Summary of the quantumsim paper [1]

Daniel Moreno Manzano

December 6, 2018

1 Error model parameters

Table 1: Main error model parameters for simulation

Parameter	Symbol	Value	Explanation and notes
Qubit relaxation time	T_1	30 μs	Only affects qubits in the excited state. Consistent set of values: [20 - 100 μs]
Qubit dephasing time (white noise)	T_ϕ	60 μs	Consistent set of values would be $2T_1$ or ∞ (all white noise dephasing eliminated)
Decay time	T_2	30 μs	$\frac{1}{T_2} = \frac{1}{T_\phi} + \frac{1}{2T_1}$
Single-qubit gate time	$T_{g,1Q}$	20 ns	
Two-qubit gate time	$T_{g,2Q}$	40 ns	
Measurement time	τ_m	300 ns	
Depletion time	τ_d	300 ns	?
Fast Measurement time	τ_m^{fast}	100 ns	
Fast Depletion time	τ_d^{fast}	100 ns	?
Readout infidelity	ϵ_{RO}	5 (-3)	
Physical qubit Fidelity	$\mathcal{F}_{phys}(t)$	-	$\mathcal{F}_{phys}(t) = \frac{1}{6}\left(1 + e^{-\frac{t}{T_1}}\right) + \frac{1}{3}\left(1 + e^{-t\left(\frac{1}{2T_1} + \frac{1}{T_\phi}\right)}\right)$
Physical qubit error rate	ϵ_{phys}	-	$\epsilon_{phys} = -\tau_{circuit}\frac{d\mathcal{F}_{phys}(t)}{dt}\|_{t=0} = \frac{\tau_{circuit}}{3T_1} + \frac{\tau_{circuit}}{3T_\phi}$
In-axis rotation error	p_{axis}	1 (-4)	Decay corresponding to shrinking along the y axis because of the single-qubit gates depolarizing noise
In-plane rotation error	p_{plane}	5 (-4)	Decay corresponding to shrinking along the x and z axis because of the single-qubit gates depolarizing noise

2 Error models

In the quantumsim module, all gates are applied in the Pauli transfer matrix representation:

$$(R_\Lambda)_{ij} = \frac{1}{2}Tr(\sigma_i \Lambda \sigma_j)$$

where σ_i are the Pauli operators: $\sigma_0 = I$, $\sigma_1 = X$, $\sigma_2 = Y$, $\sigma_3 = Z$

2.1 Qubit Idling

While idling for a time t, a transmon in $|1\rangle$ or in superposition could relax to $|0\rangle$ or acquire random quantum phase shifts due to $1/f$ noise sources (flux noise) or others. The dephasing effect only appears in a superposition state.

2.1.1 Amplitude-phase damping model

$$R_{\Lambda_{T_1}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-p_1} & 0 & 0 \\ 0 & 0 & \sqrt{1-p_1} & 0 \\ p_1 & 0 & 0 & 1-p_1 \end{bmatrix}$$

$$R_{\Lambda_{T_\phi}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-p_\phi} & 0 & 0 \\ 0 & 0 & \sqrt{1-p_\phi} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with $p_1 = 1 - e^{-\frac{t}{T_1}}$ and $p_\phi = 1 - e^{-\frac{t}{T_\phi}}$ that are the probabilities for relaxation and pure dephasing, respectively.

2.1.2 Qubit idling

Idling for a duration t:

$$R_{AP(t)} = R_{\Lambda_{T_1}} R_{\Lambda_{T_\phi}}$$

2.2 Single-qubit $R_y(\pi/2)$ rotations

"Single-qubit gates [...] errors can mostly be attributed to Markovian noise. [...] we thus model these errors as Markovian".

"Single-qubit rotations are modeled by sandwiching an instantaneous Pauli transfer matrix, representing the rotation, with periods of duration $\frac{\tau_{g,1Q}}{2}$ of amplitude and phase damping. This allows to model the gate for different T_1 and T_ϕ [...] However, [...] actual gates are more accurately described when adding a [...] depolarizing noise to the instantaneous part. In the Bloch sphere, this decay corresponds to shrinking toward the origin, with factor $1 - p_{axis}$ along the y axis and $1 - p_{plane}$ along the x- and z-axes":

$$R_{R_y(\pi/2)} = R_{AP(\frac{\tau_{g,1Q}}{2})} R'_{R_y(\pi/2)} R_{dep} R_{AP(\frac{\tau_{g,1Q}}{2})}$$

where

$$R_{dep} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-p_{plane} & 0 & 0 \\ 0 & 0 & 1-p_{axis} & 0 \\ 0 & 0 & 0 & 1-p_{plane} \end{bmatrix}$$

and $R'_{R_y(\pi/2)}$ is the Pauli transfer matrix describing the theoretical $\pi/2$ rotation along the y axis.

2.3 CZ gates

"The C-Z gate is achieved by flux pulsing a transmon into the $|11\rangle \leftrightarrow |02\rangle$ avoided crossing with another, where the 2 denotes the second-excited state of the fluxed transmon. Holding the transmons here for $\tau_{g,2Q}$ causes the probability amplitudes of $|01\rangle$ and $|11\rangle$ to acquire phases[. . .]

Our full (but simplistic) model of the CZ gate consists of an instantaneous CZ gate with single-qubit phase error $\delta_{\phi_{1Q}}$ and two-qubit phase error $\delta_{\phi_{2Q}} = \frac{\delta_{\phi_{1Q}}}{2}$, sandwiched by idling intervals of duration $\frac{\tau_{g,2Q}}{2}$."
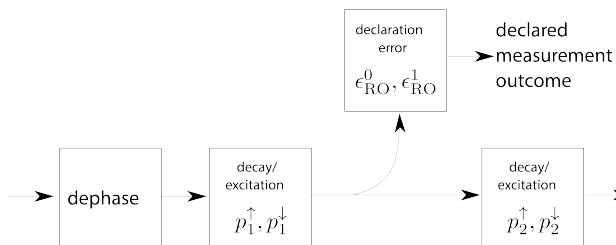
2.4 Measurement

Figure 1: The model for measurements consists of a dephasing of the qubit followed by a period of decay and excitation with probability $p_{\uparrow/\downarrow}^{(1)}$. At this point, the qubit state is sampled. The sampling result is subject to a declaration error ϵ_{RO}, and the qubit state is subject to further decay or excitation with probabilities $p_{\uparrow/\downarrow}^{(2)}$ before the end of the measurement block

The initial dephasing step in the measurement model (Fig. 1) occurs due to the photon decay effect.

"We find that the readout errors $\epsilon_{RO}^{|i\rangle}$ are almost independent of the qubit state $|i\rangle$, and so we describe them with a single readout error parameter ϵ_{RO}". The outcome-independent declaration error of $\epsilon_{RO} = \epsilon_{RO}^1 = \epsilon_{RO}^0 = 0.15\%$ is extracted from experiments.

They ignore effects leading to measurement-induced mixing and non-linearity of the readout resonator, as well as residual photon numbers.

2.5 Photon decay

In the presence of photons in a readout resonator, the coupled qubit is affected suffering a $p_{\phi,photon}$ dephasing. This dephasing is present whenever the coupled qubit is brought into superposition before the readout resonator has returned to the vacuum state following the last measurement. This dephasing is then implemented via the same Pauli transfer matrix as $R_{\Lambda_{T_\phi}}$.

2.6 Flux Noise

During a quantum algorithm, "transmons are repeatedly moved in frequency away from their sweetspot using flux pulses, either to implement a C-Z gate or to avoid one. Away from the sweetspot, transmons become first-order sensitive to flux noise, which causes an additional random phase shift."

"As this noise typically has a $1/f$ power spectrum, the largest contribution comes from low-frequency components that are essentially static for a single run, but fluctuating between different runs." "Shifting the transmon from its sweetspot $f_{q,max}$ to a lower frequency $f_q(t)$ makes it first-order sensitive to flux noise".

"In our simulation, we approximate the effect of this noise through ensemble averaging, with quasi-static phase error added to a transmon whenever it is flux pulsed."

As one could see in the figures 4 and 5 from the Supplemental information, a little over-rotation caused by inaccurate calibration of the flux pulse in a single- or two-qubit gate translates in a huge increase of the ϵ_L.

3 Effects not taking into account

They use a simple model for the CZ errors. They neglect leakage (previous experiments have reduced leakage probability per CZ to $\approx 0.3\%$). Of course this simplification is also in **quantumsim**.

4 The quantumsim simulation package

"Quantumsim performs calculations on density matrices utilizing a graphics processing unit in a standard desktop computer [. . .]

One-qubit and two- qubit gates are applied to the density matrix as completely positive, trace preserving maps represented by Pauli transfer matrices. When a gate involving a « new » qubit must be performed, the density matrix of the system is dynamically enlarged to include that one [. . .]

Qubit measurements are simulated as projective and following the Born rule, with projection probabilities given by the squared overlap of the input state with the measurement basis states. In order to capture empirical measurement errors, we implement a black-box measurement model by sandwiching the measurement between idling processes. After measuring some qubit they remove that qubit from the density matrix.

5 Observations I find Interesting

5.1 Optimization of logical error rates

As they explain in the paper and on can see in Fig. 2, they find that the optimal measuring time for the minimum ϵ_L is 280 ns.
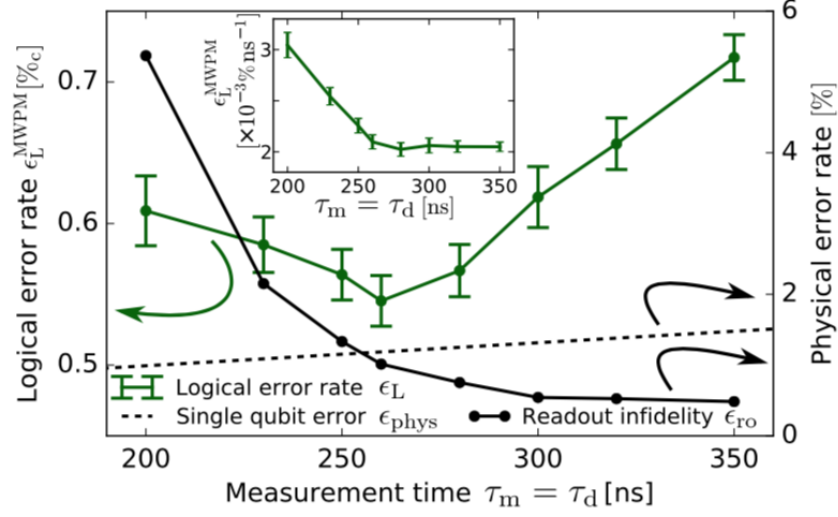
Figure 2: Measure time optimization based on the SC-17 logical error rate. Optimal $\tau_m = 280$ ns

5.2 Projected improvement with advances in quantum hardware

- Memory figure of merit $(\gamma_m = \frac{\epsilon_{phys}}{\epsilon_L})$. How close are ϵ_{phys} and ϵ_L. Metric to check how good the error correction is.

- Computational performance $(\gamma_c = \frac{\epsilon_{phys}\tau_{g,1Q}}{\epsilon_L \tau_{cycle}})$, where, at $\gamma_c = 1$ the computational break-even point is defined.

- A value of $T_1 > 80\mu s$ for planar transmons is emerging.

5.3 Other observations

The following statements are fairly general:

- "Small quasi-static qubit errors are suppressed by the repeated measurements"

- If either the ancilla error rate (ϵ_{anc}) or the ϵ_{RO} are bigger than ϵ_{phys}, ϵ_L becomes independent of both ϵ_{RO} and ϵ_{anc}

- "Optimal cycle parameters for logical error rates per cycle and per unit time are not the same. This implies that logical qubits functioning as quantum memory should be treated differently to those being used for computation"

6 Doubts

- What is the depletion time?

- I do not understand the Inset in Fig. 2

- What is the difference between qubits used for quantum memory and quantum computation? In our case we consider just computation, isn't it?

5

- Surface code is good for Quantum Memory. Which code is good for Quantum Computation?

- Why do we consider that the measurement time is 300 ns instead of 280 ns, that is the optimum time for logical qubit error rate?

- Study the optimum times for each gates to minimize the physical qubit error rate

- At some point $T_\phi = \infty$ is mentioned. Is it possible to clean all the dephasing white-noise.

- Is the Y rotation gates the only ones affected by the dephasing noise?

- ~~What is the flux noise?~~ $\rightarrow$ "Shifting the transmon from its sweetspot $f_{q,max}$ to a lower frequency $f_q(t)$ makes it first-order sensitive to flux noise"

- What are the quasi-static qubit errors?

- ~~Does the R_{dep} parameter mean that the depolarizing model is included?~~ $\rightarrow$ I would say so. But only for the single-qubit gates

- ~~Is the $p_{\phi,photon}$ summed to the p_ϕ in the $R_{\Lambda_{T_\phi}}$ or how is it done?~~ $\rightarrow$ is the dephase at the beginning of the measurement model (Fig. 1)

- What is an adiabatic gate?

- I do not understand anything in the measurement.

    - What is $\epsilon_i^{m,o}$, a and b

    - Are the ignored effects during measurements important for us? Do not think so.

- Quantumsim is able to work no taking into account the surface code, isn't it?

- Is this error model the one that they use in quantumsim? Are all the parameters ready or should I look for add some of them?

- "Ancillas are measured at the end of each cycle, and thus not entangled with the rest of the system". Is this due to the circuit they are using or quantumsim works like that in general

7   Ideas

7.1   The physical error rate is related with the depth of a circuit

Considering any quantum circuit, different from the stabilizer circuit from the Surface Code cycle with the next parameters:

$$
\begin{aligned}
&\text{Depth:} && d \\
&\text{Cycle time (minimum operation time):} && T_{cycle} \\
&\text{Circuit time:} && \tau_{circuit} = d \times T_{cycle}
\end{aligned}
$$

Taking into account the physical error rate function defined in Table 1, we could define the physical qubit error rate of the SC chips for any quantum circuit as:

$$
\epsilon_{phys} = \frac{d \times T_{cycle}}{3} \frac{T_\phi + T_1}{T_\phi T_1}
$$

7.2 Maximum depth given a ϵ_{phys}

Given a maximum error rate for the qubits ($\epsilon_{phys}^{\uparrow}$) as an upper bound, the maximum depth of the circuit would be:

$$d^{\uparrow} = \frac{3\epsilon_{phys}^{\uparrow}}{T_{cycle}} \frac{T_{\phi}T_1}{T_{\phi} + T_1}$$

Where any $d > d^{\uparrow}$ will have a physical error rate bigger than the desired one ($\epsilon_{phys} > \epsilon_{phys}^{\uparrow}$).

References

[1] T. E. O'Brien, B. Tarasinski, and L. DiCarlo. Density-matrix simulation of small surface codes under current and projected experimental noise. *npj Quantum Information*, 3(1), Sep 2017.