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Reliability of a wind turbine blade against ultimate failure in the root section.

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Introduction

The aim of this work is to compute the reliability of a wind turbine blade against ultimate failure in the root section. Both the Monte Carlo and the First Order Reliability Method will be used for this goal. The load variable (S) will be first modeled by an "unknown" function (**MysteriousLoadFunc**) which has in-built uncertainty, and then with a surrogate polynomial model. Finally, the uncertainty in wind speed estimation will be considered, and one possible way to take this into account in the reliability model will be presented.

1 Limit state equation.

Consider $g(X)$ the limit state function which identifies the state of the wind turbine blade, and X a vector of random variables with distribution $F_X(X)$. Further, $g(X) < 0$ represent the failure state. The probability of failure is thus expressed as the total probability that $g(x) < 0$. The limit state function used for the following sections is:

$$g(X) = X_{str} X_{geom} R(X) - S(X) \tag{1}$$

where X_{str} is an uncertainty variable accounting for uncertainties in the stress computations; X_{geom} is an uncertainty variable accounting for uncertainties in the blade geometry; $R(X)$ the structural resistance of the blade, which itself is a function of the material strength; $S(X)$ the extreme blade root flapwise bending moment, which is given as function of wind speed, turbulence, and wind shear. For this, an "unknown" function (**Mysterious Load-Func**) was used, which has **in-built uncertainty**. It can be considered that all variables are collected in the random variable vector

$$X = \mu, \sigma, \alpha, X_R, X_{str}, X_{geo} \tag{2}$$

where the joint distribution of X is the following:

- Mean wind speed: Weibull-distributed with $A = 11.28$ and $k = 2$.
- Turbulence: Lognormally distributed with mean $\mu = (0.75u + 3.8)$ and standard deviation $\sigma = 0.1657$.
- Wind shear exponent: Normally distributed with mean $\mu = 0.1$ and standard deviation $\sigma = min[1, 1/u]$.
- Material strength: Lognormally distributed with mean $\mu = 90e3$ and standard deviation $\sigma = 4.5e3$.
- Stress calculation method uncertainty: Lognormally distributed with mean $\mu = 1$ and standard deviation $\sigma = 0.03$.
- Blade geometry uncertainty: Lognormally distributed with mean $\mu = 1$ and standard deviation $\sigma = 0.03$.

Hence, the Rosenblatt approach can be used to sampling from the dependent random variables. In Figure 1 the histograms of the resistance R (where $R = X_{str} * X_{geo} * X_R$) and the loads S for 100.000 samples is shown to illustrate the failure zone corresponding to the intersection of the distributions.

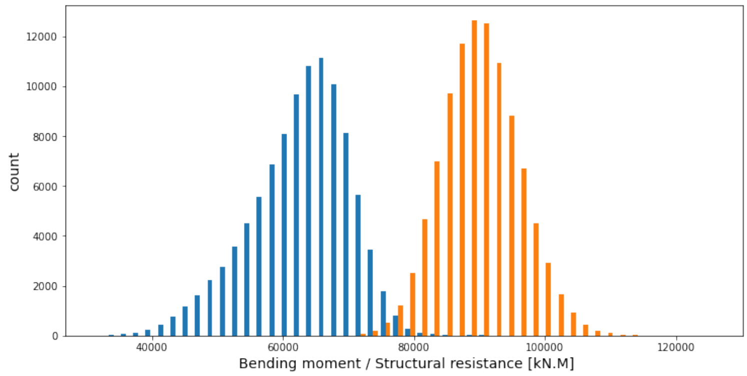

Figure 1: Histograms for R (orange) and S (blue) for 100.000 realizations, where R is a random variable ($R = X_{str} * X_{geo} * X_R$) for the resistance and S for the loads. The intersection represent the failure realizations.

2 Estimation of blade reliability via Monte Carlo.

The probability of failure given a general nonlinear safety margin $g(x)$ is found by solving the integral:

$$\int I_{g(x)\leq 0} f_x(X), dx \tag{3}$$

where I is the indication funcion, $I = 1$ for $g(X) \leq 0$ and $I = 0$ for $g(X) > 0$. For a large number of N it can be approximated as:

$$\frac{1}{N}\sum_{n=1}^{N} I(g(x) \leq 0) \tag{4}$$

Hence, taking a large number of N (100.000 in this case), N Monte Carlo realizations can be done to compute the number of fails (sum of $g(X) < 0$), thus the probability of failure ($p_f = n_{fails}/n_{total}$) can be estimated. Next, the reliability index correspond to the inverse cdf (ppf, percentile point function) evaluated at $1 - p_f$:

$$\beta = \Phi^{-1}(1 - p_f). \tag{5}$$

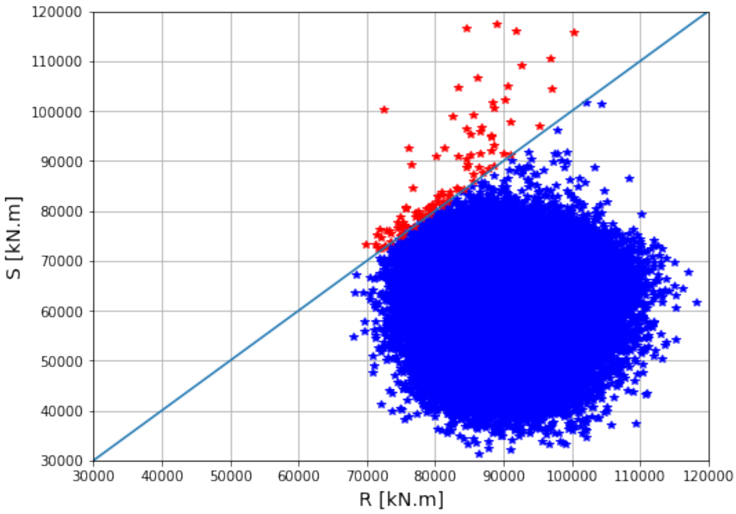

Figure 2: Estimation of the probability of failure and reliabilty index via Monte Carlo (red for failure points).

Figure 2 shows the output of the Monte Carlos approach. In Table 1 the results of the Monte Carlo realizations is displayed. As can be seen, just for 98 out of the 100.000 observations $g(x) \leq 0$, hence the need of large N to have a representative number of failures.

	Monte Carlo ($N = 100.000$) results.
Number of failure obs.	98
Prob. of failure (p_f)	9.8e-4
Reliability index (β)	3.0962

Table 1: Table containing the probability of failure observations and the reliability index estimated via Monte Carlo (**using MysteriousLoadFunc**).

3 Monte Carlo using a polynomial model to replace the MysteriousLoadFunc.

In this section the MysteriousLoadFunc is replaced by a polynomial model. The selected model has the form of $y = ax_1^n + bx_1^{n-1} + \ldots + zx_m^n$. The matrix form representation is $\mathbf{y} = \mathbf{X}\boldsymbol{\beta}$, where $\mathbf{X}$ is a design matrix containing combinations of input variable raised to various power, $\mathbf{y}$ is a vector with outputs and $\boldsymbol{\beta}$ a vector with the fitted coefficients.
Repeating the procedure of the previous section, the results in Table 2 were obtained. As can be seen, the probability of failure is now lower (almost half) and the reliability index increased roughly 7.7 %, because unlike the MysteriousLoadFunc, the selected model **does not contain any uncertainty in the predictions.**

	Monte Carlo ($N = 100.000$) results.
Number of failure obs.	40
Prob. of failure (p_f)	4.0e-4
Reliability index (β)	3.353

Table 2: Table containing the probability of failure observations and the reliability index estimated via Monte Carlo (**using a polynomial model**).

3.1 Modify the model to obtain more realistic results.

In order to obtain similar (and more realistic) results as in Table 1 when using the **MysteriousLoadFunc** uncertainty must be added to the polynomial model. A reasonable choice would be to add normal distributed uncertainty scale by the standard deviation of the residuals from the model, which is displayed in Figure 3 ($\approx 4.0e3$ kN.m). The results obtained with this uncertainty can be seen in Table 3; notice that they are very similar to those with the **MysteriousLoadFunc**.

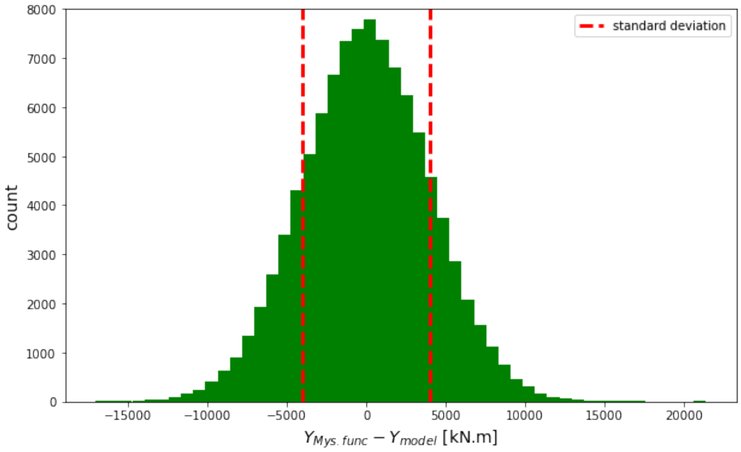

Figure 3: Residuals from the model; red dashed lines showing the standard deviation.

	Monte Carlo ($N = 100.000$) results.
Number of failure obs.	90
Prob. of failure (p_f)	9.0e-4
Reliability index (β)	3.121

Table 3: Table containing the probability of failure observations and the reliability index estimated via Monte Carlo (**using a polynomial model**).

4 First Order Reliability Method.

In this section the *First Order Reliability Method* is used to approach the previous problem. **As this method intends to find β iteratively using finite-difference approximation of the gradient of the state function, adding uncertainty to the model function (or using the MysteriousLoadFunc) will not lead to convergence**, unless the uncertainty is very low, where it may just take more iterations to converge.
In order to approach the FORM method the following iterative algorithm was used:

- Define a function which for an initial value of $u_i = u_0$ (in the normalized u-space) compute the inverse cdf, perform the Rosenblatt transformation to get values of X in the physical x-space and return $g(X)$, the state function of X.
- Next, estimate the gradient of the above defined function in the coordinate vector u_i using finite-difference approximation to get $(\nabla g(u_i))$ (*scipy.optimize.approx_fprime* in Python).
- Estimate an update value of u_{i+1} using:

$$u_{i+1} = \frac{\nabla g(u_i)^T u_i - g(u_i)}{\nabla g(u_i)^T \nabla g(u_i)} \nabla g(u_i) \tag{6}$$

- Compute an update of reliability index $\beta = \sqrt{u_{i+1}^T u_{i+1}}$. Continue iteration unless the abs. difference between $\beta_i + 1$ and β_i is less than certain tolerance.

The results for the *First Order Reliability Method* are displayed in Table 4

	First Order Reliability Method results.
Prob. of failure (p_f)	3.4e-4
Reliability index (β)	3.394

Table 4: Table containing the results using FORM and the polynomial model without any uncertainty.

5 Taking into account uncertainty in the estimation of wind speed parameters.

In this section the uncertainty in wind speed estimation, more specific in the A and k parameters from the Weibull distribution, are taking into account. In order to take that into account in the reliability model, now the value of A and k will respectively have a normal distribution with mean $A = 11.28$ and $k = 2$. For the standard deviation, both 5% and 10% of the mean values are considered. Finally, 50 realizations were made using the same procedure as in section 2 (Monte Carlo method for reliability). The results are displayed in Figure 4 and Figure 5.

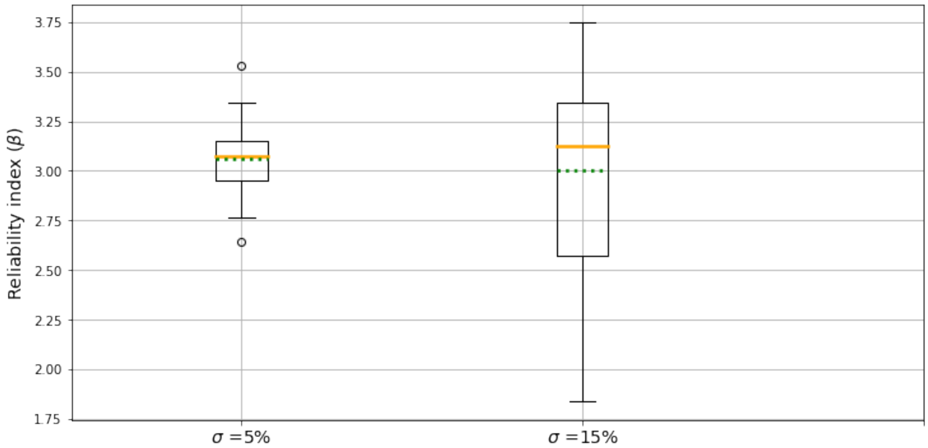

Figure 4: Effect of A and k uncertainty in the reliability index, for standard deviation equal to 5% and 15% of the expected values of k and A. Orange line represent the mean values, while green dashed the median.

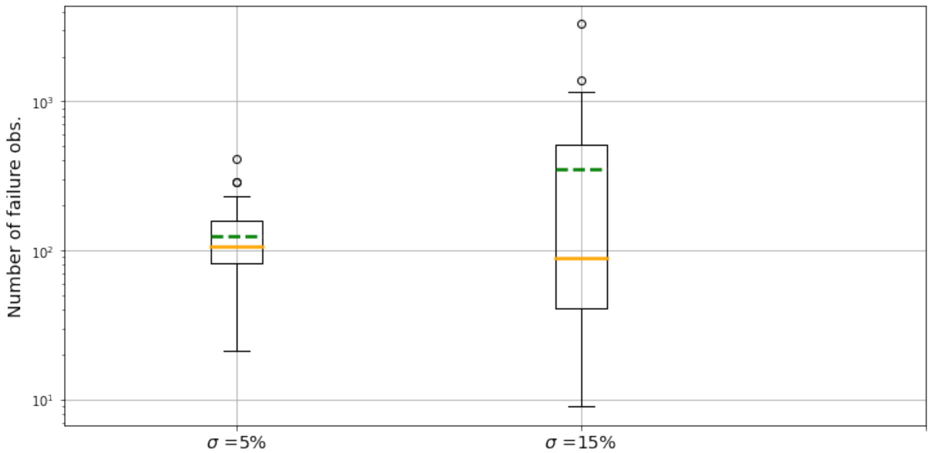

Figure 5: Number of failures observations with uncertainty of A and k in semi-log scale for better visualization. Again, orange line represent the mean values, while green dashed represents the median.

Conclusions

- In Table 5 a summary table containing the probability of failure and reliability index for the four cases is displayed. The probability of failure are plotted (Figure 6), and it can be easily observed that magnitude order is the same in all cases, but in $1-3$ (with in built uncertainty) the estimated probability of failure is more than double that in

2 − 4, namely the uncertainty considered increases the probability of failure roughly twice.

- Regarding the reliability index, the biggest difference corresponds to roughly 8.8% between the Monte Carlo method considering Mys.Func and FORM using the PM. without uncertainty.

- The effect of uncertainty in wind speed estimation can be taking into acount to the reliability model by adding uncertainty in the parameters A and k, i.e assuming a normal distribution with some arbitrary standard deviation percentage. Then, running the selected model several times will result in a graph like Figure 4. It can be observed that the main effect is a wider range of β values for higher uncertainty. Furthermore, while the mean value is just slightly affected (5.1% difference between $\mu_{\beta,\sigma=5\%} = 3.103$ and $\mu_{\beta,\sigma=15\%} = 2.943$) there is a more significant difference in the lower quantile Q1 ($\approx 9\%$) and even more in the minimum values ($\approx 38\%$) of the box plot. Whether it is too conservative to take these last values will depend on the context, but is important to be aware of it, as some uncertainty in wind speed estimation is the most frequent scenario.

	1)M.C.(MysFunc.)	2)M.C.(PM)	3)M.C.(PM w/uncer.)	4)FORM (PM)
Prob. of failure (p_f)	9.8e-4	4.0e-4	9.0e-4	3.4e-4
Reliability index (β)	3.0962	3.353	3.121	3.394

Table 5: Summary table containing the different results of prob. of failure and reliability index for Monte Carlo and FORM method; PM is an abbreviation for the Polynomial Model.

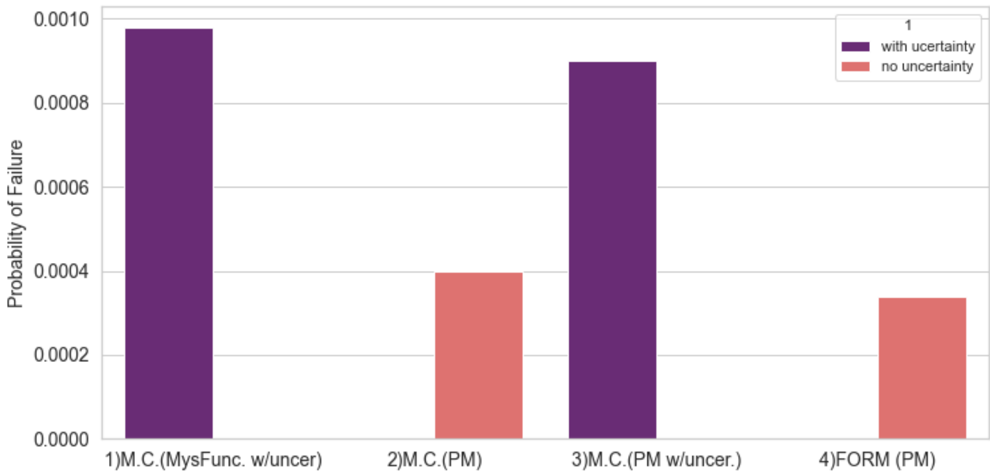

Figure 6: Probability of failure of the four cases. The hue corresponds to uncertainty presence/absence.