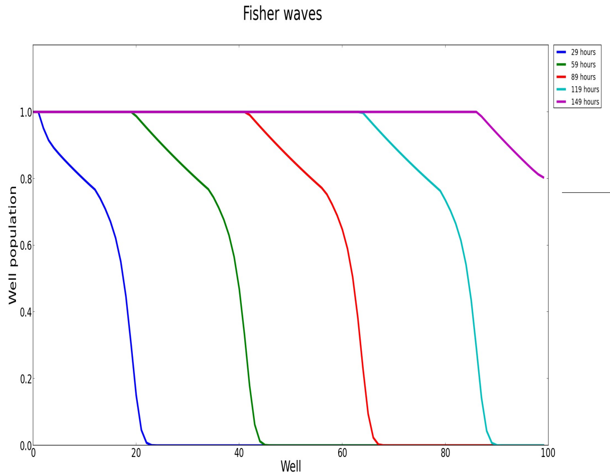
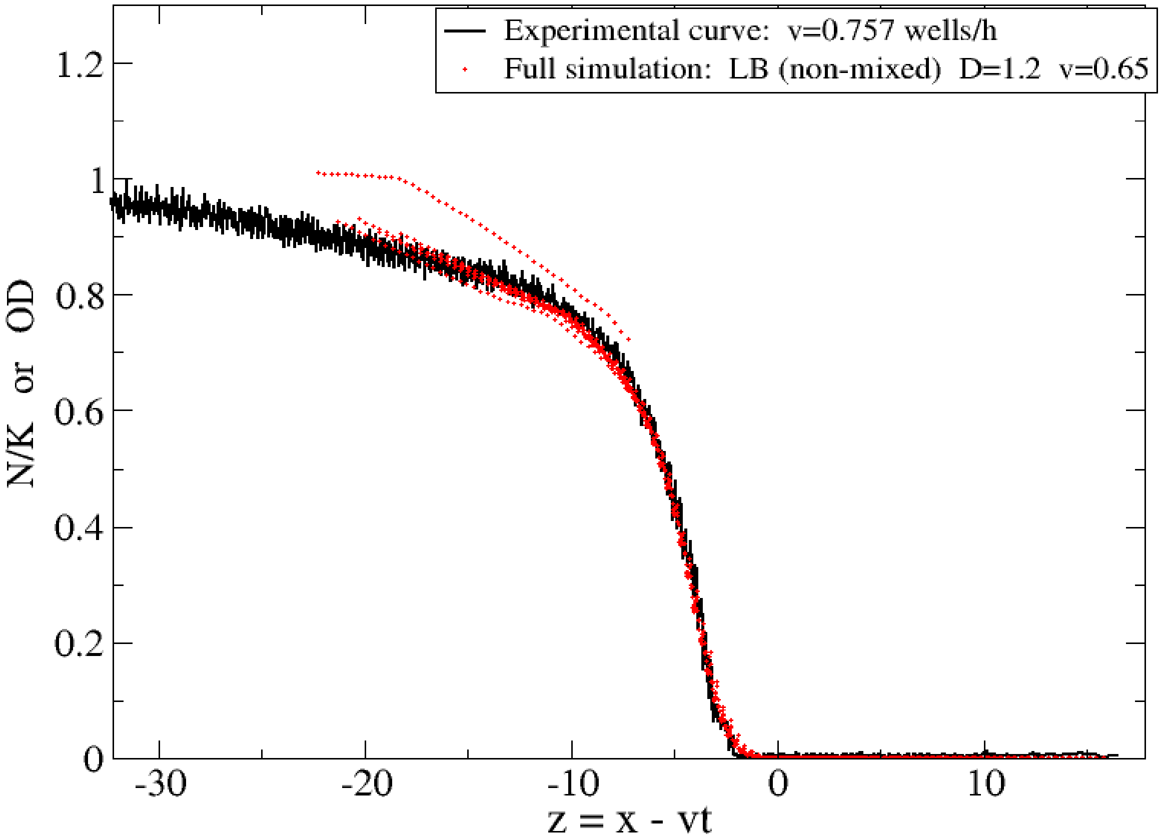
Simulating Bartek's experiment

- Full stochastic simulation of system:

  - 3D wells & connecting channels,
  - model every bacterium
  - diffusion,
  - replication with rate $g(n_i)$





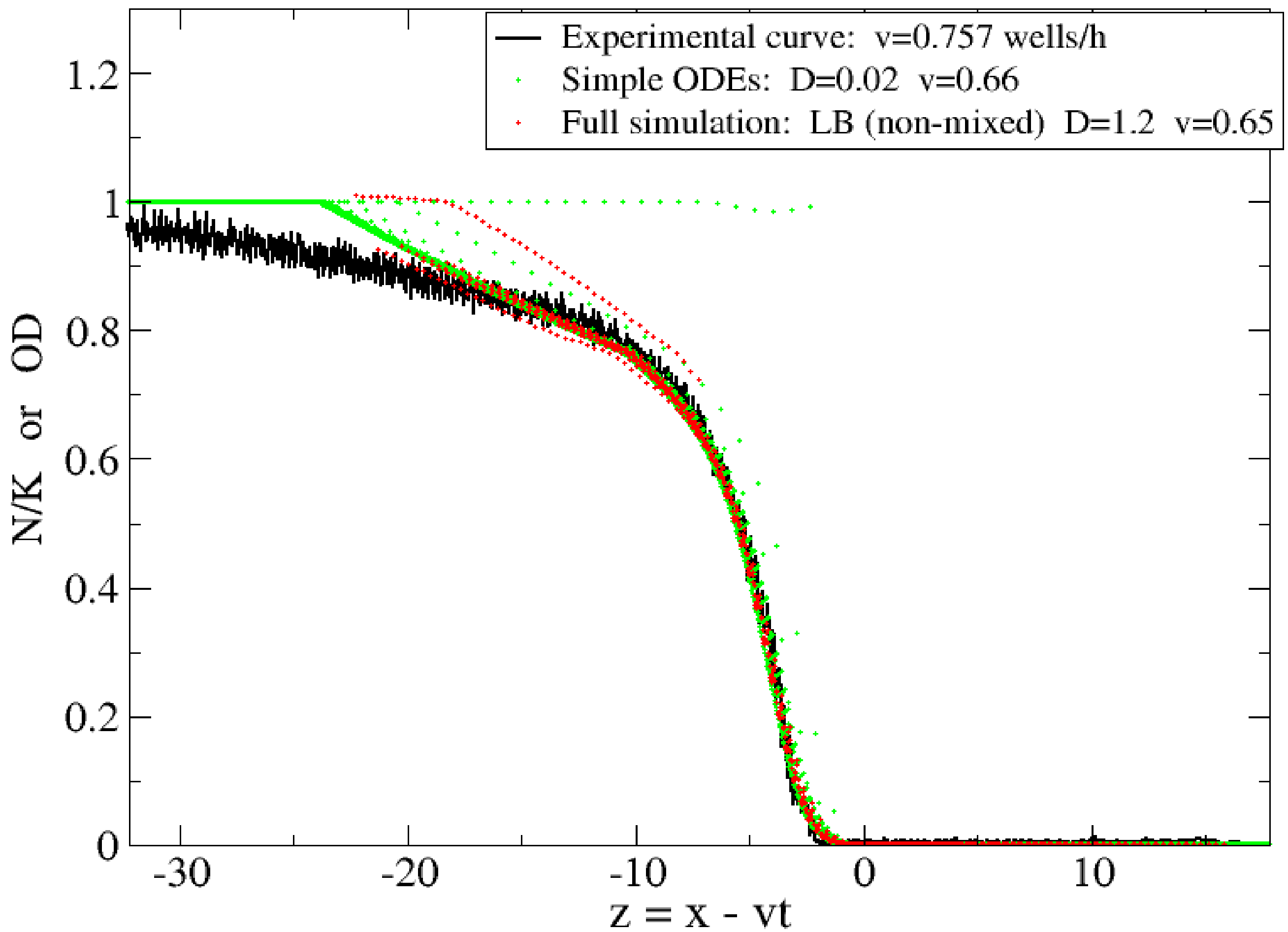
Simulating Bartek's experiment

- Solve discrete version of Fisher equation: $\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial^2 x} + n g(n)$

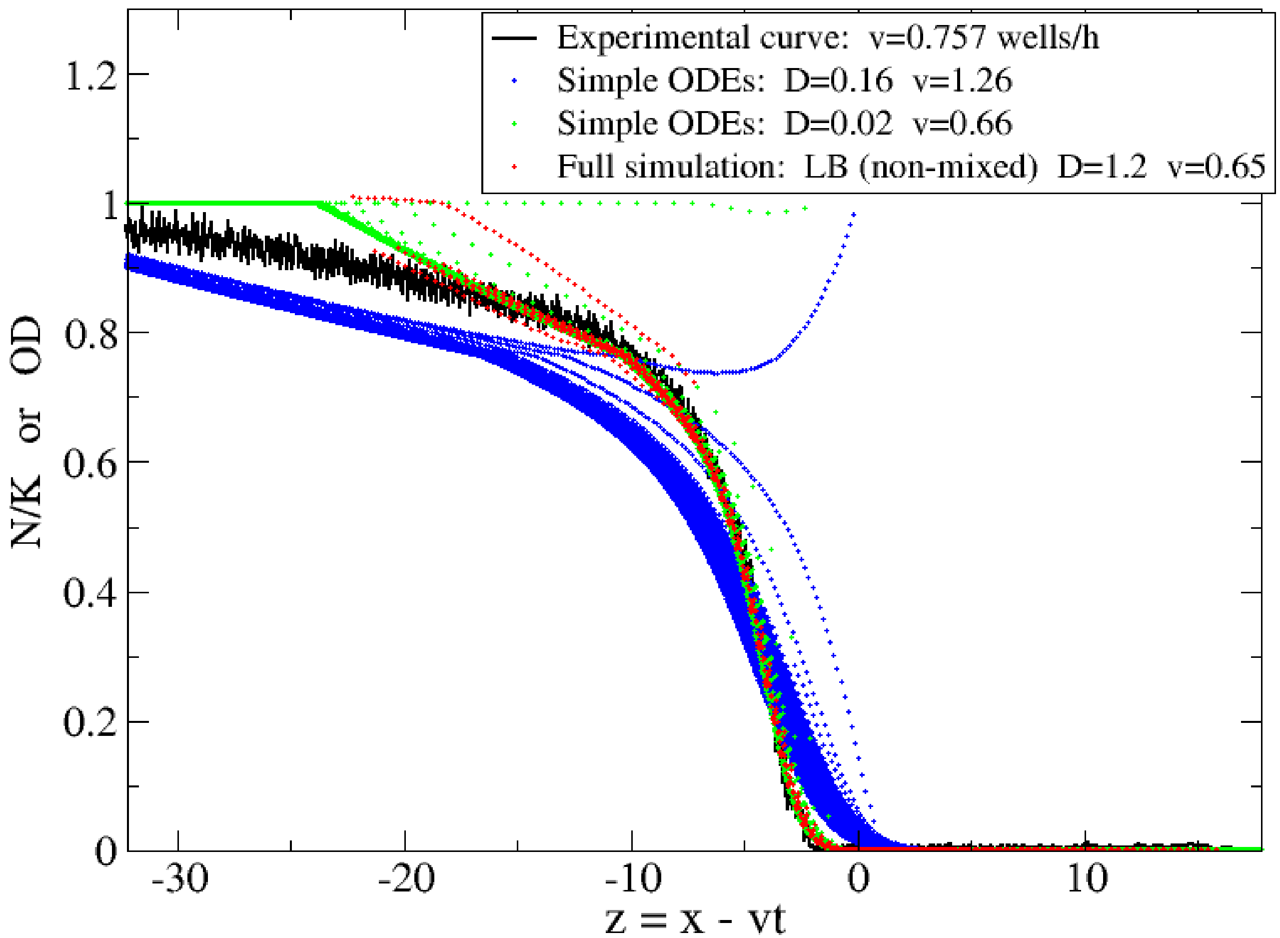
$$\frac{dn_i}{dt} = D(n_{i+1} + n_{i-1} - 2n_i) + n_i g(n_i)$$

- Ignore structure: 1D system, no well dimensions, no channels.
  No in-well diffusion.
  Bacteria not explicitly modelled.

Choose appropriate D (i.e. speed v) and correct growth function
→ Equations can reproduce simulation and experimental waveform.



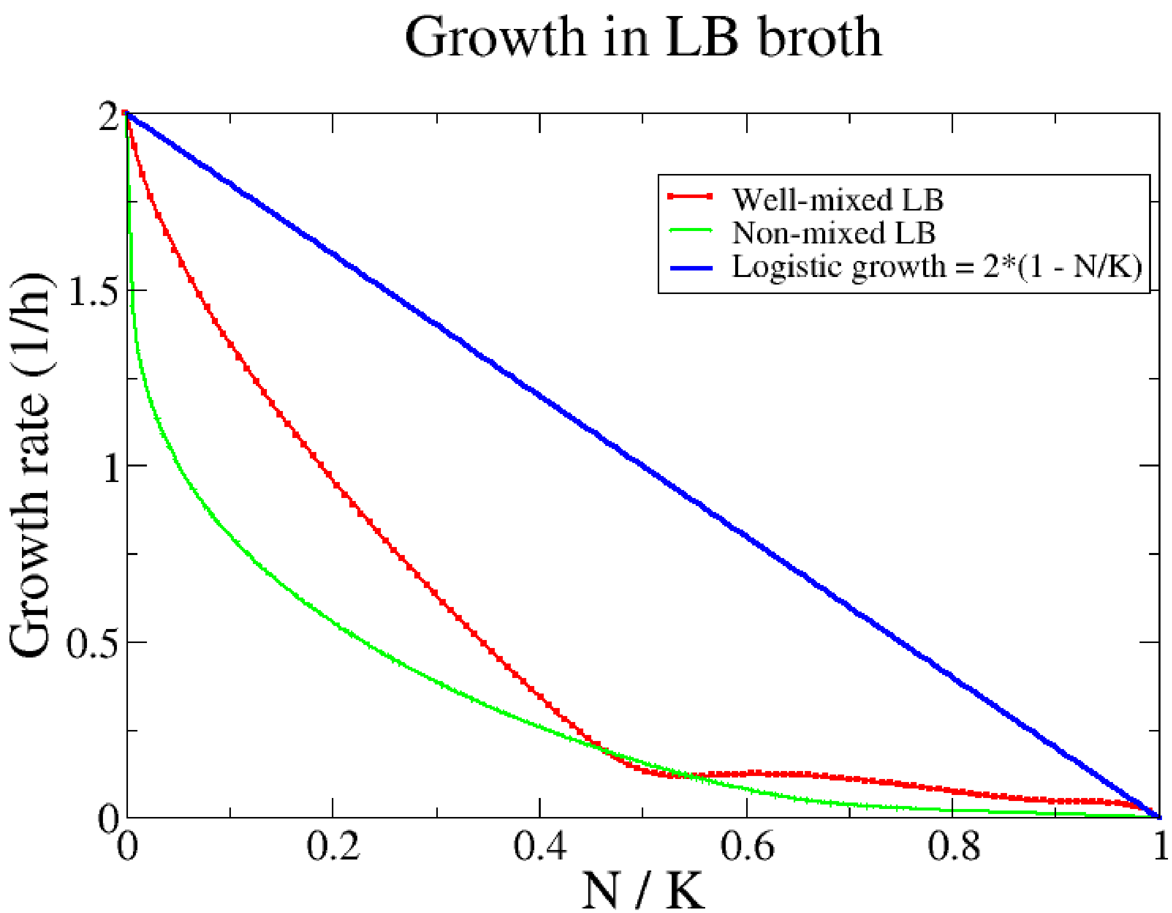
Experimental curve: v=0.757 wells/h
Simple ODEs: D=0.02 v=0.66
Full simulation: LB (non-mixed) D=1.2 v=0.65
N/K or OD
z = x - vt
1.2
1
0.8
0.6
0.4
0.2
0
-30
-20
-10
0
10



Experimental curve: v=0.757 wells/h
Simple ODEs: D=0.16 v=1.26
Simple ODEs: D=0.02 v=0.66
Full simulation: LB (non-mixed) D=1.2 v=0.65
N/K or OD
z = x - vt
0
0.2
0.4
0.6
0.8
1
1.2
-30
-20
-10
0
10

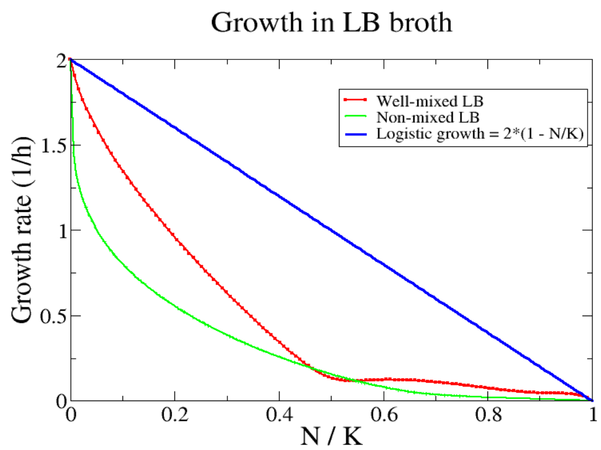
Waveshape depends on g(ni)

$$\frac{dn_i}{dt} = D\left(n_{i+1} + n_{i-1} - 2n_i\right) + n_i g\left(n_i\right)$$

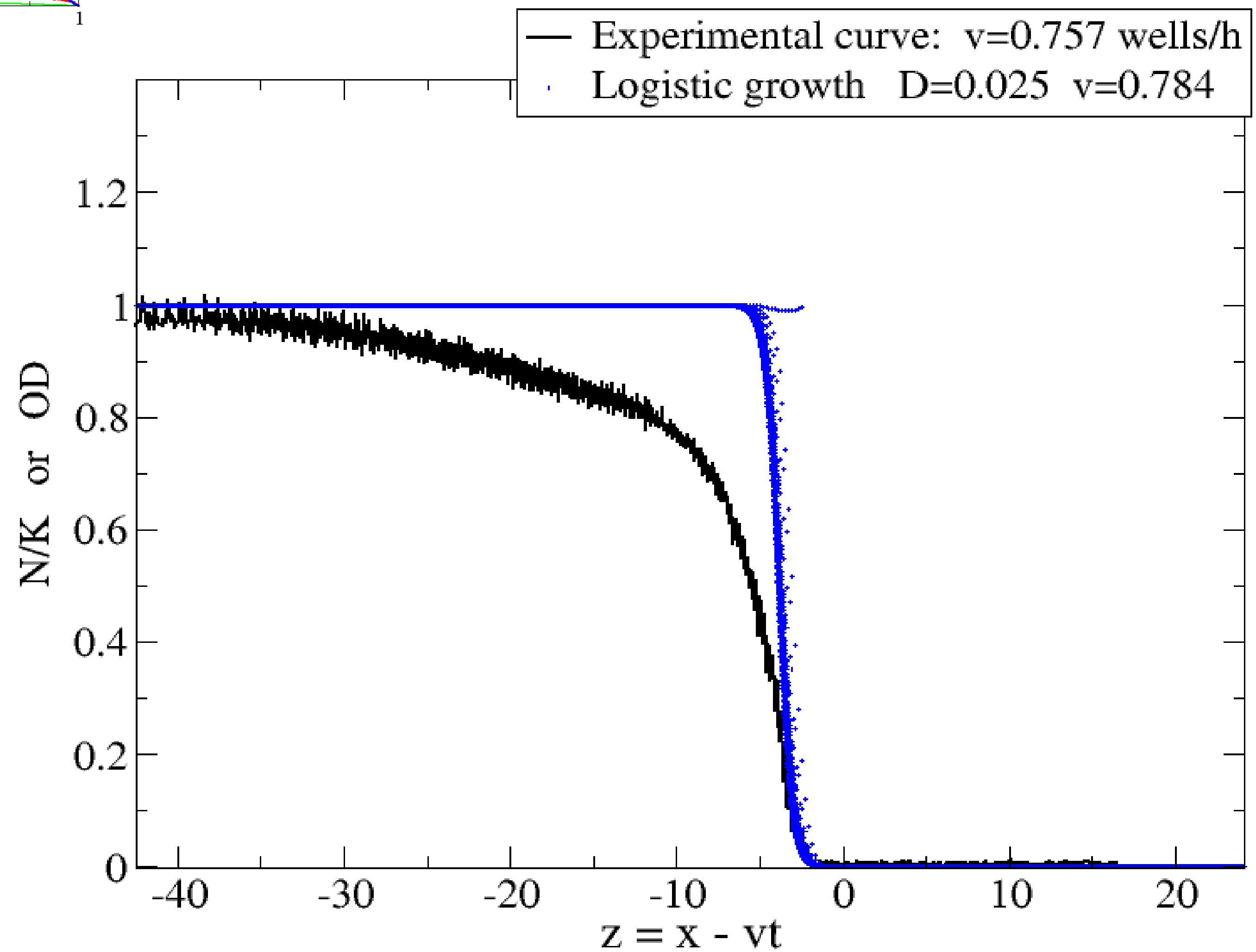


- Consider 3 growth functions:

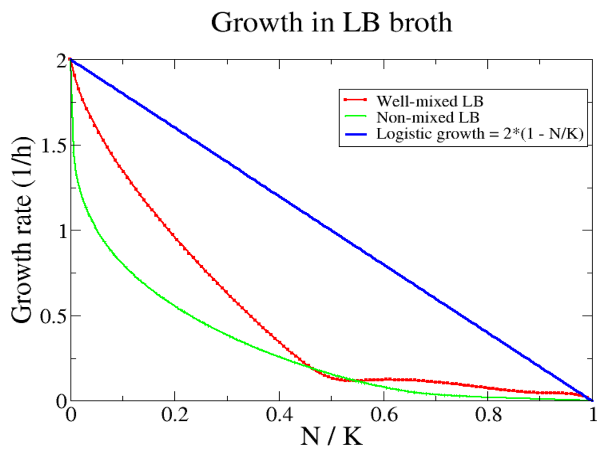
  - logistic,
  - well-mixed LB,
  - non-mixed LB

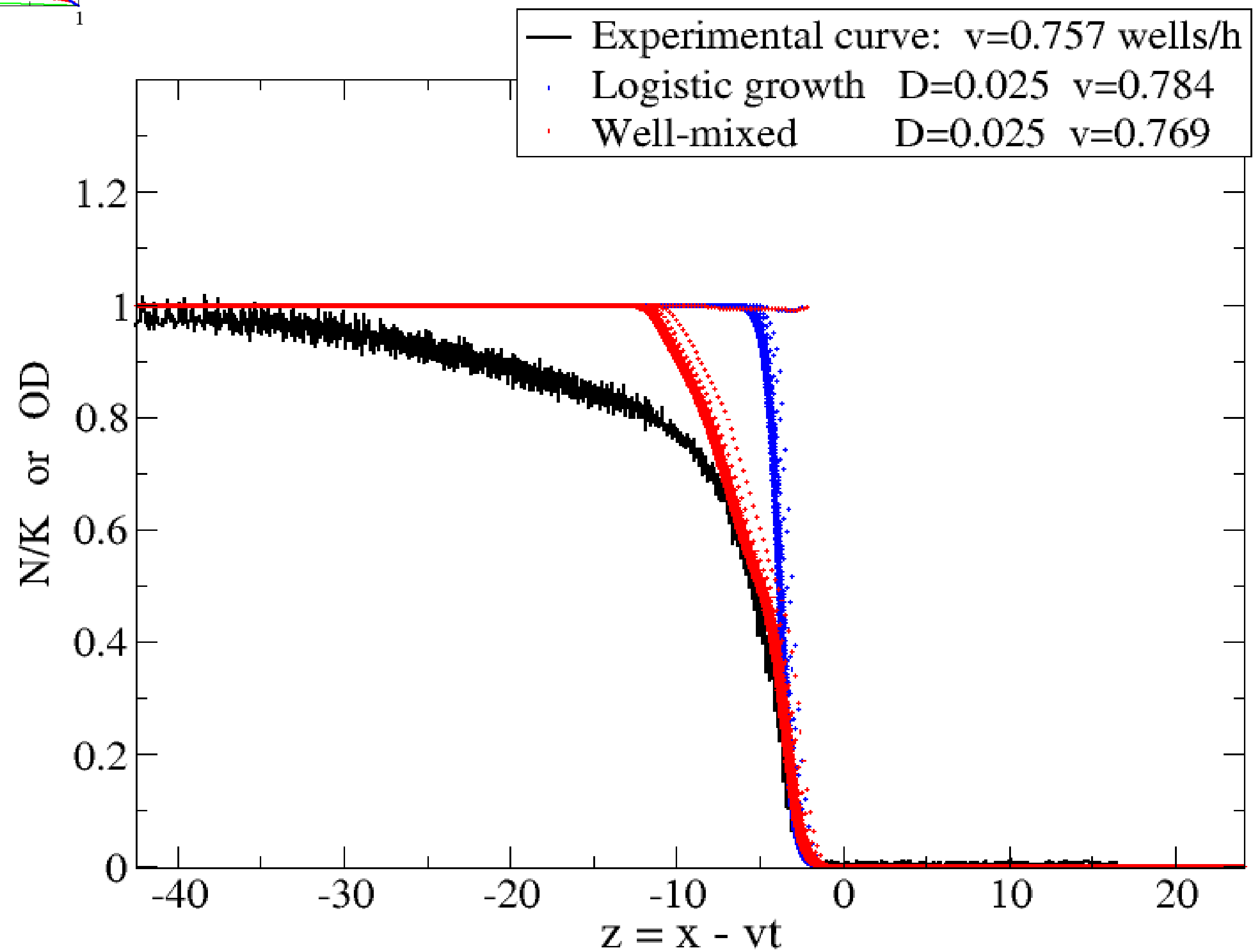


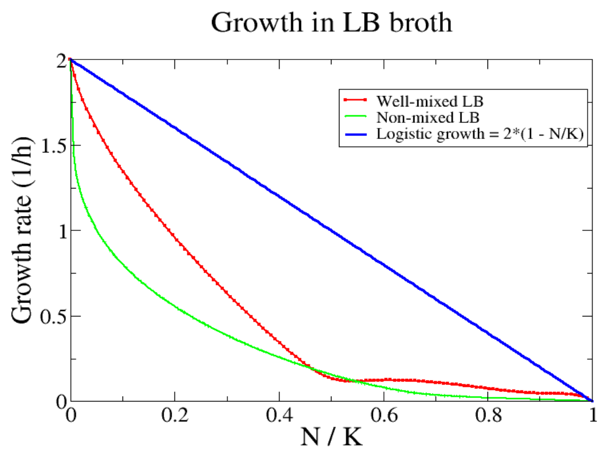
Growth in LB broth
Growth rate (1/h)
N / K
Well-mixed LB
Non-mixed LB
Logistic growth = 2*(1 - N/K)



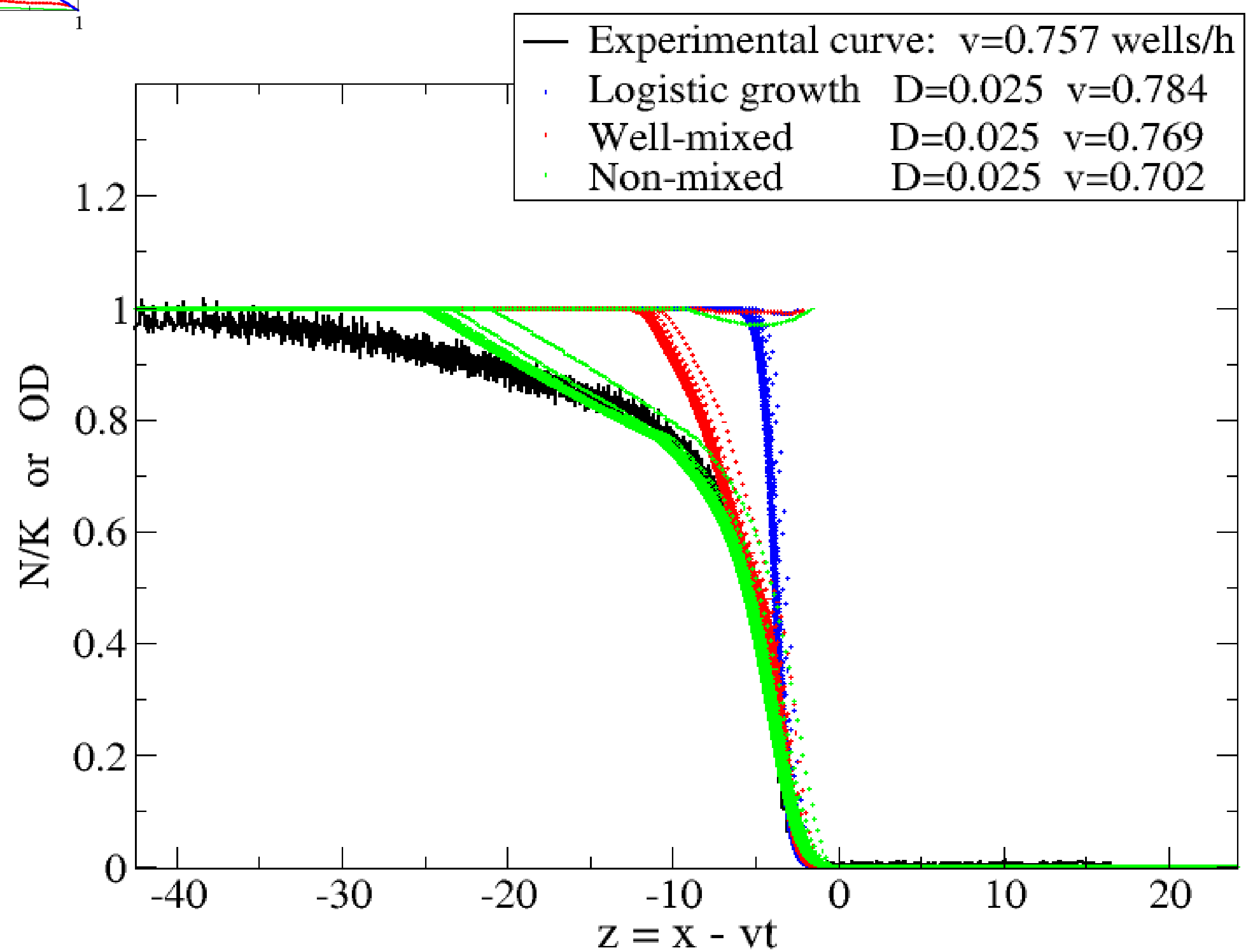
Experimental curve: v=0.757 wells/h
Logistic growth D=0.025 v=0.784
N/K or OD
z = x - vt


Growth in LB broth
Well-mixed LB
Non-mixed LB
Logistic growth = 2*(1 - N/K)
Growth rate (1/h)
N / K


Experimental curve: v=0.757 wells/h
Logistic growth D=0.025 v=0.784
Well-mixed D=0.025 v=0.769
N/K or OD
z = x - vt

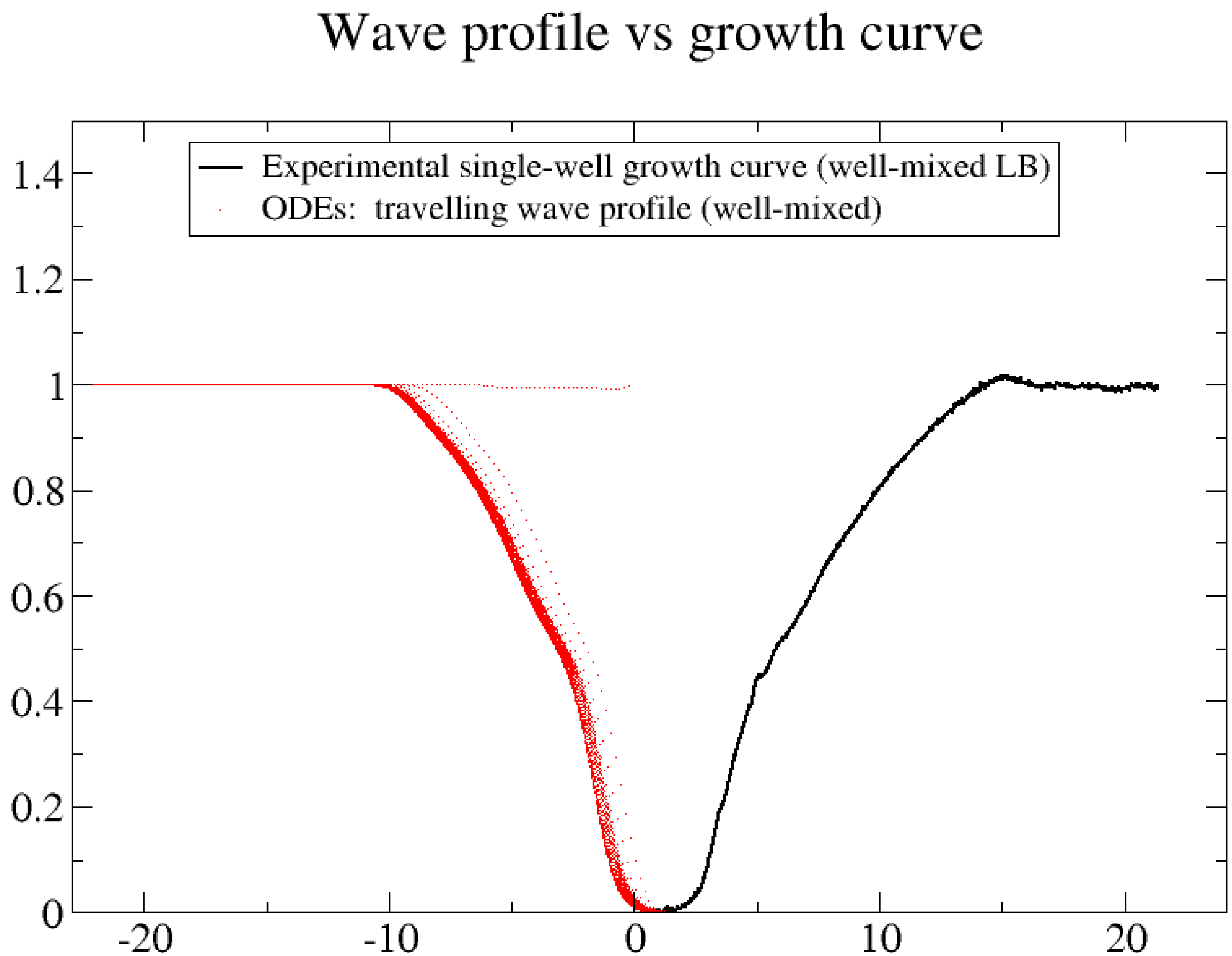


Growth in LB broth
Well-mixed LB
Non-mixed LB
Logistic growth = 2*(1 - N/K)
Growth rate (1/h)
N / K
0
0.2
0.4
0.6
0.8
1
0.5
1.5
2

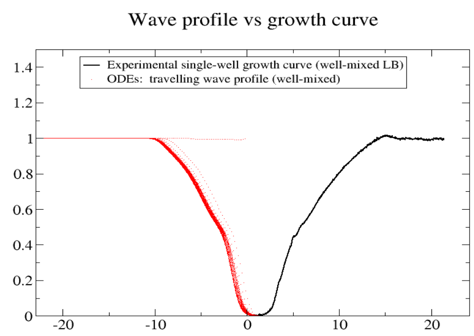


Experimental curve: v=0.757 wells/h
Logistic growth D=0.025 v=0.784
Well-mixed D=0.025 v=0.769
Non-mixed D=0.025 v=0.702
N/K or OD
z = x - vt
0
0.2
0.4
0.6
0.8
1
1.2
-40
-30
-20
-10
0
10
20

Can we predict wave profile only knowing growth curve?



Can we predict wave profile knowing only the growth curve, g(ni)?

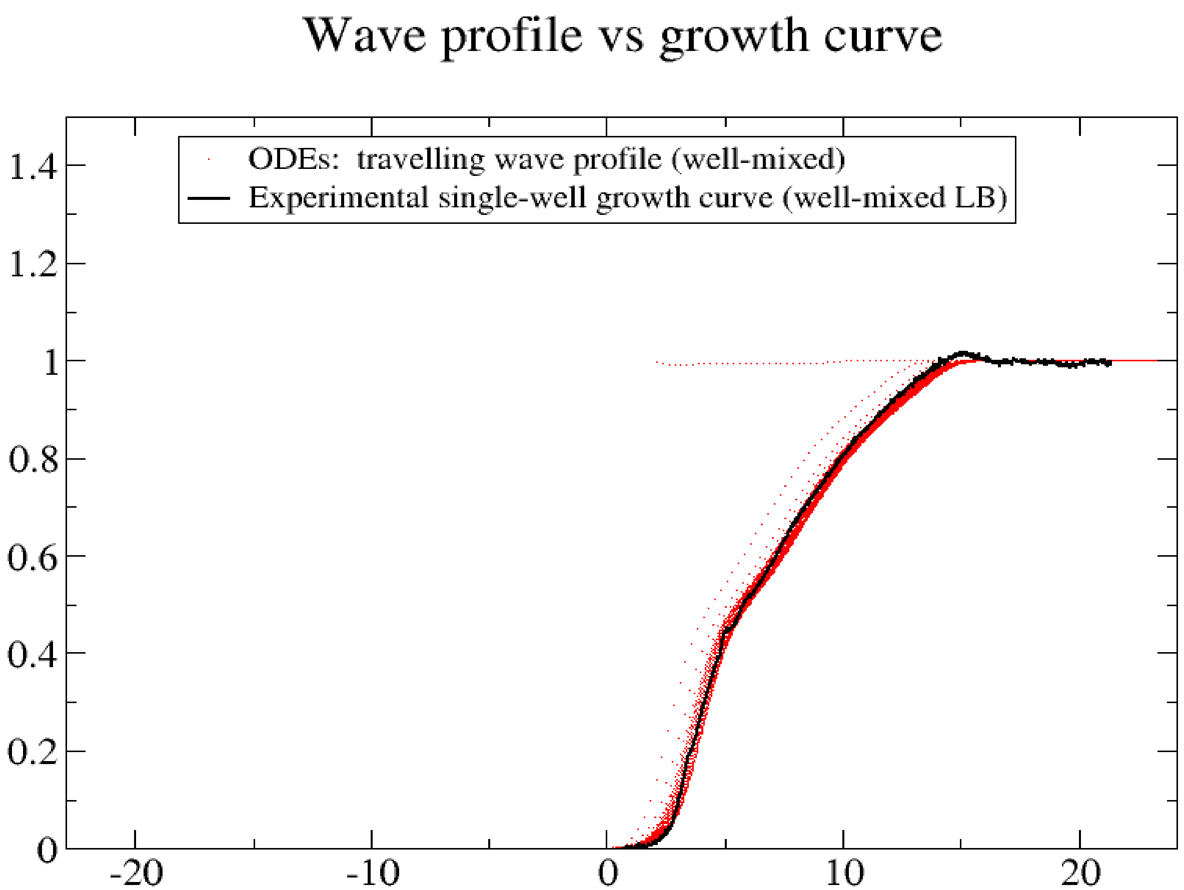


$$\frac{dn_i}{dt} = D\left(n_{i+1} + n_{i-1} - 2n_i\right) + n_i g\left(n_i\right)$$

$$n_i \equiv N\left(i - vt\right) \longrightarrow$$

$$-v\frac{dN}{dt} = D\left(\ldots\right) + Ng\left(N\right)$$

$$\boxed{\frac{dN}{dt} \simeq \frac{-1}{v} N g\left(N\right)}$$



How to predict wave speed?

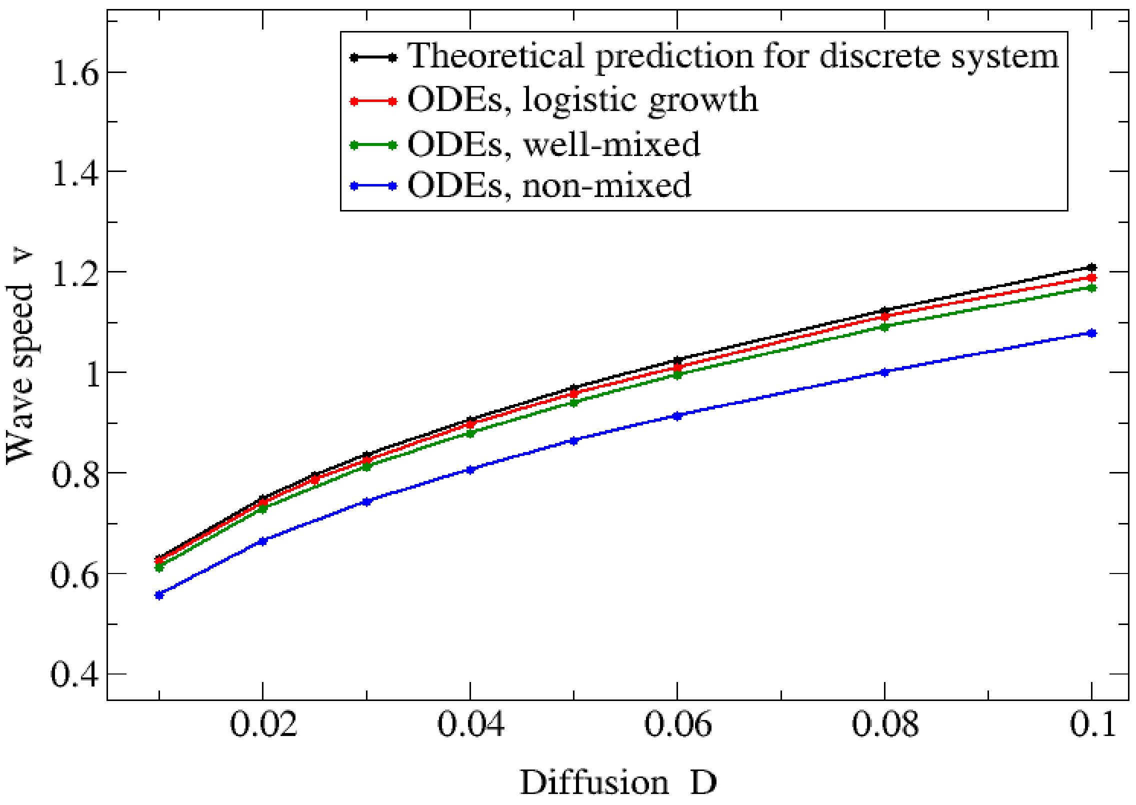
Speeds different from classic Fisher equation $v = 2\sqrt{(Dg_0)}$

Expected correction for discrete systems:

$$v = \frac{D\left(e^{-\alpha} + e^{\alpha} - 2\right)}{\alpha} + \frac{g_0}{\alpha}$$

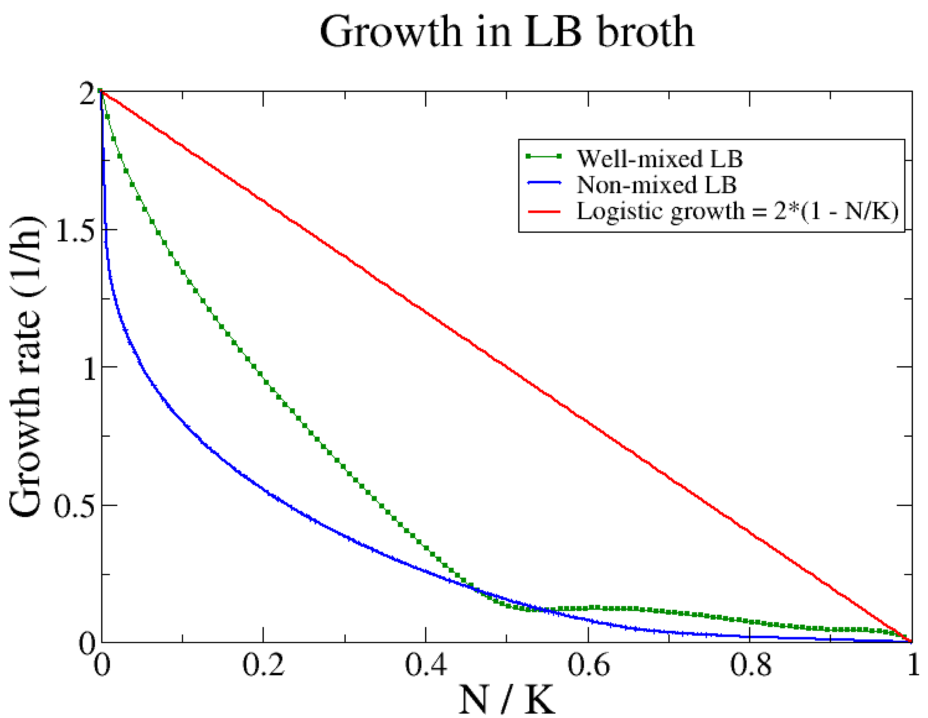
where

$$\frac{-D\left(e^{-\alpha} + e^{\alpha} - 2\right)}{\alpha^2} + \frac{D\left(e^{\alpha} - e - \alpha\right)}{\alpha} - \frac{g_0}{\alpha^2} = 0$$



How does speed depend on D?

- Dependence on D still behaves perfectly as $v \sim \sqrt{D}$
- Speeds also depend on growth function. Non-mixed LB is quite different from others due to it's very steep growth curve

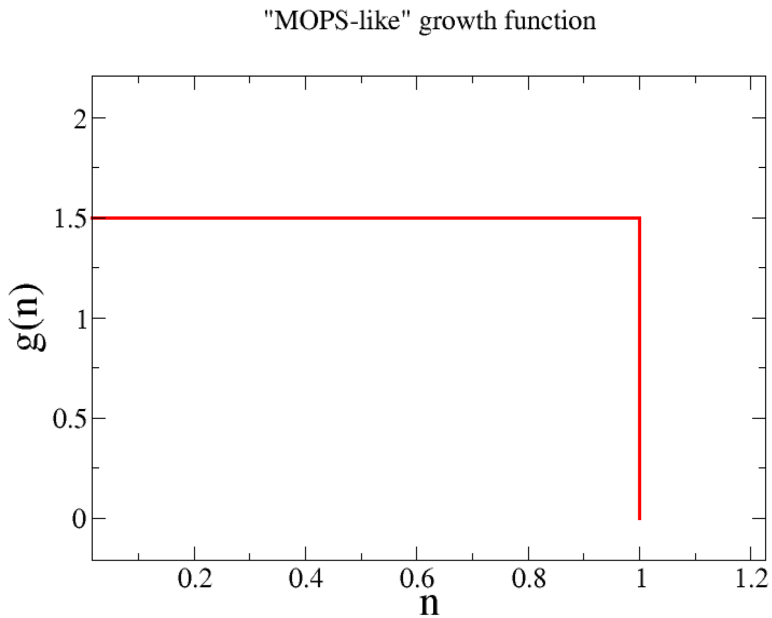


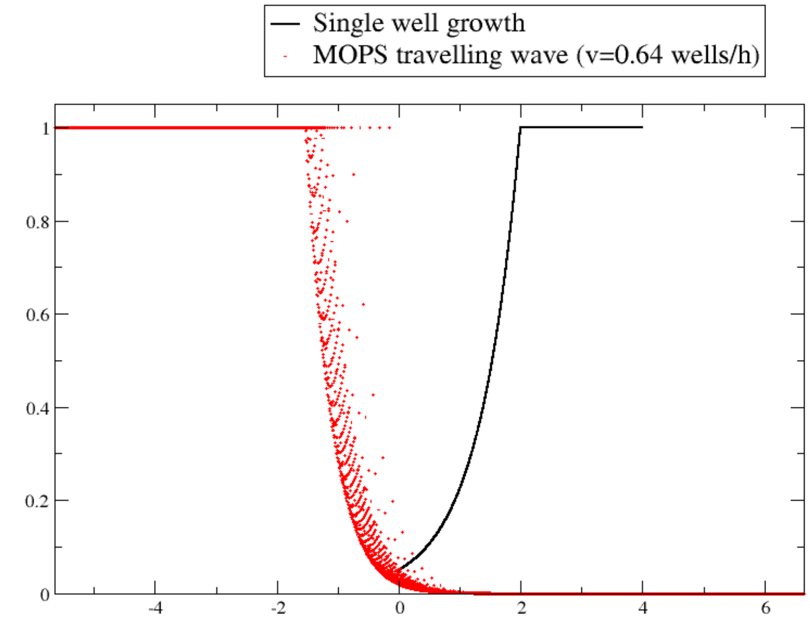
Summary

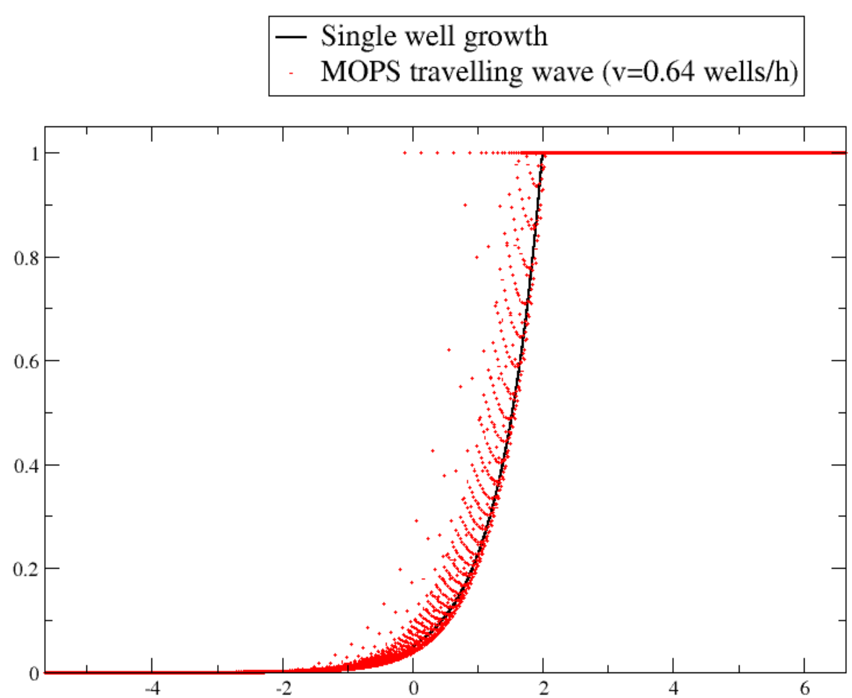
- Complicated simulation not necessary to understand experimental travelling wave

- Simple system of 1D ODEs can:
  - Predict shape of travelling wave from experimental $g(n_i)$
  - Reasonably predict speed of wave from theory of discrete system

- Test with different growth media







$$scale\ by: \frac{-1}{v}$$