CS & IT ENGINEERING

Discrete maths
Graph Theory

Lecture No 07

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01 Definition In Connectivity

02 Connected vs Disconnected

03 Range of Edges

04 Concepts of tree

05 Connectivity theorem

Connectivity in Graphs

1 ≡ 2 ≡ 3 ≡ 4
1 OR 2 OR 3 OR 4

connected Graph (k = 1)

$G = (V, E)$

Range of edges (k = 1)

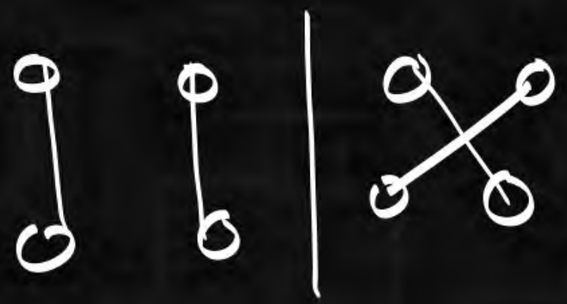
Tree

1. $\left\{\begin{array}{l} n-1 \\ + \\ \text{connected} \end{array}\right. \leq e \leq \frac{n(n-1)}{2}$ (kn)
2. connected + it does not contains cycle (acyclic Graph)
3. unique Path is available betn all pair of vertices.
4. minimally connected.

Connectivity in Graphs

n = 4

n = 4 e = 2.



n = 4 e = 1

n e = n-1

min + connected.

n = 4 e = 3

n = 5 e = 4

n = 4 e = 3

Connected

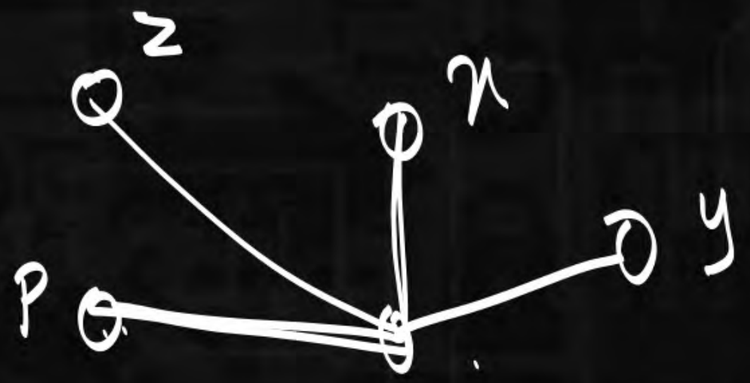
Disconnected.

Connectivity in Graphs

$n = 5 \quad e = 4$

Connected + $(n-1)$ → does not have cycle.

OR



Connectivity in Graphs

Range of edges ($k \geq 2$) ← DC

Forest

$$* \quad n-k \leq e \leq \frac{(n-k)(n-k+1)}{2} \quad *$$

$n-k$ → {min. no. of edges

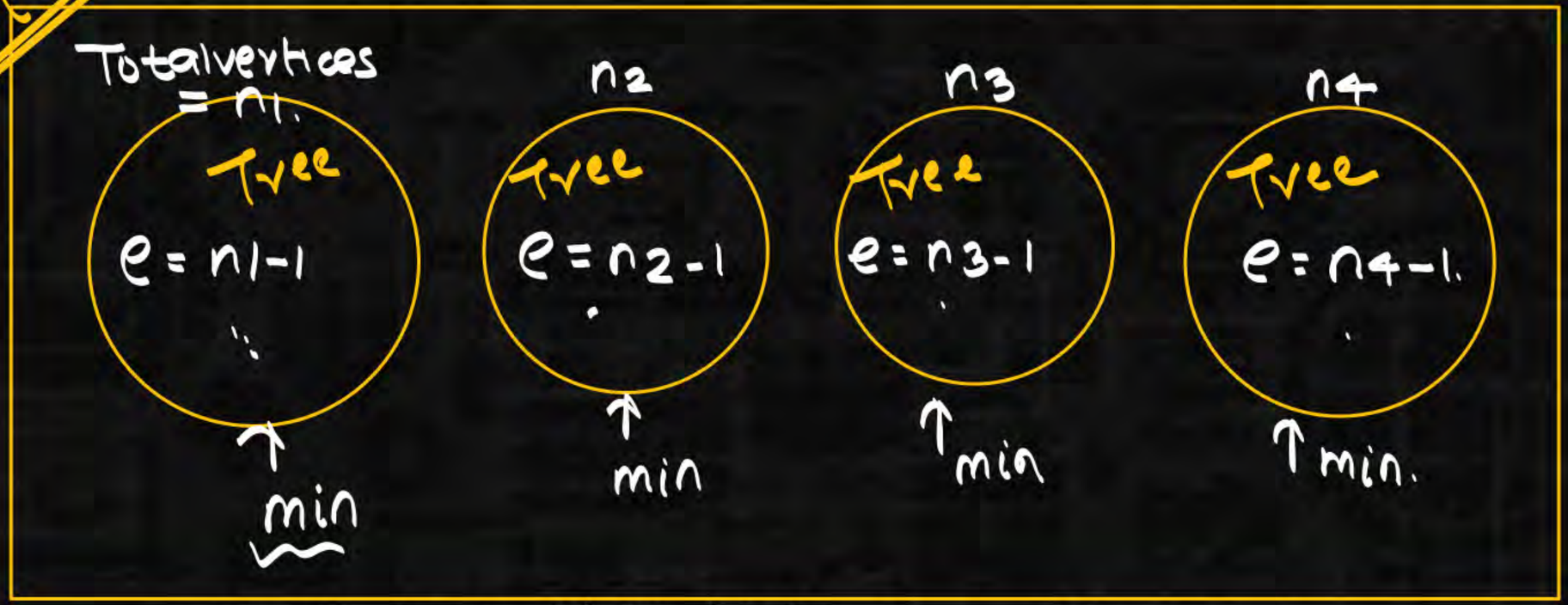
Forest : Collection of Trees.

$? = n-k$

Connectivity in Graphs

$n_1+n_2+n_3+n_4=n$. Disconnected. $K=4$.

Forest



$\Rightarrow$ min. no. of edges.

$e=n_1-1+n_2-1+n_3-1+n_4-1$

$=(n_1+n_2+n_3+n_4)-4$

$=n-4$

$e=n-4$

$e=n-K$ (min no. of edges)

Connectivity in Graphs

Disconnected Graph contains connected subparts.

$$e = n_1 - 1 + n_2 - 1 + n_3 - 1 + n_4 - 1.$$

$$= (n_1 + n_2 + n_3 + n_4) - 4.$$

$$= n - 4 = n - K.$$

Connectivity in Graphs

Consider undirected Graph having 10 vertices & 2 components.

$n = 10 \quad k = 2$

{→ min no of edges.
{→ max no of edges.

min no of edges

$e = n - k$

$= 10 - 2 = 8$

max no of edges = $e = (n-k)(n-k+1)/2$.

$= (10-2)(10-2+1)/2 = \frac{8 \cdot 9}{2} = 36$.

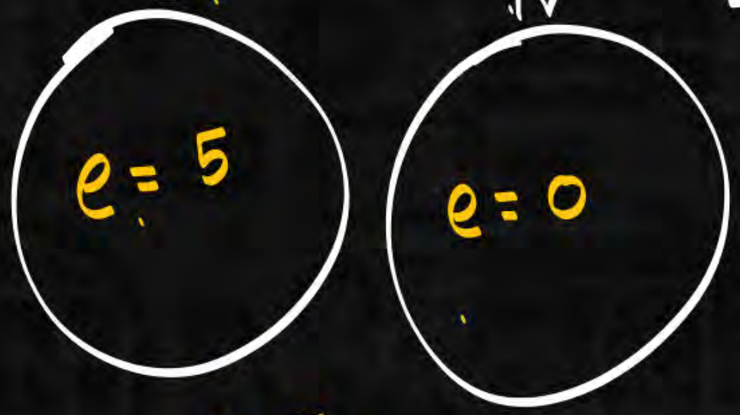
Connectivity in Graphs

Consider an undirected Graph having 7 vertices & 2 components.

$n = 7 \quad k = 2.$

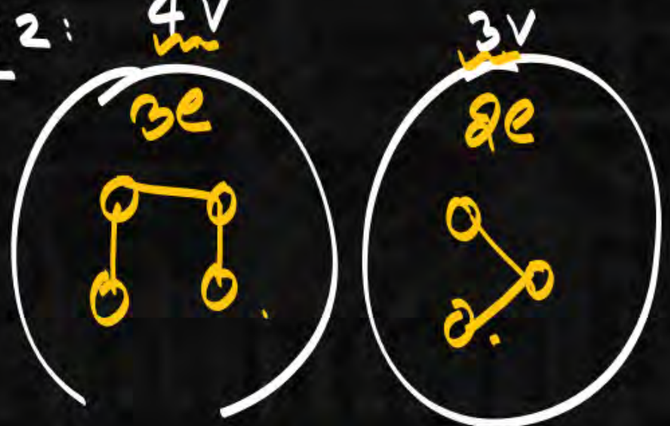
$e = n - k = 7 - 2 = 5$ (min no of edges)

Case 1:



$e = 5$

Case 2:



$= 5e$

Case 3

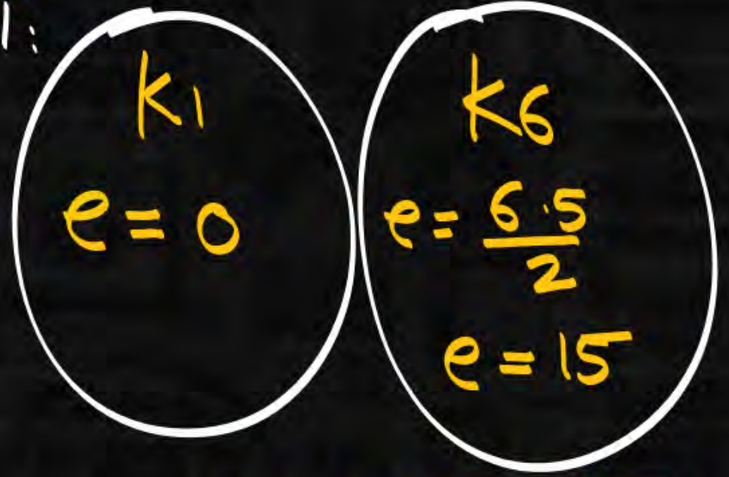
$4 + 1 = 5e.$

Connectivity in Graphs

$n = 7 \quad k = 2 \quad$ max no of edges $= e = \frac{(n-k)(n-k+1)}{2} \rightarrow K_1 \; K_6$

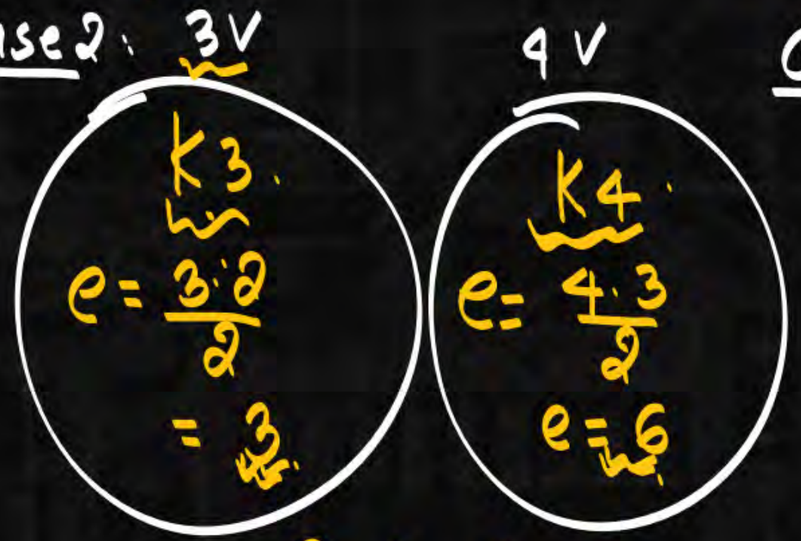
$= \frac{5 \cdot 6}{2} = 15e.$

Case1: ✓



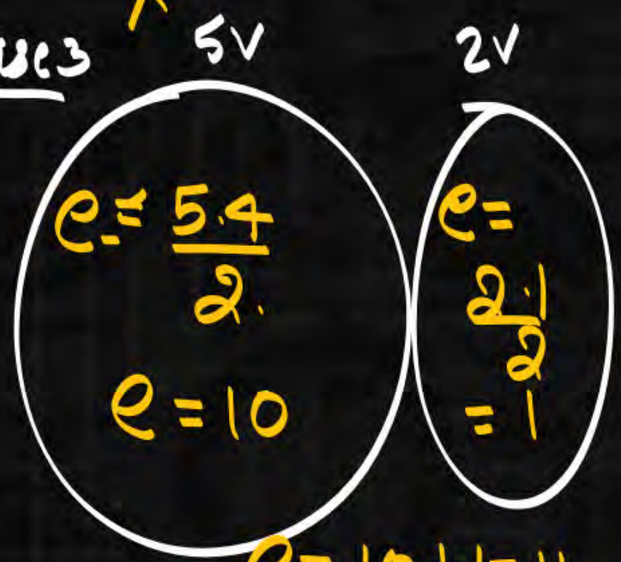
$e = 0 + 15 = 15$

Case2: 3V ✗



$e = 3 + 6 = 9$

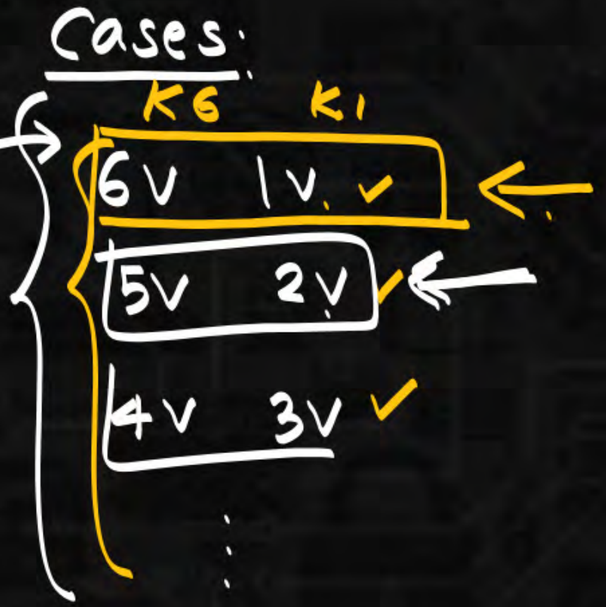
Case3 ✗



$e = 10 + 1 = 11.$

Connectivity in Graphs

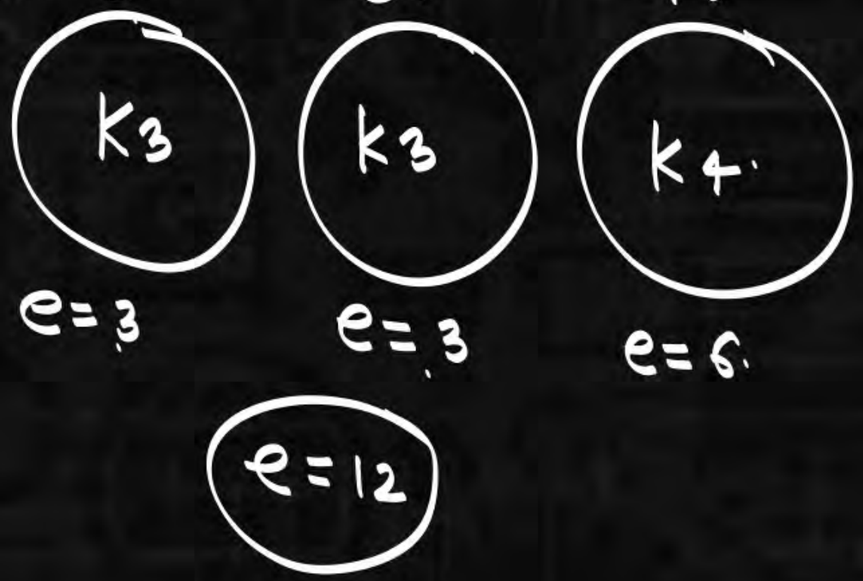
$n = 7 \quad k = 2 \quad e = 15$



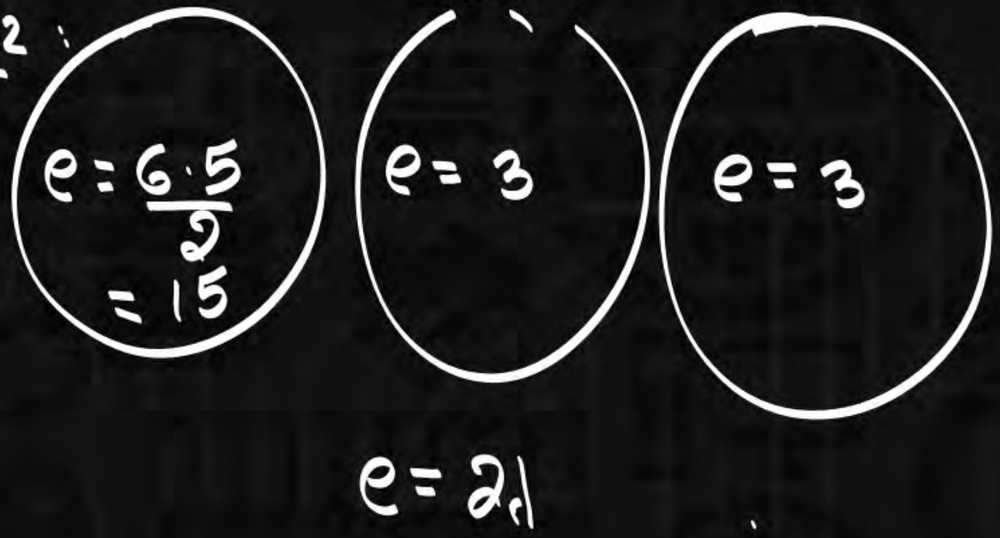
Connectivity in Graphs

$n = 10 \quad k = 3 \quad e = \frac{(n-k)(n-k+1)}{2} = \frac{(10-3)(10-3+1)}{2} = \frac{7 \cdot 8}{2} = 28e$

Case 1: 3V, 3V, 4V



Case 2: 6V, 3V, 1V



e = 21

. . .

Connectivity in Graphs

$n = 10 \quad k = 3$

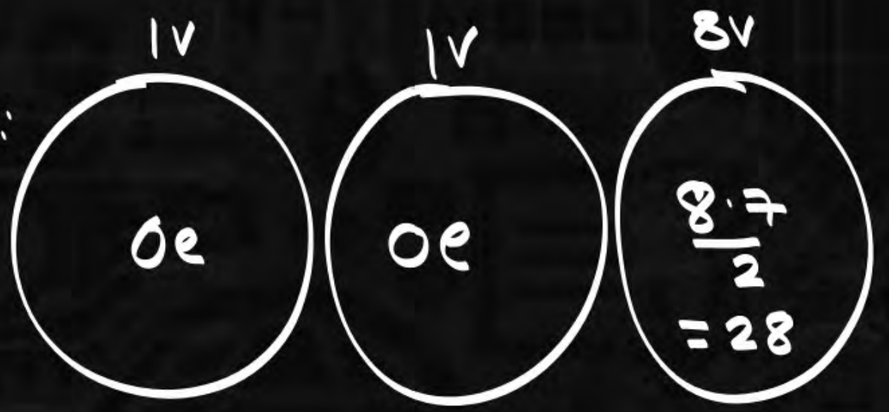
$e = 28$

2V 2V 6V

3V 3V 4V

1V 1V 8V

Case 3:



$e = 0 + 0 + 28 = 28$

Connectivity in Graphs

* Consider a disconnected Graph of 10 vertices, what will be maximum no of edges?

$(k \geq 2)$

$n = 10 \quad k \geq 2$.

missing component.

$k = 2 \quad k = 3 \quad k = 4 \quad k = 5 \ldots$

$k \uparrow \; e \downarrow$

$$e = \frac{(n-k)(n-k+1)}{2}$$

$$= (10-2)(10-2+1)/2$$

$$\frac{8 \cdot 9}{2} = 36e$$

$$e = (10-3)(10-3+1)/2$$

$$= \frac{7 \cdot 8}{2} = 28.$$

Connectivity in Graphs

K_1 K_9

* Consider a disconnected Graph of 10 vertices, what will be maximum no of edges ?

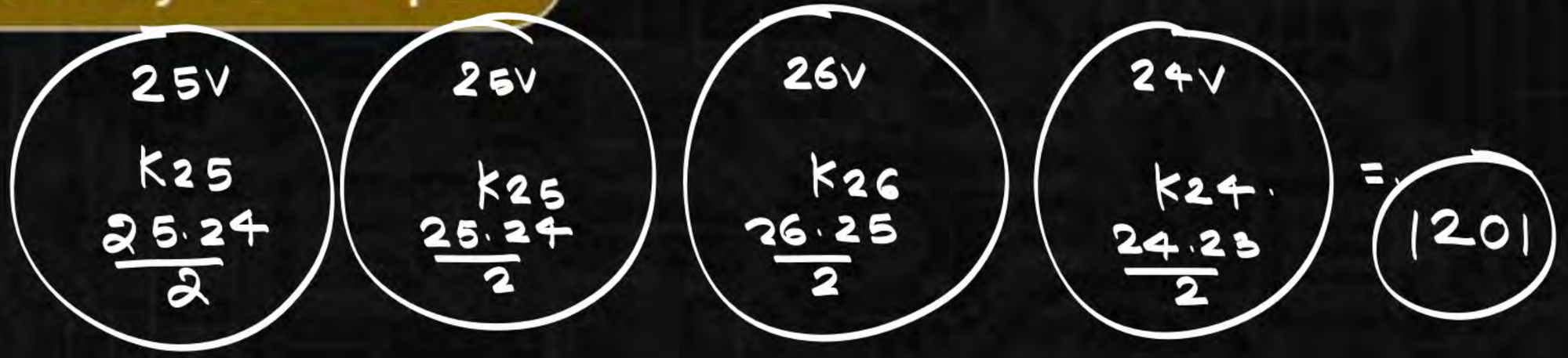
($K \geq 2$)

complement

Star Graph

$\Rightarrow K_{1,9}$

Connectivity in Graphs



* Consider a disconnected of 4 components. what will be max no of edges?

25 vertices → 1st
25 vertices → 2nd
26V → 3
24V → 4

Connectivity in Graphs

True/false?

Consider a Graph G 10 vertices if $\delta(G) \geq 5$ then it is connected?

True!

Connectivity in Graphs

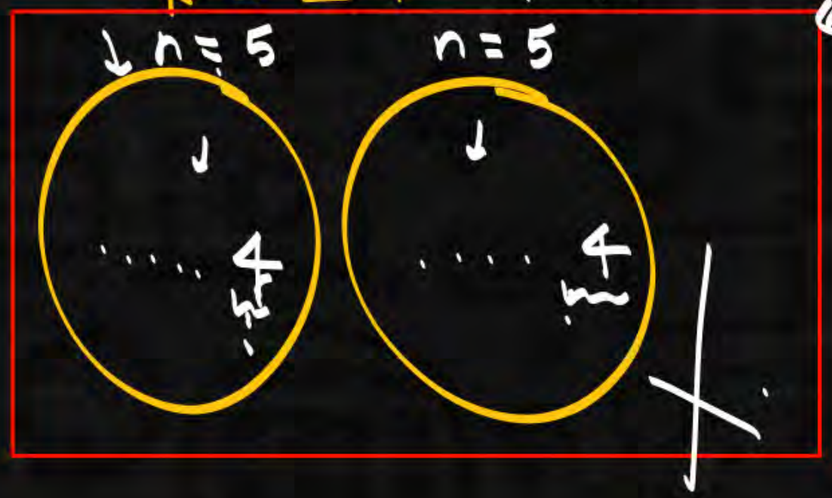
Connected ✓

$n = 10$ $\delta(G) \geq 5$ degrees of all vertices are atleast 5.

Assume: let's take disconnected. (many). $k \geq 2$.

$\delta(G) \geq 5$

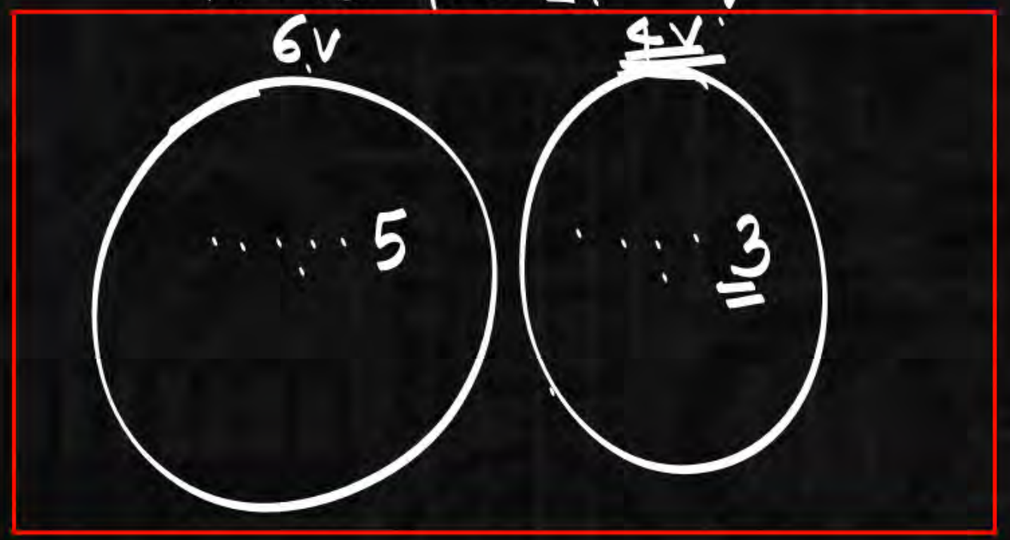
$k = 2$. $n = 10$



$\delta(G) \geq 5$ Given

Degrees should start from 5.

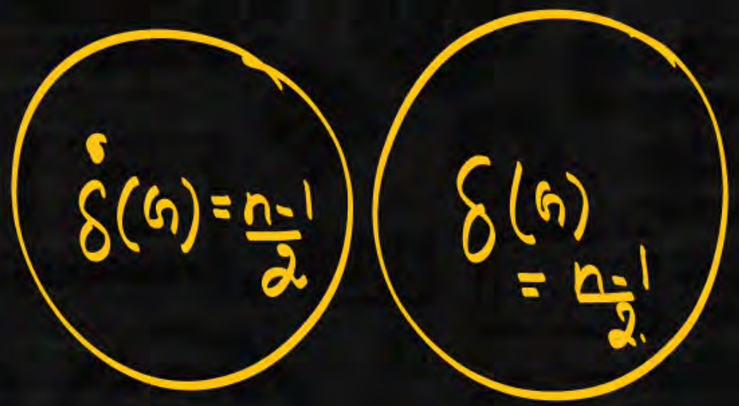
$n = 10$ $k = 2$.



Connectivity in Graphs

Thm: if $\delta(G) \geq \frac{n-1}{2}$ then it is connected Graph.

Assm: it is disconnected.



$V_1 = \frac{n-1}{2} + 1 \quad V_2 = \frac{n-1}{2} + 1$

$V = V_1 + V_2$

$= \frac{n-1}{2} + 1 + \frac{n-1}{2} + 1.$

$= \frac{2(n-1)}{2} + 2.$

$= n - 1 + 2 = (n+1)$

Total vertices $= n.$

(Ass is wrong)

it is connected.

Connectivity in Graphs

max no of edges.

$$= \frac{(n-k)(n-k+1)}{2}$$

$k = 2$ (2 component)

$$\frac{(n-2)(n-2+1)}{2}$$

$$* \quad e = \frac{(n-2)(n-1)}{2}$$

$n = 5$

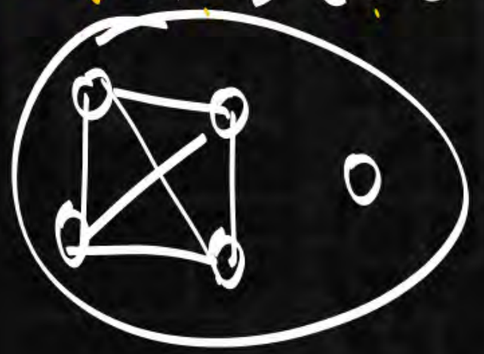
$$e = \frac{(n-1)(n-2)}{2}$$

$$e = (5-1)(5-2)/2$$

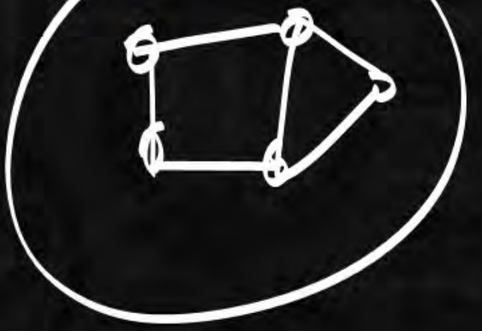
$$= 4 \cdot 3/2$$

$e = 6$

Q. C.Y $n = 5$ $e = 6$



C.Y

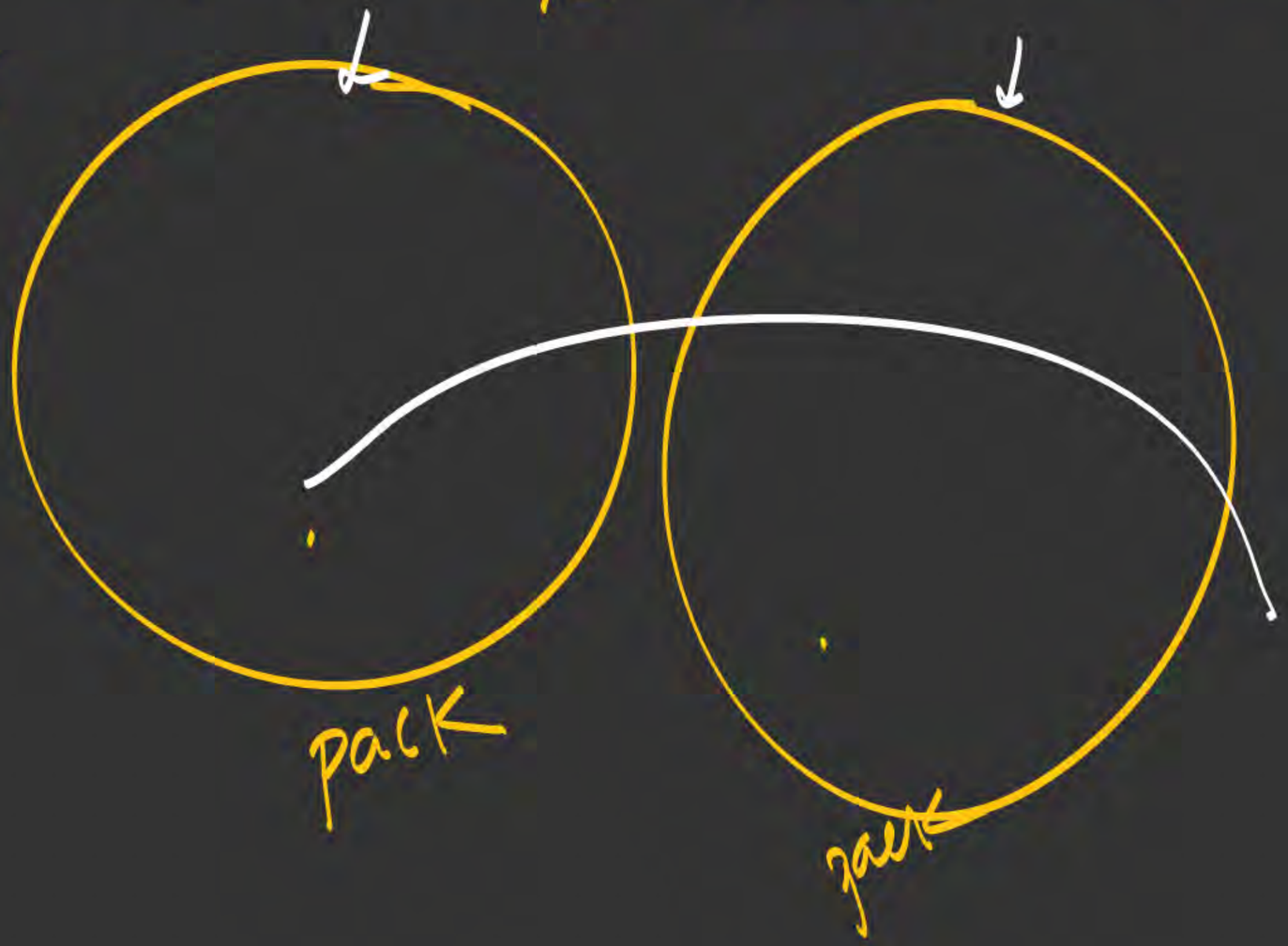


$e = (n-k)(n-k+1)/2$

$k = 2$

$e = \frac{(n-1)(n-2)}{2} + 1$

Q.C



C

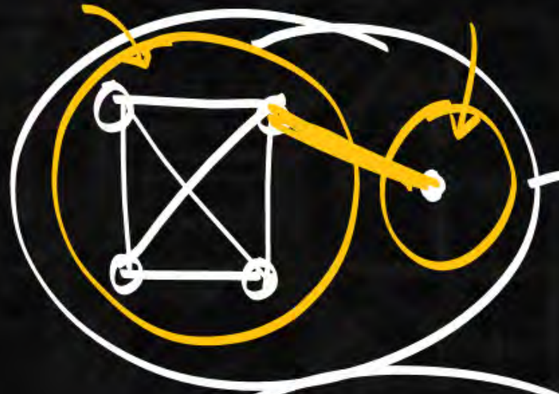
Connectivity in Graphs

if $e = \frac{(n-1)(n-2)}{2} + 1$ then it is connected (false) → may / may not.

$n = 5 \quad e = \frac{(5-1)(5-2)}{2}$

$= \frac{4 \cdot 3}{2} = 6e$

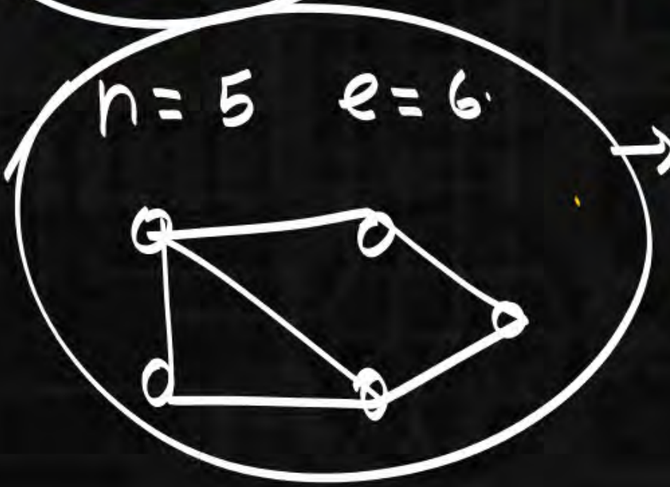
$n = 5 \quad e = 6$;



→ D.C + 1 edg

Conn.

$n = 5 \quad e = 6$



→ C. + 1 edg

Con

Connectivity in Graphs

Thm: if Graph contains more than $\frac{(n-1)(n-2)}{2}$ no.of edges it will be connected ✓.

@ satishsirP


THANK YOU GW
SOLDIERS !