Epileptic-seizure classification using phase-space representation of FBSE-EWT based EEG sub-band signals and ensemble learners

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Abstract

Electroencephalogram (EEG) signals are non-linear and non-stationary in nature. The phase-space representation (PSR) method is useful for analysing the non-linear characteristics of EEG signals. Hence non-linear features based on a phase-space representation of EEG signals are effective in epileptic-seizure classification. In the past, various machine learning methods are used for classifying seizure EEG signals. However, they might fail to classify accurately due to complex data, so with effective non-linear features, ensemble learning classifiers can be investigated to improve the accuracy of an automated epileptic-seizure detection system. In this paper, the EEG signals are first decomposed in sub-bands using empirical wavelet transform (EWT) based on the Fourier Bessel series expansion (FBSE) which is termed as FBSE-EWT. Then, these sub-bands are reconstructed into three-dimensional (3D) PSR. Next, entropy-based features like line-length (LL), log-energy-entropy (LEEnt), and norm-entropy (NEnt) are computed from Euclidean distances of 3D PSR of sub-band signals. The extracted features are ranked based on *p*-values obtained from the Kruskal-Wallis statistical test for reducing the feature space. Experiments have been conducted using obtained ranked features on different ensemble learning classifiers, and the five best-performing classifiers are reported here, which are random forest (RF), extra tree (ET), extreme gradient boosting tree (xgBT), bagged-SVM (B-SVM), and bagged-*k*-nearest neighbours (B-*k*-NN), for classifying epileptic-seizure EEG signals. The performance of the proposed framework is evaluated using a publically accessible Bonn university EEG database for classifying epileptic-seizure EEG signals on well-known classification problems such as C_1 (seizure, normal), C_2, and C_3 (seizure, normal, and seizure-free). This dataset consists recording of 100 single-EEG signals from five seizure, seizure-free, and normal (healthy) subjects each. Model is trained and tested using 10-fold cross-validation to stave off from overfitting. The performance is also compared with other state-of-art methods. Obtained results confirm the superior performance of the proposed framework by achieving maximum classification accuracy of 100 %, 98.3%, 97.8% with ET, B-SVM, and ET classifiers from each studied classification problem, respectively. Hence, the proposed framework can assist medical professionals in analysing epileptic-seizure EEG signals more accurately.

Keywords: Electroencephalogram signal, Epilepsy, FBSE-EWT method, Phase-space representation, Euclidean distances, Ensemble learning.

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1. INTRODUCTION

Epilepsy is a chronic brain disease, occurs due to an abnormal increase of neuronal synchrony in the brain. It has affected approximately 50 million people of all ages group globally [1]. Several neurological diseases are diagnosed using electroencephalogram (EEG), and epilepsy is one such disease. Usually, neurophysiologists visually examine EEG signals for detecting an epileptic-seizure. Monitoring long-duration EEG signals manually is a monotonous and tedious job. Therefore, an automated epileptic-seizure detection system using advanced signal processing and machine learning methods must be built, which can reduce the neurologist's workload for identifying epileptic-seizure EEG signals accurately within less time. Hence, designing automated epileptic-seizure detection is considered to be an active field of interest for research among researchers.

In the past few decades, numerous attempts have been made to design an automated epileptic-seizure detection algorithm in the literature, but still, it remains a challenging task to classify epileptic-seizure signals accurately. Most of the reported methods either do not perform well in all interested problems or use EEG signals acquired from multiple electrodes. However, the multiple-electrode-based algorithm performs better, but it also increases the computational complexity, making these algorithms less suitable in real-time application.

Several automated epileptic-seizure detection approaches have been presented [2]–[6] in the literature. The two most important steps involved in any automatic epileptic-seizure detection system are feature extraction and classification. Feature extraction is an initial step that can greatly impact both the accuracy and complexity of the entire system. By assuming EEG signals as stationary, several feature extraction methods based on time and frequency domain were presented to identify epileptic-seizures [7]–[10]. However, EEG signals are non-stationary in nature; hence time-frequency domain feature extraction techniques [11] are more suitable for analysing epileptic-seizure signals. Zahra et al. has used multivariate empirical mode decomposition (EMD) (MEMD) for time-frequency analysis of epileptic-seizure EEG signals, and artificial neural networks classifier for classifying signals [5]. Non-linear features were computed from analytic intrinsic mode functions (IMFs) of EMD and then fed to the C4.5 decision tree classifier, which obtains the maximum classification accuracy of 95.33% [2]. Here [12], the authors have computed the weighted multiscale Renyi permutation entropy (WMRPE) feature from the rhythms obtained by Fourier–Bessel series expansion (FBSE). Then least-square support vector machine (SVM), termed as (LS-SVM) classifier, is used for classifying seizure EEG signals. The highest classification accuracy achieved from this method is 97.33%. Hassan et al. have presented a new approach using tunable-Q wavelet transform (TQWT) to extract spectral features from seizure EEG signals [13]. Solaija et al. have presented a method that extracts curve-length as a feature based on dynamic mode decomposition (DMD). Random Under-sampling Boost (RUSBoost) decision-tree classifier is used for classifying seizure EEG signals [14]. Time, frequency, and time-frequency domain-based features are automatically extracted using a deep learning approach for classifying seizure EEG signals [15]. Mehla et al. presented a new framework [16] for detecting epileptic-seizure EEG signals using the Fourier decomposition method. They have extracted Lp norms features from Fourier intrinsic band functions (FIBFs) and then classified them using an SVM classifier. Statistical features were computed from local mean decomposition (LMD) based sub-band signals, and then these features have been classified using deep bidirectional long short-term memory (Bi-LSTM) network [17]. A comprehensive study has been carried out by Shoeibi et al. [18], in which authors have used fifty different features based on time-domain, statistical, frequency-domain, and non-linear for detecting epileptic-seizure EEG signals. Supriya et al. have presented a review report on the automatic epileptic-seizure identification approach based on graph theory [19]. For analysing epileptic-seizure EEG signals, the authors have used a significant feature based on matrix determinant [20]. In this paper [21], the authors have computed a feature vector based on L1- penalized robust regression. Then it is passed to a random forest (RF) classifier as an input, achieves the highest classification accuracy of 100% without artifacts, and while in the presence of white noise and artifacts, it achieves accuracy around 90% for classifying seizure EEG signals. Radman et al. [22] used time, frequency, and time-frequency domain features for seizure classification, with the Dempster–Shafer evidence theory (DSET) feature selection method to improve the model's performance. The authors presented a novel epilepsy detection method using discrete wavelet transform (DWT) with fuzzy entropy and associative Petri net [23]. A new approach for classifying seizure signals using the Taylor-Fourier filter-bank implemented with O-splines is presented in [24], where energy-based features are computed from Taylor-Fourier sub-band signals and are evaluated using LS-SVM classifier. Sharma et al. [25] have proposed a framework for classifying seizure EEG signals using higher-order statistics and a sparse autoencoder network. Improved eigenvalue decomposition of Hankel matrix and Hilbert transform (IEVDHM–HT), a new time-frequency approach has been introduced to classify seizure-free and seizure EEG signals. The proposed method achieves 100% of classification accuracy [26]. The 2-Dimension reconstructed phase space (RPS) based features from empirical wavelet transform (EWT) rhythms are used for classifying epileptic-seizure [27]. According to the review papers [28]–[30], non-linear features such as line-length (LL), entropy-based (Shannon, approximate, weighted-permutation, permutation, log-energy, norm entropy (NEnt) etc.) are the most effective features, which can be used for classifying epileptic-seizure EEG signals. Therefore, LL, log-energy entropy (LEEnt), and NEnt features are investigated to classify epileptic-seizure EEG signals accurately in this proposed framework.

After feature extraction, classifying discriminant features accurately is very important. Many classifiers were used to classify binary and multi-class EEG signals, with one class as epileptic-seizure in the literature. However, according to the study [29], ensemble machine learning classifiers (RF, decision tree, etc.) are more effective than single machine learning classifiers like SVM and *k*-nearest neighbours (*k*-NN). Therefore, in our proposed work, ensemble learning classifiers are used to detect epileptic-seizure EEG signals also compared them with single machine learning classifiers.

From the above literature survey, we can say that several wavelet transforms have been investigated for analysing non-stationary epileptic-seizure EEG signals in the past few years. However, most of them are non-adaptive, which is not appropriate for analysing non-stationary EEG signals. An adaptive method like EMD has the mode-mixing problem, whereas EWT has the limitation of improper segmentation of Fourier spectrum due to inference. This motivates us to use an advanced adaptive method that can resolve the above mention issues. The FBSE [31] based EWT [32] method, which is termed as FBSE-EWT [33], has recently been effective in analysing many neurological disorders [34], [35]. In this proposed framework, we have used the FBSE-EWT for classifying epileptic-seizure EEG signals. Our main contribution in this work is to extract novel features from the PSR of decomposed sub-band signals using the FBSE-EWT method. The PSR technique mainly depends on two parameters, namely the time-lag and embedding dimension. We have investigated our proposed work using a fixed value of these parameters for obtaining the PSR from each decomposed sub-band signal. Non-linear features like LL, LEEnt, and NEnt have been computed from Euclidian distances of three-dimensional (3D) PSR of extracted sub-band signals. Then, obtained features are ranked based on *p*-values of the Kruskal-Walls statistical test. Lastly, this ranked feature matrix is passed to various classifiers such as RF, extra tree (ET), extreme gradient boosting tree (xgBT), SVM, *k*-NN, bagged-SVM (B-SVM), and bagged-k-NN (B-*k*-NN) to evaluate the performance of the proposed model. The proposed methodology for classifying epileptic-seizure EEG signals is depicted in Fig. 1.

The layout of the rest of the paper is as follows. Section 2 presents a brief explanation of the dataset used in this proposed work, followed by a detailed description of FBSE-EWT, PSR, feature extraction from Euclidian distances of 3D PSR, and ensemble learning classifiers. Section 3 describes experimental results and a detailed discussion on obtained results, along with a comparison between the proposed and recent report works for automatic classification of epileptic-seizure EEG signals using the Bonn university EEG database. In the end, conclusions are provided in Section 4.

2. METHODS AND MATERIALS

Fig. 1 depicts the process of automatic detection of epileptic-seizure EEG signals. The proposed framework includes two parts. The first part consists of pre-processing step for extracting significant features, and the next part is classification using ensemble learning classifiers. The first part involves some pre-processing steps; in the first step, sub-band signals are obtained using the FBSE-EWT method applying to EEG signals. Each obtained sub-band signal is plotted in a three-dimensional PSR in the second step. In the third step, the Euclidean distances are computed between the coordinates of sub-band signals in 3D PSR and the origin point (0, 0, 0). Then non-linear features are extracted from the Euclidean distances. In the second part, the performance of the proposed methodology is evaluated using ensemble learning classifiers for classifying seizure EEG signals.

2.1. Bonn university EEG database

Dataset used in this proposed framework is provided by the Epileptology department of the University of Bonn [36]. Due to the easy availability of the dataset in the past years, various methods have been developed and studied for classifying epileptic-seizure EEG signals. The given dataset comprises of five sub-sets denoted by sets S, Z, O, N, and F, in which each set has hundred single-channel EEG signals. The acquired signals are recorded at the sampling rate of 173.61 Hz. Each segmented signal is of 23.6-second duration. Hence, each segment contains 4097 ($\approx 173.61 \times 23.6$) sample points. The sets Z and O are recorded from five healthy subjects with eyes open and closed state. The sets N, F, and S comprise EEG signals from five epileptic-seizure subjects. EEG signals of sets F and N are acquired from five subjects who have been completely recovered from seizure control after surgery of epileptic locations. Recording of set S involves EEG signals with seizure activities from an epileptogenic zone. The normal and seizure-free sets contain 200 EEG signals each, whereas seizure set S has 100 EEG signals collected during the subject's epileptic-seizures condition. A band-pass filter (0.53 Hz- 40 Hz) was applied to eliminate noise and artifacts from digitised EEG signals, which also helps in reducing power-line noise. A detailed description of the dataset is provided in [36]. Evaluation of this proposed methodology is done using three different classification problems, which are formed using the sets of this database, are denoted by C1, C2, and C3. A summary of each classification problems and its clinical relevance is provided below:

1) Classification problem C_1 (S-ZO): It is a binary class classification problem, which helps in classifying seizure and normal EEG signals. The seizure EEG signals (S) are separated from normal EEG signals (Z and O) belonging to healthy subjects.

2) Classification problem C_2 (S-Z-N): It is a ternary class classification problem, which helps in classifying seizure, normal and seizure-free EEG signals. The EEG signals belonging to the seizure subjects (S), normal EEG signals from healthy subjects (Z), and seizure-free EEG signals from (N) subjects who were having seizure earlier but now they are seizure-free are separated from each other.

3) Classification problem C_3 (S-ZO-NF): This classification problem classifies three classes, hence known as a ternary classification problem, which helps in classifying seizure, normal, and seizure-free EEG signals. The EEG signals belonging to the seizure subjects (S), normal EEG signals from the healthy subjects (Z and O) and seizure-free EEG signals from the subjects (N and F) who were having seizure earlier but now they are seizure-free are separated from each other.

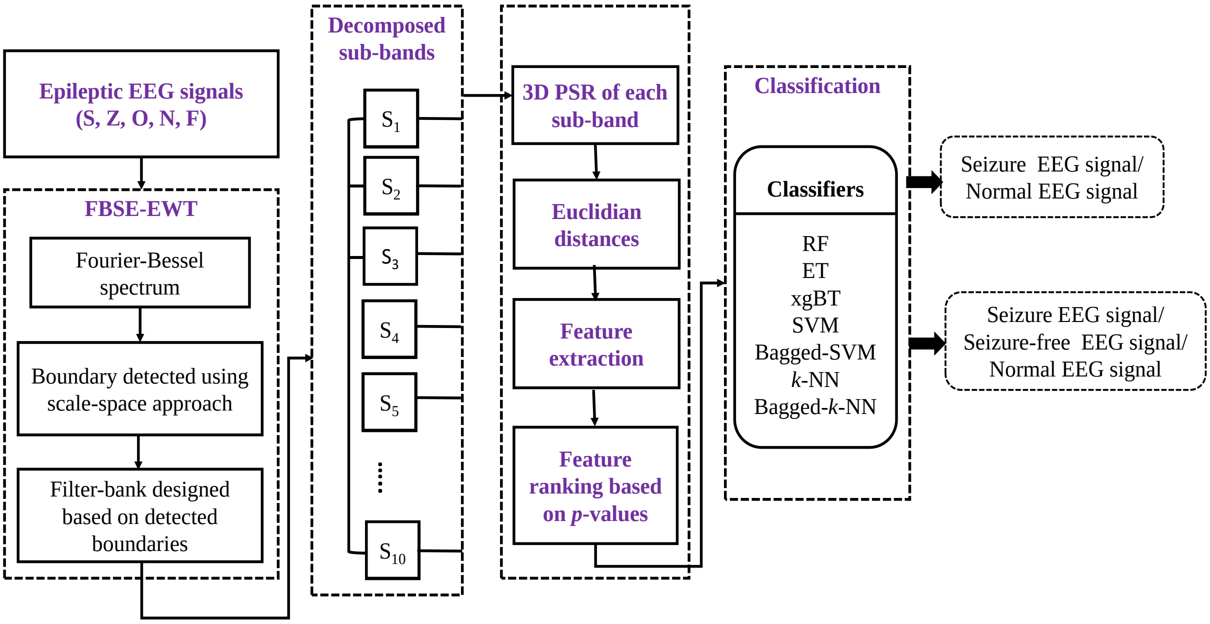

Fig. 1. Work-flow diagram for detecting epileptic-seizure EEG signals.

2.2 FBSE-EWT based decomposition

Here, the FBSE-EWT is used to decompose EEG signals into narrow sub-band signals. It is a combination of two signal processing techniques, such as FBSE [31] and EWT [32], [37] for analysing non-stationary signals [33]. The FBSE-EWT has been used here to obtain sub-band signals of EEG signals. The number of the coefficients of spectral representation based on the FBSE spectrum is the same as the length of the discrete signal, while the length of the Fourier spectrum is half of the discrete signal. Therefore, the frequency resolution of FBSE is twice that of the Fourier based representation. The FBSE-EWT method can be implemented straightforwardly compared to EMD, which is an algorithm iterative in the method [2]. Bessel functions are used as a basis function in FBSE based representation, which makes them appropriate for analysing non-stationary signals. The EWT adaptively design filter banks, which provide support to limit the analysed EEG signal on the spectrum. The FBSE-EWT have been studied in the literature for analysing the non-stationary signals, such as in alcoholic detection [34], emotion detection [38], focal seizure detection [39], ocular artifacts elimination [40], and also in detecting epileptic-seizure EEG signals [12]. A fundamental concept of the FBSE-EWT method is summarised using the under-mentioned steps as follows [33].

Step 1: The FBSE spectrum of analysed signal $x(t)$ is obtained using the FBSE method within a frequency range of [0, π]. The mathematical expression of the series expansion of analysed EEG signal $x(t)$ using zero-order Bessel functions is shown as [33],

$$x(t) = \sum_{j=1}^{M} K_j J_0\left(\frac{\alpha_j t}{M}\right), \qquad t = 0, 1, 2, 3 \ldots, M-1 \tag{1}$$

where FBSE coefficients of $x(t)$ are denoted by K_j which can be computed as follows:

$$K_j = \frac{2}{M^2\left(J_1(\alpha_j)\right)^2} \sum_{t=0}^{M-1} t x(t) J_0\left(\frac{\alpha_j t}{M}\right) \tag{2}$$

where zero and first-order Bessel functions are represented by $J_0(.)$ and $J_1(.)$, respectively. Non-negative roots of the zero-order Bessel function ($J_0(\alpha) = 0$), which are represented by α_j having $j = 1, 2, 3 \ldots, M$, The relationship between continuous time-frequency f_j (in Hertz) and corresponding j^{th} order of FBSE coefficients can be expressed as follows:

$$\alpha_j \approx \frac{2\pi f_j M}{F_S}, \text{ where } \alpha_j \approx \alpha_{j-1} + \pi \approx j\pi \tag{3}$$

F_S denotes the sampling frequency in (3). Equation (3) can be written as follows:

$$j \approx \frac{2 f_j M}{F_S} \tag{4}$$

Here, from (4), it can be concluded that in order to cover the whole bandwidth of the signal, j has to be in the range of $[1, M]$, where M represents the length of the signal.

Step 2: In this framework, the scale-space based boundary detection approach is employed to segment the FBSE spectrum into N continuous segments to obtain an optimum set of $N+1$ boundary frequencies represented by $\{\omega_j\}_{j=0,\ 1,\ 2,\ 3\ldots,N}$. Where the value of the first boundary frequency (ω_1) is 0 and last boundary frequency (ω_N) is π.

Step 3: Band-pass filters are built on each adaptive segment of the FBSE spectrum by applying empirical scaling and wavelet functions. Band-pass filters based on wavelets are created using the idea of Littlewood–Paley and Meyer's wavelets [32]. The mathematical expressions of an empirical scaling function $\phi_j(\omega)$ and wavelet function $\gamma_j(\omega)$ of EWT are represented in Table 1 [32]:

Table 1: The mathematical expressions of empirical scaling function and wavelet function

Functions	Expressions
Scaling $(\phi_j(\omega))$	$\phi_j(\omega) = \begin{cases} 1, & \text{if } \lvert\omega\rvert \le (1-v)\omega_j. \\ \cos\left[\frac{\pi * (v, \omega_j)}{2}\right], & \text{if } (1-v)\omega_j \le \lvert\omega\rvert \le (1+v)\omega_j. \\ 0, & \text{otherwise} \end{cases}$

Wavelet ($\gamma_j(\omega)$)	$\gamma_j(\omega)\begin{cases}1, & \text{if } (1+v)\omega_j \le \lvert\omega\rvert \le (1-v)\omega_{j+1}.\\ \cos\left[\frac{\pi*(v,\omega_{j+1})}{2}\right], & \text{if } (1-v)\omega_{j+1} \le \lvert\omega\rvert \le (1+v)\omega_{j+1}.\\ \sin\left[\frac{\pi*(v,\omega_j)}{2}\right], & \text{if } (1-v)\omega_j \le \lvert\omega\rvert \le (1+v)\omega_j.\\ 0, & \text{otherwise}\end{cases}$
Where, $*(v,\omega_j) = \kappa\left[\frac{(\lvert\omega\rvert-(1-u)\omega_j)}{2u\omega_j}\right]$ is the function used in above mention mathematical expressions.	

The parameter v ensures that empirical scaling and wavelet functions generate a tight frame in $L^2(\mathbb{R})$ which should satisfy the condition shown in (6) and a function $\kappa(\gamma)$ is described as [32],

$$\kappa(\gamma) = \begin{cases}0, & \text{if } \gamma \le 0.\\ \text{and } \kappa(\gamma) + \kappa(1-\gamma) = 1, & \forall\gamma \in [0,1]\\ 1, & \text{if } \gamma \ge 1.\end{cases} \tag{5}$$

$$v < \min_j\left(\frac{\omega_{j+1}-\omega_j}{\omega_{j+1}+\omega_j}\right) \tag{6}$$

Detail $d_j(s)$ and approximation coefficients $a_j(s)$ are computed from the inner product of analysed signal $x(t)$ by empirical wavelet function and scaling function, respectively, as follows [32]:

$$d_j(s) = \sum_{\varsigma=1}^{N} x(\varsigma)\overline{\omega_j(\varsigma - s)} \tag{7}$$

$$a_1(s) = \sum_{\varsigma=1}^{N} x(\varsigma)\overline{S_{j=1}(\varsigma - s)} \tag{8}$$

Lastly, the reconstruction of j^{th} order detail and approximation sub-band signals are obtained as follows [30]:

$$x_{d_j}(n) = \sum_{s=2}^{N_j} d_j(s)w_j(n-s) \tag{9}$$

$$x_{a_1}(n) = \sum_{s=1}^{N_{j=1}} a_{j=1}(s)S_{j=1}(n-s) \tag{10}$$

Where, detail sub-band signal of j^{th} (*j=1, 2, 3,…, N*) level is denoted by $x_{d_j}(n)$, approximation sub-band signal is represented by $x_{a_1}(n)$, N_j and N_1 represent the wavelet coefficient corresponding to the length of j^{th} detail and approximation sub-band signals, respectively. In this experiment, ten sub-band signals are obtained from EEG signals using the FBSE-EWT method. The EEG signals from sets S, Z, and O for an epileptic-seizure, normal with eyes open and closed conditions, are depicted in the first row of Fig. 2. The ten sub-band signals obtained from the FBSE-EWT method for these three EEG signals are also shown in fig. During the epileptic-seizure activity, there is a synchronization of neurons present in the brain, which generates spikes and sharp waves in the EEG signals [41] same is reflected in Fig. 2.

2.3 Phase-space representation (PSR)

Two parts involved in dynamical systems theory [42] are state and dynamics. The state provides the system's necessary information at a time instance, whereas the dynamics explain how the system's state is evolved with time. In order to extract non-linear dynamics of the signal, PSR is considered a very effective tool. A signal's

PSR can easily provide a clear illustration of the evolution of the dynamical behaviour of the signals over time [43]. The mathematical expression of phase space of d-dimension of EEG signal $X = \{x(1), x(2), x(3) \dots, x(M)\}$, can be expressed as follows:

$$Z(m) = (X(m),\ X(m+l), \dots, X(m+(d-1)l)), \tag{11}$$

where $m = 1, 2, \dots, M-(d-1)l$, M is the number of data points in EEG signal, our experiment M is 4097, l represents time lag, and d denotes the embedding dimension of the phase space. PSR vector Z can also be written as follows:

$$Z = \begin{bmatrix} Z(1) \\ Z(2) \\ . \\ . \\ . \\ Z(M-(d-1)l) \end{bmatrix} = \begin{bmatrix} X(1) & X(1+l) & \dots & X(1+(d-1)l) \\ X(2) & X(2+l) & \dots & X(2+(d-1)l) \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ X(M-(d-1)l) & X((M-(d-1)l)+l) & \dots & X(M) \end{bmatrix} \tag{12}$$

The PSR can easily depict the nature of non-linear signals over time, mainly when $d = 2$ or 3. In this proposed work, we have used the value of the embedding dimension $d = 3$ due to their easy visualisation, conventionally named 3D PSR. The 3D PSR using time lag $l = 1$ is studied in [43]–[45]. Two-dimension (2D) PSR has been extensively utilised to measure biomedical signal uncertainty [46], [47].

Fig. 3 demonstrates the first four sub-band signals (SB_1-SB_4) of epileptic-seizure EEG signal (Set S) in the first column, whereas their respective 3D PSR plots are presented in the middle column. In PSR plots, the x-axis represents $X(m)$ vector, whereas vectors $X(m+1)$ and $X(m+2)$ is plotted in the y-axis and z-axis.

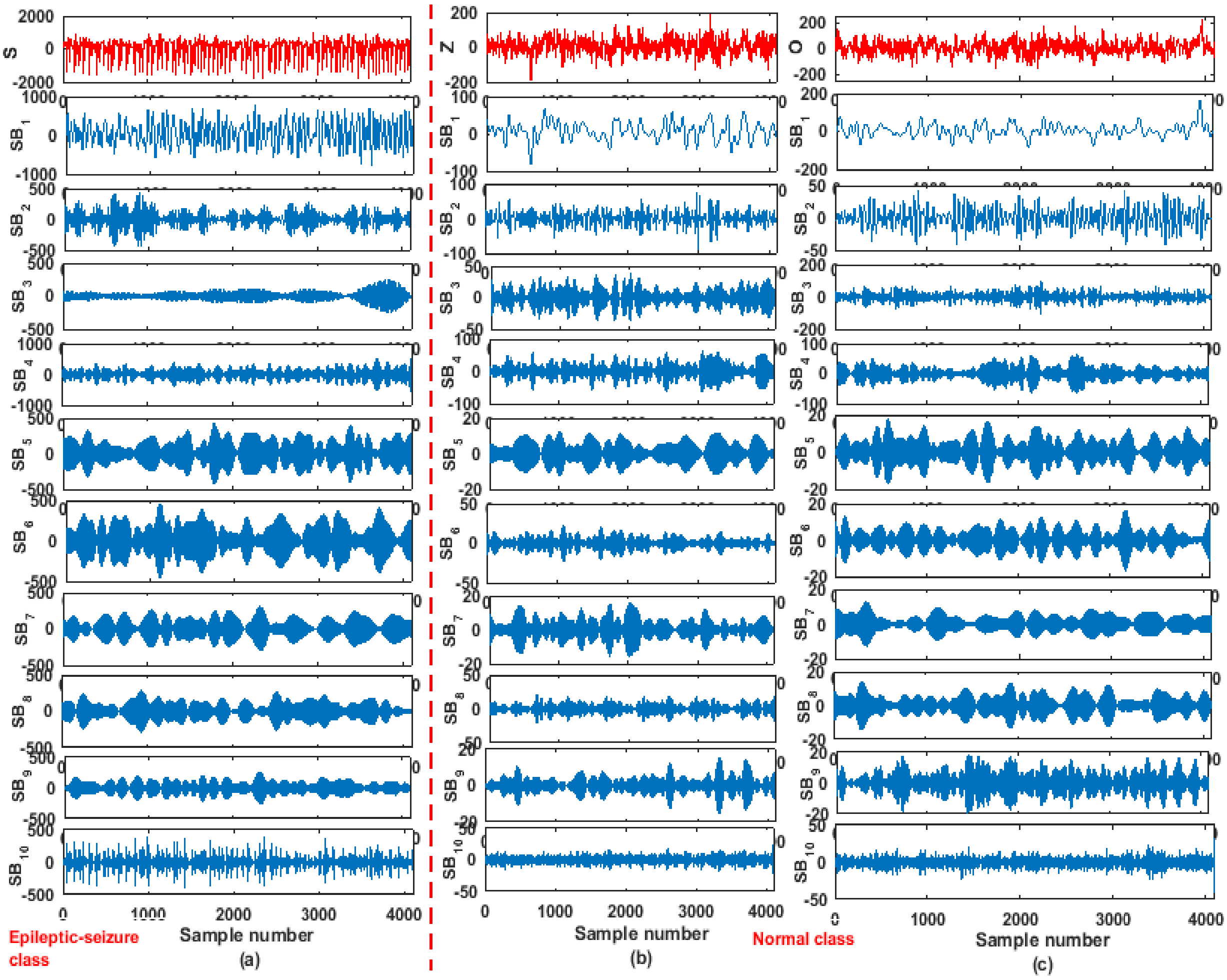

Fig. 2: The first row of the plot represents EEG signals of 4097 number of samples from sets denoted by 'S' as seizure, 'Z' as normal subjects with eyes open state, and 'O' as normal subjects with eyes closed state, respectively for C1 classification problem. Where successive plots below represent ten obtained sub-band signals (SB_1-SB_{10}) using the FBSE-EWT method from (a) S (b) Z (c) O EEG signals, respectively.

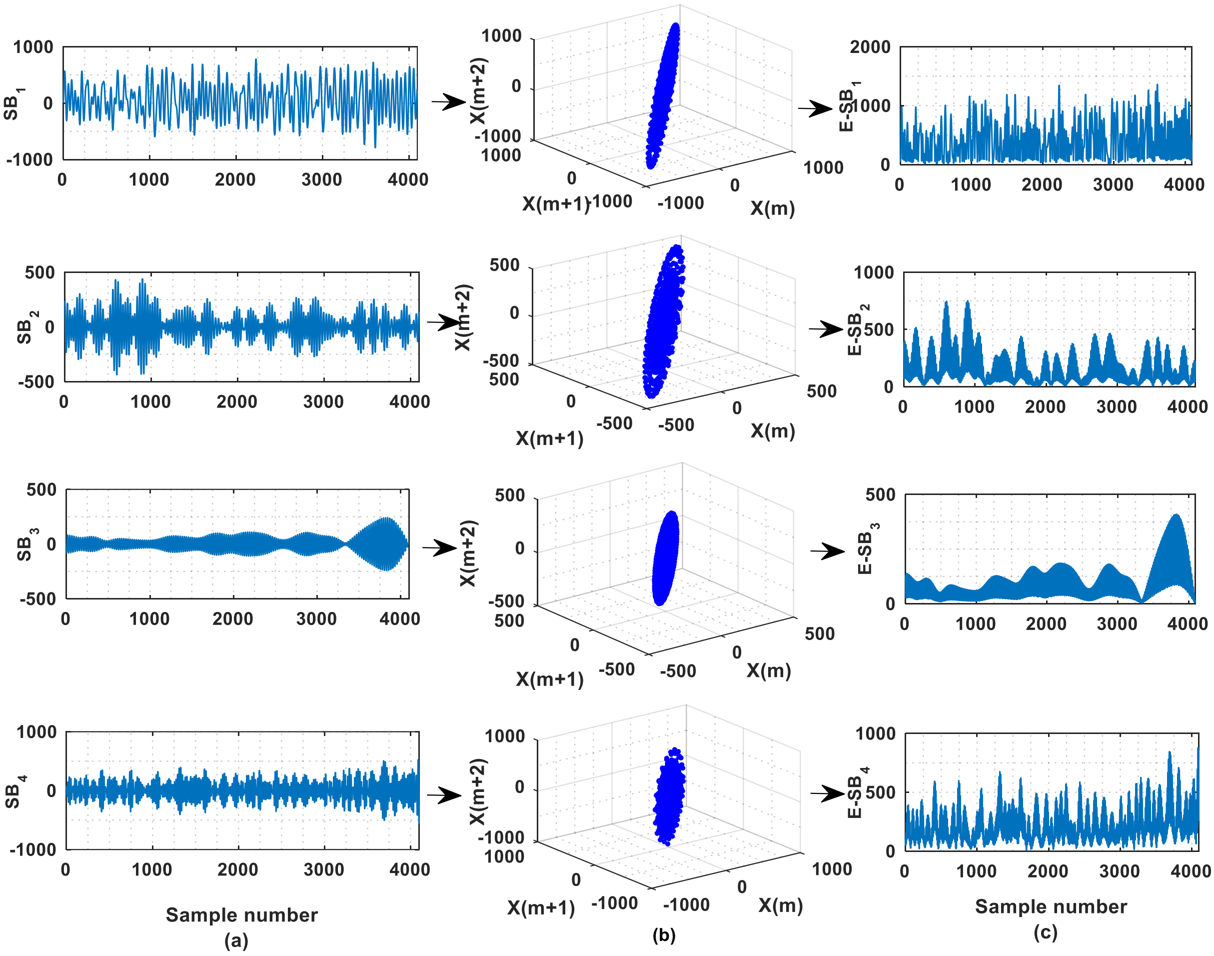

Fig. 3. (First column) represents a plot of the first four sub-band EEG signals (SB_1-SB_4) from FBSE-EWT, (Middle column) represents their 3D PSR plots, and (Last column) represents Euclidian distances of respective 3D PSR plots corresponding to epileptic-seizure EEG signal.

2.4 Feature extraction using Euclidian distances of 3D PSR

The 3D PSR is a graphical presentation of three time-delayed vectors such as $X(m),\ X(m+1),\ X(m+2)$ to view the dynamics of the system. Euclidian distances are distances computed from the delayed vector point as $(X(m),\ X(m+1),\ X(m+2))$ to origin point $(0,0,0)$ in 3D PSR. Mathematically it can be defined as follows [48]:

$$E(m) = \sqrt{X^2(m) + X^2(m+1) + X^2(m+2)} \tag{13}$$

The study presented in [49] also uses the Euclidean concept together with 3D PSR for emotion classification. In this work, the first ten sub-band signals are obtained from all sets of studied database EEG signals using the FBSE-EWT method. Based on the obtained values of Euclidian distances $E(m)$ from each sub-band's PSR vectors using (13), non-linear features like LL, LEEnt, and NEnt are computed. The Euclidian distances computed from obtained 3D PSR of epileptic-seizure EEG (set S) signal is depicted in the last column of Fig. 3. Fig. 3 represents Euclidian distances (E-SB_1 to E-SB_4) of their respective PSR plots. A summary of extracted features with their mathematical expressions are presented in Table 2.

Table 2: Mathematical formulation of all extracted features in this proposed framework

Features	Mathematical equation	Descriptions
LL [34], [50]	$LL_j = \frac{1}{K-1}\sum_{m=1}^{K-1} \text{abs}[E_j(m+1) - E_j(m)]$	abs represents an absolute value operation.
LNEnt [51]	$LEEnt_j = \sum_{m=1}^{K} \log\left([E_j(m)]^2\right)$	
NEnt [34]	$NEnt_j = \sum_{m=1}^{K} [E_j(m)]^P$	P is denoted for power, and its value in this experiment is set to be 1.
Notation: K- is denoted as total length of Euclidean distances of each sub-band and m-index of a particular sample in Euclidean distances of vector $E_j(m)$sub-band, E_j- represents Euclidian distances of j^{th} sub-band signal obtained from 3D PSR vector , here in this study j (j=10), is the total number of sub-band EEG signals obtained using the FBSE-EWT method.		

2.5 Feature ranking

In this experiment, a total of 30 (10 sub-bands×3 features attribute) features are acquired. There may be a possibility that extracted features are redundant or less significant. So, in order to eliminate such features from feature space, Kruskal–Wallis statistical test [52] is used here to analyse the significance of the features. The significant features demonstrate good discrimination, which can be determined by using the *p*-value. The features are tested at 95% of the significant level. All the features whose *p*-values are smaller than 0.05 are considered significant features in this work. It is observed during the experiment that all extracted features are significant. So, to reduce the computational complexity of the proposed method, we have used computed *p*-values to rank these features, to find an optimal set of features.

2.6 Ensemble learning classifiers

In the past, classification of normal and epileptic-seizure EEG signals have been performed using several machine learning classifiers [10], [53], [54] such as SVM, RF [35], k-NN [46], and multi-layer perceptron (MLP) etc. Here, we have experimented on different ensemble learning classifiers, and the results of the highest performing classifiers such as RF, ET, xgBT, B-SVM and B-*k*-NN are presented here, which are employed for classifying seizure EEG signals. In ensemble learning algorithms, some weak classifiers are integrated to build a strong one. Here, we have also compared conventional machine learning classifiers like SVM and *k*-NN with ensemble learning classifiers. All the classifiers used in the proposed work are described below.

2.6.1 Single classifiers

The single machine learning classifiers such as SVM and *k*-NN are commonly used and highly effective in EEG signal classification. SVM classifier works by constructing an optimal hyperplane based on the training dataset. SVM is usually used for a binary classification problem. Still, it can also be used for multi-class classification purposes base on employing the "one-versus-one" or "one-versus-rest “ approach. The *k*-NN classifier works on the principle of labelling the new dataset based on the predefined *k*. A detailed description of the SVM and *k*-NN classifiers is given in [54], [55].

2.6.2 Ensemble learning classifiers

For reducing variance and overfitting in the model, the bagging method is used where original training data is split into many small sampled datasets. After which, each sampled dataset is classified using an individual classifier, and then the final decision is made using voting the prediction of each classifier for improving the stability of the model [56].

In B-SVM, conventional SVM is used as a base learner in the ensemble bagging approach. The SVM with a bagged approach is found to be more effective than conventional SVM [57]. In B-*k*-NN, the *k*-NN classifier is used as a base learner in the ensemble bagging approach. Here *k*-NN is ensembled with bagging re-sampling. RF works on a bagging approach. The main idea behind bagging is to modify the feature matrix using the re-sampling method and train the weak-learner with a new re-sampled dataset. The ET classifier is also known as an extremely randomised trees classifier. It is a new version of bagging in which, unlike random forest classifier, whole training samples are utilised to build each random tree. It increases the randomisation of the RF classifier and also reduces the variance. The xgBT classifier works on parallel tree boosting, which improves the model's speed and performance [58]. A detailed description of the working of B-SVM, RF, ET classifiers is provided in [59]–[61].

2.7 Evaluation matrix

The assessment of the classification performance of the proposed framework is done using the accuracy parameter. Classification accuracy is defined as the rate of correctly predicted samples to the total number of the samples. The mathematical formula to computed classification accuracy is as follows:

$$\text{Classification accurcay } (\%) = \frac{\text{TPs}+\text{TNs}}{\text{TPs}+\text{TNs}+\text{FPs}+\text{FNs}}\% \tag{14}$$

Where TPs, TNs, FPs, and FNs are denoted by true-positive rate, true-negative rate, false-positive rate, and false-negative rate, respectively.

3. RESULTS AND DISCUSSIONS

We have used two different programming languages to implement and evaluate the proposed framework, where pre-processing and feature extraction is done in Matlab, whereas the classification and evaluation step is done using python. In this study, extensive experiments are performed on the publicly available Bonn university EEG database to evaluate the proposed framework using the FBSE-EWT method for classifying seizure EEG signals automatically. Here the FBSE-EWT method is used to decompose EEG signals of the Bonn university EEG database into narrow and non-overlapping sub-band signals of various bandwidth. Ten sub-band signals are obtained from all EEG signals using the FBSE-EWT method. Fig. 2 (a-c) shows all extracted sub-band signals (SB_1 to SB_{10}) from epileptic-seizure and normal EEG signal. It can be observed from Fig. 2 that the amplitude of each epileptic-seizure sub-band signals is much higher than that of normal sub-band signals. Then, Euclidean distances are computed from each 3D PSR of obtained sub-band signals. Fig. 3 shows the computed Euclidean distances from extracted sub-band signals. Non-linear features like LL, LEEnt, and NEnt are computed from the Euclidian distances of each 3D PSR of corresponding sub-band signals. A total of 30 (10 sub-bands × 3 different feature) features are obtained. We have performed the Kruskal-Wallis statistical test to determine the significance of the features to discriminate between normal and seizure EEG signals. Computed *p*-values from all investigated problems (C_1, C_2, and C_3) shows that all extracted features are statistically significant ($p<0.05$). The box-whisker plot of the twenty most statistically discriminate ranked features from the C_1 classification problem is depicted in Fig. 4. Visualisation of these plots confirms the efficacy of extracted features as most of the plots are non-overlapping. The *p*-values of all depicted box-whisker plots in Fig. 4(a)-(t) are 2.50×10^{-44}, 2.82×10^{-44}, 7.58×10^{-44}, 2.15×10^{-40}, 5.32×10^{-37}, 6.15×10^{-37}, 1.13×10^{-36}, 1.21×10^{-36}, 1.84×10^{-36}, 3.97×10^{-36}, 6.65×10^{-36}, 9.49×10^{-36}, 1.50×10^{-35}, 1.15×10^{-34}, 1.23×10^{-32}, 1.85×10^{-32}, 2.01×10^{-32}, 3.17×10^{-32}, 8.25×10^{-32}, and 1.00×10^{-31}, respectively. We can observe that the *p*-values mentioned above are almost closer to zero, which indicate that the extracted features can discriminate between normal and epileptic-seizure EEG signals. The magnitude of the normal class features is found to be lower than that of seizure class features in all box-whisker plots.

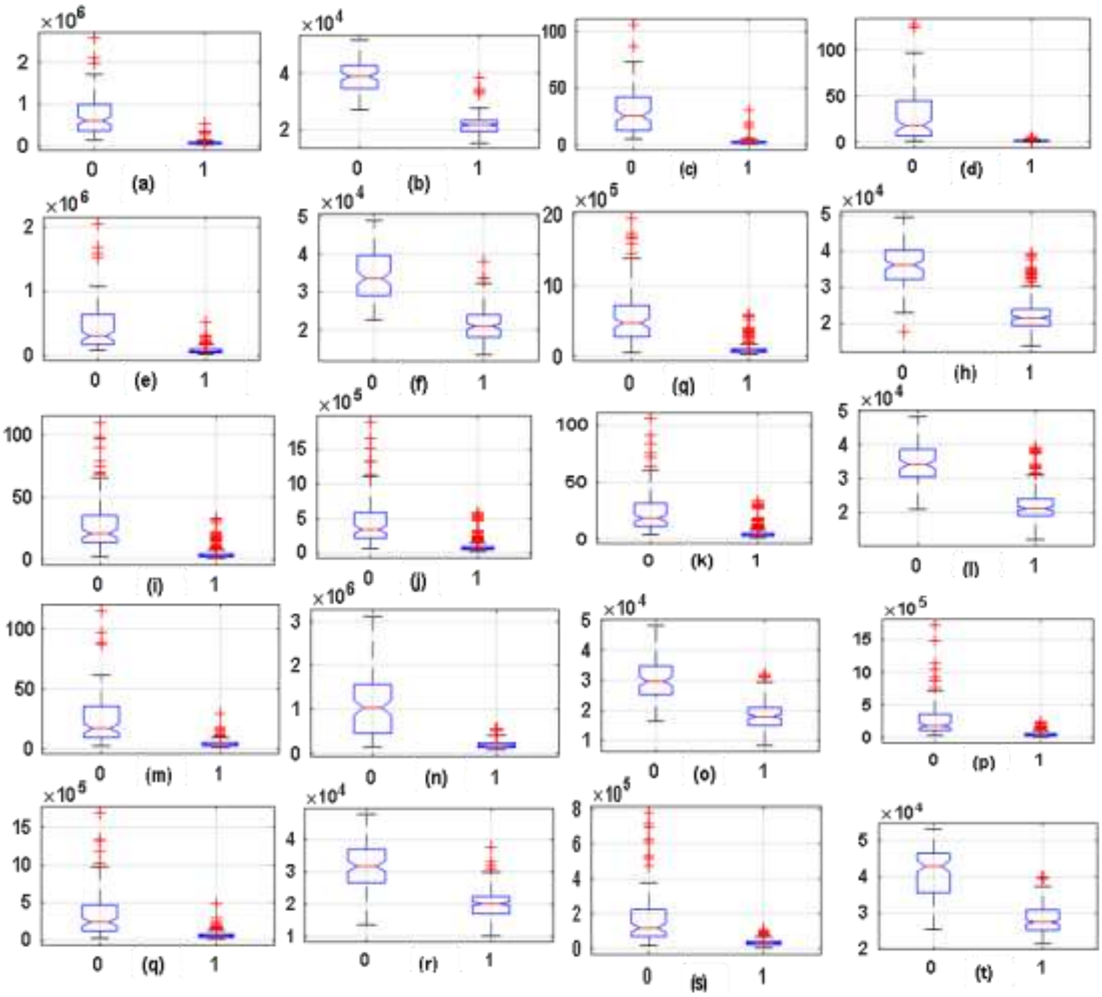

Fig. 4: Box-whisker plots from ((a)-(t)) represent the topmost ranked twenty statistically discriminant features for classifying seizure-normal classes of EEG signals from **C_1**: S-ZO corresponding to highest classification accuracy. The notation used in each box-whisker plot, such as ‘0’ denotes seizure class and ‘1’ represents a normal class.

Here, we have implemented our proposed framework for classifying seizure EEG signals in python. After the statistical test, all the discriminant features are ranked based on computed *p*-values and then obtained feature vector passed as the input to various ensemble machine learning classifiers (SVM, B-SVM, *k*-NN, B-*k*-NN, ET, RF, xgBT) for training purpose. Each classifier has some parameters that affect its performance, which must be optimised to improve the classifier's performance. Therefore, parameter optimisation of classifiers is done using a grid search with a cross-validation method. The classifier's parameters used for optimisation are mention in Table 3. Classifiers with optimised parameters are trained using a ten-fold cross-validation method to ensure robustness and the reliability of the proposed method's performance. Where obtained feature vector is randomly segmented into ten subsets in which one subset out of ten is kept for testing, and the remaining subsets are used for training the model. The whole process is iterated ten times, and in each iteration, accuracy is computed, and finally, average classification accuracy is considered as a classification performance of the classifier. The average classification accuracy parameter is used to evaluate the performance of each classifier.

Table 3: Classifier's hyper-parameters used in this experiment

Classifiers	**Parameters**
SVM	C (0.1, 1, 10, 100, 1000) , gamma (1, 0.1, 0.001, 0.001, 0.0001), decision_function_shape (one-versus-rest).
B-SVM	n_estimators (300), with radial basis function (RBF) kernel.
k-NN	k (1 to 30), leaf_size (1,50), and distance formula *p*-(1,2) here if *p*=1, then it uses Manhattan distance otherwise Euclidean.
B-*k*-NN	n_estimators (300)
RF	n_estimator (500, 1000, 1500, 2000), maximum_depth (5,10,15,20,30), min_sample_splitby (2,5,10)
ET	n_estimator (500, 1000, 1500, 3000), max_depth (10,15,20), max_features (8,12,18)).
xgBT	n_estimator (100, 200, 300, 400, 500, 1000), learning rate (0.0001, 0.001, 0.01, 0.1, 0.2, 0.3))

Fig. 5 demonstrates the classification performance of all investigated classification problems (C_1, C_2, and C_3) obtained from various ensemble learning classifiers on ranked features based on *p*-values. From Fig. 5 (a). It is observed that the maximum classification accuracy with twenty ranked features from the ET classifier is 100%, which is shown with a red colour horizontal dotted line in the figure. Fig. 5 (b) shows that B-SVM achieves the highest classification accuracy of 98.3 % from the problem (C_2) using twenty-two ranked features. Whereas Fig. 5(c) shows that from the classification problem (C_3), the highest achieved classification accuracy is 97.7% using ET classifiers with thirty ranked features. Above mentioned results are also summarized in Table 4. The highest classification accuracy achieved from each investigated classification problem is shown in bold text in Table 4. Table 4 concludes that the highest classification accuracy from investigated classification problems is 100%, 98.3% and 97.7% using ET, B-SVM and ET, respectively. Compared to all other classifiers, the performance of *k*-NN and B-k-NN are poor in all classification problems. As ensemble learning classifiers are expected to perform better than single machine learning classifiers same is also observed in this study, where B-SVM performs better than a single SVM. But it is not observed with the *k*-NN classifier. B-*k*-NN performs less than a single *k*-NN classifier because *k*-NN is a stable classifier. Any small changes in the training dataset do not influence the *k*-NN classifier's predictions [62].

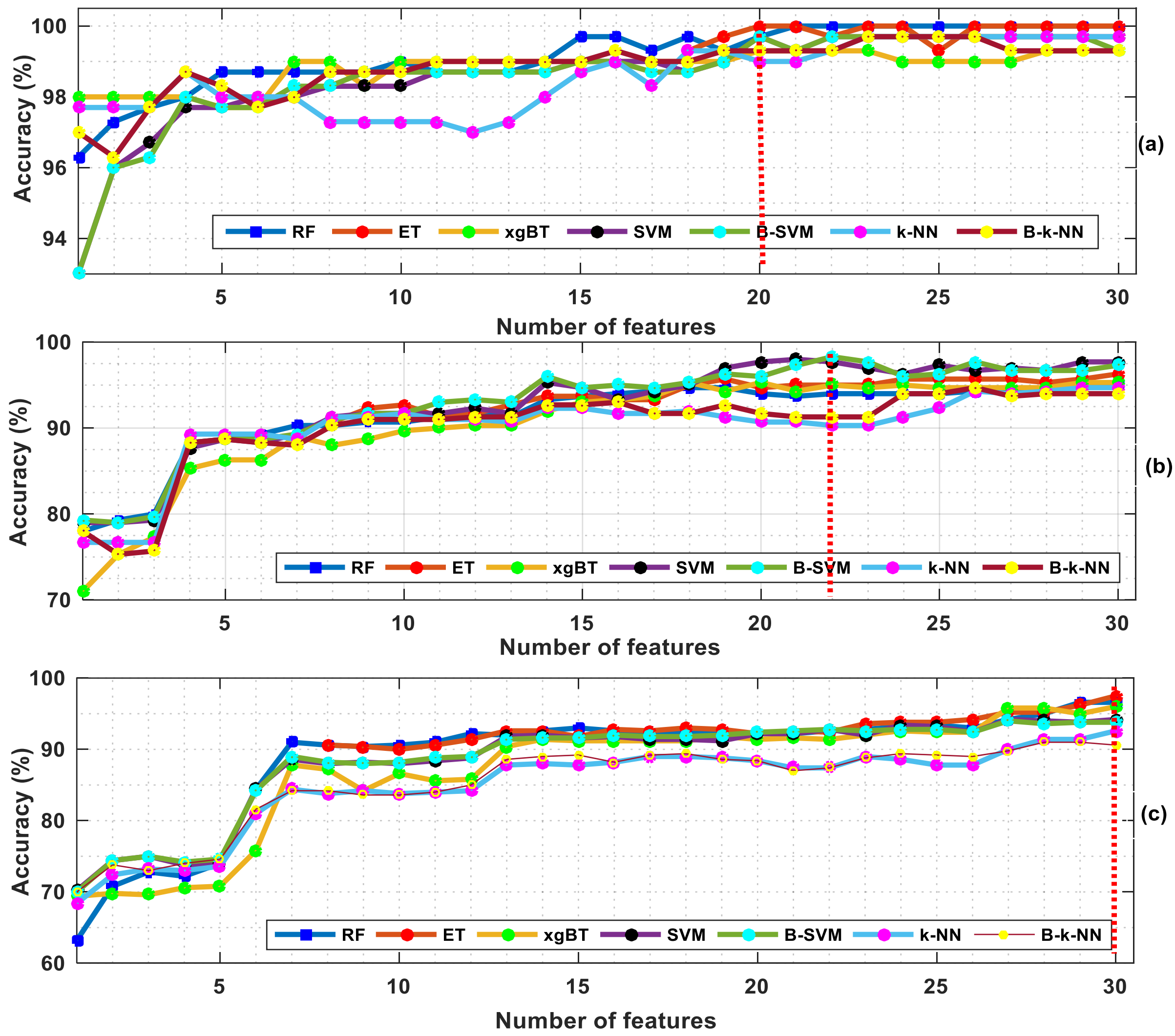

Fig. 5: Obtained average classification accuracy (%) from statistically significant ranked features from (a) **C_1** (S-ZO), (b) **C_2** (S-Z-N), and C_3 (S-ZO-NF) EEG signals classification problems, respectively.

Table 4: Highest classification accuracy (%) of the proposed framework in the classification problems (C_1, C_2, and C_3) with an optimal selected number of features (SNF).

Classification task	ET	RF	xgBT	SVM	B-SVM	*k*-NN	B-*k*-NN
C_1: (S-ZO)	**100**	100	99.3	99.7	99.7	99.7	99.7
C_1: SNF	**20**	21	20	20	20	23	23
C_2:(S-Z-N)	96.3	95.0	95.3	98.0	**98.3**	94.7	94.7
C_2: SNF	30	30	18	21	**22**	29	26
C_3: (S-ZO-NF)	**97. 7**	96.6	96	94.2	94.5	92.6	91.0
C_3: SNF	**30**	29	30	27	27	30	28

The classification accuracy is one of the most used evaluated parameters for evaluating the performance of the epileptic-seizure EEG signals detection system, whereas some other parameters can also be used to evaluate the performance of the proposed framework. To propose a perfect classification method, it should perform well in all the classification performance parameters such as overall accuracy, precision, recall, F1-score, etc. Therefore, we have also used precision, recall, and F1-score parameters, along with accuracy, to assess the proposed method's performance mentioned in Table 5. The parameter values demonstrated in Table 5 are obtained from those models, which achieve the highest classification accuracy with the minimum number of features in each classification problem (from Table 4). Table 5 consists of the confusion matrix, and the remaining columns of the table show classification accuracy, precision, recall, and F1-score of each class. The highest classification accuracy of 100% is achieved using the ET classifier from classification problem C_1. Table 5 shows that all the parameters like accuracy, precision, recall, and F1-score for classification problem C_1 achieve 100%. But, in the classification problem C_2 and C_3, the performance decreases because of the misclassification of seizure-free EEG signals. From Table 5, we can conclude that the proposed method for detecting automated epileptic-seizure EEG signals performs well in all the performance evaluated parameters. Hence, the proposed technique can be considered as a suitable tool for the diagnosis of epileptic-seizure.

The classification performance of the proposed seizure EEG signal detection method is also compared with other state-of-art methods available in the literature. For a significant comparison, results obtained from the Bonn university EEG database is only considered. Table 6 represents some of the recent methods, investigated classification problems, and their obtained accuracy values. From Table 6, it is clearly noticed that the proposed method, which uses Euclidean distances based features obtained from the 3D PSR sub-band signals using the FBSE-EWT method for classifying seizure EEG signals, performs better than other methods in all classification problems. Compared to other methods, the proposed method shows almost 3% improvement in classification accuracy of classification problem C_1, similarly 3.55% in classification problem C_2, and 2.1% in C_3. In recent works [53], [54], [63], [64], authors have used DWT to decompose EEG signals. The major drawback of DWT is that it uses a pre-fixed function for analysing signals which makes it non-adaptive. In another recent work [65], the authors have computed temporal and spectral features from EMD sub-band signals for classifying epileptic-seizure EEG signals. In EMD, frequency components are not estimated correctly because it has the problem of mode-mixing. However, the proposed methodology uses an adaptive method (FBSE-EWT) for analysing the signals, which provides better narrow sub-band EEG signals. It should be noticed that using the FBSE spectrum by replacing the conventional Fourier spectrum used in EWT provided a better spectrum representation of the EEG signals.

Table 5: Confusion matrix and classification performance of ET and B-SVM with the optimal number of features in C_1, C_2, and C_3, respectively.

ET classification performance of **C_1: (S-ZO)**							
Class	**Confusion matrix** Seizure (0)	Normal (1)		Accuracy (%)	Precision (%)	Recall (%)	F_1-score (%)
Seizure (0)	100	0		100	100	100	100
Normal (1)	0	200		100	100	100	100
				Overall accuracy=100			

B-SVM classification performance of **C_2: (S-Z-N)**							
Class	**Confusion matrix** Seizure (0)	Normal (1)	Seizure-free (2)	Accuracy (%)	Precision (%)	Recall (%)	F_1-score (%)
Seizure (0)	99	0	1	99.33	99.0	99.0	99.0
Normal (1)	0	99	1	99	99.0	98.0	99.0
Seizure-free (2)	1	2	97	98.33	97.0	98.0	97.0
				Overall accuracy=98.3			

ET classification performance of C_3: **(S-ZO-NF)**							
Class	**Confusion matrix**			Accuracy (%)	Precision (%)	Recall (%)	F_1-score (%)
	Seizure (0)	Normal (1)	Seizure-free (2)				
Seizure (0)	99	0	1	99.2	99.0	97.0	98.0
Normal (1)	0	196	4	98.6	98.0	98.0	98.0
Seizure-free (2)	3	3	194	97.8	97.0	97.0	97.0
				Overall accuracy=97.7			

Table 6: Comparative analysis of the proposed framework with other reported methodologies studied on the Bonn university EEG database.

Authors/year	**Method**	**Classification problems**	**Accuracy/ other perfomance mesures (%)**
Acharya et al. [66] / 2011	Recurrence quantification analysis (RQA) with SVM	S-Z-N	95.6
Orhan et al. [67] / 2011	DWT and MLPNN	S-Z-N S-ZO-NF	96.67 95.60
Abualsaud et al. [53] / 2015	DWT, conventional statistical features, and ensemble classifier	S-Z-N	90
Riaz et al. [65] / 2016	EMD based temporal and spectral features with SVM	S-Z-N S-ZO-NF	recall-85.0 recall-83.0
Tawfik et al. [63] / 2016	DWT, weighted permutation entropy and non-linear SVM	S-Z-N	97.50
Sharmila et al. [54] / 2016	DWT and Naive Bayes	S-ZO	99.12
Acharya et al. [68] / 2018	Convolutional neural network	S-Z-N	88.7
Gupta et al. [12] / 2019	FBSE based rhythms, weighted multiscale Renyi permutation entropy (WMRPE) and LS-SVM	S-Z-N	97.3
Raghu et al. [20] / 2019	Matrix determinant of *k*-NN and MLP	S-ZO S-Z-N S-ZO-NF	97.10 94.75 96.5
Gupta et al. [35] /2019	FBSE-EWT based Hilbert marginal spectrum (HMS), LL, log, and NEnt feature, RF	S-ZO	97.7
Nabil et al. [64] / 2020	DWT, Approximate entropy, LLE, max, min, std, mean, and multi-class SVM	S-ZO S-ZO-NF	98.50 96.80
Dash et al. [69] /2020	TQWT, entropy features, Hjorth parameter, and hidden Markov model (HMM) classifier	S-ZO	99.58
Proposed method	**LL, LEEnt, and NEnt features of Euclidian distances of 3D PSR of sub-bands using FBSE-EWT method and ensemble classifiers**	S-ZO S-Z-N S-ZO-NF	**100** **98.3** **97.7**

4. CONCLUSION

In this study, the decomposition of EEG signals in terms of narrow sub-band signals has been utilized for feature extraction because of the non-linear and non-stationary characteristic of EEG signals. It has been observed that obtained sub-band signals are suitable to generate non-linear feature-space based on Euclidean distances computed from the 3D PSR plots for classifying seizure EEG signals. We have proposed novel features based on Euclidean distances computed from 3D PSR of sub-bands of EEG signals for automatic identification of epileptic-seizure and seizure-free EEG signals. Fixed values of time lag and embedded dimension have been investigated for constructing PSR of extracted sub-band signals. Three non-linear features like LL, LEEnt, and NEnt from Euclidian distances of 3D PSR of sub-band EEG signals are considered an input feature vector for various ensemble learning classifiers for classifying epileptic-seizure EEG signals. The performance of different ensemble learning classifiers like RF, ET, xGBT, B-SVM and B-k-NN have been evaluated and also compared with single machine learning classifiers such as SVM and *k*-NN. One of the most noticeable features of this proposed method is that we have evaluated the model's performance using four classification measures such as accuracy, precision, recall, and F1-score in this study. Remarkably, it can be said that this proposed method performs well in all four measures in each class. A suitable selection of signal processing technique, the number of decomposed sub-band signals, an appropriate combination of non-linear features, and hyper-tunning of ensemble classifiers have provided good classification accuracy from all investigated classification problems, which makes it suitable for real-time implementation. Before applying this proposed method for clinical purposes, it is advisable to examine it on a big dataset. In future, it would be interesting to study this proposed framework of classifying seizure EEG signals in the context of developing a computer-aided diagnosis for classifying other biomedical signals. New features can be developed and investigated ahead with these proposed features in upcoming work to enhance the model's classification accuracy for classifying seizure EEG signals. In future studies, we can also try integrating several ensemble learning based classification models for improving algorithmic performance. Other than manual feature extraction, the deep learning approach for automatic feature extraction can also be used to enhance the performance of automatic epileptic-seizure EEG signals classification.

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