

Quantum Fault Tolerance

One-Qubit Cliffords by Measurement

Ben Criger

November 18, 2016

1 Introduction

Some fault tolerant codes don't have transversal Clifford gates, which necessitates code deformation. Sometimes (like in Bravyi's 2016 QIP tutorial) these codes have weird symmetric boundary conditions that I don't understand. I'd like to compare this to a more intuitive non-unitary implementation of Cliffords, regular old lattice surgery. To accomplish this, I need to check which gates can be obtained by a protocol where:

1. An ancilla is prepared in a Pauli +1 eigenstate
2. a two-bit joint measurement is performed, and
3. the original ancilla qubit is measured out fault-tolerantly.

To be non-trivial, the jointly-measured Pauli has to anticommute with the prepared Pauli and the measured Pauli, there are 36 such protocols. However, consider the case in which $A = D$. In this case, the tableau evolves as follows:

$$\begin{array}{c} S_{\text{init}} \\ \hline \begin{array}{cc} X & Z \end{array} \end{array} = \begin{array}{c} IA \\ \hline \begin{array}{cc} XI & ZI \end{array} \end{array} \xrightarrow{\mathcal{M}_{BC}} \begin{array}{c} BC \\ \hline \begin{array}{cc} XI(XA) & ZI(ZA) \end{array} \end{array} \xrightarrow{\mathcal{M}_{IA}} \begin{array}{c} IA \\ \hline \begin{array}{cc} XI & ZI \end{array} \end{array},$$

where either of the Paulis in brackets may be present without affecting the result. Either way, the resulting unitary is the identity, we eliminate these cases, leaving 18 remaining. I test all of them in the remainder of this document, assuming all measurements produce +1 outcomes throughout:

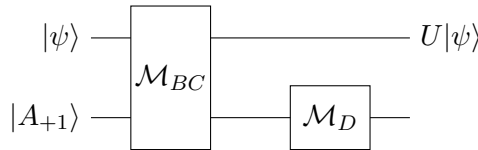
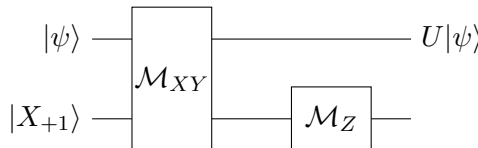
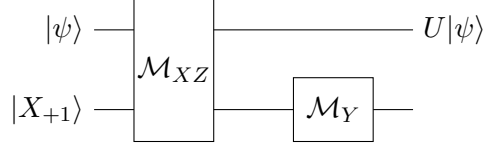


Figure 1: The basic protocol I want to analyse, with a +1 eigenstate prepared on the ancilla, a joint measurement of a two-qubit Pauli BC , and a final measurement of Pauli D .

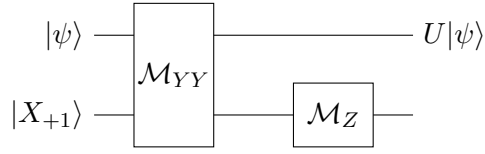
Below, I present the 18 protocols, with a little equation explaining what happens to the stabiliser and logicals throughout:



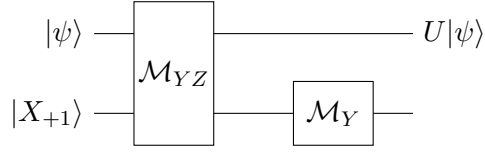
$$\frac{S_{\text{init}}}{\begin{smallmatrix} X \\ \bar{Z} \end{smallmatrix}} = \frac{IX}{XI} \xrightarrow{\mathcal{M}_{XY}} \frac{XY}{XI} \xrightarrow{\mathcal{M}_{IZ}} \frac{IZ}{XI} \therefore U \sim HPH \quad (1)$$



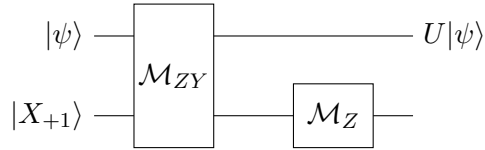
$$\frac{S_{\text{init}}}{\begin{smallmatrix} X \\ \bar{Z} \end{smallmatrix}} = \frac{IX}{XI} \xrightarrow{\mathcal{M}_{XZ}} \frac{XZ}{XI} \xrightarrow{\mathcal{M}_{IX}} \frac{IY}{XI} \therefore U \sim HPH \quad (2)$$



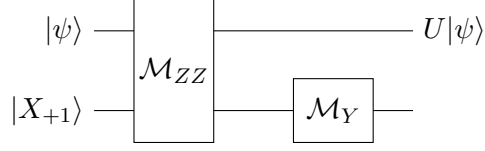
$$\frac{S_{\text{init}}}{\begin{smallmatrix} X \\ \bar{Z} \end{smallmatrix}} = \frac{IX}{XI} \xrightarrow{\mathcal{M}_{YY}} \frac{YY}{XX} \xrightarrow{\mathcal{M}_{IZ}} \frac{IZ}{ZI} \therefore U \sim H \quad (3)$$



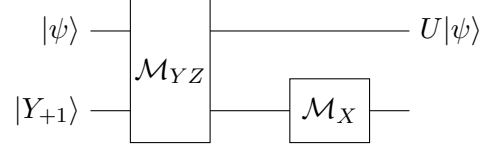
$$\frac{S_{\text{init}}}{\begin{smallmatrix} X \\ \bar{Z} \end{smallmatrix}} = \frac{IX}{XI} \xrightarrow{\mathcal{M}_{YZ}} \frac{YZ}{XX} \xrightarrow{\mathcal{M}_{IX}} \frac{IY}{ZI} \therefore U \sim H \quad (4)$$



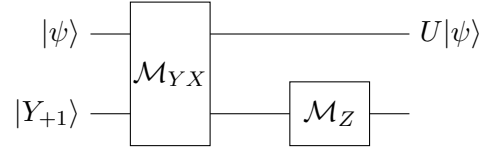
$$\frac{S_{\text{init}}}{\begin{smallmatrix} X \\ \bar{Z} \end{smallmatrix}} = \frac{IX}{XI} \xrightarrow{\mathcal{M}_{ZY}} \frac{ZY}{XX} \xrightarrow{\mathcal{M}_{IZ}} \frac{IZ}{YI} \therefore U \sim P \quad (5)$$



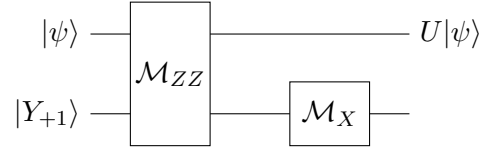
$$\frac{S_{\text{init}}}{\frac{X}{\bar{Z}}} = \frac{IX}{XI} \xrightarrow{\mathcal{M}_{ZZ}} \frac{ZZ}{XI} \xrightarrow{\mathcal{M}_{IY}} \frac{IY}{ZI} \therefore U \sim P \quad (6)$$



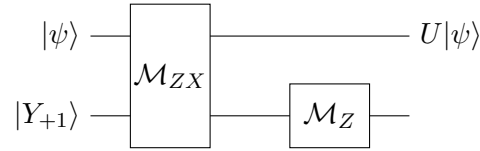
$$\frac{S_{\text{init}}}{\frac{X}{\bar{Z}}} = \frac{IY}{XI} \xrightarrow{\mathcal{M}_{YZ}} \frac{YZ}{XY} \xrightarrow{\mathcal{M}_{IX}} \frac{IX}{ZI} \therefore U \sim H \quad (7)$$



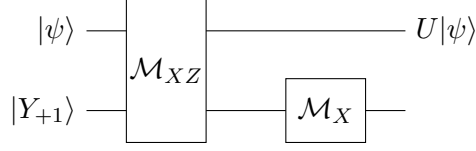
$$\frac{S_{\text{init}}}{\frac{X}{\bar{Z}}} = \frac{IY}{XI} \xrightarrow{\mathcal{M}_{YX}} \frac{YX}{XY} \xrightarrow{\mathcal{M}_{IZ}} \frac{IZ}{ZI} \therefore U \sim H \quad (8)$$



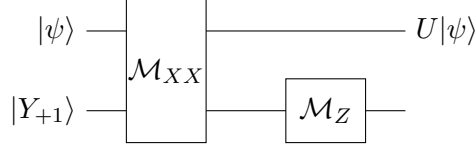
$$\frac{S_{\text{init}}}{\frac{X}{\bar{Z}}} = \frac{IY}{XI} \xrightarrow{\mathcal{M}_{ZZ}} \frac{ZZ}{XI} \xrightarrow{\mathcal{M}_{IX}} \frac{IX}{ZI} \therefore U \sim P \quad (9)$$



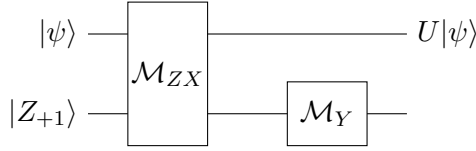
$$\frac{S_{\text{init}}}{\frac{X}{\bar{Z}}} = \frac{IY}{XI} \xrightarrow{\mathcal{M}_{ZX}} \frac{ZX}{XY} \xrightarrow{\mathcal{M}_{IZ}} \frac{IZ}{ZI} \therefore U \sim P \quad (10)$$



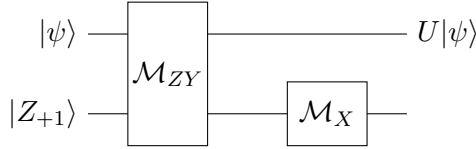
$$\frac{S_{\text{init}}}{\frac{X}{Z}} = \frac{IY}{XI} \xrightarrow{M_{XZ}} \frac{XZ}{XI} \xrightarrow{M_{IX}} \frac{IX}{XI} \therefore U \sim HPH \quad (11)$$



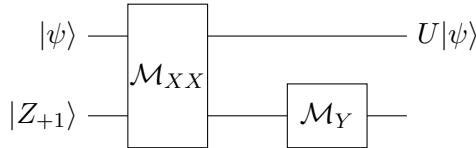
$$\frac{S_{\text{init}}}{\frac{X}{Z}} = \frac{IY}{XI} \xrightarrow{M_{XX}} \frac{XX}{XI} \xrightarrow{M_{IZ}} \frac{IZ}{XI} \therefore U \sim HPH \quad (12)$$



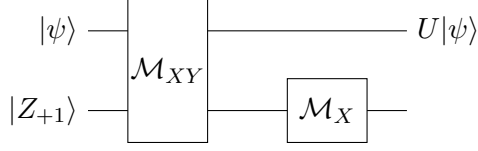
$$\frac{S_{\text{init}}}{\frac{X}{Z}} = \frac{IZ}{XI} \xrightarrow{M_{ZX}} \frac{ZX}{XZ} \xrightarrow{M_{IX}} \frac{IY}{YI} \therefore U \sim P \quad (13)$$



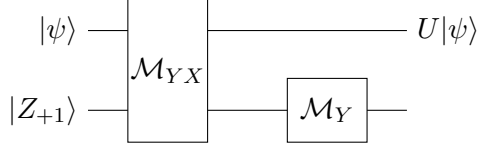
$$\frac{S_{\text{init}}}{\frac{X}{Z}} = \frac{IZ}{XI} \xrightarrow{M_{ZY}} \frac{ZY}{XZ} \xrightarrow{M_{IX}} \frac{IX}{YI} \therefore U \sim P \quad (14)$$



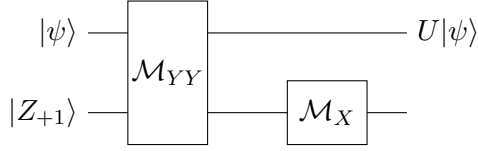
$$\frac{S_{\text{init}}}{\frac{X}{Z}} = \frac{IZ}{XI} \xrightarrow{M_{XX}} \frac{XX}{XI} \xrightarrow{M_{IX}} \frac{IY}{YI} \therefore U \sim HPH \quad (15)$$



$$\frac{S_{\text{init}}}{\frac{X}{Z}} = \frac{IZ}{XI \atop ZI} \xrightarrow{\mathcal{M}_{XY}} \frac{XY}{XI \atop ZZ} \xrightarrow{\mathcal{M}_{IX}} \frac{IX}{XI \atop YI} \therefore U \sim HPH \quad (16)$$



$$\frac{S_{\text{init}}}{\frac{X}{Z}} = \frac{IZ}{XI \atop ZI} \xrightarrow{\mathcal{M}_{YX}} \frac{YX}{XZ \atop ZZ} \xrightarrow{\mathcal{M}_{IY}} \frac{IY}{ZI \atop XI} \therefore U \sim H \quad (17)$$



$$\frac{S_{\text{init}}}{\frac{X}{Z}} = \frac{IZ}{XI \atop ZI} \xrightarrow{\mathcal{M}_{YY}} \frac{YY}{XZ \atop ZZ} \xrightarrow{\mathcal{M}_{IX}} \frac{IX}{ZI \atop XI} \therefore U \sim H \quad (18)$$

2 Two-Qubit Cliffords