

# Digital Modulation Classification Under Non-Gaussian Noise Using Sparse Signal Decomposition and Maximum Likelihood

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**Abstract**—In recent years, automatic signal detection and modulation classification play a vital role in the field of cognitive radio applications. The majority of the existing signals detection and classification methods assume that the received signal is contaminated by additive white Gaussian noise. Under impulsive noise condition, the performance of the traditional modulation classification methods may be degraded. Therefore, in this paper, we investigate the application of sparse signal decomposition using an overcomplete dictionary for detection and classification of digital modulation signals. The overcomplete hybrid dictionary consists of impulse waveform and sine and cosine waveform for effectively capturing morphological components of the impulse noise and deterministic modulated signals. The proposed modulation classification method includes the following steps: sparse signal decomposition (SSD) on hybrid dictionaries, modulated signal extraction, matched filtering, and maximum likelihood (ML) classification. The performance of the direct ML and SSD-based ML classification methods are tested and validated using different modulation techniques under different Gaussian and impulse noise conditions. The proposed system achieves a classification accuracy of 89 percent at 0 dB SNR and hence outperforms the direct ML method.

## I. INTRODUCTION

In recent years, automatic modulation classification (AMC) has taken a potential growth due to its usage in a variety of wireless communication application such as cognitive radio, electronic warfare and surveillance etc. [1]. AMC refers to identification of modulation scheme of a received signal corrupted with noise. The design of the modulation classification algorithm is very challenging as it has little or no knowledge of the signal transmitted [2].

The AMC algorithms can be commonly grouped into two broad categories: (i) likelihood-based (LB) decision-theoretic method and (ii) feature-based (FB) pattern recognition method. In likelihood-based method, the classification is viewed as a multiple hypothesis-testing decision-making problem. LB based AMC gives an optimal solution. It classifies the modulation scheme by selecting the type of modulation which maximizes the likelihood function of the received signal [3]. Most of the modulation classification algorithms considered additive white Gaussian noise in the performance analysis [3]. This assumption is not always sufficient. There are many cases where noise in the communication channel is non-Gaussian. For example, the noises from various natural and man-made sources found in radio channels are non-Gaussian

and exhibit sharp spike, i.e. they show impulsive characteristic [5]–[8]. The algorithms designed and developed for optimal results in Gaussian noise, exhibit worse performance in non-Gaussian noise environment [8]. Furthermore, it is found that the algorithms designed for white additive noise do not provide optimum results when additive noise is time correlated [6]. Most of the classification algorithms proposed in non-Gaussian environment are feature based [2], that is, they utilize modulation dependent features of the signal [9]–[11]. If the features are not robust or properly designed, their performance degrades and hence they are suboptimal. In literature, a few previously published works addressed the recognition of modulation under non-Gaussian noise environments [2], [6], [12]. In [6], the classifier uses the whitening filter to reduce complexity of ML in presence of non-Gaussian noise, where it estimates the unknown parameter and whitening filter coefficients. The estimation approach degrades the performance of classifier as it is difficult to model it precisely. In [12], the channel and noise distribution are considered to be known. This motivates us to redesign a ML classifier which can perform well in non-Gaussian environment.

Recently, sparse representation (SR) has taken a substantial growth in many signal processing applications including denoising, signal separation, compression in audio and biomedical signal processing [16]–[19]. In this paper, we propose a novel detection based classification algorithm to identify the modulation scheme with additive mixed Gaussian and impulse noise using sparse signal decomposition with an overcomplete hybrid dictionary. Firstly, using SSD impulse is removed from the received signal followed by classification of received signal using maximum likelihood based classifier.

The rest of the paper is organized as follows. In section II, we focus on the system model and motivation behind the ML classification in the presence of non-Gaussian additive noise. The proposed method is presented in section III, which discusses the impulse removal using SSD of the received signal followed by ML classification. The simulation results have been presented in section IV. Conclusions are drawn in Section V.

## II. BACKGROUND AND MOTIVATION

Let  $\mathbf{r}(t)$  be the received signal of interest whose modulation format needs to be recognized. It can be expressed as

$$\mathbf{r}(t) = \mathbf{s}(t) + \mathbf{w}(t), \quad 0 \leq t \leq LT \quad (1)$$

where  $\mathbf{s}(t)$  is the transmitted signal,  $\mathbf{w}(t)$  is the additive white Gaussian noise,  $L$  is the number of symbols and  $T$  is the symbol period.

In an amplitude phase modulated digital communication system, information is carried by utilizing phase and amplitude information during a symbol interval. As shown in [10], it can be expressed as:

$$\mathbf{s}(t) = \Re \left\{ \sum_{u=-\infty}^l A \hat{S}_u p(t - uT) e^{j(2\pi f_c t + \theta_c)} \right\}, \quad (2)$$

$$lT < t \leq (l+1)T, \quad l = 0, 1, 2, \dots$$

where  $T$  is the symbol period,  $A$  is the signal amplitude,  $f_c$  and  $\theta_c$  are the carrier frequency and phase offset respectively,  $p(t)$  is the transmitted pulse shape,  $\hat{S}_u$  is assumed to be a value from  $M$  set of complex numbers ( $S_1, S_2, \dots, S_M$ ) in the constellation of modulation scheme. By considering that it has unity average power, i.e.,  $\sum_{m=1}^M |S_m|^2 = 1$ , where  $M$  is the number of constellation points in a particular modulation scheme. It can be noted that, for this family of signals, the instantaneous frequency does not change in each symbol which provides a similarity between the complex domain and time domain representation of the signal. Due to this existing similarity, we can map a time domain received signal back into the complex domain to compare with the set of given constellation. If sufficient statistical information about the signal and communication channel is available, likelihood tests can be used to map the received signal to the library of the constellation.

The ML Classifier presumes the given ideal conditions

- Carrier frequency ( $f_c$ ), reference phase ( $\theta_c$ ), signal amplitude ( $A$ ) and symbol period ( $T$ ) is known.
- All the data symbols are independent of each other.
- The carrier frequency  $f_c$  is a multiple of symbol rate.
- The additive noise is white and Gaussian with known power density  $N_0$ .
- The received signal and noise are independent of each other.

By assuming a perfect symbol synchronization, the output of the quadrature receiver is the sufficient statistic for the maximum likelihood classifier. From [10], in-phase ( $r_{I,l}$ ) and quadrature ( $r_{Q,l}$ ) components of the quadrature receiver are

$$r_{I,l} = \int_{(l-1)T}^{lT} \mathbf{r}(t) \cos(2\pi f_c t + \theta_c) dt \quad (3)$$

$$= \frac{AT}{2} \Re\{\hat{S}_l\} + w_{I,l}$$

$$r_{Q,l} = \int_{(l-1)T}^{lT} \mathbf{r}(t) \sin(2\pi f_c t + \theta_c) dt \quad (4)$$

$$= \frac{AT}{2} \Im\{\hat{S}_l\} + w_{Q,l}$$

where

$$w_{I,l} = \int_{(l-1)T}^{lT} \mathbf{w}(t) \cos(2\pi f_c t + \theta_c) dt \quad (5)$$

$$w_{Q,l} = \int_{(l-1)T}^{lT} \mathbf{w}(t) \sin(2\pi f_c t + \theta_c) dt \quad (6)$$

Assuming noise is white and Gaussian, the noise component  $w_{I,l}$  and  $w_{Q,l}$  are zero-mean white Gaussian having variance  $\sigma^2$ . Without loss of generality, in the analysis we consider the case where  $w_{I,l}$  and  $w_{Q,l}$  are independent.

By considering  $L$  number of symbols of complex data for the time interval  $[0, LT]$ , [10]

$$x_l = r_{I,l} - jr_{Q,l}, \quad l = 1, 2, \dots, L \quad (7)$$

$$w_l = w_{I,l} - jw_{Q,l} \quad (8)$$

Here,  $\mathbf{r}(t)$  and  $\mathbf{w}(t)$  are the time-domain representation of signal and noise, whereas  $x_l$  and  $w_l$  are the complex-domain representation of the signal and noise respectively. By denoting a group of  $j$  possible constellation as

$$U_k = \{S_{k1}, S_{k2}, \dots, S_{kM_k}\}, \quad k = 1, 2, \dots, j \quad (9)$$

where  $M_k$  is the number of points in the  $U_k$  constellation. From detection theory [4], classification within group of constellations can be realized as a test on the following  $j$  hypotheses:

$$H_k : \text{the underlying constellation is } U_k, \quad k = 1, 2, \dots, j \quad (10)$$

Given a set of received data  $X_L = \{x_l = (r_{I,l}, r_{Q,l})^t\}$ , the ML method of classification selects the hypothesis whose likelihood or log-likelihood function is maximized [10], [13], [14] i.e.,

$$H_k^* = \arg \max_{H_k} \ln(p(X_L|H_k)) \quad (11)$$

In this case, the probability density function  $p(X_L|H_k)$  is expressed as

$$p(X_L|H_k) = \prod_{l=1}^L p(x_l|H_k) = \prod_{l=1}^L p(r_{I,l}, r_{Q,l}|H_k) \quad (12)$$

$$= \prod_{l=1}^L \sum_{u=1}^{M_k} P(S_{lu}|U_k) p(x_l|S_{ku}) \quad (13)$$

where  $S_{lu} = s_{I,lu} - js_{Q,lu}$ . As all constellations are equally likely in the M-ary modulation  $P(S_{lu}|U_k) = 1/M_k$ . Where  $M_k$  is the number of points in the constellation  $U_k$ . Replacing  $p(x_l|S_{ku})$  in (13) we get,

$$p(X_L|H_k) = \prod_{l=1}^L \sum_{u=1}^{M_k} \frac{1}{M_k} \frac{1}{\pi\sigma^2} e^{-\frac{1}{\sigma^2}[(r_{I,l} - s_{I,ku})^2 + (r_{Q,l} - s_{Q,ku})^2]} \quad (14)$$

Taking natural logarithm on both the side of equation (14)

$$\ln(p(X_L|H_k)) = \sum_{l=1}^L \ln \left( \sum_{u=1}^{M_k} \frac{1}{M_k} \frac{1}{\pi\sigma^2} e^{-\frac{1}{\sigma^2} [(r_{I,l} - s_{I,ku})^2 + (r_{Q,l} - s_{Q,ku})^2]} \right) \quad (15)$$

When all symbols are equi-probable, the maximum likelihood is equivalent to maximum a posterior criterion and hence results optimum performance with minimum error.

In this paper, we analyze the performance of optimum ML algorithm to recognize the modulation type in the non ideal condition (when additive noise is non-Gaussian). So, in this case received signal can be expressed as

$$\mathbf{r}(t) = \mathbf{s}(t) + \mathbf{n}(t), \quad 0 \leq t \leq LT \quad (16)$$

where  $\mathbf{n}(t) = \mathbf{w}(t) + \mathbf{i}(t)$  is the non-Gaussian noise.  $w(t)$  is the additive white Gaussian noise and  $i(t)$  is the impulsive noise added due to various natural and man-made sources [5], such as noise introduced by the power amplifier in the transmitter section. The presence of impulse and Gaussian noise together leads to the non-Gaussian noise distribution. The tail of the Gaussian distribution is widened due to addition of impulse noise and resulting distribution leads to non-Gaussian distribution. In practical applications, it is not possible to use the probabilistic method as the actual nature of the impulse noise is unknown [8]. Therefore, a common way to deal with this unknown probability density function is by assuming that it is Gaussian, which leads to result in poor performance of the ML classifier as compared to ideal ML classifier's performance. As it is evident from Fig. 3 that with non-Gaussian noise, the optimal maximum likelihood based classifier performance degrades. In this paper, we present a novel technique for recognizing the modulation type in the presence of non-Gaussian (impulse and AWGN) noise by removing the impulsive noise by SSD followed by the ML classification.

### III. PROPOSED METHOD

As mentioned earlier, maximum likelihood classifier's performance degrades in the presence of impulse noise. So, the proposed method consists of two major stages. In the first step, impulse is removed using SSD or sparse separation (SS) using an overcomplete hybrid dictionaries followed by maximum likelihood classification in the second step. The general idea is to exploit an overcomplete dictionary which composed of distinct bases [15]. Choosing the appropriate bases provide sparse representation for the two signals (impulse and modulated signal). One signal has a sparse representation in one basis, whereas not in another and vice versa. Details of sparse representation and overcomplete dictionary are provided in next subsection.

#### A. Impulse Removal using SS

1) *Sparse Representation*: Sparse representation of signal has successfully evolved in many signal processing applications such as audio, biomedical, image processing, communication etc. for classification, de-noising, and compression and a number of other applications [16]–[18].

Let  $\mathbf{r}$  be received composite signal which is sparse in an overcomplete matrix  $\Phi \in \mathbb{R}^{P \times Q}$ , where  $P < Q$ . This

overcomplete matrix is composed of different frames of bases. This provides sparse representation of the composite signal  $\mathbf{r}$ , that can be expressed as

$$\mathbf{r} = \Phi \alpha = \sum_{k=1}^Q \alpha_k \phi_k \quad (17)$$

where  $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_Q]$  is the sparse coefficients which can be obtained for an overcomplete dictionary.

2) *Design of Overcomplete Dictionaries*: In general, a geometrical prior information about temporal and spectral characteristics of the signal help to construct the dictionary. An overcomplete dictionary which should be able to provide better sparse representation of a composite signal can be constructed either non-adaptively or adaptively. This dictionary can be constructed using different hybrid fundamental waveforms from different analytical functions or by learning the dictionary from a set of training recorded signals [16].

To find suitable frames for the design of an overcomplete dictionary one has to explore the structure of the composite signal which may be sparse in one frame whereas not in other. A number of literature have been devoted for the dictionary learning which provide a best sparse representation for different data sets or signals [16].

Here, our aim is to recognize the modulation format inhibited in the received signal. So, based on this prior information one can say that these signals are periodic and will provide a good sparse representation in Fourier, cosine, sine, wavelet and in some other basis. On the other side, impulse has a spike like geometrical structure which can be captured by an identity basis (and provide sparse representation in time domain itself). It is important to design a predefined overcomplete matrix for simple and fast algorithms to exhibit sparsity of many classes of signals [16]. In this paper, for the separation of modulated signal and impulse noise we choose an identity basis, discrete cosine transform (DCT), and discrete sine transform (DST) to provide a suitable decomposition. So, the overcomplete dictionary  $P \times Q$  can be represented as

$$\Phi = [\Phi_1 | \Phi_2 | \Phi_3] \quad (18)$$

$\Phi_1$  is the identity (or impulse) dictionary with size  $P \times P$ . This impulse or identity dictionary can be a better choice to capture the peaks or impulses and provide the sparse representation in time domain itself. So, the  $\Phi_1$  can be constructed as

$$\Phi_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \quad (19)$$

$\Phi_2$  represents the discrete cosine dictionary (DCT) with size of  $P \times K$  which can be designed as

$$[\Phi_2]_{ij} = \sqrt{\frac{2}{P}} [r_i \cos(\frac{\pi(2j+1)i}{2P})] \quad (20)$$

where,  $r_i = 1/\sqrt{2}$  for  $i = 0$ , otherwise  $r_i = 1$  and  $i, j = 0, 1, 2, \dots, K-1$ .

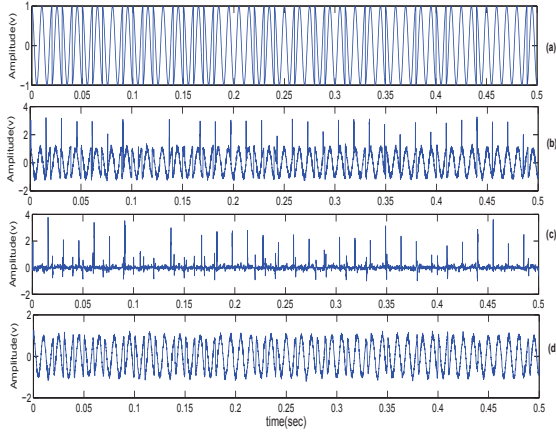


Fig. 1. (a) original BPSK signal (b) Corrupted with non-Gaussian noise (c) Non-Gaussian noise component of the signal after SS,  $\hat{\mathbf{i}}(t)$  (d) reconstructed signal  $\hat{\mathbf{s}}(t)$  after SS (approximation of original modulated signal).

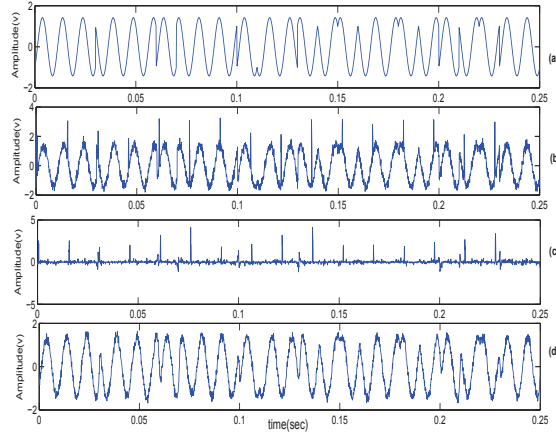


Fig. 2. (a) original QPSK signal (b) Corrupted with non-Gaussian noise (c) Non-Gaussian noise component of the signal after SS,  $\hat{\mathbf{i}}(t)$  (d) reconstructed signal  $\hat{\mathbf{s}}(t)$  after SS (approximation of original modulated signal).

And  $\Phi_3$  denotes the discrete sine matrix (DST) with size of  $P \times K$  which can be designed as

$$[\Phi_3]_{ij} = \sqrt{\frac{2}{P}} [r_i \sin(\frac{\pi(2j+1)(i+1)}{2P})] \quad (21)$$

where,  $r_i = 1/\sqrt{2}$  for  $i = P - 1$ , otherwise  $r_i = 1$  and  $i, j = 0, 1, 2, \dots, K - 1$

Modulated signals exhibit periodicity, hence sparse in frequency domain. So, the Fourier bases can be a good choice for the sparse representation of the modulated signal. It is noted that we have used DCT and DST matrix in our analysis to avoid complex term for solving the optimization problem in the following subsection. This is worth noting that one can choose only one either DCT or DST matrix because they also provide sparse representation. But, from our analysis using DCT and DST together provides a good approximation of the modulating signal with very less reconstruction error.

3) *Sparse Separation*: Let  $\mathbf{s}$  be the original modulated signal and  $\mathbf{n}_I$  is the impulse noise. Then, received signal  $\mathbf{r}$

#### Pseudocode 1 : Proposed Algorithm

**Input:** Receive the signal  $\mathbf{r}(t)$  contaminated with non-Gaussian noise.

- 1: Design overcomplete dictionary using Identity basis ( $\Phi_1$ ), DCT ( $\Phi_2$ ) and DST matrix ( $\Phi_3$ ).
- 2: Solve  $l_1$  optimization algorithm on  $\mathbf{r}(t)$  to reconstruct the sparse coefficients using (24).
- 3: Separate the impulse noise components and the modulated signal in the preprocessing stage from (28).
- 4: Do matched filtering and combine in-phase and quadrature-phase component to get complex point ( $x_l$ ) in the constellation.
- 5: Calculate  $p(\mathbf{X}_L|H_k)$  the probability of given set of modulation scheme for  $L$  number of symbols received.
- 6: Apply the ML classifier algorithm to identify the modulation type.

which is contaminated by impulse noise can be written as

$$\mathbf{r} = \mathbf{s} + \mathbf{n}_I \quad (22)$$

We are interested to find the sparse coefficients which can compactly represent the modulated signal and impulse noise components i.e.,  $\mathbf{s}$  and  $\mathbf{n}_I$ . So these sparse coefficients can be computed by solving  $l_0$  - norm convex optimization problem [16] using (17)

$$\hat{\alpha} = \underset{\alpha}{\operatorname{argmin}} \|\alpha\|_0 \quad s.t. \quad \Phi\alpha = \mathbf{r} \quad (23)$$

Due to its complexity to solve this,  $l_0$  problem is relaxed to  $l_1$  norm in the literature [19], [20] and can be solved by well known  $l_1$  norm optimization algorithm

$$\hat{\alpha} = \underset{\alpha}{\operatorname{argmin}} \|\Phi\alpha - \mathbf{r}\|_2^2 + \lambda \|\alpha\|_1 \quad (24)$$

where  $\lambda$  is the regularization parameter for sparsity to give relative weights for the different terms.  $\hat{\alpha} \in \mathbb{R}^{(P+2K) \times 1}$  is the reconstructed sparse coefficients composed of coefficients reconstructed from  $\Phi_1$ ,  $\Phi_2$ , and  $\Phi_3$ , which can be written as

$$\hat{\alpha} = [\hat{\alpha}_{\phi_1} | \hat{\alpha}_{\phi_2} | \hat{\alpha}_{\phi_3}] \quad (25)$$

Using (17) and (25), the received signal  $\mathbf{r}$  can be expressed as

$$\mathbf{r} \approx \Phi\hat{\alpha} = [\Phi_1 | \Phi_2 | \Phi_3] [\hat{\alpha}_{\phi_1} | \hat{\alpha}_{\phi_2} | \hat{\alpha}_{\phi_3}] \quad (26)$$

So the reconstructed signal (modulated and impulse separately)  $\hat{\mathbf{r}}$  can be written as

$$\hat{\mathbf{r}} = \Phi_1 \hat{\alpha}_{\phi_1} + \Phi_2 \hat{\alpha}_{\phi_2} + \Phi_3 \hat{\alpha}_{\phi_3} \quad (27)$$

So, first term denotes the impulsive noise and last two terms corresponds to the modulated signal. So it can be written as

$$\hat{\mathbf{r}} = \hat{\mathbf{s}} + \hat{\mathbf{n}}_I \quad (28)$$

where,  $\hat{\mathbf{s}} = \Phi_2 \hat{\alpha}_{\phi_2} + \Phi_3 \hat{\alpha}_{\phi_3}$  and  $\hat{\mathbf{n}}_I = \Phi_1 \hat{\alpha}_{\phi_1}$  are the reconstructed original modulated signal and impulse respectively. Extending this idea, to our problem as in (16)  $\hat{\mathbf{s}}(t)$ ,  $\hat{\mathbf{i}}(t)$  are separated using sparse separation, where  $\hat{\mathbf{s}}(t)$ ,  $\hat{\mathbf{i}}(t)$  are the reconstructed modulated signal and impulsive noise respectively. To demonstrate the effectiveness, the decomposition results for BPSK and QPSK signal corrupted by non-Gaussian noise are shown in Fig. 1 and Fig. 2. It is found that SS provides a good approximation  $\hat{\mathbf{s}}(t)$  of the modulated signal  $\mathbf{s}(t)$ . Using  $\hat{\mathbf{s}}(t)$ , modulation scheme can be identified by ML classifier.



TABLE I. CONFUSION MATRIX FOR MAXIMUM LIKELIHOOD CLASSIFIER UNDER NON-GAUSSIAN NOISE

Classifier	Modulation	SNR= 0 dB					SNR= 5 dB					SNR= 10 dB					SNR= 15 dB					SNR= 20 dB									
DIRECT ML CLASSIFIER	BPSK	78	10	4	8	0	0	84	10	2	4	0	0	96	2	0	2	0	0	98	0	2	0	0	0	100	0	0	0	0	0
	QPSK	48	36	8	8	0	0	46	54	0	0	0	0	42	58	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0
	8PSK	30	30	24	16	0	0	16	60	24	0	0	0	6	52	34	8	0	0	0	46	40	14	0	0	0	20	46	34	0	0
	16PSK	74	2	18	6	0	0	58	6	20	16	0	0	26	26	22	26	0	0	0	30	28	42	0	0	0	6	28	66	0	0
	16QAM	48	12	18	22	0	0	40	14	32	14	0	0	28	8	48	14	2	0	0	2	22	6	70	0	0	2	4	2	92	0
	64QAM	90	0	8	2	0	0	22	0	0	0	78	0	0	0	0	0	100	0	0	0	0	52	48	0	0	0	0	0	100	
ML CLASSIFIER With SS (Proposed)	BPSK	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0
	QPSK	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	0	100	0	0	0
	8PSK	10	40	50	0	0	0	0	10	90	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0
	16PSK	20	40	30	30	0	0	0	20	40	40	0	0	0	0	30	70	0	0	0	0	0	100	0	0	0	0	100	0	0	0
	16QAM	0	0	90	10	0	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	100	0	0	0	0	100	0	0	0
	64QAM	0	0	0	0	100	0	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	100	0	0	0	0	100	0	0

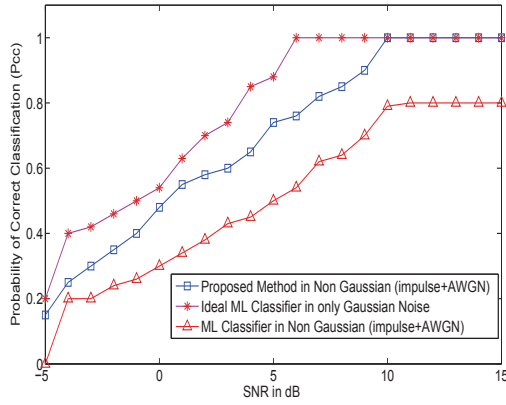


Fig. 3. Comparison of Probability of Correct Classification of ideal ML classifier in Gaussian, ML classifier and the proposed method in non-Gaussian noise for number of symbols = 120.

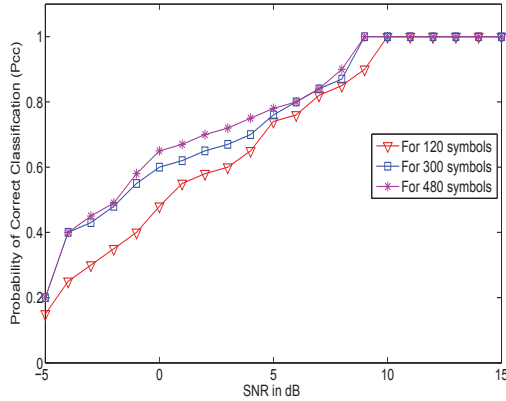


Fig. 4. Probability of Correct Classification of the proposed method for different number of symbols =120, 300 and 480 respectively.

So, our proposed method can provide optimal performance in the presence of mixed (non-Gaussian and Gaussian) noise as shown in Fig. 3.

#### IV. NUMERICAL RESULTS AND DISCUSSION

In this paper, we have considered BPSK, QPSK, 8-PSK, 16-PSK, 16-QAM and 64-QAM modulation schemes for testing the effectiveness of our proposed method. It is assumed that the modulation schemes are equally likely. In this model, we considered the random valued impulse noise having a heavy tail similar to the heavy tail in the pdf of the impulse noise. In our proposed method, proper choice of the regularization parameter plays important role in noise reduction because regularization parameter helps in balance of sparsity and reconstruction error term. The regularization parameter ( $\lambda$ ) is taken as 0.1 by considering the noise reduction capability. 50 experiments have been conducted to average out the probability of correct classification (Pcc). In our discussion, we considered  $T = 1$  and  $f_c = 1$  for easy analysis. We specify that signal to noise ratio SNR as the ratio of average received power and thermal noise variance. Fig. 3 shows the performance of ideal ML classifier in the presence of only Gaussian noise, ideal ML classifier in mixed (Gaussian and impulsive) and our proposed method at different values of SNR. The probability of correct classification is evaluated to measure the classification performance, which is expressed as

$$P_{cc} = 1/M \sum_{k=1}^M P(H_k/H_k) \quad (29)$$

where  $k = 1, 2, \dots, M$  is the number of different modulation schemes [6].  $P(H_k/H_k)$  is the conditional probability that ML classifier classify as  $H_k$  modulation scheme when  $H_k$  was transmitted.

As we can see from the Fig. 3 that ML classifier's performance degrades in the presence of mixed noise (Gaussian noise and impulse noise) as compared to the only Gaussian noise. Furthermore, Pcc of our proposed method is comparable to the ideal ML classifier and outperforms the direct ML classifier's performance in the presence of mixed noise. It is also observed that the performance of ideal and proposed ML classifier exactly matches with each other for SNR= 10 dB and above, whereas the direct ML with impulse noise provide poor performance. This extra preprocessing of the received signal by SSD increases the computational complexity of the classifier. Though the computational complexity of the proposed method is high, but one can get a high probability of correct classification at verge of little high computation. Here, the constellations are normalized to same power level. The comparison is done using 120 symbols for SNR values

ranging from  $-5$  dB to  $15$  dB. Fig. 4 shows the performance of our proposed method using different number of symbols  $N = 120, 300, 480$ . As, we can see from the Fig. 4 that classifier's performance is improved by increasing the number of symbols. Table I shows the performance of direct ML classifier and proposed ML classifier for different modulation schemes in confusion matrix. The diagonal entries of confusion matrix represent the probability of correct classification, whereas off diagonal entries represent the probability of misclassification. Simulation has been performed for 100 run for each SNR value.

## V. CONCLUSION

We have developed a novel method to classify the digital phase modulation scheme under both Gaussian noise and impulsive noise. It has been observed that the performance of the direct AMC based on ML classifier is degraded under non-Gaussian noise (or impulse noise) conditions. In the proposed scheme, reduction of impulsive noise is done using SSD on overcomplete dictionary and then the ideal ML classifier is used for classification of modulation schemes. The superiority of the proposed classification method is tested under Gaussian and non-Gaussian conditions. For different modulation techniques including BPSK, QPSK, 8-PSK, 16-PSK, 16-QAM, 64-QAM, the simulation results and confusion matrix is validated under different SNR values and magnitudes of impulse noises. Our study shows that the performance of the ML-based automatic modulation method can be further improved under impulse noise condition using the sparse signal decomposition on an overcomplete hybrid dictionary of elementary impulse, cosine and sine waveforms. The proposed system achieves a classification accuracy of 89 percent at  $0$  dB SNR under mixed noise ( Gaussian and Non-Gaussian) noise conditions.

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