# ALL BRANCHES

# ENGINEERING MATHEMATICS

## Lecture No. -10

**Probability**

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# Topics To Be Covered

- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM
- STATISTICS – I (PROBABILITY DISTRIBUTIONS)
- STATISTICS – II (CORRELATION AND REGRESSION)

- Variance $= E(x^2) - [E(x)]^2 = \sigma^2$
- Standard deviation $= \sqrt{\dfrac{\Sigma (x_i - x_m)^2}{n}} = \sqrt{Var(x)}$

## COVARIANCE :-

It is measure of how much two variables change together.
Variance is special case of cov. when two variables are identical.

$$\boxed{Cov(x,Y) = E(xY) - E(x).E(Y)}$$

$$Cov(x,x) = Var(x) = E(x^2) - [E(x)]^2$$

Properties of Var, Cov. and S.D. :-

- If x and Y are independent R.Variables and uncorelated, then $Cov(x,Y) = 0$

- If $Cov = 0$ ; $E(XY) = E(X).E(Y)$
- If $X, Y$ are independent ; $E(X^n Y^n) = E(X^n).E(Y^n)$
- $Cov(X,Y) = Cov(Y,X)$ i.e. symmetric.
- $Cov(aX, bY) = ab\, Cov(X,Y)$

$$Q = aX + bY$$

- $E[Q] = aE[X] + bE[Y]$

$$Q^2 = a^2X^2 + b^2Y^2 + 2abXY$$

- $E[Q^2] = a^2E[X^2] + b^2E[Y^2] + 2ab\,E[XY]$

- $Var(K) = 0$ ; K is constant
- $Var(KX) = K^2\sigma_x^2 = K^2\, Var(X)$
- $Var(aX+b) = a^2\sigma_x^2 = a^2\, Var(X)$

- $Var(ax+by) = a^2 Var(x) + b^2 Var(Y) + 2ab\, Cov(x, Y)$
- If X and Y are orthogonal, then $E(XY) = R_{xy} = 0$
- Coefficient of variation $= \dfrac{\sigma}{\mu} = \dfrac{S.D.}{Mean}$
- Standard error of mean $= \dfrac{\sigma}{\sqrt{n}}$, n is sample size

If X and Y are independent;

$$Var(ax+by) = a^2 Var(x) + b^2 Var(Y)$$

## 2 random variables :-

i) Minimum of 2 R.V.s .

$$P(\min(x,y) > K) = P\{(x > K) \cap (y > K)\} = P(x>K).P(y>K)$$

greater

$$P(\min(x,y) < K) = 1 - P(\min(x,y) > K)$$

$$1 - P\{(x > K) \cap (y > K)\}$$

ii) Maximum of 2 R.V.s .

$$P(\max(x,y) < K) = P\{(x < K) \cap (y < K)\} = P(x<K).P(y<K)$$

lesser

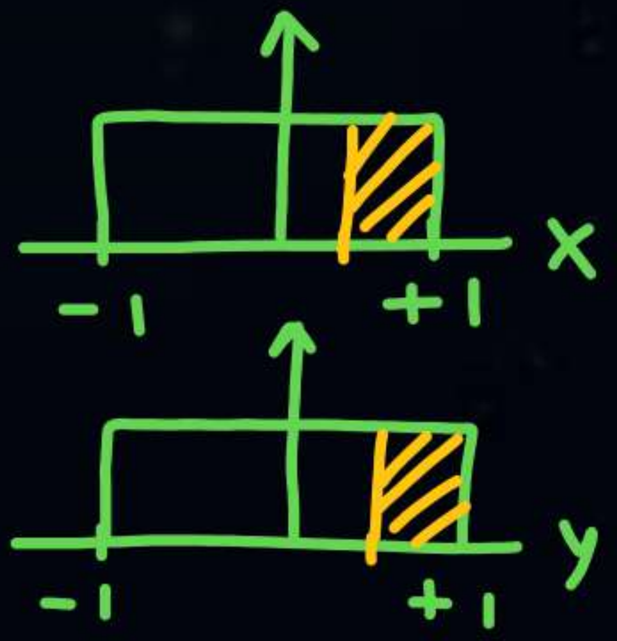
$$P(\max(x,y) > K) = 1 - P(\max(x,y) < K)$$

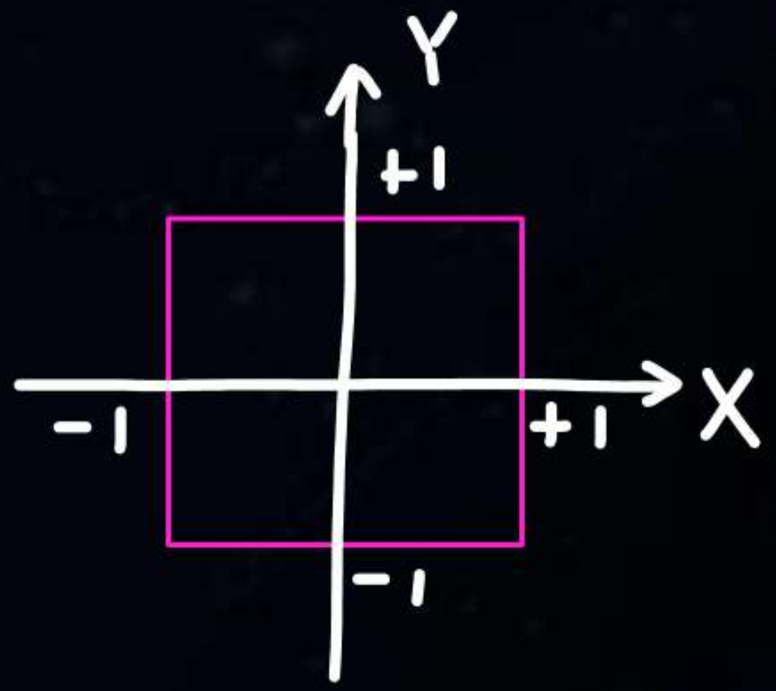
$$1 - P\{(x < K) \cap (y < K)\}$$

**Q.** If X and Y are 2 independent uniform R.V.s

$X \to (-1, +1)$

$Y \to (-1, +1)$




i) $P\{\min(x, y) > 1/2\}$

ii) $P\{\max(x, y) < 1/2\}$

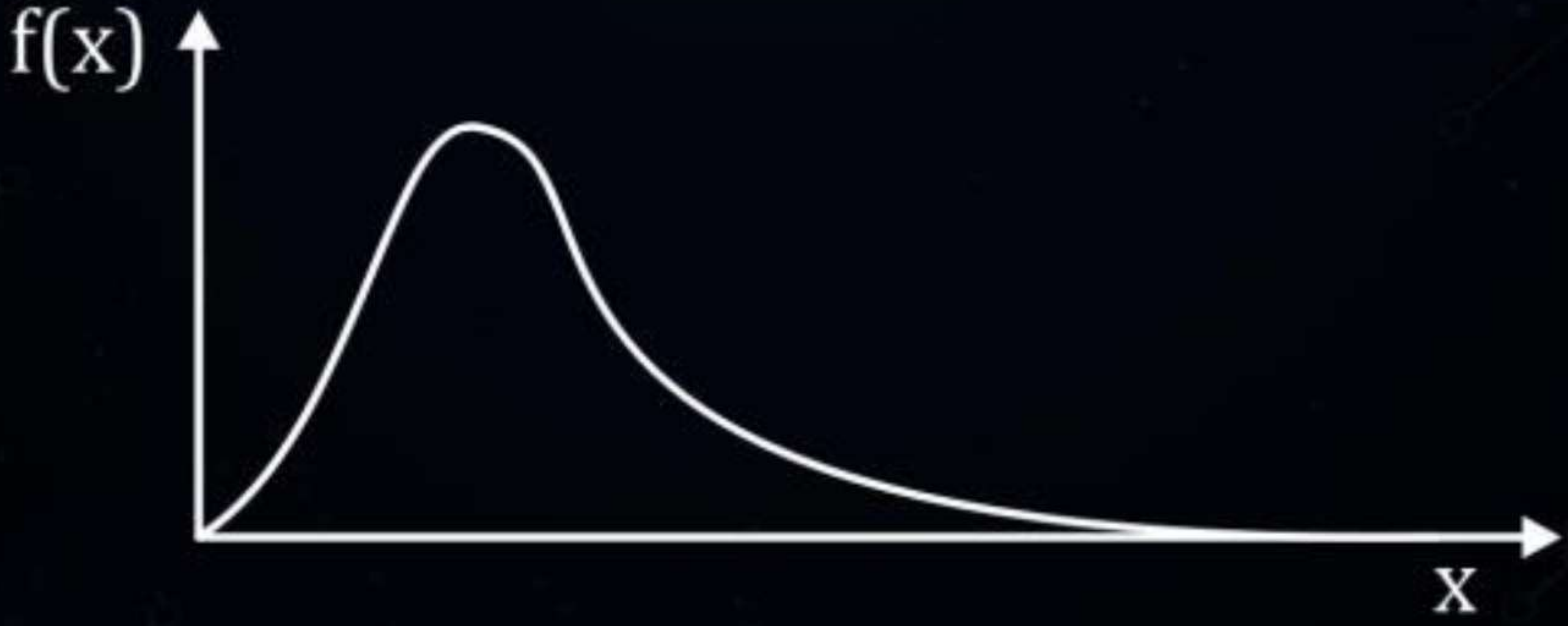
i) $P\{\min(x, y) > 1/2\} = P\{(x > 1/2) \cap (y > 1/2)\}$

$= P(x > 1/2) \cdot P(y > 1/2) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

ii) $P\{\max(x, y) < 1/2\} = P\{(x < 1/2) \cap (y < 1/2)\}$

$= P(x < \frac{1}{2}) \cdot P(y < \frac{1}{2}) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

Q. A probability distribution with right skew is shown in the figure.




The correct statement for the probability distribution is

A Mean is equal to mode

B Mean is greater than median but less than mode

C Mean is greater than median and mode

D Mode is greater than median

**Q.** **The life of a bulb(in hours) is a random variable with an exponential distribution. The probability that its value lies between 100 and 200 hours is__________. Let $\alpha$ be the parameter of the given distribution.**

Exponential C.R.V. $\quad X \rightarrow$ life of bulb


$$f(x) = \alpha e^{-\alpha x} \; ; \; x > 0$$

$$P(100 < X < 200) = \int_{100}^{200} \alpha e^{-\alpha x}\, dx = \frac{\alpha}{-\alpha}\left[e^{-\alpha x}\right]_{100}^{200}$$

$$= -\left[e^{-200\alpha} - e^{-100\alpha}\right]$$

**Q.** The random variable X takes on the values 1, 2 or 3 with probabilities $2 + 5P/5$ , $1 + 3P/5$, $1.5 + 2P/5$ respectively. The values of P and E(X) are respectively

**A** 0.05, 1.87

**B** 1.90,5.87

**C** 0.05, 1.10

**D** 0.25,1.40

**Q.** Let X be a random variable with probability density function

$$f(x)\begin{cases}0.2, & \text{for } |X| \leq 1\\ 0.1, & \text{for } 1 < |X| \leq 4\\ 0, & \text{otherwise}\end{cases}$$

The probability $P(0.5 < X < 5)$ is____.

**Q.** The graph of a function f(x) is shown in the figure


$\frac{1}{2} \times 1 \times h + \frac{1}{2} \times 1 \times 2h + \frac{1}{2} \times 1 \times 3h = 1$

$h = 1/3$

For f(x) to be a valid probability density function the value of h is

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A. $1/3$

B. $2/3$

C. $1$

D. $3$

**Q.** The number of parameters in the univariate exponential and Gaussian distributions, respectively are

Exponential $\rightarrow \lambda e^{-\lambda x}$ $(\lambda \rightarrow 1)$

Gaussian $\rightarrow \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $(\mu, \sigma \rightarrow 2)$

A) 2 and 2

B) 1 and 2

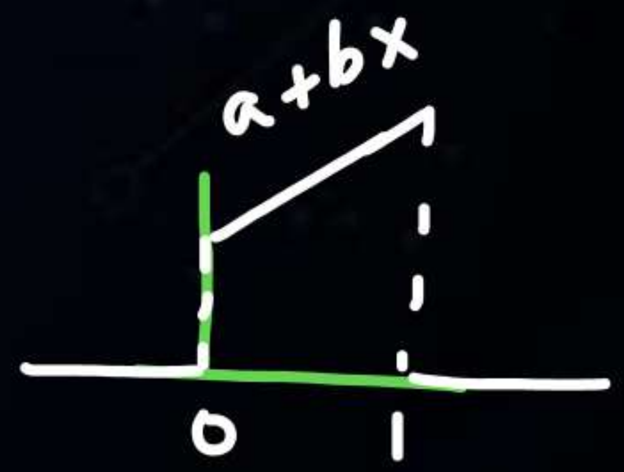
C) 2 and 1

D) 1 and 1

**Q.** For the function $f(x) = a + bx$, $0 \leq x \leq 1$, to be a valid probability density function, which one of the following statements is correct?

$$\int_0^1 (a+bx)\,dx = 1 \qquad \left[ax + b\frac{x^2}{2}\right]_0^1 = 1$$

$$a + \frac{b}{2} = 1 \quad \text{—1)}$$




$0.5 + x$

**A** $a = 1, b = 4$

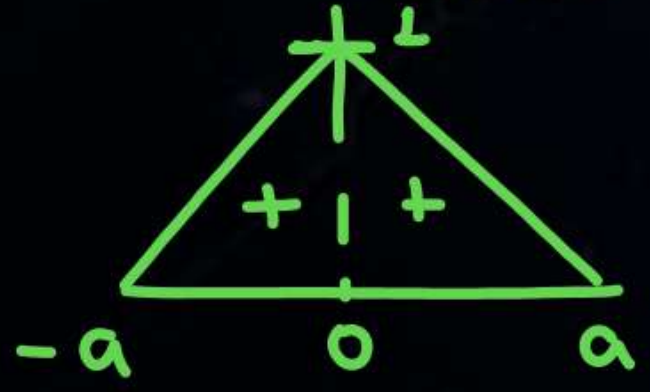
**B** $a = 0.5, b = 1$ ✓

**C** $a = 0, b = 1$

**D** $a = 1, b = -1$

**Q.** If $f(x)$ and $g(x)$ are two probability density functions,

$$f(x) = \begin{cases} \dfrac{x}{a} + 1: & -a \le x < 0 \\ -\dfrac{x}{a} + 1: & 0 \le x \le a \end{cases}$$

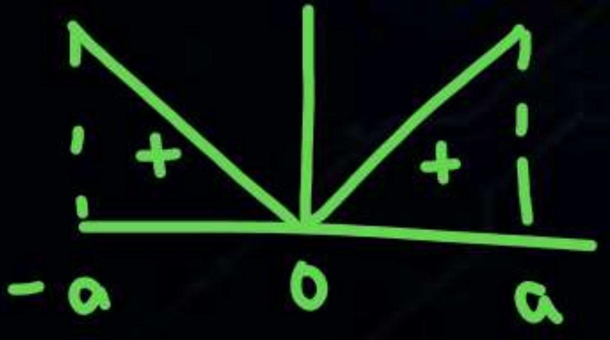



$$E(x) = \int_{-a}^{+a} \overbrace{\underbrace{x}_{\text{Odd}} \cdot \underbrace{f(x)}_{\text{Even}}}^{\text{Odd}} \, dx$$

$$= 0$$

$$Var(x) = E(x^2) = a^3/6$$

$$g(x) = \begin{cases} -\dfrac{x}{a}: & -a \le x < 0 \\ \dfrac{x}{a}: & 0 \le x \le a \\ 0: & \text{otherwise} \end{cases}$$




$$E(x) = 0$$

$$Var(x) = E(x^2) = a^3/2$$

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## Which of the following statement is true?

**A** Mean of f(x) and g(x) are same: Variance of f(x) and g(x) are same

**B** Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are different

**C** Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are same

**D** Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are different

**Q.** Given that x is a random variable in range $[0, \infty]$ with a probability density function $e^{-x/2}/K$, the value of the constant K is_____.

$$\int_0^\infty \frac{e^{-x/2}}{K}\,dx = 1$$

$$\frac{\left[e^{-x/2}\right]_0^\infty}{-\frac{1}{2}K} = -\frac{2}{K}\left[0 - 1\right] = 1$$

$$\frac{2}{K} = 1$$

$$\boxed{K = 2}$$

**Q.** $P_X(X) = Me^{(-2|x|)} + Ne^{(-3|x|)}$ is the probability density function for the real random variable X, over the entire x-axis, M and N are both positive real numbers. The equation relating M and N is

$$\int_{-\infty}^{+\infty} Me^{-2|x|} + Ne^{-3|x|}\,dx = 1$$

**A** $M + \frac{2}{3}N = 1$

$$2\frac{M}{2} + 2\frac{N}{3} = 1$$

$$M + 2\frac{N}{3} = 1$$

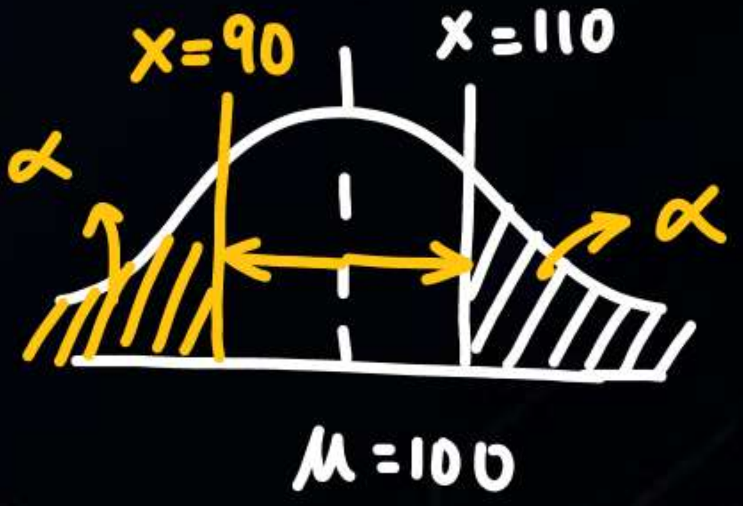
**B** $2M + \frac{1}{3}N = 1$

**C** $M + N = 1$

**D** $M + N = 3$

**Q.** For a random variable $x(-\infty < x < \infty)$ following normal distribution, the mean is $\mu = 100$. If the probability is $P = \alpha$ for $x \geq 110$. Then the probability of x lying between 90 and 110 i.e., $P(90 \leq x \leq 110)$ and equal to

$$P(90 < x < 110) = 1 - 2\alpha$$




(A) $1 - 2\alpha$

(B) $1 - \alpha$

(C) $1 - \alpha/2$

(D) $2\alpha$

# CORELATION & REGRESSION

## Curve fitting →

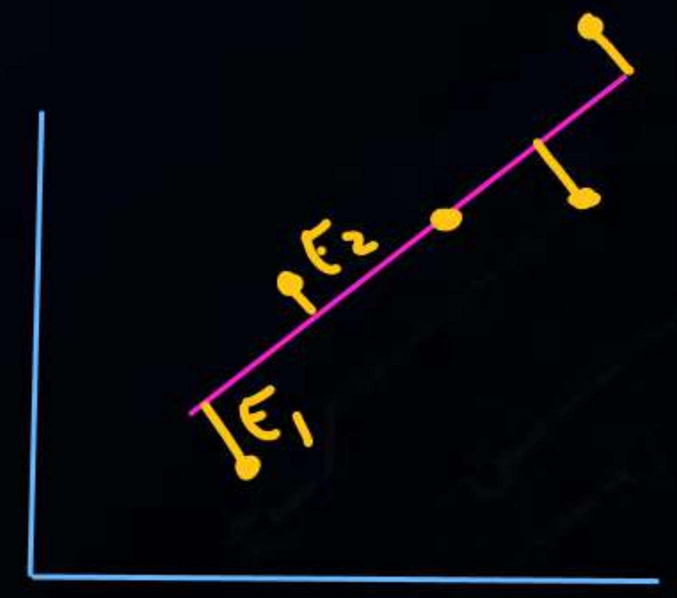
→ To form an eqn. of an approx. curve from given data.

→ To find relationship b/w two variable by an algaebric equation.

## Method of least squares →

$$E = E_1^2 + E_2^2 + E_3^2 + \dots E_n^2$$

Sum of distances of points from curve of best fit should be minimum (E should be minimum).



# Fitting of a straight line:-

Let $(x_i, y_i)$; $i = 1, 2, 3, \ldots n$ be set of observations.

let $\boxed{y = a + bx}$ be the line of best fit.

The residual at $x = x_i$ will be

$$E_i = y_i - f(x_i)$$

$$E_i = y_i - (a + bx_i)$$

Minimize this quantity $\quad u = \sum E_i^2 = \sum \left(y_i - (a + bx_i)\right)^2$

By method of least squares, u should be minimum.

$$\frac{\partial u}{\partial a} = 0, \quad \frac{\partial u}{\partial b} = 0$$

$$2\sum\{y_i-(a+bx_i)\}(-1)=0 \qquad \frac{\partial u}{\partial a}=0$$

$$2\sum\{y_i-(a+bx_i)\}(-x_i)=0 \qquad \frac{\partial u}{\partial b}=0$$

$$\boxed{\begin{aligned}\sum y &= an + b\sum x \quad &-1)\\ \sum xy &= a\sum x + b\sum x^2 \quad &-2)\end{aligned}}$$

Normal equations of line $y=a+bx$

On solving, find a and b.

Ex:- Use method of least squares, fit a straight line from following data:-

| x | y | xy | $x^2$ |
|---|---|---|---|
| 0 | 5.012 | 0 | 0 |
| 2 | 10 | 20 | 4 |
| 4 | 15 | 60 | 16 |
| 5 | 21 | 105 | 25 |
| 6 | 30 | 180 | 36 |

$n=5$

$a=?\ ,\ b=?$

$y=a+bx$

# Thank you

# GW Soldiers !