CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 09

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01 Basics of relations

02 Types of relations

03 Number Of relations

Relations

Partial order Relation: $R: \{(a,b) \mid a \leq b\}$

(RAT)
- Reflexive
- Antisymmetric
- Transitive

R: $aRa \quad a \leq a$ (T)

✓ **Anti:** $aRb \wedge bRa \rightarrow a = b$.

$a \leq b \wedge b \leq a \rightarrow a = b$.

$2 \leq 3 \wedge 3 \leq 2$

$(2,3) \in R \wedge F$

F → True.

Anti
- allows same
- no flipping.

Relations

I: $a \leq b \wedge b \leq c \rightarrow a \leq c$.

$2 \leq 3 \wedge 3 \leq 5 \rightarrow 2 \leq 5$

$(2,3) \in R \wedge (3,5) \in R \rightarrow (2,5) \in R$.

$R_1: \{(a,b) \mid a \leq b\}$ RAT.

$R_2: \{(a,b) \mid a < b\}$ R ✗ A ✓ T ✓

Relations

$R: \{(A,B) \mid A \subseteq B\}$ RAT.

R: ARA $A \subseteq A$

Anti: $A \subseteq B \wedge B \subseteq A \rightarrow A = B$.

T: $A \subseteq B \wedge B \subseteq C \rightarrow A \subseteq C$.

$R: \{(a,b) \mid gcd(a,b) = 1\}$

$\times R$: $gcd(2,2) = 2 \neq 1$

$\times$ Anti: Relation allows flipping so no anti

$\times$ Transitive:

$gcd(3,5) \wedge gcd(5,9) \rightarrow gcd(3,9)$

$= 1 \quad = 1 \quad = 3 \neq 1$.

Relations

Domain: Z^+.

$R: \{(a,b) \mid a \text{ dindes } b\}$

$R: \{(a,b) \mid a|b\}$. (RAT)

a dindes b.

$\Downarrow$

$a|b$ OR $\frac{b}{a}$

Reflexive.: $0|0$ $\frac{0}{0}$ (it is not reflexive. when Domain allows '0')

Relations

D: Z^+.

R: $\{(a,b) \mid a|b\}$.

Transitive:

$a|b \wedge b|c \rightarrow a|c$.

R: aRa $a|a$

Anti: $a|b \wedge b|a \rightarrow a=b$.

$3|6 \wedge 6|3 \rightarrow$

F

F

T

Set: $\mathbb{Z}$.

Relation: $\{(a,b) \mid a \leq b\}$

Set: $\mathbb{Z}^+$.

Relation: $\{(a,b) \mid a|b\}$

Relations

Set : { . . . a, b . . . }

aRb OR bRa.

a, b are called comparable

Relation :

Set : Z^+ : { 1, 2, 3, 4, 5, ... }

(only partial elements are comparable)

a = 3 b = 6

aRb OR bRa

3R6 OR 6R3.

3|6 OR 6∤3.

T

3, 6 are called comparable.

a = 3 b = 5

aRb OR bRa.

a|b OR b|a.

3∤5 OR 5∤3.

Relations

all elements are comparable w.r.t (≤)

Domain: $Z = \{ \ldots -2, -1, 0, 1, 2 \ldots \}$

$R: \{ (a,b) \mid a \leq b \}$

Total order Relation.

linear order Relation.

$a = 3 \quad b = 5$

$a R b$ OR $b R a$.

$a \leq b$ OR $b \leq a$.

$3 \leq 5$ OR $5 \leq 3$.

T.

$a = 100 \quad b = -1$.

$a \leq b$ OR $b \leq a$.

$100 \leq -1$ OR $-1 \leq 100$.

T.

Relations

Relation → RAT → Partial order Relation

↓

all elements are comparable.

↓ yes.

Total order Relation

Relations

Equivalence Relation : RST.

Partial order Relation.

RAT.

POR	POR.
R_1 ∪	R_2:
R ∪	R = ✓
Anti ∪	Anti = X
T_1 ∪	T_2 = X

POR	POR.
R_1:	R_2:
R ∩	R = ✓
Anti ∩	Anti = ✓
T ∩	T = ✓

Relations

A: {1, 2, 3}

R_1: {(1,1)(2,2)(3,3)(1,2)} POR.

R_2: {(11)(22)(33)(21)} POR.

Relations

Set: {1, 2, 3, 4, 6, 12}

Partial order Relation

R: { (1,1) (1,2) (1,3) (1,4) (1,6) (1,12)

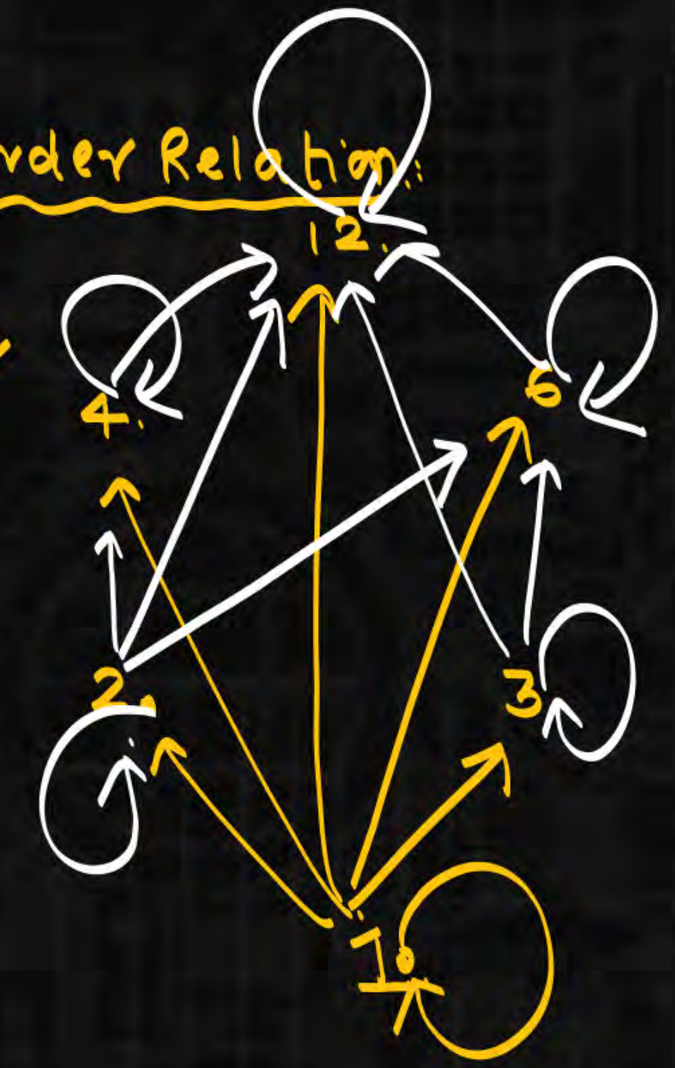
(2,2) (2,4) (2,6) (2,12)

(3,3) (3,6) (3,12)

(4,4) (4,12)

(6,6) (6,12)

(12,12) }

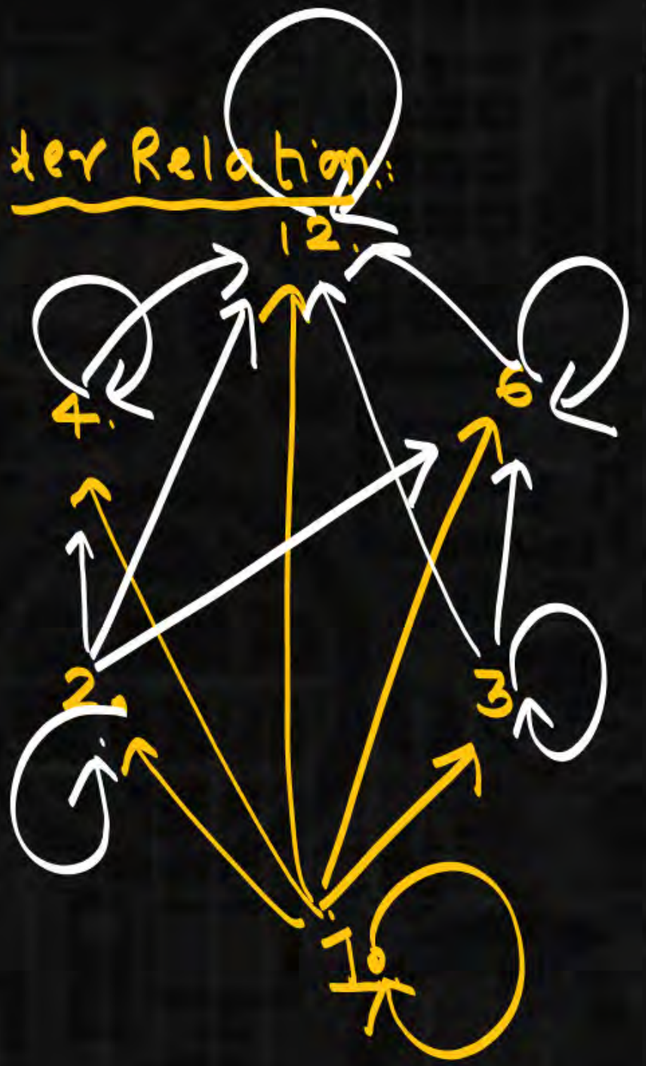


Relations

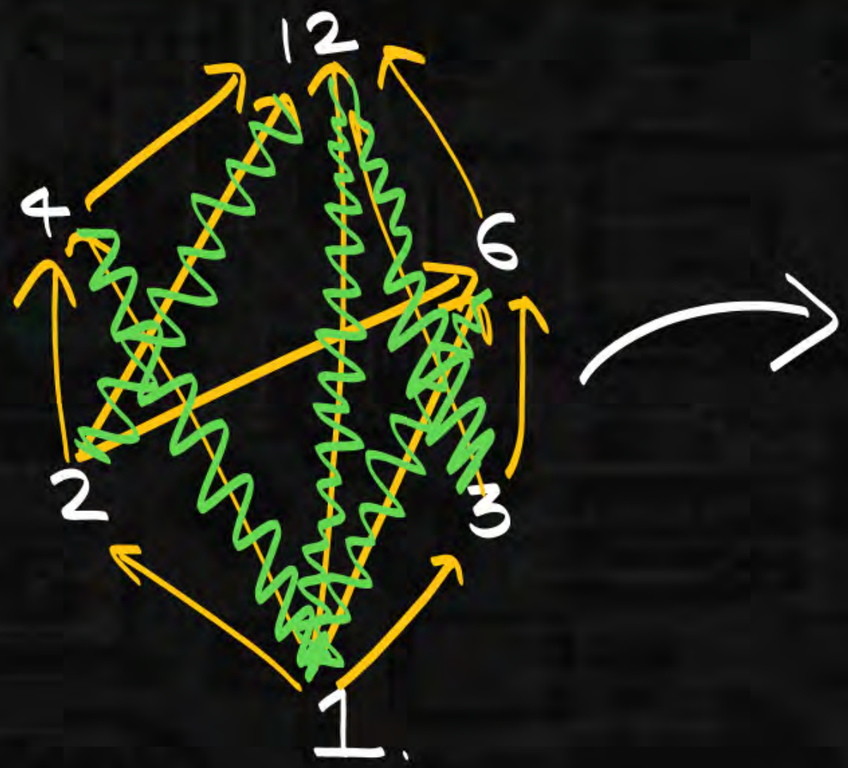
Simplification:

Remove
Self loop

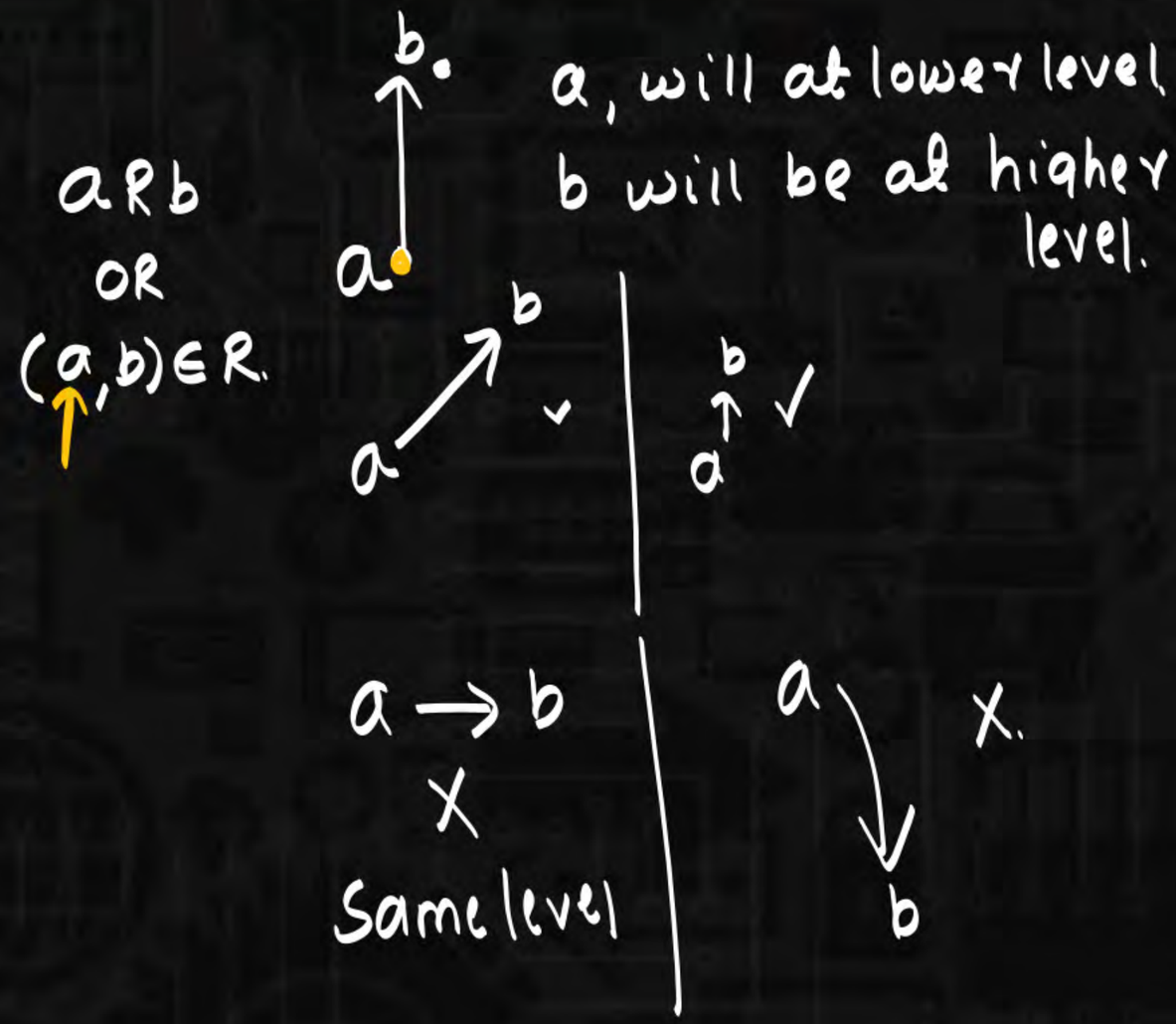
→ Remove
transitive



Relations



Relations


aRb
OR
(a,b)∈R.
b
a
a, will at lower level
b will be at higher level.
b
a
b
a
a → b
X
Same level
a
X.
b

Relations

Set: Z^+

Relation: $\leq$ POR.

(Set, Relation) (poset)

Partial order set

Poset:

$(Z, \leq)$ — $\leq$: Total order relation

↳ Total order set

(TOSET)

Chain.

Relations

Set: {1, 2, 3, 4, 6, 12}.

R: { (1,1) (1,2) (1,3) (1,4) (1,6) (1,12)
(2,2) (2,4) (2,6) (2,12)
(3,3) (3,6) (3,12)
(4,4) (4,12)
(6,6) (6,12)
(12,12) }

(Set, Relation)

({1, 2, 3, 4, 6, 12}, |)

Divisor of 12

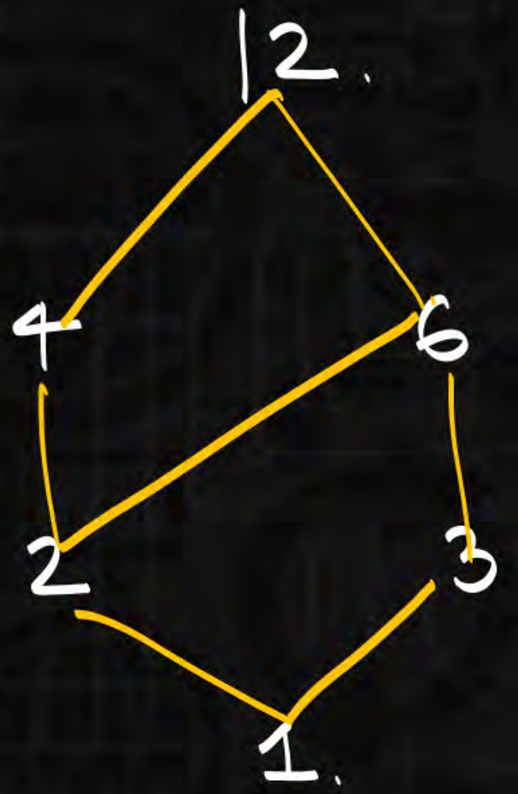
(D12, 1)

Relations

Set: {1, 2, 3, 4, 6, 12}.

R: { (1,1) (1,2) (1,3) (1,4) (1,6) (1,12)
(2,2) (2,4) (2,6) (2,12)
(3,3) (3,6) (3,12)
(4,4) (4,12)
(6,6) (6,12)
(12,12) }

(D12, 1)



Relations

(Set, Relation)

$(\{1, 2, 3\}, \leq)$

Hasse diagram.

$(D_{24}, |)$

# of edges in this?

$(D_{36}, |)$

Relations

$(D_{36}, |)$

$(D_{24}, |)$

$(\{1, 2, 3, 4, 6, 8, 12, 24\}, |)$

1 2 3 4 6 8 12 24.

edges = 10.

Relations

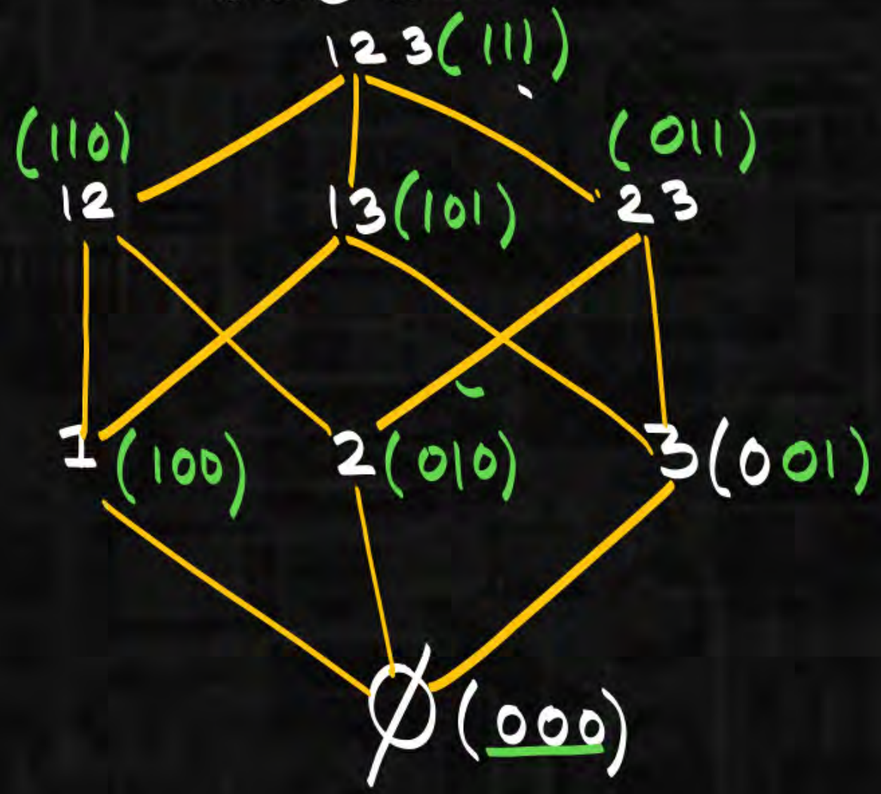
$(D_{36}, |)$

$(\{1, 2, 3, 4, 6, 9, 12, 18, 36\}, |)$

Relations

$(P(A), \subseteq)$

$A = \{1, 2, 3\}$



hypercube

Powerset: $P(A)$ / 2^A

$A = \{1, 2, 3, 4\}$ $\quad 2^n \cdot n = 2e$

$(P(A), \subseteq)$ $\quad n = 4.$

$e = n \cdot 2^{n-1}$

$= 4 \cdot 2^{4-1} = 4 \cdot 2^3 = 32.$


THANK YOU GW
SOLDIERS !