# Quantum Volume

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## 1 Introduction

Given the different hardware implementations and technologies in Quantum Computation (superconducting, ion-trap, spin qubits, ...), it is often difficult to benchmark the usefulness or power of quantum systems. A **hardware-independent measure** is required to depict whether a device is able to run a quantum circuit or not. Here is where the Quantum Volume metric appears on the scene.

The aim of Quantum Volume is to quantify the computational power of quantum devices. Consequently we will use it as a metric to measure the runnability of the quantum algorithms and the quantum devices – ***"can this algorithm be run in a given device?"***. While the device is the basis of the Quantum Volume metric, we fix our attention on the circuit. Our purpose is to assert how the mapping procedure affects the runnability of a given circuit and to study how the Quantum Volume is related to the probability of success.

## 2 Quantum Volume definition

In this section,

### 2.1 Literature review

Few studies have been published on the Quantum Volume topic [1, 2].

#### 2.1.1 Hardware parameters

The Quantum Volume metric considers that a quantum computer's performance mainly depends on the next hardware specifications:

- $N$. The number of physical qubits
- Quantum chip topology. The connectivity between qubits
- Maximum number of sequential gates with correctable errors. The number of gates that can be applied before errors or decoherence mask the result
- Gate set. Available hardware gate set

- Maximum number of parallel operations. Number of operations that can be run in parallel

### 2.1.2 Definitions and metrics

In this section we extract some required definitions [1, 2] to understand Quantum Volume.

**Model algorithm.** In the literature, Bishop uses the term *model algorithm* [1] to refer to a depth-one circuit, "constructed by random 2-qubit unitary matrix chosen uniformly over SU (4) on a random pairing of the qubits". Or, what is the same, the *model algorithm* defines any single- or two-qubit gates combination as circuit unit. The mapping requirements of the device and the mapping quality is included as well. In the case that a two-qubit gate force any qubit to be routed, the gate additions will be included in the *model algorithm*.

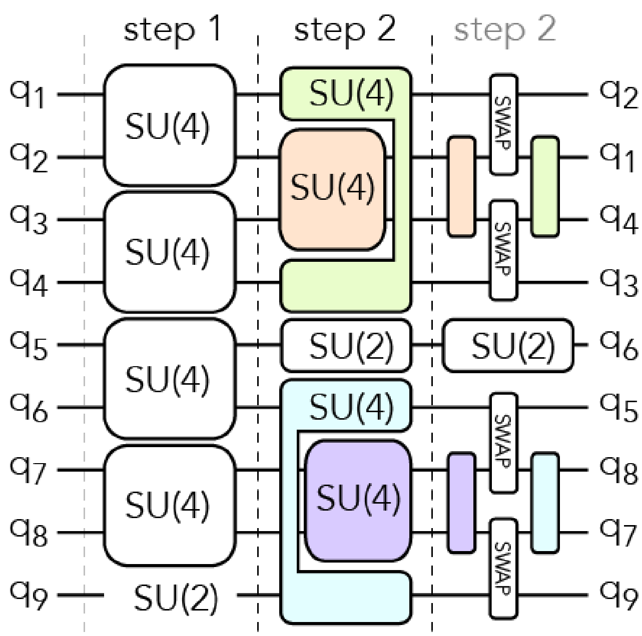


Figure 1: Model algorithm example from [2]. Each step represents a possible combination of gates considered as *model algorithm*. Notice that step 2 requires a mapping process that is shown afterwards.

**n** Number of active qubits in a device of $N$ qubits.

**Effective error rate $\epsilon_{eff} \approx 1/(dn)$.** It is the error rate per *model algorithm*, an averaged error over many realizations of depth-one circuits with random combinations of two-qubit gates. $\epsilon_{eff}$ defines how well a device can implement arbitrary pairwise interactions between qubits. As soon as it is the error of the *model algorithm*, it encapsulates errors of both single- and two-qubit gates. And it depends not only on the gate error rates and connectivity, but also on the sophistication of the scheduling algorithm responsible for mapping the model algorithm to the hardware.

**Achievable circuit depth** $d(N) \simeq \frac{1}{N\epsilon_{eff}}$ Maximum circuit depth for which the results are correctable and useful in some device.

**Quantum Volume** $\tilde{V}_Q = min(N, d(N))^2$ quantifies the space-time volume occupied by a model circuit with random two-qubit gates that can be reliably executed on a given device.

## 2.2 Insights

### 2.2.1 Runnability

Following the $$

We define runnability as

$$V_Q > V_Q^a$$

One may imagine the process of checking, whether or not, some cube with a given volume – representing the algorithm – would fit in a box – the device –.

1. Quantum Volume of a device

   Maximum Quantum Volume that a device could run

$$V_Q = \max_{n \leq N} \min \left[ n, \frac{1}{n \epsilon_{eff}(n)} \right]^2$$

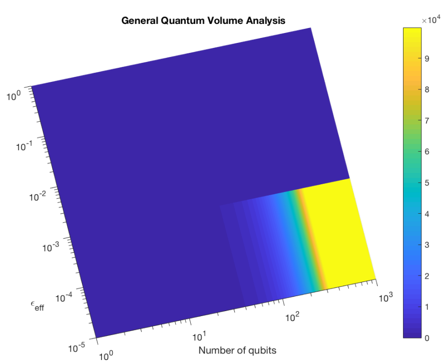


Figure 2:

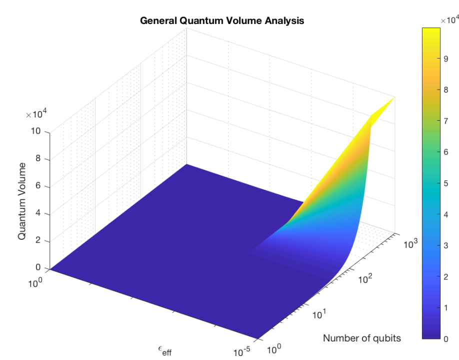


Figure 3:

2. Quantum Volume of an algorithm

$$V_Q^a = \min \left[ n, d \right]^2$$

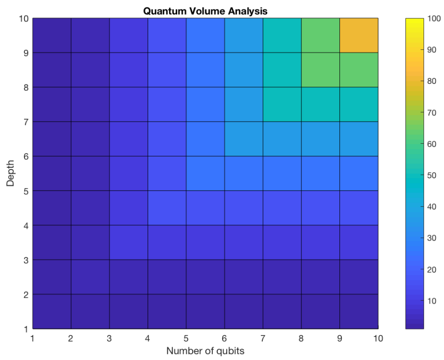


Figure 4:

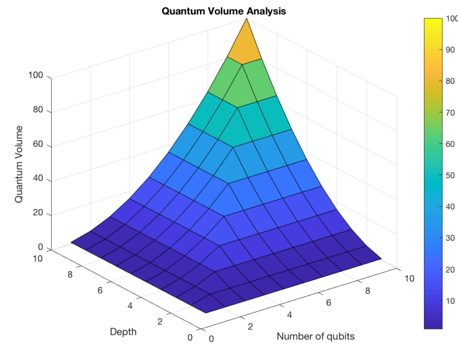


Figure 5:

#### 2.2.2 Depict $\epsilon_{eff}(n)$

*How to depict a function of $\epsilon_{eff}$ based on experiments/simulations?*

1. Bounds

   With no intelligent compiler/mapping:

$$\epsilon_{eff} > \epsilon$$

2. Averaging $\epsilon_{eff}$

   With several random circuits of just 1 cycle, check their fidelity and average. That would be the $\bar{\epsilon}_{eff}$.

3. Finding the real $\epsilon_{eff}(n)$

   *Is not this thing kind of the error model?*

#### 2.2.3 Near future

~~Quantum Volume assumes that a square circuit ($d = \frac{1}{N\epsilon_{eff}} = N$) is the maximum a quantum device could get in term of errors.~~ *Maybe is not that but the initial maximum depth calculation formula that leads you to this result* Following that reasoning, with current devices of $\epsilon_{eff} > 10^{-3}$, the maximum $N$ will be

$$N = \sqrt{\frac{1}{\epsilon_{eff}}} = 31.623$$

# 3 Methodology

# 4 TODO Probability of success relation with Quantum Volume

*How Quantum Volume is related with Probability of success?*

*How to calculate* $\epsilon_{eff}$ *with the methods of Probability of success?*

# 5 BIB [delete this HEADER]

## References


[1] Andrew Cross Jay M. Gambetta Lev S. Bishop, Sergey Bravyi and John Smolin. Quantum volume. 2017.

[2] Nikolaj Moll, Panagiotis Barkoutsos, Lev S Bishop, Jerry M Chow, Andrew Cross, Daniel J Egger, Stefan Filipp, Andreas Fuhrer, Jay M Gambetta, Marc Ganzhorn, and et al. Quantum optimization using variational algorithms on near-term quantum devices. *Quantum Science and Technology*, 3(3):030503, Jun 2018.