Quantum Fault Tolerance

Measuring Small Stabilisers onto Bare Ancillas

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1 Introduction

The toric/surface code is typically defined with weight-4 stabilisers of the form $XXXX$ and $ZZZZ$. This code detects X and Z errors independently, which can be a problem if the Y error rate is large (which is the case if the code is actually detecting amplitude damping), or if the X and Z error rates are different, potentially leading to different logical error rates (though the gates involved in syndrome measurement may 'un-bias' the noise for you). To remedy this, we can take advantage of the fact that any set of stabilisers which differs from the surface code stabilisers by a transversal unitary, which we will restrict to be a Clifford (though I don't think it has to be a Clifford, in principle).

There are two popular modifications, one which exchanges $ZZZZ$ stabilisers for $YYYY$ stabilisers, and one which acts a Hadamard on half the qubits, resulting in stabilisers of the form $XZZX$ and $XZXZ$. We might even combine these to obtain $XYYX/XYXY$ stabilisers, if X/Y error rates differ, or if it offers some advantage in terms of the independent Z error rate that's on due to dephasing. The question we address in this document is how to measure these operators onto a bare ancilla. We apply a general principle, that stabiliser measurement is equivalent to phase estimation, except that we know the phase is ± 1 in advance. The syndrome measurement circuit is therefore:

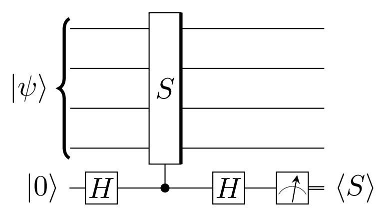

Figure 1: Stabiliser measurement circuit for whatever weight-4 stabiliser.

Using circuit identities, we can express this circuit using the experimentally available gate set. This is the origin of the Y_{90}/CPHASE-based implementations we often use in superconducting qubits. The circuit identities used in this case are simple, decomposing the Pauli X into $Y_{90}ZY_{-90}$:

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{1}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = X \tag{2}$$

We can show the same thing with the operator $Y = X_{-90} Z X_{90}$:

$$\frac{1}{2}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & -i\\ -i & 1\end{bmatrix} \tag{3}$$

$$= \frac{1}{2}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1 & -i\\ i & -1\end{bmatrix} = Y \tag{4}$$

To measure one of these stabilisers, then, all you have to do is replace the Y_{90} rounds used for X measurement with some X_{90}s. Measuring multiple stabilisers at a time might be tricky, but that's a topic for another note.

2 Open Questions

- Can you prepare a nice state using codes that differ from the toric/surface code by a transversal (perhaps non-Clifford) unitary?