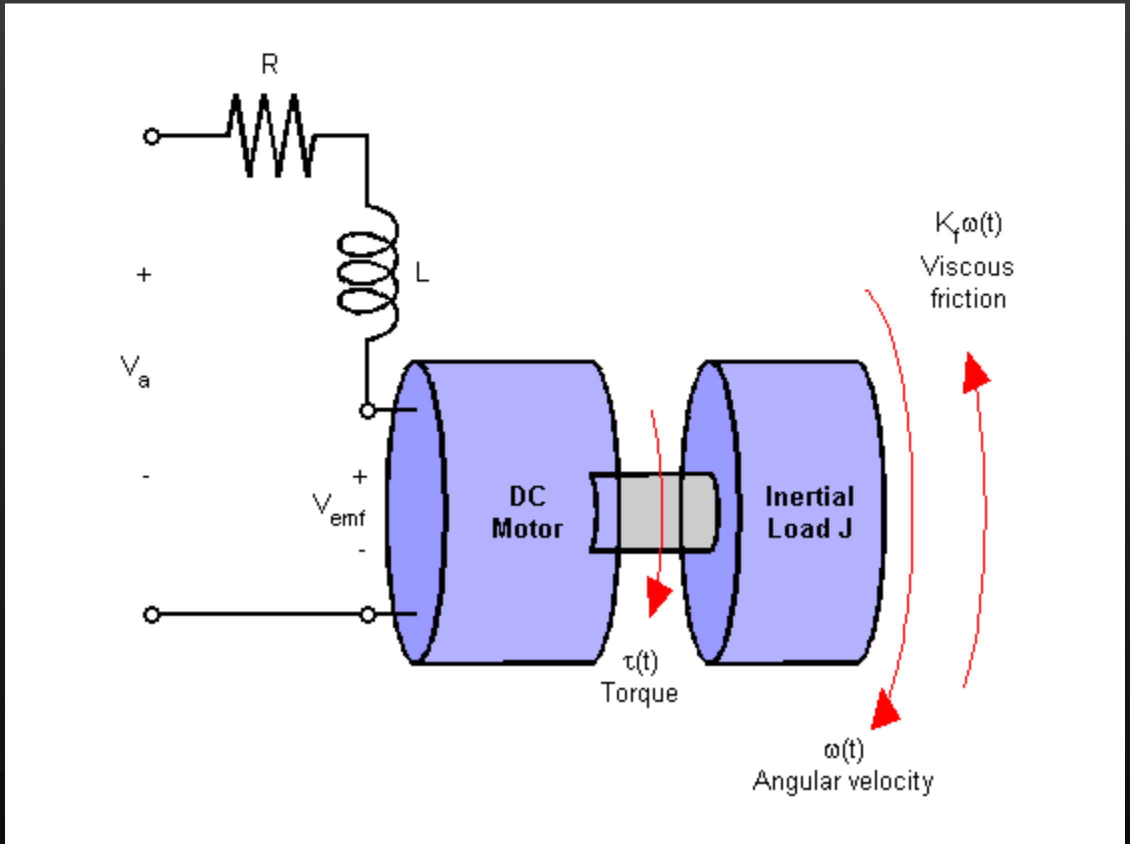
Example 5

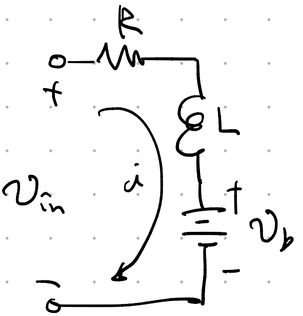
- 다음 DC모터를 모델링하여 블록다이어그램으로 나타내고, 스펙시트에 있는 수치와 비교해보자.
  (참고식)

$$\tau = K_m i$$

$$V_{emf} = K_m \omega$$

K_{emf}



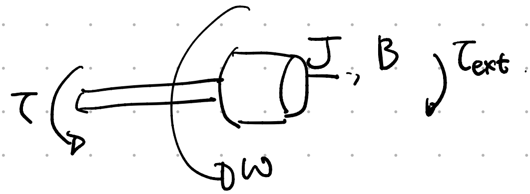


$$v_{in} - v_b = iR + L\frac{di}{dt}$$

$$\tau = Ki$$

$$V_{in} - V_b = RI + sLI$$

$$\frac{I}{V_{in} - V_b} = \frac{1}{sL + R}$$

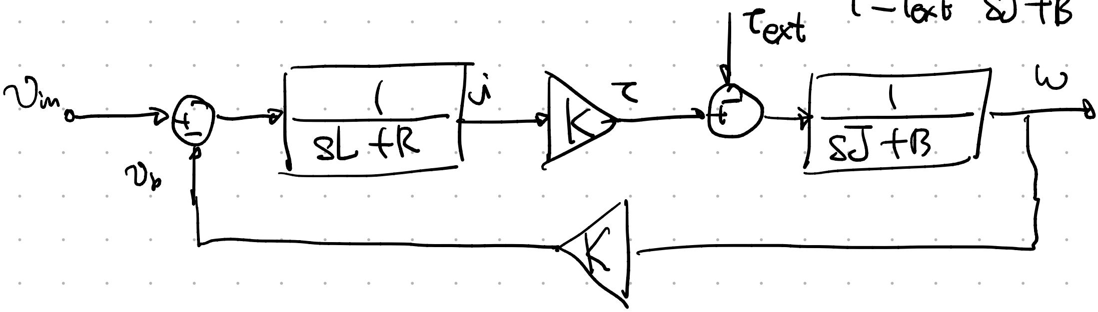


$$\tau - \tau_{ext} = J\dot{\omega} + B\omega$$

$$v_b = K\omega$$

$$T - T_{ext} = sJ\Omega + B\Omega$$

$$\frac{\Omega}{T - T_{ext}} = \frac{1}{sJ + B}$$



$$\frac{T}{V_{in} - V_b} = \frac{K}{sL + R} \qquad \frac{\Omega}{T} = \frac{1}{sJ + B} \qquad V_b = K\Omega$$

$$T = \frac{K}{sL + R}(V_{in} - V_b) \qquad \Omega = \frac{1}{sJ + B}T$$

$$\Omega = \frac{1}{sJ + B} \cdot \frac{K}{sL + R}(V_{in} - K\Omega)$$

$$\Omega\left(1 + K \cdot \frac{K}{(sJ + B)(sL + R)}\right) = \frac{K}{(sJ + B)(sL + R)}V_{in}$$

$$\Omega\left(1+K\cdot\frac{K}{(sJ+B)(sL+R)}\right)=\frac{K}{(sJ+B)(sL+R)}V_{in}$$

$$\frac{\Omega}{V_{in}}=\frac{\dfrac{K}{(sJ+B)(sL+R)}}{1+K\cdot\dfrac{K}{(sJ+B)(sL+R)}}=\frac{K}{(sJ+B)(sL+R)+K^2}$$

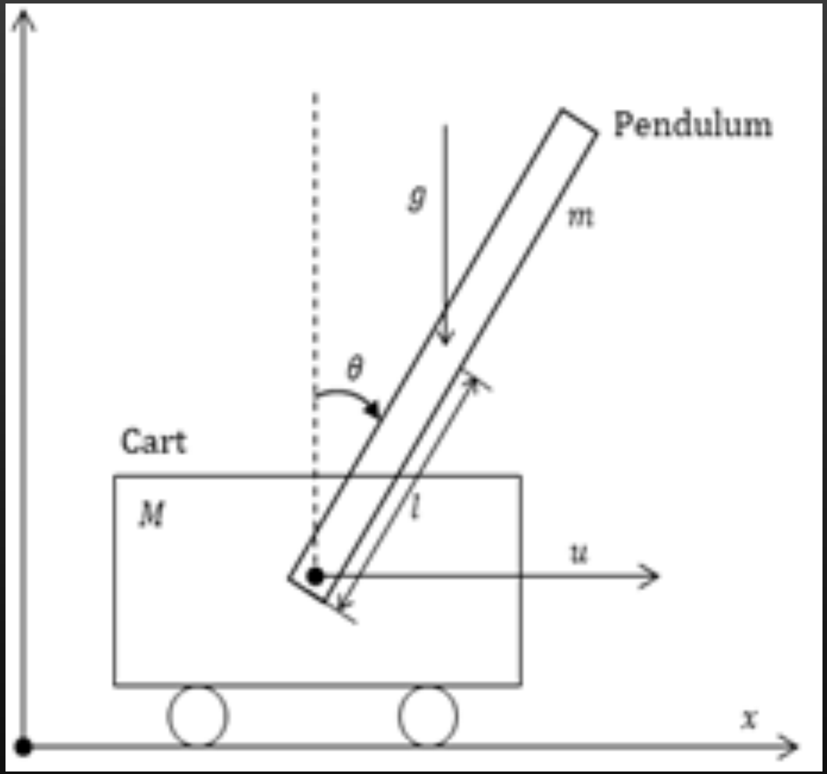
$$=\frac{K}{LJs^2+(LB+RJ)s+RB+K^2}$$

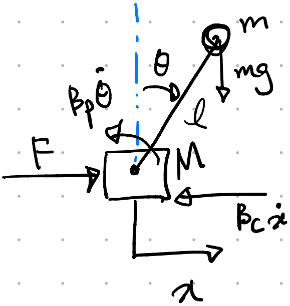
$$\frac{\theta}{V_{in}}=\frac{\Omega}{V_{in}}\times\frac{1}{s}=\frac{K}{LJs^3+(LB+RJ)s^2+(RB+K^2)s}$$

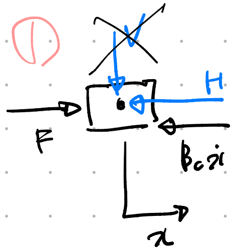
$$\frac{T}{V_{in}}=\frac{\Omega}{V_{in}}\Big/\frac{1}{(sJ+B)}=\frac{K(sJ+B)}{LJs^2+(LB+RJ)s+RB+K^2}$$

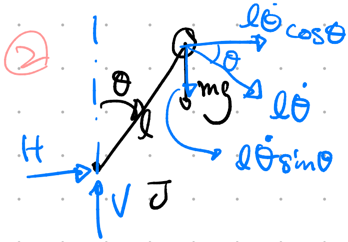
Example 6

- 다음 카트형 역진자를 모델링하여 RK를 통해 진자가 15º에서 출발했을 때의 응답을 계산해보자.
  - u: 외부에서 입력된 힘 (단위 N)
  - B_c: 카트에서 속력에 비례하는 마찰력
  - B_p: 진자의 각속도에 비례하는 마찰력









① $M\ddot{x} = F - B_c\dot{x} - H$

② $\dot{x}_p = \dot{x} + l\dot{\theta}\cos\theta$

$\dot{y}_p = -l\dot{\theta}\sin\theta$

$M\ddot{x} = F - B_c\dot{x} - m\ddot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta$

$\ddot{x}_p = \frac{d\dot{x}_p}{dt} = \ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta$

$(M+m)\ddot{x} + B_c\dot{x} + ml\cos\theta\cdot\ddot{\theta} - ml\dot{\theta}^2\sin\theta = F$

①

$\ddot{y}_p = -l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta$

$$\begin{cases} J\ddot{\theta} = l\sin\theta\, V - l\cos\theta\, H - B_p\dot{\theta} \\ m\ddot{x}_p = H \\ m\ddot{y}_p = V - mg \end{cases}$$

$$\begin{cases} H = m\ddot{x}_p = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \\ V = m\ddot{y}_p + mg = -ml\ddot{\theta}\sin\theta - ml\dot{\theta}^2\cos\theta + mg \end{cases}$$

$$\begin{aligned} J\ddot{\theta} &= l\sin\theta\{-ml\ddot{\theta}\sin\theta - ml\dot{\theta}^2\cos\theta + mg\} \\ &\quad - l\cos\theta\{m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta\} - B_p\dot{\theta} \\ &= -ml^2\ddot{\theta}\sin^2\theta - ml^2\dot{\theta}^2\sin\theta\cos\theta + mgl\sin\theta \\ &\quad - ml\ddot{x}\cos\theta - ml^2\ddot{\theta}\cos^2\theta + ml^2\dot{\theta}^2\sin\theta\cos\theta - B_p\dot{\theta} \\ &= -ml^2\ddot{\theta} - ml\ddot{x}\cos\theta + mgl\sin\theta - B_p\dot{\theta} \end{aligned}$$

②

$$\boxed{\begin{cases} (M+m)\ddot{x} + B_c\dot{x} + ml\cos\theta\cdot\ddot{\theta} - ml\dot{\theta}^2\sin\theta = F \\ ml\cos\theta\,\ddot{x} + (J+ml^2)\ddot{\theta} + B_p\dot{\theta} = mgl\sin\theta \end{cases}}$$

$$\boxed{\begin{cases} (M+m)\ddot{x} + B_c\dot{x} + ml\cos\theta\cdot\ddot{\theta} - ml\dot{\theta}^2\sin\theta = F \\ ml\cos\theta\,\ddot{x} + (J+ml^2)\ddot{\theta} + B_p\dot{\theta} = mgl\sin\theta \end{cases}}$$

$$\frac{dy}{dt} = f(t, y \cdots)$$

$$(M+m)\ddot{x} + \{ml\cos\theta\}\cdot\ddot{\theta} = F + ml\dot{\theta}^2\sin\theta - B_c\dot{x}$$

$$(ml\cos\theta)\ddot{x} + (J+ml^2)\ddot{\theta} = mgl\sin\theta - B_p\dot{\theta}$$

$$\underbrace{\begin{bmatrix} M+m & ml\cos\theta \\ ml\cos\theta & J+ml^2 \end{bmatrix}}_{A}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} F + ml\dot{\theta}^2\sin\theta - B_c\dot{x} \\ mgl\sin\theta - B_p\dot{\theta} \end{bmatrix}}_{B}$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}^{-1}\begin{bmatrix} B \end{bmatrix}$$

$$\begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix} \leftarrow \begin{pmatrix} \dot{x} \\ \dot{\theta} \end{pmatrix} \leftarrow \begin{pmatrix} x \\ \theta \end{pmatrix}$$

$$\boxed{\begin{cases} (M+m)\ddot{x} + B_c\dot{x} + ml\cos\theta\cdot\ddot{\theta} - ml\dot{\theta}^2\sin\theta = F \\ ml\cos\theta\,\ddot{x} + (J+ml^2)\ddot{\theta} + B_p\dot{\theta} = mgl\sin\theta \end{cases}}$$

$\underline{\theta \ll 1}$ $\quad \sin\theta \to \theta \quad \dot{\theta}^2 \to 0.$

$\quad \cos\theta \to 1$

$$(M+m)\ddot{x} + ml\ddot{\theta} = -B_c\dot{x} + F$$

$$ml\ddot{x} + (J+ml^2)\ddot{\theta} = -B_p\dot{\theta} + mgl\theta$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \quad & \quad \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ x \\ \theta \end{bmatrix} + \begin{bmatrix} \quad \end{bmatrix} F$$