

Algorithms

Analysis of Algorithms

DPP

[MCQ]

1. Sort the functions in ascending order of asymptotic(big-O) complexity.

$$f_1(n) = n, f_2(n) = 80, f_3(n) = n^{\log n}, f_4(n) = \log \log^2 n, f_5(n) = (\log n)^{\log n}$$

- (a) $f_2(n), f_4(n), f_1(n), f_5(n), f_3(n)$
 (b) $f_2(n), f_1(n), f_4(n), f_5(n), f_3(n)$
 (c) $f_2(n), f_1(n), f_4(n), f_3(n), f_5(n)$
 (d) $f_1(n), f_1(n), f_4(n), f_3(n), f_2(n)$

[MCQ]

2. Consider two function $f(n) = 10n + 2\log n$ and $g(n) = 5n + 2(\log n)^2$, then which of the following is correct option?

- (a) $f(n) = \theta(g(n))$ (b) $f(n) = O(g(n))$
 (c) $f(n) = \omega(g(n))$ (d) None of the above

[MCQ]

3. Consider two function $f(n) = \sqrt{n}$ and $g(n) = n \log n + n$ then $f(n)/g(n)$ is equivalent to how many of the following given below? _____.

- (i) $o(n^{-1/2})$ (ii) $O(n^{-1/2})$
 (iii) $\Omega(1/\log n)$ (iv) $\theta(n^{-1/2})$

[MCQ]

4. Consider the following C-code

```
void foo (int x)
{
    int a = 1;
    if (n == 1)
        return;
```

```
for (; a ≤ n; a++)
```

```
{
    printf("GATEWALLAH");
    break;
}
```

What is the worst time complexity of above program?

- (a) $O(1)$ (b) $O(n)$
 (c) $O(\log n)$ (d) $O(\sqrt{n})$

[MCQ]

5. Find the time complexity of the following summation, assume that k is constant, $k > 0$

$$\sum_{x=1}^n \sum_{y=x+1}^n \frac{1}{k}$$

- (a) $O(n^2)$ (b) $O(n)$
 (c) $O(n^3)$ (d) None of the above

[NAT]

6. How many of the following expressions correctly describes $T(n) = n \log(n^2)$? _____

- (a) $\theta(n^2)$ (b) $O(n)$
 (c) $\Omega(n)$ (d) $O(n^2)$

[MCQ]

7. Consider two function $f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$ then which of the following is true.

- (a) $f_1(n) = O(f_2(n))$ (b) $f_1(n) = \theta(f_2(n))$
 (c) $f_1(n) = \omega(f_2(n))$ (d) None of these

Answer Key

- | | |
|-------------|-------------|
| 1. (a) | 5. (a) |
| 2. (a) | 6. (2 to 2) |
| 3. (2 to 2) | 7. (c) |
| 4. (a) | |



Hints & Solutions

1. (a)

$$80 < n$$

$$\log \log^2 n < n$$

$$\text{put } n = 10^{100}$$

$$\log(\log n)^2 = 10^{100}$$

$$\log(100)^2 < 10^{100}$$

$$4 < 10^{100}$$

$$n < n^{\log n}$$

taking log on both side

$$\log n < \log n \log n$$

we know that $(\log n)^2 > \log n$

$$\text{now, } (\log n)^{\log n} < n^{\log n}$$

as we can see that $\log n$ in LHS and n on RHS.

$$n \quad (\log n)^{\log n}$$

taking log on both sides

$$\log n < \log n * \log \log n$$

From above we conclude that growth of $\log * \log n$ is higher than 1.

\therefore option (a) is correct.

2. (a)

As we can see in above function, 'n' is the dominating factor in these 2 functions. Which means they also have similar growth rate.

$$\therefore f(n) = O(g(n))$$

Hence option (a) is correct.

3. (2 to 2)

$$\frac{f(n)}{g(n)} = h(n)$$

Given

$$f(n) = \sqrt{n}, g(n) = n \log n + n$$

$$= \frac{\sqrt{n}}{n \log n + n}$$

$$= \frac{\sqrt{n}}{\sqrt{n}(\sqrt{n} \log n + \sqrt{n})}$$

$$= \frac{1}{\sqrt{n} \log n + \sqrt{n}}$$

$$\text{and clearly } h(n) = O(n^{-0.5}) \text{ and } h(n) = o(n^{-0.5})$$

NOTE: if small 'o' possible then Big 'O' is possible but if Big 'O' possible then small 'o' may or may not possible.

\therefore (i) and (ii) are correct.

Hence 2 expressions are correct.

4. (a)

If we see carefully, loop will execute only one time because of break statement, therefore time complexity will be $O(1)$

5. (a)

$$\sum_{x=1}^n \sum_{y=x+1}^n \frac{1}{k} = \frac{1}{k} \sum_{x=1}^n \sum_{y=x+1}^n (1)$$

$$= \frac{1}{k} \sum_{x=1}^n [1 + 1 + 1 + \dots n - (x+1) + 1 \text{ times}]$$

$$= \frac{1}{k} \sum_{x=1}^n [n - x]$$

$$= \frac{1}{k} \left[n \sum_{x=1}^n (1) - n \sum_{x=1}^n x \right] = \frac{1}{k} \left[n \cdot n - \frac{n(n+1)}{2} \right]$$

$$= \frac{1}{k} \left[n^2 - \frac{n^2 + n}{2} \right] = \frac{1}{2k} [n^2 - n]$$

$$= O(n^2)$$

6. (2 to 2)

$$\text{Given: } T(n) = n \log(n^2) = 2n \log n$$

(i) $T(n) = \theta(n^2)$, which means the value of $T(n)$ is exactly $\theta(n^2)$, but as we can see that $T(n)$ is $n \log(n^2)$ so this is incorrect.

(ii) $T(n) = O(n)$: $T(n) \leq k \cdot n$, but value of $T(n)$ is $n \log(n^2)$

So, this is also false.

(iii) $T(n) = \Omega(n)$

$T(n) \geq k \cdot n$ and the complexity given for $T(n)$ is $n \log(n^2)$, so it is correct.

(iv) $O(n^2)$

$$T(n) = O(n^2)$$

$$n \log(n^2) \leq k \cdot n^2 \text{ which is correct.}$$

Hence 2 expression out of 4 are correct.

7. (c)

$$f_1(n) = n^{2^n} \text{ and } f_2(n) = n^{n^2}$$

$$n^{2^n} = n^{n^2}$$

Taking log on both side

$$2^n \log n \quad n^2 \log n$$

as we can see that

2^n has more growth rate than

$n^2 \therefore$ we conclude

$$f_2(n) < O(f_1(n)) \text{ or}$$

$$f_1(n) = \omega(f_2(n))$$

\therefore (c) is correct.



Any issue with DPP, please report by clicking here:- <https://forms.gle/t2SzQVvQcs638c4r5>

For more questions, kindly visit the library section: Link for web: <https://smart.link/sdfez8ejd80if>



PW Mobile APP: <https://smart.link/7wwosivoicgd4>