

# CS & IT ENGINEERING

Discrete Mathematics  
Combinatorics

DPP 05

Discussion notes



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# TOPICS TO BE COVERED

01 Question

02 Discussion



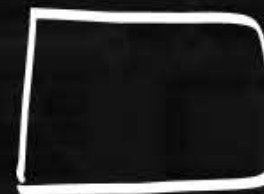
Q.1



In how many ways can 3000 identical envelopes be divided, in packages of 25, among four student groups so that each group gets at least 150, but not more than 1000, of the envelopes?

$$\frac{3000}{25} = 120 \quad \text{[MCQ]}$$

3000



25



25



25

$$\frac{150}{25}$$

6

$$\frac{1000}{6} = 40$$

A.

$$\binom{99}{96} - 4 \binom{64}{61} + 6 \binom{29}{26}$$

B.

$$\binom{99}{96} - 4 \binom{62}{61} + 6 \binom{29}{26}$$

C.

$$\binom{99}{96} - 4 \binom{62}{61} + 5 \binom{29}{26}$$

D.

None



coefficient of  $x^{20}$

$$(x^6 + x^7 + \dots + x^{40})^4$$

$$(x^6)^4 (1 + \dots + x^{34})^4$$

$$x^{24} \left( \frac{1-x^{35}}{1-x} \right)^4$$

$$x^{24} (1-x^{35})^4 (1-x)^{-4}$$

$$x^{24} (1 - 4x^{35} + 6x^{70} - \dots) (1-x)^{-4}$$

$$\frac{24}{35} \frac{1}{59}$$

$$-4C_{96} - 4x^{-4}C_{91} + 6x^{-4}C_{26}$$

$$4+96-1C_{96} - 4x^{4+91-1}C_{91} + 6x^{4+26-1}C_{26}$$

$$99C_{96} - 4x^{93}C_{91} + 6x^{29}C_{26}$$

$$\frac{24}{25} \frac{1}{94}$$



**Q.2**Determine the coefficient of  $x^{15}$  in**[NAT]**

$$f(x) = (x^2 + x^3 + x^4 + \dots)^4$$

$$(x^2)^4 (1 + x + \dots)^4$$

$$x^8 \left( \frac{1}{1-x} \right)^4$$

$$x^8 (1-x)^{-4}$$

$$(-1)^7 \cdot {}^7C_7 \cdot (-x)^7 \times x^8$$

$$4+7:1 {}^7C_7 = 10 {}^7C_7 = 10 {}^7C_3$$

Q.3

In how many ways can a police captain distribute 24 rifle shells to four police officers so that each officer gets at least three shells, but not more than eight?

[NAT]

$$(x^3 \dots x^8)^4$$

$$(x^3 + x^4 + \dots + x^8)^4$$

$$(x^3)^4 (1 + x + \dots + x^5)^4$$

$$x^{24} \cdot x^{12} \left( \frac{1-x^6}{1-x} \right)^4$$

$$x^{12} (1 - 4x^6 + 6x^{12} \dots) (1-x)^{-4}$$

$$-4C_{12} - 4 - 4C_6 + 6$$

$$15C_{12} - 4x^9C_6 + 6$$



Q.4

Determine the sequence generated by the following generating functions:

[MCQ]

$$f(x) = 1/(1 + 3x)$$

$$1 - 3x + (3x)^2 - (3x)^3 + (3x)^4 \dots$$

A.

$$1, 3, 3^2, 3^3, \dots$$

B.

$$1, 3, 3^2, -3^3, \dots$$

C.

$$1, -3, 3^2, 3^3, \dots$$

D.

$$1, -3, 3^2, -3^3, \dots$$

Q.5

In how many ways can two dozen identical robots be assigned to four assembly lines with at least three robots assigned to each line?



[NAT]

$$x^{24} \cdot \left( x^3 - \dots \right)^4$$
$$x^{12} \left( 1 + x \dots \right)^4$$

$$-4C_{12} = 4 + 12 - 1C_{12}$$
$$= \underline{15C_{12}}$$



Q.6

Find a recurrence relation, with initial condition, that uniquely determines the following geometric progressions:

[MCQ]

$7, 14/5, 28/25, 56/125$   
 $a_0 \quad a_1$

$$\frac{a_1}{a_0} = \frac{14}{5} \times \frac{1}{2}$$

A.

$$a_n = 5a_{n-1}, n \geq 1, a_0 = 2$$

B.

$$a_n = -3a_{n-1}, n \geq 1, a_0 = 6$$

C.

$$a_n = (2/5)a_{n-1}, n \geq 1, a_0 = 7$$

D.

None of these

**Q.7**

If  $a_n, n \geq 0$ , is the unique solution of the recurrence relation  $a_{n+1} - da_n = 0$ , and  $a_3 = 153/149$ ,  $a_5 = 1377/2401$ , What is  $d$ ?

**[MSQ]**

$$a_{n+1} = d^n a_n$$

$$a_2 =$$

☒ A.  $d = 3/7$

☐ B.  $d = 4/7$

☒ C.  $d = -3/7$

☐ D.  $d = -4/7$

$$\frac{d^5 a_0}{d^3 a_0} = \frac{a_5}{a_3} = \frac{1377}{2401} \times \frac{149}{153} = \frac{9}{49} = d^2$$

$$a_n = 2a_{n-1}$$

$$a_1 = 2^1 a_0$$

$$a_2 = 2^2 a_0$$

$$d = \pm \frac{3}{7}$$



Q.8



If  $a_0 = 0$ ,  $a_1 = 1$ ,  $a_2 = 4$ , and  $a_3 = 37$  satisfy the recurrence relation  $a_{n+2} + ba_{n+1} + ca_n = 0$ , where  $n \geq 0$  and  $b, c$  are constants, determine values for  $b, c$  respectively

[MCQ]

- ☒ A.  $b = -4, c = -21$
- ☐ B.  $b = -21, c = -4$
- ☐ C.  $b = 4, c = 21$
- ☐ D. None of these

$$a_{n+2} + ba_{n+1} + ca_n = 0$$

$$n=0$$

$$a_2 + ba_1 + ca_0 = 0$$

$$4 + b \cdot 1 + c \cdot 0 = 0$$

$$b = -4$$

$$n=1$$

$$a_3 + ba_2 + ca_1 = 0$$

$$37 + (-4)4 + c(1) = 0$$

$$37 - 16 + c = 0$$

$$c = -21$$

Q.9



Determine the constants  $b$  and  $c$  if  $a_n = c_1 + c_2(7^n)$ ,  $n \geq 0$ , is the general solution of the relation

$$a_{n+2} + ba_{n+1} + ca_n = 0, n \geq 0.$$

[MCQ]

→ Roots : 1, 7

$$(n-1)(n-7)=0$$

$$n^2 - 8n + 7 = 0.$$

$$b = -8 \quad c = 7.$$

A.

$$b = 7, c = -8$$

B.

$$b = -8, c = 7$$

C.

$$b = -8, c = 8$$

D.

None of these



