Algebra HW4

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1. *Page 120,3* A ring R such that $a^2 = a$ for all $a \in R$ is called a **Boolean ring**. Prove that every Boolean ring R is commutative and $a + a = 0$ for all $a \in R$.

   SOLUTION:

2. *Page 120,8* Let R be the set of all 2×2 matrices over complex field $\mathbb{C}$ of the form

   $$\begin{pmatrix} z & w \\ -\bar{w} & \bar{z} \end{pmatrix}.$$

   Then R is a division ring that is isomorphic to the division ring K of real quaternions. *Hint:* The fundamental quaternion units $1, i, j, k$ of K map to the matrices respectively,

   $$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

   SOLUTION:

3. *Page 120,11(The Freshman's Dream).* Let R be a commutative ring with identity of prime characteristic p. If $a, b \in R$, then $(a \pm b)^{p^n} = a^{p^n} \pm b^{p^n}$ for all integers $n \geq 0$. [Note that $b = -b$ if $p = 2$.]

   SOLUTION:

4. *Page 120,13* In a ring R the following conditions are equivalent.

   (a) R has no nonzero nilpotent elements.

   (b) If $a \in R$ and $a^2 = 0$, then $a = 0$.

   SOLUTION:

Probability Theory HW5

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1. *Page 46,13 Property 2.36*

   (a) If $f_n \xrightarrow{\text{a.e.}} f$, then every subsequence $\{f_{n_k}\}$ satisfies $f_{n_k} \xrightarrow{\text{a.e.}} f$.

   (b) If $f_n \xrightarrow{\text{a.e.}} f, f_n \xrightarrow{\text{a.e.}} f'$, then $f = f'$ a.e.

   (c) If $f_n \xrightarrow{\text{a.e.}} f, g_n = f_n$ a.e., $f = g$ a.e., then $g_n \xrightarrow{\text{a.e.}} g$.

   (d) If $f_n^{(k)} \xrightarrow{\text{a.e.}} f^{(k)}, k = 1, \cdots, m$, $g \in C(\bar{\mathbb{R}}^m)$, then

   $$g(f_n^{(1)}, \cdots, f_n^{(m)}) \xrightarrow{\text{a.e.}} g(f^{(1)}, \cdots, f^{(m)}).$$

   <u>Solution</u>:

2. *Page 47,14 Theorem 2.38(2)* Suppose $f, f_n, n \geq 1$ are finite measurable functions. Then $f_n - f_m \xrightarrow{\text{a.e.}} 0$ if and only if

   $$\forall \varepsilon > 0, \mu(\cap_{n=1}^{\infty} \cup_{v=1}^{\infty} \{|f_{n+v} - f_n| \geq \varepsilon\}) = 0.$$

   Specially when μ is finite, $f_n - f_m \xrightarrow{\text{a.e.}} 0$ if and only if

   $$\forall \varepsilon > 0, \mu(\cup_{v=1}^{\infty} \{|f_{n+v} - f_n| \geq \varepsilon\}) = 0 (n \to \infty).$$

   <u>Solution</u>:

3. *Page 47,16* Let $\xi_n = 1_{A_n}$, then $\xi_n \xrightarrow{\mathbb{P}}$ if and only if $\mathbb{P}(A_n) \to 0$.

   <u>Solution</u>:

4. *Page 47,22* For any random variable sequence ξ_n, there is a positive integer sequence a_n s.t. $a_n \xi_n \xrightarrow{\mathbb{P}} 0$.

   <u>Solution</u>:

5. *Page 47,24* Prove two theorems 2.49 and 2.50.

Theorem 2.49 If $\xi_n - \xi_n' \xrightarrow{\mathbb{P}} 0$ and $\xi_n' \xrightarrow{d} \xi$, then $\xi_n \xrightarrow{d} \xi$.

Theorem 2.50 If $\xi_n \xrightarrow{d} \xi, \eta_n \xrightarrow{d} a$(const), then $\xi_n + \eta_n \xrightarrow{d} \xi + a$.

   <u>Solution</u>: