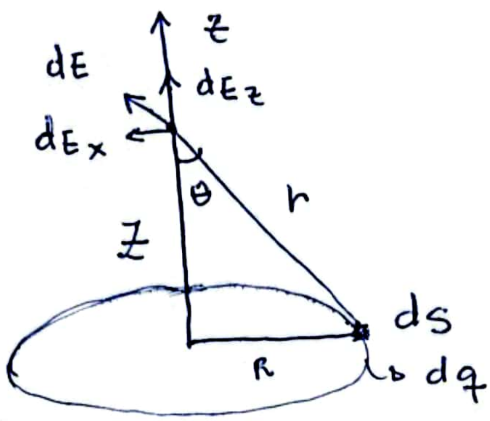
3.1. Anel carregado.



$$\lambda = \frac{Q}{L}$$

$$\cos\theta = \frac{z}{r}$$

$$r^2 = z^2 + R^2$$

$$dE = k \cdot \frac{dq}{r^2}$$

$$dE_z = dE \cdot \cos\theta$$

$$\frac{dE_z}{\cos\theta} = \frac{k\,dq}{r^2}$$

$$dE_z = \frac{k\,dq}{r^2} \cdot \frac{z}{r}$$

$$dE_z = \frac{k\,dq \cdot z}{(z^2+R^2)} \cdot \frac{1}{(z^2+R^2)^{1/2}}$$

$$\lambda = \frac{dq}{ds} \rightarrow dq = \lambda\,ds$$

$$dE_z = \frac{k z \lambda ds}{(z^2+R^2)^{3/2}}$$

$$E = \int_0^{2\pi R} \frac{k z \lambda ds}{(z^2+R^2)^{3/2}} = \frac{k z \lambda}{(z^2+R^2)^{3/2}} \int_0^{2\pi R} ds$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{z \cdot \lambda}{(z^2+R^2)^{3/2}} \cdot 2\pi R$$ USANDO λ.

$$Q = \lambda 2\pi R$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q z}{(z^2+R^2)^{3/2}}$$ USANDO Q.

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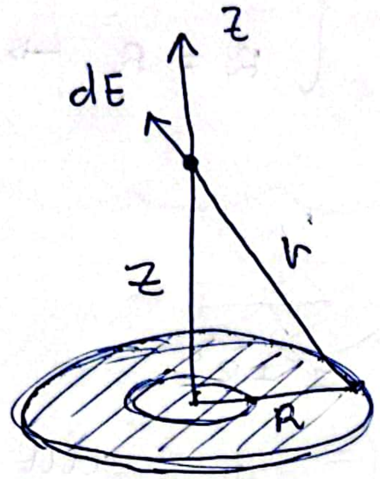
3.2. DISCO CARREGADO.



$$\sigma = \frac{dq}{dA} \rightarrow dq = \sigma\, dA$$

$$dq = \sigma . 2\pi R dR$$

ANEL:

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{z}{(z^2+R^2)^{3/2}} dq$$

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{z}{(z^2+R^2)^{3/2}} \cdot \sigma . 2\pi R . dR$$

$$E = \int_0^R \frac{1}{4\pi\varepsilon_0} \cdot \frac{z}{(z^2+R^2)^{3/2}} \cdot \sigma . 2\pi R\, dR$$

$$E = \frac{\sigma \pi z}{4\pi\varepsilon_0} \int_0^R \frac{2R\, dR}{(z^2+R^2)^{3/2}}$$

$$u = z^2 + R^2 \qquad \begin{cases} R = 0 \rightarrow u = z^2 \\ R = R \rightarrow u = z^2 + R^2 \end{cases}$$

$$du = 2R\,dR$$

$$E = \frac{\sigma z}{4\varepsilon_0} \int_{z^2}^{z^2+R^2} \frac{1}{u^{3/2}}\,du$$

$$E = \frac{\sigma z}{4\varepsilon_0} \cdot \left[-\frac{2}{u^{1/2}} \right]_{z^2}^{z^2+R^2}$$

$$E = \frac{\sigma \cdot z}{4\varepsilon_0} \left[-\frac{2}{(z^2+R^2)^{1/2}} + \frac{2}{(z^2)^{1/2}} \right]$$

$$E = -\frac{2\sigma \cdot z}{4\varepsilon_0} \left[\frac{1}{(z^2+R^2)^{1/2}} - \frac{1}{z} \right]$$

$$E = \frac{\sigma z}{2\varepsilon_0} \left[\frac{1}{z} - \frac{1}{(z^2+R^2)^{1/2}} \right]$$

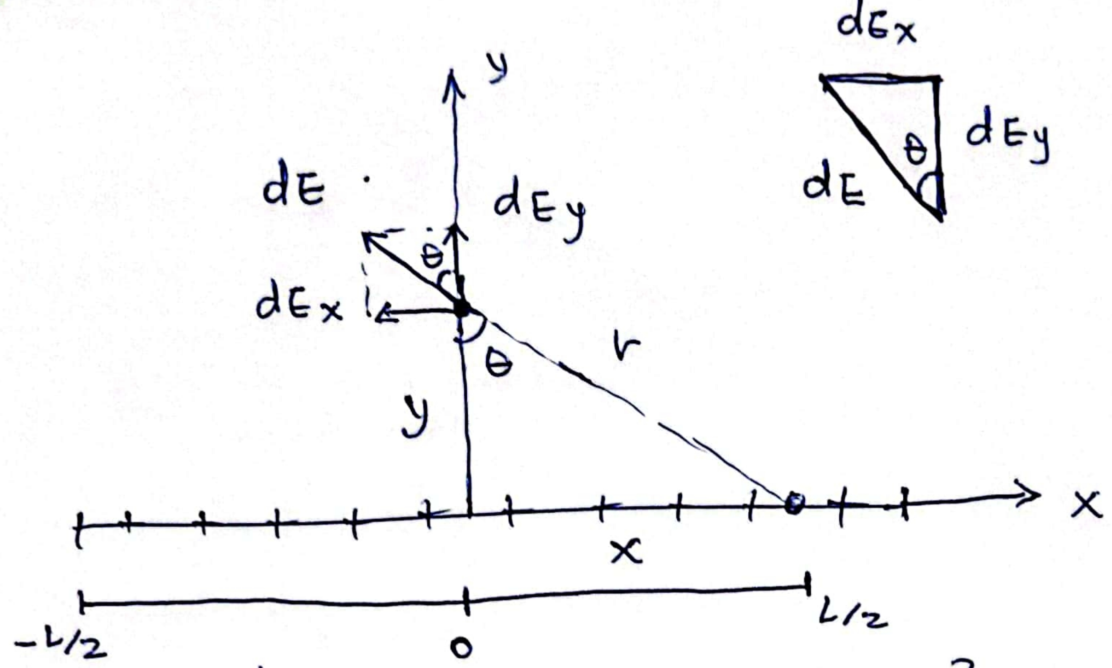
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right]}$$ $\to$ USANDO σ.

$$\sigma = \frac{Q}{A} \to \sigma = \frac{Q}{\pi R^2}$$

$$\boxed{E = \frac{Q}{2\pi\varepsilon_0 R^2} \cdot \left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right]}$$

$\hookrightarrow$ USANDO Q.

3.3 BARRA FINITA.



$$dE = \frac{k\,dq}{r^2} \quad ; \quad r^2 = y^2 + x^2$$

$$r = \sqrt{y^2 + x^2}$$

$$\cos\theta = \frac{dE_y}{dE}$$

$$dE_y = \frac{k\,dq}{r^2}\cos\theta$$

$$\cos\theta = \frac{y}{r}$$

$$dE_y = \frac{k\,y\,dq}{r^3}$$

$$dE_y = \frac{k \cdot y\,dq}{(y^2 + x^2)^{3/2}}$$

$$\lambda = \frac{Q}{L} \quad ; \quad \lambda = \frac{dq}{dx}$$

$$dq = \lambda \cdot dx$$

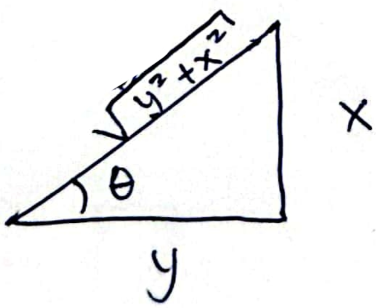
$$dE_y = \frac{K\, y\, \lambda \cdot dx}{(y^2+x^2)^{3/2}}$$

$$E = \int_{-\frac{L}{2}}^{L/2} \frac{K\, y\, \lambda\, dx}{(y^2+x^2)^{3/2}}$$

$$E = 2 \cdot K\, y\, \lambda \cdot \int_0^{L/2} \underbrace{\frac{1}{(y^2+x^2)^{3/2}}\, dx}_{\text{SUBSTITUICÃO TRIGONOMÉTRICA}}$$

$$\sqrt{y^2+x^2} \Rightarrow x = y\, tg\theta$$

$$tg\theta = \frac{x}{y}$$



$$dx = y \sec^2\theta \, d\theta$$

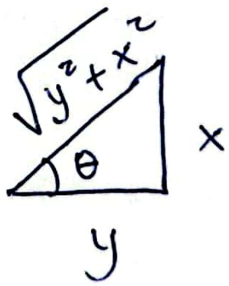
$$E = 2 k y \lambda \int_0^{L/2} \frac{1}{\left(\sqrt{y^2+x^2}\right)^3} \, dx$$

$$E = 2 k y \lambda \int_0^{L/2} \frac{y \sec^2\theta \, d\theta}{(y \sec\theta)^3}$$

$$E = \frac{2 k \lambda}{y} \int_0^{L/2} \frac{1}{\sec\theta} \, d\theta$$

$$E = \frac{2 k \lambda}{y} \int_0^{L/2} \cos\theta \, d\theta$$

$$E = \frac{2 k \lambda}{y} \Big[\operatorname{sen}\theta \Big]_0^{L/2} \; ;$$



$$E = \frac{2 k \lambda}{y} \left[\frac{x}{\sqrt{y^2+x^2}} \right]_0^{L/2}$$

$$E = \frac{2K\lambda}{y} \cdot \left[\frac{L/2}{\sqrt{y^2 + \left(\frac{L}{2}\right)^2}} - 0 \right]$$

$$E = \frac{2K\lambda}{y} \left[\frac{L}{2\sqrt{y^2 + \frac{L^2}{4}}} \right]$$

$$E = \frac{2K\lambda}{y} \cdot \left[\frac{L}{2\sqrt{\frac{4y^2 + L^2}{4}}} \right]$$

$$E = \frac{2K\lambda}{y} \cdot \frac{L}{\sqrt{4y^2 + L^2}}$$

onde $y \rightarrow$ distância da barra ao ponto

$L \rightarrow$ COMPRIMENTO DA BARRA.

$$K = \frac{1}{4\pi\varepsilon_0}$$

BARRA INFINITA :

$$L \to \infty$$

$$\lim_{L\to\infty} E = \lim_{L\to\infty} \left[\frac{2k\lambda}{y} \cdot \frac{L}{\sqrt{4y^2+L^2}} \right]$$

$$= \lim_{L\to\infty} \left[\frac{2k\lambda}{y} \cdot \frac{1}{\frac{\sqrt{4y^2+L^2}}{L}} \right]$$

$$= \lim_{L\to\infty} \left[\frac{2k\lambda}{y} \cdot \frac{1}{\sqrt{\frac{4y^2}{L} + 1}} \right] =$$

$$= \frac{2k\lambda}{y} \cdot \frac{1}{\sqrt{0+1}} = \frac{2k\lambda}{y}$$

$$E = \frac{2k\lambda}{y}$$