Multiple Attribute Group Decision Making using Interval-Valued Intuitionistic Fuzzy Soft Matrix

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Multiple Attribute Group Decision Making using Interval-Valued Intuitionistic Fuzzy Soft Matrix

Sujit Das, Mohuya B. Kar, Tandra Pal, Samarjit Kar

Abstract- **A noticeable progress has been found in decision making problems since the introduction of soft set theory by Molodtsov in 1999. It is found that classical soft sets are not suitable to deal with imprecise parameters whereas fuzzy soft sets (FSS) are proved to be useful. Use of intuitionistic fuzzy soft sets (IFSS) is more effective in environment where arguments are presented using membership and non-membership values. In this paper we propose an algorithmic approach for multiple attribute group decision making problems using interval-valued intuitionistic fuzzy soft matrix (IVIFSM). IVIFSM is the matrix representation of interval-valued intuitionistic fuzzy soft set (IVIFSS), where IVIFSS is a natural combination of interval-valued intuitionistic fuzzy set and soft set theory. Firstly, we propose the concept of IVIFSM. Then an algorithm is developed to find out the desired alternative(s) based on product interval-valued intuitionistic fuzzy soft matrix, combined choice matrix, and score values of the set of alternatives. Finally, a practical example has been demonstrated to show the effectiveness of the proposed algorithm.**

Keywords- Multiple attribute group decision making, interval-valued intuitionistic fuzzy soft matrix, interval-valued intuitionistic fuzzy soft set, choice matrix.

I. INTRODUCTION

Soft set theory [1] was introduced by Molodtsov in 1999 as a generic mathematical tool for dealing with uncertain problems which cannot be handled using traditional mathematical tools. A soft set can be used for approximate description of objects without any restriction. Due to this absence of restriction on the approximate description, soft set theory has been emerging as a convenient and easily applicable tool in practice [2]. Since its introduction, this theory has been successfully applied in many different fields such as decision making [3]-[9], data analysis [10], forecasting [11], simulation [12], optimization [13], texture classification [14], etc.

Maji et al. [15] presented some operations for soft sets and also discussed their properties. They presented the concept of fuzzy soft set (FSS) [16] which is based on a combination of the fuzzy set and soft set models. Yang et al. [17] introduced the concept of the interval-valued fuzzy soft set (IVFSS) by combining the interval-valued fuzzy set and soft set and then proposed a decision making algorithm based on IVFSS. Decision making problems were solved first by Maji and Roy [8] using soft sets. Roy and Maji, in [9], proposed a soft set theoretic approach to deal with decision making and introduced the concept of choice value. Cagman and Enginoglu [3], [4] proposed the concept of soft matrix to represent a soft set. Xu et al. [18] discussed the properties of vague soft sets. Jiang et al. [19] introduced interval-valued intuitionistic fuzzy soft sets by combining interval-valued intuitionistic fuzzy set and soft set theory and discussed their properties. Jiang et al. [6] presented an adjustable approach to intuitionistic fuzzy soft sets based decision making by using level soft sets of intuitionistic fuzzy soft sets. Feng et al. [20] extended the level soft sets method to interval-valued fuzzy soft sets. Qin et al. [21] generalized the approaches introduced by Feng et al. [20] and Jiang et al. [6]. Qin et al. [21] also defined the notion of reduct intuitionistic fuzzy soft sets and presented an adjustable approach for decision making based on interval-valued intuitionistic fuzzy soft set. Zhang et al. [22] investigated the decision making problems based on interval-valued intuitionistic fuzzy soft sets. They developed an adjustable approach to interval-valued intuitionistic fuzzy soft sets based decision making using level soft sets. Concept of fuzzy parameterized interval-valued fuzzy soft set theory has been introduced by Alkhazaleh et al. [23]. They studied their operations, then developed aggregation operator to deal with decision making problems. Mao et al. [24] presented the concept of intuitionistic fuzzy soft matrix (IFSM) and applied it in group decision making problems. Recently, Das and Kar [25], [26] have introduced the concept of intuitionistic multi fuzzy soft set and hesitant fuzzy soft set, and also applied them in decision making problem.

The goal of multiple attribute decision making (MADM) is to select better alternative(s) from a set of alternatives based on a set of attribute values. The novelty of this paper is to introduce the concept of interval-valued intuitionistic fuzzy soft matrix and present an algorithmic approach for multiple attribute group decision making using IVIFSM. In our algorithm, a group of decision makers suggest their opinions regarding a set of alternatives and their attributes using interval-valued intuitionistic fuzzy soft sets which are represented by IVIFSMs. Interval-valued intuitionistic fuzzy

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sets are found to be useful where decision makers are not sure about a particular value rather they would like to rely on an interval to represent linguistic information. To facilitate our approach we use combined choice matrix for individual decision maker by incorporating the choice parameters of the set of experts. Next, for each decision maker, we multiply the combined choice matrix with the individual IVIFSM to produce the product IVIFSM of that expert. Summation of product IVIFSMs of all decision makers compute the resultant IVIFSM, where the membership and non-membership values of each alternative are added to generate the corresponding weight. Finally, score and accuracy values are calculated to yield the desired alternative(s).

Rest of the paper is organised as follows. In section 2 we briefly review some basic notions and background of soft sets, fuzzy soft sets, interval-valued fuzzy soft sets and interval-valued intuitionistic fuzzy soft sets. Section 3 presents interval-valued intuitionistic fuzzy soft matrix followed by the proposed algorithmic approach in section 4. A case study has been illustrated in section 5 to verify the practicability and effectiveness of the proposed method. Finally, conclusions are drawn in section 6.

II. PRELIMINARIES

This section briefly reviews some basic concepts related with this article.

Definition 1 [1]. Let U be an initial universe, E be a set of parameters, P(U) be the power set of U, and $A \subset E$. A pair (F, A) is called a **soft set** over U, where F is a mapping $F : A \to P(U)$. In other words, a soft set over U is a mapping from parameters to P(U), and it is not a set, but a parameterized family of subsets of U. For any parameter $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2 [16]. Let U be an initial universe and E be a set of parameters (which are fuzzy variables). Let $\widetilde{P}(U)$ denotes the set of all fuzzy sets of U and $A \subset E$. A pair $(\widetilde{F}_A, E)$ is called a **Fuzzy Soft Set** (FSS) over U , where $\widetilde{F}_A$ is a mapping given by, $\widetilde{F}_A : E \to \widetilde{P}(U)$ such that $\widetilde{F}_A(e) = \tilde{\phi}$ if $e \notin A$, where $\tilde{\phi}$ is a null fuzzy set.

Remark 1. A fuzzy soft set is a parameterized family of fuzzy subsets of U. Its universe is the set of all fuzzy sets of U, i.e., P(U). A fuzzy soft set can be considered a special case of a soft set because it is still a mapping from parameters to a universe. The difference between fuzzy soft set and soft set is that in a fuzzy soft set, the universe to be considered is the set of fuzzy subsets of U.

Definition 3 [17]. Let U be an initial universe and E be a set of parameters. IVF (U) denotes the set of all interval-valued fuzzy sets of U. Let $A \subseteq E$. A pair $(\widehat{F}, A)$ is an **interval-valued fuzzy soft set** over U, where $\widehat{F}$ is a mapping, given by $\hat{F} : A \to IVF$ (U). An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U, thus, its universe is the set of all interval-valued fuzzy sets of U, i.e., IVF (U).

Remark 2. The combined result of fuzzy set and soft set theory is defined as fuzzy soft set. However, it is found that in many real applications, the membership degree in a fuzzy set cannot be absolutely confirmed. It is more reasonable to give an interval-valued data to describe membership degree. From such point of view, Zadeh [30] proposed the concept of interval-valued fuzzy set which is defined by combining the interval-valued fuzzy set and soft set model.

Defintion 4 [27]. Atanassov and Gargov (1989) introduced **interval-valued intuitionistic fuzzy set (IVIFS)**. Let *X* be a universal set. An interval-valued intuitionistic fuzzy set *A* in *X* can be expressed as

$$A = \{< x, [\mu_A^l(x), \mu_A^r(x)], [\nu_A^l(x), \nu_A^r(x)] > | \, x \in X\},$$

where

$[\mu_A^l(x), \mu_A^r(x)] \in [0,1]$ and $[\nu_A^l(x), \nu_A^r(x)] \in [0,1]$ are respectively the interval-valued degrees of membership and non-membership of an element $x \in X$ to *A*, which are more or less independent on each other. The only requirement is that the sum of upper bunds of these two interval-valued degrees is not greater than 1, i.e., $0 \le \mu_A^r(x) + \nu_A^r(x) \le 1.$

If $\mu_A^l(x) = \mu_A^r(x)$ and $\nu_A^l(x) = \nu_A^r(x), \ \forall \, x \in X,$ then the IVIFS

$$A = \{< x, [\mu_A^l(x), \mu_A^r(x)], [\nu_A^l(x), \nu_A^r(x)] > | \, x \in X\}$$

is reduced to Atanassov's IFS, denoted by

$$\{< x, \mu_A(x), \nu_A(x) > | \, x \in X\},$$

where

$\mu_A(x) = \mu_A^l(x) = \mu_A^r(x)$ and $\nu_A(x) = \nu_A^l(x) = \nu_A^r(x)$.

So, we can say Atanassov's IFS is a special case of IVIFS.

For a fixed $x \in X$, an object $[\mu_A^l(x), \mu_A^r(x)], [\nu_A^l(x), \nu_A^r(x)]$ is called interval-valued intuitionistic fuzzy number (IVIFN). Let $\alpha = [a,b],[c,d]$ be an IVIFN. The score function [28] S of α can be defined as $S(\alpha) = \dfrac{(a-c)+(b-d)}{2}$, where $S(\alpha) \in [0,1]$. The accuracy function [29] H of α can be defined as $H(\alpha) = \dfrac{(a+c)+(b+d)}{2}$, where $H(\alpha) \in [0,1]$.

Definition 5 [19]. Let U be an initial universe and E be a set of parameters. $IVIF$ (U) denotes the set of all interval-valued intuitionistic fuzzy sets of U. Let $A \subseteq E$. A pair $(\widehat{F}, A)$ is an **interval-valued intuitionistic fuzzy soft set** over U, where $\widehat{F}$ is a mapping, given by $\hat{F} : A \to IVIF$ (U). An interval-valued intuitionistic fuzzy soft set is a parameterized family of interval-valued intuitionistic fuzzy subsets of U, thus, its universe is the set of all interval-valued intuitionistic fuzzy sets of U, i.e., $IVIF(\mathrm{U})$. $\forall \varepsilon \in A, F(\varepsilon)$ is an interval-valued intuitionistic fuzzy set of U. $F(\varepsilon)$ can be expressed as: $F(\varepsilon)=\{<x,[\mu^{l}_{F(\varepsilon)}(x),\mu^{r}_{F(\varepsilon)}(x)],[\nu^{l}_{F(\varepsilon)}(x),\nu^{r}_{F(\varepsilon)}(x)]>| x\in X\}$.

Definition 6. Choice Matrix is a square matrix whose rows and columns both indicate parameters. If ξ is a choice matrix, then its element $\xi(i,j)$ is defined as follows:

$$\xi(i,j)^{P}=\begin{cases}(1,1),(1,1) \text{ when } \mathrm{i}^{\text{th}} \text{ and } \mathrm{j}^{\text{th}} \text{ parameters are}\\ \text{ both choice parameters of the decision makers}\\ (0,0),(0,0) \text{ otherwise, i.e., when at least one of the } \mathrm{i}^{\text{th}}\\ \text{or } \mathrm{j}^{\text{th}} \text{ parameters is not under choice of the decision makers.}\end{cases}$$

In **combined choice matrix**, rows indicate choice parameters of single decision maker, where columns indicate combined choice parameters (obtained by the intersection of parameters sets) of decision makers.

III. Interval-Valued Intuitionistic Fuzzy Soft Matrix

Let $(\widehat{F}_A,E)$ be an interval-valued intuitionistic fuzzy soft set over the initial universe U. Let E be a set of parameters and $A\subseteq E$. Then a subset of $U\times E$ is uniquely defined by $R_A=\{(u,e):e\in A,u\in \widehat{F}_A(e)\}$, which is called a relation of $(\widehat{F}_A,E)$. The membership function of R_A is written as $\mu_{R_A}:U\times E\rightarrow Int([0,1])$ and defined by

$$\mu_{R_A}(u,e)=\begin{cases}\{[\mu^{l}_{\widehat{F}A(e)}(u),\mu^{r}_{\widehat{F}A(e)}(u)],[\nu^{l}_{\widehat{F}A(e)}(u),\nu^{r}_{\widehat{F}A(e)}(u)] \; if \; e\in A\\ [0,0], \; if \; e\notin A,\end{cases}$$

where $Int([0,1])$ stands for the set of all closed subintervals of $[0,1]$ and $[\mu^{l}_{\widehat{F}A(e)}(u),\mu^{r}_{\widehat{F}A(e)}(u)]$ and $[\nu^{l}_{\widehat{F}A(e)}(u),\nu^{r}_{\widehat{F}A(e)}(u)]$ respectively denote the interval-valued intuitionistic fuzzy membership and non-membership degrees of the object u associated with the parameter e.

If $U=\{x_1,x_2,...,x_m\}$ and $E=\{e_1,e_2,...,e_n\}$, then R_A can be presented as follows:

$$\widehat{F}=\begin{pmatrix} & e_1 & e_2 & \cdots & e_n\\ x_1 & \left(\mu^{l}_{\widehat{F}(e_1)}(x_1),\mu^{r}_{\widehat{F}(e_1)}(x_1)\right), & \left(\mu^{l}_{\widehat{F}(e_2)}(x_1),\mu^{r}_{\widehat{F}(e_2)}(x_1)\right), & \cdots & \left(\mu^{l}_{\widehat{F}(e_n)}(x_1),\mu^{r}_{\widehat{F}(e_n)}(x_1)\right),\\ & \left(\nu^{l}_{\widehat{F}(e_1)}(x_1),\nu^{r}_{\widehat{F}(e_1)}(x_1)\right) & \left(\nu^{l}_{\widehat{F}(e_2)}(x_1),\nu^{r}_{\widehat{F}(e_2)}(x_1)\right) & & \left(\nu^{l}_{\widehat{F}(e_n)}(x_1),\nu^{r}_{\widehat{F}(en)}(x_1)\right)\\ x_2 & \left(\mu^{l}_{\widehat{F}(e_1)}(x_2),\mu^{r}_{\widehat{F}(e_1)}(x_2)\right), & \left(\mu^{l}_{\widehat{F}(e_2)}(x_2),\mu^{r}_{\widehat{F}(e_2)}(x_2)\right), & \cdots & \left(\mu^{l}_{\widehat{F}(e_n)}(x_2),\mu^{r}_{\widehat{F}(e_n)}(x_2)\right)\\ & \left(\nu^{l}_{\widehat{F}(e_1)}(x_2),\nu^{r}_{\widehat{F}(e_1)}(x_2)\right) & \left(\nu^{l}_{\widehat{F}(e_2)}(x_2),\nu^{r}_{\widehat{F}(e_2)}(x_2)\right) & & \left(\nu^{l}_{\widehat{F}(e_n)}(x_2),\nu^{r}_{\widehat{F}(e_n)}(x_2)\right)\\ & \vdots & \vdots & \cdots & \vdots\\ x_m & \left(\mu^{l}_{\widehat{F}(e_1)}(x_m),\mu^{r}_{\widehat{F}(e_1)}(x_m)\right), & \left(\mu^{l}_{\widehat{F}(e_2)}(x_m),\mu^{r}_{\widehat{F}(e_2)}(x_m)\right) & \cdots & \left(\mu^{l}_{\widehat{F}(e_n)}(x_m),\mu^{r}_{\widehat{F}(e_n)}(x_m)\right)\\ & \left(\nu^{l}_{\widehat{F}(e_1)}(x_m),\nu^{r}_{\widehat{F}(e_1)}(x_m)\right) & \left(\nu^{l}_{\widehat{F}(e_2)}(x_m),\nu^{r}_{\widehat{F}(e_2)}(x_m)\right) & & \left(\nu^{l}_{\widehat{F}(e_n)}(x_m),\nu^{r}_{\widehat{F}(e_n)}(x_m)\right)\end{pmatrix}$$

For simplicity, if we take the $[ij]^{th}$ entry of the relation $\widehat{F}$ as

$\widehat{a}_{ij}=\{[\mu^{l}_{\widehat{F}A(e_j)}(x_i),\mu^{r}_{\widehat{F}A(e_j)}(x_i)],[\nu^{l}_{\widehat{F}A(e_j)}(x_i),\nu^{r}_{\widehat{F}A(e_j)}(x_i)]\}$,

then the matrix can be defined as

$$[\widehat{a}_{ij}]_{m\times n}=\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{m1} & a_{m2} & \cdots & a_{mn}\end{bmatrix}.$$

The above matrix is called an **interval-valued intuitionistic fuzzy soft matrix** of order $m\times n$ corresponding to the interval-valued intuitionistic fuzzy soft set $(\widehat{F}_A,E)$ over U.

Example 1. Let U be the set of five diseases, given by, $U=\{D_1,D_2,D_3,D_4,D_5\}$ and E the set of five symptoms, given by, $E=\{S_1,S_2,S_3,S_4,S_5\}$. Let $A=\{S_1,S_2,S_3,S_5\}\subset E$. Now suppose that, $\widehat{F}_A:E\rightarrow P(U)$ describe the possible diseases corresponding to a set of symptoms and the interval-valued intuitionistic fuzzy soft set $(\widehat{F}_A,E)$ is given by

$(\widehat{F}_A,E)=\{diseases\ involved\ with\ S_1=$
$\{D_1/[.4,.6],[.3,.4],\ D_2/[.7,.8],[.1,.2],$
$D_3/[.4,.5],[.3,.4],\ D_4/[.3,.6],[.2,.4], D_5/[.2,.3],[.4,.6]\},$
$diseases\ involved\ with\ S_2=$
$\{D_1/[0,.1],[.5,.7], D_2/[.7,.9],[0,.1],$
$D_3/[.3,.4],[.3.,6], D_4/[.4,.6],[.1,.3], D_5/[.2,.5],[.1,.4]\},$

$diseases\,involved\,with\,S_3$ =
$\{D_1/[.3,.5],[.2,.5], D_2/[.4,.5],[.1,.4],$
$D_3/[.6,.8],[0,.1], D_4/[.1,.2],[.5,.8], D_5/[.3,.5],[.2,.4]\},$
$diseases\,involved\,with\,S_5$ =
$\{D_1/[.5,.7],[.2,.3], D_2/[.3,.7],[.1,.2],$
$D_3/[.1,.2],[.6,.7], D_4/[.4,.6][.2,.4], D_5/[.7,.8],[.1,.2]\}$

Hence, the IVFSM $[\widehat{a}_{ij}]$ can be written as,

$$[\widehat{a}_{ij}] = \begin{pmatrix} [.4,.6],[.3,.4] & [0,.1],[.5,.7] & [.3,.5],[.2,.5] & [0,0],[0,0] & [.5,.7],[.2,.3] \\ [.7,.8],[.1,.2] & [.7,.9],[0,.1] & [.4,.5],[.1,.4] & [0,0],[0,0] & [.3,.7],[.1,.2] \\ [.4,.5],[.3,.4] & [.3,.4],[.3,.6] & [.6,.8],[0,.1] & [0,0],[0,0] & [.1,.2],[.6,.7] \\ [.3,.6],[.2,.4] & [.4,.6],[.1,.3] & [.1,.2],[.5,.8] & [0,0],[0,0] & [.4,.6],[.2,.4] \\ [.2,.3],[.4,.6] & [.2,.5],[.1,.4] & [.3,.5],[.2,.4] & [0,0],[0,0] & [.7,.8],[.1,.2] \end{pmatrix}.$$

Product of Interval-Valued Intuitionistic Fuzzy Soft Matrix and Choice Matrix can be implemented if the number of columns of interval-valued intuitionistic fuzzy soft matrix $\widehat{A}$ be equal to the number of rows of the choice matrix β. Then $\widehat{A}$ and β are said to be conformable for the product $(\widehat{A} \otimes \beta)$ and the product $(\widehat{A} \otimes \beta)$ becomes interval-valued intuitionistic fuzzy soft matrix. The product $(\widehat{A} \otimes \beta)$ is also denoted simply by $\widehat{A}\beta$.

If $\widehat{A} = [\widehat{a}_{ij}]_{m \times n}$ and $\beta = [\widehat{\beta}_{jk}]_{n \times p}$, then $\widehat{A}\beta = [\widehat{c}_{ik}]_{m \times p}$, where

$$\widehat{c}_{ik} = \begin{Bmatrix} [\max_{j=1}^{n} \min\{\mu^l_{\widehat{a}_{ij}}, \mu^l_{\widehat{\beta}_{jk}}\}, \max_{j=1}^{n} \min\{\mu^r_{\widehat{a}_{ij}}, \mu^r_{\widehat{\beta}_{jk}}\}], \\ [\min_{j=1}^{n} \min\{\nu^l_{\widehat{a}_{ij}}, \nu^l_{\widehat{\beta}_{jk}}\}, \min_{j=1}^{n} \min\{\nu^r_{\widehat{a}_{ij}}, \nu^r_{\widehat{\beta}_{jk}}\}] \end{Bmatrix}.$$

Two interval-valued intuitionistic fuzzy soft matrices $\widehat{A}$ and $\widehat{B}$ are said to be **conformable for addition**, if they are of the same order and after addition the obtained sum is also an $IVIFSM$ of the same order. Now if both $\widehat{A} = (\widehat{a}_{ij})$ and $\widehat{B} = (\widehat{b}_{ij})$ be the same order $m \times n$, then the addition of $\widehat{A}$ and $\widehat{B}$ is denoted by $\widehat{A} \oplus \widehat{B}$ and is defined by $[\widehat{c}_{ij}] = [\widehat{a}_{ij}] \oplus [\widehat{b}_{ij}]$, *where* $\widehat{c}_{ij}$

$$= \begin{Bmatrix} [\max\{\mu^l_{a_{ij}}, \mu^l_{b_{ij}}\}, \max\{\mu^r_{a_{ij}}, \mu^r_{b_{ij}}\}], \\ [\min\{\nu^l_{a_{ij}}, \nu^l_{b_{ij}}\}, \min\{\nu^r_{a_{ij}}, \nu^r_{b_{ij}}\}] \end{Bmatrix} \forall i, j.$$

Complement of an IVIFSM $(\widehat{a}_{ij})_{m \times n}$ is denoted by $(\widehat{a}_{ij})^c_{m \times n}$, where $(\widehat{a}_{ij})_{m \times n}$ is the matrix representation of the interval-valued intuitionistic fuzzy soft set $(\widehat{F}_A, E)$. $(\widehat{a}_{ij})^c_{m \times n}$ is the matrix representation of the interval-valued intuitionistic fuzzy soft set $(\widehat{F}^c_{\neg A}, E)$ and can be defined as

$$(\widehat{a}_{ij})^c_{m \times n} = \left[\left(\mu^l_{a_{ij}}\right)^c, \left(\mu^r_{a_{ij}}\right)^c\right], \left[\left(\nu^l_{a_{ij}}\right)^c, \left(\nu^r_{a_{ij}}\right)^c\right]$$
$$= [1-\mu^l_{a_{ij}}, 1-\mu^r_{a_{ij}}], [1-\nu^l_{a_{ij}}, 1-\nu^r_{a_{ij}}].$$

IV. Algorithmic Approach

Step 1: Opinions of a set of experts / decision makers $P = \{P_1, P_2, ..., P_k\}$ for a given set of alternatives $D = \{d_1, d_2, ..., d_m\}$ and a set of attributes $S = \{s_1, s_2, ..., s_n\}$ are represented using interval-valued intuitionistic fuzzy soft matrices.

Step 2: Choice matrix $\xi(i, j)^P$ and combined choice matrix $\xi(i, j)^{P_C}$ of each of the decision makers $P = \{P_1, P_2, ..., P_k\}$ are computed in the context of interval-valued intuitionistic fuzzy set based on their choice parameters / attributes.

Step 3: Product $IVIFSM\,(P_{IVIFSM})$ for each decision maker is calculated by multiplying each $IVIFSM$ with its combined choice matrix.

Step 4: Summation of these product $IVIFSMs$ is the resultant $IVIFSM$ (R_{IVIFSM}).

Step 5: Weight $W(d_i)$ of each alternative $d_i\{i = 1, 2, ..., m\}$ is estimated by adding the membership and non-membership values of the entries of the respective row (i[th] row).

Step 6: $\forall d_i \in D$, compute the score $S(d_i)$ of d_i, such that, $S(d_i) = \{(\mu^l_i - \nu^l_i) + (\mu^r_i - \nu^r_i)\}/2,\ d_i \in D\ \forall i.$

Step 7: If $S(d_i) > S(d_j)\ \forall d_j \in D$, then alternative d_i is selected. If $\exists\, j$, such that, $S(d_i) = S(d_j)$, where $i \neq j$, for highest score value, then decision is made according to their accuracy values described in step 8.

Step 8: Accuracy value $H(d_i)\{i = 1, 2, ..., m\}$, is defined as

$$H(d_i) = \{(\mu^l_i + \nu^l_i) + (\mu^r_i + \nu^r_i)\}/2,\ d_i \in D\ \forall i.$$

If $H(d_i) > H(d_j)\ \forall j$ for which $S(d_i) = S(d_j)$, defined in step 7, alternative d_i is selected. If $H(d_i) = H(d_j)$, for any j, then d_i and d_j are selected.

V. CASE STUDY

Let $D=\{d_1,d_2,d_3,d_4,d_5\}$ be the set of five stages of heart disease (Stage ‘I’, Stage ‘II’, Stage ‘III’, Stage ‘IV’, and Stage ‘V’). Patients belonging to Stage ‘I’ are assumed not to be affected by heart disease. Patients belonging to Stage ‘II’ are in initial stage, patients belonging to Stage ‘III’ are in more unsafe stage than stage ‘II’ and so on. Patients belonging to Stage ‘V’ are in the last stage of heart disease which is unrecoverable. Let E be the set of five symptoms (Chest pain, Palpitations, Dizziness, Fainting, Fatigue) given by E $=\{s_1,s_2,s_3,s_4,s_5\}$. Suppose that a group of three experts P = {P_1, P_2, P_3} are monitoring the symptoms as per their knowledgebase to reach a consensus about which disease is more likely to appear for the patient, where expert P_1 is aware of symptoms (s_1, s_2, s_3, s_4), P_2 is aware of symptoms (s_1, s_2, s_3, s_5), and P_3 is aware of (s_1, s_2, s_4, s_5). According to the symptoms or parameters observed by the three experts, we assume to have the interval-valued intuitionistic fuzzy soft sets $(\widehat{F}_{\{P_1\}},E),(\widehat{F}_{\{P_2\}},E)$, and $(\widehat{F}_{\{P_3\}},E)$ for experts P_1, P_2, and P_3 respectively.

[Step 1] Let the interval-valued intuitionistic fuzzy soft matrices of the interval-valued intuitionistic fuzzy soft sets $(\widehat{F}_{\{P_1\}},E),(\widehat{F}_{\{P_2\}},E),(\widehat{F}_{\{P_3\}},E)$ are respectively,

$$\{p_1(i,j)\}=\begin{pmatrix}
\begin{pmatrix}(0.3.0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.3,0.4),\\(0.4,0.5)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.5),\\(0.3,0.5)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.6),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.3,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.2,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.3,0.5),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.3),\\(0.4,0.7)\end{pmatrix} & \begin{pmatrix}(0.2,0.3),\\(0.5,0.6)\end{pmatrix} & \begin{pmatrix}(0.3,0.4),\\(0.4,0.5)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}
\end{pmatrix},$$

$$\{p_2(i,k)\}=\begin{pmatrix}
\begin{pmatrix}(0.3,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.2,0.3)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.5),\\(0.3,0.5)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.5),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.5),\\(0.3,0.4)\end{pmatrix}\\
\begin{pmatrix}(0.3,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.2,0.4),\\(0.3,0.5)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.4)\end{pmatrix}\\
\begin{pmatrix}(0.2,0.6),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.3,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.3,0.4)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.5),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.3),\\(0.4,0.5)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.5),\\(0.3,0.4)\end{pmatrix}
\end{pmatrix},$$

$$\{p_3(i,l)\}=\begin{pmatrix}
\begin{pmatrix}(0.3,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.3,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.4),\\(0.5,0.6)\end{pmatrix}\\
\begin{pmatrix}(0.3,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.5),\\(0.3,0.5)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix}\\
\begin{pmatrix}(0.5,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.4),\\(0.4,0.5)\end{pmatrix}\\
\begin{pmatrix}(0.1,0.2),\\(0.5,0.7)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.2,0.3),\\(0.5,0.6)\end{pmatrix} & \begin{pmatrix}(0.1,0.4),\\(0.4,0.5)\end{pmatrix}
\end{pmatrix}.$$

Here i (1,2,...,5) be the number of alternatives (stages of heart disease) and j, k, l (1,2,...,5) be the number of attributes (symptoms).

[Step 2] The combined choice matrices for P_1, P_2, and P_3 are respectively

$$\begin{array}{cc} & s_{\{P_2\wedge P_3\}}\\ s_{\{P_1\}} & \begin{pmatrix}
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix}\\
\begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}
\end{pmatrix}\end{array}$$

$$\begin{array}{cc} & s_{\{P_1\wedge P_3\}}\\ s_{\{P_2\}} & \begin{pmatrix}
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}
\end{pmatrix}\end{array}$$

$$\begin{array}{cc} & s_{\{P_1\wedge P_2\}}\\ s_{\{P_3\}} & \begin{pmatrix}
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}
\end{pmatrix}\end{array}$$

[Step 3] The product (as defined in Section 3) of IVFSMs and combined choice matrices are given below.

$$
\{p_1(i,j)\}=\begin{pmatrix}
\begin{pmatrix}(0.3.0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.3,0.4),\\(0.4,0.5)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.5),\\(0.3,0.5)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.6),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.3,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.2,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.3,0.5),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.3),\\(0.4,0.7)\end{pmatrix} & \begin{pmatrix}(0.2,0.3),\\(0.5,0.6)\end{pmatrix} & \begin{pmatrix}(0.3,0.4),\\(0.4,0.5)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}
\end{pmatrix}\otimes s_{\{P_1\}}\overset{s_{\{P_2\wedge P_3\}}}{\begin{pmatrix}
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix}\\
\begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}
\end{pmatrix}}
$$

$$
=\begin{pmatrix}
\begin{pmatrix}(0.5.0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.5.0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5.0.7),\\(0.1,0.2)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.2)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix}\\
\begin{pmatrix}(0.3,0.5),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.3,0.5),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.3,0.5),\\(0.2,0.4)\end{pmatrix}
\end{pmatrix}
$$

$$
\{p_2(i,k)\}=\begin{pmatrix}
\begin{pmatrix}(0.3,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.2,0.3)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.5),\\(0.3,0.5)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.5),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.5),\\(0.3,0.4)\end{pmatrix}\\
\begin{pmatrix}(0.3,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.2,0.4),\\(0.3,0.5)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.4)\end{pmatrix}\\
\begin{pmatrix}(0.2,0.6),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.3,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.3,0.4)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.5),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.3),\\(0.4,0.5)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.5),\\(0.3,0.4)\end{pmatrix}
\end{pmatrix}\otimes s_{\{P_2\}}\overset{s_{\{P_1\wedge P_3\}}}{\begin{pmatrix}
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}
\end{pmatrix}}
$$

$$
=\begin{pmatrix}
\begin{pmatrix}(0.5,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}
\end{pmatrix}
$$

$$
\{p_3(i,l)\}=\begin{pmatrix}
\begin{pmatrix}(0.3,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.3,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.2,0.4),\\(0.5,0.6)\end{pmatrix}\\
\begin{pmatrix}(0.3,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.5),\\(0.3,0.5)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix}\\
\begin{pmatrix}(0.5,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.5),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.4),\\(0.4,0.5)\end{pmatrix}\\
\begin{pmatrix}(0.1,0.2),\\(0.5,0.7)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.2,0.3),\\(0.5,0.6)\end{pmatrix} & \begin{pmatrix}(0.1,0.4),\\(0.4,0.5)\end{pmatrix}
\end{pmatrix}\otimes s_{\{P_3\}}\overset{s_{\{P_1\wedge P_2\}}}{\begin{pmatrix}
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}\\
\begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(1,1),\\(1,1)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix} & \begin{pmatrix}(0,0),\\(0,0)\end{pmatrix}
\end{pmatrix}}
$$

$$
=\begin{pmatrix}
\begin{pmatrix}(0.3,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.5,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}\\
\begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix}
\end{pmatrix}
$$

[Step 4] The sum of these product interval-valued intuitionistic fuzzy soft matrices is

$$\begin{pmatrix} \begin{pmatrix}(0.5.0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.5.0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5.0.7),\\(0.1,0.2)\end{pmatrix} \\ \begin{pmatrix}(0.4,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.2)\end{pmatrix} \\ \begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} \\ \begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} \\ \begin{pmatrix}(0.3,0.5),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.3,0.5),\\(0.2,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.3,0.5),\\(0.2,0.4)\end{pmatrix} \end{pmatrix} \oplus \begin{pmatrix} \begin{pmatrix}(0.5,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \\ \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \\ \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \\ \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \\ \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \end{pmatrix} \oplus$$

$$\begin{pmatrix} \begin{pmatrix}(0.3,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \\ \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \\ \begin{pmatrix}(0.5,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \\ \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \\ \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} & \begin{pmatrix}(0.0,0.0),\\(0.0,0.0)\end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.3,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} \\ \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.2)\end{pmatrix} \\ \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.7,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.2,0.3)\end{pmatrix} \\ \begin{pmatrix}(0.5,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.5,0.8),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.2,0.3)\end{pmatrix} & \begin{pmatrix}(0.5,0.7),\\(0.1,0.2)\end{pmatrix} & \begin{pmatrix}(0.4,0.8),\\(0.1,0.2)\end{pmatrix} \\ \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.4,0.6),\\(0.3,0.4)\end{pmatrix} & \begin{pmatrix}(0.4,0.7),\\(0.1,0.3)\end{pmatrix} & \begin{pmatrix}(0.3,0.5),\\(0.2,0.4)\end{pmatrix} \end{pmatrix}.$$

[Step 5] Now the weights of various stages of heart disease $W(d_i), i = 1, 2, \ldots, 5$ are calculated as follows:

$$W(d_1) = \begin{bmatrix} [0.5 + 0.5 + 0.3 + 0.5 + 0.5, \\ 0.7 + 0.7 + 0.7 + 0.7 + 0.7], \\ [0.1 + 0.1 + 0.1 + 0.2 + 0.1, \\ 0.2 + 0.2 + 0.3 + 0.3 + 0.2] \end{bmatrix} = \begin{bmatrix} [2.3, 3.5], \\ [0.6, 1.2] \end{bmatrix}.$$

Similarly, $W(d_2) = \begin{bmatrix} [2.9, 4.5], \\ [0.6, 1.1] \end{bmatrix}$, $W(d_3) = \begin{bmatrix} [3.0, 3.7], \\ [0.6, 1.2] \end{bmatrix}$,

$W(d_4) = \begin{bmatrix} [2.3, 3.8], \\ [0.6, 1.1] \end{bmatrix}$, $W(d_5) = \begin{bmatrix} [1.9, 3.2], \\ [0.8, 1.7] \end{bmatrix}$.

[Step 6] Scores of various stages of heart disease can be computed as follows:

$$S(d_1) = \{(2.3 - 0.6) + (3.5 - 1.2)\}/2 = 2$$
$$S(d_2) = \{(2.9 - 0.6) + (4.5 - 1.1)\}/2 = 2.85$$
$$S(d_3) = \{(3.0 - 0.6) + (3.7 - 1.2)\}/2 = 2.45$$
$$S(d_4) = \{(2.3 - 0.6) + (3.8 - 1.1)\}/2 = 2.2$$
$$S(d_5) = \{(1.9 - 0.8) + (3.2 - 1.7)\}/2 = 1.3$$

[Step 7] Since score of d_2 is maximum, the patient under consideration belongs to Stage 'II', i.e., initial stage of heart disease as per the collective opinions of the group of experts.

VI. Conclusions

This paper presents an algorithmic approach for multiple attribute group decision making using interval-valued intuitionistic fuzzy soft matrix. Firstly we propose interval-valued intuitionistic fuzzy soft matrix and define some of its relevant operations. Next we propose an algorithm using combined choice matrix, score function, and accuracy function. The proposed method yields optimal alternative(s) which reflects the collective opinion of a group of decision makers. We have presented a case study related with medical diagnosis. In this study, we use opinions of a group of experts about a common set of symptoms where opinions of some experts about a subset of symptoms might be missing due to lack of knowledge or experience. Experts provide their opinions using interval-valued intuitionistic fuzzy soft sets which are represented in terms of interval-valued intuitionistic fuzzy soft matrices. Individual interval-valued intuitionistic fuzzy soft matrices are multiplied with respective combined choice matrices. The resultant individual product matrices are summed up to find out the final matrix where membership values of each alternative are added to generate the weights of alternatives. Next, score values are calculated. More score will lead to better alternative. When there is a tie in score values, then accuracy values are used for final decision. Alternative with more accuracy value will be the optimal choice. Future scope of this research work might be to investigate the various properties of interval-valued intuitionistic fuzzy soft matrix and apply them to suitable uncertain decision making problems.

References

[1] D. Molodtsov, "Soft set theory – first results," *Comput. Math. Appl.* vol. 37, no. (4–5), pp. 19–31, 1999

[2] A. Kharal, "Distance and similarity measures for soft sets," *New Mathematics and Natural Computation*, vol. 6, no. 3, pp. 312–334, 2010.

[3] N. Cagman and S. Enginoglu, "Soft matrix theory and its decision making," *Computers and Mathematics with Applications*, vol. 59, no. 10, pp. 3308–3314, 2010.

[4] N. Cagman and S. Enginoglu, "Soft set theory and uni–int decision making," *European Journal of Operational Research*, vol. 207, no. 2, pp. 848–855, 2010.

[5] F. Feng, Y. B. Jun, X. Liu, and L. Li, "An adjustable approach to fuzzy soft set based decision making," *Journal of Computational and Applied Mathematics*, vol. 234, no. 1, pp. 10–20, 2010.

[6] Y. Jiang, Y. Tang, and Q. Chen, "An adjustable approach to intuitionistic fuzzy soft sets based decision making," *Applied Mathematical Modelling*, vol. 35, no. 2, pp. 824–836, 2011.

[7] Z. Kong, L. Gao, and L. Wang, "Comment on A fuzzy soft set theoretic approach to decision making problems," *Journal of Computational and Applied Mathematics*, vol. 223, no. 2, pp. 540–542, 2009.

[8] P. K. Maji, A. R. Roy, and R. Biswas, "An application of soft sets in a decision making problem," *Computers and Mathematics with Applications*, vol. 44, no. (8-9), pp. 1077–1083, 2002.
[9] A. R. Roy and P. K. Maji, "A fuzzy soft set theoretic approach to decision making problems," *Journal of Computational and Applied Mathematics*, vol. 203, no. 2, pp. 412–418, 2007.
[10] Y. Zou and Z. Xiao, "Data analysis approaches of soft sets under incomplete information," *Knowledge-Based Systems*, vol. 21, no. 8, pp. 941–945, 2008.
[11] Z. Xiao, K. Gong, and Y. Zou, "A combined forecasting approach based on fuzzy soft sets," *Journal of Computational and Applied Mathematics*, vol. 228, no. 1, pp. 326–333, 2009.
[12] S. J. Kalayathankal and G. S. Singh, "A fuzzy soft flood alarm model," *Mathematics and Computers in Simulation*, vol. 80, no. 5, pp. 887–893, 2010.
[13] D. V. Kovkov, V. M. Kolbanov, and D. A. Molodtsov, "Soft sets theory-based optimization," *Journal of Computer and Systems Sciences International*, vol. 46, no. 6, pp. 872–880, 2007
[14] M. M. Mushrif, S. Sengupta, and A. K. Ray, "Texture classification using a novel, soft set theory based classification algorithm," *in: P.J. Narayanan, S.K. Nayar, H.Y. Shum (Eds.), Proceedings of the 7th Asian Conference on Computer Vision*, 2006. Lecture Notes in Computer Science, vol. 3851, pp. 246–254.
[15] P. K. Maji, R. Biswas, and A. R. Roy, "Soft sets theory," *Comput. Math. Appl.*, vol. 45, pp. 555–562, 2003.
[16] P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 589–602, 2001.
[17] X. B. Yang, T. Y. Lin, J. Y. Yang, Y. Li, and D. Y. Yu, "Combination of interval-valued fuzzy set and soft set," *Comput. Math. Appl.*, vol. 58, pp. 521–527, 2009.
[18] W. Xu and J. Ma, "Vague soft sets and their properties," *Comput. Math. Appl.*, vol. 59, pp. 787–794, 2010.
[19] Y. Jiang and Y. Tang, "Interval-valued intuitionistic fuzzy soft sets and their properties," *Comput. Math. Appl.*, vol. 60, pp. 906–918, 2010.
[20] F. Feng, Y. Li, and V. Leoreanu-Fotea, "Application of level soft sets in decision making based on interval-valued fuzzy soft sets," *Comput. Math. Appl.*, vol. 60, pp. 1756–1767, 2010.
[21] H. Qin, X. Ma, T. Herawan, and J. M. Zain, "An adjustable approach to interval-valued intuitionistic fuzzy soft sets based decision making," *in: N. T. Nguyen, C. G. Kim, and A. Janiak (Eds.), Proceedings of ACIIDS*, 2011, LNAI, vol. 6592, pp. 80–89, Springer-Verlag Berlin Heidelberg.
[22] Z. Zhang, C. Wang, D. Tian, and K. Li, "A novel approach to interval-valued intuitionistic fuzzy soft set based decision making," *Applied Mathematical Modelling*, 2013, (in press).
[23] S. Alkhazaleh, A. R. Salleh, and N. Hassan, "Fuzzy parameterized interval-valued fuzzy soft set," *Appl. Math. Sci.*, vol. 67, pp. 3335–3346, 2011.
[24] J. Mao, D. Yao, and C. Wang, "Group decision making methods based on intuitionistic fuzzy soft matrices," *Applied Mathematical Modelling*, vol. 37, pp. 6425–6436, 2013.
[25] S. Das and S. Kar, "Intuitionistic Multi Fuzzy Soft Set and its Application in Decision Making," *in: P. Maji et al. (Eds.), Proceedings of Fifth International Conference on Pattern Recognition and Machine Intelligence (PReMI)*, Kolkata, December 10-14, 2013, Lecture Notes in Computer Science, vol. 8251, pp. 587–592.
[26] S. Das and S. Kar, "The Hesitant Fuzzy Soft Set and its Application in Decision Making," *in Proceedings of International Conference on Facets of Uncertainties and Applications (ICFUA)*, Kolkata, December 5-7, 2013 (in press).
[27] K. Atanassov and G. Gargov, "Interval-valued intuitionistic fuzzy sets." *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.
[28] S. Chen and J. Tan, "Handling multicriteria fuzzy decision-making problems based on vague set theory," *Fuzzy Sets and Systems*, vol. 67, no.2, pp. 163-172, 1994.
[29] Z. Xu, "Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making," *Control and Decision*, vol. 22, pp. 215-219, 2007.
[30] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning—I," *Information Sciences*, vol. 8, no. 3, pp. 199–249, 1975.