Blind Channel Length Estimation for OFDM Systems using Cumulant Features

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) systems are now a part of all major wireless standards, because of its potential to offer high data rate. Cyclic prefix (CP) in OFDM system converts a multipath channel to a flat fading channel, thus simplifies the design of equalizer. However, there is a lot of ambiguity in choosing the length of CP. In this paper, we propose a new method for calculation of length of CP using cumulant features. The merits of proposed method are verified by computer simulation.**

***Keywords*—Orthogonal frequency division multiplexing (OFDM), Cyclic prefix (CP), Blind channel length estimation.**

I. Introduction

Increasing in the number of wireless applications and users demands for reliable, high data rate communication. High data rate can be achieved by effective utilization of the spectrum. Over the past decade, dynamic spectrum access (DSA) has been seen as a potential solution for utilizing the spectrum effectively. The research efforts in DSA are further fueled by the advent of cognitive radio (CR) [1], due to its potential platform for implementing DSA. High data rates can also be achieved by employing multiple input multiple output (MIMO) and orthogonal frequency division multiplexing (OFDM) based systems. CR combined with MIMO and OFDM has the potential to achieve reliability and high data rate. In fact, utilization of CR along with MIMO and OFDM is included in the 4G standards.

OFDM provides high data rate by exploiting the orthogonality in the frequency domain. One of the key issue in OFDM is to choose the length of cyclic prefix (CP). CP enables OFDM to convert multipath channel to flat fading channel and hence simplifies the design of equalizer. In general, the length of CP is decided greater than delay spread of the channel. For practical implementation, generally we consider CP length as one fourth of the OFDM symbol [2]. The way of choosing the fixed CP length by this traditional method leads to the wastage of transmission bandwidth. As CP does not carry any extra information, it is required to optimize the length of CP without disturbing the original data. The length of CP should be longer than the length of the channel impulse response (CIR). However, the length of the CIR is not known at the transmitter. Hence, it has to be estimated at the receiver end and then conveyed to the transmitter through some control channel (Refer Fig. 1).

Some of the existed work on calculation of channel length and CP length for OFDM system is summarized as follows: In [3], Jong Bu Lim *et al.* proposed iterative CP reconstruction procedure based on residual inter symbol interference cancellation (RISIC). In [4], D. Salvatore *et al.* introduced bit loading algorithms for an adaptive CP length to enhance the performance of system. In [5], Bin Sheng *et al.* presented blind algorithm to estimate the channel length based on minimum description length (MDL) for OFDM system.

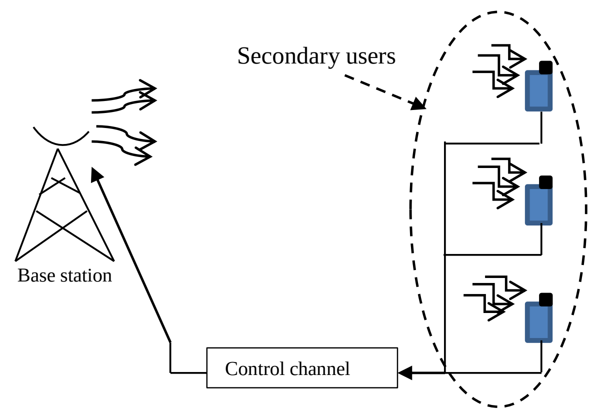

Fig. 1: Proposed system representation for OFDM transmission

In this paper, we propose a novel method to choose the CP length. The proposed approach is blind, that is no pilot or training sequence is required and hence further saves bandwidth. The proposed method is well suited for a DSA setup as illustrated in Fig. 1. In Fig. 1, the secondary users estimate the ideal CP length, and communicate it to the base station via control channel. The base station then adjusts the CP length for the future transmission. Therefore, by estimating the channel length, CP length can be adjusted. Moreover, the bandwidth can be utilized effectively by avoiding the pilot sequence transmission. In the proposed method, we first estimate the channel length and then use this information to choose the CP length. For estimating the channel length, we used the algorithm presented in our previous published work [6]. The method is based on cumulant features. Cumulant features have been widely used in the field of blind signal processing [7]–[11]. In this paper, we exploit some unique properties of cumulant features that make it suitable for channel length estimation.

The rest of this paper is structured as follows: Section II presents the problem statement. Section III introduces mathematical background on cumulant features whereas Section

IV describes the proposed method. Section V presents the simulation results followed by conclusion in Section VI.

II. Problem Statement

In this paper, we consider an OFDM system with N subcarriers. The m^{th} OFDM block or symbol is denoted as $\bar{X}_m = \{X_m(0), X_m(1), X_m(2), \dots X_m(N-1)\}$, where $X_m(k)$ is a complex valued alphabet drawn from a QAM constellation. The inverse discrete Fourier transform (IDFT) $\mathscr{F}^{-1}$ of the m^{th} block is denoted as $\bar{x}_m = \{x_m(0), x_m(1), x_m(2), \dots x_m(N-1)\}$, i.e.,

$$\bar{x}_m = \mathscr{F}^{-1}\left\{\bar{X}_m\right\} \tag{1}$$

The transmitted symbol is subjected to multi-path fading. The multi-path channel can be modeled as a finite impulse response (FIR) filter

$$h(z^{-1}) = h(0) + h(1)z^{-1} + \dots + h(M)z^{-M} \tag{2}$$

where M is the length of the multipath channel and $h(i)$(for $i = 0, 1 \dots M$) is the multi-path gains vector. CP is added before transmitting the time domain block $\bar{x}_m$. As mentioned earlier, CP is added to avoid intersymbol interference (ISI). Let, the length of CP is denoted as P, and hence the total length of the m^{th} block is $(N+P)$. The m^{th} received OFDM block after removing CP is denoted as $\bar{y}_m = \{y_m(0), y_m(1), y_m(2), \dots, y_m(N-1)\}$. The corresponding frequency domain block is given by $\bar{Y}_m = \{Y_m(0), Y_m(1), Y_m(2), \dots Y_m(N-1)\}$ where

$$\bar{Y}_m = \mathscr{F}\left\{\bar{y}_m\right\}, \tag{3}$$

where $\mathscr{F}$ denotes the DFT.

The relationship between the transmitted and received symbols of m^{th} block is given by

$$Y_m(K) = H(K)X_m(K), \quad K = 0, 1, \dots N-1 \tag{4}$$

where $H(K) = H(0), H(1), \dots, H(N-1)$ is the N point DFT of the channel impulse response in (3).

Selection of the length of CP is a challenging problem. The CP length must be greater than the length of the channel impulse response (let M) in order to avoid ISI. However, length of the channel impulse response is not known. In this paper, we propose a novel blind algorithm to estimate the CP length P at the receiver. The block diagram of the proposed method is shown in Fig. 2. In the proposed algorithm, we first estimate the channel length and then use this information to choose the CP length. The algorithm for estimating the channel length is based on exploiting the properties of n^{th} order cumulant features.

III. Background Theory on Cumulant Features

In this paper, n^{th} order cumulants features are used. For a complex random process $v(n)$, the n^{th} order moment is defined as [7], [8], [11], [12]

$$R_{v(n,m)}(\tau) = E[\prod_{j=1}^{n} v^{*_j}(\tau_j)] \tag{5}$$

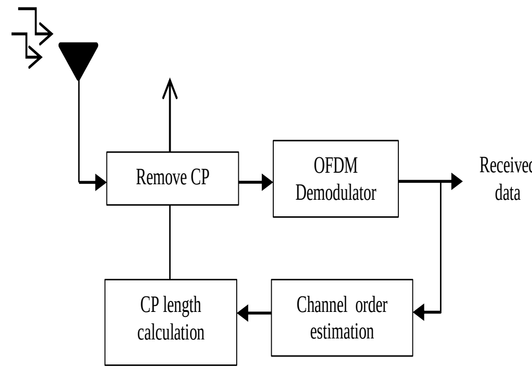

Fig. 2: Block diagram of proposed system at the receiver

where n is the order, m is the number of conjugate factors and $\tau = [\tau_1, \tau_2, \dots, \tau_n]$ is the delay vector. The n^{th} order cumulant function is defined as [7], [8], [11], [12].

$$C_{v(n,m)}(\tau) = \sum_{pn} F(p) \prod_{j=1}^{p} R_{v(n_j,m_j)}(\tau) \tag{6}$$

where the sum is over distinct partition of indexed set $1, 2, \dots, n$ and $F(p) = (-1)^{p-1}(p-1)!$. The normalized n^{th} order cumulant values are defined as

$$\tilde{C}_{v(n,m)}(\tau) = \frac{|C_{v(n,m)}(\tau)|}{[C^2_{v(2,1)}(0)]} \quad \text{for} \quad n = 4, 6 \tag{7}$$

The rationale behind choosing cumulant features, is the following two properties of cumulants:
Additive property: Let $x(k)$ and $y(k)$ be two independent random processes. If $z(k) = x(k) + y(k)$ then the n^{th} order cumulant value of $z(k)$ is the sum of those of $x(k)$ and $y(k)$. That is,

$$C_{z(n,m)}(\tau) = C_{x(n,m)}(\tau) + C_{y(n,m)}(\tau) \tag{8}$$

Scaling property: Let $x = ay$. Then the n^{th} order cumulant value of $x(k)$ is $|a|^n$ times the cumulant value of y(k).

IV. Proposed Method

Before illustrating the proposed approach, we first study the effect of multipath channel over n^{th} order cumulant values of the received OFDM symbols. The relationship between the transmitted and received symbols of the m^{th} block is given by $Y_m(K) = H(K)X_m(K) \quad \text{for} \quad k = 0, 1, \dots, N-1$ where

$$H(K) = \sum_{i=0}^{M-1} h(i) e^{-\frac{(i2\Pi k)}{N}} \tag{9}$$

Using the scaling property of the cumulant features, it can be easily shown that

$$\tilde{C}_{Y_m(K)(n,m)}(\tau, M) = \beta(M)\tilde{C}_{X_m(K)(n,m)} \tag{10}$$

where $\tilde{C}_{Y_m(K)(n,m)}$ is the normalized cumulant value of k^{th} OFDM symbol in m^{th} block and

$$\beta(M) = \frac{\sum_{K=0}^{M-1} |H(K)|^n}{(\sum_{K=0}^{M-1} |H(K)|^2)^{n/2}} \tag{11}$$

where M is the order or length of channel. By observing (9) and (11), and by using the fact that $|e^{-\frac{(i2\Pi k)}{N}}| = 1$, it can be easily shown that $\beta(M) < 1$. Hence, it is clear that the length of channel is inversely proportional to the magnitude of cumulant values of the received OFDM symbols. The magnitude of the cumulant values of the received OFDM symbols shrinks by increasing the length of channel. Also, the magnitude of shrinkage depends on the value of M, i.e.,

$$\beta(M) < \beta(M') \quad \text{for} \quad M > M' \tag{12}$$

and hence

$$\tilde{C}_{Y_m(K)(n,m)}(\tau, M) < \tilde{C}_{Y_m(K)(n,m)}(\tau, M') \quad \text{for} \quad M > M' \tag{13}$$

The pseudocode of the proposed algorithm is shown as below:

Algorithm 1 Pseudocode of the proposed algorithm

```
Input: Received data Y_m(K) with N subcarriers
Estimate the received cumulant C~_{Y_m(K)} values of the
received data using the method in [7], [8], [11], [12]
for i=1,2, ...M do
    if (C~_{Y_m(K)}(i+1) + C~_{Y_m(K)}(i))/2 < C~_{Y_m(K)}
        < (C~_{Y_m(K)}(i-1) + C~_{Y_m(K)}(i))/2 then
        if M=M' then
            Terminate loop
        endif
    else
        i=i+1;
    endif
endfor
EndProcedure
```

The estimated channel length M', is used to calculate the ideal CP length. Typically, CP length P is chosen such that $P \geq M$. In this work, we choose CP length P to be

$$P = k \times M' \tag{14}$$

where k is an integer greater than two. The value of P is chosen in such a way that it is at least double of the estimated channel length.

V. Simulation Results

MATLAB simulations are performed to test the proposed algorithm and the results are summarized. In this case, probability of correctness (P_c) is taken as performance metric and can be defined as P_c= prob(detection says M' is true | M is M'). In other words, $P_c = P(M'/M')$ is the probability that the channel length is estimated as M', given the channel length is M'. Cumulant values are calculated at the output of the OFDM demodulator. For the simulations, 4^{th} and 6^{th} order cumulants $|C_{Y_m(4,0)}|$ and $|C_{Y_m(6,0)}|$ features are considered, as they are robust to phase rotation. The following experiments are performed with BPSK constellation.

Experiment-1: In this experiment, we considered fourth order cumulant features. Channel order considered as 1,2. That is the proposed algorithm has to estimate whether the tap of the channel is 1 or 2. Here, we have used $N = 1024$ bits per OFDM symbol to estimate the cumulants. The results are summarized in Fig. 3. In Fig. 3, $M = 1$ denotes the probability of correctly estimating the 1 tap channel and $M = 2$ for 2 tap channel. From Fig. 3, it can be observed that the probability of correctly estimating the channel (P_c) is more than 75% even at negative SNR (in dB scale).

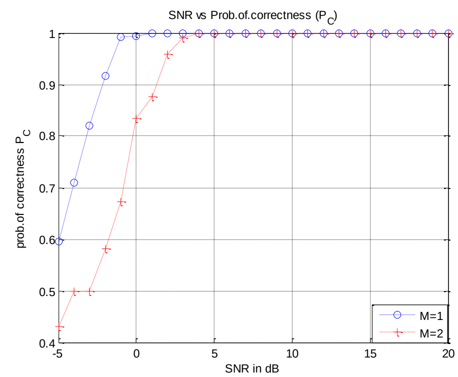

Fig. 3: SNR vs P_c with $N = 1024$

Experiment-2: This experiment is similar to previous one, except that four possible conditions are considered. That is the algorithm need to identify whether the channel is a 1 tap, 2 taps, 3 taps or 4 taps. The results are summarized in Fig. 4. In Fig. 4, $M = i$ (for $i = 1, 2, 3, 4$), has the same meaning as previous experiment. It can be seen that the algorithm performs well in estimating 1, 2 and 3 tap channel. However, in the case of 4 tap channel the performance of the method is poor. This is because, the shrinkage in the cumulant values is high and more number of bits or samples are required to estimate it. We repeat the experiment with more number of samples ($N = 4096$) and the results are summarized in Fig. 5. From the Fig. 5, it can be seen that the performance of estimating 4 tap channel is improved (70% at 0 dB) when N is increased.

Experiment-3: In this experiment, we consider the same case as experiment 1, here we considered sixth order cumulants instead of fourth order and channel length is indicated with L. From Fig. 6, it can be seen that the robustness of correct detection. The reason is sixth order cumulants are more robust than fourth order cumulants.

TABLE I: CP Length Comparison

Channel order	CP length by traditional method [2] (bits per OFDM symbol)	CP length by proposed method (bits per OFDM symbol)
1	256	2
2	256	4
3	256	6
4	256	8

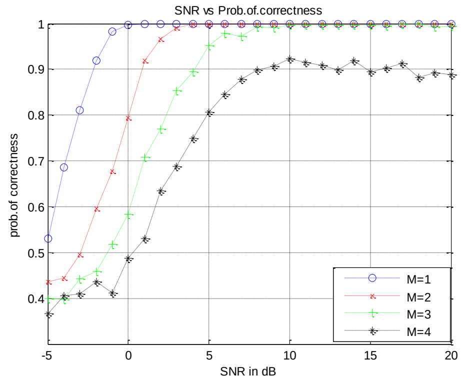

Fig. 4: SNR vs P_c with $N = 1024$

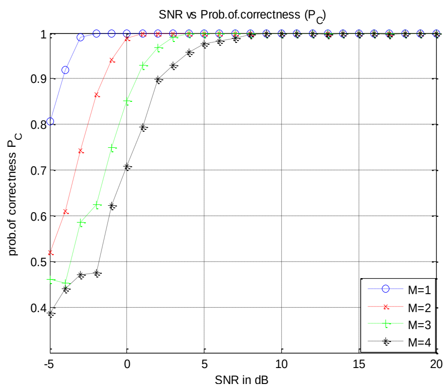

Fig. 5: SNR vs P_c with $N = 4096$

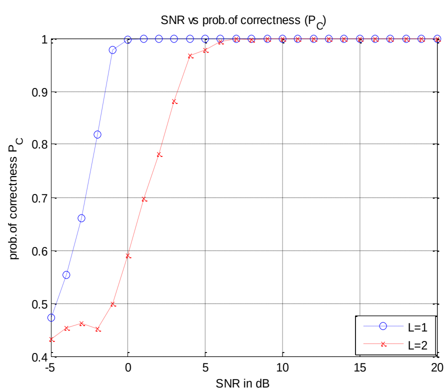

Fig. 6: SNR vs P_c by sixth order cumulants with $N = 1024$

From the experiments, it can be seen that our proposed algorithm estimates the channel length even at low SNR, compared to MDL method [5], whose performance degrades at low SNR. In the proposed method, once the channel length is estimated the CP length is chosen to be twice the channel length. This results in a significant reduction in bandwidth as illustrated in Table I. Here, we consider $N = 1024$ subcarriers per OFDM symbol. In the traditional method, CP length is chosen as one fourth of the OFDM symbol length, i.e., 256 bits per OFDM symbol. However, in our proposed method CP length is chosen according to channel length.

VI. Conclusion

In this paper, a new method is introduced for calculating the ideal CP length in OFDM system. The significant reduction in CP length leads to bandwidth efficient OFDM system. As a part of work a novel algorithm for estimating the channel length has been proposed. The proposed algorithm is blind and does not require any training sequence to estimate channel length. Using simulation results, it was shown that the proposed algorithm outperforms other algorithms even at low SNR.

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