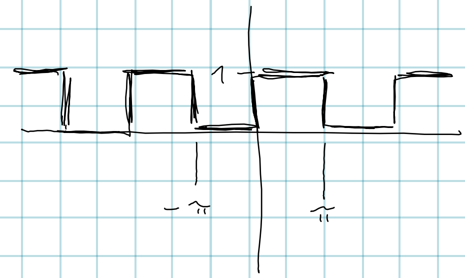
1

HCS Übung 05

Aufgabe 5



$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\left(\int_{-\pi}^{0} f(x)\,dx - \int_{0}^{\pi} f(x)\,dx\right)$$

$$= \frac{1}{2\pi}\left(\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} 1\,dx\right) = \frac{1}{2\pi}\int_{0}^{\pi} 1\,dx$$

$$= \frac{1}{2\pi}\Big[x\Big]_0^{\pi} = \frac{1}{2\pi}(\pi - 0) = \frac{\pi}{2\pi} = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} f(x)\cos(nx)\,dx + \int_{0}^{\pi} f(x)\cos(nx)\,dx\right)$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{0} 0\cdot\cos(nx)\,dx + \int_{0}^{\pi} 1\cdot\cos(nx)\,dx\right) = \frac{1}{\pi}\int_{0}^{\pi}\cos(nx)\,dx$$

$$= \frac{1}{\pi}\left[\frac{\sin(nx)}{n}\right]_0^{\pi} = \frac{1}{\pi}\left(\frac{\sin(\pi n) - \sin(0)}{n}\right) = \frac{1}{\pi n}\cdot\sin(\pi n)$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin(nx)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} f(x)\sin(nx)\,dx + \int_{0}^{\pi} f(x)\sin(nx)\,dx\right)$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{0} 0\cdot\sin(nx)\,dx + \int_{0}^{\pi} 1\cdot\sin(nx)\,dx\right) = \frac{1}{\pi}\int_{0}^{\pi}\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[\frac{-\cos(nx)}{n}\right]_0^{\pi} = \frac{1}{\pi}\left(\frac{-\cos(\pi n)}{n} - \frac{-\cos(0\cdot n)}{n}\right)$$

$$= \frac{1}{\pi n}\left(-\cos(\pi n) \underbrace{- -\cos(0)}_{+1}\right) = \frac{1}{\pi n}\left(1 - \cos(\pi n)\right)$$

<u>Fall $n \bmod 2 = 0$</u>: $(\cos(\pi n) = 1)$

$$b_n = \frac{1}{\pi n}(1-1) = \frac{1}{\pi n}\cdot 0 = 0$$

<u>Fall $n \bmod 2 = 1$</u>: $(\cos(\pi n) = -1)$

$$b_n = \frac{1}{\pi n}(1-(-1)) = \frac{2}{\pi n}$$

$$\Rightarrow f(x) = \frac{1}{2} + \frac{2}{\pi}\sin(x) + \frac{2}{3\pi}\sin(3x) + \frac{2}{5\pi}\sin(5x) + \dots$$