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## Contents

1	src/string/z.cpp
2	src/string/suffixarray.cpp
3	src/string/aho-corasick.cpp
4	src/string/suffixautomaton.cpp
5	src/string/lcparray.cpp
6	src/graph/eulertour.cpp
7	src/graph/stronglyconnected.cpp
8	src/graph/cutvertices.cpp
9	src/graph/mincostflow.cpp
10	src/graph/dynamicconnectivity.cpp
11	src/graph/scalingflow.cpp
12	src/geometry/basic.cpp
13	src/geometry/convexhull.cpp
14	src/geometry/anglesort.cpp
15	src/geometry/minkowskisum.cpp
16	src/geometry/closestpoints.cpp
17	src/datastructure/orderedset.cpp
18	src/datastructure/treap.cpp
19	src/math/fft.cpp
20	src/math/primitiveroot.cpp
21	src/math/miller-rabin.cpp
22	src/math/diophantine.cpp
23	src/math/fftmod.cpp
24	src/math/gaussjordan.cpp

## 25 src/math/crt.cpp

20

## 1 src/string/z.cpp

```

3
3 // TCR
4 // Computes the Z array in linear time
5 // z[i] is the length of the longest common prefix of substring
5 // starting at i and the string
5 // You can use string s instead of vector<int> s
5 // z[0]=0 by definition
6 #include <bits/stdc++.h>
6 using namespace std;

6 vector<int> zAlgo(vector<int> s) {
7     int n=s.size();
7     vector<int> z(n);
8     int l=0;
8     int r=0;
9     for (int i=1;i<n;i++) {
9         z[i]=max(0, min(z[i-1], r-i));
10        while (i+z[i]<n&& s[z[i]]==s[i+z[i]]) z[i]++;
10        if (i+z[i]>r) {
11            l=i;
11            r=i+z[i];
12        }
12    }
12    return z;
12 }
13

```

## 2 src/string/suffixarray.cpp

```

14
14 // TCR
14 // Suffix array in O(n log^2 n)
14 // ~300ms runtime for 10^5 character string, ~2000ms for 5*10^5
15 // You can change vector<int> s to string s
15 #include <bits/stdc++.h>
15 #define F first
16 #define S second
16 using namespace std;

17 vector<int> suffixArray(vector<int> s) {
17     int n=s.size();
17     vector<int> k(n);
18     for (int i=0;i<n;i++) {
18         k[i]=s[i];
19     }

```

```

vector<pair<pair<int, int>, int> > v(n);
for (int t=1; t<=n; t*=2) {
    for (int i=0; i<n; i++) {
        int u=-1;
        if (i+t<n) u=k[i+t];
        v[i]={k[i], u, i};
    }
    sort(v.begin(), v.end());
    int c=0;
    for (int i=0; i<n; i++) {
        if (i>0 && v[i-1].F!=v[i].F) c++;
        k[v[i].S]=c;
    }
    if (c==n-1) break;
}
vector<int> sa(n);
for (int i=0; i<n; i++) sa[k[i]]=i;
return sa;
}

```

### 3 src/string/aho-corasick.cpp

```

// TCR
// Aho-Corasick algorithm
// Building of automaton is O(L) where L is total length of dictionary
// Matching is O(n + number of matches), O(n sqrt(L)) in the worst case
// Add dictionary using addString and then use pushLinks
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

struct AhoCorasick {
    vector<map<char, int> > g;
    vector<int> link;
    vector<int> tlink;
    vector<int> te;

    // Use 1-indexing in id
    void addString(const string& s, int id) {
        int tn=0;
        for (int i=0; i<(int)s.size(); i++) {
            if (g[tn][s[i]]==0) {
                g[tn][s[i]]=g.size();
                g.push_back(map<char, int>());
                link.push_back(0);
            }
            tn=g[tn][s[i]];
        }
        tlink[tn]=id;
    }
};

```

```

        tlink.push_back(0);
        te.push_back(0);
    }
    tn=g[tn][s[i]];
}
te[tn]=id;
}

void pushLinks() {
    queue<int> bfs;
    bfs.push(0);
    while (!bfs.empty()) {
        int x=bfs.front();
        bfs.pop();
        for (auto nx:g[x]) {
            int l=link[x];
            while (l!=-1 && g[l].count(nx.F)==0) l=link[l];
            if (l!=-1) link[nx.S]=g[l][nx.F];
            bfs.push(nx.S);
            if (te[link[nx.S]]) {
                tlink[nx.S]=link[nx.S];
            }
            else {
                tlink[nx.S]=tlink[link[nx.S]];
            }
        }
    }
}

// Returns matches {id, endpos}
vector<pair<int, int> > match(const string& s) {
    int tn=0;
    vector<pair<int, int> > re;
    for (int i=0; i<(int)s.size(); i++) {
        while (tn!=-1 && g[tn].count(s[i])==0) tn=link[tn];
        if (tn!=-1) tn=g[tn][s[i]];
        int f=tlink[tn];
        if (te[tn]) re.push_back({te[tn], i});
        while (f) {
            re.push_back({te[f], i});
            f=tlink[f];
        }
    }
    return re;
}

```

```

AhoCorasick() {
    g.push_back(map<char, int>());
    link.push_back(-1);
    tlink.push_back(0);
    te.push_back(0);
}
};

```

#### 4 src/string/suffixautomaton.cpp

```

// TCR
// Online suffix automaton construction algorithm
// Time complexity of adding one character is amortized O(1)
#include <bits/stdc++.h>
using namespace std;

```

```

struct SuffixAutomaton {
    vector<map<char, int> > g;
    vector<int> link;
    vector<int> len;
    int last;
    void addC(char c) {
        int p=last;
        int t=link.size();
        link.push_back(0);
        len.push_back(len[last]+1);
        g.push_back(map<char, int>());
        while (p!=-1&&g[p].count(c)==0) {
            g[p][c]=t;
            p=link[p];
        }
        if (p!=-1) {
            int q=g[p][c];
            if (len[p]+1==len[q]) {
                link[t]=q;
            }
            else {
                int qq=link.size();
                link.push_back(link[q]);
                len.push_back(len[p]+1);
                g.push_back(g[q]);
                while (p!=-1&&g[p][c]==q) {
                    g[p][c]=qq;
                    p=link[p];
                }
            }
        }
    }
};

```

```

        link[q]=qq;
        link[t]=qq;
    }
    last=t;
}
suffixAutomaton() : suffixAutomaton("") {}
suffixAutomaton(string s) {
    last=0;
    g.push_back(map<char, int>());
    link.push_back(-1);
    len.push_back(0);
    for (int i=0;i<(int)s.size();i++) {
        addC(s[i]);
    }
};

```

#### 5 src/string/lcparray.cpp

```

// TCR
// Constructs LCP array from suffix array in O(n) time
// You can change vector<int> s to string s
#include <bits/stdc++.h>
using namespace std;

vector<int> lcpArray(vector<int> s, vector<int> sa) {
    int n=s.size();
    int k=0;
    vector<int> ra(n), lcp(n);
    for (int i=0;i<n;i++) ra[sa[i]]=i;
    for (int i=0;i<n;i++) {
        if (k) k--;
        if (ra[i]==n-1) {
            k=0;
            continue;
        }
        int j=sa[ra[i]+1];
        while (k<n&&s[(i+k)%n]==s[(j+k)%n]) k++;
        lcp[ra[i]]=k;
        if (ra[(sa[ra[i]]+1)%n]>ra[(sa[ra[j]]+1)%n]) k=0;
    }
    return lcp;
}

```

## 6 src/graph/eulertour.cpp

```
// TCR
// Finds Euler tour of graph in O(E) time

// Parameters are the adjacency list, number of nodes,
// return value vector, and d=1 if the graph is directed
// Return array contains E+1 elements, the first and last
// elements are same

// Undefined behavior if Euler tour doesn't exist

// Note that Eulerian path can be reduced to Euler tour
// by adding an edge from the last vertex to the first

// In bidirectional graph edges must be in both direction
// Be careful to not add loops twice in case of bidirectional graph
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

struct EulerTour {
    int dir;
    vector<vector<pair<int, int> > > g;
    vector<int> used;

    void dfs(int x, vector<int>& ret) {
        int t=x;
        vector<int> c;
        while (1) {
            while (used[g[t].back().S]) g[t].pop_back();
            auto nx=g[t].back();
            g[t].pop_back();
            used[nx.S]=1;
            t=nx.F;
            c.push_back(t);
            if (t==x) break;
        }
        for (int a:c) {
            ret.push_back(a);
            while (g[a].size()>0&&used[g[a].back().S]) g[a].pop_back();
            if (g[a].size()>0) dfs(a, ret);
        }
    }
};
```

```
EulerTour(vector<int>*& og, int n, vector<int>& ret, int d=0) : dir(d),
g(n+1) {
    int i2=0;
    for (int i=1;i<=n;i++) {
        for (int nx:og[i]) {
            if (d==1||nx<=i) {
                if (d==0&&nx<i) {
                    g[nx].push_back({i, i2});
                }
                g[i].push_back({nx, i2++});
            }
        }
    }
    used.resize(i2);
    for (int i=1;i<=n;i++) {
        if (g[i].size()>0) {
            ret.push_back(i);
            dfs(i, ret);
            break;
        }
    }
};
```

## 7 src/graph/stronglyconnected.cpp

```
// TCR
// Kosaraju's algorithm for strongly connected components O(V+E)
// Components will be returned in topological order
// Uses 1-indexing
#include <bits/stdc++.h>
using namespace std;

struct SCC {
    vector<int> used;
    vector<vector<int> > g2;

    void dfs1(vector<int>*& g, int x, vector<int>& ns) {
        if (used[x]==1) return;
        used[x]=1;
        for (int nx:g[x]) {
            g2[nx].push_back(x);
            dfs1(g, nx, ns);
        }
        ns.push_back(x);
    }
};
```

```

void dfs2(int x, vector<int>& co) {
    if (used[x]==2) return;
    used[x]=2;
    co.push_back(x);
    for (int nx:g2[x]) {
        dfs2(nx, co);
    }
}

// Returns strongly connected components of the graph in vector ret
// n is the size of the graph, g is the adjacency list
SCC(vector<int>* g, int n, vector<vector<int>>& ret) : used(n+1),
g2(n+1) {
    vector<int> ns;
    for (int i=1;i<=n;i++) {
        dfs1(g, i, ns);
    }
    for (int i=n-1;i>=0;i--) {
        if (used[ns[i]]!=2) {
            ret.push_back(vector<int>());
            dfs2(ns[i], ret.back());
        }
    }
};

```

## 8 src/graph/cutvertices.cpp

```

// TCR
// Finds cutvertices and 2-vertex-connected components of graph
// 2-vertex-connected components are stored in bg
// Uses 1-indexing
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

struct Biconnected {
    vector<int> cut;
    vector<int> h;
    vector<int> d;
    vector<map<int, vector<int>>>> bg;
    vector<pair<int, int>> es;
    vector<int> used;

```

```

    int cc;
    void dfs1(vector<int>* g, int x, int p) {
        h[x]=d[x];
        int f=0;
        for (int nx:g[x]) {
            if (nx!=p) {
                if (!used[nx]) es.push_back({x, nx});
                if (d[nx]==0) {
                    f++;
                    d[nx]=d[x]+1;
                    int ts=es.size();
                    dfs1(g, nx, x);
                    h[x]=min(h[x], h[nx]);
                    if (h[nx]>=d[x]) {
                        cut[x]=1;
                        while ((int)es.size()>=ts) {
                            auto e=es.back();
                            bg[e.F][cc].push_back(e.S);
                            bg[e.S][cc].push_back(e.F);
                            used[e.S]=1;
                            used[e.F]=1;
                            es.pop_back();
                        }
                        used[x]=0;
                        cc++;
                    }
                }
                h[x]=min(h[x], d[nx]);
            }
        }
        if (p==0) {
            if (f>1) cut[x]=1;
            else cut[x]=0;
        }
    }

    Biconnected(vector<int>* g, int n) : cut(n+1), h(n+1), d(n+1), bg(n+1),
used(n+1) {
        cc=1;
        for (int i=1;i<=n;i++) {
            if (d[i]==0) {
                d[i]=1;
                dfs1(g, i, 0);
            }
        }
    }
}

```

```
};

int main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);
}
```

## 9 src/graph/mincostflow.cpp

```
// TCR
// Finds minimum-cost k-flow
//  $O(V E^2 \log U)$ , where  $U$  is maximum possible flow
// Finding augmenting path is  $O(V E)$ , usually faster
// Uses scaling flow and finds augmenting path with SPFA
// Only 1-directional edges allowed
// Doesn't work if graph contains negative cost cycles
// Uses 1-indexing
```

```
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
typedef long long ll;
typedef long double ld;
```

```
const ll inf=1e18;
```

```
struct MinCostFlow {
    vector<vector<ll> > f;
    vector<vector<int> > g;
    vector<vector<ll> > c;
    vector<ll> d;
    vector<int> from;
    vector<int> inq;
    queue<int> spfa;

    void relax(int x, ll di, int p) {
        if (di>=d[x]) return;
        d[x]=di;
        from[x]=p;
        if (!inq[x]) {
            spfa.push(x);
            inq[x]=1;
        }
    }
}
```

```
ll augment(ll x, ll s, ll fl) {
    if (x==s) return fl;
    ll r=augment(from[x], s, min(fl, f[from[x]][x]));
    f[from[x]][x]-=r;
    f[x][from[x]]+=r;
    return r;
}

pair<ll, ll> flow(int s, int t, ll miv, ll kf) {
    int n=g.size()-1;
    for (int i=1;i<=n;i++) {
        d[i]=inf;
        inq[i]=0;
    }
    relax(s, 0, 0);
    while (!spfa.empty()) {
        int x=spfa.front();
        spfa.pop();
        inq[x]=0;
        for (int nx:g[x]) {
            if (f[x][nx]>=miv) relax(nx, d[x]+c[x][nx], x);
        }
    }
    if (d[t]<inf) {
        ll fl=augment(t, s, kf);
        return {fl, fl*d[t]};
    }
    return {0, 0};
}

// maxv is maximum possible flow on a single augmenting path
// kf is inteded flow, set infinite for maxflow
// returns {flow, cost}
pair<ll, ll> getKFlow(int source, int sink, ll maxv, ll kf) {
    int n=g.size()-1;
    for (int i=1;i<=n;i++) {
        g[i].clear();
        for (int ii=1;ii<=n;ii++) {
            if (f[i][ii]!=0||f[ii][i]!=0) g[i].push_back(ii);
        }
    }
    ll r=0;
    ll k=1;
    ll co=0;
    while (k*2<=maxv) k*=2;
    for (;k>0&&kf>0;k/=2) {
```



```

        while (1) {
            pair<ll, ll> t=flow(source, sink, k, kf);
            r+=t.F;
            kf-=t.F;
            co+=t.S;
            if (kf==0||t.F==0) break;
        }
    }
    return {r, co};
}

void addEdge(int a, int b, ll capa, ll cost) {
    f[a][b]=capa;
    c[a][b]=cost;
    c[b][a]=-cost;
}

MinCostFlow(int n) : f(n+1), g(n+1), c(n+1), d(n+1), from(n+1), inq(n+1) {
    for (int i=1;i<=n;i++) {
        f[i]=vector<ll>(n+1);
        c[i]=vector<ll>(n+1);
    }
};

```

## 10 src/graph/dynamicconnectivity.cpp

```

// TCR
// O(n log n) offline solution for dynamic connectivity problem
// ? count the number of connected components
// + A B add edge between A and B
// - A B remove edge between A and B
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

struct e{
    int a,b,l,r;
};

int qqs[603030];
int qv[603030];
int is[603030];
int uf[603030];
int id[603030];

```

```

int getu(int a){
    if (uf[a]==a) return a;
    return uf[a]=getu(uf[a]);
}

void un(int a, int b){
    a=getu(a);
    b=getu(b);
    if (a!=b) uf[a]=b;
}

void go(int l, int r, int uc, int n, vector<e> es){
    for (int i=1;i<=n;i++){
        is[i]=0;
    }
    int i2=1;
    vector<pair<int, int> > te;
    vector<e> ce;
    for (e ee:es){
        if (ee.a!=ee.b&&(!(ee.l>r||ee.r<l))){
            if (is[ee.a]==0){
                is[ee.a]=i2;
                ee.a=i2++;
            }
            else{
                ee.a=is[ee.a];
            }
            if (is[ee.b]==0){
                is[ee.b]=i2;
                ee.b=i2++;
            }
            else{
                ee.b=is[ee.b];
            }
            if (ee.l<=l&&r<=ee.r){
                te.push_back({ee.a, ee.b});
            }
            else{
                ce.push_back(ee);
            }
        }
    }
    for (int i=1;i<=n;i++){
        if (is[i]==0){
            uc++;
        }
    }
}

```

```

    }
    for (int i=1;i<i2;i++){
        uf[i]=i;
        id[i]=0;
    }
    for (auto ee:te){
        un(ee.F, ee.S);
    }
    int i3=1;
    for (int i=1;i<i2;i++){
        if (id[getu(uf[i])]==0){
            id[getu(uf[i])]=i3++;
        }
    }
    for (e&ee:ce){
        ee.a=id[getu(ee.a)];
        ee.b=id[getu(ee.b)];
    }
    if (l==r){
        qv[l]=uc+i3-1;
    }
    else{
        int m=(l+r)/2;
        go(l, m, uc, i3-1, ce);
        go(m+1, r, uc, i3-1, ce);
    }
}

int main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n,k;
    cin>>n>>k;
    int qs=0;
    vector<e> es;
    map<pair<int, int>, int> ae;
    for (int i=1;i<=k;i++){
        char t;
        cin>>t;
        if (t=='?'){
            qqs[qs++]=i;
        }
        else{
            int a,b;
            cin>>a>>b;
            if (t=='+'){

```

```

                pair<int, int> lol={min(a, b), max(a, b)};
                ae[lol]=i;
            }
            else{
                pair<int, int> lol={min(a, b), max(a, b)};
                int s=ae[lol];
                ae[lol]=0;
                es.push_back({a, b, s, i});
            }
        }
    }
    for (auto t:ae){
        if (t.S>0){
            es.push_back({t.F.F, t.F.S, t.S, k});
        }
    }
    go(0, (1<<19)-1, 0, n, es);
    for (int i=0;i<qqs;i++){
        cout<<qv[qqs[i]]<<'\n';
    }
}

```

## 11 src/graph/scalingflow.cpp

```

// TCR
// Scaling flow algorithm for maxflow
//  $O(E^2 \log U)$ , where  $U$  is maximum possible flow
// In practice  $O(E^2)$ 
// Uses 1-indexing

#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
typedef long long ll;

struct MaxFlow {
    vector<vector<ll> > f;
    vector<vector<int> > g;
    vector<int> used;
    int cc;

    ll flow(int x, int t, ll fl, ll miv) {
        if (x==t) return fl;
        used[x]=cc;
        for (int nx:g[x]) {

```

```

        if (used[nx]!=cc&&f[x][nx]>=miv) {
            ll r=flow(nx, t, min(fl, f[x][nx]), miv);
            if (r>0) {
                f[x][nx]-=r;
                f[nx][x]+=r;
                return r;
            }
        }
    }
    return 0;
}

// maxv is maximum expected maxflow
ll getMaxFlow(int source, int sink, ll maxv) {
    int n=g.size()-1;
    for (int i=1;i<=n;i++) {
        g[i].clear();
        for (int ii=1;ii<=n;ii++) {
            if (f[i][ii]!=0||f[ii][i]!=0) g[i].push_back(ii);
        }
    }
    cc=1;
    ll r=0;
    ll k=1;
    while (k*2<=maxv) k*=2;
    for (;k>0;k/=2) {
        while (ll t=flow(source, sink, maxv, k)) {
            r+=t;
            cc++;
        }
        cc++;
    }
    return r;
}

void addEdge(int a, int b, ll c) {
    f[a][b]+=c;
}

MaxFlow(int n) : f(n+1), g(n+1), used(n+1) {
    for (int i=1;i<=n;i++) {
        f[i]=vector<ll>(n+1);
    }
}
};

```

## 12 src/geometry/basic.cpp

```

// TCR
// Basic geometry functions using complex numbers
// Mostly copied from https://github.com/ttalvitie/libcontest/
/* Useful functions of complex number class
    CT abs(co x): Length
    CT norm(co x): Square of length
    CT arg(co x): Angle
    co polar(CT length, CT angle): Complex from polar components
*/
#include <bits/stdc++.h>
#define X real()
#define Y imag()
using namespace std;
typedef long double ld;
typedef long long ll;
// Coordinate type
typedef ld CT;
typedef complex<CT> co;

// Return true iff points a, b, c are CCW oriented.
bool ccw(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y>0;
}

// Return true iff points a, b, c are collinear.
// Note: doesn't make much sense with non-integer CT.
bool collinear(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y==0;
}

// Rotate x with angle ang
co rotate(co x, CT ang) {
    return x*polar((CT)1, ang);
}

// Check whether segments [a, b] and [c, d] intersect.
// The segments must not be collinear. Doesn't handle edge cases (endpoint of
// a segment on the other segment) consistently.
bool intersects(co a, co b, co c, co d) {
    return ccw(a, d, b)!=ccw(a, c, b)&&ccw(c, a, d)!=ccw(c, b, d);
}

// Interpolate between points a and b with parameter t.
co interpolate(CT t, co a, co b) {

```

```

    return a+t*(b-a);
}

// Return interpolation parameter between a and b of projection of v to the
// line defined by a and b.
// Note: no rounding behavior specified for integers.
CT projectionParam(co v, co a, co b) {
    return ((v-a)/(b-a)).X;
}

// Compute the distance of point v from line a..b.
// Note: Only for non-integers!
CT pointLineDistance(co p, co a, co b) {
    return abs(((p-a)/(b-a)).Y)*abs(b-a);
}

// Compute the distance of point v from segment a..b.
// Note: Only for non-integers!
CT pointSegmentDistance(co p, co a, co b) {
    co z=(p-a)/(b-a);
    if(z.X<0) return abs(p-a);
    if(z.X>1) return abs(p-b);
    return abs(z.Y)*abs(b-a);
}

// Return interpolation parameter between a and b of the point that is also
// on line c..d.
// Note: Only for non-integers!
CT intersectionParam(co a, co b, co c, co d) {
    co u=(c-a)/(b-a);
    co v=(d-a)/(b-a);
    return (u.X*v.Y-u.Y*v.X)/(v.Y-u.Y);
}

```

### 13 src/geometry/convexhull.cpp

```

// TCR
// Computes the convex hull of given set of points in O(n log n)
// Uses Andrew's algorithm
// The points on the edges of the hull are not listed
// Change > to >= in ccw function to list the points on the edges
// Returns points in counterclockwise order
#include <bits/stdc++.h>
#define X real()
#define Y imag()
using namespace std;

```

```

typedef long double ld;
typedef long long ll;
// Coordinate type
typedef ll CT;
typedef complex<CT> co;

bool ccw(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y>0;
}

vector<co> convexHull(vector<co> ps) {
    auto cmp = [](co a, co b) {
        if (a.X==b.X) return a.Y<b.Y;
        else return a.X<b.X;
    };
    sort(ps.begin(), ps.end(), cmp);
    ps.erase(unique(ps.begin(), ps.end(), ps.end()));

    int n=ps.size();
    if (n<=2) return ps;

    vector<co> hull;
    hull.push_back(ps[0]);
    for (int d=0;d<2;d++) {
        if (d) reverse(ps.begin(), ps.end());
        int s=hull.size();
        for (int i=1;i<n;i++) {
            while ((int)hull.size()-s&&!ccw(hull[hull.size()-2],
hull.back(), ps[i])) {
                hull.pop_back();
            }
            hull.push_back(ps[i]);
        }
        hull.pop_back();
        return hull;
    }
}

```

### 14 src/geometry/anglesort.cpp

```

// TCR
// Comparasion function for sorting points around origin
// Points are sorted in clockwise order
/*
122
143

```

```

443*/
#include <bits/stdc++.h>
#define X real()
#define Y imag()
#define F first
#define S second
using namespace std;
typedef long double ld;
typedef long long ll;
// Coordinate type
typedef ld CT;
typedef complex<CT> co;

bool ccw(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y>0;
}

int ar(co x) {
    if (x.Y>=0&&x.X<0) return 1;
    if (x.X>=0&&x.Y>0) return 2;
    if (x.Y<=0&&x.X>0) return 3;
    return 4;
}

bool cp(co p1, co p2) {
    if (ar(p1)!=ar(p2)) {
        return ar(p1)<ar(p2);
    }
    return ccw({0, 0}, p2, p1)>0;
}

```

## 15 src/geometry/minkowskisum.cpp

```

// TCR
// Computes the Minkowski sum of 2 convex polygons in O(n+m log n+m)
// Returns convex polygon in counterclockwise order
// The points on the edges of the hull are listed
// The convex hulls must be in counterclockwise order
#include <bits/stdc++.h>
#define X real()
#define Y imag()
#define F first
#define S second
using namespace std;
typedef long double ld;
typedef long long ll;

```

```

// Coordinate type
typedef ll CT;
typedef complex<CT> co;

bool ccw(co a, co b, co c) {
    return ((c-a)*conj(b-a)).Y>0;
}

int ar(co x) {
    if (x.Y>=0&&x.X<0) return 1;
    if (x.X>=0&&x.Y>0) return 2;
    if (x.Y<=0&&x.X>0) return 3;
    return 4;
}

bool cp(pair<co, pair<int, int> > p1, pair<co, pair<int, int> > p2) {
    if (ar(p1.F)!=ar(p2.F)) {
        return ar(p1.F)<ar(p2.F);
    }
    return ccw({0, 0}, p2.F, p1.F)>0;
}

vector<co> minkowski(vector<co>&a, vector<co>&b) {
    int n=a.size();
    int m=b.size();
    if (n==0) return b;
    if (m==0) return a;
    if (n==1) {
        vector<co> ret(m);
        for (int i=0;i<m;i++) {
            ret[i]=b[i]+a[0];
        }
        return ret;
    }
    if (m==1) {
        vector<co> ret(n);
        for (int i=0;i<n;i++) {
            ret[i]=a[i]+b[0];
        }
        return ret;
    }
    vector<pair<co, pair<int, int> > > pp;
    for (int i=0;i<n;i++) {
        pp.push_back({a[(i+1)%n]-a[i], {1, i}});
    }
    for (int i=0;i<m;i++) {

```

```

        pp.push_back({b[(i+1)%m]-b[i], {2, i}});
    }
    sort(pp.rbegin(), pp.rend(), cp);
    co s={0, 0};
    co ad={0, 0};
    for (int i=0;i<(int)pp.size();i++) {
        s+=pp[i].F;
        if (pp[i].S.F!=pp[i+1].S.F) {
            if (pp[i].S.F==1) ad=a[(pp[i].S.S+1)%n]+b[(pp[i+1].S.S)%m];
            else ad=b[(pp[i].S.S+1)%m]+a[(pp[i+1].S.S)%n];
            ad-=s;
            break;
        }
    }
    s=ad;
    vector<co> ret(pp.size());
    for (int i=0;i<(int)pp.size();i++) {
        ret[i]=s;
        s+=pp[i].F;
    }
    return ret;
}

```

## 16 src/geometry/closestpoints.cpp

```

// TCR
// Returns square of distance between closest 2 points
// O(n log n)
#include <bits/stdc++.h>
#define X real()
#define Y imag()
#define F first
#define S second
using namespace std;
typedef long long ll;
typedef complex<ll> co;

const ll inf=2e18;

ll csqrt(ll x) {
    ll r=sqrt(x);
    while (r*r<x) r++;
    while (r*r>x) r--;
    return r;
}

```

```

ll sq(ll x) {
    return x*x;
}

ll closestPoints(vector<co> points) {
    int n=points.size();
    vector<pair<ll, ll> > ps(n);
    for (int i=0;i<n;i++) {
        ps[i]={points[i].X, points[i].Y};
    }
    sort(ps.begin(), ps.end());
    int i2=0;
    ll d=inf;
    set<pair<ll, ll> > pss;
    for (int i=0;i<n;i++) {
        while (i2<i&&sq(ps[i].F-ps[i2].F)>d) {
            pss.erase({ps[i2].S, ps[i2].F});
            i2++;
        }
        auto it=pss.lower_bound({ps[i].S-csqrt(d), -inf});
        for (;it!=pss.end();it++) {
            if (sq(it->F-ps[i].S)>d) break;
            d=min(d, sq(it->F-ps[i].S)+sq(it->S-ps[i].F));
        }
        pss.insert({ps[i].S, ps[i].F});
    }
    return d;
}

```

## 17 src/datastructure/orderedset.cpp

```

// TCR
// Sample code on how to use g++ ordered set
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
using namespace std;
using namespace __gnu_pbds;
//using namespace pb_ds;

typedef tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update>
ordered_set;

int main() {
    ordered_set X;
}

```

```

X.insert(1);
X.insert(4);
cout<<*X.find_by_order(1)<<endl; // 4
cout<<X.order_of_key(3)<<endl;   // 1
}

```

## 18 src/datastructure/treap.cpp

```

// TCR
// Treap implementation with pointers
// Expected running time of split and merge is O(log n)
#include <bits/stdc++.h>
using namespace std;

typedef struct node* pnode;
struct node {
    pnode l,r;
    int pr,c;
    node() {
        l=0;
        r=0;
        c=1;
        pr=rand();
    }
};

// Returns the size of the subtree t
int cnt(pnode t) {
    if (!t) return 0;
    return t->c;
}

// Updates the size of the subtree t
void upd(pnode t) {
    if (!t) return;
    t->c=cnt(t->l)+cnt(t->r)+1;
}

// Put lazy updates here
void push(pnode t) {
    if (!t) return;
    // Do lazy update
}

// Merges trees l and r into tree t
void merg(pnode& t, pnode l, pnode r) {

```

```

    push(l);
    push(r);
    if (!l) t=r;
    else if (!r) t=l;
    else {
        if (l->pr>r->pr) {
            merg(l->r, l->r, r);
            t=l;
        }
        else {
            merg(r->l, l, r->l);
            t=r;
        }
    }
    upd(t);
}

// Splits tree t into trees l and r
// Size of tree l will be k
void split(pnode t, pnode& l, pnode& r, int k) {
    if (!t) {
        l=0;
        r=0;
        return;
    }
    else {
        push(t);
        if (cnt(t->l)>=k) {
            split(t->l, l, t->l, k);
            r=t;
        }
        else {
            split(t->r, t->r, r, k-cnt(t->l)-1);
            l=t;
        }
    }
    upd(t);
}

```

## 19 src/math/fft.cpp

```

// TCR
// Fast Fourier transform and convolution using it
// O(n log n)
#include <bits/stdc++.h>
using namespace std;

```

```

typedef long double ld;
typedef long long ll;
typedef complex<ld> co;
const ld PI=atan2(0, -1);

vector<co> fft(vector<co> x, int d) {
    int n=x.size();
    for (int i=0;i<n;i++) {
        int u=0;
        for (int j=1;j<n;j*=2) {
            u*=2;
            if (i&j) u++;
        }
        if (i<u) {
            swap(x[i], x[u]);
        }
    }
    for (int m=2;m<=n;m*=2) {
        co wm=exp(co{0, d*2*PI/m});
        for (int k=0;k<n;k+=m) {
            co w=1;
            for (int j=0;j<m/2;j++) {
                co t=w*x[k+j+m/2];
                co u=x[k+j];
                x[k+j]=u+t;
                x[k+j+m/2]=u-t;
                w*=wm;
            }
        }
    }
    if (d==1) {
        for (int i=0;i<n;i++) {
            x[i]/=n;
        }
    }
    return x;
}

vector<ll> conv(vector<ll> a, vector<ll> b) {
    int as=a.size();
    int bs=b.size();
    vector<co> aa(as);
    vector<co> bb(bs);
    for (int i=0;i<as;i++) {
        aa[i]=a[i];
    }
}

```

```

        for (int i=0;i<bs;i++) {
            bb[i]=b[i];
        }
        int n=1;
        while (n<as+bs-1) n*=2;
        aa.resize(n*2);
        bb.resize(n*2);
        aa=fft(aa, 1);
        bb=fft(bb, 1);
        vector<co> c(2*n);
        for (int i=0;i<2*n;i++) {
            c[i]=aa[i]*bb[i];
        }
        c=fft(c, -1);
        c.resize(as+bs-1);
        vector<ll> r(as+bs-1);
        for (int i=0;i<as+bs-1;i++) {
            r[i]=(ll)round(c[i].real());
        }
        return r;
    }
}

int main() {
    // Shoud print 12 11 30 7
    vector<ll> a={3, 2, 7};
    vector<ll> b={4, 1};
    vector<ll> c=conv(a, b);
    for (ll t:c) {
        cout<<t<<endl;
    }
}

```

## 20 src/math/primitiveroot.cpp

```

// TCR
// Computes primitive root
// O(sqrt(n))
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;

ll pot(ll x, ll p, ll mod) {
    if (p==0) return 1;
    if (p%2==0) {
        x=pot(x, p/2, mod);
        return (x*x)%mod;
    }
}

```



```

    }
    return (x*pot(x, p-1, mod))%mod;
}

ll primitiveRoot(ll p) {
    vector<ll> fact;
    ll phi=p-1;
    ll n=phi;
    for (ll i=2;i*i<=n;i++) {
        if (n%i==0) {
            fact.push_back(i);
            while (n%i==0) n/=i;
        }
    }
    if (n>1) fact.push_back(n);
    for (ll res=2;res<=p;res++) {
        bool ok = true;
        for (int i=0;i<(int)fact.size()&&ok;i++) ok&=pot(res, phi/fact[i],
p)!=1;
        if (ok) return res;
    }
    return -1;
}

int main() {
    cout<<primitiveRoot(1000000007)<<endl;// should print 5
}

```

## 21 src/math/miller-rabin.cpp

```

// TCR
// Deterministic Miller-Rabin primality test
// Works for all 64 bit integers
// Support of 128 bit integers is required to test over 32 bit integers
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef __int128 lll;

lll powmod(lll a, lll p, lll mod) {
    if (p==0) return 1;
    if (p%2==0) {
        a=powmod(a, p/2, mod);
        return (a*a)%mod;
    }
    return (a*powmod(a, p-1, mod))%mod;
}

```

```

}

bool is_w(ll a, ll even, ll odd, ll p) {
    lll u = powmod(a, odd, p);
    if (u==1) return 0;
    for (ll j=1;j<even;j*=2) {
        if (u==p-1) return 0;
        u*=u;
        u%=p;
    }
    return 1;
}

bool isPrime(ll p) {
    if (p==2) return 1;
    if (p<=1||p%2==0) return 0;
    ll odd=p-1;
    ll even=1;
    while (odd%2==0) {
        even*=2;
        odd/=2;
    }
    ll b[7]={2, 325, 9375, 28178, 450775, 9780504, 1795265022};
    for (ll i=0;i<7;i++) {
        ll a=b[i]%p;
        if (a==0) return 1;
        if (is_w(a, even, odd, p)) return 0;
    }
    return 1;
}

```

## 22 src/math/diophantine.cpp

```

// TCR
// Solves ax+by=c in O(log a+b) time
// Returns {is, {x, y}}, is=0 if there is no solution
// Use __int128 for 64 bit numbers
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
typedef long long ll;

ll ee(ll a, ll b, ll ca, ll cb, ll xa, ll xb, ll&x, ll&y) {
    if (cb==0) {
        x=xa;

```

```

        if (b==0) y=0;
        else y=(ca-a*xa)/b;
        return ca;
    }
    else return ee(a, b, cb, ca%cb, xb, xa-(ca/cb)*xb, x, y);
}

pair<int, pair<ll, ll> > solve(ll a, ll b, ll c) {
    if (c==0) return {1, {0, 0}};
    if (a==0&&b==0) return {0, {0, 0}};
    ll x,y;
    ll g=ee(a, b, a, b, 1, 0, x, y);
    if (abs(c)%g>0) return {0, {0, 0}};
    return {1, {x*(c/g), y*(c/g)}};
}

```

## 23 src/math/fftmod.cpp

```

// TCR
// Precise FFT modulo mod
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
// luku muotoa (2^20)*k+1
const ll mod=1045430273;
// luku jonka order 2^20
const ll root=363;
// sen kaanteisluku
const ll root_1=296637240;
const ll root_pw=1<<20;

ll pot(ll x, ll p) {
    if (p==0) return 1;
    if (p%2==0) {
        x=pot(x, p/2);
        return (x*x)%mod;
    }
    return (x*pot(x, p-1))%mod;
}

ll inv(ll x) {
    return pot(x, mod-2);
}

vector<ll> fft (vector<ll> a, int d) {
    int n=(int)a.size();

```

```

    for (int i=1,j=0;i<n;i++) {
        int bit=n>>1;
        for (;j>=bit;bit>>=1) {
            j-=bit;
        }
        j+=bit;
        if (i<j) swap (a[i], a[j]);
    }
    for (int len=2;len<=n;len<<=1) {
        ll wlen=root;
        if (d==1) {
            wlen=root_1;
        }
        for (int i=len;i<root_pw;i<<=1) wlen=(wlen*wlen)%mod;
        for (int i=0;i<n;i+=len) {
            ll w = 1;
            for (int j=0;j<len/2;j++) {
                ll u = a[i+j];
                ll v = (a[i+j+len/2]*w)%mod;
                if (u+v<mod) {
                    a[i+j]=u+v;
                }
                else {
                    a[i+j]=u+v-mod;
                }
                if (u-v>=0) {
                    a[i+j+len/2]=u-v;
                }
                else {
                    a[i+j+len/2]=u-v+mod;
                }
                w=(w*wlen)%mod;
            }
        }
    }
    if (d==1) {
        ll nrev=inv(n);
        for (int i=0;i<n;i++) a[i]=(a[i]*nrev)%mod;
    }
    return a;
}

vector<ll> conv(vector<ll> a, vector<ll> b) {
    int as=a.size();
    int bs=b.size();
    vector<ll> aa(as);

```

```

vector<ll> bb(bs);
for (int i=0;i<as;i++) {
    aa[i]=a[i];
}
for (int i=0;i<bs;i++) {
    bb[i]=b[i];
}
int n=1;
while (n<as+bs-1) n*=2;
aa.resize(n*2);
bb.resize(n*2);
aa=fft(aa, 1);
bb=fft(bb, 1);
vector<ll> c(2*n);
for (int i=0;i<2*n;i++) {
    c[i]=(aa[i]*bb[i])%mod;
}
c=fft(c, -1);
c.resize(as+bs-1);
return c;
}

int main() {
    // Shoud print 12 11 30 7
    vector<ll> a={3, 2, 7};
    vector<ll> b={4, 1};
    vector<ll> c=conv(a, b);
    for (ll t:c) {
        cout<<t<<endl;
    }
}

```

## 24 src/math/gaussjordan.cpp

```

// TCR
// Solves system of linear equations in  $O(n \cdot m^2)$ 
// Using doubles or mod 2
// Using doubles might have large precision errors or overflow
// Returns 0 if no solution exists, 1 if there is one solution
// or 2 if infinite number of solutions exists
// If at least one solution exists, it is returned in ans
// You can modify the general algorithm to work mod p by using modular inverse
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef long double ld;

```

```

const ld eps=1e-12;

// Using doubles
int gaussD (vector<vector<ld> > a, vector<ld>& ans) {
    int n=(int)a.size();
    int m=(int)a[0].size()-1;

    vector<int> where(m,-1);
    for (int col=0,row=0;col<m&&row<n;col++) {
        int sel=row;
        for (int i=row;i<n;i++) {
            if (abs(a[i][col])>abs(a[sel][col])) sel=i;
        }
        if (abs(a[sel][col])<eps) continue;
        for (int i=col;i<=m;i++) {
            swap (a[sel][i], a[row][i]);
        }
        where[col]=row;

        for (int i=0;i<n;i++) {
            if (i!=row) {
                ld c=a[i][col]/a[row][col];
                for (int j=col;j<=m;j++) {
                    a[i][j]-=a[row][j]*c;
                }
            }
        }
        row++;
    }

    ans.assign(m, 0);
    for (int i=0;i<m;i++) {
        if (where[i]!=-1) ans[i]=a[where[i]][m]/a[where[i]][i];
    }
    for (int i=0;i<n;i++) {
        ld sum=0;
        for (int j=0;j<m;j++) {
            sum+=ans[j]*a[i][j];
        }
        if (abs(sum-a[i][m])>eps) return 0;
    }

    for (int i=0;i<m;i++) {
        if (where[i]==-1) return 2;
    }
    return 1;
}

```

```

}

// mod 2
// n is number of rows m is number of variables
const int M=4;
int gaussM(vector<bitset<M> > a, int n, int m, bitset<M-1>& ans) {
    vector<int> where (m, -1);
    for (int col=0,row=0;col<m&&row<n;col++) {
        for (int i=row;i<n;i++) {
            if (a[i][col]) {
                swap (a[i], a[row]);
                break;
            }
        }
        if (!a[row][col]) continue;
        where[col]=row;

        for (int i=0;i<n;i++) {
            if (i!=row&&a[i][col]) {
                a[i]^=a[row];
            }
        }
        row++;
    }
    ans=0;
    for (int i=0;i<m;i++) {
        if (where[i]!=-1) ans[i]=a[where[i]][m];
    }
    for (int i=0;i<n;i++) {
        int sum=0;
        for (int j=0;j<m;j++) {
            sum^=ans[j]*a[i][j];
        }
        if (sum!=a[i][m]){
            return 0;
        }
    }
    for (int i=0;i<m;i++){
        if (where[i]==-1) return 2;
    }
    return 1;
}

int main() {
    // Should output 2, 1 2 0
    vector<vector<ld> > d(3);

```

```

d[0]={3, 3, -15, 9};
d[1]={1, 0, -2, 1};
d[2]={2, -1, -1, 0};
vector<ld> da;
cout<<gaussD(d, da)<<endl;
cout<<da[0]<<" "<<da[1]<<" "<<da[2]<<endl;

// Should output 1, 110
// Note that bitsets are printed in reverse order
bitset<M> r1("0110");
bitset<M> r2("1101");
bitset<M> r3("0111");
vector<bitset<M> > m={r1, r2, r3};
bitset<M-1> ma;
cout<<gaussM(m, 3, 3, ma)<<endl;
cout<<ma<<endl;
}

```

## 25 src/math/crt.cpp

```

// TCR
// Solves x from system of equations x == a_i (mod p_i)
// Overflows only if p_1*...*p_n overflows
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;

ll pot(ll x, ll p, ll mod) {
    if (p==0) return 1;
    if (p%2==0) {
        x=pot(x, p/2, mod);
        return (x*x)%mod;
    }
    return (x*pot(x, p-1, mod))%mod;
}

ll inv(ll x, ll mod) {
    return pot(x%mod, mod-2, mod);
}

ll solve(vector<ll> a, vector<ll> p) {
    vector<ll> x(a.size());
    ll r=0;
    ll k=1;
    for (int i=0;i<(int)a.size();i++) {
        x[i]=a[i];

```

```
    for (int j=0;j<i;j++) {  
        x[i]=inv(p[j], p[i])*(x[i]-x[j]);  
        x[i]=x[i]%p[i];  
        if (x[i]<0) x[i]+=p[i];  
    }  
    r+=k*x[i];  
    k*=p[i];  
}  
return r;  
}
```