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Synchronization of delayed chaotic systems with parameter mismatches by using intermittent linear state feedback

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Abstract
This paper investigates the synchronization of coupled chaotic systems with time delay in the presence of parameter mismatches by using intermittent linear state feedback control. Quasi-synchronization criteria are obtained by means of a Lyapunov function and the differential inequality method. Numerical simulations on the chaotic systems are presented to demonstrate the effectiveness of the theoretical results.

Mathematics Subject Classification: 92A09, 34C15, 58F40

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In nonlinear science, chaos implies extreme sensitivity to initial conditions, which is known as the 'butterfly effect'. Accordingly, for a long time, to synchronize two chaotic systems was mistakenly considered to be impossible. However, since the pioneering work of Pecora and Carroll [1] on chaos control, the analysis of synchronization phenomena [1–32, 34, 36, 42–45] has been an active topic in nonlinear science, due to its many applications in secure communication [40], control theory, telecommunications, biological networks and artificial neural networks, etc. There are several types of synchronization of coupled chaotic oscillators that have been described theoretically and observed experimentally. The most commonly discussed configuration for the synchronization of chaotic systems consists of two parts: a master (drive) system and a slave (response) system. The slave system can be described as a duplicate of a part or the whole of the master system. So far, many effective methods have been proposed for synchronization of chaotic systems such as adaptive control [12, 41–43],

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fuzzy control [13], feedback control [9–11], switch control [5], intermittent control [3, 4, 6, 7] and impulsive control [16, 17].

In comparison with continuous control of chaos, the discontinuous control method, which includes impulsive control and intermittent control, has attracted more interest recently due to its easy implementation in engineering control. In some cases, as mentioned in [3, 6], it may be impossible to use only synchronization at all times and, moreover, to use intermittent control may prove to be more cost effective than using control at all times. Synchronization of chaotic systems without delay was investigated in [3, 4, 6, 7]. In our recent paper [20], we studied the synchronization of identically chaotic systems with delays using intermittent linear state feedback.

In this paper, we investigate the synchronization of non-identical systems in the presence of parameter mismatches. In reality, parameter mismatches between the drive system and the response system are unavoidable in practical synchronization implementations. Naturally, we need to consider the effect of parameter mismatches on chaos synchronization. In certain cases parameter mismatches are harmful to synchronization. In general, in the presence of parameter mismatches, the synchronization error does not decay to zero with time, but fluctuates about zero or even a non-zero mean value. It will even lead to the loss of synchronization [22–25]. Normally, the synchronization error does not approach zero asymptotically if drive-response systems are not identical. In other words, to achieve complete synchronization is impossible in the presence of parameter mismatches. However, it is important to know whether synchronization error between the transmitter and the receiver is bounded within a small region around zero. There have been some reports on synchronization in the presence of parameter mismatches [22–33]. Most of these studies focus only on the coupled systems *without time delay*, but the study of synchronization phenomena in time-delayed systems [38–40, 43, 44] is of critical importance since they are ubiquitous in nature, technology and society due to finite signal transmission times, switching speeds and memory effects, etc. However, it appears that in reality the relation between chaos synchronization in time-delayed systems and parameter mismatches is quite intricate and complex [22]. In this paper, we study the time-delayed chaotic systems in the presence of parameter mismatch using intermittent control. Some criteria for synchronization of the drive-response chaotic systems with delays up to a relatively small error bound are derived by means of a Lyapunov function, differential inequality and linear matrix inequality. In addition, numerical simulations are presented to validate the effectiveness of the theoretical results.

The rest of the paper is organized as follows. In the next section, the problem to be studied is formulated and some preliminaries are presented. In section 3, some criteria for quasi-synchronization for the delayed chaotic systems in the presence of parameter mismatch are obtained by intermittent control. In section 4, two numerical examples are given to demonstrate the effectiveness of this method by applying it to several well-known chaotic systems with time-delays. Finally, some conclusions are drawn in section 5.

2. Problem formulation and preliminaries

Consider a class of chaotic systems with delay:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = A_1 x(t) + B_1 f(x(t)) + C_1 g(x(t-\tau)), & t > 0 \\ x(t) = \phi(t), & -\tau \leqslant t \leqslant 0. \end{cases} \tag{1}$$

Using the intermittent feedback control to synchronize system (1), the slave (response) system is designed as

$$\begin{cases} \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = A_2 y(t) + B_2 f(y(t)) + C_2 g(x(t-\tau)) + K(t)(x(t) - y(t)), & t > 0 \\ y(t) = \psi(t), & -\tau \leqslant t \leqslant 0, \end{cases} \tag{2}$$

where $x, y \in R^n$ are the state vectors of systems (1) and (2) respectively, $f, g : R^n \to R^n$ are nonlinear functions satisfying $f(0) = 0$ and $g(0) = 0$, τ is the time delay and $K(t)$ is the intermittent linear state feedback control gain defined as follows:

$$K(t) = \begin{cases} K & k\omega \leqslant t \leqslant k\omega + \delta, \\ 0 & k\omega + \delta < t \leqslant (k+1)\omega, \end{cases}$$

where $K \in R^{n\times n}$ is a constant control gain, $\omega > 0$ is the control period and $\delta > 0$ is called the control width (control duration). $A_1, A_2, B_1, B_2, C_1, C_2 \in R^{n\times n}$, in the case $A_1 \neq A_2$ or $B_1 \neq B_2$ or $C_1 \neq C_2$; in other words, there exist parameter mismatches between the drive and the response systems. In this paper, we consider the synchronization between chaotic system (1) and system (2) in the presence of parameter mismatches by means of intermittent linear feedback control. Let $\Delta A = A_2 - A_1$, $\Delta B = B_2 - B_1$ and $\Delta C = C_2 - C_1$ denote the parameter mismatch errors.

Let $e(t) = y(t) - x(t)$ be the synchronization error between the states of drive system (1) and response system (2). Then, for $t \in (0, \infty)$, we have the following:

$$\begin{aligned} \frac{\mathrm{d}e(t)}{\mathrm{d}t} &= A_2 y + B_2 f(y) + C_2 g(y(t-\tau)) + K(t)(x(t) - y(t)) - (A_1 x + B_1 f(x) \\ &\quad + C_1 g(x(t-\tau))) = A_2 y - A_2 x + A_2 x - A_1 x + B_2 f(y) - B_2 f(x) + B_2 f(x) \\ &\quad - B_1 f(x) + C_2 g(y(t-\tau)) - C_2 g(x(t-\tau)) + C_2 g(x(t-\tau)) \\ &\quad - C_1 g(x(t-\tau)) + K(t)(x(t) - y(t)) = A_2 e(t) + (A_2 - A_1)x \\ &\quad + B_2(f(y) - f(x)) + (B_2 - B_1) f(x) \\ &\quad + C_2(g(y(t-\tau)) - g(x(t-\tau))) + (C_2 - C_1) g(x(t-\tau)) - K(t)e(t). \end{aligned}$$

Namely, the error system is governed by the following:

$$\begin{cases} \dfrac{\mathrm{d}e(t)}{\mathrm{d}t} = A_2 e(t) + \Delta A x & k\omega \leqslant t \leqslant k\omega + \delta \\ + B_2(f(y) - f(x)) + \Delta B f(x) + C_2(g(y(t-\tau)) & \\ \quad - g(x(t-\tau))) + \Delta C g(x(t-\tau)) - K e(t); & \\ \dfrac{\mathrm{d}e(t)}{\mathrm{d}t} = A_2 e(t) + \Delta A x + B_2(f(y) - f(x)) + \Delta B f(x) & k\omega + \delta < t \leqslant (k+1)\omega \\ \quad + C_2(g(y(t-\tau)) - g(x(t-\tau))) + \Delta C g(x(t-\tau)). & \end{cases} \tag{3}$$

It is a differential equation with discontinuous terms. In the presence of parameter mismatches, the origin $e = 0$ is not an equilibrium point of the error system (3), so the complete synchronization is impossible. However, it is possible to synchronize the drive-response systems up to a relatively small error bound which is dependent on the differences in the parameters between the two systems. In this paper, we investigate two delayed chaotic systems in the presence of parameter mismatches in terms of the synchronization with error bound ε using the intermittent control.

We adopt the uniform synchronization with error bound in [29] to the non-identical chaotic systems with delay.

Definition 1. *The synchronization schemes (1) and (2) uniformly synchronize with error bound* ε *if there exist* $\delta_0 > 0$ *and* $T \geqslant 0$ *such that if* $||y(0) - x(0)||_\tau \leqslant \delta_0$ *then* $||y(t) - x(t)|| \leqslant \varepsilon$ *for all* $t \geqslant T$, *where* $||y(0) - x(0)||_\tau = \max\limits_{-\tau \leqslant s \leqslant 0} ||y(s) - x(s)||$.

Definition 2 ([32]). *Let* Ω *denote a region of interest in the phase space that contains the chaotic attractor of system (1). The synchronization schemes (1) and (2) are said to be uniformly quasi-synchronized with error bound* $\varepsilon > 0$ *if there exists a* $T \geqslant t_0$ *such that* $||y(t) - x(t)|| \leqslant \varepsilon$ *for all* $t \geqslant T$ *starting from any initial values* $x(t_0) \in \Omega$ *and* $y(t_0) \in \Omega$.

The uniform synchronization concept given in definition 1 is local since it depends on the initial values, while the quasi-synchronization concept given in definition 2 is global since it does not depend on the initial value. In this paper, synchronization results are in terms of quasi-synchronization.

In this paper, we assume that $f, g : R^n \to R^n$ are Lipschitz continuous functions: there exist positive constants L_f, L_g such that, for all $x, y \in R^n$,

$$\begin{aligned} ||f(x) - f(y)|| &\leqslant L_f ||x - y||, \\ ||g(x) - g(y)|| &\leqslant L_g ||x - y||. \end{aligned} \tag{4}$$

The following two cited lemmas on matrix inequalities will be used later.

Lemma 1 (Sanchez and Perez [37]). *For any vectors* $x, y \in R^n$ *and a positive-definite matrix* $Q \in R^{n\times n}$, *the following matrix inequality holds:*

$$2x^T y \leqslant x^T Q x + y^T Q^{-1} y.$$

Lemma 2 ([38]). *The following LMI:*

$$\begin{bmatrix} Q & S \\ S^{\mathrm{T}} & R \end{bmatrix} > 0$$

is equivalent to one of the following:

(1) $R > 0, Q - SR^{-1}S^{\mathrm{T}} > 0$;
(2) $Q > 0, R - SQ^{-1}S^{\mathrm{T}} > 0$.

Throughout this paper, let $P > 0 (<0, \leqslant 0, \geqslant 0)$ denote a symmetrical positive (negative, semi-negative, semi-positive) definite matrix P and $P^{\mathrm{T}}, \lambda_{\mathrm{M(m)}}(P)$ be the transpose and the maximum (minimum) eigenvalue of a square matrix P, respectively. The vector (or matrix) norm is taken to be Euclidian, denoted by $\|\cdot\|$. For function $y(t)$, $|y(t)|_\tau = \max\limits_{t-\tau \leqslant s \leqslant t} |y(s)|$.

3. Criteria for synchronization

To derive criteria for the synchronization of chaotic systems (1) and (2) with parameter mismatches, first we obtain the following three lemmas that play critical roles in the proof of the main results.

Lemma 3. *If the non-negative function* $y(t)$, $t \in [t_0 - \tau, +\infty)$ *satisfies the following*

$$y'(t) < ay(t) + by(t - \tau) + \varepsilon, \qquad t \in [t_0, +\infty), \tag{5}$$

where a, b *are positive, then the following holds:*

$$y(t) \leqslant \left(|y(t_0)|_\tau + \frac{\varepsilon}{a+b}\right) e^{(a+b)(t-t_0)} - \frac{\varepsilon}{a+b}, \qquad \text{for } t \in [t_0, +\infty),$$

where $|y(t_0)|_\tau = \max\limits_{t_0-\tau \leqslant s \leqslant t_0} |y(s)|$.

Proof. Let $z(t) = (|y(t_0)|_\tau + \varepsilon/(a+b))\mathrm{e}^{(a+b)(t-t_0)} - \varepsilon/(a+b)$ for $t \in [t_0, +\infty)$. We just need to prove $y(t) \leqslant z(t)$ for $t \in [t_0, +\infty)$, or $y(t) - z(t) \leqslant 0$ for $t \in [t_0, +\infty)$. Suppose it is not true, there exists a $t_1 \in (t_0, +\infty)$ such that

$$\begin{aligned} &y(t) - z(t) \leqslant 0, t \in (t_0, t_1],\\ &z(t_1) - y(t_1) = 0,\\ &y'(t_1) - z'(t_1) \geqslant 0. \end{aligned} \tag{6}$$

It is clear that for any $t \in (t_0, t_1]$, $y(t-\tau) \leqslant z(t_1)$ using the fact that $z(t)$ is increasing on $[t_0, t_1]$, since in the case $t - \tau \in [t_0 - \tau, t_0]$ $\quad y(t-\tau) \leqslant |y(t_0)|_\tau = z(t_0) \leqslant z(t_1)$, the other case $t - \tau \in [t_0, t_1]$, $y(t-\tau) \leqslant z(t-\tau) \leqslant z(t) \leqslant z(t_1)$.

By condition (5), we are able to evaluate and estimate $y'(t_1) - z'(t_1)$.

$$\begin{aligned} &y'(t_1) - z'(t_1)\\ &< ay(t_1) + by(t_1-\tau) + \varepsilon - (a+b)\left(|y(t_0)|_\tau + \frac{\varepsilon}{a+b}\right)\mathrm{e}^{(a+b)(t-t_0)}\\ &\leqslant (a+b)z(t_1) - (a+b)\left(|y(t_0)|_\tau + \frac{\varepsilon}{a+b}\right)\mathrm{e}^{(a+b)(t-t_0)} + \varepsilon\\ &\leqslant (a+b)z(t_1) - (a+b)\left(z(t_1) + \frac{\varepsilon}{a+b}\right) + \varepsilon\\ &= 0 \end{aligned}$$

This is contradictory to (6). Thus, $y(t) \leqslant z(t)$ for $t \in [t_0, +\infty)$**.** We have completed the proof.

Lemma 4. . *Suppose that function $y(t)$ is non-negative when $t \in (t_0 - \tau, \infty)$ and satisfies the following*

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} \leqslant -ay(t) + by(t-\tau) + \varepsilon, \qquad t \geqslant t_0, \tag{7}$$

where a, b, ε are positive constants, and $a > b$. Then we have the following inequality

$$y(t) \leqslant |y(t_0)|_\tau \mathrm{e}^{-r(t-t_0)} + \frac{\varepsilon}{r}, \qquad t \geqslant t_0, \tag{8}$$

where $|y(t_0)|_\tau = \max\limits_{t_0-\tau\leqslant s\leqslant t_0} |y(s)|$and r is the unique positive solution of

$$-r = -a_1 + b\mathrm{e}^{r\tau}. \tag{9}$$

Proof. Let $f(r) = -a + b\mathrm{e}^{r\tau} + r$. Since $f(0) = -a + b < 0$, $\lim\limits_{r\to+\infty} f(r) = +\infty$ and for positive r, $f'(r) = br\mathrm{e}^{r\tau} + 1 > 0$, equation (9) has a unique positive solution.

Let $z(t) = |y(t_0)|_\tau \mathrm{e}^{-r(t-t_0)} + \varepsilon/r$, $t \geqslant t_0 - \tau$. We need to prove that $y(t) \leqslant z(t)$, $t \geqslant t_0$. It is clear that $y(t) < z(t)$, $t \in [t_0 - \tau, t_0]$. Assume it is not true. Since $y(t_0) - z(t_0) < 0$, there exists a $t_1 \geqslant t_0$ such that $y(t) - z(t) < 0$, $t_0 \leqslant t < t_1$, and $y(t_1) - z(t_1) = 0$, while $y'(t_1) - z'(t_1) \geqslant 0$.

In the following, we evaluate $y'(t_1) - z'(t_1)$

$$\begin{aligned} &y'(t_1) - z'(t_1)\\ &\leqslant -ay(t_1) + by(t_1-\tau) + \varepsilon - z'(t_1)\\ &< -az(t_1) + bz(t_1-\tau) + \varepsilon - z'(t_1) \end{aligned}$$

$$= -a\left(|y(t_0)|_\tau \mathrm{e}^{-r(t_1-t_0)} + \frac{\varepsilon}{r}\right) + b\left(|y(t_0)|_\tau \mathrm{e}^{-r(t_1-t_0-\tau)} + \frac{\varepsilon}{r}\right) + \varepsilon - z'(t_1)$$

$$= -a|y(t_0)|_\tau \mathrm{e}^{-r(t_1-t_0)} + b|y(t_0)|_\tau \mathrm{e}^{-r(t_1-t_0-\tau)} - a\frac{\varepsilon}{r} + b\frac{\varepsilon}{r} + \varepsilon - z'(t_1)$$

$$= |y(t_0)|_\tau \mathrm{e}^{-r(t_1-t_0)}(-a + b\mathrm{e}^{r\tau}) - a\frac{\varepsilon}{r} + b\frac{\varepsilon}{r} + \varepsilon - z'(t_1)$$

$$= |y(t_0)|_\tau \mathrm{e}^{-r(t_1-t_0)}(-r) - a\frac{\varepsilon}{r} + b\frac{\varepsilon}{r} + \varepsilon - z'(t_1)$$

$$= z'(t_1) - a\frac{\varepsilon}{r} + b\frac{\varepsilon}{r} + \varepsilon - z'(t_1)$$

$$\leqslant z'(t_1) + (-a + b\mathrm{e}^{r\tau})\frac{\varepsilon}{r} + \varepsilon - z'(t_1)$$

$$= z'(t_1) + (-r)\frac{\varepsilon}{r} + \varepsilon - z'(t_1)$$

$$= 0.$$

Thus, we have $y'(t_1) - z'(t_1) < 0$, which is contradictory to $y'(t_1) - z'(t_1) \geqslant 0$. Therefore, we have $y(t) \leqslant z(t), t \geqslant t_0$ and the proof of the lemma is complete.

Lemma 5. *If non-negative function $y(t)$ satisfies the following conditions:*

$$y(t) \leqslant |y(k\omega)|_\tau \mathrm{e}^{-r(t-k\omega)} + \alpha, \qquad \text{for } k\omega \leqslant t \leqslant k\omega + \delta, \tag{10}$$

and

$$y(t) \leqslant (|y(k\omega + \delta)|_\tau + \beta)\mathrm{e}^{s(t-k\omega-\delta)} - \beta, \qquad \text{for } k\omega + \delta < t \leqslant (k+1)\omega, \tag{11}$$

also, $\rho = \rho_1 - \rho_2 > 0$, where $\rho_1 = r(\delta - \tau)$, $\rho_2 = s(\omega - \delta)$, then the following inequality holds:

$$y(t) \leqslant |y(0)|_\tau \mathrm{e}^{\rho}\mathrm{e}^{-(\rho/\omega)t} + \frac{\upsilon}{1 - \mathrm{e}^{-\rho}} + \alpha, \qquad \text{for } t \geqslant 0, \text{ where } \upsilon = (\alpha + \beta)\mathrm{e}^{\rho_2} - \beta.$$

Proof. By (10), we are able to obtain the following:

$$|y(\delta)|_\tau$$
$$= \max_{\delta-\tau\leqslant t\leqslant\delta} y(t)$$
$$\leqslant \max_{\delta-\tau\leqslant t\leqslant\delta} (|y(0)|_\tau \mathrm{e}^{-rt} + \alpha)$$
$$= |y(0)|_\tau \mathrm{e}^{-r(\delta-\tau)} + \alpha$$
$$= |y(0)|_\tau \mathrm{e}^{-\rho_1} + \alpha.$$

By (11) and noticing that $(|y(k\omega + \delta)|_\tau + \beta)\mathrm{e}^{s(t-k\omega-\delta)} - \beta$ is increasing in the interval $[k\omega + \delta, (k+1)\omega]$, we have the following:

$$|y(\omega)|_\tau$$
$$= \max_{\omega-\tau\leqslant t\leqslant\omega} y(t)$$
$$\leqslant \max\{|y(\delta)|_\tau, \max_{\delta\leqslant t\leqslant\omega} (|y(\delta)|_\tau + \beta)\mathrm{e}^{s(t-\delta)} - \beta\}$$
$$= (|y(\delta)|_\tau + \beta)\mathrm{e}^{s(\omega-\delta)} - \beta$$
$$= (|y(\delta)|_\tau + \beta)\mathrm{e}^{\rho_2} - \beta.$$

By the above estimations on $|y(\delta)|_\tau$, $|y(\omega)|_\tau$, we have the following:

$$\begin{aligned}|y(\omega)|_\tau &\leqslant [(|y(0)|_\tau e^{-\rho_1} + \alpha) + \beta]e^{\rho_2} - \beta\\ &\leqslant |y(0)|_\tau e^{-\rho_1+\rho_2} + (\alpha+\beta)e^{\rho_2} - \beta.\end{aligned}$$

Namely,

$$|y(\omega)|_\tau \leqslant |y(0)|_\tau e^{-\rho_1+\rho_2} + \upsilon,$$

Similarly,

$$\begin{aligned}|y(2\omega)|_\tau &\leqslant [((|y(0)|_\tau e^{-(\rho_1-\rho_2)} + \upsilon)e^{-\rho_1} + \alpha) + \beta]e^{\rho_2} - \beta\\ &\leqslant |y(0)|_\tau e^{-2(\rho_1-\rho_2)} + e^{-(\rho_1-\rho_2)}\upsilon + \upsilon.\end{aligned}$$

By mathematical induction, we have the following:

$$\begin{aligned}|y(k\omega)|_\tau \leqslant{}& |y(0)|_\tau e^{-k(\rho_1-\rho_2)} + \upsilon[1 + e^{-(\rho_1-\rho_2)}\\ &+\cdots+e^{-(k-2)(\rho_1-\rho_2)} + e^{-(k-1)(\rho_1-\rho_2)}]\end{aligned}$$

for any non-negative integers.

For any $t > 0$, there is a non-negative k, such that $k\omega < t \leqslant (k+1)\omega$. When $t \in (k\omega, k\omega+\delta]$, by condition (10) and the above inequality,

$$\begin{aligned}y(t) &\leqslant |y(k\omega)|_\tau e^{-r(t-k\omega)} + \alpha\\ &\leqslant (|y(0)|_\tau e^{-k(\rho_1-\rho_2)} + \upsilon[1 + e^{-(\rho_1-\rho_2)}\\ &\quad +\cdots+e^{-(k-2)(\rho_1-\rho_2)} + e^{-(k-1)(\rho_1-\rho_2)}])e^{-r(t-k\omega)} + \alpha\\ &\leqslant |y(0)|_\tau e^{-k(\rho_1-\rho_2)} + \frac{\upsilon}{1-e^{-(\rho_1-\rho_2)}} + \alpha\\ &= |y(0)|_\tau e^{\rho} e^{-(k+1)\omega\times\rho/\omega} + \frac{\upsilon}{1-e^{-(\rho_1-\rho_2)}} + \alpha\\ &\leqslant |y(0)|_\tau e^{\rho} e^{-(\rho/\omega)t} + \frac{\upsilon}{1-e^{-\rho}} + \alpha.\end{aligned}$$

Thus, it is clear that when $t \in (k\omega, k\omega+\delta]$, the conclusion holds. Also, from the first part of the above inequality, we have

$$\begin{aligned}|y(k\omega+\delta)|_\tau &\leqslant \max_{k\omega+\delta-\tau\leqslant t\leqslant k\omega+\delta}\{|y(k\omega)|_\tau e^{-r(t-k\omega)} + \alpha\}\\ &= |y(k\omega)|_\tau e^{-\rho_1} + \alpha\\ &\leqslant (|y(0)|_\tau e^{-k(\rho_1-\rho_2)} + \upsilon[1 + e^{-(\rho_1-\rho_2)}\\ &\quad +\cdots+e^{-(k-2)(\rho_1-\rho_2)} + e^{-(k-1)(\rho_1-\rho_2)}])e^{-\rho_1} + \alpha.\end{aligned}$$

When $t \in (k\omega+\delta, (k+1)\omega]$, by condition (11) and the above estimation,

$$\begin{aligned}y(t) &\leqslant (|y(k\omega+\delta)|_\tau + \beta)e^{s(t-k\omega-\delta)} - \beta\\ &\leqslant (((|y(0)|_\tau e^{-k(\rho_1-\rho_2)} + \upsilon[1 + e^{-(\rho_1-\rho_2)}\\ &\quad +\cdots+e^{-(k-1)(\rho_1-\rho_2)}])e^{-\rho_1} + \alpha) + \beta)e^{s(t-k\omega-\delta)} - \beta\\ &\leqslant (((|y(0)|_\tau e^{-k(\rho_1-\rho_2)} + \upsilon[1 + e^{-(\rho_1-\rho_2)}\\ &\quad +\cdots+e^{-(k-1)(\rho_1-\rho_2)}])e^{-\rho_1} + \alpha) + \beta)e^{\rho_2} - \beta\end{aligned}$$

$$= |y(0)|_\tau \mathrm{e}^{-(k+1)(\rho_1-\rho_2)} + \upsilon[1 + \mathrm{e}^{-(\rho_1-\rho_2)} + \cdots + \mathrm{e}^{-k(\rho_1-\rho_2)}]$$

$$\leqslant |y(0)|_\tau \mathrm{e}^{-(k+1)(\rho_1-\rho_2)} + \frac{\upsilon}{1-\mathrm{e}^{-(\rho_1-\rho_2)}}$$

$$\leqslant |y(0)|_\tau \mathrm{e}^{-\frac{\rho}{\omega}t} + \frac{\upsilon}{1-\mathrm{e}^{-\rho}}.$$

This implies that the conclusion holds for $t \in (k\omega + \delta, (k+1)\omega]$. The proof is complete.

In the following, we present the criteria of synchronization of the chaotic systems with parameter mismatches by intermittent feedback control. Let us consider the following positive definite quadratic Lyapunov function:

$$V(e(t)) = e(t)^{\mathrm{T}} Pe(t), \qquad P = P^{\mathrm{T}} > 0,$$

where P is a positive symmetric definite matrix.

Theorem 1. *Suppose that $\Psi = \{x \in R^n |\ ||x|| \leqslant \psi\}$ is the phase range of the interested chaotic systems, and the parameter-mismatches satisfy $||\Delta A|| + L_f||\Delta B|| + L_g||\Delta C|| \leqslant \varepsilon$. Also, suppose that there exist a symmetric positive definite matrix P >0 and positive scalars $\mu_3 > \mu_1 > \mu_2, \gamma, \mu$, such that the following conditions hold:*

(a) $\Gamma_1 = PA_2 + A_2P - K^{\mathrm{T}}P - PK + \gamma PB_2B_2^{\mathrm{T}}P + \gamma^{-1}L_f^2 I + \eta PC_2C_2^{\mathrm{T}}P + \lambda P^2 + \mu_1 P < 0,$

(b) $\Gamma_2 = \eta^{-1}L_g^2 I - \mu_2 P \leqslant 0,$

(c) $\Gamma_3 = PA_2 + A_2P + \gamma PB_2B_2^{\mathrm{T}}P + \gamma^{-1}L_f^2 I + \eta PC_2C_2^{\mathrm{T}}P + \lambda P^2 - (\mu_3 - \mu_1)P \leqslant 0$

(d) $\rho = \rho_1 - \rho_2 = r(\delta - \tau) - (\mu_3 - \mu_1 + \mu_2)(\omega - \delta) > 0,$

where r is the unique positive solution of $-r = -\mu_1 + \mu_2 \mathrm{e}^{r\tau}$, then the synchronization error system (3) converges exponentially to a small region D containing the origin, where

$$D = \left\{ e \in R^n |\, ||e|| \leqslant \sqrt{\frac{\varepsilon_1}{\lambda_m(P)}} \right\}, \qquad \text{in which } \varepsilon_1 = \frac{\upsilon}{1-\mathrm{e}^{-\rho}} + \alpha$$

and

$$\upsilon = (\alpha + \beta)\mathrm{e}^{\rho_2} - \beta,$$

where

$$\alpha = \frac{\lambda^{-1}\psi^2\varepsilon^2}{r}, \qquad \beta = \frac{\lambda^{-1}\psi^2\varepsilon^2}{\mu_1 - \mu_2 + \mu_3}.$$

Consequently, the quasi-synchronization between system (1) and system (2) is achieved with error bound $\varepsilon_2 + \sqrt{\varepsilon_1/(\lambda_{\mathrm{m}}(P))}$ for any arbitrary small positive number ε_2.

Proof. Consider the following Lyapunov-like function:

$$V(t) = e(t)^{\mathrm{T}} Pe(t). \tag{12}$$

The derivative of V with respect to time along the solution of equation (3)

(i) when $k\omega \leqslant t \leqslant k\omega + \delta$,
where k is a non-negative integer, is

$$\dot{V}(t) = 2e(t)^{\mathrm{T}} P[A_2 e(t) + \Delta A x + B_2(f(y) - f(x)) + \Delta B f(x) + C_2(g(y(t-\tau)) - g(x(t-\tau))) + \Delta C g(x(t-\tau)) - K e(t)],$$

$$\begin{aligned}\dot{V}(t) = {} & e(t)^{\mathrm{T}}[P A_2 + A_2 P - K^T P - P K] e(t) \\ & + 2e(t)^{\mathrm{T}} P B_2(f(y(t)) - f(x(t))) \\ & + 2e(t)^{\mathrm{T}} P C_2(g(y(t-\tau)) - g(x(t-\tau))) \\ & + 2e(t)^{\mathrm{T}} P[\Delta A x + \Delta B f(x(t)) + \Delta C g(x(t-\tau))]. \end{aligned} \tag{13}$$

Using lemma 1 and (5), we have the following estimation:

$$\begin{aligned} & 2e(t)^{\mathrm{T}} P B_2(f(y(t)) - f(x(t))) \\ & \leqslant \gamma e(t)^{\mathrm{T}} P B_2 B_2^{\mathrm{T}} P e(t) + \gamma^{-1}(f(y(t)) - f(x(t)))^{\mathrm{T}}(f(y(t)) - f(x(t))) \\ & \leqslant \gamma e(t)^{\mathrm{T}} P B_2 B_2^{\mathrm{T}} P e(t) + \gamma^{-1} L_f^2 e(t)^{\mathrm{T}} e(t) \\ & = e(t)^{\mathrm{T}}(\gamma P B_2 B_2^{\mathrm{T}} P + \gamma^{-1} L_f^2 I) e(t). \end{aligned} \tag{14}$$

Similarly, we have the estimation for another term: $2e(t)^{\mathrm{T}} P C_2(g(y(t-\tau)) - g(x(t-\tau)))$,

$$\begin{aligned} & 2e(t)^{\mathrm{T}} P C_2(g(y(t-\tau)) - g(x(t-\tau))) \\ & \leqslant \eta e(t)^{\mathrm{T}} P C_2 C_2^{\mathrm{T}} P e(t) + \eta^{-1} L_g^2 e(t-\tau)^T e(t-\tau). \end{aligned} \tag{15}$$

Also, we are able to get the following inequality using lemma 1 and conditions for this theorem:

$$\begin{aligned} & 2e(t)^{\mathrm{T}} P[\Delta A x + \Delta B f(x(t)) + \Delta C g(x(t-\tau))] \\ & \leqslant \lambda e(t)^{\mathrm{T}} P^2 e(t) + \lambda^{-1}[\Delta A x + \Delta B f(x(t)) + \Delta C g(x(t-\tau))]^{\mathrm{T}} \\ & \times[\Delta A x + \Delta B f(x(t)) + \Delta C g(x(t-\tau))] \\ & \leqslant \lambda e(t)^{\mathrm{T}} P^2 e(t) + \lambda^{-1}[(||\Delta A|| + ||\Delta B|| L_f + ||\Delta C|| L_g)\psi]^2 \\ & \leqslant \lambda e(t)^{\mathrm{T}} P^2 e(t) + \lambda^{-1}\psi^2\varepsilon^2. \end{aligned} \tag{16}$$

Substituting these into equation (13), we have the following:

$$\begin{aligned} & \dot{V}(t) \\ & \leqslant e(t)^{\mathrm{T}}[P A_2 + A_2 P - K^{\mathrm{T}} P - P K] e(t) \\ & \quad + e(t)^{\mathrm{T}}(\gamma P B_2 B_2^{\mathrm{T}} P + \gamma^{-1} L_f^2 I) e(t) \\ & \quad + \eta e(t)^{\mathrm{T}} P C_2 C_2^{\mathrm{T}} P e(t) + \eta^{-1} L_g^2 e(t-\tau)^{\mathrm{T}} e(t-\tau) \\ & \quad + \lambda e(t)^{\mathrm{T}} P^2 e(t) + \lambda^{-1}\psi^2\varepsilon^2 \\ & = e(t)^{\mathrm{T}}[P A_2 + A_2 P - K^{\mathrm{T}} P - P K + \gamma P B_2 B_2^{\mathrm{T}} P + \gamma^{-1} L_f^2 I \\ & \quad + \eta P C_2 C_2^{\mathrm{T}} P + \lambda P^2] e(t) + \eta^{-1} L_g^2 e(t-\tau)^{\mathrm{T}} e(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2 \\ & = e(t)^{\mathrm{T}}[P A_2 + A_2 P - K^{\mathrm{T}} P - P K + \gamma P B_2 B_2^{\mathrm{T}} P + \gamma^{-1} L_f^2 I \\ & \quad + \eta P C_2 C_2^{\mathrm{T}} P + \lambda P^2 + \mu_1 P] e(t) - \mu_1 e(t)^{\mathrm{T}} P e(t) \\ & \quad + e(t-\tau)^{\mathrm{T}}[\eta^{-1} L_g^2 I - \mu_2 P] e(t-\tau) + e(t-\tau)^{\mathrm{T}} \mu_2 P e(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2 \\ & = e(t)^{\mathrm{T}} \Gamma_1 e(t) - \mu_1 e(t)^{\mathrm{T}} P e(t) + e(t-\tau)^{\mathrm{T}} \Gamma_2 e(t-\tau) + \mu_2 V(t-\tau) \\ & \quad + \lambda^{-1}\psi^2\varepsilon^2. \end{aligned}$$

By conditions (a) and (b), we have the following:

$$\dot{V}(t) \leqslant -\mu_1 e(t)^{\mathrm{T}} P e(t) + e(t-\tau)^{\mathrm{T}} \mu_2 P e(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2,$$

$$\dot{V}(t) \leqslant -\mu_1 V(t) + \mu_2 V(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2, \qquad \text{for } k\omega \leqslant t \leqslant k\omega + \delta.$$

By lemma 4, we have,

$$V(t) \leqslant ||V(k\omega)||_\tau \mathrm{e}^{-r(t-k\omega)} + \frac{\lambda^{-1}\psi^2\varepsilon^2}{r} \qquad \text{for } k\omega \leqslant t \leqslant k\omega + \delta, \tag{17}$$

where r is the unique solution of the equation: $-r = -\mu_1 + \mu_2 \mathrm{e}^{r\tau}$.
(ii) When $k\omega + \delta < t \leqslant (k+1)\omega$,

where k is a non-negative integer. In this case, the error system (3) is without feedback control. We are able to do similar estimation as we did for the case $k\omega \leqslant t \leqslant k\omega + \delta$. By (14)–(16), thus, we have the following inequality:

$$\dot{V}(t) = 2e(t)^{\mathrm{T}} P[A_2 e(t) + \Delta A x + B_2(f(y) - f(x)) + \Delta B f(x)$$
$$+ C_2(g(y(t-\tau)) - g(x(t-\tau))) + \Delta C g(x(t-\tau))].$$

By (14)–(16), thus, we have the following inequality:

$$\begin{aligned}
&\dot{V}(t)\\
&\leqslant e(t)^{\mathrm{T}}(\Gamma_1 + K^{\mathrm{T}}P + PK)e(t) - \mu_1 e(t)^{\mathrm{T}} P e(t)\\
&\quad + e(t-\tau)^{\mathrm{T}}\Gamma_2 e(t-\tau) + \mu_2 V(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2\\
&= e(t)^{\mathrm{T}}\Gamma_1 e(t) + e(t)^{\mathrm{T}}(K^{\mathrm{T}}P + PK)e(t) - \mu_1 e(t)^{\mathrm{T}} P e(t)\\
&\quad + e(t-\tau)^{\mathrm{T}}\Gamma_2 e(t-\tau) + \mu_2 V(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2\\
&< e(t)^{\mathrm{T}}(K^{\mathrm{T}}P + PK)e(t) - \mu_1 V(t) + \mu_2 V(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2\\
&= e(t)^{\mathrm{T}}(K^{\mathrm{T}}P + PK - \mu_3 P)e(t) + \mu_3 e(t)^{\mathrm{T}} P e(t)\\
&\quad - \mu_1 V(t) + \mu_2 V(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2\\
&\leqslant (\mu_3 - \mu_1)V(t) + \mu_2 V(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2.
\end{aligned}$$

Thus, we have obtained that

$$\dot{V}(t) < (\mu_3 - \mu_1)V(t) + \mu_2 V(t-\tau) + \lambda^{-1}\psi^2\varepsilon^2, \qquad \text{for } k\omega + \delta < t \leqslant (k+1)\omega.$$

By lemma 3, we have, for $k\omega + \delta < t \leqslant (k+1)\omega$,

$$V(t) \leqslant \left[||V(k\omega+\delta)||_\tau + \frac{\lambda^{-1}\psi^2\varepsilon^2}{\mu_3 - \mu_1 + \mu_2}\right]\mathrm{e}^{(\mu_3-\mu_1+\mu_2)(t-k\omega-\delta)} - \frac{\lambda^{-1}\psi^2\varepsilon^2}{\mu_3 - \mu_1 + \mu_2}. \tag{18}$$

By the results of (17) and (18), we know that the non-negative function $V(t)$ for $t \geqslant 0$ satisfies the conditions of lemma 5. Using lemma 5, we have obtained the following quasi-synchronization result:

$$V(t) \leqslant |V(0)|_\tau \mathrm{e}^{\rho}\mathrm{e}^{-(\rho/\omega)t} + \varepsilon_1, \tag{19}$$

where $|V(0)|_\tau = \max\limits_{-\tau \leqslant s \leqslant 0} (\phi(s) - \psi(s))^{\mathrm{T}} P(\phi(s) - \psi(s))$,

$$\rho = r(\delta - \tau) - (\mu_3 - \mu_1 + \mu_2)(\omega - \delta) \qquad \text{and} \qquad \varepsilon_1 = \frac{\upsilon}{1 - \mathrm{e}^{-\rho}} + \alpha,$$

where

$$\alpha = \frac{\lambda^{-1}\psi^2\varepsilon^2}{r}, \qquad \beta = \frac{\lambda^{-1}\psi^2\varepsilon^2}{\mu_1 - \mu_2 + \mu_3}$$

and

$$\upsilon = (\alpha + \beta)\mathrm{e}^{\rho_2} - \beta.$$

Note that the Lyapunov-like function (11) satisfies

$$\lambda_m(P)||e(t)||^2 \leqslant V(t) \leqslant \lambda_{\mathrm{M}}(P)||e(t)||^2, \tag{20}$$

where $\lambda_{\mathrm{m(M)}}$ is the minimal (maximal) eigenvalue of the square positive matrix P. By inequalities (19) and (20), we have

$$\lambda_{\mathrm{m}}(P)||e(t)||^2 \leqslant V(t) \leqslant |V(0)|_\tau \mathrm{e}^{\rho}\mathrm{e}^{-(\rho/\omega)t} + \varepsilon_1.$$

Thus, we have

$$||e(t)|| \leqslant \sqrt{\frac{1}{\lambda_{\mathrm{m}}(P)}(|V(0)|_\tau \mathrm{e}^{\rho}\mathrm{e}^{-(\rho/\omega)t} + \varepsilon_1)} \leqslant \sqrt{\frac{\mathrm{e}^{\rho}}{\lambda_{\mathrm{m}}(P)}|V(0)|_\tau}\,\mathrm{e}^{-(\rho/2\omega)t} + \sqrt{\frac{\varepsilon_1}{\lambda_{\mathrm{m}}(P)}}. \tag{21}$$

The synchronization error system (3) converges exponentially to a small region D containing the origin, where

$$D = \left\{x \in R^n \,\middle|\, ||x|| \leqslant \sqrt{\frac{\varepsilon_1}{\lambda_{\mathrm{m}}(P)}}\right\}.$$

For any arbitrary small positive number ε_2, from inequality (21), there is a positive T such that for any $t \geqslant T$,

$$||e(t)|| \leqslant \varepsilon_2 + \sqrt{\frac{\varepsilon_1}{\lambda_{\mathrm{m}}(P)}}.$$

Thus, the quasi-synchronization between system (1) and system (2) is achieved with error bound $\varepsilon_2 + \sqrt{\varepsilon_1/(\lambda_{\mathrm{m}}(P))}$, for any arbitrary small positive number ε_2. This completes the proof.

Remark 1. The conditions (a)–(b) in this theorem imply that the controlled subsystem of (3) is exponentially stabilized with the exponential convergence degree, r, satisfying $-r = -\mu_1 + \mu_2 \mathrm{e}^{r\tau}$. This fact can be easily observed from the first part of the above proof.

Remark 2. From the second part of the above proof. Condition (c) is an estimation of the exponential divergence degree of the uncontrolled subsystem of (3).

Remark 3. Condition (d) characterizes the aggregated effects of the exponential convergence degree of the controlled subsystem, the exponential divergence degree of the uncontrolled subsystem, control period and control duration on the stabilization of the synchronization error system (3).

Remark 4. It is clear that when the parameter mismatch vanishes, complete synchronization will occur. In other words, conditions (a)–(d) above can also guarantee exponentially complete synchronization between two coupled identical chaotic systems by using intermittent feedback control. Thus, it contains the existing results as a special case.

In the following, we shall establish two numerically tractable quasi-synchronization conditions. To this end, in theorem 1 we let $P = I$, $K = kI$ and

$$\gamma = \frac{L_f}{\sqrt{\lambda_{\mathrm{M}}\left(B_2 B_2^{\mathrm{T}}\right)}}, \qquad \eta = \frac{L_g}{\sqrt{\lambda_{\mathrm{M}}(C_2 C_2^{\mathrm{T}})}},$$

$$\xi = \lambda_{\mathrm{M}}(A_2 + A_2^{\mathrm{T}}) + 2L_f\sqrt{\lambda_{\mathrm{M}}(B_2 B_2^{\mathrm{T}})} + L_g\sqrt{\lambda_{\mathrm{M}}(C_2 C_2^{\mathrm{T}})} + \lambda,$$

$\mu_1 = 2k - \xi$, $\mu_2 = L_g\sqrt{\lambda_{\mathrm{M}}(C_2 C_2^{\mathrm{T}})}$, $\mu_3 = 2k$. Then, from theorem 1 the following corollary is immediate.

Corollary 1. Given τ, δ, ω, and λ. If there exist control gain k and positive constant r such that $2k - \xi = r + L_g e^{r\tau}\sqrt{\lambda_M(C_2C_2^T)}$ and $r(\delta - \tau) - (\xi + L_g\sqrt{\lambda_M(C_2C_2^T)})(\omega - \delta) > 0$. Then the quasi-synchronization between system (1) and system (2) is achieved with error bound $\varepsilon_2 + \sqrt{\varepsilon_1}$, for any arbitrary small positive number ε_2.

From corollary 1, we can determine the control strength k, as stated in the following corollary.

Corollary 2. Given τ, δ, ω, and λ. For any arbitrary small positive number ε_2, the quasi-synchronization between system (1) and system (2) is achieved with error bound $\varepsilon_2 + \sqrt{\varepsilon_1}$ provided that $k > k^* = \Phi(r^*)$, where $\Phi(r) = \frac{1}{2}(r + \xi + L_g e^{r\tau}\sqrt{\lambda_M(C_2C_2^T)})$ and $r^* = (\omega - \delta)/(\delta - \tau)(\xi + L_g\sqrt{\lambda_M(C_2C_2^T)}) > 0$.

Remark 5. From corollary 2, one can determine the control strength k given δ and ω on the one hand, while on the other hand, corollary 2 also characterizes the functional relationship between the low critical bound k^* of the control strength k and the control duration δ if the control period ω is already known.

4. Numerical examples

In this section, we present two examples to show the effectiveness of the proposed results.

Example 1. We take a scalar time-delay system, namely, the Ikeda oscillator of the form

$$\dot{x}(t) = -a_1 x(t) + b_1 \sin(x(t - \tau)). \tag{22}$$

This system exhibits chaotic behaviour when $\tau = 2$, $a_1 = 1$ and $b_1 = 4$, as shown in figure 1. For numerical simulations, we assume that the response system associated with (22) is of the form

$$\dot{y}(t) = -a_2 y(t) + b_2 \sin(y(t - \tau)) + u, \tag{23}$$

where $a_2 = 0.999$, $b_2 = 4.001$ and

$$u = \begin{cases} k(x - y) & m\omega \leqslant t \leqslant m\omega + \delta, \\ 0 & m\omega + \delta < t \leqslant (m+1)\omega, \quad m = 0, 1, \cdots. \end{cases}$$

Note that the chaotic attractor of (22) is contained in the space $\Psi = \{x \in R \mid |x| \leqslant 4\}$ and parameter mismatches satisfy that $|a_1 - a_2| + |b_1 - b_2| \leqslant 0.002$. From corollary 2, we calculate that $k^* = 10.143$ when $\omega = 4$ and $\delta = 3.9$. Fixing $\omega = 4$, one can plot the relationship curve between k^* and δ, as shown in figure 1. For numerical simulation, we select $\omega = 4$, $\delta = 3.9$ and $k = 10.5$ and plot the quasi-synchronization error curve, as shown in figure 2. In this case, we also estimate the convergence region D of quasi-synchronization error system (3) by corollary 1, i.e. $D = \{e \in R^n \mid \|e\| \leqslant 0.02645\}$.

Example 2. Consider the neural oscillator described by the following delayed differential equation [35]

$$\dot{x}(t) = -C_1 x(t) + A_1 f(x(t)) + B_1 f(x(t - 1)), \tag{24}$$

where

$$C_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad A_1 = \begin{pmatrix} 3.0 & 5.0 \\ 0.1 & 2.0 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} -2.5 & 0.2 \\ 0.1 & -1.5 \end{pmatrix},$$

and $f_i(t) = \tanh(x_i(t))$.

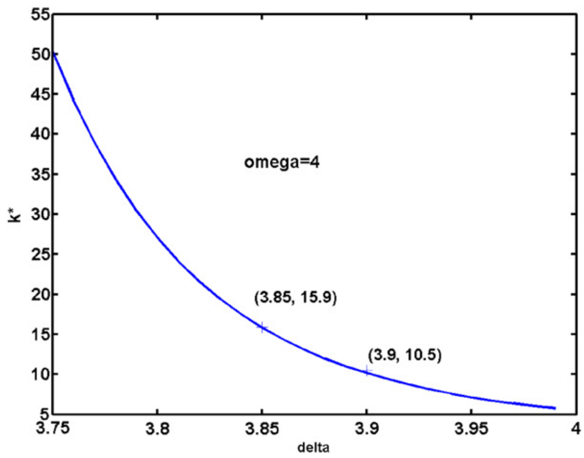

Figure 1. Given $\omega = 4$, the relationship between k^* and δ.

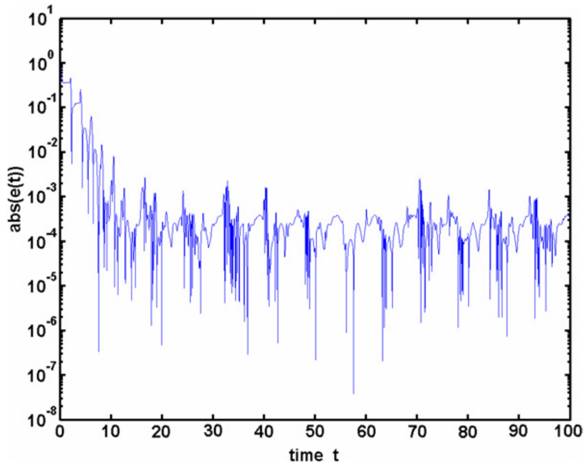

Figure 2. Synchronization error curve in terms of absolute value with the control parameters $\omega = 4$, $\delta = 3.9$ and $k = 10.5$. The initial values for drive and response systems are $x(\theta) = -2$ and $y(\theta) = 3, \theta \in [-2, 0]$ respectively.

This model was investigated by Lu in [35], where it is shown to be chaotic with $\Psi = \{x \in R^2 | |x| \leqslant 4\}$. In this example, one observes that $L_f = L_g = 1$ and $\tau = 1$. For numerical simulations, we assume that the response system associated with (24) is of the form

$$\dot{y}(t) = -C_2 y(t) + A_2 f(y(t)) + B_2 f(y(t-1)) + u(t), \tag{25}$$

where

$$C_2 = \begin{pmatrix} 1.001 & 0 \\ 0 & 1.001 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 2.999 & 4.999 \\ 0.0999 & 2.001 \end{pmatrix}, \quad B_2 = \begin{pmatrix} -2.499 & 0.2001 \\ 0.0999 & -1.5001 \end{pmatrix},$$

and

$$u = \begin{cases} k(x-y) & m\omega \leqslant t \leqslant m\omega + \delta, \\ 0 & m\omega + \delta < t \leqslant (m+1)\omega, \quad m = 0, 1, \cdots. \end{cases}$$

Note that parameter mismatches satisfy that $\|\Delta A\| + L_f \|\Delta B\| + L_g \|\Delta C\| \leqslant 0.0028$.

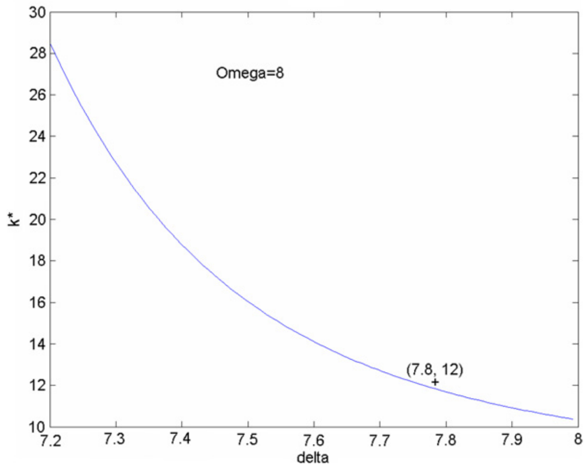

Figure 3. Given $\omega = 8$, the relationship between k^* and δ.

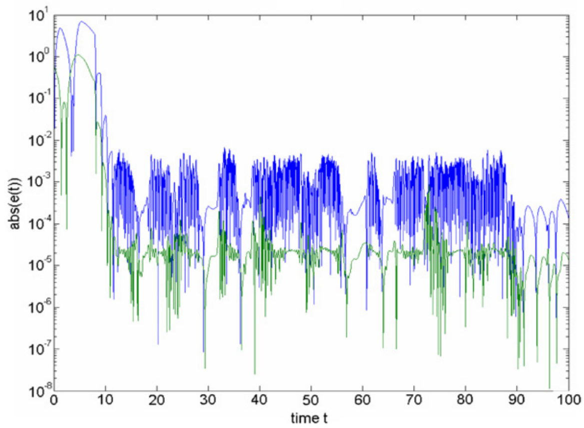

Figure 4. Synchronization error curve in terms of absolute value with the control parameters $\omega = 8$, $\delta = 7.8$ and $k = 12$.

Fixing $\omega = 8$, from corollary 2, one can plot the relationship curve between k^* and δ, as shown in figure 3. For numerical simulation, we select$\omega = 8, \delta = 7.8$ and $k = 12$, and plot the quasi-synchronization error curve, as shown in figure 4. In this case, we also estimate the convergence region D of quasi-synchronization error system (3) by corollary 1, i.e. $D = \{e \in R^n | ||e|| \leqslant 0.1494\}$.

5. Conclusions

The results in this paper have a strong constraint on the control duration, which arises from the theoretical analysis. Namely, we require that the control duration should be larger than the system delay. In practice, this constraint may not always be necessary. Therefore, it will be interesting to obtain some modified criteria releasing this constraint.

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