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A New Crossover Operator in Differential Evolution for Numerical Optimization

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Abstract: **The first pioneering study on Evolutionary Computation was studied by Holland (1975). Since the inception of Evolutionary Computation, researchers are constantly focusing on developing new methodologies, techniques, operators etc. for better convergence of the evolutionary algorithms. In the present paper, we propose a new crossover operator in Differential Evolution (DE) which has an effective convergence and enhances the efficiency of the optimization strategy, used for numerical data.**

***Keywords* -- Differential Evolution (DE), Crossover operator, Numerical Optimization, Optimization.**

I. INTRODUCTION

In the evolutionary approach, the dataset that needs to be clustered is seen as an optimization problem and is solved by using evolutionary search heuristic such as genetic algorithm [1] which is inspired by Darwinian Theory of '*survival of the fittest*'. The key idea is to create a population of candidate solutions to an optimization problem which is iteratively refined by alteration and selection of good solutions for the next iteration. A fitness function evaluates the quality of candidate solutions with respect to the optimization problem. In *GA's* the alteration in the population is achieved with the help of mutation and crossover that helps in exploring large solution space. An important advantage of these algorithms is their ability to cope with local optima by maintaining, recombining, and comparing several candidate solutions simultaneously. Very few studies considered alternative stochastic search heuristics other than *GA's* or simulated annealing. Two promising algorithms for numerical optimization, which are hardly known outside the search heuristic field, are particle swarm optimization (PSO) [2, 3, 4] and differential evolution [5, 6]. Studies [6, 7] show that DE is clearly and consistently superior as compared to GA's and PSO. Along with good performance, *DE* is easy to implement, requires less parameter tuning as compared to other stochastic approaches [6, 8, 9]. In the present work a new crossover operator for differential evolution has been proposed and validated against low dimensional numerical data, for which it has shown promising results. The objective function defined for the numerical dataset is a minimization problem.

The rest of this paper is organized as follows: Section II includes a brief introduction to Differential Evolution and an introduction to the problem of optimization. Section III discusses the working of DE. In section IV the new crossover operator is introduced. Section V and VI illustrates the problem statement and the experimental results. Finally in section VII & VII, the results are concluded and future work is presented, respectively.

II. DIFFERENTIAL EVOLUTION

In 1995, Storn & Price [5] proposed a new floating point encoded evolutionary algorithm for global optimization and named it Differential evolution (DE) owing to a special kind of differential operator, which they invoked to create new offspring from parent chromosomes, instead of classical crossover or mutation. The algorithm gained popularity because of its ease of implementation and negligible parameter tuning. DE is suitable for problems where objective functions are non-linear, non-differentiable, noisy, flat, multi-dimensional, have many local minima, multiple constraints or stochasticity [10].

Optimization is a very important phase of any data processing model. Since its inception in 1995, DE has drawn the attention of many researchers all over the world resulting in a lot of variants of the basic algorithm with improved performance. Differential Evolution is one of the most powerful stochastic real- valued optimization algorithms in current use. Unlike traditional evolutionary algorithms, Differential Evolution variants induce variety by scaling the differences of randomly selected population vectors and do not use any separate probability distribution for generating offspring. The optimization problem that is considered here is in the following form, where a *local minimum* x^* is defined as a point for which there exists some $\delta > 0$ so that for all x such that

$$\| x - x^* \| \le \delta;$$
$$\text{the expression}$$
$$f(x^*) \le f(x)$$

Holds on some region around x^* all of the function values are greater than or equal to the value at that point.

The aim of optimization is to determine the best suited solution to a problem under a given set of constraints. Several researchers over the decades have come up with different solutions to linear and non-linear optimization problems. Mathematically, an optimization problem involves a fitness function describing the problem, under a set of constraints representing the solution space of the problem. Most of the traditional optimization techniques revolve around evaluating the first derivatives to locate the optima on a given constrained surface. To locate the optima for many rough and discontinuous optimization surfaces, several derivative free optimization algorithms have emerged. Differential evolution is one of those intelligent optimization strategies which employ one or more agents to determine the optima on a search landscape, representing constrained surface for the optimization problem [5].

Use of differential evolution as the optimization technique successfully fulfills the requirements of a practical minimization technique [5,6]. These requirements are as follows:

1. Ability to handle non-differentiable, non-linear and multimodal cost functions.
2. Parallelizability to cope with computation intensive cost functions.
3. Ease of use, i.e., few control variables to steer the minimization. These variables should also be robust and easy to choose.
4. Good convergence properties, i.e., consistent convergence to the global minimum in consecutive independent trials.

The motivation behind the present work is to study the convergence of Differential Evolution for the optimization of low dimensional numerical data. In the present paper we discuss Differential Evolution as a numerical optimization technique with improved crossover operator that helps in inducing variety and selection of pseudo-random population vector that affects the convergence.

III. DIFFERENTIAL EVOLUTION WORKING

DE is a population based stochastic meta-heuristic for global optimization on continuous domains. The characteristic of DE is that at every generation it constructs a *mutant vector*, for each element of the given population,. The mutant vector is constructed through a mutation operation based on adding differences between randomly selected elements of the same population to another element. For instance, in classical DE, a *mutant vector "y"* is constructed from a current population $\{x_1, x_2, x_3, \ldots, .x_m)$ in the following manner:

$$y = x_{r1} + f_i * (x_{r2} - x_{r3})$$

where x_{r1} , x_{r2} and x_{r3} are distinct random indices selected from the current population $\{1, \ldots.., m\}$ where f_i is a scalar factor usually $\in$ [0,1] is varied as follows [8]:

$$f_i = 0.5 * (1 + \mathrm{rand}(0,1))$$

Based on the mutant vector, a *trial vector* is constructed through a crossover operation which combines the components of the present vector and the mutant vector, according to a control parameter $Cr \in [0,1]$ called *crossover rate*. The trial vector is compared with the current population element and the best one, with respect to the objective function, is admitted to the next generation [11].

Differential evolution is selected for optimization procedure. Differential evolution is one of those intelligent optimization strategies which employ one or more agents to determine the optima on a search landscape, representing constrained surface for the optimization problem [5]. Use of differential evolution as the optimization technique successfully fulfills the requirements of a practical minimization technique.

IV. PROPOSED CROSSOVER OPERATOR

For exploring a large solution space for a given vector the crossover operator has been modified. The variation in the crossover operator, in successive iterations, is determined by the mean value and the current value of the scale factor at the i^{th} year. The variation is also controlled by a random number *(rand)* in the range [0, 1] .The *crossover rate Cr* is varied in a random manner which is determined by using the relation

$$Cr = f_i * \mathrm{rand}(1 - f_{min}) \qquad (1)$$

where, f_i is the value of the scale factor F at i^{th} year , *rand* random number in the range [0,1] and f_{min} is the minimum value of the scale factor F.

Crossover increases the potential diversity in a population. It enables the system to turn up with a new generation of solution that inherits some of the functionalities/behavior from the parent generation and some from the random behavior of the optimization technique.

V. PROBLEM STATEMENT

In this paper, Differential Evolution has been applied for optimization of numerical data. Most of the traditional optimization techniques revolve around evaluating the first derivatives to locate the optima on a given constrained surface. A practical minimization problem of optimizing the numerical data is thus used. The car road accidents from year 1974-2004 in Belgium [12] have been used to test the efficiency & convergence of the proposed crossover operator in Differential Evolution.

VI. EXPERIMENTAL RESULTS

In this paper, the *optimization of numerical data* on real world data sets of Car road accidents in Belgium has been studied. In DE, the parameters such as population size, number of iterations, crossover rate and scaling factor f are set. The population size is almost twice the number of dimensions for any particular application. The number of iterations was initially set to 30 that did not give promising results. So the final number of iterations was set to 40. Furthermore, the scale factor f is varied in random manner as explained in Section III. Table I provides the numeric data of yearly car road accidents in Belgium from year 1974-2004. To measure the accuracy, mean square error is used as a descriptive measure to monitor the performance of the model. It provides a single, easily interpreted measure of model's accuracy. The mean square error (MSE) is computed as follows:

$$\mathrm{MSE} = \frac{\sum_{i=1}^{n}(A_i - F_i)^2}{n} \qquad (2)$$

Where, A_i denotes the actual value and F_i denotes the optimized value of year (i), respectively. The computed mean square error is 247.9 that effectively show a good optimization, i.e., the accuracy of optimization is nearly 90%. A comparison of the actual data with the optimized values can be figured out from figure 1. The crossover operator has certainly proved efficient for optimizing the *numerical* data values. The cause of concern is mainly the initial value that loses its bound when the population is randomly initialized during the procedure of differential evolution.

Table I: Car road accidents from 1974-2004 in Belgium

Year	Number of Accidents
2004	953
2003	1035
2002	1145
2001	1288
2000	1253
1999	1173
1998	1224
1997	1150
1996	1122
1995	1228
1994	1415
1993	1346
1992	1380
1991	1471
1990	1574
1989	1488
1988	1432
1987	1390
1986	1456
1985	1308
1984	1369
1983	1479
1982	1464
1981	1564
1980	1616
1979	1572
1978	1644
1977	1597
1976	1536
1975	1460
1974	1574

Table I shows the numerical data set of the actual number of car road accidents in Belgium from year 1974-2004.

Since Differential Evolution is a population based stochastic technique, in this paper, we have tried to optimize these data values so as to reach the maximum amount of accuracy with the proposed crossover operator (section IV). The Differential evolution has the usual phases of initially randomizing the initial population (initialization). Then for each individual data vector a mutant vector (mutation) is generated and then a trial vector is computed using the proposed crossover operator (section III).

Table II shows the optimized numerical data obtained after the optimization using the new crossover operator from year 1974-2004.

Table II: Optimized values from year 1974-2004

Year	Actual Number	Optimized Number
2004	953	1000
2003	1035	1035
2002	1145	1145
2001	1288	1288
2000	1253	1253
1999	1173	1173
1998	1224	1224
1997	1150	1150
1996	1122	1122
1995	1228	1228
1994	1415	1415
1993	1346	1346
1992	1380	1380
1991	1471	1471
1990	1574	1574
1989	1488	1488
1988	1432	1432
1987	1390	1390
1986	1456	1456
1985	1308	1308
1984	1369	1369
1983	1479	1479
1982	1464	1464
1981	1564	1564
1980	1616	1616
1979	1572	1572
1978	1644	1644
1977	1597	1597
1976	1536	1536
1975	1460	1460
1974	1574	1500

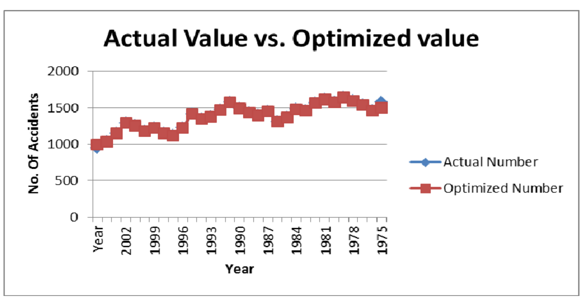

Figure 1: Actual value vs. Optimized values

Figure 1 shows the graphical representation of the optimization technique with effective results.

VII. CONCLUSION

In this paper, a new crossover operator in Differential Evolution has been proposed and its convergence with low dimensional numerical data (number of car road accidents in Belgium from year 1974-2004) has been studied .The performance of the proposed crossover operator can be accounted with the resultant value of MSE = 247.9. The new crossover operator has also been applied for the optimization of other numerical data set of the Population of India from year 1930-2000 [13] & the population of Delhi with two dimensions i.e. for males and females. The new crossover operator has certainly improved optimization which is validated against low dimensional numerical data sets. The results can be observed from figure 2, 3&4 respectively. The purpose of using yearly dataset is to study the effect of the proposed crossover operator for optimization of numerical data. The convergence of the operator can be studied on other exponential data as well.

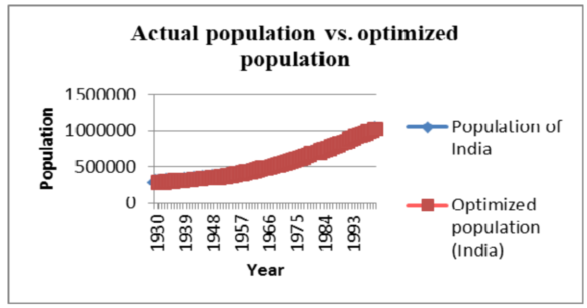

Figure 2: Population of India vs. optimized population

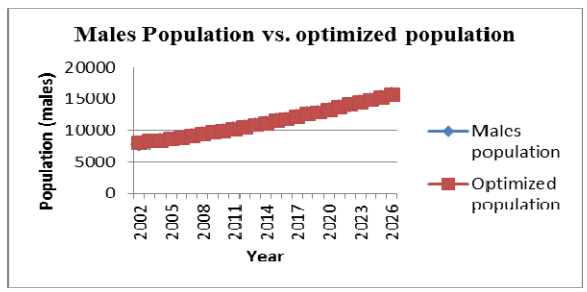

Figure 3: males population (Delhi) vs. optimized population (males)

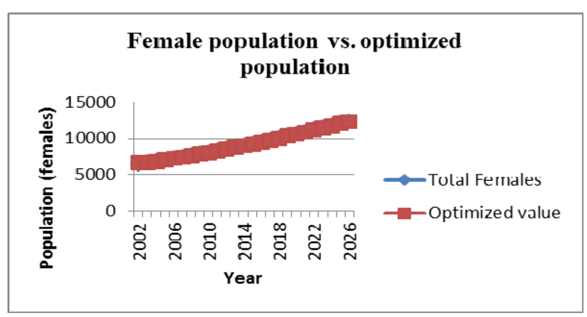

Figure 4: Female population (Delhi) vs. optimized population (females)

VIII. FUTURE WORK

Although DE has shown promising results, the factor affecting the convergence can be investigated further. In this paper the effect of random value of scale factor is taken into account that effectively improves the performance of crossover rate. DE's vector generation and the variation in control parameters (crossover rate, *Cr* and scale factor, *F*) can lead to faster convergence and can be investigated more deeply. The data used in this approach is uni-dimensional and the approach can be extended to study the working of the proposed crossover operator with multi-dimensional data.

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