Univariate and multivariate linear regression

May 8, 2019

1 Univariate Linear Regression Programming Excercise

[1]:
```
# %load ../../standard_import.txt
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

from sklearn.linear_model import LinearRegression
from mpl_toolkits.mplot3d import axes3d

pd.set_option('display.notebook_repr_html', False)
pd.set_option('display.max_columns', None)
pd.set_option('display.max_rows', 150)
pd.set_option('display.max_seq_items', None)

#%config InlineBackend.figure_formats = {'pdf',}
%matplotlib inline

import seaborn as sns
sns.set_context('notebook')
sns.set_style('white')
```

1.1 Linear Regression with one variable

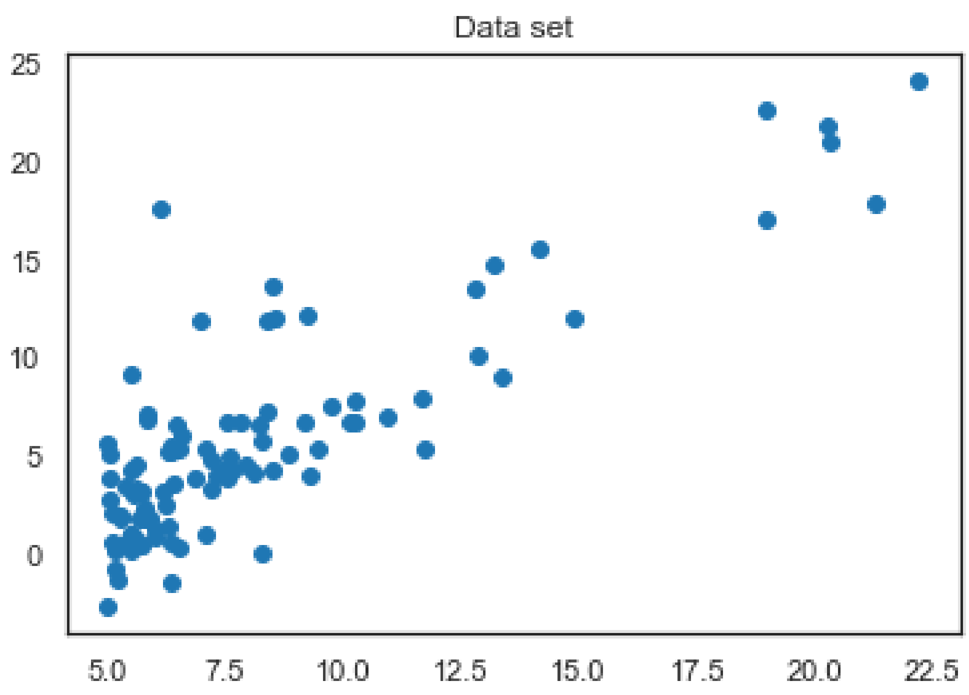
[2]:
```
data = np.loadtxt('data/ex1data1.txt', delimiter=',')
print(data.shape)
```

```
(97, 2)
```

[3]:
```
x = np.c_[np.ones(data.shape[0]), data[:, 0]]
y = np.c_[data[:, 1]]
```

[4]:
```
plt.scatter(x[:,1],y)
plt.title('Data set')
```

[4]:
```
Text(0.5, 1.0, 'Data set')
```



1.2 Cost function

A cost function has to be used to track how are training phase is going and as a metric of goodness. The cost function has the following form:

$$J(\theta) = \frac{1}{2 * m} \sum_{j=1}^{m} (h_\theta(x^{(j)}) - y^{(j)})^2$$

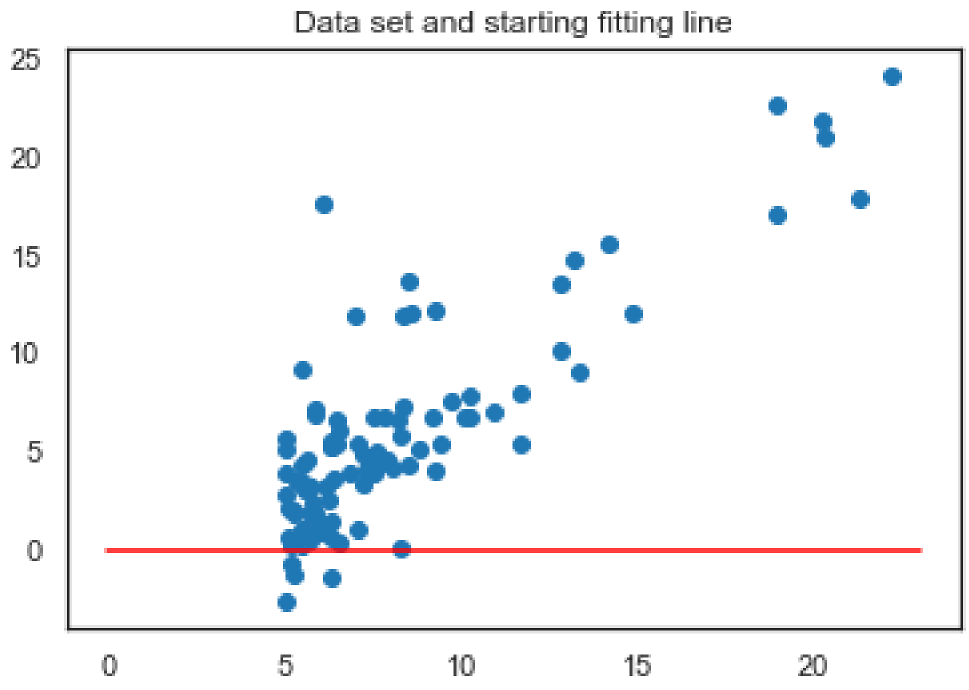
where: - $h_\theta(x^{(j)})$ represents the prediction that our model do - $y^{(j)}$ is the true value. In the best case, we'd like our model to predict this value - m is the number of sample at our disposal in the training set

[5]:
```
theta = [[0],[0]]
```

[6]:
```
def calculate_cost(x, y, theta = [[0],[0]]):
    h = x.dot(theta)
    return 1/(2 * data.shape[0]) * np.sum(np.square(h - y))
```

[7]:
```
plt.scatter(x[:,1],y)
plt.plot(np.arange(0, 24) * theta[1] + theta[0], 'r')
plt.title('Data set and starting fitting line')
```

[7]:
```
Text(0.5, 1.0, 'Data set and starting fitting line')
```



In the graph above it is possibile to see the line representing the parameters drawn with the red color. **Our model is not trained yet, so it doesn't follow the data**.

Batch Gradient Descent Let's use the *batch gradient descent*, wich uses the whole training set to predict the parameters that approximate the data in the best possible way. The problem with this approach is that it becomes slower as the dataset becomes bigger. For this reason, it is best to use this approach only with small training sets.

Since we're trying to reduce the cost function, we will consider the gradient of this function, which can be written in two version: - Scalar version - Vectorized version

1.2.1 Gradient function

Scalar version

$$\frac{\partial J}{\partial \theta_i} = \frac{1}{m}(h_\theta x^{(i)} - y^{(i)})x^{(i)}$$

Vectorized version

$$\frac{\partial J}{\partial \theta}(\theta) = \frac{1}{m}x^T(x\theta - y)$$

Gradient descent The gradient just introduced has to be used in a method (an iterative one). We will implement an algorithm that uses the following formula:

$$\theta^{(k)} = \theta^{(k-1)} - \frac{\alpha}{m}\frac{\partial J}{\partial \theta_i}(\theta^{(k)})$$

At each itearation, the current estimatate of the parameters get updated using the last estimate and a term that depends on a learning rate α and the gradient value.

```
[8]: iterations_to_do = 1500
     initial_cost = calculate_cost(x, y)
     costs = [initial_cost]
```

```
[9]: def gradient_descent(x, y, alpha = .1, theta=[[0],[0]], iterations = 1500):
         for i in range(0, iterations):
             theta = theta - (alpha / data.shape[0]) * (x.T).dot(x.dot(theta) - y)
             costs.append(calculate_cost(x, y, theta))
         return theta
```

```
[10]: final_theta = gradient_descent(x, y, alpha = .01, theta=[[0],[0]], iterations =␣
       ↪iterations_to_do)

      print("The estimated parameters that fit our date are: \n{}".
       ↪format(final_theta))
```

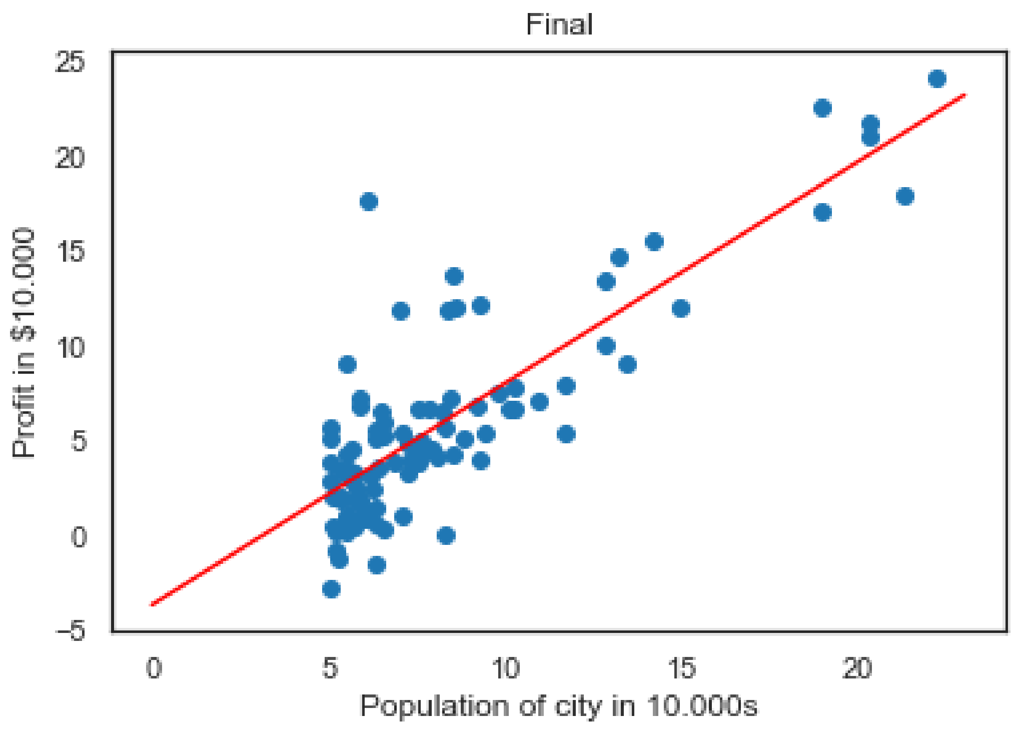
```
The estimated parameters that fit our date are:
[[-3.63029144]
 [ 1.16636235]]
```

1.3 Final results

1.3.1 Fitted lines

```
[11]: plt.scatter(x[:,1],y)
      plt.plot(np.arange(0, 24) * final_theta[1] + final_theta[0], 'r')
      plt.title('Final')
      plt.xlabel('Population of city in 10.000s')
      plt.ylabel('Profit in $10.000')
```

```
[11]: Text(0, 0.5, 'Profit in $10.000')
```



It is now possible to see the line that we can use to do predictions. It is the best (linear) line possible that approximate the sample in the training set used. ### Errors We can see how the error is affected when the parameters change. Let's first see the differences between the first and the last iteration. #### First iteration

[12]:
```
print("The error at the first iteration is: {}".format(initial_cost))
```

```
The error at the first iteration is: 32.072733877455676
```

Last iteration

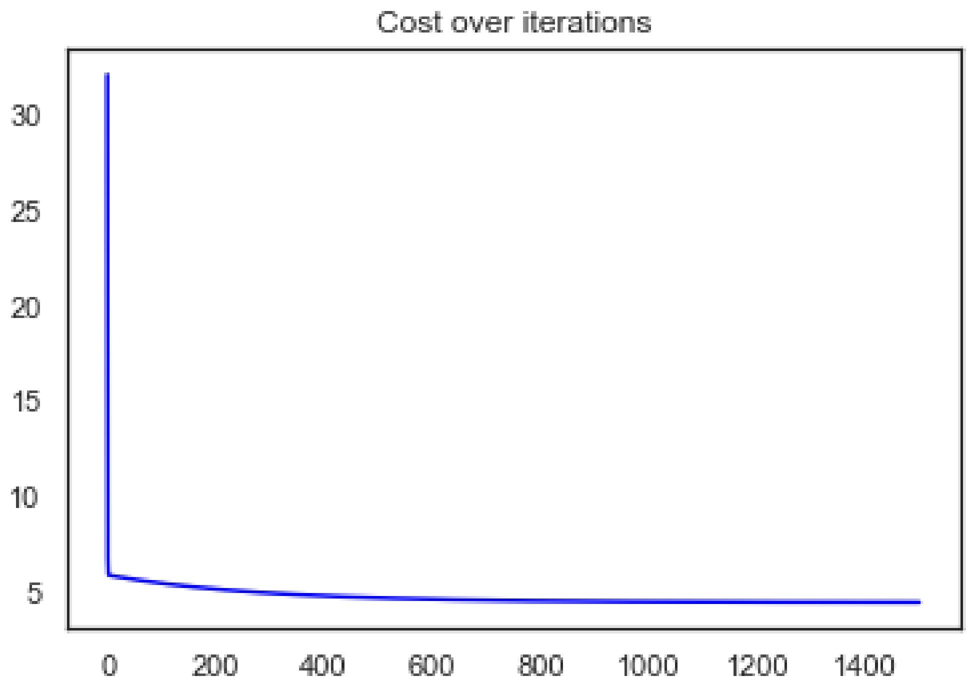
[13]:
```
print("The error at the last iteration is: {}".format(calculate_cost(x, y,␣
  ↪final_theta))
```

```
The error at the last iteration is: 4.483388256587725
```

Overall cost on different iteration Let's draw how the cost changes at each iteration

[14]:
```
plt.plot(np.arange(0, iterations_to_do + 1, 1), costs, 'b')
plt.title('Cost over iterations')
```

[14]:
```
Text(0.5, 1.0, 'Cost over iterations')
```



1.3.2 Prediction from learned parameters

We'll now use the parameters found to predict: 1. The profit for a population of 3.5k people 2. The profit for a population of 7k people

[15]:
```
print("The profit for a population of 3.5k people is: {}".format(final_theta.T.
  ↪dot([1, 3.5])*10000))
print("The profit for a population of 7k people is: {}".format(final_theta.T.
  ↪dot([1, 7])*10000))
```

```
The profit for a population of 3.5k people is: [4519.7678677]
The profit for a population of 7k people is: [45342.45012945]
```

1.3.3 Show if the optimization has gone right

In order to see if the optimization of the cost function worked, we can use the contour plot to see where the parameter are with respect to the variation of the parameters and, consequently, the cost function.

[16]:
```
# Create grid coordinates for plotting
B0 = np.linspace(-10, 10, 50)
B1 = np.linspace(-1, 4, 50)
xx, yy = np.meshgrid(B0, B1, indexing='xy')
Z = np.zeros((B0.size,B1.size))

# Calculate Z-values (Cost) based on grid of coefficients
```

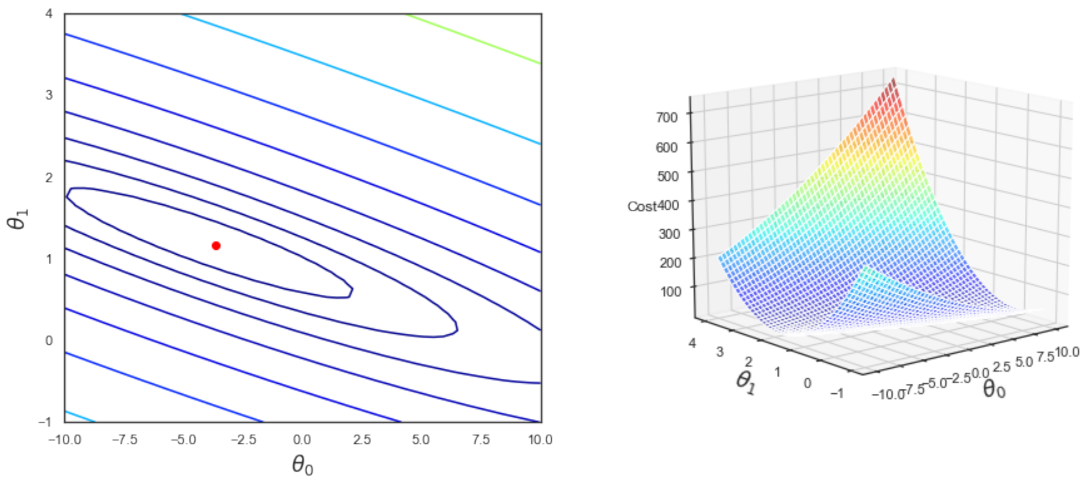
```
for (i,j),v in np.ndenumerate(Z):
    Z[i,j] = calculate_cost(x,y, theta=[[xx[i,j]], [yy[i,j]]])

fig = plt.figure(figsize=(15,6))
ax1 = fig.add_subplot(121)
ax2 = fig.add_subplot(122, projection='3d')

# Left plot
CS = ax1.contour(xx, yy, Z, np.logspace(-2, 3, 20), cmap=plt.cm.jet)
ax1.scatter(final_theta[0],final_theta[1], c='r')

# Right plot
ax2.plot_surface(xx, yy, Z, rstride=1, cstride=1, alpha=0.6, cmap=plt.cm.jet)
ax2.set_zlabel('Cost')
ax2.set_zlim(Z.min(),Z.max())
ax2.view_init(elev=15, azim=230)

# settings common to both plots
for ax in fig.axes:
    ax.set_xlabel(r'$\theta_0$', fontsize=17)
    ax.set_ylabel(r'$\theta_1$', fontsize=17)
```



In the left image the red dot represents the parameters found using the gradient descent algorithm. It is possible to see that it is near the center of the smaller ellipsis, which represent the zone where the cost function value is the smallest.

In the right image, it is possible to see a 3D plot og the cost function.

2 Multivariate Linear Regression

The dataset has the squared meters of the house and the number of bathrooms, while the third column represents the price of the houses. The problem here is the different scale of the features.

The problem can be solved by **feature scaling**.

2.1 Load data

[17]:
```
data = np.loadtxt('data/ex1data2.txt', delimiter=',')

print(data.shape)
```

```
(47, 3)
```

[18]:
```
x = data[:, :2]
y = data[:, -1]

# from n-dimensional vector to m x 1 matrix
y = np.reshape(y, (y.shape[0], 1))

m = x.shape[0]
print("Training examples: {}".format(m))

n = x.shape[1]
print("Features: {}".format(n))

print("10 examples: \n", x[:10, :])
```

```
Training examples: 47
Features: 2
10 examples:
 [[2.104e+03 3.000e+00]
 [1.600e+03 3.000e+00]
 [2.400e+03 3.000e+00]
 [1.416e+03 2.000e+00]
 [3.000e+03 4.000e+00]
 [1.985e+03 4.000e+00]
 [1.534e+03 3.000e+00]
 [1.427e+03 3.000e+00]
 [1.380e+03 3.000e+00]
 [1.494e+03 3.000e+00]]
```

2.2 Features normalization

The **feature normalization** is necessary because the features have not the same order of magnitude. This is a bad thing as our classifiers will give more weight to the larger features instead of treating all of them equally. This phase will help removing this problem by reducing the differences in the order of magnitude while keeping the same informations on the feature being normalized.

From all the features, the normalization can be done in two steps: 1. Subtract the mean value 2. Didive by the standard deviation

[19]:
```
def feature_normalize(x):
    x_norm = x

    mu = np.zeros((1, x.shape[1]))
    sigma = np.zeros((1, x.shape[1]))

    mu = np.mean(x, axis = 0) # mean value
    sigma = np.std(x, axis = 0) # std deviation value

    for i in range(x.shape[1]):
        x_norm[:,i] = (x[:,i] - mu[i])/sigma[i]

    return x_norm, mu, sigma
```

[20]:
```
x_norm, mu, sigma = feature_normalize(x)
print("10 normalized examples: \n", x_norm[:10, :])
print("Mean value: {}".format(mu))
print("Standard deviation value: {}".format(sigma))
```

```
10 normalized examples:
 [[ 0.13141542 -0.22609337]
 [-0.5096407  -0.22609337]
 [ 0.5079087  -0.22609337]
 [-0.74367706 -1.5543919 ]
 [ 1.27107075  1.10220517]
 [-0.01994505  1.10220517]
 [-0.59358852 -0.22609337]
 [-0.72968575 -0.22609337]
 [-0.78946678 -0.22609337]
 [-0.64446599 -0.22609337]]
Mean value: [2000.68085106    3.17021277]
Standard deviation value: [7.86202619e+02 7.52842809e-01]
```

2.3 Add a column for dealing with all the parameters

As the hypothesis function is as follow:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

we need to add a new column of *1* to the dataset in order to deal with the θ_0 parameter.

[21]:
```
x_norm = np.concatenate([np.ones((m,1)), x_norm], axis = 1) # Add the column of␣
 ↪ones to the data examples (for $\theta_0$)

print(x_norm.shape)
```

```
(47, 3)
```

2.4 Introduce functions to compute cost and gradient descent

[22]:
```
def gradient_descent(x, y, theta, alpha = 0.1, iterations = 1500):
    costs = []
    for i in range(0, iterations):
        theta = theta - (alpha / x.shape[0]) * (x.T).dot(x.dot(theta) - y)
        costs.append(calculate_cost(x, y, theta))
    return theta, costs

def calculate_cost(x, y, theta = [[0],[0],[0]]):
    h = x.dot(theta)
    return 1/(2 * x.shape[0]) * np.sum(np.square(h - y))
```

2.5 Running gradient descent

[23]:
```
alpha = 0.1
num_iters = 400

theta = np.zeros((3, 1))
theta, cost_history = gradient_descent(x_norm, y, theta, alpha, num_iters)

theta_no_norm = np.zeros((3, 1))
x = np.concatenate([np.ones((m,1)), x], axis = 1)# Add column of ones to non␣
 ↪normalized dataset
theta_no_norm, cost_history_no_norm = gradient_descent(x, y, theta_no_norm,␣
 ↪alpha, num_iters)

print("Non normalized theta:\n {}\n\n".format(theta_no_norm))
print("Normalized theta:\n {}\n\n".format(theta))
```

```
Non normalized theta:
 [[340412.65957447]
 [109447.79558639]
 [ -6578.3539709 ]]


Normalized theta:
 [[340412.65957447]
 [109447.79558639]
 [ -6578.3539709 ]]
```

[24]:
```
# Plot the convergence graph
plt.plot([i for i in range(num_iters)], cost_history, '-b', label = 'Normalized␣
 ↪cost')
```