# Simple Median based information fusion in wireless sensor network

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# Simple Median Based Information Fusion in Wireless Sensor network

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***Abstract*—The accuracy of a system is measured by the deviation of the system's results from the actual results. Information fusion deals with the combination of information from same source or different sources to obtain improved fused estimate with greater quality or greater relevance. As larger amount of sensors are deployed in harsher environment, it is important that sensor fusion techniques are robust and fault-tolerant, so that they can handle uncertainty and faulty sensor readouts. The sensor nodes in Wireless Sensor Network (WSN) are constrained with computation and communication resources, and efforts are required to increase the performance measures of the network. Thus sensor fusion techniques should be simple with less computation complexity. In this paper we propose a novel Median based sensor fusion function named *D* function. It is shown that the proposed *D* function satisfies the lipschitz condition. Paper also presents some of the ideas which can open new areas for research in fusion problem.**

***Index Terms* - Wireless Sensor Network, Sensor Fusion, Me- dian.**

## I. Introduction

Information fusion problems are centuries old. There are many applications in which information fusion methods are employed, and information fusion in sensor network is one of them. Wireless Sensor Networks (WSNs) are used to perform distributed sensing in various fields, such as health, military, home etc, in order to have a better understanding of the behavior of the monitored entity or to monitor an environment for the occurrence of a set of possible events, so that the proper action may be taken whenever necessary. WSN consists of a set of sensor nodes that are deployed in a field and interconnected with a wireless communication network. Each of these scattered sensor nodes has the capabilities to collect data, fuse that data and route the data back to the sink/base station [1], [2]. To collect data, each of these sensor nodes makes decision based on its observation of a part of the environment and on partial a-priori information. As larger amount of sensors are deployed in harsher environment, it is important that sensor fusion techniques are robust and fault-tolerant. The redundancy in the sensor readouts is used to provide error tolerance in fusion. The need for transferring information to locally disparate sensors and the need to associate their data both require a mechanism for transporting data of different structure at minimal costs. WSN are limited with the battery life constraints of the nodes, and thus in WSNs, information fusion techniques are applied for accuracy improvement while taking care of energy of the nodes. The data fusion using large multiple sensor agents, its network structure and performance is discussed in paper [3].

The terminology related to fusion of data from multiple sources is not uniform. Different terms have been adopted, usually associated with specific aspects that characterize the fusion. Before we understand the techniques for sensor fusion, it is important to understand the relationship among the fusion related terminologies. The term *data fusion* and *information fusion* can be used interchangeably. Information fusion is the merging of information from disparate sources with differing conceptual, contextual and typographical representations. It is used for consolidation of data from unstructured or semi-structured resource. On the other hand, *multi-sensor integration* is a slightly different term in the sense that it applies information fusion to make inferences using sensory devices and associated information (e.g., from database systems) to interact with the environment. According to [4] Multi-sensor integration is the synergistic use of information provided by multiple sensory devices to assist in the accomplishment of a task by a system; and multisensory fusion deals with the combination of different sources of sensory information into one representational format during any stage in the integration process. Multi-sensor integration is a broader term than multi-sensor fusion. Thus, *sensor/multi-sensor fusion* is fully contained in the intersection of multi-sensor integration and information/data fusion. *Data aggregation* defines another subset of information fusion that aims to reduce the data volume (typically, summarization), which can manipulate any type of data/information, including sensory data. Figure 1 depicts the relationship among

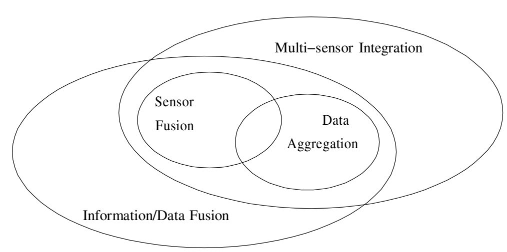


Fig. 1. Relations ship among fusion Terms

the concepts of sensor/multi-sensor fusion, multi-sensor integration, data aggregation, data fusion, and information fusion.

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We may regard a sensor an entity through which we are able to view a physical property. In general the physical property is evolving continuously in time and value. However, the sensor only provides us with a picture of the process: typically the output of a sensor is reduced to a single crisp value. The output of a sensor includes Entity-Name, Spatial Location, Time Instant (t), Measurement (y), and Uncertainty in the measurement ($\Delta y$ [5]. Thus the sensor observation is an interval as $(y - \Delta y, y + \Delta y)$. This kind of interval fusion is first introduced by Marzullo in [6], and is known as abstract sensor fusion. An abstract sensor is a sensor that reads a physical parameter and gives out an abstract interval estimate which is a bounded and connected subset of the real line. This means an abstract sensor *A* will give the interval of real numbers $[l_A, u_A]$, where $l_A < u_A$; $l_A$ is the lower bound and $u_A$ is the upper bound of the interval. Abstract sensors can be classified into correct sensors and faulty sensors. A correct sensor is an abstract sensor whose interval estimate contains the actual value of the parameter being measured. Otherwise, it is a faulty sensor. A faulty sensor is tamely faulty if it overlaps with a correct sensor, and is wildly faulty if it does not overlap with any correct sensor [7].

So, typically the physical parameter is a crisp value, like in the case of temperature sensing, the value of temperature sensed is a crisp value. The process of arriving at the abstract interval estimate from the physical parameter or adding the uncertainty, can be done in two ways:

(a). By adding equal left and right tolerance to the sensed crisp value / physical parameter.

(b). By adding unequal left and right tolerance to the sensed crisp value / physical parameter.

Thus in sensor fusion problem can be considered as a problem of giving a fused (interval or crisp) estimate of the intervals generated with equal left and right tolerance and unequal left and right tolerance, as well as defining the architectural technique of fusion, i.e. what all node will do the information fusion. In this paper, abstract sensor fusion problem for Wireless Sensor network is discussed and a simple solution is proposed. Paper also gives an improved solution of existing fusion function in the case of uncertain number of faulty sensor nodes. The rest of the paper is organized as follows: Section II covers the literature survey related to abstract sensor fusion and the existing fusion techniques. Section III discusses some of the ideas which can open new areas for research in fusion problem. In section IV, proposed solution approach is discussed. It also presents the comparison between the proposed solution and the existing solutions. Section **??** proposes a hybrid fusion function. Finally in section V conclusion of paper is drawn.

## II. Literature Survey

According to paper [6], sensor failures can be classified by:

- Fail-stop failures, in which a failed abstract sensor can be detected
- Arbitrary Failures with bounded inaccuracy
- Arbitrary failures, in which an abstract sensor can fail arbitrary.

The fault tolerant fusion algorithm works with the assumption that no more than $f$ sensors are faulty out of the total $n$ sensors. Table I summarizes the maximum number of faulty sensors that can be tolerated.

TABLE I
Maximum Faulty Sensors

| **Failure Model** | $f_{max}$ |
|---|---|
| Arbitrary failure, unbounded inaccuracy | $\lfloor \frac{n-1}{3} \rfloor$ |
| Arbitrary failure, bounded inaccuracy | $\lfloor \frac{n-2}{2} \rfloor$ |
| Fail-stop failures | $n - 1$ |

All the solutions discussed in this section, work with the assumption of maximum number of faulty sensors with bounded inaccuracy. Thus one can assume $f_{max} = (n - 2)/2$.
This means that with $(n - 2)/2$ faulty sensors algorithm will work properly. Let $I_1, I_2, \ldots, I_n$ be the interval estimates from $n$ abstract sensors, and maximum $f$ of them could be faulty. Four functions were developed representing four milestones in this area discussed in [7], these four functions are shown in figure 2.

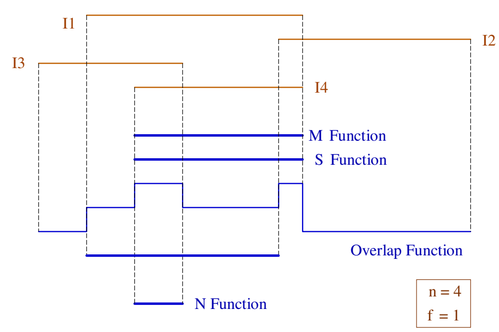


Fig. 2. Existing Fusion Functions

*M* function [6] is defined as the smallest interval that contains all the intersections of $(n - f)$ intervals. It is guaranteed to contain the true value provided the number of faulty sensors is at most $f$, i.e. $f_{max} = f$. However, *M* function exhibits an unstable behavior in the sense that a slight difference in the input may produce a quite different output. This behavior was formalized as violating Lipschitz condition [8].

The $\Omega$ function [9] is also called the overlap function. $\Omega(x)$ gives the number of intervals overlapping at x. $\Omega$ function results in an integration interval with the highest peak and the widest spread at a certain resolution. The $\Omega$ function is also robust, satisfying Lipschitz condition, which ensures that minor changes in the input intervals cause only minor changes in the integrated result.

The *N* function [10] improves the $\Omega$ function to only generate the interval with the overlap function ranges $[n - f, n]$. It also satisfies Lipschitz condition.

*S* function of Schmid and Schossmaier [11] returns a closed interval $[a, b]$ where $a$ is the $(f + 1)^{th}$ maximum left end point and $b$ is the $(f + 1)^{th}$ minimum right end point of the intervals i.e. there are exactly $f$ left end points to the right of $a$ when the left end points are sorted in increasing order and similarly there are $f$ right end points to the left of $b$ when right end points are sorted in increasing order. This function also satisfies the lipschitz condition [11]. Schimd *et al* also presented that S function is an optimal function from the listed functions.

## III. Some New Ideas for Fusion Functions

This section discusses some of the ideas which can open new areas for research in fusion problem. The below subsections discusses different ideas:

### A. Fusion Estimate as Rough Set

In all the existing fusion functions, fused estimate of interval values is also an interval value. The fused estimate can also be declared as an Rough Set. In computer science, a rough set, first described by a Polish computer scientist Zdzislaw I. Pawlak, is a formal approximation of a crisp set in terms of a pair of sets which give the lower and the upper approximation of the original set. In the standard version of rough set theory [12], the lower- and upper-approximation sets are crisp sets, but in other variations, the approximating sets may be fuzzy sets or interval valued sets. Here for sensor fusion problem, the lower- and upper-approximation sets are interval values.

Let us specifically consider the "*S*" function for example. The logic of *S* function is to arrange the left end points in increasing order, and pick the $(f + 1)^{th}$ left end point counting from maximum left end point. This is the best one can do, as the goal is to make length of the fused estimate as small as possible, respecting the fact that $f$ sensors are faulty. But in the worst case the fusion estimate, which maximize the length of the fused estimate, is also the correct fused estimate. The best case and worst case fused estimates are shown in figure 3. Thus the final fused estimate can be declared as a rough set of $\{I, I\}$, where $I = \mathrm{BestCaseInterval}$ and $I = \mathrm{WostCaseInterval}$. Depending upon the application of the network, the fused value can be utilized from the rough set, like in the case where minimum or maximum value is of interest. The rough set defined here, follow $I \subseteq I$ property.

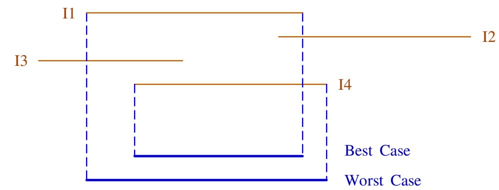


Fig. 3. Rough Set Fusion Estimate

### B. Uncertainty in the Number of Faulty Sensors

In this section we propose an algorithm for sensor fusion where the number of faulty sensors, $f$, is not fixed and can take value between $f_{min}$ & $f_{max}$. It should be noted that in the existing functions discussed in section II, the number of faulty sensors are assumed to be $f_{max}$. If the minimum and maximum number of faulty sensors are known, then also the fused estimate can be represented as a Rough Set of fused estimate with $f_{min}$ and $f_{max}$ as a faulty sensors. That means the final fused estimate is represented as $\{I, I\}$, where $I = \mathrm{Interval}_{f_{min}}$ and $I = \mathrm{Interval}_{f_{max}}$. Here $\mathrm{Interval}_{f_{min}}$ and $\mathrm{Interval}_{f_{max}}$ can be calculated with *S* function.

Based on application, if one want to declare a fused estimate as an interval, then one need to consider all the possibilities of number of faulty sensors, i.e. $\{f_{min}, f_{min} + 1, \ldots, f_{max} - 1, f_{max}\}$. The number of faulty sensors can be considered as a Discrete Random Variable with the associated probability mass function. Let the probability that the number of faulty sensors is i, is given by $p_i$, where $i = \{f_{min}, f_{min} + 1, \ldots, f_{max} - 1, f_{max}\}$. The algorithm for improvement over existing fusion is:

1) Calculate the fused interval estimates, $J_i$, corresponding to each i, i.e. for $\{f_{min}, f_{min} + 1, \ldots, f_{max} - 1, f_{max}\}$.
2) Let $L_i$ and $R_i$ represent the left and right end points of the $J_i$ fused interval estimates respectively.
3) Calculate the final left end point of the fused estimate by $\sum_i p_i L_i, i = \{f_{min}, \ldots, f_{max}\}$.
4) Calculate the final right end point of the fused estimate by $\sum_i p_i R_i, i = \{f_{min}, \ldots, f_{max}\}$.
5) The final fused interval estimates is given by $\{\sum_i p_i L_i, \sum_i p_i R_i\}, i = \{f_{min}, \ldots, f_{max}\}$.

Here the left and right end points of the final fused estimate is calculated using probabilistic average of the left and right end points of fused intervals with $f = \{f_{min}, f_{min} + 1, \ldots, f_{max} - 1, f_{max}\}$. If the length of the fused estimate with $f_i$ faulty sensors is $n_i$, then the length of final fused estimate is given by:

$$n = \sum_{i=f_{min}}^{f_{max}} (R_i - L_i)p_i = \sum_{i=f_{min}}^{f_{max}} n_i p_i \qquad (1)$$

### C. Relation to the Entropy

In information theory, entropy is a measure of the uncertainty associated with a random variable. Entropy is usually expressed by the average number of units needed for storage or communication. The term by itself usually refers to the Shannon entropy [13], which quantifies, an expected value, the information contained in a message. Equivalently, the Shannon entropy is a measure of the average information content one is missing when one does not know the value of the random variable. Shannon denoted the entropy, $H$, of a discrete random variable $X$ with possible values $\{x_1, ..., x_n\}$ as $H(X) = E(I(X))$. Here $E$ is the expected value, and $I$ is the information content of $X$, and $I(X)$ is itself a random variable. The larger the entropy is, the more uncertainties the information has. If p denotes the probability mass function of $X$ then the entropy can explicitly be written as

$$H(X) = \sum_{i=1}^{n} p(x_i) I(x_i) = -\sum_{i=1}^{n} p(x_i) \log_b p(x_i), \quad (2)$$

where b is the base of the logarithm used, and common value of b is 2.

Lets take uncertainty in the number of faulty sensors case where the number of faulty sensors, $f$, is not fixed and can take value between $f_1, f_2, \ldots, f_n$. Here the number of faulty sensors can be seen as a discrete random variable $F$ with possible values $f_1, f_2, \ldots, f_n$. The probability mass function of $F$ is represented by $\rho$. So the entropy of $F$ is given by $H(F) = -\sum_{i=1}^{n} \rho(x_i) \log_b \rho(x_i)$. If there are fixed 'one value' of $f$, i.e. $\rho(f_j) = 1$, then entropy is zero and hence uncertainty is zero. If all the $\rho_i$ are equal, $\rho_i = \frac{1}{n}$, then entropy is $\log_b n$ which is maximum and hence uncertainty is maximum. And in this case, entropy is a monotonic increasing function of n. With equally likely events there is more choice, or uncertainty, when there are more possible events.

The algorithm discussed in subsection III-B, can be applied to calculate the final fused estimate. The final fused estimate will be given by:

- **Maximum Entropy Case:** In this case, left and right end points of the final fused interval estimates can be calculated by simple average of left and right end points of the fused interval estimates with number of faulty sensor as $f_1, f_2, \ldots, f_n$. Thus the final fused interval estimates is given by:

$$\left[ \frac{1}{n} \sum_{i=f_1}^{f_n} L_i , \frac{1}{n} \sum_{i=f_1}^{f_n} R_i \right] \quad (3)$$

- **Minimum Entropy Case:** In this case, only one fixed value of $f$ is known, thus this case is same as considered by the existing fusion functions, discussed in section II.

## IV. Proposed Approach

As discussed earlier, abstract sensor is a piecewise continuous function from the physical state to a dense interval of real numbers. This physical state is the crisp value, and the tolerance on left and right side define the interval in the abstract sensor. In this paper, we are proposing a simple median based fusion function called *D* function. Few definitions related to solution are given below:

**Definition 1:** The length of an abstract sensor 'A' is given by $(u_A - l_A)$

**Definition 2:** The midpoint of abstract sensor 'A' is given by $Mid_A = l_A + \frac{u_A - l_A}{2}$

**Definition 3:** The left and right tolerance of an abstract sensor 'A' is given by $(Mid_A - l_A)$ and $(u_A - Mid_A)$,
respectively.
Generally left and right tolerance of an abstract sensor are equal, i.e. $LTolerance_A = RTolerance_A$.

**Definition 4:** A correct abstract sensor is an abstract sensor whose interval contains the actual physical value of interest.

Based on the definitions given above the fusion problem in sensor network can be classified in two cases:

- **Case 1:** Where the length of all the abstract sensors are same.
- **Case 2:** Where length of abstract sensors can vary by sensor to sensor.

It can also be seen as tolerances of all the sensors are same for case one and for case two tolerances are variable. The algorithm for proposed fusion function, *D* function, in same tolerance case is:

1) Let $I_1, I_2, \ldots, I_n$ be the interval estimates from $n$ abstract sensors, then calculate the midpoint for each abstract sensor.
2) Calculate the median of the mid points of abstract sensors. The resulting value, $Mid_{sol}$, is the midpoint of the estimated interval.
3) Calculate the tolerance of an abstract sensor using by $Tolerance_A = (Mid_A - l_A) = (u_A - Mid_A)$ of any abstract sensor A.
4) Calculate the estimated interval $\{(Mid_{sol} - Tolerance_A), (Mid_{sol} + Tolerance_A)\}$.

The algorithm for proposed fusion function, *D* function, in varying tolerance case is:

1) Let $I_1, I_2, \ldots, I_n$ be the interval estimates from $n$ abstract sensors, then calculate the midpoint for each abstract sensor.
2) Calculate the median of the mid points of abstract sensors. The resulting value, $Mid_{sol}$, is the midpoint of the estimated interval.

3) Calculate the tolerance of all the abstract sensor using by $Tolerance_A = (Mid_A - l_A) = (u_A - Mid_A)$.
4) Calculate average tolerance, $Tolerance_{AVG}$, of all the sensors.
5) Now temporary estimated interval is $\{(Mid_{sol} - Tolerance_A), (Mid_{sol} + Tolerance_A)\}$.
6) Now recalculate the average tolerance considering only the sensors which has their midpoints in this temporary estimated interval, and which has $Tolerance <= Tolerance_{AVG}$
7) Calculate the estimated interval $\{(Mid_{sol} - Tolerance_{AVG}), (Mid_{sol} + Tolerance_{AVG})\}$

The proposed fusion function, *D* function, also satisfies the monotonicity property and lipschitz condition. Monotonicity property and lipschitz condition for *D* function is proved by following lemma:

***Lemma 1:*** The median based fusion function, *D* function, satisfies monotonicity property, means it satisfy the following relations:

(i) $D_n^f(I) \subseteq D_n^{f+k}(I)$ for any integer k with $0 \leq k \leq (n - f)$.
(ii) $D(I) \subseteq D(J)$ for any $J = \{J_1, J_2, \ldots, J_n\}$ with $I_l \subseteq J_l$ for $1 \leq l \leq n$.

**Proof:** Item (i) of lemma is trivial as the proposed fusion function, *D* function, does not take number of faulty sensors, $f$, in calculating the fused estimate. Thus $D_n^f(I)$ is equal to $D_n^{f+k}(I)$, and hence $D_n^f(I) \subseteq D_n^{f+k}(I)$.

For item (ii), it should be noted that typically the output of sensors are the physical estimate (crisp values). They are converted into interval valued output through the process of associating tolerance with the crisp sensed value. Thus if interval $I_l$ and $J_l$ measure the same physical entity with same (type) sensor, then the mid point of both $I_l$ and $J_l$ is same. And thus, the mid point of the fused estimate is same for both $I$ and $J$. If $I_l \subseteq J_l$ for $1 \leq l \leq n$, then the length of $I_l$ is smaller or equal to $J_l$ for $1 \leq l \leq n$. Thus average tolerance of $I$ is smaller or equal to the average length of $J$. Hence, $D(I) \subseteq D(J)$ for any $J = \{J_1, J_2, \ldots, J_n\}$ with $I_l \subseteq J_l$ for $1 \leq l \leq n$.

***Lemma 2:*** The median based fusion function *D* satisfies Lipschitz condition for the uniform metrics $\mu$ which means that for any $\delta > 0$ and any two sets of sensor interval readings $I = I_1, I_2, \ldots, I_n, I' = I'_1, I'_2, \ldots, I'_n$ of non compatible intervals, $\mu(D(I), D(I')) < \delta$ provided that $\mu(I_i, I'_i) < \delta$, for $1 \leq i \geq n$.

**Proof:** Let $I_i = [l_i, u_i], l_i < u_i$ and $I'_i = [l'_i, u'_i], l'_i < u'_i$. So, Mid point of interval $I$ and $I'$ are $m_i = \frac{u_i - l_i}{2}$ and $m'_i = \frac{u'_i - l'_i}{2}$, respectively. And midpoint of interval from $mid(D(I')) = median\{m'_1, m'_2, \ldots, m'_n\}$. Paper [8] defines two important pseudo-metrics as:

- The midpoint pseudo-metric $\mu_m$, where $\mu_m(I, I')$ equals the distance between the midpoint of $I$ and $I'$.
- The uniform metric $\mu_u$, where $\mu_u([x, y], [v, w])$ equals the maximum of $|v - x|$ and $|w - y|$.

Paper also states that if any function F satisfies the Lipschitz condition for the pseudo-metrics $\mu_m$, then it also satisfies the condition for the metrics $\mu_u$. Thus we need to prove

that $\mu_m(D(I), D(I')) < \delta$ provided that $\mu_m(I_i, I'_i) < \delta$, for $1 \leq i \geq n$. In simple words we need to prove that $|mid(D(I)) - mid(D(I'))| < \delta$ provided $|m_i - m'_i| < \delta$ for $1 \leq i \geq n$. Here we consider two cases, n is odd or n is even.

**Case 1:** n is odd. For this case, $|mid(D(I)) - mid(D(I'))| < \delta \Rightarrow |m_j - m'_j| < \delta$, where $j$ is the middle index. It is given that $|m_i - m'_i| < \delta, \forall i$. Thus $|mid(D(I)) - mid(D(I'))| < \delta$.

**Case 2:** n is even. For this case, $|mid(D(I)) - mid(D(I'))| < \delta \Rightarrow \left|\frac{m_j + m_{j+1}}{2} - \frac{m'_j + m'_{j+1}}{2}\right| < \delta$, where $j$ and $j + 1$ are the index of two middle variables.

$|m_i - m'_i| < \delta, \forall i$
$\Rightarrow |m_j - m'_j| < \delta$ and $|m_{j+1} - m'_{j+1}| < \delta$
$\Rightarrow \left|\frac{m_j + m_{j+1}}{2} - \frac{m'_j + m'_{j+1}}{2}\right| < \delta$.
Thus $|mid(D(I)) - mid(D(I'))| < \delta$.
And hence $\mu(D(I), D(I')) < \delta$.

Now, we will compare the fused estimated for both the *S* function from literature and the proposed *D* function. Figure 4 shows the fused estimates with tamely faulty sensors. It is

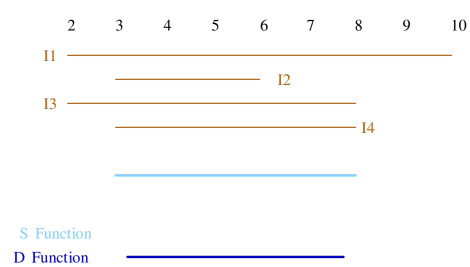


Fig. 4. Fusion Functions with tamely faulty sensors

shown in the figure that the fused estimates are acceptable for both the functions. The length of fused estimated of *D* function is smaller to the fused estimate of *S* function.

Figure 5 shows the fused estimates with wildly faulty sensors. Similar to tamely faulty sensors, with wildly faulty sensors also the fused estimates are acceptable for both the functions. The length of fused estimated of *D* function is smaller to the fused estimate of *S* function.

function D is $mid(D(I)) = median\{m_1, m_2, \ldots, m_n\}$ and

Advantages of proposed solution are as follows:

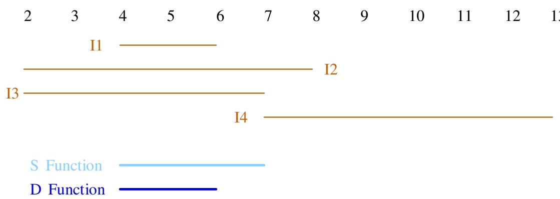


Fig. 5. Fusion Functions with wildly faulty sensors

- The solution is simple; it is based on simple mathematical operation (median).
- The solution doesn't take number of faulty sensor into account while calculating fused estimate. In case of *S* function if actual faulty sensors are less than $f$, the solution may not be accurate. But in case of *D* function, $f$ is not considered and hence solution does not depend on $f$.
- Computation of *S* function require sorting of left and right interval values. One side sorting will take minimum of $N \log_2 N$ comparisons. While in *D* function complexity of calculating median is $N \log_2 N$ comparisons only. Thus computation complexity is reduced by the factor of $1/2$.

## V. Hybrid Fusion Function

For further research in this area, the exiting fusion functions can be combined and the final results and the properties of these hybrid fusion functions can be studied. Here in this section, one of the hybrid fusion function of *S* function and symmetric median based function is proposed. The approach is simple and can be implemented based on the application requirements.

In this hybrid fusion function, for calculating left end point of fused estimate, first throw away $f$ left end points counting from the maximum left end points as in *S* function, and then compute the median of the remaining $(n - f)$ left end points. Declare the result as a left end point of fused estimate. Similarly, for calculating right end point of fused estimate, first throw away $f$ right end points counting from the minimum right end points as in S function, and then compute the median of the remaining $(n - f)$ right end points. Declare the result as a right end point of fused estimate. The properties such as monotonicity property and satisfaction to Lipschitz condition, of hybrid function are out of the scope of this paper.

## VI. Conclusion

The paper presented a simple median based information fusion function, which provides comparable results with respect to the existing optimal function. Paper also shows that the proposed fusion function satisfies the lipschitz condition, and is utilizing computation resource of sensor nodes in efficiently manner by reducing the computation complexity. And thus, the proposed solution is suitable for wireless sensor network. Paper also proposes several ideas which can open new areas for research in fusion problem.

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