Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics
Set Theory

DPP-06

[NAT]

1. Consider the following statement about relations on a set A, where | A | = n. How many of the following statements are TRUE?

I. If R is a relation on A and $|R| \geq n$, then R is reflexive.

II. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_1 is reflexive $\Rightarrow$ R_2 is reflexive

III. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_1 is symmetric $\Rightarrow$ R_2 is symmetric

IV. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_1 is anti-symmetric $\Rightarrow$ R_2 is anti-symmetric

V. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_1 is transitive $\Rightarrow$ R_2 is transitive

VI. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_2 is reflexive $\Rightarrow$ R_1 is reflexive

VII. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_2 is symmetric $\Rightarrow$ R_1 is symmetric

VIII. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_2 is anti-symmetric $\Rightarrow$ R_1 is anti-symmetric

IX. If R_1, R_2 are relations on A and $R_2 \supseteq R_1$, then R_2 is transitive $\Rightarrow$ R_1 is transitive

X. If R is an equivalence relation A, then $n \leq |R| \leq n^2$

[MSQ]

2. Consider A = {w, x, y, z}, then which of the following options is/are correct for the given set?

(a) The number of symmetric relations is 2^{10}

(b) The number of symmetric relations which contain (x, y) is 2^9

(c) The number of relations which is anti-symmetric and contain (x, y) is $2^4 \cdot 3^5$

(d) The number of relations which is reflexive, symmetric and anti-symmetric is 1

[MCQ]

3. Let A be a set with |A| = n, and let R be a relation on A that is antisymmetric, then which of the following option is correct?

(a) The maximum value for | R | is $(n^2 + n)/2$.

(b) The number of anti-symmetric relations have size | R | is $2^{(n^2+n)/2}$.

(c) Both a and b

(d) None of these.

[MCQ]

4. Let A be a set with |A| = n, and let R be an equivalence relation on A with |R| = r. Which of the following is TURE for the given relation R?

(a) r – n will be always even.

(b) r – n will be always odd.

(c) Both a and b

(d) None of these.

[NAT]

5. Let A = {1, 2, 3, 4, 5, 6, 7, 8}. How many symmetric relations on A contain exactly four ordered pairs?

Answer Key

1. (3)
2. (a, b, c, d)
3. (c)
4. (a)
5. (1232)

Hints and Solutions

1. **(3)**

(a) False: Let A = {1, 2} and R = {(1, 2), (2, 1)}.

(b) (i) Reflexive: True

(ii) Symmetric: False. Let A = {1, 2}, R_1 = {(1, 1)}, R_2 = {(1, 1), (1, 2)}.

(iii) Antisymmetric and Transitive: False. Let A = {1, 2}, R_1 = {(1, 2)}, R_2 = {(1, 2), (2, 1)}.

(c) (i) Reflexive: False. Let A = {l, 2}, R_1 = {(1, 1)}, R_2 = {(1, 1), (2, 2)}.

(ii) Symmetric: False. Let A = {1, 2}, R_1 = {(1, 2)}, R_2 = {(1, 2), (2, 1)}.

(iii) Antisymmetric: True

(iv) Transitive: False. Let A = {1, 2}, R_1 = {(1, 2), (2, 1)}, R_2 = {(1, 1), (1, 2), (2, 1), (2, 2)}.

2. **(a, b, c, d)**

Option a: True.

The number of symmetric relations is 2^{10}

Option b: True.

The number of symmetric relations which contain (x, y) is 2^9

Option c: True.

The number of relations which is anti-symmetric and contain (x, y) is $2^4 \cdot 3^5$

Option d: True.

The number of relations which is reflexive, symmetric and anti-symmetric is 1

3. **(c)**

Option a:

There are n ordered pairs of the form (x, x), x $\in$ A. For each of the $(n^2 - n)/2$ sets {(x, y), (y, x)} of ordered pairs where x, y $\in$ A, x $\neq$ y, one element is chosen. This results in a maximum value of $n + (n^2 - n)/2 = (n^2 + n)/2$.

Option b:

The number of anti-symmetric relations have size | R | is $2^{(n^2+n)/2}$.

Hence, both option a and b is correct.

4. **(a)**

r – n counts the elements in R of the form (a, b), a $\neq$ b. Since R is symmetric, r – n will be always even.

5. **(1232)**

An element of your relation can come from one of the following two ways:

1. Choice of an unordered pair {i, j}, which gives you two elements of your relation, namely the ordered pairs (i, j) and (j, i) (to maintain symmetry). The number of unordered pairs to choose from is $\binom{8}{2} = 28$

2. A singleton {i}, which gives you only one element of your relation, namely (i, i). The number of singletons to choose from is obviously 8.

One can get 4 elements in the relation in one of the following three ways:

1. Choice of 2 unordered pairs, 0 singletons in $\binom{28}{2}\binom{8}{0}$ ways
2. Choice of 1 unordered pair, 2 singletons in $\binom{28}{1}\binom{8}{2}$ ways
3. Choice of 0 unordered pairs, 4 singletons in $\binom{28}{0}\binom{8}{4}$ ways

So, the total is $\binom{28}{2}\binom{8}{0} + \binom{28}{1}\binom{8}{2} + \binom{28}{0}\binom{8}{4}$

= 1232