

Effect of Decoherence During Gates

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July 5, 2016

1 Introduction

The purpose of this note is to determine the effect of biased (read: mostly dephasing) noise during the operation of certain gates. I'm going to start with the Y_{90} gate, implemented by a nice square pulse. There are two limits that can be handled succinctly, the perfectly-Markovian limit (where we can't decrease the error rate without a quantum error-correcting code), and the 'unknown constant Hamiltonian' limit, where we can apply pulse sequences, reversing the effect of the unknown Hamiltonian for $\sim 1/2$ the pulse duration. I begin with the Markovian limit, as I'm not so familiar with DD pulse sequences.

2 Helpful Math

Let's vectorize the density matrix, in the Pauli basis:

$$\rho = \frac{\hat{1}}{2} + \rho_x \sigma_x + \rho_y \sigma_y + \rho_z \sigma_z \quad (1)$$

To express the equation of motion in this basis, I calculate a few commutators and dissipators.

$$\mathcal{D}[A](B) = ABA^\dagger - \frac{1}{2} \{A^\dagger A, B\} \quad (2)$$

$$\mathcal{D}[A](\hat{1}) = A\hat{1}A^\dagger - \frac{1}{2} \{A^\dagger A, \hat{1}\} = [A, A^\dagger] \quad (3)$$

$$\therefore \mathcal{D}[\sigma_z](\hat{1}) = [\sigma_z, \sigma_z] = 0 \quad (4)$$

$$\mathcal{D}\sigma_z = 0 \quad (5)$$

$$\mathcal{D}[\sigma_z](\sigma_{x,y}) = -2\sigma_{x,y} \quad (6)$$

$$[\sigma_y, \sigma_x] = -2i\sigma_z \quad [\sigma_y, \sigma_z] = 2i\sigma_x \quad (7)$$

With these in hand, we can start expressing the equation of motion for a noisy Y_{90} in this operator basis.

3 Y_{90} with Markovian Noise

A simple master equation for a Y_{90} , subject to dephasing is:

$$\dot{\rho} = -i\frac{\omega}{2} [\sigma_y, \rho] + \frac{\gamma}{2} \mathcal{D}[\sigma_z](\rho), \quad (8)$$

where the factors of two are included to make the matrix description look nice, as we will see in a minute. I express the commutator and dissipator in matrix form:

$$\frac{\gamma}{2} \mathcal{D}[\sigma_z](\cdot) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & 0 \\ 0 & 0 & -\gamma & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot -i\frac{\omega}{2} [\sigma_y, \cdot] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega \\ 0 & 0 & 0 & 0 \\ 0 & -\omega & 0 & 0 \end{bmatrix} \cdot \quad (9)$$

The Lindbladian is just the sum of these two terms:

$$\dot{\vec{\rho}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & \omega \\ 0 & 0 & -\gamma & 0 \\ 0 & -\omega & 0 & 0 \end{bmatrix} \vec{\rho} = \hat{L}\vec{\rho} \quad (10)$$

We take the matrix exponent $\exp(\hat{L}t)$ to get the superoperator S :

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-\frac{\gamma t}{2}} \left(\cosh(\beta t/2) - \frac{\gamma}{\beta} \sinh(\beta t/2) \right) & 0 & -\frac{2\omega}{\beta} e^{-\frac{\gamma t}{2}} \sinh(\beta t/2) \\ 0 & 0 & e^{-\frac{\gamma t}{2}} & 0 \\ 0 & \frac{2\omega}{\beta} e^{-\frac{\gamma t}{2}} \sinh(\beta t/2) & 0 & e^{-\frac{\gamma t}{2}} \left(\cosh(\beta t/2) + \frac{\gamma}{\beta} \sinh(\beta t/2) \right) \end{bmatrix} \quad (11)$$

where $\beta = \sqrt{\gamma^2 - 4\omega^2}$.

If ω is large, and γ is small (as we hope will be the case in low-noise systems), then β will be imaginary, and the hyperbolic functions will become regular trigonometric functions:

$$\beta \equiv i\nu \quad (12)$$

$$\cosh\left(\frac{\beta t}{2}\right) = \cos\left(\frac{\nu t}{2}\right) \quad (13)$$

$$\frac{\sinh(\frac{\beta t}{2})}{\beta} = \frac{\sin(\frac{\nu t}{2})}{\nu} \quad (14)$$

$$S \mapsto \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-\frac{\gamma t}{2}} \left(\cos(\nu t/2) - \frac{\gamma}{\nu} \sin(\nu t/2) \right) & 0 & -\frac{2\omega}{\nu} e^{-\frac{\gamma t}{2}} \sin(\nu t/2) \\ 0 & 0 & e^{-\frac{\gamma t}{2}} & 0 \\ 0 & \frac{2\omega}{\nu} e^{-\frac{\gamma t}{2}} \sin(\nu t/2) & 0 & e^{-\frac{\gamma t}{2}} \left(\cos(\nu t/2) + \frac{\gamma}{\nu} \sin(\nu t/2) \right) \end{bmatrix} \quad (15)$$

We have control over t , and we'd like to determine how to set it in order to obtain the maximum-fidelity Y_{90} . To separate the noise from the gate we'd like to perform, we express the total superoperator as the product of a desired Y_{90} superoperator and a noise operator:

$$S = S_{\text{Noise}} S_{\text{Id}} \quad (16)$$

$$\therefore S_{\text{Noise}} = S_{\text{Id}}^{-1} S \quad (17)$$

$$S_{\text{Id}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad (18)$$

$$\therefore S_{\text{Noise}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{2\omega}{\nu} \exp(-\frac{\gamma t}{2}) \sin(\frac{\nu t}{2}) & 0 & \exp(-\frac{\gamma t}{2}) \left(\frac{\gamma}{\nu} \sin(\frac{\nu t}{2}) - \cos(\frac{\nu t}{2}) \right) \\ 0 & 0 & \exp(-\gamma t) & 0 \\ 0 & \exp(-\frac{\gamma t}{2}) \left(\frac{\gamma}{\nu} \sin(\frac{\nu t}{2}) + \cos(\frac{\nu t}{2}) \right) & 0 & \frac{2\omega}{\nu} \exp(-\frac{\gamma t}{2}) \sin(\frac{\nu t}{2}) \end{bmatrix} \quad (19)$$

To find the channel fidelity $F_\Lambda = \langle \Omega | \Lambda \otimes \hat{1} (|\Omega\rangle\langle\Omega|) | \Omega \rangle$ (where $|\Omega\rangle$ is a Bell state), we take the trace of this superoperator and divide by 4 (I won't prove this here, but leave it as an exercise):

$$F_{S_{\text{Noise}}} = \frac{1}{4} \left[1 + \frac{4\omega}{\nu} \exp\left(-\frac{\gamma t}{2}\right) \sin\left(\frac{\nu t}{2}\right) + \exp(-\gamma t) \right] \quad (20)$$

4 Questions

1. Show that "shorting" the gate time optimally (maximizing the channel fidelity) doesn't appreciably raise the fidelity over just setting $\omega t = \frac{\pi}{2}$.
2. How does all this change when we add amplitude damping?