Lattice Surgery

Effect of Mismeasurement

Ben Criger

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1 Introduction

There's an argument floating around that a certain lattice surgery technique won't be fault-tolerant, which I heard from Christophe Vuillot, who (I think) heard it from Earl Campbell. I've since seen the technique pop up in other places, so I figure I'd summarise the argument here.

2 High-Weight Operators and Mismeasurement

The technique we're going to discuss is the direct measurement of high-weight operators in lattice surgery (here, 'high' means greater than 2). The objection will be clear if we just discuss a weight-3 example at two different distances, so I'll begin at distance 3.

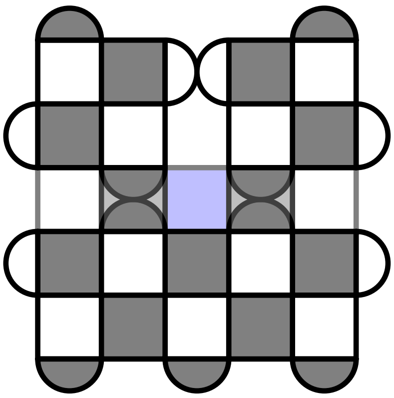

Figure 1: Distance-3 layout for measuring $\overline{X}^{\otimes 3}$ through lattice surgery. Merge operators are drawn semi-transparently, the operator drawn in blue is the *central operator*, to be discussed shortly.

If all goes well, we should be able to merge and split in order to learn the eigenvalue of the operator $\overline{XXX}$, but what if we don't measure the central operator correctly? Specifically, what if we accidentally measure a weight-2 'ear' operator? Then, we would have a situtation where the measurement is telling us about $\overline{XX}$ instead of the weight-3 $\overline{XXX}$, which is a logical failure mode not present in the weight-2 logical measurement.

The real bad news, though, comes when we discuss how this changes with distance. Consider the distance-5 case:

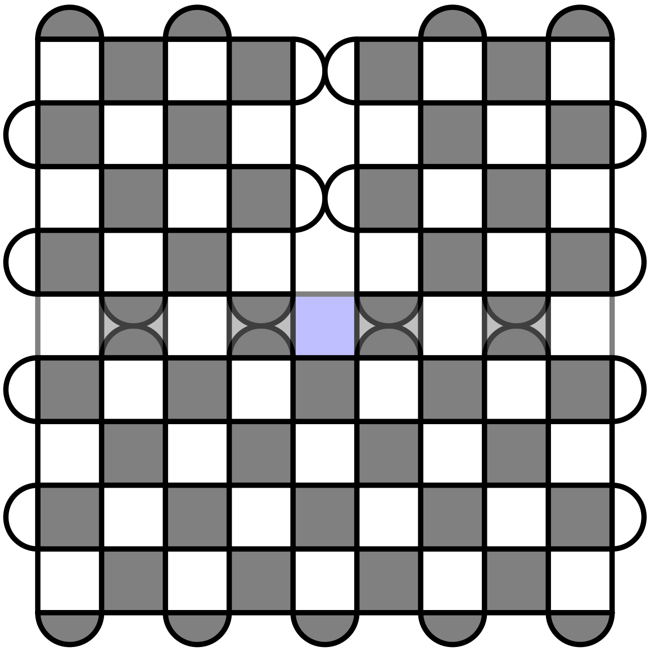

Figure 2: Distance-5 layout for measuring $\overline{X}^{\otimes 3}$ through lattice surgery. The central operator is unchanged by the increase in distance.

The central operator is in the same place as before, at the same size. Thus, the same mismeasurement as before produces the same logical error as before, with no increase in weight. This implies that the technique is not fault-tolerant.

3 Open Questions

In reality, we need to repeat the measurement $O(d)$ times in order to infer the eigenvalue of a logical operator with accuracy. Does this imply that we'd have to mismeasure the central operator $O(d)$ times in order to actually realise the logical failure imagined above?