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Sparse Decomposition Framework for Maximum Likelihood Classification under Alpha-Stable Noise

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Abstract*—Recently, automatic modulation classification has gained a lot of attention in the area of cognitive radio (CR), signal detection, electronic warfare and surveillance etc. Most of the existing modulation classification algorithms are developed based on the assumption that the received signal to be identified is corrupted by only additive white Gaussian noise. The performances of these conventional algorithms degrade significantly by addition of impulse noise. In this paper, we propose a robust algorithm using sparse signal decomposition which comprises of an overcomplete dictionary for detection and classification of modulated signals. In this work, an overcomplete dictionary is constructed using the identity basis, cosine and sine elementary waveforms to capture morphological components of the impulse noise and deterministic modulated signals effectively. The proposed method of modulation classification consists of the three major steps: sparse signal decomposition (SSD) on hybrid dictionaries, modulated signal extraction, and maximum likelihood (ML) based classification. The testing and validation of both direct ML and SSD-based ML classification methods are carried out under different Gaussian and impulse noise conditions for modulation classification. Our proposed method achieves a classification accuracy of 85% at 5 dB SNR and outperforms the conventional classification methods.

***Keywords*—Modulation classification, Maximum likelihood, Impulse noise, Sparse representation, Overcomplete dictionary.**

I. Introduction

Automatic modulation classification (AMC) is an indispensable component in the field of wireless communication because of its vast usage in various application such as cognitive radio, electronic warfare and surveillance, etc. [1]. It is an intermediate step between signal interception and demodulation. Furthermore, it helps in identifying the modulation scheme of a noisy received signal. AMC has little or no prior knowledge of the signal transmitted hence designing a modulation classification algorithm is quite challenging task [2].

The automatic modulation classification algorithms can be categorized into two broad classes: (i) likelihood-based (LB) decision theoretic methods and (ii) feature matching based (FB) pattern recognition methods. In decision theoretic approaches, the ML criterion is implemented to either the received signal directly or to a certain transform of it, which results in a likelihood ratio or a set of likelihood functions. The decision of classification is performed by comparing the likelihood ratio or the likelihood functions with a threshold. As this approach minimizes the probability of misclassification, hence, the solution obtained by LB algorithm is optimal [3]. On the other hand, FB algorithm extracts one or several features from the received signal for decisions making. Most of these conventional classification algorithms assumed noise as additive white Gaussian in nature in the performance analysis [3]. However, this assumption is not realistic in a non ideal condition. In many situations, the additive noise is non-Gaussian in nature. For example, noises from various natural and man-made sources exhibit sharp spike and hence, impulsive in characteristic [4]–[7]. This non-Gaussian noise is one of the prime source of error in digital transmission system [8], [9]. Hence, there is a need of more realistic approach to implement noise model that comprises additive mixture of Gaussian noise and non-Gaussian impulsive noise. Alpha-stable noise is one of noise model to satisfy these requirements [10]. This noise model can generate impulsive non-Gaussian distribution by varying the value of characteristic exponent α, where $0 < \alpha \leq 2$. Smaller value of α generates more sharp impulsive noise.

As the traditional algorithms are designed and developed for getting optimal results in Gaussian distribution, performance degrades severely in non-Gaussian noise environment [7]. In [5], it is found that the traditional LB algorithms do not give optimum results in time correlated additive noise. It is found from literature that most of the classification algorithms are developed by extracting modulation dependent features from the received signal under non-Gaussian environment [2], [10]–[13]. If these feature are not robust or properly designed, performance of classification algorithms degrade. Few literature have been addressed the modulation classification in presence of non-Gaussian noise using LB classifiers [2], [5], [8]. The classifier implements the whitening filter to minimize the complexity of ML in presence of impulse noise, where the unknown parameter and whitening filter coefficients are estimated [5]. The performance of classifier is affected due to this extra estimation step, since precise modeling is difficult in this approach. In [8], the channel and noise distribution are assumed to be known. This motivates us to redesign a ML classifier which can achieve better classification accuracy in non-Gaussian environment.

Recently, sparse representation (SR) has achieved a great deal of attention in many signal processing applications including compression, biomedical signal processing, de-noising and signal separation [9], [14]–[17]. In [9], application of SSD has been investigated to remove the impulse noise followed by classification. This paper presents similar approach for classification of the modulation type with additive mixed Gaussian and impulse noise after modelling it as α-stable noise by using sparse signal decomposition (SSD) with an overcomplete hybrid dictionary. The proposed algorithm utilizes SSD to remove the impulse from the received signal followed by classification using ML classifier.

This paper is organized as follows: System model and motivation behind the ML classification under non-Gaussian additive noise are highlighted in Section II. In Section III, the proposed method is presented for removal of impulse noise by incorporating SSD method and then signal classification using ML classifier. Section IV discusses the simulation results. Conclusions are presented in Section V.

II. Background and Motivation

Let $\mathbf{y}(t)$ be the received signal of interest whose modulation format needs to be identified, can be represented as

$$\mathbf{y}\left(t\right)=\mathbf{x}\left(t\right)+\mathbf{w}\left(t\right),\quad 0\leq t\leq LT_0 \tag{1}$$

where $\mathbf{x}(t)$ is the transmitted signal, $\mathbf{w}(t)$ is the additive white Gaussian noise, T_0 is symbol period and L is the number of symbols.

In digital amplitude and phase modulated communication, information is transmitted by utilizing phase and amplitude information during a symbol interval. This can be expressed as [12]:

$$\begin{aligned}\mathbf{x}\left(t\right)=\Re\left\{\sum_{u=-\infty}^{l}A\hat{X}_u p\left(t-uT_0\right)e^{j\left(2\pi f_c t+\theta_c\right)}\right\},\\ lT_0<t\leq\left(l+1\right)T_0,\quad l=0,1,2,.....\end{aligned} \tag{2}$$

where f_c and θ_c are the carrier frequency and phase offset respectively, T_0 is the symbol period, A is the signal amplitude, $p(t)$ is the transmitted pulse shape, $\hat{X}_u$ is assumed to be a value from M set of complex numbers $(X_1, X_2, ...X_M)$ in the constellation of modulation scheme. The power is normalized, i.e., $\sum_{m=1}^{M}|X_m|^2 = 1$, where the number of constellation points of each modulation type is represented by M. The similarity between the time domain and complex domain helps to map a time domain received signal back into the complex domain and then compare with the given constellation set. The likelihood based testing can be utilized for mapping the received modulated signal to the library of given constellation set when sufficient statistical information about the signal and communication channel is available.

The ML based modulation classification algorithms perform under the following conditions

- The signal amplitude (A), symbol period (T_0), carrier frequency (f_c) and reference phase (θ_c) are known.
- The transmitted data symbols are independent.
- The carrier frequency is a multiple of symbol rate.
- The additive noise is white and Gaussian in nature having known power density N_0.
- Both received modulated signal and noise added are independent of each other.

It is assumed that symbols are in perfect synchronization, hence, the output of the quadrature receiver provides sufficient statistic for ML classifier. The in-phase $(y_{I,l})$ and quadrature $(y_{Q,l})$ components of the quadrature receiver are [12],

$$\begin{aligned}y_{I,l}&=\int_{(l-1)T}^{lT}\mathbf{y}(t)\ \cos(2\pi f_c t+\theta_c)\ dt\\&=\frac{AT}{2}\Re\{\hat{X}_l\}+w_{I,l}\end{aligned} \tag{3}$$

$$\begin{aligned}y_{Q,l}&=\int_{(l-1)T}^{lT}\mathbf{y}(t)\ \sin(2\pi f_c t+\theta_c)\ dt\\&=\frac{AT}{2}\Im\{\hat{X}_l\}+w_{Q,l}\end{aligned} \tag{4}$$

where

$$w_{I,l}=\int_{(l-1)T}^{lT_0}\mathbf{w}(t)\ \cos(2\pi f_c t+\theta_c)\ dt \tag{5}$$

$$w_{Q,l}=\int_{(l-1)T}^{lT_0}\mathbf{w}(t)\ \sin(2\pi f_c t+\theta_c)\ dt \tag{6}$$

Assuming white and Gaussian noise, the quadrature noise component ($w_{I,l}$ and $w_{Q,l}$) are zero-mean white Gaussian with σ^2 as the variance. In the following analysis, it is assumed that the noise components $w_{I,l}$ and $w_{Q,l}$ are independent. The complex-domain representation of the received signal and noise can be expressed as follows [12],

$$r_l=y_{I,l}-jy_{Q,l},\quad l=1,2,.....L \tag{7}$$

$$w_l=w_{I,l}-jw_{Q,l} \tag{8}$$

Where L is the number of symbols of complex data for the time interval $[0, LT_0]$ and time-domain representation of signal and noise is given by $\mathbf{y}(t)$ and $\mathbf{w}(t)$, and complex-domain representation of the signal and noise is expressed as r_l and w_l respectively. Denoting a group of k possible constellation as

$$U_j=\{X_{j1},X_{j2},.....X_{jM_j}\},\quad j=1,2,.....k \tag{9}$$

where M_j is the number of points in the U_j constellation. Digital modulation classification within group of constellations can be evaluated from detection theory [18] as a test on the following k hypotheses

$$H_j:\text{the underlying constellation is } U_j,\quad j=1,2,.....k \tag{10}$$

Given a set of received data $R_L = \{r_l = (y_{I,l}, y_{Q,l})^t\}$, the classification algorithm based on maximum likelihood principle selects the hypothesis whose likelihood or log-likelihood function is maximum [12], [19], [20] i.e.,

$$H_j^*=arg\quad \max_{H_j}\ln(p(R_L|H_j)) \tag{11}$$

In this case, the probability density function $p(R_L|H_j)$ is represented as

$$p(R_L|H_j)=\prod_{l=1}^{L}p(r_l|H_j)=\prod_{l=1}^{L}p(y_{I,l},y_{Q,l}|H_j) \tag{12}$$

$$=\prod_{l=1}^{L}\sum_{u=1}^{M_j}P(X_{lu}|U_j)p(r_l|X_{ju}) \tag{13}$$

where $X_{lu} = s_{I,lu} - js_{Q,lu}$. In the M-ary modulation $P(X_{lu}|U_j)$= $1/M_j$ as all constellations are equally likely. Where, M_j is the number of points in the constellation U_j. Replacing $p(r_l|X_{ju})$ in (13) we get,

$$p(R_L|H_j) = \prod_{l=1}^{L}\sum_{u=1}^{M_j}\frac{1}{M_j}\frac{1}{\pi\sigma^2}e^{-\frac{1}{\sigma^2}[(y_{I,l}-s_{I,ju})^2+(y_{Q,l}-s_{Q,ju})^2]} \tag{14}$$

By taking natural logarithm on both the side of equation (14)

$$ln(p(R_L|H_j)) = \sum_{l=1}^{L} ln\left(\sum_{u=1}^{M_j}\frac{1}{M_j}\frac{1}{\pi\sigma^2}e^{-\frac{1}{\sigma^2}[(y_{I,l}-s_{I,ju})^2+(y_{Q,l}-s_{Q,ju})^2]}\right) \tag{15}$$

As the maximum likelihood is equivalent to maximum a posterior criterion when all symbols are equi-probable, the ML based classification algorithm provides optimum performance with minimum error.

As the AMC always function in the non ideal condition, the low-frequency atmospheric noise and man-made noise are likely to exhibit sharp spikes and hence, introduce impulsive characteristics [4]. This paper analyzes the behavior of optimum ML classification algorithm to detect the modulation scheme in this non-ideal situation, i.e., additive noise consists of mixture of impulse noise and Gaussian noise. Therefore, the modulated received signal is expressed as,

$$\mathbf{y}(t) = \mathbf{x}(t) + \mathbf{n}(t), \quad 0 \le t \le LT_0 \tag{16}$$

where $\mathbf{n}(t) = \mathbf{w}(t) + \mathbf{q}_0(t)$ is the mixed non-Gaussian noise, $w(t)$ is the additive white Gaussian noise, $q_0(t)$ is the impulse noise, such as noise introduced by the power amplifier in the transmitter section.

The main characteristic of impulsive or impulse process is that it attains a high spiky event in the signal. A mathematical model to represent the impulsive noise is called α-stable noise [10], where $0 < \alpha \le 2$. Due to the addition of the impulse noise, the tail of the Gaussian distribution is widened and hence, resulting non-Gaussian distribution. The noise is more impulsive for a smaller value of α. In this paper, we assume that the impulse noise obeys α-stable distribution, which can be characterized by (a, β, α) where a is the location parameter, β is the index of skewness and α is the characteristic exponent which measures the "thickness" of tails of the density function. The distribution is symmetric about the center a, when $\beta = 0$. In practical scenario, the probabilistic approach is not useful as the actual behavior of the impulsive noise is not known [7]. Therefore, in the conventional algorithms a common way to attend this unknown probability density function is to assume that it is Gaussian in nature, which results poor performance of the ML classifier. It is evident from Fig.1 that the optimal maximum likelihood based classifier performance degrades with non-Gaussian noise. This paper presents a novel approach for identifying the modulation scheme in the presence of mixed noise (impulse and AWGN) to eliminate the impulse noise by SSD followed by the ML classification.

III. Proposed Method

As discussed earlier, ML classifiers' performance deteriorates tremendously under impulse noise. The proposed method of removing the impulse from the contaminated noisy signal consists of two major stages. The first stage is to remove impulse using SSD or sparse separation (SS) on an overcomplete hybrid dictionaries followed by ML based classification in the second step. In this approach, it is required to exploit an overcomplete dictionary which comprises of distinct bases [21]. Appropriate bases can be selected which can provide sparse representation for the two signals, i.e., impulse and modulated signal. In this case, each signal has a sparse representation in one basis, whereas not in another and vice versa. In the next subsection, details of sparse representation and overcomplete dictionary are discussed.

A. Impulse Removal using SS

1) Sparse Representation: Sparse representation of signal has taken a potential growth in many signal processing applications such as biomedical, image processing, communication and audio, etc. for de-noising, compression, and classification, etc. [14], [15], [17].

Let $\mathbf{y}$ be received composite signal, which is sparse in an overcomplete matrix $\mathbf{\Psi} \in \mathbb{R}^{P\times Q}$, where $P < Q$. This overcomplete matrix is composed of different frames of bases. This provides sparse representation of the composite signal $\mathbf{y}$, that can be expressed as

$$\mathbf{y} = \mathbf{\Psi}\alpha = \sum_{k=1}^{Q}\alpha_k\psi_k \tag{17}$$

where $\alpha = [\alpha_1, \alpha_2, \cdots \alpha_Q]$ is the sparse coefficients vector, which can be obtained for an overcomplete dictionary.

2) Overcomplete Dictionaries Construction: Generally, a prior information about temporal and spectral characteristics helps in constructing the dictionary. An overcomplete dictionary should provide an efficient sparse representation of a composite signal either non-adaptively or adaptively. This dictionary can be constructed using different elementary waveform from their analytical functions or learning by trained recorded signals [17]. One has to explore structure for sparseness of composite signal to find suitable elementary waveform for the design of an overcomplete dictionary. This dictionary learning has been exploited to provide a best sparse representation for different signals in many literature [17].

Here, the recognition of modulation format of the received signal is main goal. Hence, based on prior knowledge, one can assume that these signals are periodic and provide a good sparse representation in Fourier, cosine, sine, wavelet and in some other basis. On the contrary, impulse has a spiky geometrical structure which can be captured effectively by an identity basis (and provides sparse representation in time domain itself). It is important to design a predefined effective overcomplete matrix which can exploit sparsity of many classes of signals [17]. In this paper, our goal is to separate the modulated signal and impulse noise. Hence, we select an identity basis, discrete cosine transform (DCT), and discrete sine transform (DST) to provide appropriate decomposition. So, the overcomplete dictionary $P \times Q$ can be defined as

$$\mathbf{\Psi} = [\mathbf{\Psi_1}|\mathbf{\Psi_2}|\mathbf{\Psi_3}] \tag{18}$$

$\mathbf{\Psi_1}$ denotes the identity (or impulse) matrix with size $P \times P$. This identity matrix provides the sparse representation in time domain itself to capture the peaks or impulses. Hence, the $\mathbf{\Psi_1}$ can be designed as

$$\mathbf{\Psi_1} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{19}$$

$\mathbf{\Psi_2}$ denotes the discrete cosine dictionary (DCT) with size of $P \times K$ which can be constructed as

$$[\mathbf{\Psi_2}]_{ij} = \sqrt{\frac{2}{P}}[a_i cos(\frac{\pi(2j+1)i}{2P})] \tag{20}$$

where, $a_i = 1/\sqrt{2}$ for $i = 0$, otherwise $a_i = 1$ and $i, j = 0, 1, 2, \cdots K-1$.
And $\mathbf{\Psi_3}$ represents the discrete sine matrix (DST) with size of $P \times K$ which can be designed as

$$[\mathbf{\Psi_3}]_{ij} = \sqrt{\frac{2}{P}}[a_i sin(\frac{\pi(2j+1)(i+1)}{2P})] \tag{21}$$

where, $a_i = 1/\sqrt{2}$ for $i = P-1$, otherwise $a_i = 1$ and $i, j = 0, 1, 2, \cdots K-1$.

Modulated signals are periodic in nature, hence exhibit sparsity in frequency domain. Hence, Fourier bases provide a good choice to capture modulated signal. It is worth noting that we selected DCT and DST matrix in our analysis to avoid complex term for solving the optimization problem in the following subsection. One can select either DCT or DST matrix for exploiting sparsity. Moreover, from our analysis using DCT and DST together provide a better approximation of the modulating signal with very less reconstruction error.

Pseudocode 1 : Proposed Algorithm

Input: Receive the modulated signal $\mathbf{y}(t)$ corrupted with additive non-Gaussian noise.

1: Design an overcomplete dictionary using Identity basis ($\mathbf{\Psi}_1$), DCT ($\mathbf{\Psi}_2$) and DST matrix ($\mathbf{\Psi}_3$).
2: Reconstruct the sparse coefficients using (24) by solving l_1 optimization algorithm on $\mathbf{y}(t)$.
3: Separate the non-Gaussian impulsive noise components and the modulated signal component from (28).
4: Apply matched filtering and combine the in-phase component and quadrature-phase component to derive complex point (r_l) in the constellation.
5: Calculate $p(\mathbf{R}_L|H_j)$ for given set of modulation scheme for L number of received symbols.
6: Do classification using the ML classifier algorithm to recognize the modulation scheme.

3) Sparse Separation: Let $\mathbf{s}$ be the modulated signal and $\mathbf{q_0}$ is the impulse noise. Then, received signal $\mathbf{y}$ corrupted from impulse noise can be written as

$$\mathbf{y} = \mathbf{s} + \mathbf{q_0} \tag{22}$$

Our goal is to find the sparse coefficients which can represent the modulated signal and impulse noise components, i.e., $\mathbf{s}$ and $\mathbf{q_0}$ compactly. These sparse coefficients can be calculated by solving l_0 - norm convex optimization problem [17] using (17)

$$\hat{\alpha} = argmin._\alpha \|\alpha\|_0 \qquad s.t. \quad \mathbf{\Psi}\alpha = \mathbf{y} \tag{23}$$

This l_0 problem is relaxed to l_1 norm due to its complexity [16], [22] and can be solved by well known l_1 norm optimization algorithm

$$\hat{\alpha} = argmin.\|\mathbf{\Psi}\alpha - \mathbf{y}\|_2^2 + \lambda\|\alpha\|_1 \tag{24}$$

where λ is the regularization parameter for adjusting the reconstruction fidelity and sparsity by relative weights for the different terms. $\hat{\alpha} \in \mathbb{R}^{(P+2K)\times 1}$ is the reconstructed sparse coefficients consists of coefficients reconstructed from $\mathbf{\Psi}_1$, $\mathbf{\Psi}_2$, and $\mathbf{\Psi}_3$, which can be written as

$$\hat{\alpha} = [\hat{\alpha}_{\psi_1}|\hat{\alpha}_{\psi_2}|\hat{\alpha}_{\psi_3}] \tag{25}$$

Using (17) and (25), the received signal $\mathbf{y}$ can be described as

$$\mathbf{y} \approx \mathbf{\Psi}\hat{\alpha} = [\mathbf{\Psi}_1|\mathbf{\Psi}_2|\mathbf{\Psi}_3][\hat{\alpha}_{\psi_1}|\hat{\alpha}_{\psi_2}|\hat{\alpha}_{\psi_3}] \tag{26}$$

So the reconstructed signal $\hat{\mathbf{y}}$ can be written as

$$\hat{\mathbf{y}} = \mathbf{\Psi}_1\hat{\alpha}_{\psi_1} + \mathbf{\Psi}_2\hat{\alpha}_{\psi_2} + \mathbf{\Psi}_3\hat{\alpha}_{\psi_3} \tag{27}$$

Here, first term represents the impulsive noise and last two terms correspond to the modulated signal. So it can be described as

$$\hat{\mathbf{y}} = \hat{\mathbf{s}} + \hat{\mathbf{q_0}} \tag{28}$$

where, $\hat{\mathbf{s}} = \mathbf{\Psi}_2\hat{\alpha}_{\psi_2} + \mathbf{\Psi}_3\hat{\alpha}_{\psi_3}$ and $\hat{\mathbf{q_0}} = \mathbf{\Psi}_1\hat{\alpha}_{\psi_1}$ are the reconstructed original modulated signal and impulse respectively.

Extending this idea, to our problem as in (16) $\hat{\mathbf{s}}(t)$, $\hat{\mathbf{q}}_0(t)$ are segregated using sparse separation, where $\hat{\mathbf{s}}(t)$, $\hat{\mathbf{q}}_0(t)$ are the reconstructed modulated signal and impulsive noise respectively. After removing the impulse SS provides a good approximation $\hat{\mathbf{s}}(t)$ of the modulated signal $\mathbf{s}$(t). Then, $\hat{\mathbf{s}}(t)$ can be employed to identify the modulation scheme by ML classifier. So, our proposed method can achieve optimal performance in the presence of mixed (non-Gaussian and Gaussian) noise as shown in Fig.1.

IV. Numerical Results and Discussion

The efficiency of our proposed method is validated through extensive simulations. The proposed algorithm can efficiently classify the modulation scheme under mixed additive non-Gaussian noise. The digital phase modulation considered to examine the effectiveness of the algorithm are BPSK, QPSK, 8-PSK, 16-PSK, 16-QAM and 64-QAM modulation schemes. All the modulation schemes are considered equally likely and their constellations are normalized to equal power level. The characteristic exponent (α) is taken as 1.2 and 1.8, which quantify the "thickness" of tails of the density function. The index of skewness parameter (β) = 0, hence the distribution is symmetric about center of the location parameter a. In the proposed method, impulsive components are removed from the contaminated signal by selecting proper value of the regularization parameter (λ) as it helps in balancing sparsity and reconstruction error term. In this simulation, we have considered $\lambda = 0.01$ as the noise reduction capability. For simple analysis, we have taken $T = 1$ and $f_c = 1$. Probability of correct classification (P_{cc}) has been calculated to evaluate the performance of the proposed algorithm. Total number of 100 experiments have been performed to average out the

TABLE I. CONFUSION MATRIX FOR MAXIMUM LIKELIHOOD CLASSIFIER UNDER α- STABLE NOISE AT $\alpha = 1.2$

Classifier	Modulation	SNR= 0 dB						SNR= 5 dB						SNR= 10 dB						SNR= 15 dB						SNR= 20 dB					
Direct ML Classifier	BPSK	0	0	0	0	6	40	0	4	0	0	0	26	0	0	0	2	14	42	0	0	0	6	8	60	0	1	4	2	14	48
	QPSK	0	0	0	2	2	40	0	0	0	4	0	42	0	0	0	0	2	60	0	2	2	2	0	70	0	4	0	8	0	74
	8-PSK	0	0	0	14	8	42	0	0	0	24	8	44	0	0	0	18	6	42	0	0	12	14	10	42	0	10	22	16	2	32
	16-PSK	0	0	0	8	16	36	0	0	0	4	2	46	0	0	2	18	2	50	0	0	4	24	6	36	0	4	20	22	6	30
	16-QAM	0	0	0	0	0	28	0	0	0	0	0	48	0	0	0	0	8	72	0	0	0	0	20	68	0	0	0	0	60	34
	64-QAM	0	0	0	0	0	36	0	0	0	0	66	0	0	0	0	0	74	0	0	0	0	0	82	0	0	0	0	0	0	90
ML Classifier With SS (Proposed)	BPSK	98	2	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0
	QPSK	4	96	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0
	8-PSK	0	36	44	20	0	0	0	0	66	34	0	0	0	2	74	24	0	0	0	0	96	4	0	0	0	0	100	0	0	0
	16-PSK	0	64	16	20	0	0	0	22	44	34	0	0	0	0	24	74	0	0	0	0	0	100	0	0	0	0	0	100	0	0
	16-QAM	0	0	90	10	0	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0
	64-QAM	0	0	0	0	2	98	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100	0	0	0	0	0	100

probability of correct classification. The expression of the probability of correct classification is given by [5]

$$P_{cc} = 1/M_c \sum_{m=1}^{M_c} P(H_m/H_m) \tag{29}$$

where m is the number of different digital modulation schemes used, M_c is the total number of modulation schemes to be verified and $m = 1, 2, ..., M_c$. $P(H_m/H_m)$ is the conditional probability that ML classifier classify as H_m modulation scheme when H_m was transmitted.

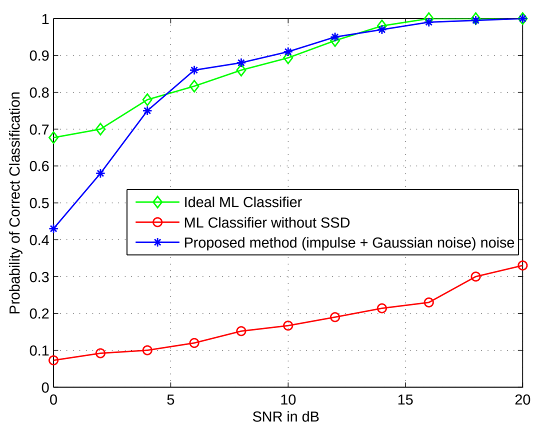

Fig. 1. Comparison of probability of correct classification of ideal ML classifier in Gaussian, ML classifier without SS under non-Gaussian noise and the proposed method in non-Gaussian noise, for $\alpha = 1.2$

The performance of the ideal ML classifier under only Gaussian noise, ideal ML classifier in presence of mixed (Gaussian and impulsive) and our proposed algorithm results are demonstrated in Fig. 1 and Fig. 2 for different values of SNR with different characteristic exponent α value. The SNR values ranging from 0 dB to 20 dB have been used to measure the P_{cc}. It is evident from the Fig.1 that the direct ML classifier behaves poorly in presence of mixed noise (Gaussian noise as well as impulse noise) when $\alpha = 1.2$. In this experimental result, it can be noticed that the direct ML modulation classifier is not able to classify modulation type due to the presence of high impulsive noise. The signal, which is contaminated with this non-Gaussian noise when applied to ideal ML classifier

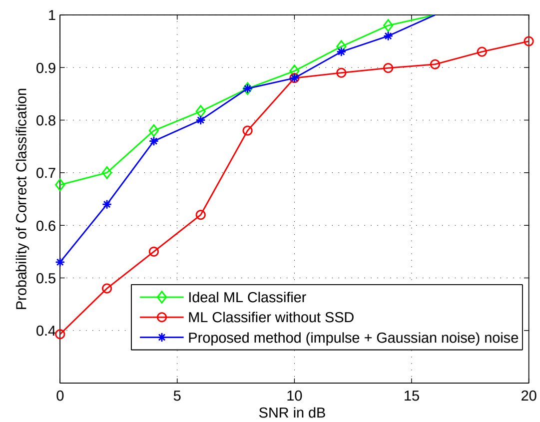

Fig. 2. Comparison of probability of correct classification of ideal ML classifier in Gaussian, ML classifier without SS under non-Gaussian noise and the proposed method in non-Gaussian noise, for $\alpha = 1.8$.

for classification, in most of the cases it fails and not able to classify. Whereas, Fig.1 shows that P_{cc} of ideal ML classifier is greatly enhanced after implementing our proposed method. Both ideal and proposed ML classifier's performance almost matches each other when SNR $= 10$ dB and above. In Fig.2, since $\alpha = 1.8$, the distribution is more Gaussian and hence, performance of the classifier without SSD improves because of less impulsive noise. It is evident from the Fig.1 that our algorithm outperforms the direct ML classifier's performance under highly impulsive condition and P_{cc} is comparable to the ideal ML classifier performance in presence of only Gaussian noise. Hence, the proposed method is able to separate the impulse noise from the signal contaminated with mixed noise and hence, leaving only Gaussian noise. This novel method of removing the impulse noise requires extra preprocessing of the received signal by SSD which increases the computational complexity of the classifier. However, high probability of correct classification can be achieved at verge of this extra computation. The confusion matrix is presented in the Table I, which gives the performance of direct ML classifier and proposed ML classifier under mixed noise for different modulation schemes. Probability of correct classification is highlighted in the diagonal entries. The diagonal entries of confusion matrix represent the probability of correct classification, whereas off diagonal entries provide the probability of misclassification.

V. Conclusion

In this paper, we have addressed a problem of digital phase modulated signal contaminated with mixed noise, i.e., alpha-stable impulsive noise with Gaussian noise. We have proposed a novel method of modulation classification under non-Gaussian noise. Automatic modulation classifier performance based on direct ML classification algorithm degrades severely in presence of mixed noise. In the first step, impulse noise is removed by implementing sparse signal decomposition on overcomplete dictionary. In the next step, the modulation type is identified using ideal ML classifier. The proposed method is validated for various modulation schemes including BPSK, QPSK, 8-PSK, 16-PSK, 16-QAM and 64-QAM under different SNR values. The simulation results and the confusion matrix provide the evidence of superiority of the proposed method. From the experimental results, it can be concluded that the performance of automatic modulation classifier based on ML classifier can be further improved under non-Gaussian noise by incorporating sparse decomposition on an overcomplete hybrid dictionary comprising impulse, cosine and sine waveforms. The classification accuracy of 85% achieved using the proposed scheme at 5 dB SNR under alpha-stable noise.

References

[1] H. C. Wu, M. Saquib and Z. Yun, "Novel automatic modulation classification using cumulant features for communications via multipath channels," *IEEE Trans. Wireless Commun.* , vol. 7, no. 8, pp. 3098-3105, Aug. 2008.

[2] V. G. Chavali and C. R. da Silva, "Maximum-likelihood classification of digital amplitude-phase modulated signals in flat fading non-Gaussian channels," *IEEE Trans. Commun.*, vol. 59, no. 8, pp. 2051-2056, Aug. 2011.

[3] O. A. Dobre, A. Abdi, Y. Bar-Ness and W. Su, "Survey of automatic modulation classification techniques: classical approaches and new trends," *IET Communications*, vol. 1, no. 2, pp. 137-156, Apr. 2007.

[4] K. L. Blackard, T. S. Rappaport and C. W. Bostian,"Measurements and models of radio frequency impulsive noise for indoor wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 11, no. 7, pp. 991-1001, Sep. 1993.

[5] V. G. Chavali and C. R. da Silva, "Classification of Digital Amplitude-Phase Modulated Signals in Time-Correlated Non-Gaussian Channels," *IEEE Trans. Commun.*, vol. 61, no. 6, pp.2408-2419, June 2013.

[6] K. J. Sangston and K. R. Gerlach, "Coherent detection of radar targets in a non-Gaussian background," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 30, no. 2, pp. 330-340, Apr. 1994.

[7] D. Middleton, "Statistical-physical models of electromagnetic interference," *IEEE Trans. Electromagn. Compat.*, vol. 19, no. 3, pp. 106-127, Aug. 1977.

[8] A. E. El-Mahdy, "Classification of MFSK signals over time-varying flat correlated fading channels under class-A impulsive noise environment," *IEE Proc. Commun.*, vol. 151, no. 6, pp. 619-626, Dec. 2004.

[9] M. Mohanty, U. Satija, B. Ramkumar and M. S. Manikandan, "Digital modulation classification under non-gaussian noise using sparse signal decomposition and maximum likelihood," *IEEE 21st Nat. Conf. Commun.*, 2015, Accepted.

[10] H. Tao, and Z. Zheng'ou, "Modulation classification in alpha-stable noise," *IEEE Int. Conf. Commun. Circuits and Syst., ICCCAS 2007.*, pp. 193-196, July. 2007.

[11] B. M. Sadler, "Detection in correlated impulsive noise using fourth-order cumulants," *IEEE Trans. Signal Process.*, vol. 44, no. 11, pp. 2793-2800, Nov. 1996.

[12] W. Wei and J. M. Mendel, "A fuzzy logic method for modulation classification in nonideal environments," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 3, pp. 333-344, 1999.

[13] A. Hazza, M. Shoaib, A. Saleh and A. Fahd, "Classification of digitally modulated signals in presence of non-Gaussian HF noise," *IEEE 7th Int. Symposium on Wireless Communication Systems (ISWCS)*, pp. 815-819, Sep. 2010.

[14] M. S. Manikandan and B.Ramkumar, "Straightforward and robust QRS detection algorithm for wearable cardiac monitor," *Healthcare Technol. Lett.*, vol. 1, no. 1, pp 40-44, Mar. 2014.

[15] M. S. Manikandan, S. Samantaray and I. Kamwa, "Detection and Classification of Power Quality Disturbances Using Sparse Signal Decomposition on Hybrid Dictionaries," *IEEE Trans. Instrum. Meas.*, 2014.

[16] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489-509, Feb. 2006.

[17] M. Aharon, M. Elad and A. Bruckstein, "K-SVD: An algorithm for designing overcomplete dictionaries for sparse representation," *IEEE Trans. Signal Process.*, vol. 54, no. 11, pp. 4311-4322, Nov. 2006.

[18] S.M. Kay., "Fundamental of Statistical Signal Processing: Detection Theory," *Prentice-Hall*, 1993.

[19] J. L. Xu, W. Su and M. Zhou, "Likelihood-ratio approaches to automatic modulation classification," *IEEE Trans. Syst. Man Cybern. C, Appl. Rev.*, vol. 41, no. 4, pp. 455-469, July 2011.

[20] W. Wei and J. M. Mendel, "Maximum-likelihood classification for digital amplitude-phase modulations," *IEEE Trans. Commun.*, vol. 48, no. 2, pp. 189-193, Feb. 2000.

[21] D. L. Donoho and X. Huo, "Uncertainty principles and ideal atomic decomposition," *IEEE Trans. Inform. Theory*, vol. 47, no. 7, pp. 2845-2862, 2001.

[22] D. L. Donoho and M. Elad, "Optimally sparse representation from overcomplete dictionaries via L1-norm minimization," *Proc. Nat. Acad. Sci.*, vol. 100, no. 5, pp. 2197-3002, 2002.