Algebra HW7

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1. *Page 148,1* Determine the complete ring of quotients of the ring Z_n for each $n \geq 2$.

   Solution: For any $r \in R$, $i, j \in J$, $k, l \in K$,

$$\begin{aligned} (ri)a &= r(ia) = r0 = 0, \\ (i-j)a &= ia - ja = 0 - 0 = 0; \\ a(kr) &= (ak)r = 0r = 0, \\ a(k-l) &= ak - al = 0 - 0 = 0. \end{aligned}$$

   Thus $ri \in J, i - j \in J$, J is a left ideal; $kr \in K, k - l \in K$, K is a right ideal. □

2. *Page 133,3*

   (a) The set E of positive even integers is a multiplicative subset of Z such that $E - I(Z)$ is the field of rational numbers.

   (b) State and prove condition(s) on a multiplicative subset S of Z which insure that S-IZ is the field of rationals.

   Solution:

   (a) For any $a, b \in E$, ab is still positive and even. Then E is a multiplicative subset of Z.

   □

3. *Page 148,5* Let R be an integral domain with quotient field F. If T is an integral domain such that $R \subset T \subset F$, then F is (isomorphic to) the quotient field of T.

   Solution:

4. *Page 148,8* Let R be a commutative ring with identity, I an ideal of R and $\pi : R \to R/I$ the canonical projection.

   (a) If S is a multiplicative subset of R, then $\pi S = \pi(S)$ is a multiplicative subset of R/I.

   (b) The mapping $\theta : S^{-1}R \to (\pi S)^{-1}(R/I)$ given by $r/sf \mapsto \pi(r)/\pi(s)$ is a welldefined function.

   (c) θ is a ring epimorphism with kernel $S^{-1}I$ and hence induces a ring isomorphism $S^{-1}R/S^{-1}I \cong (\pi S)^{-1}(R/I)$.

   Solution:

5. *Page 148,10* Let R be an integral domain and for each maximal ideal M (which is also prime, of course), consider R_M as a subring of the quotient field of R. Show that $\cap R_M = R$, where the intersection is taken over all maximal ideals M of R.

   Solution: