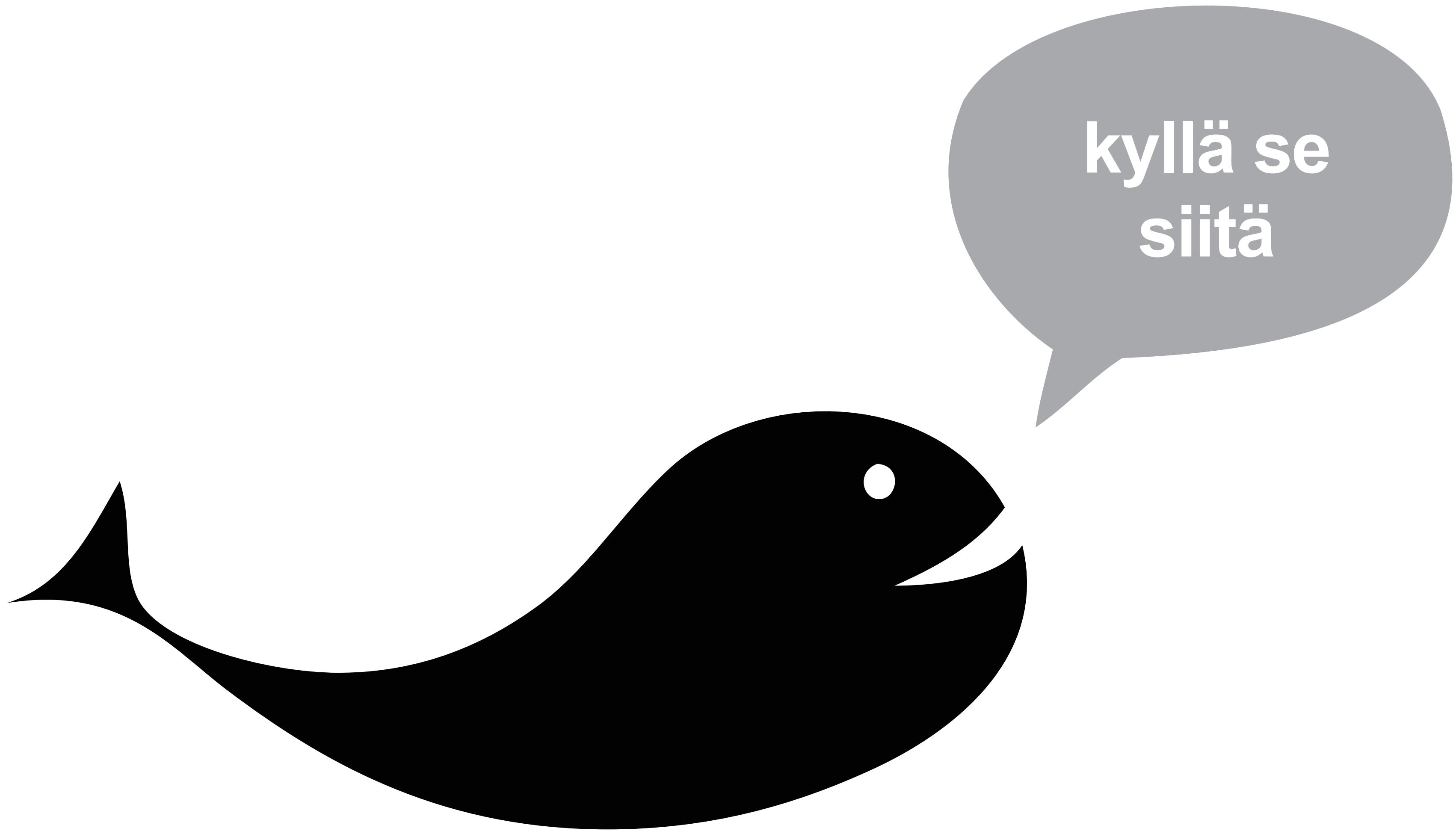
UNIVERSITY OF HELSINKI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Contents

1 src/string/suffixarray.cpp

```
// TCR
// Suffix array in O(n log^2 n)
// ~300ms runtime for 10^5 character string, ~2000ms for 5*10^5
// You can change vector<int> s to string s
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

vector<int> suffixArray(vector<int> s) {
        int n=s.size();
        vector<int> k(n);
        for (int i=0;i<n;i++) {
                k[i]=s[i];
        }
        vector<pair<pair<int, int>, int> > v(n);
        for (int t=1;t<=n;t*=2) {
                for (int i=0;i<n;i++) {
                        int u=-1;
                        if (i+t<n) u=k[i+t];
                        v[i]={{k[i], u}, i};
                }
                sort(v.begin(), v.end());
                int c=0;
                for (int i=0;i<n;i++) {
                        if (i>0&&v[i-1].F!=v[i].F) c++;
                        k[v[i].S]=c;
                }
                if (c==n-1) break;
        }
        vector<int> sa(n);
        for (int i=0;i<n;i++) sa[k[i]]=i;
        return sa;
}
```

2 src/string/lcparray.cpp

```
// TCR
```

```
// Constructs LCP array from suffix array in O(n) time
// You can change vector<int> s to string s
#include <bits/stdc++.h>
using namespace std;

vector<int> lcpArray(vector<int> s, vector<int> sa) {
	int n=s.size();
	int k=0;
	vector<int> ra(n), lcp(n);
	for (int i=0;i<n;i++) ra[sa[i]]=i;
	for (int i=0;i<n;i++) {
		if (k) k--;
		if (ra[i]==n-1) {
			k=0;
			continue;
		}
		int j=sa[ra[i]+1];
		while (k<n&&s[(i+k)%n]==s[(j+k)%n]) k++;
		lcp[ra[i]]=k;
		if (ra[(sa[ra[i]]+1)%n]>ra[(sa[ra[j]]+1)%n]) k=0;
	}
	return lcp;
}
```

3 src/string/aho-corasick.cpp

```
// TCR
// Aho-Corasick algorithm
// Building of automaton is O(L) where L is total length of dictionary
// Matching is O(n + number of matches), O(n sqrt(L)) in the worst case
// Add dictionary using addString and then use pushLinks
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

struct AhoCorasick {
	vector<map<char, int> > g;
	vector<int> link;
	vector<int> tlink;
	vector<int> te;

	// Use 1-indexing in id
	void addString(const string& s, int id) {
		int tn=0;
		for (int i=0;i<(int)s.size();i++) {
			if (g[tn][s[i]]==0) {
				g[tn][s[i]]=g.size();
				g.push_back(map<char, int>());
				link.push_back(0);
				tlink.push_back(0);
				te.push_back(0);
			}
			tn=g[tn][s[i]];
		}
		te[tn]=id;
	}

	void pushLinks() {
		queue<int> bfs;
		bfs.push(0);
		while (!bfs.empty()) {
			int x=bfs.front();
			bfs.pop();
			for (auto nx:g[x]) {
				int l=link[x];
				while (l!=-1&&g[l].count(nx.F)==0) l=link[l];
				if (l!=-1) link[nx.S]=g[l][nx.F];
				bfs.push(nx.S);
				if (te[link[nx.S]]) {
					tlink[nx.S]=link[nx.S];
				}
				else{
					tlink[nx.S]=tlink[link[nx.S]];
				}
			}
		}
	}

	// Returns matches {id, endpos}
	vector<pair<int, int> > match(const string& s) {
		int tn=0;
		vector<pair<int, int> > re;
		for (int i=0;i<(int)s.size();i++) {
			while (tn!=-1&&g[tn].count(s[i])==0) tn=link[tn];
			if (tn==-1) tn=0;
			tn=g[tn][s[i]];
			int f=tlink[tn];
			if (te[tn]) re.push_back({te[tn], i});
			while (f) {
				re.push_back({te[f], i});
				f=tlink[f];
```

```
                }
            }
            return re;
        }

        AhoCorasick() {
            g.push_back(map<char, int>());
            link.push_back(-1);
            tlink.push_back(0);
            te.push_back(0);
        }
};
```

4 src/string/z.cpp

```
// TCR
// Computes the Z array in linear time
// z[i] is the length of the longest common prefix of substring
// starting at i and the string
// You can use string s instead of vector<int> s
// z[0]=0 by definition
#include <bits/stdc++.h>
using namespace std;

vector<int> zAlgo(vector<int> s) {
        int n=s.size();
        vector<int> z(n);
        int l=0;
        int r=0;
        for (int i=1;i<n;i++) {
                z[i]=max(0, min(z[i-l], r-i));
                while (i+z[i]<n&&s[z[i]]==s[i+z[i]]) z[i]++;
                if (i+z[i]>r) {
                        l=i;
                        r=i+z[i];
                }
        }
        return z;
}
```

5 src/string/suffixautomaton.cpp

```
// TCR
// Online suffix automaton construction algorithm
// Time complexity of adding one character is amortized O(1)
#include <bits/stdc++.h>
using namespace std;

struct SuffixAutomaton {
        vector<map<char, int> > g;
        vector<int> link;
        vector<int> len;
        int last;
        void addC(char c) {
                int p=last;
                int t=link.size();
                link.push_back(0);
                len.push_back(len[last]+1);
                g.push_back(map<char, int>());
                while (p!=-1&&g[p].count(c)==0) {
                        g[p][c]=t;
                        p=link[p];
                }
                if (p!=-1) {
                        int q=g[p][c];
                        if (len[p]+1==len[q]) {
                                link[t]=q;
                        }
                        else {
                                int qq=link.size();
                                link.push_back(link[q]);
                                len.push_back(len[p]+1);
                                g.push_back(g[q]);
                                while (p!=-1&&g[p][c]==q) {
                                        g[p][c]=qq;
                                        p=link[p];
                                }
                                link[q]=qq;
                                link[t]=qq;
                        }
                }
                last=t;
        }
        SuffixAutomaton() : SuffixAutomaton("") {}
        SuffixAutomaton(string s) {
                last=0;
                g.push_back(map<char, int>());
                link.push_back(-1);
                len.push_back(0);
                for (int i=0;i<(int)s.size();i++) {
                        addC(s[i]);
                }
```

```
        }
        vector<int> terminals() {
                vector<int> t;
                int p=last;
                while (p>0) {
                        t.push_back(p);
                        p=link[p];
                }
                return t;
        }
};
```

6 src/geometry/convexhull.cpp

```
// TCR
// Computes the convex hull of given set of points in O(n log n)
// Uses Andrew's algorithm
// The points on the edges of the hull are not listed
// Change > to >= in ccw function to list the points on the edges
// Returns points in counterclockwise order
#include <bits/stdc++.h>
#define X real()
#define Y imag()
using namespace std;
typedef long double ld;
typedef long long ll;
// Coordinate type
typedef ll CT;
typedef complex<CT> co;

bool ccw(co a, co b, co c) {
        return ((c-a)*conj(b-a)).Y>0;
}

vector<co> convexHull(vector<co> ps) {
        auto cmp = [](co a, co b) {
                if (a.X==b.X) return a.Y<b.Y;
                else return a.X<b.X;
        };
        sort(ps.begin(), ps.end(), cmp);
        ps.erase(unique(ps.begin(), ps.end()), ps.end());

        int n=ps.size();
        if (n<=2) return ps;

        vector<co> hull;
        hull.push_back(ps[0]);
        for (int d=0;d<2;d++) {
                if (d) reverse(ps.begin(), ps.end());
                int s=hull.size();
                for (int i=1;i<n;i++) {
                        while ((int)hull.size()>s&&!ccw(hull[hull.size()-2],
hull.back(), ps[i])) {
                                hull.pop_back();
                        }
                        hull.push_back(ps[i]);
                }
        }
        hull.pop_back();
        return hull;
}
```

7 src/geometry/anglesort.cpp

```
// TCR
// Comparasion function for sorting points around origin
// Points are sorted in clockwise order
/*
122
143
443*/
#include <bits/stdc++.h>
#define X real()
#define Y imag()
#define F first
#define S second
using namespace std;
typedef long double ld;
typedef long long ll;
// Coordinate type
typedef ld CT;
typedef complex<CT> co;

bool ccw(co a, co b, co c) {
        return ((c-a)*conj(b-a)).Y>0;
}

int ar(co x) {
        if (x.Y>=0&&x.X<0) return 1;
        if (x.X>=0&&x.Y>0) return 2;
        if (x.Y<=0&&x.X>0) return 3;
        return 4;
```

```
}

bool cp(co p1, co p2) {
	if (ar(p1)!=ar(p2)) {
		return ar(p1)<ar(p2);
	}
	return ccw({0, 0}, p2, p1)>0;
}
```

8 src/geometry/minkowskisum.cpp

```
// TCR
// Computes the Minkowski sum of 2 convex polygons in O(n+m log n+m)
// Returns convex polygon in counterclockwise order
// The points on the edges of the hull are listed
// The convex hulls must be in counterclockwise order
#include <bits/stdc++.h>
#define X real()
#define Y imag()
#define F first
#define S second
using namespace std;
typedef long double ld;
typedef long long ll;
// Coordinate type
typedef ll CT;
typedef complex<CT> co;

ll ccw(co a, co b, co c) {
	return ((c-a)*conj(b-a)).Y;
}

int ar(co x) {
	if (x.Y>=0&&x.X<0) return 1;
	if (x.X>=0&&x.Y>0) return 2;
	if (x.Y<=0&&x.X>0) return 3;
	return 4;
}

bool cp(pair<co, pair<int, int> > p1, pair<co, pair<int, int> > p2) {
	if (ar(p1.F)!=ar(p2.F)) {
		return ar(p1.F)<ar(p2.F);
	}
	assert((ccw({0, 0}, p1.F, p2.F)==0)==(ccw({0, 0}, p2.F, p1.F)==0));
	if (ccw({0, 0}, p1.F, p2.F)==0){
		return p1.S>p2.S;
	}
	return ccw({0, 0}, p2.F, p1.F)>0;
}

vector<co> minkowski(vector<co>&a, vector<co>&b) {
	int n=a.size();
	int m=b.size();
	if (n==0) return b;
	if (m==0) return a;
	if (n==1) {
		vector<co> ret(m);
		for (int i=0;i<m;i++) {
			ret[i]=b[i]+a[0];
		}
		return ret;
	}
	if (m==1) {
		vector<co> ret(n);
		for (int i=0;i<n;i++) {
			ret[i]=a[i]+b[0];
		}
		return ret;
	}
	vector<pair<co, pair<int, int> > > pp;
	int f1=0;
	int f2=0;
	for (int i=0;i<n;i++) {
		if (ccw(a[(i-1+n)%n], a[i], a[(i+1)%n])!=0) {
			f1=i;
			break;
		}
	}
	for (int i=0;i<n;i++) {
		pp.push_back({a[(i+1+f1)%n]-a[(i+f1)%n], {1, i}});
	}
	for (int i=0;i<m;i++) {
		if (ccw(b[(i-1+m)%m], b[i], b[(i+1)%m])!=0) {
			f2=i;
			break;
		}
	}
	for (int i=0;i<m;i++) {
		pp.push_back({b[(i+1+f2)%m]-b[(i+f2)%m], {2, i}});
	}
	sort(pp.rbegin(), pp.rend(), cp);
	co s={0, 0};
```

```
	co ad={0, 0};
	for (int i=0;i<(int)pp.size();i++) {
		s+=pp[i].F;
		if (pp[i].S.F!=pp[i+1].S.F) {
			if (pp[i].S.F==1) ad=a[(pp[i].S.S+1+f1)%n]+b[(pp[i+1].S.S+f2)%m];
			else ad=b[(pp[i].S.S+1+f2)%m]+a[(pp[i+1].S.S+f1)%n];
			ad-=s;
			break;
		}
	}
	s=ad;
	vector<co> ret(pp.size());
	for (int i=0;i<(int)pp.size();i++) {
		ret[i]=s;
		s+=pp[i].F;
	}
	return ret;
}
```

9 src/geometry/basic.cpp

```
// TCR
// Basic geometry functions using complex numbers
// Mostly copied from https://github.com/ttalvitie/libcontest/
/* Useful functions of complex number class
	CT abs(co x): Length
	CT norm(co x): Square of length
	CT arg(co x): Angle
	co polar(CT length, CT angle): Complex from polar components
*/
#include <bits/stdc++.h>
#define X real()
#define Y imag()
using namespace std;
typedef long double ld;
typedef long long ll;
// Coordinate type
typedef ld CT;
typedef complex<CT> co;

// Return true iff points a, b, c are CCW oriented.
bool ccw(co a, co b, co c) {
	return ((c-a)*conj(b-a)).Y>0;
}

// Return true iff points a, b, c are collinear.
// Note: doesn't make much sense with non-integer CT.
bool collinear(co a, co b, co c) {
	return ((c-a)*conj(b-a)).Y==0;
}

// Rotate x with agle ang
co rotate(co x, CT ang) {
	return x*polar((CT)1, ang);
}

// Check whether segments [a, b] and [c, d] intersect.
// The segments must not be collinear. Doesn't handle edge cases (endpoint of
// a segment on the other segment) consistently.
bool intersects(co a, co b, co c, co d) {
	return ccw(a, d, b)!=ccw(a, c, b)&&ccw(c, a, d)!=ccw(c, b, d);
}

// Interpolate between points a and b with parameter t.
co interpolate(CT t, co a, co b) {
	return a+t*(b-a);
}

// Return interpolation parameter between a and b of projection of v to the
// line defined by a and b.
// Note: no rounding behavior specified for integers.
CT projectionParam(co v, co a, co b) {
	return ((v-a)/(b-a)).X;
}

// Compute the distance of point v from line a..b.
// Note: Only for non-integers!
CT pointLineDistance(co p, co a, co b) {
	return abs(((p-a)/(b-a)).Y)*abs(b-a);
}

// Compute the distance of point v from segment a..b.
// Note: Only for non-integers!
CT pointSegmentDistance(co p, co a, co b) {
	co z=(p-a)/(b-a);
	if(z.X<0) return abs(p-a);
	if(z.X>1) return abs(p-b);
	return abs(z.Y)*abs(b-a);
}

// Return interpolation parameter between a and b of the point that is also
// on line c..d.
```

```
// Note: Only for non-integers!
CT intersectionParam(co a, co b, co c, co d) {
	co u=(c-a)/(b-a);
	co v=(d-a)/(b-a);
	return (u.X*v.Y-u.Y*v.X)/(v.Y-u.Y);
}
```

10 src/geometry/closestpoints.cpp

```
// TCR
// Returns square of distance between closest 2 points
// O(n log n)
#include <bits/stdc++.h>
#define X real()
#define Y imag()
#define F first
#define S second
using namespace std;
typedef long long ll;
typedef complex<ll> co;

const ll inf=2e18;

ll csqrt(ll x) {
	ll r=sqrt(x);
	while (r*r<x) r++;
	while (r*r>x) r--;
	return r;
}

ll sq(ll x) {
	return x*x;
}

ll closestPoints(vector<co> points) {
	int n=points.size();
	vector<pair<ll, ll> > ps(n);
	for (int i=0;i<n;i++) {
		ps[i]={points[i].X, points[i].Y};
	}
	sort(ps.begin(), ps.end());
	int i2=0;
	ll d=inf;
	set<pair<ll, ll> > pss;
	for (int i=0;i<n;i++) {
		while (i2<i&&sq(ps[i].F-ps[i2].F)>d) {
			pss.erase({ps[i2].S, ps[i2].F});
			i2++;
		}
		auto it=pss.lower_bound({ps[i].S-csqrt(d), -inf});
		for (;it!=pss.end();it++) {
			if (sq(it->F-ps[i].S)>d) break;
			d=min(d, sq(it->F-ps[i].S)+sq(it->S-ps[i].F));
		}
		pss.insert({ps[i].S, ps[i].F});
	}
	return d;
}
```

11 src/datastructure/orderedset.cpp

```
// TCR
// Sample code on how to use g++ ordered set
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
using namespace std;
using namespace __gnu_pbds;
//using namespace pb_ds;

typedef tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update>
ordered_set;

int main() {
	ordered_set X;
	X.insert(1);
	X.insert(4);
	cout<<*X.find_by_order(1)<<endl; // 4
	cout<<X.order_of_key(3)<<endl;   // 1
}
```

12 src/datastructure/treap.cpp

```
// TCR
// Treap implementation with pointers
// Expected running time of split and merge is O(log n)
#include <bits/stdc++.h>
using namespace std;

typedef struct node* pnode;
struct node {
```

```
	pnode l,r;
	int pr,c;
	node() {
		l=0;
		r=0;
		c=1;
		pr=rand();
	}
};

// Returns the size of the subtree t
int cnt(pnode t) {
	if (t) return t->c;
	return 0;
}

// Updates the size of the subtree t
void upd(pnode t) {
	if (t) t->c=cnt(t->l)+cnt(t->r)+1;
}

// Put lazy updates here
void push(pnode t) {
	if (t) {
		// Do lazy update
	}
}

// Merges trees l and r into tree t
void merg(pnode& t, pnode l, pnode r) {
	push(l);
	push(r);
	if (!l) t=r;
	else if(!r) t=l;
	else {
		if (l->pr>r->pr) {
			merg(l->r, l->r, r);
			t=l;
		}
		else {
			merg(r->l, l, r->l);
			t=r;
		}
	}
	upd(t);
}

// Splits tree t into trees l and r
// Size of tree l will be k
void split(pnode t, pnode& l, pnode& r, int k) {
	if (!t) {
		l=0;
		r=0;
		return;
	}
	else {
		push(t);
		if (cnt(t->l)>=k) {
			split(t->l, l, t->l, k);
			r=t;
		}
		else {
			split(t->r, t->r, r, k-cnt(t->l)-1);
			l=t;
		}
	}
	upd(t);
}
```

13 src/math/crt.cpp

```
// TCR
// (Generalised) Chinese remainder theorem (for arbitrary moduli):
// Solves x from system of equations x == a_i (mod m_i), giving answer modulo m =
lcm(m_1,...,m_n)
// Runs in O(log(m)+n) time
// Overflows only if m overflows
// Returns {1, {x, m}} if solution exists, and {-1, {0,0}} otherwise
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef __int128 lll;

ll ee(ll ca, ll cb, ll xa, ll xb, ll&x) {
	if (cb) return ee(cb, ca%cb, xb, xa-(ca/cb)*xb, x);
	x = xa;
	return ca;
}

pair<int, pair<ll, ll>> crt(vector<ll> as, vector<ll> ms) {
	ll aa = 0, mm = 1, d, a, x;
	for (int i = 0; i < (int) as.size(); i++) {
```

```
			d = ee(ms[i], mm, 1, 0, x);
			if ((aa-as[i])%d) return {-1,{0,0}};
			a = ms[i]/d;
			mm *= a;
			aa = (as[i] + (aa-as[i])*(((lll)a*x)%mm))%mm;
		}
		if (aa < 0) aa += mm;
		return {1, {aa, mm}};
}
```

14 src/math/fftmod.cpp

```
// TCR
// Precise FFT modulo mod
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
// luku muotoa (2^20)*k+1
const ll mod=1045430273;
// luku jonka order 2^20
const ll root=363;
// sen kaanteisluku
const ll root_1=296637240;
const ll root_pw=1<<20;

ll pot(ll x, ll p) {
	if (p==0) return 1;
	if (p%2==0) {
		x=pot(x, p/2);
		return (x*x)%mod;
	}
	return (x*pot(x, p-1))%mod;
}

ll inv(ll x) {
	return pot(x, mod-2);
}

vector<ll> fft (vector<ll> a, int d) {
	int n=(int)a.size();
	for (int i=1,j=0;i<n;i++) {
		int bit=n>>1;
		for (;j>=bit;bit>>=1) {
			j-=bit;
		}
		j+=bit;
		if (i<j) swap (a[i], a[j]);
	}
	for (int len=2;len<=n;len<<=1) {
		ll wlen=root;
		if (d==-1) {
			wlen=root_1;
		}
		for (int i=len;i<root_pw;i<<=1) wlen=(wlen*wlen)%mod;
		for (int i=0;i<n;i+=len) {
			ll w = 1;
			for (int j=0;j<len/2;j++) {
				ll u = a[i+j];
				ll v = (a[i+j+len/2]*w)%mod;
				if (u+v<mod) {
					a[i+j]=u+v;
				}
				else {
					a[i+j]=u+v-mod;
				}
				if (u-v>=0) {
					a[i+j+len/2]=u-v;
				}
				else {
					a[i+j+len/2]=u-v+mod;
				}
				w=(w*wlen)%mod;
			}
		}
	}
	if (d==-1) {
		ll nrev=inv(n);
		for (int i=0;i<n;i++) a[i]=(a[i]*nrev)%mod;
	}
	return a;
}

vector<ll> conv(vector<ll> a, vector<ll> b) {
	int as=a.size();
	int bs=b.size();
	vector<ll> aa(as);
	vector<ll> bb(bs);
	for (int i=0;i<as;i++) {
		aa[i]=a[i];
	}
	for (int i=0;i<bs;i++) {
		bb[i]=b[i];
```

```
	}
	int n=1;
	while (n<as+bs-1) n*=2;
	aa.resize(n*2);
	bb.resize(n*2);
	aa=fft(aa, 1);
	bb=fft(bb, 1);
	vector<ll> c(2*n);
	for (int i=0;i<2*n;i++) {
		c[i]=(aa[i]*bb[i])%mod;
	}
	c=fft(c, -1);
	c.resize(as+bs-1);
	return c;
}

int main() {
	// Shoud print 12 11 30 7
	vector<ll> a={3, 2, 7};
	vector<ll> b={4, 1};
	vector<ll> c=conv(a, b);
	for (ll t:c) {
		cout<<t<<endl;
	}
}
```

15 src/math/gaussjordan.cpp

```
// TCR
// Solves system of linear equations in O(n m^2)
// Using doubles or mod 2
// Using doubles might have large precision errors or overflow
// Returns 0 if no solution exists, 1 if there is one solution
// or 2 if infinite number of solutions exists
// If at least one solution exists, it is returned in ans
// You can modify the general algorithm to work mod p by using modular inverse
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef long double ld;
const ld eps=1e-12;

// Using doubles
int gaussD (vector<vector<ld> > a, vector<ld>& ans) {
	int n=(int)a.size();
	int m=(int)a[0].size()-1;
	vector<int> where(m,-1);
	for (int col=0,row=0;col<m&&row<n;col++) {
		int sel=row;
		for (int i=row;i<n;i++) {
			if (abs(a[i][col])>abs(a[sel][col])) sel=i;
		}
		if (abs(a[sel][col])<eps) continue;
		for (int i=col;i<=m;i++) {
			swap (a[sel][i], a[row][i]);
		}
		where[col]=row;

		for (int i=0;i<n;i++) {
			if (i!=row) {
				ld c=a[i][col]/a[row][col];
				for (int j=col;j<=m;j++) {
					a[i][j]-=a[row][j]*c;
				}
			}
		}
		row++;
	}

	ans.assign(m, 0);
	for (int i=0;i<m;i++) {
		if (where[i]!=-1) ans[i]=a[where[i]][m]/a[where[i]][i];
	}
	for (int i=0;i<n;i++) {
		ld sum=0;
		for (int j=0;j<m;j++) {
			sum+=ans[j]*a[i][j];
		}
		if (abs(sum-a[i][m])>eps) return 0;
	}

	for (int i=0;i<m;i++) {
		if (where[i]==-1) return 2;
	}
	return 1;
}

// mod 2
// n is number of rows m is number of variables
const int M=4;
int gaussM(vector<bitset<M> > a, int n, int m, bitset<M-1>& ans) {
```

```
	vector<int> where (m, -1);
	for (int col=0,row=0;col<m&&row<n;col++) {
		for (int i=row;i<n;i++) {
			if (a[i][col]) {
				swap (a[i], a[row]);
				break;
			}
		}
		if (!a[row][col]) continue;
		where[col]=row;

		for (int i=0;i<n;i++) {
			if (i!=row&&a[i][col]) {
				a[i]^=a[row];
			}
		}
		row++;
	}
	ans=0;
	for (int i=0;i<m;i++) {
		if (where[i]!=-1) ans[i]=a[where[i]][m];
	}
	for (int i=0;i<n;i++) {
		int sum=0;
		for (int j=0;j<m;j++) {
			sum^=ans[j]*a[i][j];
		}
		if (sum!=a[i][m]){
			return 0;
		}
	}
	for (int i=0;i<m;i++){
		if (where[i]==-1) return 2;
	}
	return 1;
}

int main() {
	// Should output 2, 1 2 0
	vector<vector<ld> > d(3);
	d[0]={3, 3, -15, 9};
	d[1]={1, 0, -2, 1};
	d[2]={2, -1, -1, 0};
	vector<ld> da;
	cout<<gaussD(d, da)<<endl;
	cout<<da[0]<<" "<<da[1]<<" "<<da[2]<<endl;
	// Should output 1, 110
	// Note that bitsets are printed in reverse order
	bitset<M> r1("0110");
	bitset<M> r2("1101");
	bitset<M> r3("0111");
	vector<bitset<M> > m={r1, r2, r3};
	bitset<M-1> ma;
	cout<<gaussM(m, 3, 3, ma)<<endl;
	cout<<ma<<endl;
}
```

16 src/math/fft.cpp

```
// TCR
// Fast Fourier transform and convolution using it
// O(n log n)
#include <bits/stdc++.h>
using namespace std;
typedef long double ld;
typedef long long ll;
typedef complex<ld> co;
const ld PI=atan2(0, -1);

vector<co> fft(vector<co> x, int d) {
	int n=x.size();
	for (int i=0;i<n;i++) {
		int u=0;
		for (int j=1;j<n;j*=2) {
			u*=2;
			if (i&j) u++;
		}
		if (i<u) {
			swap(x[i], x[u]);
		}
	}
	for (int m=2;m<=n;m*=2) {
		co wm=exp(co{0, d*2*PI/m});
		for (int k=0;k<n;k+=m) {
			co w=1;
			for (int j=0;j<m/2;j++) {
				co t=w*x[k+j+m/2];
				co u=x[k+j];
				x[k+j]=u+t;
				x[k+j+m/2]=u-t;
				w*=wm;
```

```
                        }
                }
        }
        if (d==-1) {
                for (int i=0;i<n;i++) {
                        x[i]/=n;
                }
        }
        return x;
}

vector<ll> conv(vector<ll> a, vector<ll> b) {
        int as=a.size();
        int bs=b.size();
        vector<co> aa(as);
        vector<co> bb(bs);
        for (int i=0;i<as;i++) {
                aa[i]=a[i];
        }
        for (int i=0;i<bs;i++) {
                bb[i]=b[i];
        }
        int n=1;
        while (n<as+bs-1) n*=2;
        aa.resize(n*2);
        bb.resize(n*2);
        aa=fft(aa, 1);
        bb=fft(bb, 1);
        vector<co> c(2*n);
        for (int i=0;i<2*n;i++) {
                c[i]=aa[i]*bb[i];
        }
        c=fft(c, -1);
        c.resize(as+bs-1);
        vector<ll> r(as+bs-1);
        for (int i=0;i<as+bs-1;i++) {
                r[i]=(ll)round(c[i].real());
        }
        return r;
}

int main() {
        // Shoud print 12 11 30 7
        vector<ll> a={3, 2, 7};
        vector<ll> b={4, 1};
        vector<ll> c=conv(a, b);
        for (ll t:c) {
                cout<<t<<endl;
        }
}
```

17 src/math/miller-rabin.cpp

```
// TCR
// Deterministic Miller-Rabin primality test
// Works for all 64 bit integers
// Support of 128 bit integers is required to test over 32 bit integers
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef __int128 lll;

lll powmod(lll a, lll p, lll mod) {
        if (p==0) return 1;
        if (p%2==0) {
                a=powmod(a, p/2, mod);
                return (a*a)%mod;
        }
        return (a*powmod(a, p-1, mod))%mod;
}

bool is_w(ll a, ll even, ll odd, ll p) {
        lll u = powmod(a, odd, p);
        if (u==1) return 0;
        for (ll j=1;j<even;j*=2) {
                if (u==p-1) return 0;
                u*=u;
                u%=p;
        }
        return 1;
}

bool isPrime(ll p) {
        if (p==2) return 1;
        if (p<=1||p%2==0) return 0;
        ll odd=p-1;
        ll even=1;
        while (odd%2==0) {
                even*=2;
                odd/=2;
        }
        ll b[7]={2, 325, 9375, 28178, 450775, 9780504, 1795265022};
```

```
	for (ll i=0;i<7;i++) {
		ll a=b[i]%p;
		if (a==0) return 1;
		if (is_w(a, even, odd, p)) return 0;
	}
	return 1;
}
```

18 src/math/primitiveroot.cpp

```
// TCR
// Computes primitive root
// O(sqrt(n))
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;

ll pot(ll x, ll p, ll mod) {
	if (p==0) return 1;
	if (p%2==0) {
		x=pot(x, p/2, mod);
		return (x*x)%mod;
	}
	return (x*pot(x, p-1, mod))%mod;
}

ll primitiveRoot(ll p) {
	vector<ll> fact;
	ll phi=p-1;
	ll n=phi;
	for (ll i=2;i*i<=n;i++) {
		if (n%i==0) {
			fact.push_back(i);
			while (n%i==0) n/=i;
		}
	}
	if (n>1) fact.push_back (n);
	for (ll res=2;res<=p;res++) {
		bool ok = true;
		for (int i=0;i<(int)fact.size()&&ok;i++) ok&=pot(res, phi/fact[i],
p)!=1;
		if (ok)  return res;
	}
	return -1;
}

int main() {
	cout<<primitiveRoot(1000000007)<<endl;// should print 5
}
```

19 src/math/diophantine.cpp

```
// TCR
// Solves ax+by=c in O(log a+b) time
// Returns {is, {x, y}}, is=0 if there is no solution
// Use __int128 for 64 bit numbers
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
typedef long long ll;

ll ee(ll a, ll b, ll ca, ll cb, ll xa, ll xb, ll&x, ll&y) {
	if (cb==0) {
		x=xa;
		if (b==0) y=0;
		else y=(ca-a*xa)/b;
		return ca;
	}
	else return ee(a, b, cb, ca%cb, xb, xa-(ca/cb)*xb, x, y);
}

pair<int, pair<ll, ll> > solve(ll a, ll b, ll c) {
	if (c==0) return {1, {0, 0}};
	if (a==0&&b==0) return {0, {0, 0}};
	ll x,y;
	ll g=ee(a, b, a, b, 1, 0, x, y);
	if (abs(c)%g>0) return {0, {0, 0}};
	return {1, {x*(c/g), y*(c/g)}};
}
```

20 src/graph/stronglyconnected.cpp

```
// TCR
// Kosaraju's algorithm for strongly connected components O(V+E)
// Components will be returned in topological order
// Uses 1-indexing
#include <bits/stdc++.h>
using namespace std;

struct SCC {
	vector<int> used;
```

```
	vector<vector<int> > g2;

	void dfs1(vector<int>* g, int x, vector<int>& ns) {
		if (used[x]==1) return;
		used[x]=1;
		for (int nx:g[x]) {
			g2[nx].push_back(x);
			dfs1(g, nx, ns);
		}
		ns.push_back(x);
	}

	void dfs2(int x, vector<int>& co) {
		if (used[x]==2) return;
		used[x]=2;
		co.push_back(x);
		for (int nx:g2[x]) {
			dfs2(nx, co);
		}
	}

	// Returns strongly connected components of the graph in vector ret
	// n is the size of the graph, g is the adjacency list
	SCC(vector<int>* g, int n, vector<vector<int> >& ret) : used(n+1),
g2(n+1) {
		vector<int> ns;
		for (int i=1;i<=n;i++) {
			dfs1(g, i, ns);
		}
		for (int i=n-1;i>=0;i--) {
			if (used[ns[i]]!=2) {
				ret.push_back(vector<int>());
				dfs2(ns[i], ret.back());
			}
		}
	}
};
```

21 src/graph/eulertour.cpp

```
// TCR
// NOT TESTED PROPERLY
// Finds Euler tour of graph in O(E) time

// Parameters are the adjacency list, number of nodes,
// return value vector, and d=1 if the graph is directed
// Return array contains E+1 elements, the first and last
// elements are same

// Undefined behavior if Euler tour doesn't exist

// Note that Eulerian path can be reduced to Euler tour
// by adding an edge from the last vertex to the first

// In bidirectional graph edges must be in both direction
// Be careful to not add loops twice in case of bidirectional graph
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

struct EulerTour {
	int dir;
	vector<vector<pair<int, int> > > g;
	vector<int> used;

	void dfs(int x, vector<int>& ret) {
		int t=x;
		vector<int> c;
		while (1) {
			while (used[g[t].back().S]) g[t].pop_back();
			auto nx=g[t].back();
			g[t].pop_back();
			used[nx.S]=1;
			t=nx.F;
			c.push_back(t);
			if (t==x) break;
		}
		for (int a:c) {
			ret.push_back(a);
			while (g[a].size()>0&&used[g[a].back().S]) g[a].pop_back();
			if (g[a].size()>0) dfs(a, ret);
		}
	}

	EulerTour(vector<int>* og, int n, vector<int>& ret, int d=0) : dir(d),
g(n+1) {
		int i2=0;
		for (int i=1;i<=n;i++) {
			for (int nx:og[i]) {
				if (d==1||nx<=i) {
```

```
                                if (d==0&&nx<i) g[nx].push_back({i, i2});
                                g[i].push_back({nx, i2++});
                            }
                        }
                    }
                    used.resize(i2);
                    for (int i=1;i<=n;i++) {
                        if (g[i].size()>0) {
                            ret.push_back(i);
                            dfs(i, ret);
                            break;
                        }
                    }
        }
};
```

22 src/graph/cutvertices.cpp

```
// TCR
// Finds cutvertices and 2-vertex-connected components of graph
// 2-vertex-connected components are stored in bg
// Uses 1-indexing
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

struct Biconnected {
        vector<int> cut, h, d, used;
        vector<map<int, vector<int> > > bg;
        vector<pair<int, int> > es;
        int cc;
        void dfs(vector<int>* g, int x, int p) {
                h[x]=d[x];
                int f=0;
                for (int nx:g[x]) {
                        if (nx!=p) {
                                if (!used[nx]) es.push_back({x, nx});
                                if (d[nx]==0) {
                                        f++;
                                        d[nx]=d[x]+1;
                                        int ts=es.size();
                                        dfs(g, nx, x);
                                        h[x]=min(h[x], h[nx]);
                                        if (h[nx]>=d[x]) {
                                                cut[x]=1;
                                                while ((int)es.size()>=ts) {
                                                        auto e=es.back();
                                                        bg[e.F][cc].push_back(e.S);
                                                        bg[e.S][cc].push_back(e.F);
                                                        used[e.S]=1;
                                                        used[e.F]=1;
                                                        es.pop_back();
                                                }
                                                used[x]=0;
                                                cc++;
                                        }
                                }
                                h[x]=min(h[x], d[nx]);
                        }
                }
                if (p==0) {
                        if (f>1) cut[x]=1;
                        else cut[x]=0;
                }
        }

        Biconnected(vector<int>* g, int n) : cut(n+1), h(n+1), d(n+1), used(n+1),
bg(n+1) {
                cc=1;
                for (int i=1;i<=n;i++) {
                        if (d[i]==0) {
                                d[i]=1;
                                dfs(g, i, 0);
                        }
                }
        }
};
```

23 src/graph/linkcut.cpp

```
// TCR
// Link/cut tree. All operations are amortized O(log n) time
#include <bits/stdc++.h>
using namespace std;
struct Node {
        Node* c[2], *p;
        int id, rev;
        int isr() {
                return !p||(p->c[0]!=this&&p->c[1]!=this);
        }
        int dir() {
```

```
		return p->c[1]==this;
	}
	void setc(Node* s, int d) {
		c[d]=s;
		if (s) s->p=this;
	}
	void push() {
		if (rev) {
			swap(c[0], c[1]);
			if (c[0]) c[0]->rev^=1;
			if (c[1]) c[1]->rev^=1;
			rev=0;
		}
	}
	Node(int i) : id(i) {
		c[0]=0;
		c[1]=0;
		p=0;
		rev=0;
	}
};
struct LinkCut {
	void rot(Node* x) {
		Node* p=x->p;
		int d=x->dir();
		if (!p->isr()) {
			p->p->setc(x, p->dir());
		}
		else {
			x->p=p->p;
		}
		p->setc(x->c[!d], d);
		x->setc(p, !d);
	}
	void pp(Node* x) {
		if (!x->isr()) pp(x->p);
		x->push();
	}
	void splay(Node* x) {
		pp(x);
		while (!x->isr()) {
			if (x->p->isr()) rot(x);
			else if(x->dir()==x->p->dir()) {
				rot(x->p);
				rot(x);
			}
			else {
				rot(x);
				rot(x);
			}
		}
	}
	Node* expose(Node* x) {
		Node* q=0;
		for (;x;x=x->p) {
			splay(x);
			x->c[1]=q;
			q=x;
		}
		return q;
	}
	void evert(Node* x) {
		x=expose(x);
		x->rev^=1;
		x->push();
	}
	void link(Node* x, Node* y) {
		evert(x);
		evert(y);
		splay(y);
		x->setc(y, 1);
	}
	void cut(Node* x, Node* y) {
		evert(x);
		expose(y);
		splay(x);
		x->c[1]=0;
		y->p=0;
	}
	int rootid(Node* x) {
		expose(x);
		splay(x);
		while(x->c[0]) {
			x=x->c[0];
			x->push();
		}
		splay(x);
		return x->id;
	}
};
```

24 src/graph/scalingflow.cpp

```
// TCR
// Scaling flow algorithm for maxflow
// O(E^2 log U), where U is maximum possible flow
// In practice O(E^2)
// Uses 1-indexing

#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
typedef long long ll;

struct MaxFlow {
        // Use vector<map<int, ll> > for sparse graphs
        vector<vector<ll> > f;
        vector<vector<int> > g;
        vector<int> used;
        int cc;

        ll flow(int x, int t, ll fl, ll miv) {
                if (x==t) return fl;
                used[x]=cc;
                for (int nx:g[x]) {
                        if (used[nx]!=cc&&f[x][nx]>=miv) {
                                ll r=flow(nx, t, min(fl, f[x][nx]), miv);
                                if (r>0) {
                                        f[x][nx]-=r;
                                        f[nx][x]+=r;
                                        return r;
                                }
                        }
                }
                return 0;
        }

        // maxv is maximum expected maxflow
        ll getMaxFlow(int source, int sink, ll maxv) {
                cc=1;
                ll r=0;
                ll k=1;
                while (k*2<=maxv) k*=2;
                for (;k>0;k/=2) {
                        while (ll t=flow(source, sink, maxv, k)) {
                                r+=t;
                                cc++;
                        }
                        cc++;
                }
                return r;
        }

        void addEdge(int a, int b, ll c) {
                if (f[a][b]==0&&f[b][a]==0) {
                        g[a].push_back(b);
                        g[b].push_back(a);
                }
                f[a][b]+=c;
        }

        MaxFlow(int n) : f(n+1), g(n+1), used(n+1) {
                for (int i=1;i<=n;i++) {
                        f[i]=vector<ll>(n+1);
                }
        }
};
```

25 src/graph/bridges.cpp

```
// TCR
// Finds bridges and 2-edge connected components of graph
// Component of vertex x is c[x]
// Edge is bridge iff its endpoints are in different components
// Graph in form {adjacent vertex, edge id}
// Uses 1-indexing
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

struct Bridges {
        vector<int> c, h;
        void dfs(vector<pair<int, int> >* g, int x, int pe, int d, vector<int>&
ns) {
                if (h[x]) return;
                h[x]=d;
                ns.push_back(x);
                for (auto nx:g[x]) {
                        if (nx.S!=pe) {
                                dfs(g, nx.F, nx.S, d+1, ns);
                                h[x]=min(h[x], h[nx.F]);
```

```
                }
            }
            if (h[x]==d) {
                while (ns.size()>0) {
                    int t=ns.back();
                    c[t]=x;
                    ns.pop_back();
                    if (t==x) break;
                }
            }
        }
        Bridges(vector<pair<int, int> >* g, int n) : c(n+1), h(n+1) {
            vector<int> ns;
            for (int i=1;i<=n;i++) {
                dfs(g, i, -1, 1, ns);
            }
        }
};
```

26 src/graph/mincostflow.cpp

```
// TCR
// Finds minimum-cost k-flow
// O(V E^2 log U), where U is maximum possible flow
// Finding augmenting path is O(V E), usually faster
// Uses scaling flow and finds augmenting path with SPFA
// Only 1-directional edges allowed
// Doesn't work if graph contains negative cost cycles
// Uses 1-indexing

#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;
typedef long long ll;
typedef long double ld;

const ll inf=1e18;

struct MinCostFlow {
        // Use vector<map<int, ll> > for sparse graphs
        vector<vector<ll> > f, c;
        vector<vector<int> > g;
        vector<ll> d;
        vector<int> from, inq;
        queue<int> spfa;
        void relax(int x, ll di, int p) {
                if (di>=d[x]) return;
                d[x]=di;
                from[x]=p;
                if (!inq[x]) {
                        spfa.push(x);
                        inq[x]=1;
                }
        }

        ll augment(ll x, ll s, ll fl) {
                if (x==s) return fl;
                ll r=augment(from[x], s, min(fl, f[from[x]][x]));
                f[from[x]][x]-=r;
                f[x][from[x]]+=r;
                return r;
        }

        pair<ll, ll> flow(int s, int t, ll miv, ll kf) {
                int n=g.size()-1;
                for (int i=1;i<=n;i++) {
                        d[i]=inf;
                        inq[i]=0;
                }
                relax(s, 0, 0);
                while (!spfa.empty()) {
                        int x=spfa.front();
                        spfa.pop();
                        inq[x]=0;
                        for (int nx:g[x]) {
                                if (f[x][nx]>=miv) relax(nx, d[x]+c[x][nx], x);
                        }
                }
                if (d[t]<inf) {
                        ll fl=augment(t, s, kf);
                        return {fl, fl*d[t]};
                }
                return {0, 0};
        }

        // maxv is maximum possible flow on a single augmenting path
        // kf is inteded flow, set infinite for maxflow
        // returns {flow, cost}
        pair<ll, ll> getKFlow(int source, int sink, ll maxv, ll kf) {
                ll r=0;
```

```
		ll k=1;
		ll co=0;
		while (k*2<=maxv) k*=2;
		for (;k>0&&kf>0;k/=2) {
			while (1) {
				pair<ll, ll> t=flow(source, sink, k, kf);
				r+=t.F;
				kf-=t.F;
				co+=t.S;
				if (kf==0||t.F==0) break;
			}
		}
		return {r, co};
	}

	void addEdge(int a, int b, ll capa, ll cost) {
		if (f[a][b]==0&&f[b][a]==0) {
			g[a].push_back(b);
			g[b].push_back(a);
		}
		f[a][b]=capa;
		c[a][b]=cost;
		c[b][a]=-cost;
	}

	MinCostFlow(int n) : f(n+1), c(n+1), g(n+1), d(n+1), from(n+1), inq(n+1) {
		for (int i=1;i<=n;i++) {
			f[i]=vector<ll>(n+1);
			c[i]=vector<ll>(n+1);
		}
	}
};
```

27 src/graph/dynamicconnectivity.cpp

```
// TCR
// O(n log n) offline solution for dynamic connectivity problem.
// Query types:
// {1, {a, b}} add edge. If edge already exists nothing happns.
// {2, {a, b}} remove edge. If no edge exists nothing happens.
// {3, {0, 0}} count number of connected components.
// Uses 1-indexing
#include <bits/stdc++.h>
#define F first
#define S second
using namespace std;

struct DynamicConnectivity {
	struct Edge {
		int a, b, l, r;
	};
	vector<int> ret, tq, id, is;
	vector<vector<int> > g;
	int dfs(int x, int c) {
		id[x]=c;
		int r=is[x];
		for (int nx:g[x])
			if (!id[nx]) r|=dfs(nx, c);
		return r;
	}
	void go(int l, int r, int n, int out, vector<Edge>& es) {
		vector<Edge> nes;
		for (int i=1;i<=n;i++) {
			g[i].clear();
			id[i]=0;
			is[i]=0;
		}
		for (auto e:es) {
			if (e.l>r||e.r<l||e.a==e.b) continue;
			if (e.l<=l&&r<=e.r) {
				g[e.a].push_back(e.b);
				g[e.b].push_back(e.a);
			}
			else {
				nes.push_back(e);
				is[e.a]=1;
				is[e.b]=1;
			}
		}
		int i2=1;
		for (int i=1;i<=n;i++) {
			if ((int)g[i].size()>0||is[i]) {
				if (!id[i]) {
					int a=dfs(i, i2);
					if (!a) out++;
					else i2++;
				}
			}
			else {
				out++;
			}
		}
```

```
		for (auto&e:nes) {
			e.a=id[e.a];
			e.b=id[e.b];
		}
		if (l==r) {
			if (tq[l]) ret[tq[l]-1]=out+i2-1;
		}
		else {
			int m=(l+r)/2;
			go(l, m, i2-1, out, nes);
			go(m+1, r, i2-1, out, nes);
		}
	}
	vector<int> solve(int n, vector<pair<int, pair<int, int> > > queries) {
		map<pair<int, int>, int> ae;
		tq.resize(queries.size());
		id.resize(n+1);
		is.resize(n+1);
		g.resize(n+1);
		int qs=0;
		vector<Edge> es;
		for (int i=0;i<(int)queries.size();i++) {
			auto q=queries[i];
			if (q.S.F>q.S.S) swap(q.S.F, q.S.S);
			if (q.F==1) {
				if (ae[q.S]==0) ae[q.S]=i+1;
			}
			else if(q.F==2) {
				if (ae[q.S]) {
					es.push_back({q.S.F, q.S.S, ae[q.S]-1, i});
					ae[q.S]=0;
				}
			}
			else if (q.F==3) {
				tq[i]=1+qs++;
			}
		}
		for (auto e:ae) {
			if (e.S) es.push_back({e.F.F, e.F.S, e.S-1,
(int)queries.size()});
		}
		ret.resize(qs);
		if ((int)queries.size()>0) go(0, (int)queries.size()-1, n, 0, es);
		return ret;
	}
};
```