Quantum Fault Tolerance

One-Qubit Cliffords by Measurement

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1 Introduction

Some fault tolerant codes don't have transversal Clifford gates, which necessitates code deformation. Sometimes (like in Bravyi's 2016 QIP tutorial) these codes have weird symmetric boundary conditions that I don't understand. I'd like to compare this to a more intuitive non-unitary implementation of Cliffords, regular old lattice surgery. To accomplish this, I need to check which gates can be obtained by a protocol where:

1. An ancilla is prepared in a Pauli $+1$ eigenstate
2. a two-bit joint measurement is performed, and
3. the original ancilla qubit is measured out fault-tolerantly.

To be non-trivial, the jointly-measured Pauli has to anticommute with the prepared Pauli and the measured Pauli, there are 36 such protocols. However, consider the case in which $A = D$. In this case, the tableau evolves as follows:

$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IA \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{BC}}{\longmapsto} \begin{array}{c} BC \\ \hline XI(XA) \\ ZI(ZA) \end{array} \overset{\mathcal{M}_{IA}}{\longmapsto} \begin{array}{c} IA \\ \hline XI \\ ZI \end{array},$$

where either of the Paulis in brackets may be present without affecting the result. Either way, the resulting unitary is the identity, we eliminate these cases, leaving 18 remaining. I test all of them in the remainder of this document, assuming all measurements produce $+1$ outcomes throughout:

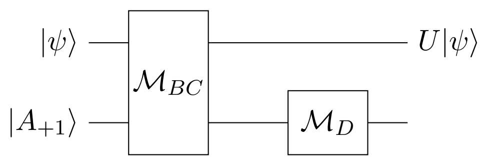
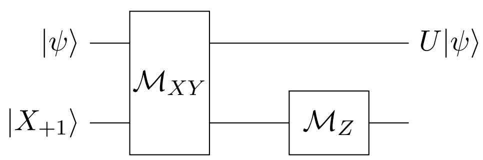
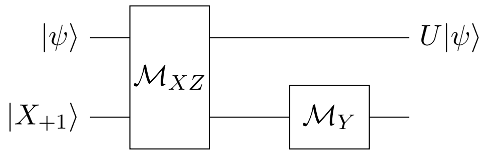


Figure 1: The basic protocol I want to analyse, with a $+1$ eigenstate prepared on the ancilla, a joint measurement of a two-qubit Pauli BC, and a final measurement of Pauli D.

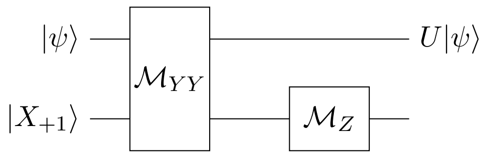
Below, I present the 18 protocols, with a little equation explaining what happens to the stabiliser and logicals throughout:



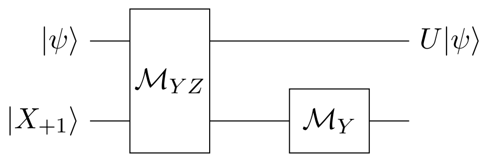
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IX \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{XY}}{\longmapsto} \begin{array}{c} XY \\ \hline XI \\ ZX \end{array} \overset{\mathcal{M}_{IZ}}{\longmapsto} \begin{array}{c} IZ \\ \hline XI \\ YI \end{array} \therefore U \sim HPH \tag{1}$$



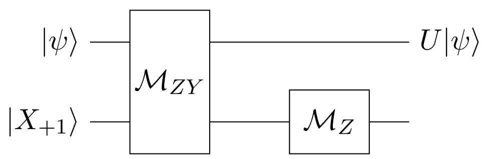
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IX \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{XZ}}{\longmapsto} \begin{array}{c} XZ \\ \hline XI \\ ZX \end{array} \overset{\mathcal{M}_{IY}}{\longmapsto} \begin{array}{c} IY \\ \hline XI \\ YI \end{array} \therefore U \sim HPH \tag{2}$$



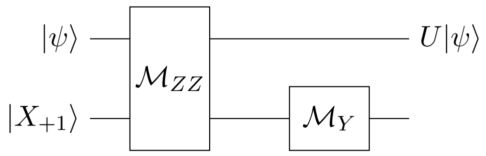
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IX \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{YY}}{\longmapsto} \begin{array}{c} YY \\ \hline XX \\ ZX \end{array} \overset{\mathcal{M}_{IZ}}{\longmapsto} \begin{array}{c} IZ \\ \hline ZI \\ XI \end{array} \therefore U \sim H \tag{3}$$



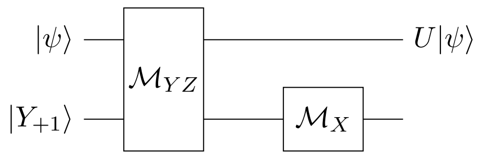
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IX \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{YZ}}{\longmapsto} \begin{array}{c} YZ \\ \hline XX \\ ZX \end{array} \overset{\mathcal{M}_{IY}}{\longmapsto} \begin{array}{c} IY \\ \hline ZI \\ XI \end{array} \therefore U \sim H \tag{4}$$



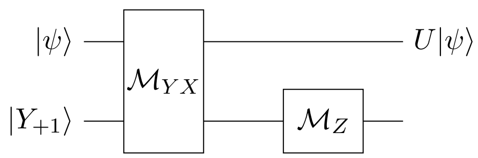
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IX \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{ZY}}{\longmapsto} \begin{array}{c} ZY \\ \hline XX \\ ZI \end{array} \overset{\mathcal{M}_{IZ}}{\longmapsto} \begin{array}{c} IZ \\ \hline YI \\ ZI \end{array} \therefore U \sim P \tag{5}$$



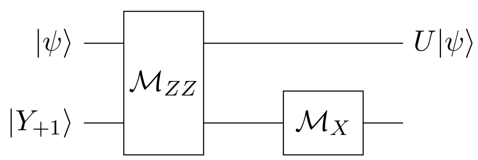
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IX \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{ZZ}}{\longmapsto} \begin{array}{c} ZZ \\ \hline XX \\ ZI \end{array} \overset{\mathcal{M}_{IY}}{\longmapsto} \begin{array}{c} IY \\ \hline YI \\ ZI \end{array} \therefore U \sim P \tag{6}$$



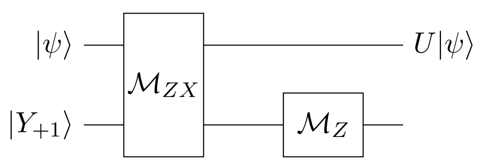
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IY \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{YZ}}{\longmapsto} \begin{array}{c} YZ \\ \hline XY \\ ZY \end{array} \overset{\mathcal{M}_{IX}}{\longmapsto} \begin{array}{c} IX \\ \hline ZI \\ XI \end{array} \therefore U \sim H \tag{7}$$



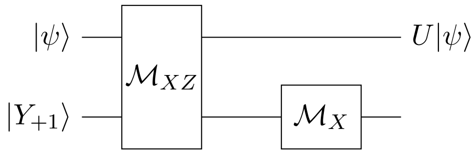
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IY \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{YX}}{\longmapsto} \begin{array}{c} YX \\ \hline XY \\ ZY \end{array} \overset{\mathcal{M}_{IZ}}{\longmapsto} \begin{array}{c} IZ \\ \hline ZI \\ XI \end{array} \therefore U \sim H \tag{8}$$



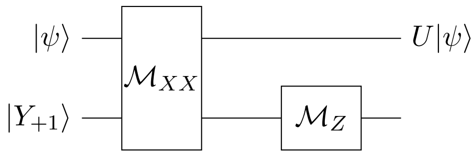
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IY \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{ZZ}}{\longmapsto} \begin{array}{c} ZZ \\ \hline XY \\ ZI \end{array} \overset{\mathcal{M}_{IX}}{\longmapsto} \begin{array}{c} IX \\ \hline YI \\ ZI \end{array} \therefore U \sim P \tag{9}$$



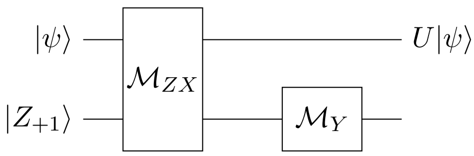
$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IY \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{ZX}}{\longmapsto} \begin{array}{c} ZX \\ \hline XY \\ ZI \end{array} \overset{\mathcal{M}_{IZ}}{\longmapsto} \begin{array}{c} IZ \\ \hline YI \\ ZI \end{array} \therefore U \sim P \tag{10}$$



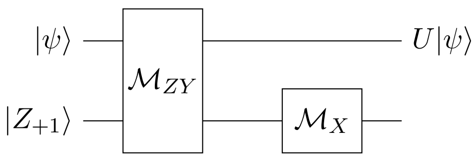
$$\begin{array}{c}S_{\text{init}}\\ \hline \overline{X}\\ \overline{Z}\end{array} = \begin{array}{c}IY\\ \hline XI\\ ZI\end{array} \xmapsto{\mathcal{M}_{XZ}} \begin{array}{c}XZ\\ \hline XI\\ ZY\end{array} \xmapsto{\mathcal{M}_{IX}} \begin{array}{c}IX\\ \hline XI\\ YI\end{array} \therefore U \sim HPH \tag{11}$$



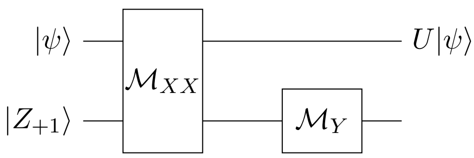
$$\begin{array}{c}S_{\text{init}}\\ \hline \overline{X}\\ \overline{Z}\end{array} = \begin{array}{c}IY\\ \hline XI\\ ZI\end{array} \xmapsto{\mathcal{M}_{XX}} \begin{array}{c}XX\\ \hline XI\\ ZY\end{array} \xmapsto{\mathcal{M}_{IZ}} \begin{array}{c}IZ\\ \hline XI\\ YI\end{array} \therefore U \sim HPH \tag{12}$$



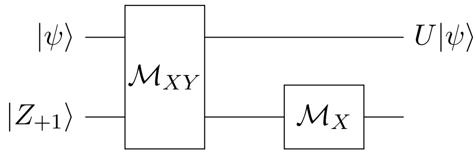
$$\begin{array}{c}S_{\text{init}}\\ \hline \overline{X}\\ \overline{Z}\end{array} = \begin{array}{c}IZ\\ \hline XI\\ ZI\end{array} \xmapsto{\mathcal{M}_{ZX}} \begin{array}{c}ZX\\ \hline XZ\\ ZI\end{array} \xmapsto{\mathcal{M}_{IY}} \begin{array}{c}IY\\ \hline YI\\ ZI\end{array} \therefore U \sim P \tag{13}$$



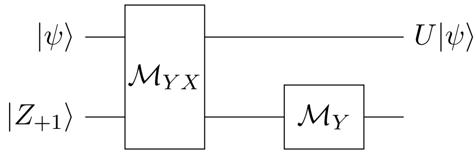
$$\begin{array}{c}S_{\text{init}}\\ \hline \overline{X}\\ \overline{Z}\end{array} = \begin{array}{c}IZ\\ \hline XI\\ ZI\end{array} \xmapsto{\mathcal{M}_{ZY}} \begin{array}{c}ZY\\ \hline XZ\\ ZI\end{array} \xmapsto{\mathcal{M}_{IX}} \begin{array}{c}IX\\ \hline YI\\ ZI\end{array} \therefore U \sim P \tag{14}$$



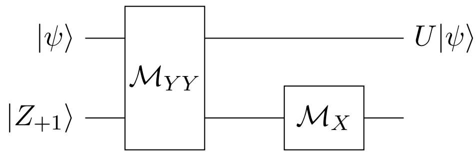
$$\begin{array}{c}S_{\text{init}}\\ \hline \overline{X}\\ \overline{Z}\end{array} = \begin{array}{c}IZ\\ \hline XI\\ ZI\end{array} \xmapsto{\mathcal{M}_{XX}} \begin{array}{c}XX\\ \hline XI\\ ZZ\end{array} \xmapsto{\mathcal{M}_{IY}} \begin{array}{c}IY\\ \hline XI\\ YI\end{array} \therefore U \sim HPH \tag{15}$$



$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IZ \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{XY}}{\longmapsto} \begin{array}{c} XY \\ \hline XI \\ ZZ \end{array} \overset{\mathcal{M}_{IX}}{\longmapsto} \begin{array}{c} IX \\ \hline XI \\ YI \end{array} \therefore U \sim HPH \tag{16}$$



$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IZ \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{YX}}{\longmapsto} \begin{array}{c} YX \\ \hline XZ \\ ZZ \end{array} \overset{\mathcal{M}_{IY}}{\longmapsto} \begin{array}{c} IY \\ \hline ZI \\ XI \end{array} \therefore U \sim H \tag{17}$$



$$\begin{array}{c} S_{\text{init}} \\ \hline \overline{X} \\ \overline{Z} \end{array} = \begin{array}{c} IZ \\ \hline XI \\ ZI \end{array} \overset{\mathcal{M}_{YY}}{\longmapsto} \begin{array}{c} YY \\ \hline XZ \\ ZZ \end{array} \overset{\mathcal{M}_{IX}}{\longmapsto} \begin{array}{c} IX \\ \hline ZI \\ XI \end{array} \therefore U \sim H \tag{18}$$

2 Two-Qubit Cliffords

It would be interesting to generalise this to two-qubit Cliffords (CNOTs)