Quantum Fault Tolerance

Y-Eigenstate Preparation

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1 Introduction

People always act like fault-tolerant state preparation by measurement is limited to the X or Z eigenstates. This implies that, in order to prepare a Y eigenstate, we have to use state distillation. Let's confirm that this is the case by attempting some naïve measurement-based preparation and showing that it doesn't work.

2 Surface Code

We begin with nine qubits in Y eigenstates. The checks to be measured are:

$$X_{01},\ X_{1245},\ X_{3467},\ X_{78},\ Z_{25},\ Z_{4578},\ Z_{0134},\ Z_{36} \tag{1}$$

We measure in the X/Z boundary operators first:

$$
\begin{aligned}
S &\mapsto \langle Y_4,\, (-1)^{x_{01}}X_{01},\, Y_{01},\, (-1)^{x_{78}}X_{78},\, Y_{78},\, (-1)^{z_{25}}Z_{25},\, Y_{25},\, (-1)^{z_{36}}Z_{36},\, Y_{36}\rangle & (2)\\
&\underset{M_{X_{1245}}}{\mapsto} \langle (-1)^{x_{1245}}X_{1245},\, Y_{014},\, (-1)^{x_{01}}X_{01},\, (-1)^{x_{78}}X_{78},\, Y_{78},\, (-1)^{z_{25}}Z_{25},\, Y_{25},\, (-1)^{z_{36}}Z_{36},\, Y_{36}\rangle & (3)\\
&\underset{M_{X_{3467}}}{\mapsto} \langle (-1)^{x_{1245}}X_{1245},\, (-1)^{x_{3467}}X_{3467},\, (-1)^{x_{01}}X_{01},\, (-1)^{x_{78}}X_{78},\, (-1)^{z_{25}}Z_{25},\, (-1)^{z_{36}}Z_{36},\\
&\qquad Y_{01478},\, Y_{25},\, Y_{36}\rangle & (4)\\
&\underset{M_{Z_{0134}}}{\mapsto} \langle (-1)^{x_{1245}}X_{1245},\, (-1)^{z_{0134}}Z_{0134},\, (-1)^{x_{3467}}X_{3467},\, (-1)^{x_{01}}X_{01},\, (-1)^{x_{78}}X_{78},\, (-1)^{z_{25}}Z_{25},\, (-1)^{z_{36}}Z_{36},\\
&\qquad Y_{0134678},\, Y_{25}\rangle & (5)
\end{aligned}
$$

Under the last stabiliser measurement, this maps to a joint eigenstate of the surface code stabilisers and a transversal Y operator. This transversal operator is a logical Y for any odd-distance surface code, since it is the product of an odd number of Z columns and an odd number of X rows.