Probability Theory HW4

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1. Does any distribution function never decreases? Prove or give a counter example.

   SOLUTION: No, the distribution functions never decrease.

   Proof. For any $a \leq b$, $F(b)$ □

2. The measure which has finite value on finite interval of $(\mathbb{R}^n, \mathscr{B}^n)$ is called L-S measure. Prove that every L-S measure is a Lebesgue-Stieljes measure generated by some distribution function.

   SOLUTION: Set μ an L-S measure and construct the function F on $\mathbb{R}^n$. Let $F(0) = 0$, $F(x) = \mu([0, x])$ for $x \geq 0$. For other $x = (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$, let $\bar{x} = (|x_1|, |x_2|, \cdots, |x_n|) \geq 0$. Let $F(x) = F(\bar{x})$ and we get a function on $\mathbb{R}^n$.

   F is continue since μ is continue for the endpoints. For $a \leq b$, $\Delta_{b,a} F =$

3. If $F(x) = \mathbb{P}(\xi < x)$ is continue, then $\eta = F(\xi)$ has a uniform distribution on $(0, 1)$.

   Solution:

4. Are the chararcteristic functions and simple functions measurable? Prove or give counter examples.

   Solution: For any chararcteristic function 1_A where $A \subset \Omega$,

$$\begin{aligned}\sigma(1_A) &= \sigma(\{E(x) = \{1_A(\xi) < x \mid \xi \in \Omega\} \mid x \in \Omega\}) \\ &= \sigma(\{\varnothing, A^c, \Omega\}) = \sigma(\{A\}).\end{aligned}$$

Then $\sigma(1_A) \subset \mathscr{A}$ if and only if $A \in \mathscr{A}$. The chararcteristic function 1_A is measurable if and only if A is measurable.

For any simple function $f = \sum_{k=1}^{n} a_k 1_{A_k}$, assume $a_1 \leq a_2 \leq \cdots \leq a_n$.

$$\begin{aligned}\sigma(\sum_{k=1}^{n} a_k 1_{A_k}) &= \sigma(\{E(x) = \{\sum_{k=1}^{n} a_k 1_{A_k}(\xi) < x \mid \xi \in \Omega\} \mid x \in \Omega\}) \\ &\subset \sigma(\{\varnothing, A_1, A_1 \cup A_2, \cdots, \sum_{k=1}^{n} A_k\}) \\ &\subset \mathscr{A}.\end{aligned}$$

Since every $A_k, k = 1, 2, \cdots, n$ is in $\mathscr{A}$, $\sigma(f) \subset \mathscr{A}$ is measurable.