Assignment - 2

1. k_1, k_2 are valid kernels

(a) $\Rightarrow \int f(x) k_1(x,z) f(z) dx \cdot dz \geq 0$ — ① & $\int f(x) k_2(x,z) f(z) dx \cdot dz \geq 0$ — ②

for any $f(x)$ s.t $\int f^2(x)$ is finite

① + ②

$\Rightarrow \int f(x) \left(k_1(x,z) + k_2(x,z)\right) f(z) dx \cdot dz \geq 0$

$\Rightarrow \int f(x)\, k(x,z) \cdot f(z) \cdot dx \cdot dz \geq 0$

for any $f(x)$ s.t $\int f^2(x)$ is finite

$\Rightarrow$ k is valid kernel

(b) $k(x,z) = k_1(x,z) \cdot k_2(x,z)$

k_1, k_2 are valid kernels

$\Rightarrow$ $k_1(x,z) = \phi_1^T(x)\, \phi_1(z)$ — let ϕ_1 produce m dimensional vector,

$k_2(x,z) = \phi_2^T(x) \cdot \phi_2(z)$ — ϕ_2 produce n dimensional vector

$$k_1(x,z) \cdot k_2(x,z) = \left(\sum_{i=1}^{m} \phi_1(x)_i \phi_1(z)_i\right)\left(\sum_{j=1}^{n} \phi_2(x)_j \cdot \phi_2(z)_j\right)$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\phi_1(x)_i \phi_2(x)_j\right) \cdot \left(\phi_1(z)_i \cdot \phi_2(z)_j\right)$$

$$= \phi_3(x)^T \cdot \phi_3(z)$$

(ϕ_3 - produces m·n dimensional vector)

$\Rightarrow$ k is valid kernel

(c) $k(x,z) = h(k_1(x,z))$

h - polynomial function with positive coeffs

$= \sum_{i=0}^{n} a_i k_1^{i}$ (form)

from (b), k_1^{i} is valid (i.e. replacing k_2 with k_1) - provided k_1 is valid

from (a), $a_i k_1$ is valid - provided k_1 is valid

$\Rightarrow$ polynomial function is also valid

$\Rightarrow$ k is valid kernel

(d) $k(x,z) = \exp(k_1(x,z))$

$\exp(x) = \lim_{n \to \infty} \left(1 + x + \cdots + \frac{x^n}{n!}\right)$

$\downarrow$

This is same as a polynomial function

$\Rightarrow$ from (c), $k(x,z)$ is a valid kernel.

(e) $k(x,z) = \exp\left(-\frac{\|x-z\|_2^2}{\sigma^2}\right)$

$= \exp\left(\frac{-\|x\|_2^2 - \|z\|_2^2 + 2x^T \cdot z}{\sigma^2}\right)$

$= \underbrace{\exp\left(\frac{-\|x\|_2^2}{\sigma^2}\right) \cdot \exp\left(\frac{-\|z\|^2}{\sigma^2}\right)}_{\downarrow} \cdot \underbrace{\exp\left(\frac{2x^T z}{\sigma^2}\right)}_{\downarrow}$

~~valid~~ Kernel say $k_2(x,z)$

It is of the form $\exp(k_1(x,z))$ $\Rightarrow$ valid from d

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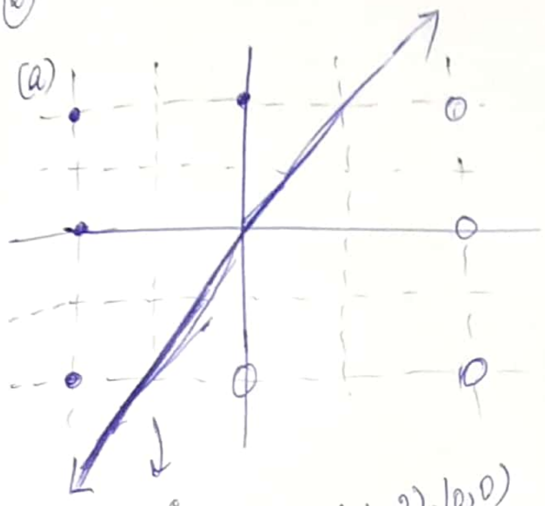
$\Rightarrow k(x,z) = k_2(x,z) \cdot \exp(k_1(x,z))$

$\Rightarrow k$ is valid (from (b))

(2)

(a)



passing through $(1,2), (-1,-2), (0,0)$

w_1, w_2 are weights

b - bias

$w_1 x + w_2 y + b = 0$: Equation of line

$\Rightarrow w_1(0) + w_2(0) + b = 0$

$w_1(1) + w_2(2) + b = 0$

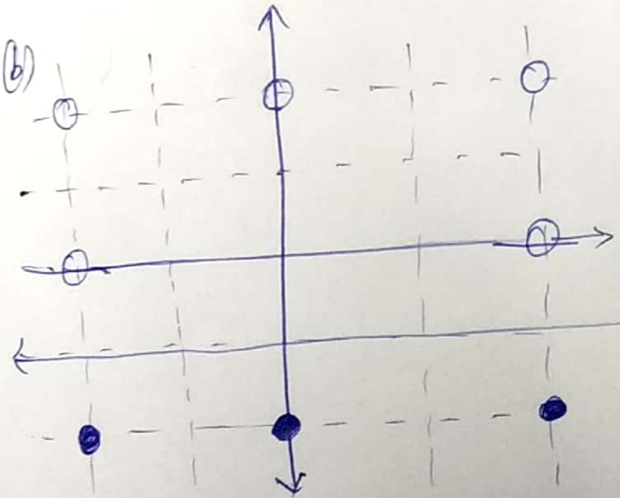
$w_1(-1) + w_2(-2) + b = 0$

$\Rightarrow b = 0$

$w_1 = 2 ; w_2 = -1$

$b = 0, [w_1 \ w_2] = [2 \ -1]$

(b)



decision boundary $(y = -1)$

$w_1 x + w_2 y + b = 0$

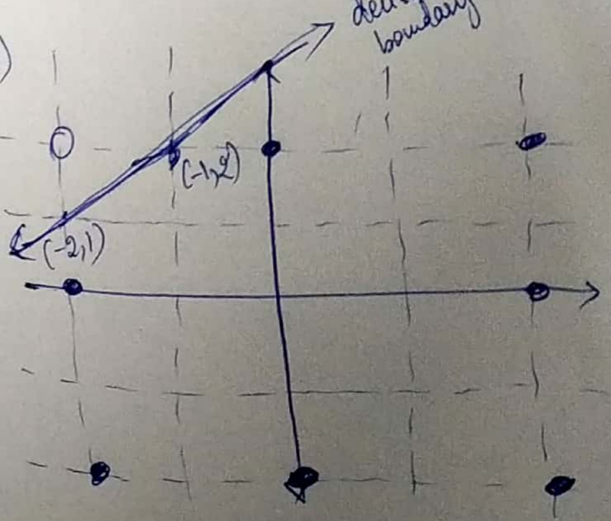
$w_1 = 0$

$w_2 = 1$

$b = 1$

$\Rightarrow b = 1 ; [w_1 \ w_2] = [0 \ 1]$

(c)



$w_1 x + w_2 y + b = 0$

$-w_1 + 2w_2 + b = 0$

$-2w_1 + w_2 + b = 0$

line equation:

$y = x + 3$

$x - y + 3 = 0$

$w_1 x + w_2 y + b = 0$

$b = 3, w_1 = 1 ; w_2 = -1$

$\therefore b = 3 ; [w_1 \ w_2] = [1 \ -1]$

(3) $x_1 = [1 \ \ 1]$; $y_1 = 1$

$x_2 = [1 \ \ -1]$; $y_2 = -1$

$$\text{Error } (E) = \sum_{\forall i} \frac{1}{2}(d_i - y)^2$$

$$= \frac{1}{2}[w_1 + w_2 - 1]^2 + \frac{1}{2}[w_1 - w_2 + 1]^2$$

$$= w_1^2 + w_2^2 - 2w_2 + 1$$

(a) Error ~~of~~ surface - convex

Minimum at $w_2 = 1$, $w_1 = 0$

(b) Hessian of $E = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$\frac{\partial E}{\partial w_1} = 2w_1 \qquad \frac{\partial^2 E}{\partial w_1^2} = 2$$

$$\frac{\partial E}{\partial w_2} = 2w_2 - 2 \qquad \frac{\partial^2 E}{\partial w_2^2} = 2$$

$$\frac{\partial^2 E}{\partial w_1 \partial w_2} = 0 \ ; \ \frac{\partial^2 E}{\partial w_2 \partial w_1} = 0$$

Eigen values $= (2, 2)$

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