Axisymmetric Supersonic Nozzle Model for Inference Problem DARPA EQUiPS SEQUOIA Team

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1 Introduction

The axisymmetric fluid flow analyses required by the inverse problem previously specified by the DARPA EQUiPS SEQUOIA team can be implemented using the `MULTI-F` software (accessed on GitHub at `<https://github.com/vmenier/MULTIF>`. A quickstart tutorial for running the pertinent fluid analyses can be found at `https://github.com/vmenier/MULTIF/wiki/Quickstart-%28Inverse-Problem%29`. This document describes in more detail the inputs and outputs of the analyses and corresponding inverse problem.

2 Model

The `MULTI-F` code is used to model an axisymmetric supersonic nozzle with random inlet stagnation temperature T_{stag}, inlet stagnation pressure P_{stag}, wall temperature $T_w(x)$, and atmospheric temperature T_∞ and pressure P_∞. Geometry is fixed at a baseline shape. A quasi-1D fluid flow analysis or a RANS analysis can be performed to determine the properties of the flow inside the nozzle as well as estimate thrust.

2.1 Inputs

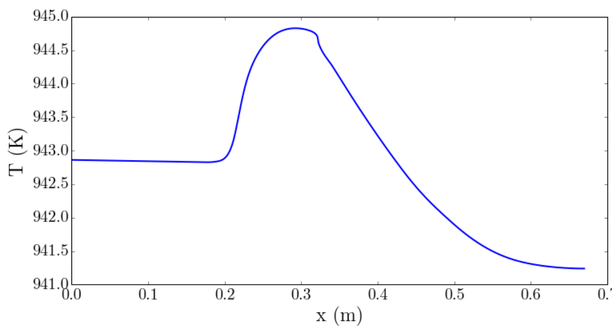
The fixed deterministic inputs to the model include the shape of the nozzle's inner wall. Wall layers, baffle and stringer geometry, material constants, and the heat transfer coefficient to the ambient atmosphere do not contribute to this problem's quantities of interest. Distributions and baseline values for P_{stag}, T_{stag}, T_∞, and P_∞ have been previously specified for the top-of-climb mission (see figure 1).

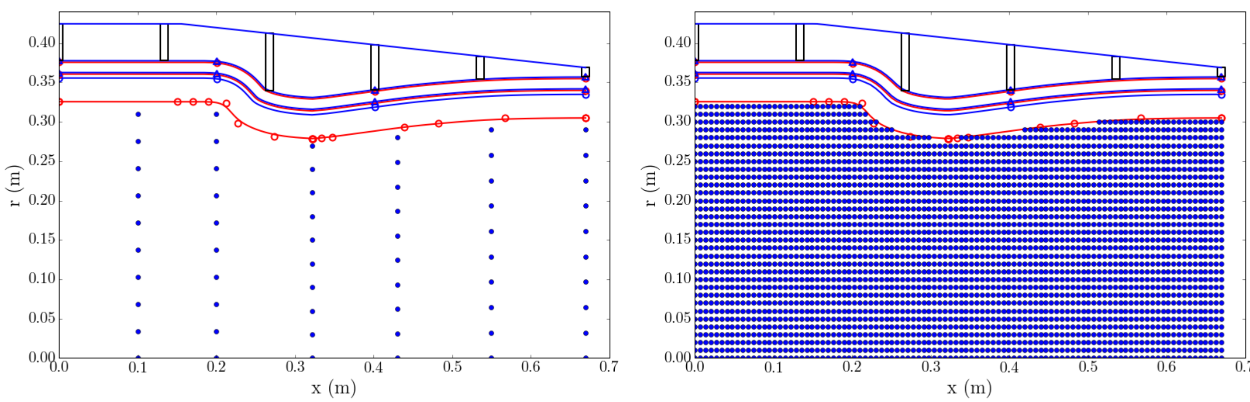
The wall temperature $T_w(x)$ is parameterized as a piecewise linear function with 100 evenly-spaced nodes. Figure 1 shows the baseline values for $T_w(x)$. No distribution has been specified for each $T_w(x_i)$ for $i \in 1...100$. A similar distribution to that specified for T_{stag} may be useful with a certain covariance kernel. Lastly, it is important that T_{stag} and $T_w(x_1 = 0)$ are close in value, perhaps with means within 10 K of each other.

Table 1: Mission parameters.

Parameter	**Units**	**Nominal Value**	**Distribution**
Altitude	ft (km)	40,000 (12.192)	N/A
Mach		0.511	N/A
Required Thrust	N	21,500	N/A
P_{stag}	Pa	97,585	$\ln\mathcal{N}(11.5010, 0.0579^2)$
T_{stag}	K	955.0	$\ln\mathcal{N}(6.8615, 0.0119^2)$
P_∞	Pa	18,754	$\ln\mathcal{N}(9.8386, 0.0323^2)$
T_∞	K	216.7	$\ln\mathcal{N}(5.3781, 0.0282^2)$

Figure 1: Baseline values for $T_w(x)$ (perhaps to be used as prior).





(a) Locations of static pressure probe measurements.

(b) Locations of velocity measurements via PIV.

Figure 2: Pressure and velocity measurement locations.

2.2 Outputs

The set of experimental data that can be obtained for the nozzle internal flow includes pressures, velocities, and net forces. The outputs described below can be obtained in some cases at considerable expense and labor, but are representative of the type of experimental data that may be obtained. Although experiments are often performed at sea level atmospheric conditions, here we maintain the top-of-climb condition for our simulation of experimental data (corresponding to the altitude and Mach in table 1).

$F(\xi)$ In practice, the static thrust produced by the nozzle can be measured using a force balance.

$P_w(x,\xi)$ In practice, the static pressure at the nozzle wall can be measured using a series of static pressure taps located on the wall. Static pressure is measured at 100 locations evenly spaced in the x-coordinate along the nozzle wall.

$P(x,r,\xi)$ In practice, the static pressure inside the nozzle can be measured laboriously at several points using a static pressure probe. Figure 2 shows the location of 60 measurement points.

$\vec{U}(x,r,\xi)$ In practice, the velocity inside the nozzle can be measured at a few points using pitot tubes, or in 2-D planes using particle image velocimetry (PIV) techniques. Figure 2 shows the location of 3,095 points where velocity could be extracted in the x-r plane at 1 cm intervals using PIV techniques.