taller #1

Algebra lineal

1)

R) $T + 80C = 39$

$T + 55C = 31.5$

$T = 39 - 80C$

$(39 - 80C) + 55C = 31.5$

$39 - 80C + 55C = 31.5$

$39 - 25C = 31.5$

$-25C = 7.5$

$C = 0.3 \longrightarrow T = 39 - 80C \quad T = 39 - 80(0.3) \quad T = 39 - 24 \quad T = 15$

matriz

$\begin{bmatrix} 1 & 80 \\ 1 & 55 \end{bmatrix} \begin{bmatrix} T \\ C \end{bmatrix} \begin{bmatrix} 34 \\ 31.5 \end{bmatrix}$
$A = \begin{bmatrix} 1 & 80 \\ 1 & 55 \end{bmatrix}$
$A = (1 \times 55) - (1 \times 80) = 55 - 80 = -25$

$A_T \begin{bmatrix} 1 & 39 \\ 1 & 31.5 \end{bmatrix}$

$A_C \begin{bmatrix} 39 & 80 \\ 31.5 & 55 \end{bmatrix}$

$|A_T| = (39 \cdot 55) - (31.5 \cdot 80) = 2145 - 2520 = -375$

$|A_C| = (1 \times 31.5) - (1 \times 39) = 31.5 - 39 = -7.5$

Regla de cramer

$T = \dfrac{|A_T|}{|A|} = \dfrac{-375}{-25} = 15$

$C = \dfrac{|A_C|}{|A|} = \dfrac{-7.5}{-25} = 0.3$

②

① $30x + 20y + 30y = 0$

② $-30x + 10y + 10z = 0$

③ $-x + y + 10z = 100$

$$\begin{vmatrix} 30 & 20 & -30 \\ -30 & 10 & 10 \\ -1 & 1 & 0 \end{vmatrix} \begin{matrix} 30 & 20 \\ -30 & 10 \\ -1 & 1 \end{matrix}$$

$= + (30)(10)(0) + (20)(10)(-1) + (-30)(-30)(1)$
$ - (-1)(10)(-30) - (1)(10)(20) - (0)(-30)(30)$
$= 0 + (-200) + (900)$
$= (-300) - (200) - (0) = 100$

$$\Delta_1 \begin{vmatrix} 0 & 20 & -30 \\ 0 & 10 & 10 \\ 100 & 1 & 0 \end{vmatrix} \begin{matrix} 0 & 20 \\ 0 & 10 \\ 100 & 1 \end{matrix}$$

$[0](10)(0) + (20)(10)(100) + (-30) 0 1$
$- (100)(10)(-30) - (1)(10)(0) - (0)(0)(20$
$20.000 + 30.000$

$$\Delta_1 \begin{vmatrix} 30 & 0 & -30 \\ -30 & 0 & 10 \\ -1 & 100 & 0 \end{vmatrix} \begin{matrix} 30 & 0 \\ -30 & 0 \\ -1 & 100 \end{matrix}$$

$= + (50)(0)(0) + (0)(10)(-1) + (-30)(-30)(-30)(100)$
$= (-1)(0)(-30) - (100)(10)(30) - (0)(-30)(0)$
$= 0 + 0 + 90.000 - 0 - 30.000$
$= 60.000$

$$\Delta_3 \begin{vmatrix} 30 & 20 & 0 \\ -30 & 10 & 0 \\ -1 & 1 & 100 \end{vmatrix} \begin{matrix} 30 & 20 \\ -30 & 10 \\ -1 & 1 \end{matrix}$$

$= + (30)(10)(100) + (20)(0)(-1) + (0)(-30)(1)$
$= - (-1)(10)(0) - (1)(0)(30) - (100)(-30)(20)$
$= 30.000 + 0 + 0 - 0 - 0 - (-60.000)$
$= 90.000$

$x_1 = \Delta_1/\Delta = \dfrac{50000}{100} = 500$

$x_2 = \Delta_2/\Delta = \dfrac{60000}{100} = 600$

$x_3 = \Delta_3/\Delta = \dfrac{90000}{100} = 900$

③

$a_n = 4 \cdot 2^{(10-1)} = 4 \cdot 2^9 = 4 \cdot 512 = 2048$ R) En 10 días había 2048 personas enfermas y se necesita 22 días para que 6.800.000 personas se enfermen

$6.500.000 = 4 \cdot 2^{(n-1)}$

$2^{(n-1)} = \dfrac{6.500.000 - 1.625.000}{4}$

$(n-1) = \log(1.625.000)$ $\log(1.625.000) = 20.6$ $n = 21.6$

④ $x^2 + 5x - 3000 = 0$

x 300
y $x + 5$
$x(x+5) = 300$ $x^2 + 5x - 3000 = 0$

$x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(-800)}}{2(1)} \rightarrow x = \dfrac{-5 \pm \sqrt{25 + 1200}}{2}$

$x = \dfrac{-5 \pm \sqrt{1225}}{2}$ $x = \dfrac{30}{2} = 15$ $x = \dfrac{-40}{2} = -20$

$x = \dfrac{-5 \pm 35}{2}$ $15 \times 20 \times 5 = 1500$ R) El largo es 15cm y el ancho 20cm

$y = x + 5 = 15 + 5 = 20$

5) Matriz de cometa   matriz vento

$$C = \begin{bmatrix} 1000 & 500 & 300 \\ 2000 & 1000 & 600 \end{bmatrix} \qquad V = \begin{bmatrix} 1100 & 550 & 350 \\ 2200 & 1100 & 700 \end{bmatrix}$$

$$T = C + V = \begin{bmatrix} 1000+1100 & 500+550 & 300+350 \\ 2000 & 2200 & 1000 & 1100 & 600+700 \end{bmatrix} \qquad T = \begin{bmatrix} 2100 & 1050 & 650 \\ 4200 & 2100 & 1300 \end{bmatrix}$$

6) Primera tienda

compra: (1000 500 300)
venta: (1100 550 300)

Segunda tienda
compra: (2000 1000 600)
venta: (2200 1100 700)

Compra = (1000+2200 550+1100 350+700)
= (3300 1650 1050) = 60.000

7) Pasto (2017 y 2018)

$$\begin{bmatrix} 250 & 300 & 20 & 5 \\ 300 & 200 & 10 & 16 \end{bmatrix}$$

Sede Pasto: 2017: 250 + 300 + 20 + 15 = 685
            2018: 300 + 200 + 10 + 10 = 520

Sede Ipiales: 2017: 150 + 220 + 22 + 18 = 410
              2018: 305 + 269 + 25 + 12 = 611

Ipiales 2017 y 2018

$$\begin{bmatrix} 150 & 220 & 22 & 18 \\ 305 & 269 & 25 & 12 \end{bmatrix}$$

8)

$$A = \begin{bmatrix} 100 & 96 & 50 & 119 \\ 111 & 115 & 59 & 124 \end{bmatrix} \qquad B \begin{bmatrix} 15 & 29 & 38 & 54 \\ 29 & 39 & 34 & 25 \end{bmatrix}$$

$$C = \begin{bmatrix} 100 & 96 & 50 & 119 \\ 111 & 115 & 59 & 124 \end{bmatrix} \cdot \begin{bmatrix} 15 & 29 & 38 & 54 \\ 29 & 39 & 34 & 25 \end{bmatrix}$$

Primera tienda
. 100·5 + 96·29 + 50·38 + 119·59
= 1500 + 2784 + 1900 + 7021 = 12.105 → total tienda 1
. 100·29 + 96·37 + 50·54 + 119·25
= 2900 + 3744 + 7100 + 2945 = 10419 → total tienda 2

Segunda tienda

. 111·29 + 115·39 + 69·34 + 129·25
= 3216 + 4485 + 2006 + 3225 = 11935

$$C = \begin{pmatrix} 12105 & 10419 \\ 14855 & 11985 \end{pmatrix}$$

a)

$$\begin{pmatrix} 0.05 & 0.06 & 0.07 & 0.04 \\ 0.95 & 0.94 & 0.93 & 0.96 \end{pmatrix} \quad \begin{pmatrix} 150 & 220 \\ 410 & 150 \\ 125 & 80 \\ 303 & 150 \end{pmatrix}$$

Bombillas transparentes

- $0.05 \cdot 150 + 0.06 \cdot 410 + 0.07 \cdot 25 + 0.04 \cdot 303$
  $= 7.5 + 246 + 8.75 + 12.12 = 83.91 \longrightarrow$ Defectuosa

- $0.95 \cdot 150 + 0.94 \cdot 410 + 0.93 \cdot 125 + 0.96 \cdot 303$
  $= 142.5 + 384.4 + 116.25 + 290.88 = 933.03 \longrightarrow$ aceptable

Bombillas opacas y buenas

- $0.05 \cdot 220 + 0.06 \cdot 150 + 0.07 \cdot 8 + 0.04 \cdot 150$
  $= 11 + 9 + 5.6 + 6 = 31.6$ Defectuosa

- $0.95 \cdot 220 + 0.94 \cdot 150 + 0.93 \cdot 80 + 0.96 \cdot 150$
  $= 206 + 141 + 174.4 + 144 = 588.4$

$$\begin{pmatrix} 53.91 & 81.6 \\ 903.03 & 568.4 \end{pmatrix}$$

10)

$x = $ tarifa mensual (Dolares)

$y = $ costo por minuto

$$\begin{cases} x = 80y = 39 \\ x + 55y = 31.5 \end{cases}$$

$A \cdot x = B$

$A = \begin{pmatrix} 1 & 80 \\ 1 & 55 \end{pmatrix}$ $x = \begin{pmatrix} x \\ y \end{pmatrix}$ $y = \begin{pmatrix} 39 \\ 31.5 \end{pmatrix}$

$|A| = (1)(55) - (1)(80) = 55 - 80 = -25$

$adj(A) = \begin{pmatrix} 55 & -80 \\ -1 & 1 \end{pmatrix}$

$A^{-1} = \dfrac{1}{det(A)}$ $adj(A) = \dfrac{1}{-25} \begin{pmatrix} 55 & -80 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} \dfrac{-55}{25} & \dfrac{80}{25} \\ \dfrac{1}{25} & \dfrac{-1}{25} \end{pmatrix}$

$A^{-1} \approx \begin{pmatrix} -2.2 & 3.2 \\ 0.04 & -0.04 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} 39 \\ 31.5 \end{pmatrix}$

$x = (-2.2) \cdot 39 + (3.2) \cdot 31.5 = -85.8 + 108.8 = 15$

$y = (0.04) \cdot 39 + (-0.04) \cdot 31.5 = 1.56 - 1.26 = 0.30$

La tarifa mensual es $ 15 dolares

El costo por minuto es $ 0.30 dolares

- Videoclub

$I =$ Numero de peliculas infantiles

$O =$ Numero de peliculas oeste

$T =$ Numero de peliculas terror

- $0.60 \, I + 0.50 \, O = 0.30$
- $0.20 \, I + 0.60 \, O + 0.70 \, T = 0.50$
- $0.10 \, I + 0.30 \, O + 0.80 \, T = 10$

$$x = \begin{pmatrix} I \\ O \\ T \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$$

$$A = \begin{pmatrix} 0.80 & 0.20 & -0.80 \\ -0.90 & 0.10 & 0.20 \\ 0.10 & 0.90 & 0.90 \end{pmatrix} \quad AX = B$$

1) $3I + 2O - 3T = 0$

2) $-3I + O + 2T = 0$

3) $I + 3O + 3T = 100$

- suma ecuación 1 y 2 para eliminar I

$(3I - 3I) + (2O + O) + (-3T + 2T) = 0 \Rightarrow 3O - T = 0$

$$T = 30$$

- de la 1 ecuación sustituimos T

$(3I + 2O - 3(30)) = 0 \Rightarrow 3I + 2O - 90 = 0 \Rightarrow 3I - 70 = 0$

$$3I = 70$$

$$I = \frac{70}{3}$$

- de la 3 sustituimos T y I

$\left(\frac{70}{3}\right) + 3O + 3(30) = 100$

$\frac{70}{3}O + 30 + 90 = 100$

$$\frac{7}{3}O + \frac{9}{3}O + \frac{27}{3}O = \frac{7+9+27}{3} \qquad O = \frac{43}{3}$$

$$O = 100$$

$$O = \frac{100 \cdot 3}{43} = \frac{300}{43} \approx 6.98$$

$$T = 3O = \frac{3 \times 300}{43} = \frac{900}{43} \approx 20.93$$

$$I = \frac{7}{3}O = \frac{7}{3}O \times \frac{300}{43} = \frac{2100}{129} = \frac{700}{43} \approx 16.28$$

$$I = 16.28 \qquad O = 6.98 \qquad T = 20.93$$