Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics
Mathematical Logic

DPP-02

[MSQ]

1. Which of the following is/are logical equivalence?

I. $\sim(p \to q)$

II. $\sim(p \to q) \wedge (q \to r)$

III. $p \wedge \sim q$

IV. $(p \vee q) \to r$

(a) I and II (b) I and III

(c) II and IV (d) II and III

[MSQ]

2. Consider the following statement

S_1: $(p \to q) \wedge (p \to r)$

S_2: $p \to (q \wedge r)$

Which of the following is True?

(a) S_1 is tautology

(b) S_1 is contingency

(c) S_1 is logically equivalence to S_2

(d) None of these

[MSQ]

3. Which of the following is logically equivalence?

(a) $(p \to r) \vee (q \to r)$

(b) $(p \leftrightarrow q) \vee (q \to r)$

(c) $(p \wedge q) \wedge r$

(d) $(p \leftrightarrow r) \wedge (q \leftrightarrow r)$

[MCQ]

4. Consider the following statement

S_1: $\sim (p \leftrightarrow q)$

S_2: $p \leftrightarrow \sim q$

Which of the following is correct?

(a) S_1 is tautology

(b) S_2 is contradiction

(c) S_1 is equivalence to S_2

(d) None of these

[MCQ]

5. Consider the following statement

S_1: $\sim (p \vee (\sim p \wedge q))$

S_2: $\sim p \wedge \sim q$

Which of the following is correct?

(a) S_1 is tautology

(b) S_2 is contradiction

(c) S_1 is equivalence to S_2

(d) S_1 is not equivalence to S_2

Answer Key

1. (b, c)
2. (b, c)
3. (a, b, c)
4. (c)
5. (c)

Hints and solutions

1. (b, c,)

Two statements forms are logical equivalent if and only if their resulting truth values are identical for each variation of statement variables.

I. $\sim (p \to q)$

$= \sim(\sim p \vee q)$

$= p \wedge \sim q$

Hence, I is logically equivalent to III.

II. $(p \to r) \wedge (q \to r)$

$= (\bar{p} + r) \wedge (\bar{q} + r)$

$= \bar{p}\,\bar{q} + \bar{p}\,r + \bar{q}\,r + r$

$= \bar{p}\,\bar{q} \mid \bar{p}\,r + r$

$= \bar{p}\,\bar{q} + r$

$= (\overline{p \vee q}) + r \equiv (p \vee q) \to r$

Hence, II and IV are logically equivalence.

2. (b, c)

Statement S_1: $(p \to r) \wedge (p \to r)$

$= (\bar{p} + q) \wedge (\bar{p} + r)$

$= \bar{p} + \bar{p}\,r + \bar{p}\,q + qr$

$= \bar{p} + \bar{p}\,q + qr$

$= \bar{p} + qr$

$= p \to (q \wedge r) \neq 41$

Hence, S_1 is not tautology and S_1 is logically equivalent to S_2.

Statement S_2: $p \to (q \wedge r)$

$= \bar{p} + (q \wedge r)$

$= \bar{p} + qr \neq 1 \text{ or } 0$

Hence, statement S_2 is contingency.

3. (a, b, c)

Option A: $(p \to r) \vee (q \to r)$

$= (\bar{p} + r) \vee (\bar{q} + r)$

$= \bar{p} + r + \bar{q} + r$

$= \bar{p} + \bar{q} + r$

$= \overline{pq} + r \equiv (\overline{p \wedge q}) + r$

$\equiv (p \wedge q) \to r$

So, option A is logically equvalence to option C.

Option B: $(p \leftrightarrow r) \vee (q \to r)$

$= \bar{p}\,\bar{r} + pr + \bar{q} + r$

$= \bar{p}\,\bar{r} + \bar{q} + pr + r$

$= \bar{p}\,\bar{r} + \bar{q} + r$

$= \bar{p}\,\bar{r} + r + \bar{q}$

$= \bar{p} + r + \bar{q}$

$= \bar{p} + \bar{q} + r$

$= (\overline{p \wedge q}) + r = (p \wedge q) \to r$

So, option B is also logically equvalence to option A.

4. (c)

Statement S_1: $\sim (p \leftrightarrow q)$

$= \sim (\bar{p}\,\bar{q} + pq)$

$= (p + q)(\bar{p} + \bar{q})$

$= p\,\bar{q} + q\,\bar{p}$

Statement S_2: $p \leftrightarrow \sim q$

$= \bar{p}\,q + p\,\bar{q}$

Hence, S_1 and S_2 are equivalence to each other.

5. (c)

Statement S_1: $\sim (p \vee (\sim p \wedge q))$

$= \sim p \wedge [\sim (\sim p \wedge q)]$

$= \sim p \wedge [\sim (\sim p \vee \sim q)]$

$= \sim p \wedge [p \vee \sim q)]$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= F \vee (\sim p \wedge \sim q)$

$= (\sim p \wedge \sim q)$

$= \sim p \wedge \sim q)$

Hence, S_1 is equivalence to S_2.