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Article *in* Soft Computing · May 2017

DOI: 10.1007/s00500-015-1941-9

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Soft Comput (2017) 21:2297–2306
DOI 10.1007/s00500-015-1941-9

METHODOLOGIES AND APPLICATION

A solid transportation model with product blending and parameters as rough variables

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Published online: 23 November 2015

Abstract In this paper, we formulate a practical solid transportation problem with product blending which is a common issue in many operational and planning models in the chemical, petroleum, gasoline and process industries. In the problem formulation, we consider that raw materials from different sources with different quality (or purity) levels are to be transported to some destinations so that the materials received at each destination can be blended together into the final product to meet minimum quality requirement of that destination. The parameters such as transportation costs, availabilities, demands are considered as rough variables in designing the model. We construct a rough chance-constrained programming (RCCP) model for the problem with rough parameters based on trust measure. This RCCP model is then transformed into deterministic form to solve the problem. Numerical example is presented to illustrate the problem model and solution strategy. The results are obtained using the standard optimization solver LINGO.

Keywords Solid transportation problem · Blending · Rough variable · Trust measure · Chance-constrained programming

Communicated by V. Loia.

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1 Introduction

In many industries like oil refining, chemical, petroleum and process; shipping of final products involves a common issue called product blending, i.e., blending raw materials with different attributes (purity, concentration levels, etc.) into desired homogeneous intermediate or end products. As an example, suppose a particular product of different qualities are transported to different demand points from various sources, provided that the products are blended at each destination must maintain the desired quality. In other words, the amount of products with different qualities will reach at a destination so that they can be blended together to meet the desired quality of product at that demand point. Such type of blending in the petroleum industry is very often observed (Rigby et al. 1995). The optimal scheduling of crude oil has been shown to lead to high benefits (Kelly and Mann 2003). An interesting history of blending in the petroleum industry is mentioned in DeWitt et al. (1989), Romo et al. (2009), Méndez et al. (2006) and Misener and Floudas (2009). Recently, Papageorgiou et al. (2012) studied a uncapacitated fixed-charge transportation problem with product blending.

The classical transportation problem (TP) deals with transportation of goods from some sources to certain destinations. Solid transportation problem (STP) is an extension of classical TP. In STP, three kinds of constraints are taken into consideration instead of two (sources and destination constraints) in a TP. This extra constraint may be due to modes of transportation (conveyances), types of goods, etc. Traditionally, STP (Haley 1962; Jiménez and Verdegay 1998, 1999; Liu 2006; Yang and Liu 2007; Nagarjan and Jeyaraman 2010; Ojha et al. 2009; Kundu et al. 2013a, 2014) is modeled considering total supply capacity of all the conveyances and it is assumed that this total capacity is available for utilization from all sources to destinations routes. However, in many practical situations, this may not always be true. Practically, most of the time full vehicles, e.g., trucks, rail coaches, etc. are to be booked and the availability of each type of conveyances at each source may not be the same. Fulfillment of the capacity of a vehicle affects the optimal transportation policy, as generally the unit transportation costs are determined on utilization of capacity of the vehicle. To deal with such practical situations, we formulate a useful solid transportation model.

Future transportation planning always depends upon previous experiences. But the available data in a transportation system, such as transportation costs, supplies, demands, conveyance capacities are not always available precisely but are uncertain in nature, due to insufficient information, fluctuating financial market, lack of evidence, data collected from multiple sources, etc. Many researchers studied STP with different uncertain parameters such as fuzzy (e.g., Chanas and Kuchta 1996; Jiménez and Verdegay 1999; Yang and Liu 2007; Ojha et al. 2009; Kaur and Kumar 2012; Kundu et al. 2013a, 2014), stochastic (e.g., Yang and Feng 2007; Nagarjan and Jeyaraman 2010). Rough set theory is one of the most convenient and accepted tool to deal with uncertainty. Though transportation problems in various types of uncertain environments such as fuzzy, random are studied by many researchers, there are few research papers (Tao and Xu 2012; Kundu et al. 2013b) in rough uncertain environment. Rough set theory was first proposed by Pawlak (1982). Later it was developed by many researchers (Pawlak 1991; Pawlak and Skowron 2007; Polkowski 2002). Liu (2002) proposed the concept of rough variable which is a measurable function from rough space to the set of real numbers and developed trust theory to study the behavior of rough events. Liu (2002, 2004) studied some rough programming models with rough variables as parameters. Liu and Zhu (2007) introduced X-valued rough variable. Xu and Yao (2010) studied a two-person zero-sum matrix game with payoffs as rough variables. Xu et al. (2009) proposed a rough DEA model to solve a supply chain performance evaluation problem with rough parameters. Mondal et al. (2013) considered a production–repairing inventory model with fuzzy rough variables.

In this paper, we formulate a solid transportation problem with product blending (STPwB). There are several sources of a certain petroleum or chemical product with different quality levels, and the product has demand at several demand points with different quality levels. Each demand point (destination) has quality checking process and they receive the product only if its quality satisfies a minimum desired quality predetermined at the demand point. The problem is to transport different quality products from different sources to different destinations so that the products received at each destination can be blended together to meet the required quality as well as to satisfy the demanded amount at that destination. For blending purpose, it is assumed that each destination has blending headers or charging points. The objective of this problem is minimization of total logistics (transportation) costs. The parameters of this problem such as transportation costs, availabilities, demands are taken as rough variables to deal with increasing uncertainties in decision making problems. We formulate a rough chance-constrained programming model for the STPwB with parameters as rough variables and its crisp form is obtained to solve the problem. The constrained cost minimization problem is solved by gradient based non-linear optimization technique-Generalized Reduced Gradient (GRG) method (using Lingo 13.0).

The rest of the paper is organized as follows. In Sect. 2, some basic concepts and properties of rough set are reviewed. Section 3 proposes the single objective transportation model for STPwB. Section 4 describes the solution methodology of STPwB with parameters as rough variables. In Sect. 5, a numerical example is presented to illustrate the model. Finally, Sect. 6 provides conclusions and directions for further research.

2 Preliminaries

Here, we present some basic concepts on rough variables. Rough set theory, introduced by Pawlak (1982), has been proved to be an efficient mathematical tool dealing with vague/imprecise description of objects. The idea of rough set theory comes from approximation of a subset of a certain universe by means of lower and upper approximations using the information contained in an another known subset. The lower approximation is a definable (exact) subset containing the objects surely belonging to the set, whereas the upper approximation is a superset containing the objects possibly belonging to the set. In other words a rough set is defined by a pair of crisp sets, called lower and upper approximations those are originally produced by an equivalent relation. To deal with vast complexities of the real-world problems, many researchers considered some generalizations of basic rough sets so that rough set theory has wide applications. In this study, we mainly concentrate on the concepts of rough variable (Liu 2002, 2004) and trust theory which is a branch of mathematics that studies the behavior of rough events.

2.1 Rough variable

The concept of rough variable was introduced by Liu (2002). The following definitions are based on Liu (2002, 2004).

Definition 2.1.1 (*Rough space*) Let Λ be a nonempty set, $\mathcal{A}$ be a σ-algebra of subsets of Λ, Δ be an element in $\mathcal{A}$, and π be a nonnegative, real-valued and additive set function on $\mathcal{A}$. Then $(\Lambda, \Delta, \mathcal{A}, \pi)$ is called a rough space.

Definition 2.1.2 (*Rough variable*) A rough variable ξ on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$ is a measurable function from Λ to the set of real numbers $\Re$ such that for every Borel set B of $\Re$, we have $\{\lambda \in \Lambda \mid \xi(\lambda) \in B\} \in \mathcal{A}$.

Then the lower and upper approximations of the rough variable ξ, denoted by $\underline{\xi}$ and $\overline{\xi}$, respectively, are defined as follows:

$$\underline{\xi} = \{\xi(\lambda) \mid \lambda \in \Delta\} \; and \; \overline{\xi} = \{\xi(\lambda) \mid \lambda \in \Lambda\}.$$

Definition 2.1.3 Let ξ be a rough vector on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$, and $f_j : \Re^n \to \Re$ be continuous functions, $j = 1, 2, \ldots, m$. Then the upper and lower trusts of the rough event characterized by $f_j(\xi) \le 0$; $j = 1, 2, \ldots, m$ are, respectively, defined by

$$T\bar{r}\{f_j(\xi) \le 0\} = \frac{\pi\{\lambda \in \Lambda | f_j(\xi(\lambda)) \le 0\}}{\pi(\Lambda)},$$

and

$$T\underline{r}\{f_j(\xi) \le 0\} = \frac{\pi\{\lambda \in \Delta | f_j(\xi(\lambda)) \le 0\}}{\pi(\Delta)}, \; j = 1, 2, \ldots, m.$$

If $\pi(\Delta) = 0$, then the upper and lower trusts of the rough event are assumed to be equivalent, i.e., $T\bar{r}\{f_j(\xi) \le 0\} \equiv T\underline{r}\{f_j(\xi) \le 0\}$, $j = 1, 2, \ldots, m$.

The trust of the rough event is defined as the average value of the lower and upper trusts, i.e.,

$$Tr\{f_j(\xi) \le 0\} = \frac{1}{2}(T\bar{r}\{f_j(\xi) \le 0\} + T\underline{r}\{f_j(\xi) \le 0\}), \quad j = 1, 2, \ldots, m.$$

Definition 2.1.4 Let ξ be a rough variable on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$ and $\alpha \in (0, 1]$, then

$$\xi_{sup}(\alpha) = \sup\{r | Tr\{\xi \ge r\} \ge \alpha\}$$

is called α-optimistic value to ξ and

$$\xi_{inf}(\alpha) = \inf\{r | Tr\{\xi \le r\} \ge \alpha\}$$

is called α-pessimistic value to ξ.

Definition 2.1.5 Let ξ be a rough variable on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$. The expected value of ξ is defined as follows

$$E[\xi] = \int_0^{\infty} Tr\{\xi \ge r\}dr - \int_{-\infty}^{0} Tr\{\xi \le r\}dr.$$

Example 2.1 Let $\xi = ([a, b], [c, d])$ be a rough variable with $c \le a < b \le d$, where $[a, b]$ and $[c, d]$ are, respectively, the lower and upper approximations of ξ. It implies that the elements in $[a, b]$ are certainly members of the variable and the elements of $[c, d]$ are possible members of the variable. Here, $\Delta = \{\lambda | a \le \lambda \le b\}$ and $\Lambda = \{\lambda | c \le \lambda \le d\}$, $\xi(x) = x$ for all $x \in \Lambda$, $\mathcal{A}$ is the Borel algebra on Λ and π is the Lebesgue measure.

As a practical example, consider the possible transportation cost of unit product to be transported from a source i to a destination j through a conveyance k for a certain time period. However, as transportation cost depends upon fuel price, labor charges, tax, road condition, etc., and each of which fluctuates with time, it is not always possible to determine or predict exact value of transportation cost. Suppose four experts give the possible unit transportation cost for the route $i - j$ through conveyance k, determined in a certain time period, as intervals [3,5], [3.5,6], [4,5] and [4,6], respectively. Let c_{ijk} denote 'the possible value of the unit transportation cost for the route $i - j$ through conveyance kk'. Then c_{ijk} cannot have an exact value and can be approximated by means of lower and upper approximations. It is clear that [4,5] is the lower approximation of c_{ijk} as it is the longest definable (exact) set that c_{ijk} contains, i.e., every member of [4,5] is certainly a value of c_{ijk}. Here, [3,6] is the upper approximation, as members of [3,6] may or may not be possible transportation cost according to all experts. So, c_{ijk} can be represented as a rough variable ([4,5], [3,6]).

For a given value r and $\xi = ([a, b], [c, d])$, trust of rough events characterized by $\xi \le r$ and $\xi \ge r$ (Liu, 2002, 2004) is, respectively, given by the following expressions

$$Tr\{\xi \le r\} = \begin{cases} 0, & \text{if } r \le c; \\ \frac{r-c}{2(d-c)}, & \text{if } c \le r \le a; \\ \frac{1}{2}(\frac{r-a}{b-a} + \frac{r-c}{d-c}), & \text{if } a \le r \le b; \\ \frac{1}{2}(\frac{r-c}{d-c} + 1), & \text{if } b \le r \le d; \\ 1, & \text{if } r \ge d. \end{cases} \quad (1)$$

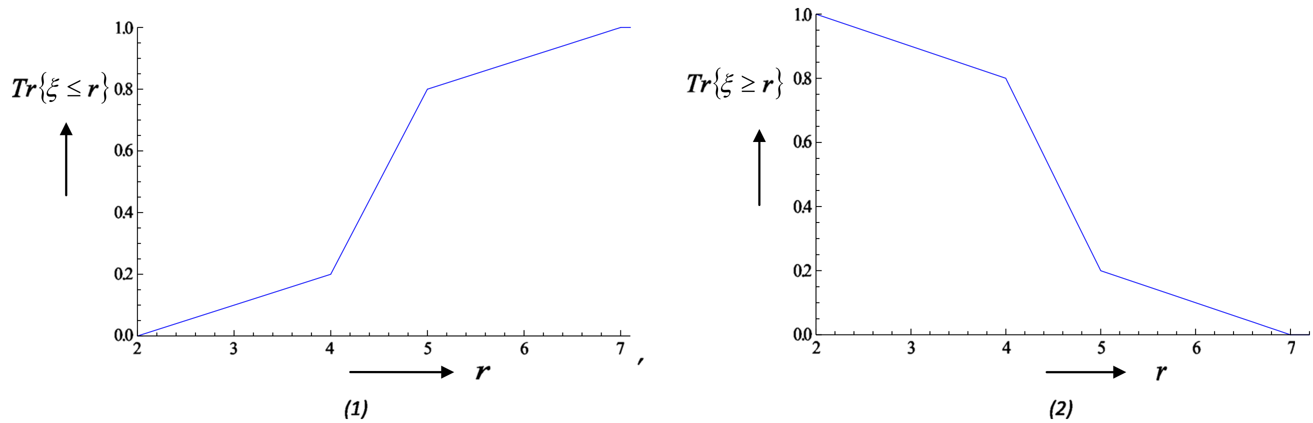

Fig. 1 The trust of the rough event defined in (1) $\xi \leq r$ and (2) $\xi \geq r$

$$Tr\{\xi \geq r\} = \begin{cases} 0, & \text{if } r \geq d; \\ \frac{d-r}{2(d-c)}, & \text{if } b \leq r \leq d; \\ \frac{1}{2}(\frac{d-r}{d-c} + \frac{b-r}{b-a}), & \text{if } a \leq r \leq b; \\ \frac{1}{2}(\frac{d-r}{d-c} + 1), & \text{if } c \leq r \leq a; \\ 1, & \text{if } r \leq c. \end{cases} \tag{2}$$

Rough variable $\xi = ([4, 5], [2, 7])$, $Tr\{\xi \leq r\}$ and $Tr\{\xi \geq r\}$ are depicted in Fig. 1.

For $\xi = ([a, b], [c, d])$, using Definition 2.1.4 and Eq. (2), α-optimistic value to ξ is obtained as

$$\xi_{sup}(\alpha) = \begin{cases} (1-2\alpha)d + 2\alpha c, & \text{if } \alpha \leq ((d-b)/2(d-c)); \\ 2(1-\alpha)d + (2\alpha-1)c, & \text{if } \alpha \geq ((2d-a-c)/2(d-c)); \\ \frac{d(b-a)+b(d-c)-2\alpha(b-a)(d-c)}{(b-a)+(d-c)}, & \text{otherwise.} \end{cases} \tag{3}$$

and using Eq. (1), α-pessimistic value to ξ is

$$\xi_{inf}(\alpha) = \begin{cases} (1-2\alpha)c + 2\alpha d, & \text{if } \alpha \leq ((a-c)/2(d-c)); \\ 2(1-\alpha)c + (2\alpha-1)d, & \text{if } \alpha \geq ((b+d-2c)/2(d-c)); \\ \frac{c(b-a)+a(d-c)+2\alpha(b-a)(d-c)}{(b-a)+(d-c)}, & \text{otherwise.} \end{cases} \tag{4}$$

The expected value of ξ, $E(\xi) = \frac{1}{4}(a + b + c + d)$.

2.2 Theorem 1

If $\xi = ([a, b], [c, d])$ be a rough variable with $c \leq a < b \leq d$, then for a predetermined α, $0 < \alpha \leq 1$, $Tr\{\xi \leq r\} \geq \alpha$ is equivalent to

(i) $(1-2\alpha)c + 2\alpha d \leq r,\ when\ \alpha \leq \frac{a-c}{2(d-c)}$,
(ii) $2(1-\alpha)c + (2\alpha-1)d \leq r,\ when\ \alpha \geq \frac{b+d-2c}{2(d-c)}$,
(iii) $\frac{c(b-a)+a(d-c)+2\alpha(b-a)(d-c)}{(b-a)+(d-c)} \leq r,\ otherwise.$

Proof For a given value r and $\xi = ([a, b], [c, d])$, $Tr\{\xi \leq r\}$ is given by (1).
Case I: For $c \leq r \leq a$ and predetermined α, from (1) we have

$$\begin{aligned} Tr\{\xi \leq r\} \geq \alpha &\Rightarrow \frac{r-c}{2(d-c)} \geq \alpha \\ &\Rightarrow (1-2\alpha)c + 2\alpha d \leq r. \end{aligned}$$

However, in this case, maximum possible value of $Tr\{\xi \leq r\}$ can be $((a-c)/2(d-c))$ and minimum possible value is 0 so that the value of α must be less or equal to $((a-c)/2(d-c))$.
Case II: For $b \leq r \leq d$ and predetermined α,

$$\begin{aligned} Tr\{\xi \leq r\} \geq \alpha &\Rightarrow \frac{1}{2}(\frac{r-c}{d-c}+1) \geq \alpha \\ &\Rightarrow 2(1-\alpha)c + (2\alpha-1)d \leq r \end{aligned}$$

In this case, maximum possible value of $Tr\{\xi \leq r\}$ can be 1 and minimum possible value is $\frac{1}{2}(\frac{b-c}{d-c} + 1)$ so that the value of α must be greater or equal to $\frac{1}{2}(\frac{b-c}{d-c} + 1)$, which implies $\alpha \geq \frac{b+d-2c}{2(d-c)}$.
Case III: For $a \leq r \leq b$ and predetermined α,

$$\begin{aligned} Tr\{\xi \leq r\} \geq \alpha &\Rightarrow \frac{1}{2}(\frac{r-a}{b-a} + \frac{r-c}{d-c}) \geq \alpha \\ &\Rightarrow \frac{c(b-a)+a(d-c)+2\alpha(b-a)(d-c)}{(b-a)+(d-c)} \leq r. \end{aligned}$$

In this case (i.e., $a \leq r \leq b$), minimum possible value of $Tr\{\xi \leq r\}$ can be $((a-c)/2(d-c))$ and maximum possible value is $\frac{1}{2}(1 + \frac{b-c}{d-c}) = \frac{b+d-2c}{2(d-c)}$ and hence the proof is complete. □

2.3 Theorem 2

If $\xi = ([a, b], [c, d])$ be a rough variable with $c \leq a < b \leq d$, then for a predetermined α, $0 < \alpha \leq 1$, $Tr\{\xi \geq r\} \geq \alpha$ is equivalent to

(i) $(1-2\alpha)d + 2\alpha c \geq r,\ when\ \alpha \leq \frac{d-b}{2(d-c)}$,
(ii) $2(1-\alpha)d + (2\alpha - 1)c \geq r,\ when\ \alpha \geq \frac{2d-a-c}{2(d-c)}$,
(iii) $\frac{d(b-a)+b(d-c)-2\alpha(b-a)(d-c)}{(b-a)+(d-c)} \geq r,\ otherwise.$

Proof Using expression of $Tr\{\xi \geq r\}$ as given in (2), the proof is similar to the proof of Theorem 1. □

3 Description of the problem and model formulation

In traditional STP, total transportation capacity of all types of conveyances is assumed to be utilized for all routes irrespective of the allocation of products in the routes. However, in many real transportation systems, different types of vehicles (e.g., trucks for road transportation, coaches for rail transportation, etc.) are to be booked and number of vehicles are determined according to the amount of products to be transported through a particular route. The difficulty, in this case, arises when the amount of allocated product is not sufficient to utilize the full capacity of the vehicle. As the unit transportation cost is generally determined on the total capacity of the vehicle, some extra cost is incurred if the vehicle is not filled up. To deal with these situations, we formulate a solid transportation model with vehicle capacity.

Suppose, q_k be the capacity of a single vehicle of kth type conveyance. Let z_{ijk} be the frequency (number of vehicles required) of conveyance k for transporting goods from source i to destination j and x_{ijk} (decision variable) be the corresponding amount of goods to be transported. Then z_{ijk} is also a decision variable which takes only positive integer or zero and if the vehicle capacity is q_k, then we have

$$x_{ijk} \leq z_{ijk} \cdot q_k.$$

For this type of vehicle transportation system, unit transportation cost depends upon the utilization of the capacity of the vehicle. Let us denote the route from source i to destination j through the conveyance k as $i-j-k$. Then for the route $i-j-k$, if the unit transportation cost c_{ijk} is determined on full utilization of the vehicle capacity q_k, then an extra cost (penalty) will be added if the capacity q_k is not fully utilized. Additional cost for deficit amount depends upon the corresponding transportation authority. Two cases may arise, either the authority does not compromise with variation of deficit amount and so a direct cost c_{ijk} is considered as the additional cost for unit deficit amount, or the authority agrees to compromise and fixes an additional cost for unit amount of deficit. In the first case, transportation cost of fully loaded vehicle will be charged whatever be the actual amount of transportation. To determine the additional cost, we first calculate the deficit amount of goods to the total capacity of the vehicles for each route. This can be done in two ways, either by calculating the deficit amount for $i-j-k$ route directly as $(z_{ijk} \cdot q_k - x_{ijk})$, or by calculating the empty ratio of vehicle of kth type conveyance for transporting goods from source i to destination j as follows:

$$d_{ijk} = \begin{cases} 0, & \text{if } \frac{x_{ijk}}{q_k} = [\frac{x_{ijk}}{q_k}]; \\ 1 - (\frac{x_{ijk}}{q_k} - [\frac{x_{ijk}}{q_k}]), & \text{otherwise.} \end{cases}$$

so that the deficit amount for $i-j-k$ route is given by $q_k \cdot d_{ijk}$. Now if u_{ijk} represents additional cost for unit amount of deficit from source i to destination j via conveyance k, then additional cost ϵ_{ijk} for this route is given by

$$\epsilon_{ijk} = u_{ijk}(z_{ijk} \cdot q_k - x_{ijk})\ or\ \epsilon_{ijk} = u_{ijk} \cdot q_k \cdot d_{ijk}.$$

The total additional (penalty) cost for the problem is

$$C(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \epsilon_{ijk}$$

and so the total transportation cost becomes

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (c_{ijk}\, x_{ijk} + \epsilon_{ijk}). \tag{5}$$

In many operational and planning models within the chemical, petroleum, gasoline and process industries, a common issue involves blending raw materials with varying attributes and concentration levels into homogeneous intermediate or end products. Blending raw materials affords an organization the opportunity to realize sizable cost savings while meeting demand for a set of final products, and satisfying predetermined quality requirements for each type of product. The inherent flexibility of the blending process can be exploited to optimize the allocation and transportation of raw materials to production facilities. So, STP with this blending incorporates an additional proportionality requirement on the quality of the product. Specifically, let p_i denotes the nominal quality (or purity) of product available from supplier (source) i, and let $p_j^{\min}$ denotes the minimum quality required at destination (consumer/demand point) j. Then, the additional constraint, which we refer to as a linear blending constraint, requires that the average quality of all products received by consumer j must be at least $p_j^{\min}$, where we assume that products received at a destination can be blended together to meet the requirement. Now the average quality of all the products received at destination j is as follows:

$$\frac{\sum_{i=1}^{m}\sum_{k=1}^{K} p_i\, x_{ijk}}{\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}}, \quad j=1,2,\ldots,n.$$

As $p_j^{\min}$ is the minimum quality of the product required at destination j, the constraint on the quality requirement of the product can be represented as given in (6).

$$\frac{\sum_{i=1}^{m}\sum_{k=1}^{K} p_i\, x_{ijk}}{\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}} \geq p_j^{\min}, \quad \forall\, j$$
$$\Rightarrow \sum_{i=1}^{m}\sum_{k=1}^{K} (p_i - p_j^{\min})x_{ijk} \geq 0, \quad \forall\, j. \tag{6}$$

Let each source i have minimum and maximum amounts of supplies of the given product, denoted by a_i^l and a_i^u, respectively. Similarly, each consumer j has a minimum and maximum demand for the product, denoted b_j^l and b_j^u, respectively. Finally, the mathematical model of the solid transportation problem with product blending (STPwB) is given below in (7)–(12).

$$\text{Min } Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} (c_{ijk}\, x_{ijk} + \epsilon_{ijk}) \tag{7}$$

$$\text{s.t. } \sum_{i=1}^{m}\sum_{k=1}^{K} (p_i - p_j^{\min})x_{ijk} \geq 0, \quad j=1,2,\ldots,n, \tag{8}$$

$$a_i^l \leq \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \leq a_i^u, \quad i=1,2,\ldots,m, \tag{9}$$

$$b_j^l \leq \sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \leq b_j^u, \quad j=1,2,\ldots,n, \tag{10}$$

$$x_{ijk} \leq z_{ijk}\cdot q_k, \quad i=1,2,\ldots,m;\; j=1,2,\ldots,n;\, k=1,2,\ldots,K \tag{11}$$

$$x_{ijk} \geq 0,\; z_{ijk} \in \mathbb{N}^0 \quad \forall\, i,j,k, \tag{12}$$

where $\mathbb{N}^0$ denotes the set of nonnegative integers.

4 STPwB with parameters as rough variables

Consider the unit transportation costs c_{ijk}, unit additional costs u_{ijk}, supplies a_i^l, a_i^u and demands b_j^l, b_j^u as rough variables for the model (7)–(12) and represented by $c_{ijk} = ([c_{ijk}^2, c_{ijk}^3], [c_{ijk}^1, c_{ijk}^4])$, $u_{ijk} = ([u_{ijk}^2, u_{ijk}^3], [u_{ijk}^1, u_{ijk}^4])$, $a_i^l = ([a_i^{l2}, a_i^{l3}], [a_i^{l1}, a_i^{l4}])$, $a_i^u = ([a_i^{u2}, a_i^{u3}], [a_i^{u1}, a_i^{u4}])$, $b_j^l = ([b_j^{l2}, b_j^{l3}], [b_j^{l1}, b_j^{l4}])$ and $b_j^u = ([b_j^{u2}, b_j^{u3}], [b_j^{u1}, b_j^{u4}])$, respectively, where $c_{ijk}^1 \leq c_{ijk}^2 < c_{ijk}^3 \leq c_{ijk}^4$, $u_{ijk}^1 \leq u_{ijk}^2 < u_{ijk}^3 \leq u_{ijk}^4$ and so on.

Since c_{ijk} and u_{ijk} are rough variables and $x_{ijk} \geq 0$ for all i, j, k, then the objective function of the model (7)–(12), i.e.,

$Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} (c_{ijk}\; x_{ijk} + \epsilon_{ijk})$, $\epsilon_{ijk} = u_{ijk}(z_{ijk}\cdot q_k - x_{ijk})$ becomes a rough variable defined as $Z = ([Z^2, Z^3], [Z^1, Z^4])$, where

$$Z^r = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} (c_{ijk}^r\, x_{ijk} + \epsilon_{ijk}^r), \quad r=1,2,3,4, \tag{13}$$

$$\epsilon_{ijk}^r = u_{ijk}^r(z_{ijk}\cdot q_k - x_{ijk}), \quad r=1,2,3,4.$$

Rough chance-constrained programming: In chance-constrained programming (CCP), the uncertain constraints are allowed to vary such that constraints must be satisfied at some chance (/confidence) level. Liu and Iwamura (1998) presented chance-constrained programming with fuzzy parameters. Yang and Liu (2007), Kundu et al. (2014) applied chance-constrained programming to solve solid transportation problem in fuzzy environment. Applying the idea of CCP technique, we formulate rough chance-constrained programming (CCP) using trust measure for the model (7)–(12) with rough parameters. Since the problem is a minimization problem, we minimize the smallest possible objective $\bar{Z}$ satisfying $Tr\{Z \leq \bar{Z}\} \geq \alpha$, where $\alpha \in (0, 1]$ is a specified trust (confidence) level, i.e., we minimize the α-pessimistic value $Z_{\inf}(\alpha)$ of Z. Now we formulate the rough CCP as follows:

$$\text{Min (Min } \bar{Z}) \tag{14}$$

$$\text{s.t. } Tr\{Z \leq \bar{Z}\} \geq \alpha, \tag{15}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} \left(p_i - p_j^{\min}\right) x_{ijk} \geq 0, \quad j=1,2,\ldots,n, \tag{16}$$

$$Tr\left\{\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \leq a_i^u\right\} \geq \alpha_i^u, \quad i=1,2,\ldots,m, \tag{17}$$

$$Tr\left\{\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \geq a_i^l\right\} \geq \alpha_i^l, \quad i=1,2,\ldots,m, \tag{18}$$

$$Tr\left\{\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \geq b_j^l\right\} \geq \beta_j^l, \quad j=1,2,\ldots,n, \tag{19}$$

$$Tr\left\{\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \leq b_j^u\right\} \geq \beta_j^u, \quad j=1,2,\ldots,n, \tag{20}$$

$$x_{ijk} \leq z_{ijk}\cdot q_k, \quad i=1,2,\ldots,m;\; j=1,2,\ldots,n; \quad k=1,2,\ldots,K, \tag{21}$$

$$x_{ijk} \geq 0,\; z_{ijk} \in \mathbb{N}^0\; \forall\, i,j,k, \tag{22}$$

where α_i^u, α_i^l, β_j^l and β_j^u are the predetermined trust levels of the source and destination constraints. Here $\mathbb{N}^0$ denotes the set of nonnegative integers.

4.1 Crisp equivalence

The objective function of the above CCP model, i.e., Min $\bar{Z}$ s.t. $Tr\{Z \leq \bar{Z}\} \geq \alpha$ is actually the α-pessimistic value $Z_{\inf}(\alpha)$ of Z and so equal to Z' (say) which is obtained using (4) as

$$Z' = Z_{inf}(\alpha) = \begin{cases} (1-2\alpha)Z^1 + 2\alpha Z^4, & \text{if } \alpha \leq \frac{Z^2-Z^1}{2(Z^4-Z^1)}; \\ 2(1-\alpha)Z^1 + (2\alpha-1)Z^4, & \text{if } \alpha \geq \frac{Z^3+Z^4-2Z^1}{2(Z^4-Z^1)}; \\ \frac{Z^1(Z^3-Z^2)+Z^2(Z^4-Z^1)+2\alpha(Z^3-Z^2)(Z^4-Z^1)}{(Z^3-Z^2)+(Z^4-Z^1)}, & \text{otherwise.} \end{cases} \tag{23}$$

In the constraint, given in (17), a_i^u represents a rough variable. Using Theorem 2 (Sect. 2.3), crisp form of this constraint, i.e., $Tr\{a_i^u \geq \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}\} \geq \alpha_i^u$ becomes

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \leq F_{a_i^u}, \quad i = 1, 2, \ldots, m, \tag{24}$$

where

$$F_{a_i^u} = \begin{cases} (1-2\alpha_i^u)a_i^{u4} + 2\alpha_i^u a_i^{u1}, & \text{if } \alpha_i^u \leq \frac{a_i^{u4}-a_i^{u3}}{2(a_i^{u4}-a_i^{u1})}; \\ 2(1-\alpha_i^u)a_i^{u4} + (2\alpha_i^u-1)a_i^{u1}, & \text{if } \alpha_i^u \geq \frac{2a_i^{u4}-a_i^{u2}-a_i^{u1}}{2(a_i^{u4}-a_i^{u1})}; \\ \frac{a_i^{u4}(a_i^{u3}-a_i^{u2})+a_i^{u3}(a_i^{u4}-a_i^{u1})-2\alpha_i^u(a_i^{u3}-a_i^{u2})(a_i^{u4}-a_i^{u1})}{(a_i^{u3}-a_i^{u2})+(a_i^{u4}-a_i^{u1})}, & \text{otherwise.} \end{cases} \tag{25}$$

In the constraint given in (18), a_i^l represents a rough variable. Using Theorem 1 (Sect. 2.2), crisp form of this constraint, i.e., $Tr\{a_i^l \leq \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}\} \geq \alpha_i^l$ becomes

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \geq F_{a_i^l}, \quad i = 1, 2, \ldots, m, \tag{26}$$

where

$$F_{a_i^l} = \begin{cases} (1-2\alpha_i^l)a_i^{l1} + 2\alpha_i^l a_i^{l4}, & \text{if } \alpha_i^l \leq \frac{a_i^{l2}-a_i^{l1}}{2(a_i^{l4}-a_i^{l1})}; \\ 2(1-\alpha_i^l)a_i^{l1} + (2\alpha_i^l-1)a_i^{l4}, & \text{if } \alpha_i^l \geq \frac{a_i^{l3}+a_i^{l4}-2a_i^{l1}}{2(a_i^{l4}-a_i^{l1})}; \\ \frac{a_i^{l1}(a_i^{l3}-a_i^{l2})+a_i^{l2}(a_i^{l4}-a_i^{l1})+2\alpha_i^l(a_i^{l3}-a_i^{l2})(a_i^{l4}-a_i^{l1})}{(a_i^{l3}-a_i^{l2})+(a_i^{l4}-a_i^{l1})}, & \text{otherwise.} \end{cases} \tag{27}$$

Similarly the crisp forms of the constraints (19) and (20), obtained using Theorem 1 and Theorem 2, are respectively as given below

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \geq F_{b_j^l}, \quad j = 1, 2, \ldots, n, \tag{28}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \leq F_{b_j^u}, \quad j = 1, 2, \ldots, n, \tag{29}$$

where

$$F_{b_j^l} = \begin{cases} (1-2\beta_j^l)b_j^{l1} + 2\beta_j^l b_j^{l1}, & \text{if } \beta_j^l \leq \frac{b_j^{l2}-b_j^{l1}}{2(b_j^{l4}-b_j^{l1})}; \\ 2(1-\beta_j^l)b_j^{l1} + (2\beta_j^l-1)b_j^{l4}, & \text{if } \beta_j^l \geq \frac{b_j^{l3}+b_j^{l4}-2b_j^{l1}}{2(b_j^{l4}-b_j^{l1})}; \\ \frac{b_j^{l1}(b_j^{l3}-b_j^{l2})+b_j^{l2}(b_j^{l4}-b_j^{l1})+2\beta_j^l(b_j^{l3}-b_j^{l2})(b_j^{l4}-b_j^{l1})}{(b_j^{l3}-b_j^{l2})+(b_j^{l4}-b_j^{l1})}, & \text{otherwise.} \end{cases} \tag{30}$$

$$F_{b_j^u} = \begin{cases} (1-2\beta_j^u)b_j^{u4} + 2\beta_j^u b_j^{u1}, & \text{if } \beta_j^u \le \frac{b_j^{u4}-b_j^{u3}}{2(b_j^{u4}-b_j^{u1})}; \\ 2(1-\beta_j^u)b_j^{u4} + (2\beta_j^u - 1)b_j^{u1}, & \text{if } \beta_j^u \ge \frac{2b_j^{u4}-b_j^{u2}-b_j^{u1}}{2(b_j^{u4}-b_j^{u1})}; \\ \frac{b_j^{u4}(b_j^{u3}-b_j^{u2})+b_j^{u3}(b_j^{u4}-b_j^{u1})-2\beta_j^u(b_j^{u3}-b_j^{u2})(b_j^{u4}-b_j^{u1})}{(b_j^{u3}-b_j^{u2})+(b_j^{u4}-b_j^{u1})}, & \text{otherwise.} \end{cases} \tag{31}$$

So crisp form of the CCP model (14)–(22) can be written as

$$\text{Min } Z' \tag{32}$$

$$\text{s.t.} \sum_{i=1}^{m}\sum_{k=1}^{K} (p_i - p_j^{\min})x_{ijk} \ge 0, \quad j = 1, 2, \ldots, n, \tag{33}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le F_{a_i^u}, \quad i = 1, 2, \ldots, m, \tag{34}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \ge F_{a_i^l}, \quad i = 1, 2, \ldots, m, \tag{35}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge F_{b_j^l}, \quad j = 1, 2, \ldots, n, \tag{36}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \le F_{b_j^u}, \quad j = 1, 2, \ldots, n, \tag{37}$$

$$x_{ijk} \le z_{ijk} \cdot q_k, \quad i = 1, \ldots, m;\ j = 1, \ldots, n;\ k = 1, \ldots, K, \tag{38}$$

$$x_{ijk} \ge 0,\ z_{ijk} \in \mathbb{N}^0 \quad \forall\, i, j, k. \tag{39}$$

5 Numerical experiment

Consider the STPwB model (7)–(12) with three sources ($i = 1, 2, 3$), three destinations ($j = 1, 2, 3$), and two types of conveyances ($k = 1, 2$). The unit transportation costs are rough variables and given in Tables 1 and 2, respectively, for two types of conveyances. The minimum and maximum availabilities at each source, and demand of each destination are represented by rough variables, which are given below.

$a_1^l = ([20, 22], [19, 23])$, $a_1^u = ([26, 27], [24, 28])$, $a_2^l = ([14, 15], [12, 16])$, $a_2^u = ([19, 20], [18, 21])$, $a_3^l = ([24, 25], [23, 27])$, $a_3^u = ([31, 34], [30, 36])$, $b_1^l = ([11, 13], [10, 14])$, $b_1^u = ([16, 18], [15, 19])$, $b_2^l = ([24, 26], [23, 27])$, $b_2^u = ([27, 28], [25, 30])$, $b_3^l = ([19, 20], [18, 21])$, $b_3^u = ([23, 24], [22, 26])$.

Capacities of each type of conveyances are $q_1 = 2.45$ and $q_2 = 3.78$. For convenience, let additional cost for unit deficit amount be $u_{ijk} = 0.8 \cdot c_{ijk}$.

The nominal quality (or purity) of product available from each supplier are given by $p_1 = 0.9$ (i.e., 90 % purity), $p_2 = 0.8$, $p_3 = 0.9$ and the minimum qualities required at each of the three destinations (consumer) are $p_1^{\min} = 0.85$, $p_2^{\min} = 0.85$ and $p_3^{\min} = 0.9$.

Now to solve the proposed rough CCP problem, as formulated in (14)–(22), with trust level $\alpha = 0.9$ for the objective function and $\alpha_i^l = \alpha_i^u = \beta_j^l = \beta_j^u = 0.9$, $i, j = 1, 2, 3$ for the respective constraints, we have corresponding deterministic form using (32)–(39), given below:

$$Min\ Z' \tag{40}$$

$$\text{s.t.} \sum_{i=1}^{3}\sum_{k=1}^{2} (p_i - p_j^{\min})x_{ijk} \ge 0, \quad j = 1, 2, 3, \tag{41}$$

$$\sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk} \le F_{a_i^u}, \quad i = 1, 2, 3, \tag{42}$$

$$\sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk} \ge F_{a_i^l}, \quad i = 1, 2, 3, \tag{43}$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{ijk} \ge F_{b_j^l}, \quad j = 1, 2, 3, \tag{44}$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{ijk} \le F_{b_j^u}, \quad j = 1, 2, 3, \tag{45}$$

$$x_{ijk} \le z_{ijk} \cdot q_k, \quad i = 1, 2, 3;\ j = 1, 2, 3;\ k = 1, 2, \tag{46}$$

$$x_{ijk} \ge 0,\ z_{ijk} \in \mathbb{N}^0 \quad \forall\, i, j, k, \tag{47}$$

Table 1 Unit transportation costs c_{ij1}

$i \backslash j$	1	2	3
1	([7,9], [6,10])	([10,11], [8,12])	([11,13], [10,12])
2	([6,8], [5,9])	([9,10], [7,11])	([5,7], [4,8])
3	([8,10], [7,11])	([13,15], [12,16])	([8,10], [7,11])

Table 2 Unit transportation costs c_{ij2}

$i \backslash j$	1	2	3
1	([10,12], [9,13])	([8,10], [7,11])	([12,14], [11,15])
2	([11,12], [9,13])	([6,8], [5,9])	([9,10], [7,11])
3	([11,12], [10,13])	([10,11], [9,12])	([8,9], [7,11])

Table 3 Optimum solution of the problem, defined in (40)–(47)

$x_{111} = 7.09$, $x_{211} = 3.86$, $x_{311} = 4.85$, $x_{331} = 2.45$, $x_{122} = 15.11$, $x_{222} = 11.34$, $x_{322} = 18.9$,
Min $Z' = 620.4556$, $z_{111} = 3$, $z_{211} = 2$, $z_{311} = 2$, $z_{331} = 1$, $z_{122} = 4$, $z_{222} = 3$, $z_{332} = 5$

where Z' is obtained using (23) as

$$Z' = \begin{cases} -0.8Z^1 + 1.8Z^4, & \text{if } 0.9 \leq ((Z^2 - Z^1)/2(Z^4 - Z^1)); \\ 0.2Z^1 + 0.8Z^4, & \text{if } 0.9 \geq ((Z^3 + Z^4 - 2Z^1)/2(Z^4 - Z^1)); \\ \frac{Z^1(Z^3-Z^2)+Z^2(Z^4-Z^1)+1.8(Z^3-Z^2)(Z^4-Z^1)}{(Z^3-Z^2)+(Z^4-Z^1)}, & \text{otherwise.} \end{cases}$$

$$Z^r = \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{2} (c^r_{ijk}\, x_{ijk} + \epsilon^r_{ijk}), \quad r = 1, 2, 3, 4,$$

$\epsilon^r_{ijk} = 0.8c^r_{ijk}(z_{ijk} \cdot q_k - x_{ijk})$, $r = 1, 2, 3, 4$, and the values of $F_{a^u_i}$, $F_{a^l_i}$, $F_{b^l_j}$ and $F_{b^u_j}$ for $i, j = 1, 2, 3$ are calculated using the equations (25), (27), (30) and (31), respectively, as $F_{a^u_1} = 24.8$, $F_{a^u_2} = 18.6$, $F_{a^u_3} = 31.07$, $F_{a^l_1} = 22.2$, $F_{a^l_2} = 15.2$, $F_{a^l_3} = 26.2$, $F_{b^l_1} = 13.2$, $F_{b^l_2} = 26.2$, $F_{b^l_3} = 20.4$, $F_{b^u_1} = 15.8$, $F_{b^u_2} = 26.83$ and $F_{b^u_3} = 22.8$.

Solving this problem we get the solution as presented in Table 3.

6 Conclusion and future extension

In this paper, we study a solid transportation problem with product blending. The presented model has importance in various ways, as it is applicable for transportation system where full vehicle capacities are to be used for transportation and also blending is a related issue in shipment of the products. The rough chance-constrained programming (RCCP) is formulated to solve this problem, where the parameters are used as rough variables. There is a lack of appropriate method in the literature to deal with constraints involving rough variables. The presented RCCP model can also be applied to solve many others decision making problems, like inventory control, supply chain, etc.

In problem formulation, we consider a single product having single attribute. Practically, there could be multiple products each with multiple attributes (e.g., purity, concentration levels, etc.) making the demands for multiple products with multiple qualities. Thus, the presented problem can be extended to a multi-item shipment problem with products having multiple attributes.

Acknowledgements Pradip Kundu sincerely acknowledges the support received from IISER Kolkata to carry out this research work.

Compliance with ethical standards

Conflict of interest None.

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