Robust Consensus: A New Measure for Multicriteria Robust Group Decision Making Problems Using Evolutionary Approach

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Robust Consensus: A new Measure for Multicriteria Robust Group Decision Making Problems using Evolutionary Approach

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Abstract. In fuzzy group decision making problems, we often use multiobjective evolutionary optimization. The optimizers search through the whole search space and provide a set of nondominated solutions. But, sometimes the decision makers express their prior preferences using fuzzy numbers. In this case, the optimizers search in the preferred *soft* region and provide solutions with higher *consensus*. If perturbation in the decision variable space is unavoidable, we also need to search for *robust* solutions. Again, this perturbation affects the degree of consensus of the solutions. This leads to search for solutions those are robust to their degree of consensus. In this work, we address these issues by redefining consensus and proposing a new measure called *robust consensus*. We also provide a reformulation mechanism for multiobjective optimization problems. Our experimental results show that the proposed method is capable of finding robust solutions having robust consensus in the specified soft region.

Keywords: Consensus, evolutionary algorithms, fuzzy group decision making, multiobjective optimization, robustness.

1 Introduction

Most of the real world optimization problems have multiple conflicting objectives. That is why, in the last few decades multiobjective optimization (MOO) has drawn a lot of research interests. Multiobjective optimizers, e.g., genetic algorithms (GAs), after completion of their search process, usually provide a set of nondominated solutions, such that, without additional knowledge further reduction of the solution set is not possible. It is left to decision makers' (DMs') choice to pick the right solution from the solution set.

It is hard from DMs' point of view to find the appropriate solution from a set of nondominated solutions. Besides, we cannot rely on a single DM due to her lack of knowledge about all the objectives. As an example, in an interview board, a set of experts from different knowledge domains makes a consensus decision. Another example is how a decision is made by an organization - here the board of directors makes the decision. Group decision making (GDM), thus, is a point of interest. In most cases, the DMs cannot express their specific choices a priori. Rather they provide a rough idea about their choices. Fuzzy group decision making (FGDM) is one of the popular ways to address this issue. A popular FGDM strategy is that each DM expresses her approximate prior opinion by providing a fuzzy reference point for each objective. Again, the weights of all the DMs may not necessarily be the same. In this case, a suitable aggregation operator is used to find the optimal solution.

In the above mentioned FGDM strategy, a problem associated with the DMs is that their opinions often change. For example, the board of directors of a company may change. Even, individual DM's choice evolves depending on her past experiences. In this case, it becomes important to find solutions, which will be acceptable by the DMs even if some of DMs change their individual preference. *Consensus* is a measure to address this issue. In FGDM, usually consensus is used to find the closeness among the DMs' choices [3], [18]. It is expected that the set of solutions chosen finally should be as close as possible to the collective decision.

There are several unavoidable circumstances when the solutions perturb in decision variable space. In those cases, we prefer the solutions which are *robust* to such perturbations. In the literature, there are several definitions of *robustness* [4], [2] in multiobjective optimization. Robustness is defined either in objective space or in variable space. In this work, we find solutions which are robust to their perturbation in the variable space. Again, when a solution gets perturbed in the variable space, it is likely to be shifted in the objective space. As a consequence, the consensus of the solution also changes. So, we want to find solutions which will be robust with respect to its consensus.

In this work, we assume that the DMs provide some *soft constraints* to the multiobjective optimizer to restrict the search process to a set of specific regions of the search space, and the optimizer provides *robust* solutions from this *roughly* specified preference regions. We reformulate the optimization problem to obtain robust solutions from these specified regions, and find solutions which are robust with respect to their degree of consensus. For this purpose we define a new measure called *robust consensus.*

2 Preliminary Concepts

2.1 Multiobjective Optimization

In a multiobjective optimization problem (MOP), we intend to optimize more than one conflicting objectives, sometimes trying to satisfy also a set of equality and inequality constraints. In this work, however, we consider only unconstrained MOPs (UMOPs). An UMOP can always be restated as an unconstrained multiobjective minimization problem (UMMP) and throughout this paper, unless mentioned specifically, we always consider UMMPs. To be more specific about UMMPs, below we formally define several basics of it.

Definition 1. Formally an UMMP can be defined as in (1).

$$\text{minimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_m(\mathbf{x})), \ \mathbf{x} \in \mathbf{\Omega}, \tag{1}$$

where $\mathbf{\Omega} \subset \mathcal{R}^{\mathbf{n}}$ is the variable space, $\mathbf{f} : \mathcal{R}^{\mathbf{n}} \rightarrow \mathcal{R}^{\mathbf{m}}$, and the functions $f_i(i = 1, 2, \cdots, m)$ are called objective functions.

Definition 2. A solution $\mathbf{f}(\mathbf{x_1}), \mathbf{x_1} \in \mathbf{\Omega}$, is said to *dominate* another solution $\mathbf{f}(\mathbf{x_2}), \mathbf{x_2} \in \mathbf{\Omega}$, denoted by $\mathbf{f}(\mathbf{x_1}) \preceq \mathbf{f}(\mathbf{x_2})$, if $\forall i, f_i(\mathbf{x_1}) \leq f_i(\mathbf{x_2})$, and $\exists j$, s.t., $f_j(\mathbf{x_1}) < f_j(\mathbf{x_2})$.

Definition 3. A solution $\mathbf{f}(\mathbf{x}^*), \mathbf{x}^* \in \mathbf{\Omega}$ is called a *Pareto optimal solution*, if $\nexists \mathbf{x} \in \mathbf{\Omega}$, s.t., $\mathbf{f}(\mathbf{x}) \preceq \mathbf{f}(\mathbf{x}^*)$. A set of all such solutions in the objective space is called *Pareto front*. The corresponding set of points in the decision variable space is called the *Pareto Set*.

Definition 4. A set of solutions $\mathcal{S}$ is called a *nondominated* set of solutions, if $\forall \mathbf{u} \in \mathcal{S}$, $\nexists \mathbf{v} \in \mathcal{S}$, s.t., $\mathbf{v} \preceq \mathbf{u}$.

There are many multiobjective evolutionary algorithms (MOEAs) in the literature to solve MOPs. SPEA2 [21], NSGA-II [5] etc. are some of the popular MOEAs. The default goal of these algorithms is to provide the DMs a set of non-dominated solutions, which is close to, and well spread along the Pareto front. If the DMs want to provide some prior *hard* preferences, they represent them as constraints. In many cases, however, the DMs want to search in a particular region of the search space corresponding to some particular area of the Pareto front. Sometimes, they want to stop searching much before the Pareto front is reached. Again, DMs may like to search in *robust* regions, as well as the regions where they have experiences. In these cases, the searching needs to be guided to those specific regions. In our work, we address this issue by embedding consensus in the search process.

2.2 Fuzzy Group Decision Making

Group Decision Making (GDM) has been proven to be useful in many disciplines, like emergency management [20], situation assessment [16], product development [15], and accident evaluation [14]. In most cases, GDM is performed in two steps (processes): *consensus process* and *selection process* [18]. In consensus process, the target is to find the maximum degree of agreement among the DMs. The selection process is used to obtain the solution set of alternatives in accordance with the collective opinions of DMs. For this model it is desirable to obtain the maximum degree of consensus before applying the selection process. In our work, however, we propose an *embedded* model, where these two processes are considered in an integrated manner.

The DMs often prefer to provide some prior *soft* preferences regarding their choices. In fuzzy group decision making (FGDM), one way to represent DM's choices is to express their preferences by fuzzy numbers. Often these numbers are far away from the Pareto front. In that case we need to restrict the search in that specified region. Again, if there are perturbations in the decision variable space, it is possible that though a solution is robust in objective space, its degree of consensus varies highly. In this case, we want to get solutions those are robust with respect to their consensus.

2.3 Related Works

There are few works on FGDM problems with consensus and/or robustness. However, there is no work, as per our knowledge, that have incorporated robustness and consensus in an integrated manner in the FGDM using MOOs. Works on robustness in MOEAs can be found in [2], [4]. Some works related to consensus are there in [9], [10], [12], [13]. A work, somewhat similar to us, can be found in [18]. Nevertheless, the authors, in [18], did not deal with *robust consensus*. They used another definition of robustness. At first their search procedure would reach the Pareto front, and then, a solution selection scheme based on robustness and consensus is used. It makes their system always trying to provide some solutions from the Pareto front, which may not be the desirable solutions with respect to consensus as DMs' preferences may be far away from it. So, essentially they select consensus solution from the Pareto front. In our work, we evolve solutions from the *soft* regions expressed by the DMs as their preferred region in the objective space. In [18], the authors have worked on preference robustness, which is defined by the minimum transition cost in the decision space when a solution is perturbed in the objective space.

3 Problem Formulation

Let there be d DMs, denoted as D_j $(j = 1, 2, \cdots, d)$. The weight vector associated with the DMs is represented as $\mathbf{w} = (w_1, w_2, \cdots, w_d)$, s.t. $\sum_{j=1}^{d} w_j = 1$. To express their preferences, DMs provide reference points in the objective space, denoted by $R_j = (r_{j1}, r_{j2}, \cdots, r_{jm})$, where r_{ji} $(i = 1, 2, \cdots, m)$ is the reference value of the i^{th} objective provided by j^{th} DM. In this work, we consider that the DMs provide reference values as triangular fuzzy numbers [19], where each value is represented as triplet, $r_{ji} = (r_{ji}^{\text{lower}}, r_{ji}^{\text{most}}, r_{ji}^{\text{upper}})$. The membership value of a point r is defined in (2).

$$\mu_{r_{ji}}(r)=\begin{cases}\dfrac{\left(r-r_{ji}^{\text{lower}}\right)}{\left(r_{ji}^{\text{most}}-r_{ji}^{\text{lower}}\right)}, r_{ji}^{\text{lower}}\leq r\leq r_{ji}^{\text{most}}\\ \dfrac{\left(r_{ji}^{\text{upper}}-r\right)}{\left(r_{ji}^{\text{upper}}-r_{ji}^{\text{most}}\right)}, r_{ji}^{\text{most}}\leq r\leq r_{ji}^{\text{upper}}\\ 0, \text{otherwise.}\end{cases} \tag{2}$$

Here r_{ji}^{lower}, r_{ji}^{most}, and r_{ji}^{upper} are respectively the lower bound, most desirable value, and the upper bound of the DM's preference fuzzy number r_{ji}.

4 Robustness, Consensus, and Problem Reformulation

4.1 Robustness

In the literature, robustness has been defined in many ways according to the application areas, such as life science, engineering, mathematics, statistics, optimization, and decision science [2]. In multiobjective optimization, two of the major contributions on robustness can be found in [2], [4]. Among the several definitions of robustness, in this work, we use the one defined in [18].

Definition 5. A solution $\mathbf{x}^{\circledast}$ is called a multiobjective robust solution, if it is a Pareto optimal solution to the multiobjective minimization problem as defined in (3).

$$\begin{aligned}&\text{minimize } \mathbf{f}^e(\mathbf{x})=(f_1^e(\mathbf{x}),f_2^e(\mathbf{x}),\cdots,f_m^e(\mathbf{x})),\mathbf{x}\in\mathbf{\Omega},\\ &\text{subject to } f_j^e(\mathbf{x})=\frac{1}{|\mathcal{B}_\delta(\mathbf{x})|}\int_{\mathbf{y}\in\mathcal{B}_\delta(\mathbf{x})}f_j(\mathbf{y})d\mathbf{y}, j=1,\cdots,m.\end{aligned} \tag{3}$$

The above formulation is defined with respect to a δ-neighborhood $\mathcal{B}_\delta(\mathbf{x})$ of a solution $\mathbf{x}$. A solution which is robust as per the above definition, according to the literature, is called *multiobjective robust solution of type-I* [4].

4.2 Consensus

There are several definitions of consensus in the literature [9], [11], [18]. The drawback in the definition of consensus presented in [18] is that even if the value of the objective function $f_i(\mathbf{x})$ matches exactly with r_{ji}^{most}, the peak of the membership function $\forall i$ and $\forall j$, then also there will be a substantial value of d_{ji} (fuzzy distance between the solution and the reference point of the j^{th} DM on i^{th} objective [18]) suggesting a mismatch between the computed solution and preferred solution and thereby reducing the consensus. To overcome this drawback, we define consensus in the following way.

Definition 6. Let $\mathbf{w} = (w_1, w_2, \cdots, w_d)$ be the weight vector associated with d DMs. Then *consensus* of a solution $\mathbf{x} \in \mathbf{\Omega}$ is defined in (4) as follows.

$$consensus(\mathbf{x}) = \sum_{j=1}^{d} w_j \hat{\mu}_j(\mathbf{x}), \tag{4}$$

where $\hat{\mu}_j(\mathbf{x})$ is defined in (5).

$$\hat{\mu}_j(\mathbf{x}) = \phi(\mu_{r_{j1}}(f_1(\mathbf{x})),\ \mu_{r_{j2}}(f_2(\mathbf{x})), \cdots,\ \mu_{r_{jm}}(f_m(\mathbf{x}))). \tag{5}$$

Here, $\hat{\mu}_j(\cdot)$ is basically a multidimensional membership function; m is the number of objectives; $\mu_{r_{ji}}(\,\cdot\,)$, $i = 1, 2, \cdots, m$, is already defined in (2); $f_i(\,\cdot\,)$, $i = 1, 2, \cdots, m$, is the i^{th} objective function; and $\phi(\,\cdot\,)$ is a t-norm aggregation operator which in this work is taken as the $min(\cdot)$.

There is a problem with this definition of consensus: it assumes that a robust solution will always be robust to its degree of consensus. But, this may not always be true. To demonstrate this scenario with an example, let us consider Fig. 1. In this figure, a robust solution $\mathbf{x} \in \mathbf{\Omega}$ in the variable space and its mapping $\mathbf{f}(\mathbf{x})$ in the objective space are shown respectively in the left panel and in the right panel. The preference points in the objective space provided by two DMs, D_1 and D_2, are shown by + symbol. Let, the weights of the DMs be w_1 and w_2 respectively, and $w_1 > w_2$. Since $w_1 > w_2$ and $\mathbf{f}(\mathbf{x})$ is closer to D_1, the robust solution $\mathbf{x}$ is also a solution with good consensus. But when $\mathbf{x}$ is perturbed, $\mathbf{f}(\mathbf{x})$ no longer is a solution with good consensus. Note that the degree of consensus is not only dependent on the weights but also on membership functions. The δ neighborhood of $\mathbf{x}$ is shown in the left panel and the corresponding perturbation in the objective space is shown in the right panel by the shaded regions. Due to the perturbation of $\mathbf{x}$ the objectives get shifted towards D_2's reference point. In this case, the consensus should decrease. In other words, although $\mathbf{x}$ is a robust solution, it is not robust to its degree of consensus. To overcome this problem, we need to find solutions which are robust to their degree of consensus. To address this issue, we define a new measure, *robust consensus*, bellow in (6).

Definition 7. Robust consensus of a solution is defined in (6).

$$robust\ consensus(\mathbf{x}) = \frac{1}{|\mathcal{B}_\delta(\mathbf{x})|} \int_{\mathbf{z} \in \mathcal{B}_\delta(\mathbf{x})} consensus(\mathbf{z}) d\mathbf{z} \tag{6}$$

The above formulation is defined with respect to a δ-neighborhood ($\mathcal{B}_\delta(\mathbf{x})$) of a solution $\mathbf{x}$. Higher value of this measure indicates higher *robust consensus* of the solution.

4.3 Problem Reformulation for MOEA-FGDM

We could use any multiobjective evolutionary algorithm (MOEA) [5], [6], [17], [21], [22] for this task. We have, however, used NSGA-II [5] as the multiobjective

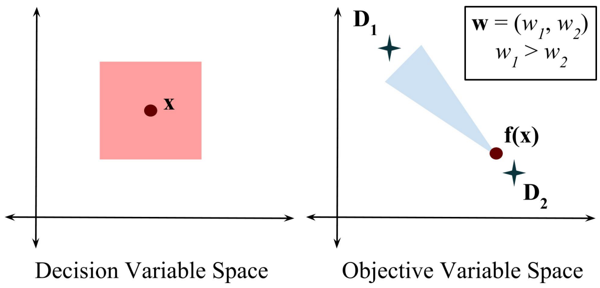

Fig. 1. Showing a solution x, its δ neighborhood, its mapping (perturbed) in objective space, and two DMs' preferred reference points D_1 and D_2

optimizer. To find robust solutions, which are also robust consensus, we reformulate the m objective UMMP defined in (1) as an $(m+1)$ objective UMMP as described in (7).

$$\begin{aligned} &\text{minimize } \hat{\mathbf{f}}^e(\mathbf{x}) = (f_1^e(\mathbf{x}), f_2^e(\mathbf{x}), \cdots, f_m^e(\mathbf{x}), -\ \textit{robust consensus}(\mathbf{x})), \\ &\qquad\qquad\qquad \text{subject to } \mathbf{x} \in \mathbf{\Omega}, \end{aligned} \tag{7}$$

where $f_j^e(\mathbf{x})$s are the same as in (3).

By solving this problem, we can obtain the desired robust solutions. There is, however, a concern: how to computationally integrate the consensus over a region? We address this problem in the following way. Let, $\mathbf{x} \in \mathbf{\Omega}$ be a solution. A set of H random points $\mathbf{x}_k$ $(k = 1, 2, \cdots, H)$ is chosen such that $\forall k$ $(k = 1, 2, \cdots, H), \forall i (i = 1, 2, \cdots, n)$, $\left(\mathbf{x}^{(i)} - \delta^{(i)}\right) \le \mathbf{x}_k^{(i)} \le \left(\mathbf{x}^{(i)} + \delta^{(i)}\right)$, where, $x_k^{(i)}$ is the i^{th} component of x_k, $\delta^{(i)} \ge 0$ is the maximum allowed perturbation along i^{th} variable, and n is the number of variables. For simplicity, we consider $\forall i, \delta^{(i)} = \tilde{\delta}$. However, one can use different values of $\delta^{(i)}$s, and that may be more appropriate for real life problems. For all randomly chosen points we compute the objective (or consensus) values and find their arithmetic mean. With an increase in H the accuracy level increases. To decrease computational cost, nonetheless, one can choose smaller value of H. The $\delta^{(i)}$s are very important parameters. The results vary significantly with the choice of this parameter value. When $\tilde{\delta} = 0$, the proposed robust consensus reduces to consensus, and this problem formulation will not provide robust solutions.

5 Test Problem, Experimentation, and Discussions

5.1 Test Problem

BINH [1] is a well known UMMP test problem defined in (8).

$$\begin{aligned} \text{minimize } f_1(x_1, x_2) &= x_1^2 + x_2^2, \\ \text{minimize } f_2(x_1, x_2) &= (x_1 - 5)^2 + (x_2 - 5)^2, \end{aligned} \tag{8}$$

Table 1. DMs' fuzzy reference points for modified M-BINH problem

Decision Maker (D_j)	r_{j1}	r_{j2}
D1	(10.0, 15.0, 20.0)	(16.0, 21.0, 26.0)
D2	(8.5, 14.0, 19.5)	(16.0, 22.0, 28.0)
D3	(11.0, 16.0, 21.0)	(13.0, 20.0, 27.0)
D4	(8.0, 13.0, 18.0)	(15.0, 19.0, 23.0)
D5	(9.0, 17.0, 25.0)	(14.0, 18.0, 22.0)

where $-5 \leq x_1, x_2 \leq 10$. The authors [18] have modified this problem to make it suitable for the robustness-consensus FGDM problem. They call it M-BINH. We use the same modified formulation. It is described in (9).

$$
\begin{gathered}
x_p^c = \frac{x_p^{\max} + x_p^{\min}}{2}, \; r_{1,p} = 0.2, \; r_{2,p} = \frac{x_p}{x_p^{\max}}, \\
x_p = \begin{cases} x_p, \text{ if } x_p \leq 1 \\ x_p^{\min} + \text{floor}\left(\dfrac{x_p - x_p^{\min}}{r_{1,p}}\right) r_{1,p}, \text{ if } x_p < x_p^c \\ x_p^c + \text{floor}\left(\dfrac{x_p - x_p^c}{r_{2,p}}\right) r_{2,p}, \text{ else,} \end{cases} \\
p = 1, 2, \\
\text{minimize } f_1(x_1, x_2) = x_1^2 + x_2^2, \\
\text{minimize } f_2(x_1, x_2) = (x_1 - 5)^2 + (x_2 - 5)^2,
\end{gathered} \tag{9}
$$

where $0 \leq x_1, x_2 \leq 5$, $x_p^{\min}$ and $x_p^{\max} (p = 1, 2)$ indicate respectively the lower and upper bounds of the variable space.

We assume that there are five DMs and the corresponding weight vector is $\mathbf{w} = (0.20, 0.20, 0.20, 0.20, 0.20)$, i.e., all the DMs are equally important. Their fuzzy preference points are presented in Table 1. These parameters are not the same as in [18]. We have changed them to make the problem more suitable to show the effectiveness of our approach.

We reformulate the M-BINH problem in (10).

$$
\begin{gathered}
\text{minimize } \mathbf{f}_{\text{M-BINH}}^{\text{robust}}(\mathbf{x}) = \left(f_1^e(\mathbf{x}), f_2^e(\mathbf{x}), -\, \textit{robust consensus}(\mathbf{x})\right), \\
\text{where } f_1^e(\mathbf{x}) = \frac{1}{|\mathcal{B}_\delta(\mathbf{x})|} \int_{\mathbf{y} \in \mathcal{B}_\delta(\mathbf{x})} f_1(\mathbf{y}) d\mathbf{y}, \\
f_2^e(\mathbf{x}) = \frac{1}{|\mathcal{B}_\delta(\mathbf{x})|} \int_{\mathbf{y} \in \mathcal{B}_\delta(\mathbf{x})} f_2(\mathbf{y}) d\mathbf{y}.
\end{gathered} \tag{10}
$$

Here $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ are defined as in (9), and *robust consensus* is defined in (6).

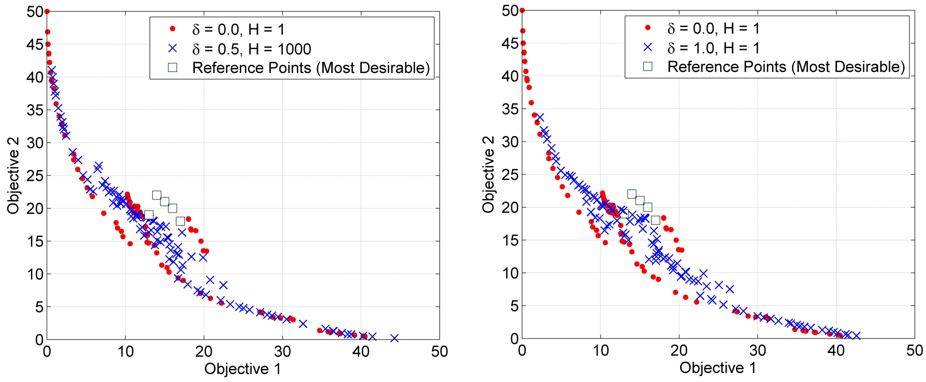

Fig. 2. Showing nondominated sets $\mathcal{S}_1^0$, $\mathcal{S}_{1000}^{0.5}$, and $\mathcal{S}_{1000}^{1}$

5.2 Common Parameter Settings

We have set population size to 100. Simulated binary crossover (SBX) has been used, where crossover probability $p_c = 0.95$ and distribution index for crossover $\eta_c = 20$. We have used polynomial mutation where mutation probability $p_m = 1/n$ and distribution index for mutation $\eta_m = 10$. Here $n (= 2)$ is the number of variables. We have executed NSGA-II for 1000 generations. The reason for choosing such a high number of generations is to allow the searching algorithm enough chance to converge. We have used jMetal 4.4 [7], [8] for the simulation purpose.

5.3 Experiments, Results, and Discussions

At first we execute the algorithm for three pairs of $(H, \tilde{\delta})$ parameter sets: $(1, 0)$, $(1000, 0.5)$, and $(1000, 1)$. Let us denote the sets of nondominated solutions obtained for these three parameter sets by $\mathcal{S}_1^0$, $\mathcal{S}_{1000}^{0.5}$, and $\mathcal{S}_{1000}^{1}$ respectively. When $H = 1$, and $\tilde{\delta} = 0$, our problem searches for solutions which are not robust both in terms of the objectives and the consensus. To reduce the error in computing robustness, we have used a high value of H. It is worth mentioning that for the parameter pair $(1, 0)$, our problem formulation reduces to simple consensus optimization problem, i.e., in that specific case, we are searching for consensus solutions which may not be robust in terms of their objectives. However, to observe how the output changes with the change of $\tilde{\delta}$, we plot the objective values of $\mathcal{S}_1^0$ and $\mathcal{S}_{1000}^{0.5}$ in the left panel, and the objective values of $\mathcal{S}_1^0$ and $\mathcal{S}_{1000}^{1}$ in the right panel in Fig. 2.

From Fig. 2, we observe that when $\tilde{\delta}$ increases, the obtained set of solutions moves away from the set $\mathcal{S}_1^0$. Basically, with the increase of $\tilde{\delta}$, i.e., when we search for more robust solutions, the robust consensus of the solutions decreases. In Fig. 2, we have also shown the most desirable points suggested by each DM. Around the coordinate $(15, 15)$ in the objective space, there is a region with more crowded solutions which is close to the region where most of the desirable points suggested by the DMs lie. We observe that with the increase of $\tilde{\delta}$, this region becomes wider as well as one of the end points of the solution set gets drifted towards the middle region. With the increase of $\tilde{\delta}$, the system stops in a region which is preferred by the *soft* choices of the DMs and the obtained solutions are away from the Pareto front of the unaltered UMMP. When δ changes the t-norm operator $\phi(\,\cdot\,)$ and the weight vector $\mathbf{w}$ also play important roles on the direction of the drift of the solution sets.

6 Conclusions

In this work, we redefine consensus and define a new measure called *robust consensus*, which indicates the robustness of a multiobjective solution with respect to its consensus among the preferences provided by a set of DMs. We have also shown a reformulation mechanism for multiobjective fuzzy decision making problems. It provides the DMs a set of solutions from their preferred search regions. The DMs express their prior preferences by providing reference points for each objective. Using the multiobjective genetic algorithm NSGA-II, we have successfully solved a modified test problem, M-BINH and shown that the proposed method is capable of providing solutions from the region desired by the DMs. Further, we have shown that the proposed definition of *robust consensus* is sensitive to its parameter δ. The effect of the formulation for different aggregation operators is not studied in this work. We intend to do this in our future work.

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