Universidade do Minho Academic Year 2020/2021

Mestrado Integrado em Engenharia Informática / Mestrado em Informática

Algoritmos Paralelos

Work 2: numerical solution of the Poisson equation

Introduction

Our problem is to solve the Poisson equation in two dimensions:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = g(x, y),$$

on the unit square $\{(x, y) : 0 < x, y < 1\}$ with values of u being kown at the boundaries of the square. We discretize at the grid points in (x_i, y_j) with $x_i = ih$ and $y_j = jh$, with $i, j = 1, ..., N$ and $h = \frac{1}{N-1}$. Values of i and j equal to 1 or N do correspond to boundary points. Approximating the second order derivatives with centered differences, with a truncation error which is $\mathcal{O}(h^2)$, and with $u_{ij} = u(x_i, y_j)$, one gets a system with $(N-2)^2$ linear equations

$$u_{i-1,j} - 4u_{ij} + u_{i+1,j} + u_{i,j-1} + ui, j + 1 = h^2 g(x_i, y_j),\ i, j = 2, ..., N-1 \tag{1}$$

This system may be solved by different iterative algorithms (see below). For simplicity, we will assume that $g(x, y) = 0$.

Matlab codes The following Matlab codes are included.

- PoissonJac solves the system (1) using the Jacobi method
- PoissonGS solves the same system using the Gauss-Seidel method ordering the unknows from bottom to top and from left to right;
- PoissonGSRB solves the system using the Gauss-Seidel method using the Red-Black ordering which is well-suited to parallel computing;
- PoissonSORRB.m is an implementation of the SOR method with Red-Black ordering.

The wok to be done by the students

To develop sequential and parallel implementations in C/C++ of PoissonGS, PoissonGSRB and PoissonSORRB and test them in the cluster SEARCH. Use different values of N and different number of nodes. Write a report with about 10 pages. The report should include the statement of the problem, a brief description of the numerical algorithms implemented, the parallelization strategy, the results obtained and a brief discussion of the scalability of the algorithm.

Deadline: May, 7th 2021