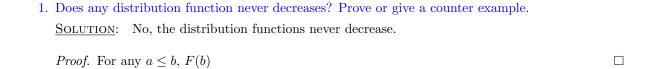
## Probability Theory HW4

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2. The measure which has finite value on finite interval of  $(\mathbb{R}^n, \mathscr{B}^n)$  is called L-S measure. Prove that every L-S measure is a Lebesgue-Stieljes measure generated by some distribution function.

SOLUTION: Set  $\mu$  an L-S measure and construct the function F on  $\mathbb{R}^n$ . Let F(0)=0,  $F(x)=\mu([0,x])$  for  $x\geq 0$ . For other  $x=(x_1,x_2,\cdots,x_n)\in\mathbb{R}^n$ , let  $\bar{x}=(|x_1|,|x_2|,\cdots,|x_n|)\geq 0$ . Let  $F(x)=F(\bar{x})$  and we get a function on  $\mathbb{R}^n$ .

F is continue since  $\mu$  is continue for the endpoints. For  $a \leq b, \, \Delta_{b,a}F =$ 

3. If  $F(x) = \mathbb{P}(\xi < x)$  is continue, then  $\eta = F(\xi)$  has a uniform distribution on (0,1). Solution:

4. Are the characteristic functions and simple functions measurable? Prove or give counter examples.

Solution: For any characteristic function  $1_A$  where  $A \subset \Omega$ ,

$$\sigma(1_A) = \sigma(\{E(x) = \{1_A(\xi) < x \mid \xi \in \Omega\} \mid x \in \Omega\})$$
  
=  $\sigma(\{\varnothing, A^c, \Omega\}) = \sigma(\{A\}).$ 

Then  $\sigma(1_A) \subset \mathscr{A}$  if and only if  $A \in \mathscr{A}$ . The characteristic function  $1_A$  is measurable if and only if A is measurable.

For any simple function  $f = \sum_{k=1}^{n} a_k 1_{A_k}$ , assume  $a_1 \le a_2 \le \cdots \le a_n$ .

$$\sigma(\sum_{k=1}^{n} a_k 1_{A_k}) = \sigma(\{E(x) = \{\sum_{k=1}^{n} a_k 1_{A_k}(\xi) < x \mid \xi \in \Omega\} \mid x \in \Omega\})$$

$$\subset \sigma(\{\varnothing, A_1, A_1 \cup A_2, \cdots, \sum_{k=1}^{n} A_k\})$$

$$\subset \mathscr{A}.$$

Since every  $A_k, k=1,2,\cdots,n$  is in  $\mathscr{A}, \sigma(f)\subset \mathscr{A}$  is measurable.