

Discrete Mathematics

Set Theory

DPP-01

[NAT]

1. Which of the following statements are true?

- I. $\phi \in \phi$
- II. $\phi \subset \phi$
- III. $\phi \subseteq \phi$
- IV. $\phi \in \{\phi\}$
- V. $\phi \subset \{\phi\}$
- VI. $\phi \subseteq \{\phi\}$

[NAT]

2. If a set A has 63 proper subsets, then what is the cardinality of A?

[MCQ]

3. If a set A has 64 subsets of odd cardinality, then what is $|A|$?

- (a) 6
- (b) 63
- (c) 7
- (d) 128

[NAT]

4. How many subset of $\{1, 2, 3, \dots, 11\}$ contain at-least one even integer?

[NAT]

5. Let $A = \{1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18\}$
How many subsets of A contain six elements?

[NAT]

6. Let $A = \{2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15\}$
How many six-elements subsets of A contain four even integers and two odd integers?

[NAT]

7. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
How many subsets of A contain only odd integers?

Answer Key

1. (4)
2. (6)
3. (c)
4. (1984)

5. (924)
6. (225)
7. (63)



Hints and Solutions

1. (4)

- I. $\phi \in \phi$ is false. The empty set has no members.
 II. $\phi \subset \phi$ is false. The empty set is not a proper subset of itself.
 III. $\phi \subseteq \phi$ is true. The empty set is a subset of every set
 \therefore subset of itself
 IV. $\phi \in \{\phi\}$ is true. ϕ is a member here.
 V. $\phi \subset \{\phi\}$ is true. The empty set is a proper subset of itself.
 VI. $\phi \subseteq \{\phi\}$ is true. The empty set is a subset of every set.

2. (6)

If a set has n elements then the number of subsets will be 2^n and the number of proper subsets will be $2^n - 1$.

A has 63 proper subsets, so $2^n - 1 = 63$

$$2^n = 63 + 1$$

$$2^n = 64$$

$$2^n = 2^6$$

$$\therefore n = 6$$

The cardinality of A is 6

3. (c)

The number of subsets for $\{1, 2, 3, \dots, n\}$ with odd cardinality is 2^{n-1} .

Number of subsets with cardinality $i = {}^nC_i$

So, the number of subsets with odd cardinality

$$\sum_{i=1, 3, \dots, n-1} {}^nC_i = 2^{n-1}.$$

Now, given

$$2^{n-1} = 64$$

$$2^{n-1} = 2^6$$

$$n - 1 = 6$$

[Bases are same, so equating power]

$$n = 7$$

4. (1984)

2^{11} subset for $\{1, 2, 3, \dots, 11\}$

2^6 subset for $\{1, 3, 5, 7, 9, 11\}$ contains none of the even integers $\{2, 4, 6, 8, 10\}$.

Hence, there are $2^{11} - 2^6 = 1984$ subsets that contain at least one even integer.

5. (924)

If we choose 6 elements from a set of 12 elements where order does not matter. Then we can do it in ${}^{12}C_6$ ways.

For example consider a set = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ out of this set we have to choose a subset of 6 elements. This can be done in following ways:

$\{1, 2, 3, 4, 5, 6\}, \{2, 3, 4, 5, 6, 7\}, \{3, 4, 5, 6, 7, 8\}, \{4, 7, 8, 3, 11, 12\}, \{5, 7, 9, 10, 11, 12\} \dots$ and so on.

That means the arrangement or order of elements does not matter, therefore we can do it using combinations.

$$\begin{aligned} {}^{12}C_6 &= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \\ &= 11 \times 12 \times 7 \\ &= 77 \times 12 \\ &= 924 \end{aligned}$$

6. (225)

Out of 6 element subsets, we can choose 4 even integers in 6C_4 ways.

Similarly to find 2 odd integers out of 6 element subset, can be done in 6C_2 ways.

$$\begin{aligned} \text{Therefore } {}^6C_4 * {}^6C_2 &= \frac{6 \times 5 \times 4 \times 3}{4 \times 3 \times 2 \times 1} \times \frac{6 \times 5 \times 4 \times 3}{4 \times 3 \times 2 \times 1} \\ &= 15 * 15 = 225 \end{aligned}$$

7. (63)

In the given set, there are 6 odd integers, we have two choices for each odd integer to be included or

not included, therefore total possibilities = $2^6 - 1 = 63$.



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