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ABSTRACT

Pipeline generally extends over long distances traversing through wide variety of different soils, geological conditions and regions with different seismicity. Majority of the past works in the area of pipeline subjected to fault motion is restricted in several ways. There were many analytical models developed in the past for pipeline fault crossing, however, they are of limited usage, for example analytical model developed for pipeline fault crossing can be useful for strike slip fault crossing only. Likewise incorporating the large geometric changes in analytical study is a tricky task; however pipeline subjected to the fault motion itself is a phenomenon of large geometric changes. Especially when pipeline subjected to compression, where in addition to material deformation it also undergoes general as well as local buckling with bending, contradictorily past work mostly assumed that pipeline is under tension.

With day by day increasing capacity of computation and advancement in numerical modeling, one can find more facts for pipeline subjected fault motions including cases of pipe under compression as well. In this paper, past work is reviewed for pipeline subjected to large fault motion. A three dimensional FE based numerical model is suggested to carry out pipeline performance of buried pipeline subjected to fault motion. A proposed model includes material nonlinearity, as well as effect of the large geometric changes. For this purpose, three dimensional FE program is developed in MATLAB. Displacement controlled Arc-length technique is implemented to solve the nonlinear behavior. To reduce the computation time of analysis here parallelization tool kit of MATLAB is utilized.

Keywords: Buried continuous pipeline, Fault motion, Nonlinear-large deformation FEM, Displacement controlled Arc-length technique.

1. INTRODUCTION

Pipelines are common transportation means for oil and natural gas, which always act as an important lifeline facility for any nation. Generally, these pipelines laid underground for economic, aesthetic, safety, and environmental reasons. While running through the length and breadth of country pipeline expose to diverse soil conditions. Seismic hazard of pipeline is well demonstrated and documented during past several earthquakes all over the world. Predominant study for seismic hazard of pipeline started after the 1971 San Fernando earthquake. Newmarks and Hall (1975) did the pioneer work for pipeline crossing the fault by assuming pipe as cable in their analytical study. The only force considered acting on the pipeline is the friction force at the pipe-soil interface along the longitudinal direction without lateral force offered by the soil. This model further modified by the Kennedy et al. (1977) and by incorporating the lateral pressure offered by the soil. In 1985 Wang and Yeh further modified model by dividing pipe in to three regions depending up on the curvature in the pipeline with I region near fault plane. It was also assumed that strain in region II and III are elastic while the strain in region I is inelastic. For straight portion in region III they used the theory of beam on elastic foundation. In a model they notify that maximum bending strain is in the region II and crucial combination of axial and bending strain will at junction of II and III region hence concluded that the pipe would fail at this junction, which seems counterintuitive since one expects tensile ruptures at or very near to the fault crossing. Newly Karamitros et al (2007) introduced a number of refinements in the method proposed by Wang and Yeh (1985). Previous method overlooked the effect of axial force on bending stiffness. Karamitros et al suggested most unfavourable combination of axial and bending would not necessarily take place at the end of high curvature portion but within the zone, closer to the fault crossing point.

In addition to above, analytical models there are several numerical models proposed, which includes beam on nonlinear Winkler foundation. In which pipe modelled with beam/shell elements and soil with springs (Takada et al 1998). Nodes of the shell elements of the pipe are attached to soil that is modelled as springs.

2. SCOPE

Though improved analytical models provide a good result but models are developed with fundamental assumptions that that curvature of the pipeline on either side of fault plane is symmetrical. In case of strike slip fault-pipeline crossing pipe essentially deforms in the horizontal plane were soil on either side of the pipeline extends to very large or for infinite distance. This offers the symmetric resistance to the pipe on either side in the plane of pipe deformation. This symmetry also takes care of point of contra-flexure to draw it closer to the pipeline fault crossing. Hence, the analytical models developed in past are applicable to strike slip fault motions case only. For dip slip fault motion, the pipe-soil interaction forces along the fault motion are dissimilar due to great variation in the depth. Lesser depth of the soil above the pipe offer less resistance compare to bottom soil for deformed pipe. In addition to this deformation of the pipe is greatly depends on the soil movement of the upper layer which usually differs in hanging wall and footwall. Hence, assumptions for identical curvature on either side of the fault plane no longer valid. Analytical study also restrict for pipeline under tension cases. Pipeline under compression usually involves general as well as local geometric instability issues (e.g. pipeline during 1999, Izmit, Turkey earthquake (EERI, 1999)). Handling complex geometric stability is always hard to models in analytical studies. Faulting itself is phenomenon of large geometric changes hence theory of small deformation no longer suitable for pipeline fault crossing which were used in past. Hence, study of the pipeline under compression needs appropriate understanding as it involves both material as well as geometric failure.

However numerical models proposed by Takada et al (1998), LIU Ai-wen et al (2004) for buried pipeline using shell element and nonlinear springs for pipe and soil respectively can perform well for pipeline under compression. Though, post yielding of soil spring gives higher strain value in pipe. This could be the result of the inadequacy of the spring models to incorporate the actual soil behaviour during soil yielding. In addition, these models do not consider the large geometric changes of upper soil layer, which has significant effect on pipeline performance. Stiffness of each individual spring is independent i.e. each spring behaves independently which disregard the effect of lateral soil confinement.

Considering limitation of previous models and day-by-day increasing capacity, speed and powerfulness of the computer, the computer can make it possible to solve field problems by doing more realistic numerical analysis. Here more realistic numerical program using three dimensional FEM is developed for buried continuous pipelines. This program is developed using isoparametric brick element. The developed model takes care of material as well as large geometric changes to comprise fault motion.

3. METHODOLOGY

3.1 Numerical Modeling

The governing nonlinear finite element equation for of solid continuum can be obtained from principle of virtual work. Eq 1 is adaptation from the one presented by K. J. Bathe (1996) and J.N. Reddy (2004) for updated Lagrangian approach.

$$\left({}_{t}^{t}K_{L}+{}_{t}^{t}K_{NL}\right)\Delta U^{(i)}={}^{t+\Delta t}R-{}_{t+\Delta t}^{t+\Delta t}F^{(i-1)} \tag{1}$$

Where

$${}_{t}^{t}K_{L}=\int_{{}^{t}V}{}_{t}^{t}B_{L}^{T}\,{}_{t}D\,{}_{t}^{t}B_{L}\,d^{t}V$$

$${}_{t}^{t}K_{NL}=\int_{{}^{t}V}{}_{t}^{t}B_{NL}^{T}\,{}^{t}\tau\,{}_{t}^{t}B_{NL}\,d^{t}V$$

$${}_{t+\Delta t}^{t+\Delta t}F^{(i-1)}=\int_{{}^{t}V}{}_{t}^{t}B_{NL}^{T}\,{}^{t}\hat{\tau}\,{}_{t}^{t}B_{NL}\,d^{t}V$$

${}^{t}\hat{\tau}$ = Cauchy stress vector

B = Transformation matrix

${}^{t+\Delta t}R$ = vector of externally applied loads at time $t+\Delta t$

The numerical integration is performed according to Gaussian quadrature rule. A code is developed in MATLAB-7.9 for three dimensional FEA using 8 nodded isoparametric elements.

The successes of any nonlinear analysis is primarily depends on the accuracy, convergence, efficiency and stability of nonlinear solution technique. The nonlinear Eq 1 can be solved by various nonlinear solution techniques available in the literature. Among this full or modified Newton-Raphson, method is simple to understand, implement and it generally converges in few iterations. However, this method fails to trace the nonlinear equilibrium path through the limit or bifurcation points, in vicinity of limit points, tangent stiffness matrix becomes singular and the iteration procedure diverges. This is common in buckling and strain softening nonlinear material behaviour type of the analysis. The displacement boundary condition in nonlinear analysis needs linearization of the prescribed boundary displacement, which can be easily incorporated in other methods like arc-length method. Hence, more robust arc-length method is employed for this work. Arc-length method originally developed by Riks (1972; 1979) and Wempner (1971) and later modified by several researchers.

3.2 Validation of Code

For the validation of developed code, tests have been performed on the 3D cantilever beam. Load-deflection curve is compared with commercially available finite element package ANSYS-12. Material behaviour assumed for the test is same as pipe material. Beam dimensions, meshing and point load considered at free end are given in Table 1. Figure 1 shows perfectly matching load-deflection curve obtained from developed code and Ansys.

Table 1: Comparison of Code and ANSYS Results

L x D x B (m)	Element size (m)	Point load (kN)	U_{max} (m)	
			Model	Ansys
3.0 x 0.2 x 0.05	0.05 x 0.05 x 0.05	80	0.130	0.135

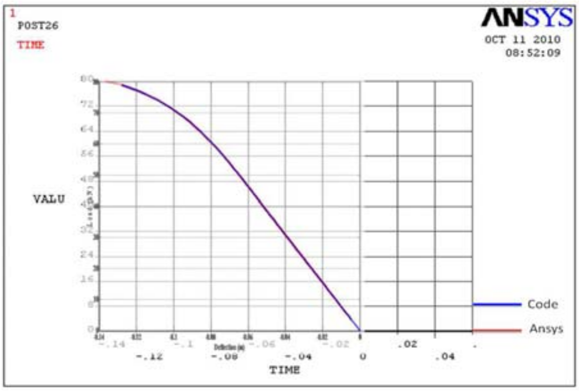
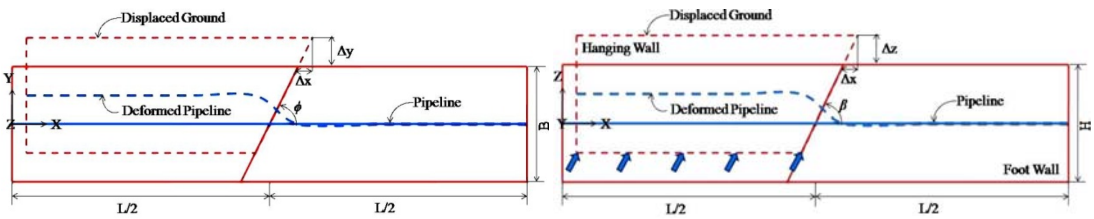


Fig. 1: Comparison of Results between ANSYS and Developed Code

3.3 Model Dimensions

The coordinate system and notations used for this work are shown in Figure 2. In reality, soil media does not have any fixed boundaries or can be assumed at infinite distance, which is virtually impossible to incorporate in numerical model, hence model dimensions are determined for boundary effect and it considered as 80m (L) x 12m (H) x 15m (W).



(a) **(b)**

Fig. 2: (a) Plan View of Buried Pipeline Model for Strike-slip Fault Motion. (b) Sectional View of Buried Pipeline Model for Dip-slip Fault Motion

The pipe near the fault is usually suffers large deformation, which is not so long, about 10m ~ 30m, and the damaged point of pipe also occurred in this pipe segment (LIU Ai-wen, 2004). For this, meshing of varying element lengths is considered to optimize the memory and time usage. Finer meshing of 0.5 m element lengths are used near the fault region for 20m distance on either side. Then for remaining length 1m, element size is used. Pipe is divided in eight equal divisions in circumferential direction and single division is made for wall thickness. Near the pipe, soil is meshed by small elements with varying size for the square region of 1.2m (Figure 3). Elements size of soil, which is far from the pipe, is taken as 1.5m.

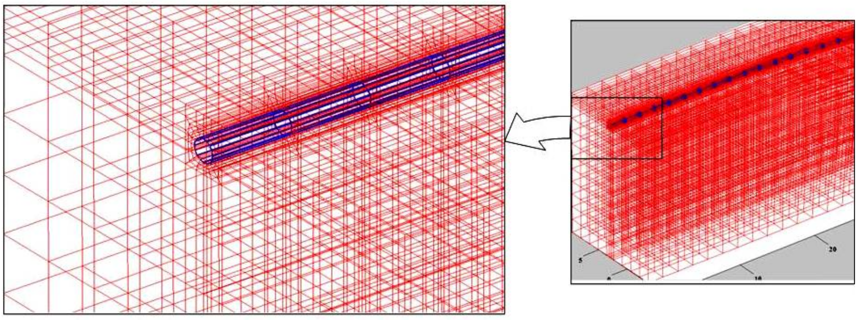


Fig. 3: Proposed Finite Element Model of Buried Pipeline

Default parameter in evaluation of pipeline performance are considered as maximum fault offset ($\Delta_{\max}$) = 0.6m, pipeline fault crossing angle (ϕ) = 90^0, diameter of the pipeline (D) = 0.61m (24 inches), pipeline wall thickness (t_p) = 0.0095m (0.375 inches) and depth of the buried pipeline s(d_b) = 0.91m (3feet). Performance is evaluated for no internal pressure condition. Few assumptions are made in the development of the models. It assumed that there is perfect bond exist between the soil and pipe material.

3.4 Boundary Condition

The target ground displacements are applied at the bottom, with the free top boundary. While for side boundaries, all nodal degree of freedoms other than, in the direction of the components of targeted displacements are constrained. The total soil mass block is divided in to two parts, on either side of the fault plane. The fault displacement (Δ) is applied to first block by keeping other one fixed.

3.5 Material Modeling

The For pipe Ramberg-Osgood relationship is one of the most widely used models (M. O'Rourke(1999), IITK-GSDMA GUIDELINES (2007)), while for soil hyperbolic is common (S.L Kramer (2007)). Hence, the same are used in this study which are summarised below.

$$\varepsilon = \frac{\sigma}{E}\left[1 + \frac{n}{1+r}\left(\frac{\sigma}{\sigma_y}\right)^r\right] \tag{2}$$

Where

ε= Engineering strain

σ= Stress in the pipe

E_i= Initial Young's modulus

σ_y = Yield strain of the pipe material

r, n=Ramberg - Osgood parameters adopted as r = 31.50 &

n = 38.32 Karamitros (2007)

$$\tau = \frac{G_{\max}\gamma}{1+\left(G_{\max}/\tau_{\max}\right)|\gamma|} \tag{3}$$

Where

τ=shear stress,

γ= shear strain,

G_{max} = maximum shear modulus and

τ_{max}=maximum shear stress

The API5L-X 65 steel pipe is frequently used in literature (Newmark-Hall (1975), Karamitros (2007) etc) hence the same is adopted for this study. Table 2 show the properties used for API5L-X 65 pipe material. While soil is assumed as typical sand with initial Young's modulus as E_i = 50Mpa and Poisson's ratio 0.4. Table 3 show constants used in hyperbolic model.

Table 2: Properties of AP15L-X 65 Pipe

Properties for API5L X-65 Pipe	**Magnitude**
Initial Young's Modulus (E_i)	210 Gpa
Yield Stress (σ_y)	490 Mpa
Failure Stress (σ_f)	513 Mpa
Failure Strain (ε_f)	4%
Poisson's ratio (μ)	0.3
Density (ρ_p)	7.8g/cm³

Table 3: Constants for Hyperbolic Model

Medium Density sand Properties	**Magnitude**
Maximum Shear Modulus (G_{max})	60 Mpa
Maximum shear Strength (τ_{max})	0.0216 Map
Failure Stress (σ_f)	513 Mpa
Density (ρ_s)	1.44/cm³

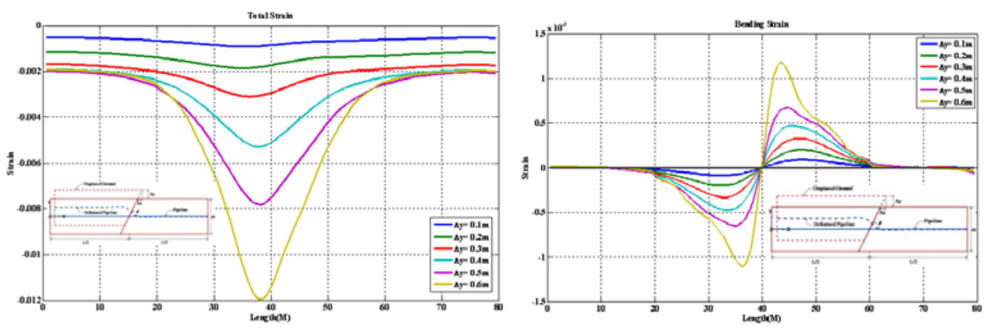
4. RESULTS AND DISCUSSION

The performance of continues buried pipeline crossing active fault is studied using proposed finite element models. The developed model can implemented for all sort of fault motion with variation in other geometric parameters. Here case of strike slip is taken to determine the influencing on the performance of pipeline for the fault offset (Δ), pipeline-fault crossing angle (β), wall thickness to diameter ratio (t_p/D), depth of the buried pipeline (d_b) and their combinations. In the nonlinear numerical analysis of soil maximum ground displacements is generally restricted up to 5% of the total depth of the model beyond which results usually deviate and are unrealistic, hence maximum component of fault offset is limited to 0.6m. Total fault offset of 0.6 is applied with an increment of 0.1m. Pipeline fault plane angle is varied from 40^0 to 140^0 with an increment of 10^0. The pipeline wall-thickness 0.0095, 0.0103 and 0.0190 are considered, while depth is varied 2 to 4 feet.

4.1 Pipeline Subjected to Strike Slip Fault Motion

4.1.1 Effect of the Fault offset

Hence for $\phi = 90^0$ pipeline mainly subjected to the bending; while for positive ϕ small angle pipeline will be under pure compression. Hence, for determining the effect of the fault offset 60^0 angles is chosen, where effect of both the direct and bending strain can be seen. Figure 4 shows the effect of the fault offset on total and bending strain distribution in the pipeline. Maximum fault offset 0.6m is applied with an increment of 0.1m. Generally two kind of failure are associated with pipeline, first material failure when pipe material strained beyond sustain limit in general yield strain and geometric failure in which geometry of the pipe is so distort that pipeline becomes inadequate to pass the fluid. From figure 4 (a) it can be seen that maximum strain (ε_{xx}= 0.00186) generating in the pipeline for fault offset 0.2m is just below the yield strain (ε_y= 0.002). From which it can be said that pipe do not have any serious damage upto 0.2m fault offset. Thereafter the difference of the strain curve, near the fault plane continuously increases. This indicates that pipe material entered in plastic stage. While discussing about the pipeline damage there are two points, which are most significant. First, how much material has yielded and secondly how much length of the pipe enters in the plastic stage. This has great significance in case of post event repair and maintains. This large longitudinal strain in the pipe material further causes reduction in the wall thickness (developing upon Poisson's ratio), which may not be safe design thickness for the internal pressure and other load. From the figure 4(a) one can see that for the 0.3m fault offset only near the fault plane about 10m pipe length is beyond the yield strain. While majority of the pipe length is just crosses the yield strain. There after both length of pipeline crossing yield point and maximum strain beyond the yield strain increasing seriously. For the considered cases no large geometric failure is observed. From bending strain distribution curves (figure 4b) one can observe that bending strain in the pipe is smoothly increasing up to the 0.4 fault offset. After that bending strain distribution curve slightly disturbing for 0.5 and 0.6m fault offset at 10m on either side of the fault plane. This kind of disorder mainly signifies the local buckling on the pipe.



(a) (b)

Fig. 4: Effect of Fault Offset on Strain Distribution for Strike Slip with Δy = 0.1m to0.6m and $\phi = 60^0$

4.1.2 Effect of the Pipeline Fault Angle

The pipeline fault angle is second most vital parameter related to the pipeline performance. Hence effect of the pipeline-fault angle (ϕ) is studied by varying the pipeline-fault angle from 40^0, 60^0, 80^0, 90^0, 100^0, 120^0 to 140^0 for fault offset of 0.6m. The performance analysis of pipeline becomes more complex due to unlike behaviour of the pipeline under compression and tension. In general, under tension pipe is fail due to excessive straining of pipe material, while in case of compression in addition to the material failure geometric failure is also takes place. The foremost point that can be observe in the strain distribution curve plotted in figure 5 is maximum strain developing for negative pipeline fault angle (ϕ<90^0) is much higher than the positive pipeline fault crossing angle (ϕ>90^0). The reason for this can be understood as, when pipeline is subjected to the compression pipe has a chance of bending and/or buckling and hence the fault offset is accommodate by the geometric change without much internal deformation, which leads to lesser internal deformation in the pipe. There are two fundament troubles associated with the pipeline buckling, firstly it is a sudden phenomenon and may have an adversely affect the operational pipelines. Secondly, the large geometric distortion during buckling further causes pressure loss in the pipeline, which is the foremost significant parameter for the petroleum pipeline. In case of pipeline is under tension whole fault displacement at the pipe fault crossing is needed to accommodate by the internal deformation of pipeline material. From figure 5 it can also observe that for $\pm 80^0$ angle total normal strain distribution are similar on the opposite side of zero strain axis. This indicates that the buckling of the pipe does not take place for all pipeline fault angle.

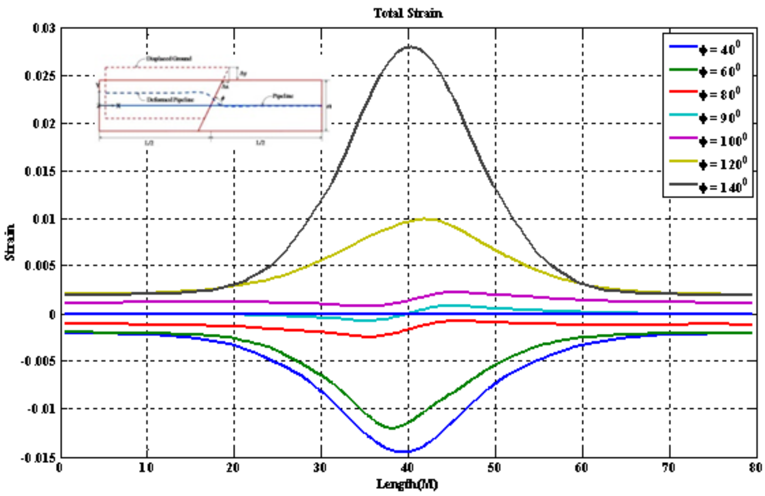


Fig. 5: Effect of Pipeline Fault Angle on Total Normal Strain Distribution for Strike Slip with Δy = 0.6m

4.1.3 Effect of Pipeline Wall Thickness to Diameter Ratio

In general design of the wall thickness is the mainly function of internal pressure. Where it is design for hoop and longitudinal stresses and checked for secondary loads like overburden and live loads. Nevertheless, the present study shows that thickness to diameter ratio has great hold on the pipeline performance crossing strike slip fault especially when pipeline subjected to the compression. To understand the effect of the wall thickness here 0.0095, 0.0136 and 0.0190 are the three wall-thicknesses to diameter ratios considered. To have an effect of geometric failure under compression parametric study is performed for $\phi = 40^0$ where pipe can be subjected to sufficient compression. The maximum fault offset here could able applied is 0.47m after which pipe subjected to large geometric changes which further causes soil failure and diverges numerical analysis.

Geometric failure of the pipe can be more clearly understood by observing the deform pipe hence for deformed shapes of the pipes with different wall thickness to diameter ratios are plotted in Figure 6. From figure, it can be clearly seen that pipe with thicker wall thickness subject to more geometric changes than the pipe with thinner wall thickness. However, thicker wall pipe has higher internal deformation capacity, which can be observer in the strain distribution Figure 7. Nevertheless, for less strain thicker pipe got more damage this clearly indicates geometric failure of the thick wall pipe. The reason for this is quite understandable that thinner wall pipe has lesser moment of inertia can be easily bent and deform to accommodate the fault displacement. On other hand thicker wall pipe, which subjected to less strain indicate that lesser internal deformation, therefore thick wall pipe needs to accommodate fault displacement by large geometric deformations. From the above discursion, it is clear that when pipe is subjected strike-slip fault with ϕ< 90^0, thicker pipe are more vulnerable to geometric failure.

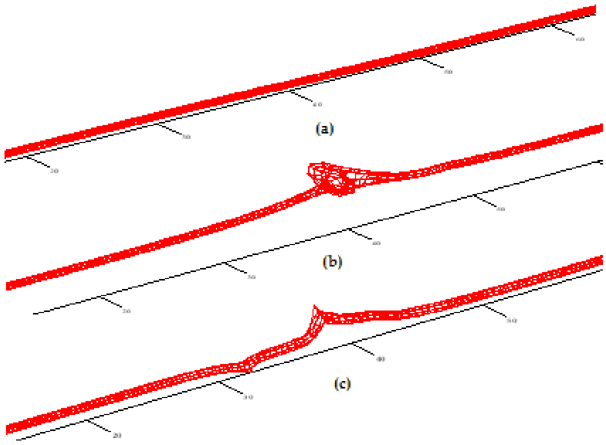


Fig. 6: Deformed Shape of Pipe with t_w/D = (a) 0.0095, (b) 0.0136, and (c) 0.0190 for Strike Slip with Δy = 0.47m and $\phi = 40^0$

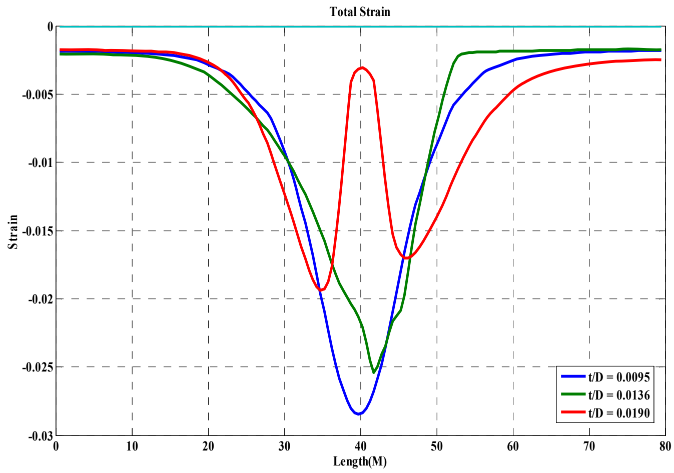


Fig. 7: Effect of Pipeline Wall Thickness to Diameter Ration for Strike Slip with Δy = 0.46m and $\phi = 40^0$

If we observe the deformed shape of the pipe in Figure 6, it can be seen that pipe with 0.0095 wall thickness to diameter pipe is deformed only in the horizontal plane which indicates bending of the pipe with strike-slip fault motion. While in case of t_w/D = 0.0136 pipe is look like little moved with fault and then bulged in horizontal plane indicates geometric failure in plastic stage. Finally in case t_w/D = 0.0190 pipe is purely buckled in vertical plane, which is catastrophic geometric failure. Much lesser depth the depth of top soil compares to three remaining direction offer lesser resistance to the buckling pipe. That could be the reason why strong pipe buckled in vertical plane.

5. CONCLUSION

Main conclusions of this study can be stated as following:

- Numerical modeling of physical problem if implemented with latest updated methods could yield much better results.
- Apart from material behaviour and its failure, geometrical behaviour becomes important when studies are done on pipes subjected to thrust fault motions.
- Compression failure behaviour of the pipelines is catastrophic in nature as it leads to sudden buckling. Which crucially depends on the pipe wall thickness.
- A strike-slip numerical study of buried pipelines with different parameters has shown the effect of both direct and bending strain
- Though here developed model is implemented on the strike slip fault motion but the same can be implemented for other kind of ground motions.

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