All Branches Hinglish

Subject: Engineering Mathematics

DPP-02

Chapter: Vector Calculus

Topic : Gradient of scalar function & directional derivative

1. The directional derivative of the function $f(x, y, z) = x + y$ at the point P(1, 1, 0) along the direction $\hat{i} + \hat{j}$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) $-\sqrt{2}$ (d) 2

2. The derivative of $f(x, y)$ at point (1, 2) in the direction of vector i + j is $2\sqrt{2}$ and in the direction of the vector –2j is –3. Then the derivative of $f(x, y)$ in direction $-i - 2j$ is

(a) $2\sqrt{2} + \frac{3}{2}$ (b) $\frac{-7}{\sqrt{5}}$
(c) $-2\sqrt{2} - \frac{3}{2}$ (d) $\frac{1}{\sqrt{5}}$

3. The directional derivative $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at point P(2, 1, 3) in the direction of the vector $\vec{a} = \vec{i} - \overrightarrow{2k}$ is

(a) $\frac{4}{\sqrt{5}}$ (b) $-\frac{4}{\sqrt{5}}$
(c) $\frac{\sqrt{5}}{4}$ (d) $-\frac{\sqrt{5}}{4}$

4. The maximum value of the directional derivative of the function $\phi = 2x^2 + 3y^2 + 5z^2$ at a point (1, 1, –1) is

(a) 10 (b) –4
(c) $\sqrt{152}$ (d) 152

5. The directional derivative of the scalar function $f(x, y, z) = x^2 + 2y^2 + z$ at the point P = (1, 1, 2) in the direction of the vector $\vec{a} = 3\vec{i} - 4\vec{j}$ is

(a) –4 (b) –2
(c) –1 (d) 1

6. For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, the magnitude of the gradient at the point (1, 3) is

(a) $\sqrt{\frac{13}{9}}$ (b) $\sqrt{\frac{9}{2}}$
(c) $\sqrt{5}$ (d) $\frac{9}{2}$

7. A scalar field is given by $f = x^{2/3} + y^{2/3}$, where x and y are the Cartesian coordinates. The derivative of 'f' along the line $y = x$ directed away from the origin at the point (8, 8) is

(a) $\frac{\sqrt{2}}{3}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{\sqrt{2}}$

8. The magnitude of the gradient of the function $f = xyz^3$ at (1, 0, 2) is

(a) 0 (b) 3
(c) 8 (d) ∞

9. For the function $\phi = ax^2y - y^3$ to represent the velocity potential of an ideal fluid, $\nabla^2\phi$ should be equal to zero. In that case, the value of 'a' has to be

(a) –1 (b) 1
(c) –3 (d) 3

10. The gradient of field $f = y^2 x + xyz$ is

(a) $y(y + z)i + x(2y + z)j + xyk$
(b) $y(2x + z)i + x(x + z)j + xyk$
(c) $y^2 i + 2yxj + xyk$
(d) $y(2y + z)i + x(2y + z)j + xyk$

11. The magnitude of the gradient of the function $f = xyz^3$ at (1, 0, 2) is

(a) 0 (b) 3
(c) 8 (d) ∞

12. Let $f:R^2 \to R$ be defined by

$$f(x,y)=\begin{cases}\dfrac{x^2y}{x^4+y^2} & \text{for}(x,y)\neq(0,0)\\ 0 & \text{for }(x,y)=(0,0)\end{cases}$$

The directional derivative of f at (0, 0) in the direction of the vector $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ is

(a) 0

(b) $\frac{1}{2}$

(c) $\frac{1}{2\sqrt{2}}$

(d) $\frac{1}{4\sqrt{2}}$

Answer Key

1. (b)
2. (b)
3. (b)
4. (c)
5. (b)
6. (c)
7. (a)
8. (c)
9. (d)
10. (a)
11. (c)
12. (a)