Derivation of Equations used in ArduPilots Autonomous Autorotation Flight Mode

1 Introduction

This short report gives an overview of the equations used within the autonomous autorotation flight mode implimented in ArduPilot, for use with traditional helicopters.

The purpose of this document is to provide a guide to developers, giving the derivation of the equations used, making the code implementation easier to follow, and preserving the intent and rational as to why the controller is written as it is.

In an attempt to keep this document brief, where possible, equations from other resources have been used as a starting point, to avoid duplication of already available and well documented methods. All references and links are provided.

2 Nomenclature

C_{l_α}	(rad^{-1}) lift-curve slope.
r	Non-dimensional radial distance.
λ	Rotor inflow ratio (normalised by tip speed).
λ_i	Induced inflow ratio.
λ_h	Induced inflow ratio in the hover.
λ_c	Climb inflow ratio.
σ	Rotor solidity ratio.
θ	(rad) Blade pitch angle.

TODO: Add a few diagrams of rotor geometry to help introduce the nomenclature.

3 Flare Hight Estimate

In order to approximate what height the aircraft will begin to flare, Blade Element Momentum Theory (BEMT) is used to construct an approximation of the decceleration that can be expected from the helicopter in the flare, hence at what height the flare must begin to achieve a successful landing

TODO: Give a little flow diagram of the calculation order to provide context

3.1 Approximation of Hover Inflow Ratio

Starting from equation (3.60) of Leishman (2006) (Page 127), the local rotor inflow ratio (λ) that is a function of the non-dimensional rotor radius (r), $\lambda = \lambda(r)$, can be computed from the following quadratic:

$$\lambda^2 + \left(\frac{\sigma\, C_{l_\alpha}}{8} - \lambda_c \right) - \frac{\sigma\, C_{l_\alpha}}{8}\, \theta\, r = 0 \tag{1}$$

For cleanliness in the equations, the solidity ratio ($sigma$) and lift-curve slope (C_{l_α}) are grouped into one variable $b = \sigma\, C_{l_\alpha}$, giving:

$$\lambda^2 + \left(\frac{b}{8} - \lambda_c\right)\lambda - \frac{b}{8}\,\theta\,r = 0 \tag{2}$$

In this application the hover case is of specific interest, hence the induced flow ratio in the hover (λ_h) can be obtained by setting $\lambda_c = 0$, and recalling that the definition of $\lambda = \lambda_i + \lambda_c$, thus:

$$\lambda_h = \lambda_i = \lambda - \lambda_c = \lambda,$$
$$\lambda_h^2 + \frac{b}{8}\lambda_h - \frac{b}{8}\,\theta\,r = 0 \tag{3}$$

Solving for the roots of the quadratic in equation (3) to obtain:

$$\lambda_h = \frac{1}{2}\left(-\frac{b}{8} \pm \sqrt{\left(\frac{b}{8}\right)^2 + 4\,\frac{b}{8}\,\theta\,r}\right)$$
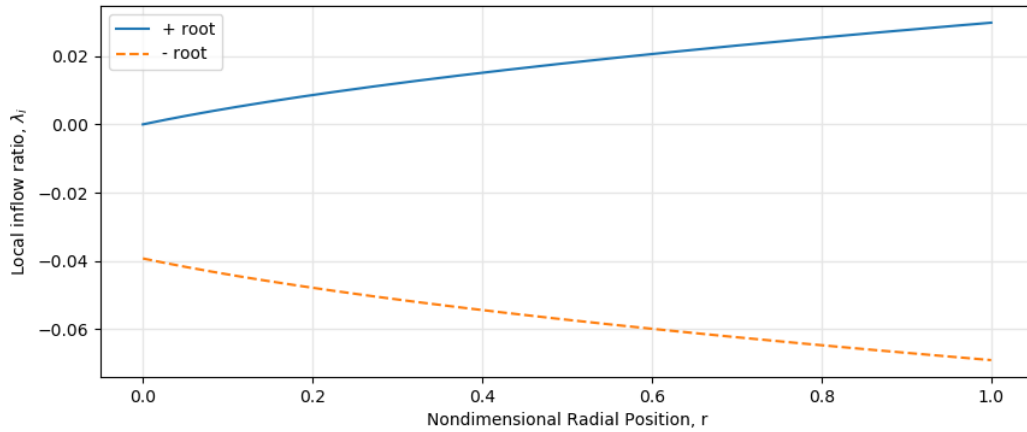$$\lambda_h = -\frac{b}{16} \pm \sqrt{\frac{b^2}{256} + \frac{b}{8}\,\theta\,r} \tag{4}$$

Figure 1: Local inflow distribution obtained by the roots of equation 1.

References

Leishman. *Principles of Helicopter Aerodynamics*. Cambridge Aerospace Series, 2nd edition, 2006.