Magic States for Multi-Qubit Operations

Ben Criger

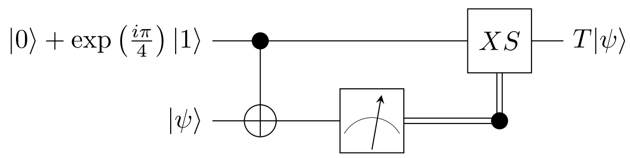
July 12, 2016

1 Introduction

The purpose of this note is to determine the utility of single-qubit magic states for performing multi-qubit operations. All this may already be known, but I'm not looking it up, for whatever reason. I start with a review of single-qubit gate teleportation, then try to phrase this protocol as a joint measurement protocol. I conclude (I hope) by generalising this protocol to multi-qubit measurements.

2 Review

Here's the well-known protocol for implementing a T gate using Clifford operations and state preparation:



To show that this circuit does what it's supposed to, we can do a little math:

$$|\psi_{\mathrm{init}}\rangle = \left(|0\rangle + \exp\left(\frac{i\pi}{4}\right)|1\rangle\right) \otimes (\alpha|0\rangle + \beta|1\rangle) = \alpha|00\rangle + \beta|01\rangle + \alpha\exp\left(\frac{i\pi}{4}\right)|10\rangle + \beta\exp\left(\frac{i\pi}{4}\right)|11\rangle \quad (1)$$

$$\begin{aligned}\mathrm{CNOT}|\psi_{\mathrm{init}}\rangle &= \alpha|00\rangle + \beta|01\rangle + \alpha\exp\left(\frac{i\pi}{4}\right)|11\rangle + \beta\exp\left(\frac{i\pi}{4}\right)|10\rangle \\ &= \left(\alpha|0\rangle + \beta\exp\left(\frac{i\pi}{4}\right)\right)\otimes|0\rangle + \left(\alpha\exp\left(\frac{i\pi}{4}\right)|1\rangle + \beta|0\rangle\right)\otimes|1\rangle \quad (2) \\ &= \left(\alpha|0\rangle + \beta\exp\left(\frac{i\pi}{4}\right)\right)\otimes|0\rangle + S^{-1}X\left(\exp\left(\frac{-i\pi}{4}\right)\alpha|0\rangle + \beta|1\rangle\right)\otimes|1\rangle \quad (3)\end{aligned}$$

If we measure the bottom qubit, obtaining 0 or 1, the post-selected states are equivalent up to a global phase after applying the classically-controlled XS.

To try to generalise this protocol, it might be useful to phrase it in terms of joint Pauli measurements.

3 Joint Measurement Protocol

If we propagate the measured operator Z_1 back through the CNOT, we get a joint Z_0Z_1. Measuring this operator projects either into the space spanned by $\{|00\rangle, |11\rangle\}$ or $\{|01\rangle, |10\rangle\}$. At this point, we could do the classically-controlled Clifford before we do the CNOT, but it's tough to see the general principle at work.

Though, maybe you could disentangle by measuring an X or something.

4 Multi-Qubit Measurement

5 Questions

1. Taking a look at Daniel Gottesman's thesis, near equation 5.23, it looks like there's a state that can be prepared from post-selected joint Pauli measurement that facilitates a Toffoli gate. It would be interesting to see if this can be extended to other fixed-parity multi-qubit states, and what the resource count looks like.

2. Can you get a T gate from a Toffoli gate? I remember there being some conversion in Nielsen/Chuang, but it was between the Toffoli gate and some other fancy angle.