with size 2016. Suppose after deletion of 2 vertices from the above graph, the modified graph have x number of edges and y number of vertices. Find the value of x – y ?

[MSQ]

2. Which of the following options is/are True?

(a) Some k–regular graph can be complete graph.

(b) A graph with more than 2 vertices, it must have at least 2 vertices with same degree.

(c) The degree sum of odd degree vertices must be even.

(d) The degree sum of odd degree vertices must be odd.

[MCQ]

3. Consider a wheel graph (w_n) with $n \geq 4$. Which of the following is minimum number of edges added to the above wheel graph to make it complete graph?

(a) $n(n-1)$

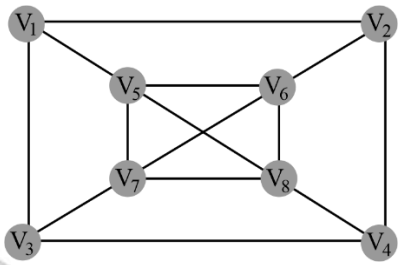
(b) $\frac{\left(n^2-5n+4\right)}{2}$

(c) $\frac{n(n-1)}{4}$

(d) None of these.

[NAT]

4. Consider the given graph G(V,E) with order is 8 $\{V_1, V_2, V_3, \ldots V_8\}$. Find the minimum number of edges to be deleted from the graph, such that the graph become bipartite graph _____?



[MSQ]

5. Which of the following options is/are correct?

(a) Every NULL graph is always bipartite graph.

(b) Some cycle graph is complete graph.

(c) A cyclic graph is different from cycle graph.

(d) A graph G is bipartite graph if and only if it has even cycle.

[MCQ]

6. Consider a regular graph with order 6 and size 12. Which of the following is the minimum degree(δ) and maximum degree (Δ)?

(a) $\delta = 3, \Delta = 4$

(b) $\delta = 4, \Delta = 3$

(c) $\delta = 4, \Delta = 4$

(d) None of these

Answer Key

1. **(1829)**
2. **(a, b, c)**
3. **(b)**
4. **(2)**
5. **(b, c)**
6. **(c)**

Hints and solutions

1. (1829)

As we know that the number of edges in the complete graph is:

Number of edges = $\frac{n\,(n-1)}{2}$

Where n = Number of vertices

Now,

In the problem size of the complete graph is given that is number of edges is 2016.

$\therefore \quad 2016 = \frac{n(n-1)}{2}$

So, the number of vertices (n) = 64.

Now, after detetion of 2 vertices, the modified complete graph would have 62 vertices

$\therefore$ Number of edges = $\frac{62*61}{2} = 1891$

So, x = 1891 and y = 62

Hence, the find value x – y = 1891 – 62

= 1829.

2. (a, b, c)

Option a : true

Every complete graph is regular graph.

Thus, a regular graph in which every vertex have degree n – 1 is k_n complete graph.

Hence, option A is correct.

Option b : True

In a simple connected undirected graph (with more than 2 vertices), at least 2 vertices must have same degree, since if this is not true, then all vertices would have different degree. A graph with all vertices having different degrees is not possible to construct (By 'Havel-Hakimi theorem).

Option c: True

No graph have odd nubmer of vertices with odd degree.

$\therefore \quad \sum \deg(v) = 2\,|E|$

$\sum$ even deg (v) + $\sum$ odd deg(v) = Even

$\therefore$ $\sum$ odd deg (v) = Even.

Hence, the sum of degree of odd degree vertices must be enve.

3. (b)

I. The total number of edges in the wheel graph (w_n) is 2(n – 1) and the number of edges in the complete graph is $\frac{n(n-1)}{2}$

II. Now, the number of edges need to odd into the wheel graph is:

$\frac{n(n-1)}{2} - 2(n-1)$

$\Rightarrow \quad \frac{(n^2-n)}{2} - (2n-2)$

$\Rightarrow \quad \frac{(n^2-n)-2(2n-2)}{2}$

$\Rightarrow \quad \frac{n^2-n-4n+4}{2}$

$\Rightarrow \quad \frac{(n^2-5n+4)}{2}$ edges.

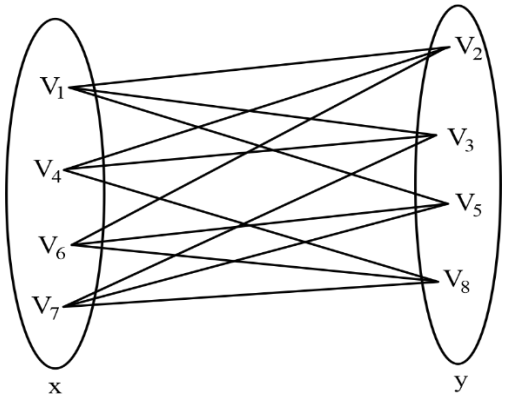
4. (2)

I. We know that bipartite graph consists of two sets of vertices.

If we delete the edge $\{V_6, V_7\}$ and $\{V_5, V_8\}$ then it is possible to divide the modified graph into two sets of vertices

x = $\{V_1, V_4, V_6, V_7\}$ and y = $\{V_2, V_3, V_5, V_8\}$

II. The vertices of set x are joined only with the vertices of set y and vice – versa.



Hence, we need to delete 2 edges to make it bipartite graph.

5. (b, c)

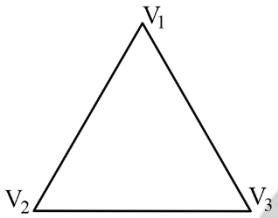
Option a: Incorrect

A null graph with number of vertices ≥ 2 is always bipartite graph but null graph with number of vertex either 0 or 1 is not bipartite graph.

Hence, every null can not be bipartite.

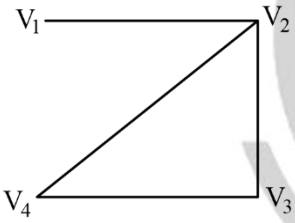
Option b: Correct

A cycle graph with number of vertices n = 3 is also a complete graph (K_3)

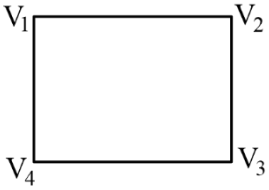


Option c: Correct

I. A cyclic graph is a graph in which a cycle is present.



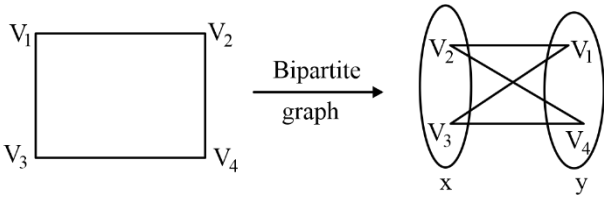
II. A cycle graph itself is one single cycle, where degree of each vertex is 2.



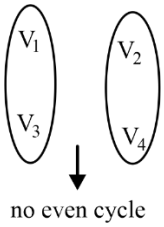
III. A cycle graph is always cyclic but every cyclic graph need not be cycle graph.

Option d: incorrect

A bipartite graph does not consist of odd length cycle but it may or may not have even length cycle.



II. Null graph with $n \geq 2$



Hence, the statement is incorrect.

6. (c)

A graph in which degree of all vertices is same is called as regular graph.

Hence,

$$n.\delta(G) = 2 \mid E \mid = n \cdot \Delta(G)$$

So, $6 \cdot \delta(G) = 2 * 12 = 6 \cdot \Delta(G)$

$$\therefore \ \delta(G) = 4 = \Delta(G)$$

Hence, the minimum and maximum degree of the graph is 4.