More on Fisher waves

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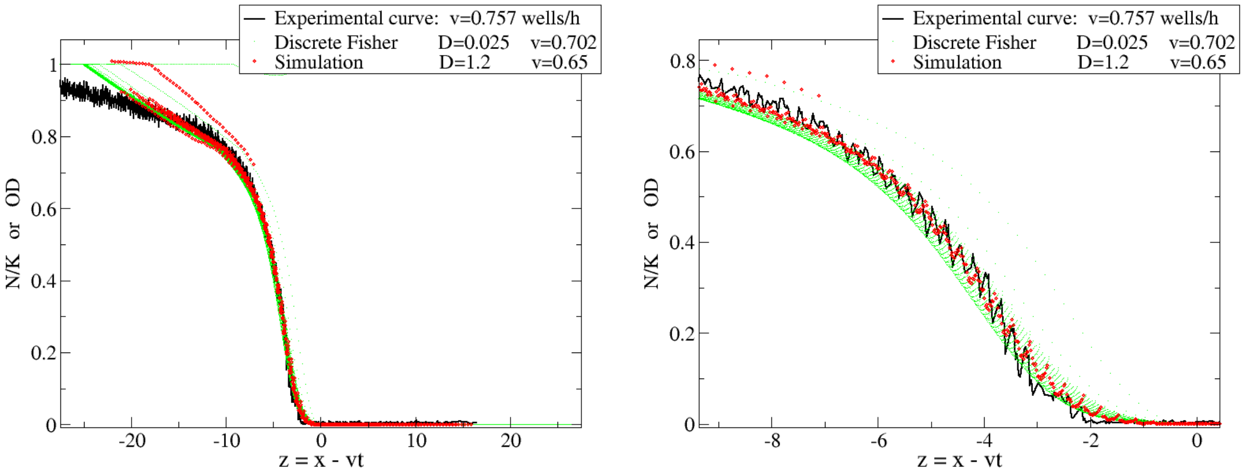
1 Summary

- We corrected our units of speed (mm/h vs wells/h) and found that we can actually predict the wave profiles.
- Full stochastic simulation predicts similar profile to that found by solving the set of coupled-ODEs corresponding to the discrete version of Fisher's equation: Fig. 1. (Left hand tail doesn't match so well though; due to poor quality of growth data? Also need to bear in mind that Bartek produces the growth function by assuming linear dependence on OD and N, so maybe this breaks down here too).
- By taking the non-mixed growth curve $N(t)$, we can (almost) reproduce the wave profile simply by rescaling the time variable $t \to -vt$ where v is the speed of the wave, Figs. 2 and 3. Though, oddly, the well-mixed data seems to match the profile better (at least in the right-hand tail).
- Rosalind's idea to use a local density measurement when calculating the growth rate did not allow the experimental wave to be reproduced when using the well-mixed LB growth data. Implies we probably need to model nutrients explicitly. (Though check how initial distribution of bacteria affects this).
- The speed of the waves (from discrete Fisher ODEs) does not obey $v = 2\sqrt{Dg(0)}$ but is close to the (non-analytic) form that we can calculate from discrete equation, Fig. 5 (though I don't understand exactly how this is produced). However, this form is not consistent with $v \sim \sqrt{D}$. A "free-fit" of $v = a + bD^c$ in xmgrace gave me $c = 0.45$ but actually when I plot v vs D in a ln-ln plot it does not even produce a straight line. Also, the fit is progressively worse for the logistic, well-mixed and non-mixed cases, Fig. 6. I think this is due to the fact that the decrease from the maximum rate $g(0) = 2$ h^{-1} is progressively steeper for these different $g(n)$'s, and so the linearisation performed in the approximation becomes less justified.

2 To do

- Are wave profiles reaching their steady state? Fig. 1 suggests not. Get profiles from 24 well, 100 well and 500 well simulations using discrete Fisher equation.
- Explicitly model nutrients to reproduce non-mixed growth curve and wave profile when using the well-mixed data.
- Re-do speed vs carrying capacity K with non-mixed growth data. CURRENTLY RUNNING.

3 Simulation and discrete Fisher profiles

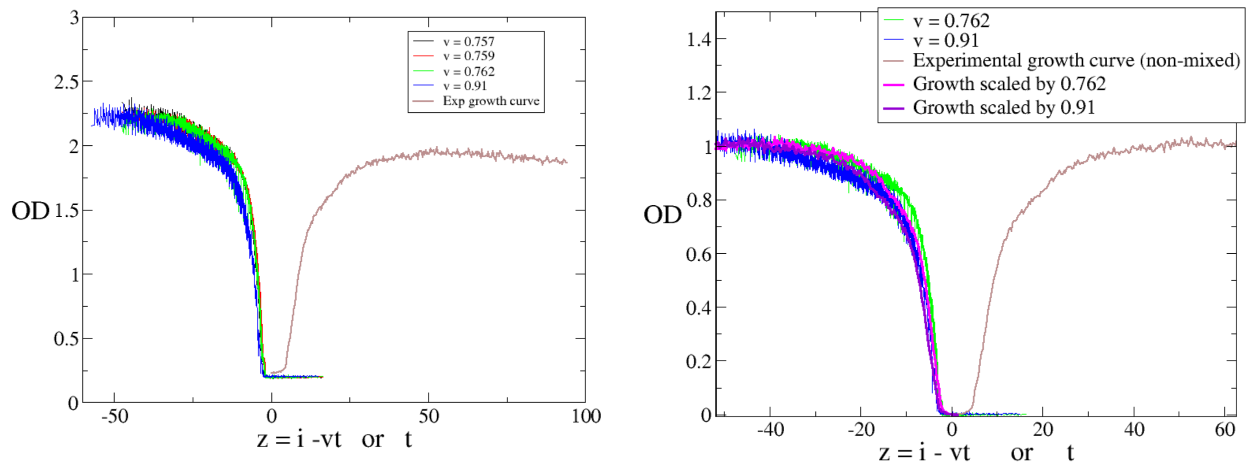


(a) (b)

Figure 1: The travelling wave profiles produced from the full stochastic simulation (red) and the discrete Fisher equation (green). Black curve is an experimental profile. In each case, the growth function $g(n)$ used is that from the non-mixed LB growth experiment. (b) is same plot but zoomed in. Arguably the profile predicted by the discrete Fisher equation is not quite as good as from the full stochastic simulation.

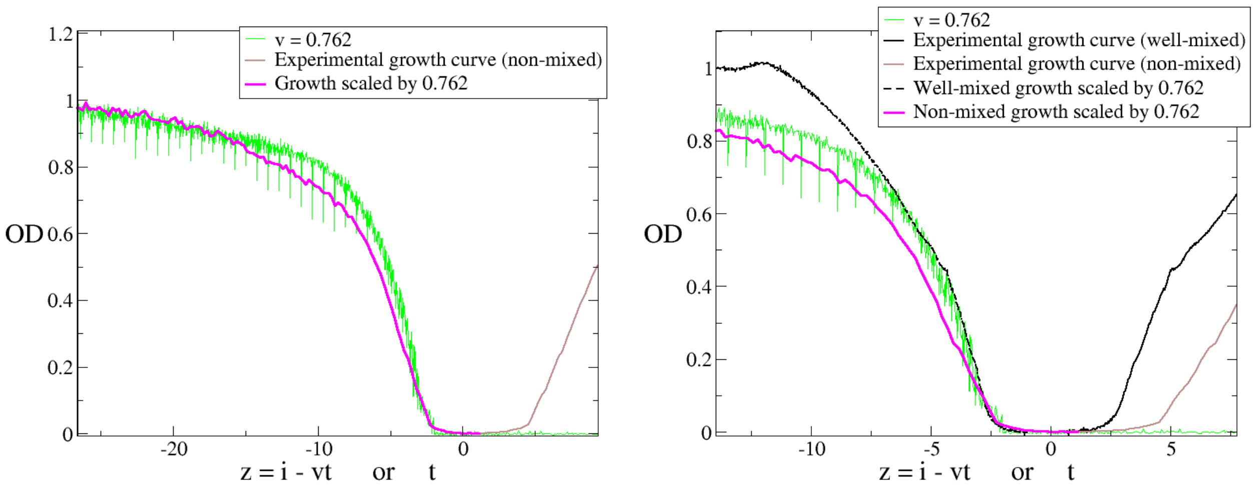
4 Rescaling growth curve

We predicted that (if diffusion is negligible) we should be able to reproduce the wave profiles simply by rescaling the growth curve by $-v$ where v is the speed of the wave.



(a) (b)

Figure 2: (a) Experimental travelling waves (blue, green, red, black) and growth curve for non-mixed LB. (b) Two experimental profiles with different speeds (green and blue) and an experimental growth curve (brown). All curves have been shifted and scaled so that the OD range from 0 to 1. The magenta and violet curves are the result of scaling the time-coordinate (x-axis) of the growth curve by $-v$ for the two different wave speeds. They look like reasonable matches, but are not perfect and are not as good as the actual profiles predicted from solving the ODEs or the full stochastic simulations (Fig. 1).



(a) (b)

Figure 3: (a) shows a close-up view of the $(-v)$-scaled non-mixed growth curve and we see that it's not an exact match. (b) shows that rescaling the well-mixed LB growth curve actually does better at predicting the shape at the front of the wave.

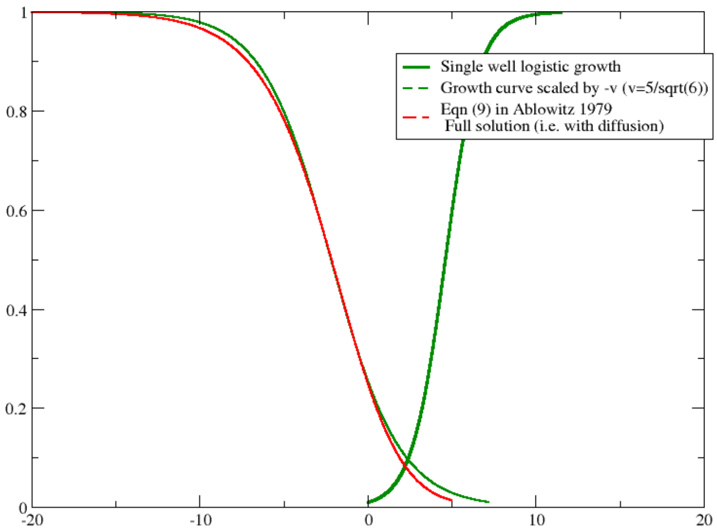


Figure 4: Here I just wanted to check the validity of scaling the growth curve by $(-v)$ and ignoring D. In Albowitz, 1979 "Explicit solutions of Fisher's equation for a special wave speed", an exact solution of the travelling wave is given: $u(z) = (1+e^{z/\sqrt{6}})^{-2}$, which holds for a logistic growth term and the special wave speed of $v_s = 5/\sqrt{6}$. The red curve plotted above is this exact form, while the solid green line is the standard logistic growth curve. The dashed green line is this growth curve scaled by $-v_s$ and we see that it only differs slightly (for $D = 1$?).

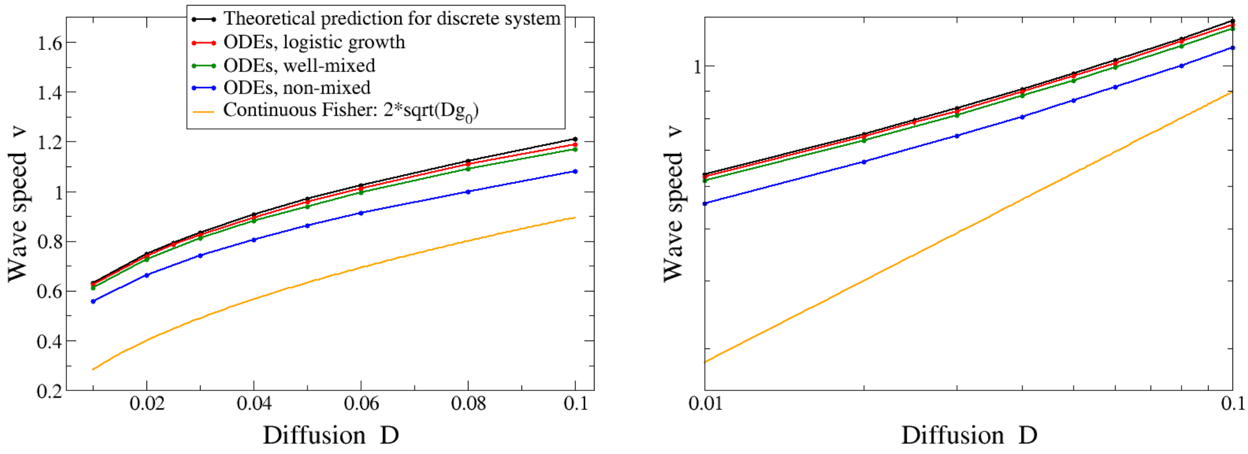
5 Wave speeds

A linearising approximation of the discrete Fisher wave predicts a speed of:

$$v = \frac{D}{\alpha}\left(e^{-\alpha} + e^{\alpha} - 2\right) + \frac{g(0)}{\alpha} \tag{1}$$

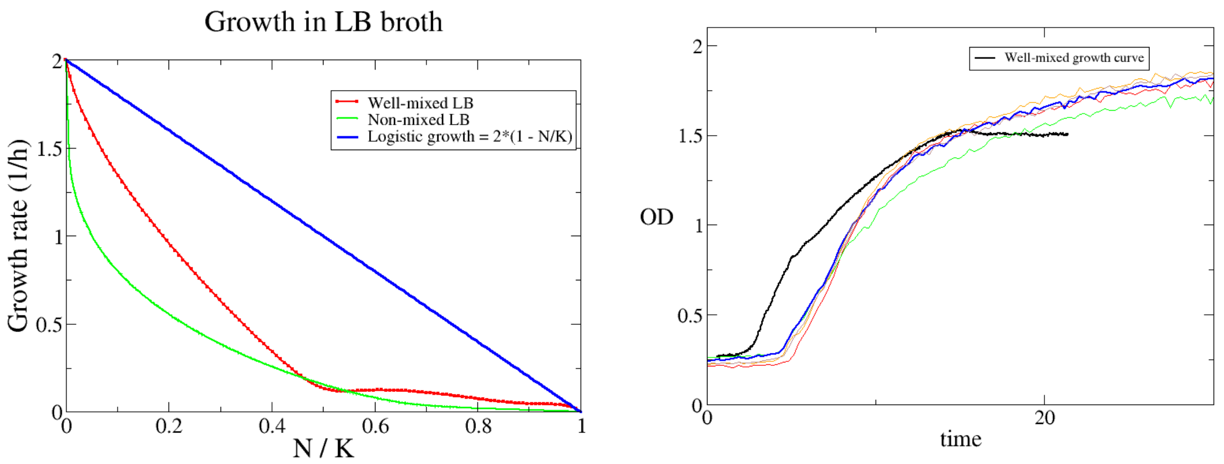
where α is the solution of (I don't understand why $dv/d\alpha$=0):

$$\frac{-D}{\alpha^2}\left(e^{-\alpha} + e^{\alpha} - 2\right) + \frac{D}{\alpha}\left(e^{\alpha} - e^{-\alpha}\right) - \frac{g(0)}{\alpha^2} = 0 \tag{2}$$



(a) (b)

Figure 5: (a) Black curve shows theoretical prediction of speed for discrete Fisher equation (Eq. 1) and yellow the continuous Fisher equation ($v = 2\sqrt{Dg(0)}$). Red, green and blue lines show how speed varies with D when using logistic, well-mixed and non-mixed growth data respectively. The worsening fit is probably due to the steeper fall offs from the maximal growth rates for these curves (Fig. 6) which makes the linearisation inappropriate. (b) Note that in this ln-ln plot the speed does not behave as $v \sim D^k$ for any curves except the yellow (continuous Fisher prediction) in which $v \sim D^{0.5}$.



(a) (b)

Figure 6: (a) The growth functions $g(n)$ for the well-mixed LB and non-mixed LB as extracted from Bartek's growth curves. The steep fall off from $g(0) = 2$ for the well-mixed and non-mixed LB curves reduces the quality of the speed-D fit to the linearised prediction. (b) The difference in Bartek's experimental growth curves when well-mixed (black) and non-mixed LB (all others) are used. Why do they reach different OD values?