RLT: Moody's Analytics ESG Model and Calibration Training

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School of Mathematics

Session 2019

Introduction

Assignment 1 : Risk Neutral Valuation

- Quick view of generating scenario
- Implement the answer of the Assignment 1

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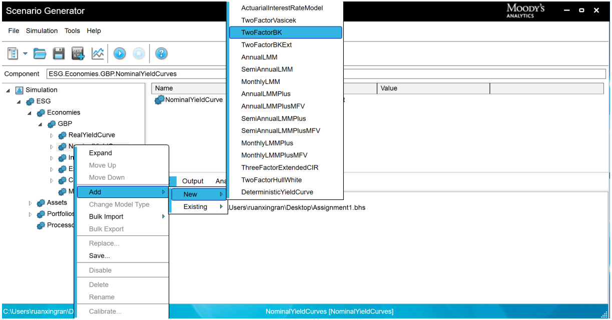
Quick view of generating scenario

GBP-TwoFactorBK Model



Quick view of generating scenario

GBP-TwoFactorBK Model

- The risk-neutral process for the short rate r is :

$$d\log(r) = \alpha_1[\log(m) - \log(r)]dt + \sigma_1 dZ_1$$

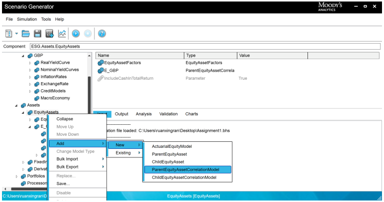
- Where the mean-reversion level m follows the stochastic process :

$$d\log(m) = \alpha_2[\mu - \log(m)]dt + \sigma_2 dZ_2$$

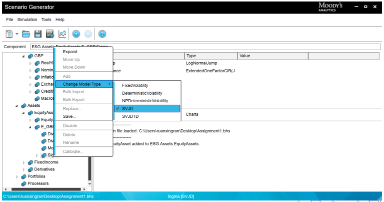
Quick view of generating scenario

Equity Asset-Parant equity correlation model and SVJD

- Using Parant equity correlation model



- Using SVJD model



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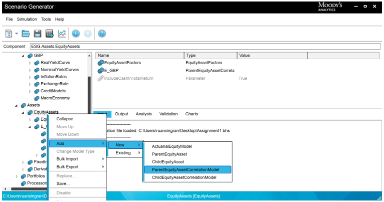

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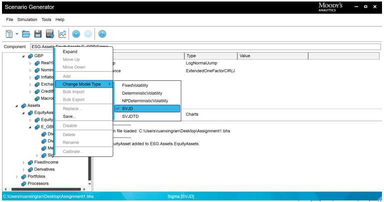

Quick view of generating scenario

Equity Asset-Parant equity correlation model and SVJD

- The log-return of asset $P(t)$ is :

$$\log\left(\frac{P(t+\Delta t)}{P(t)}\right) = \left(r(t) + \mu_p - \pi_p - \frac{1}{2}\sigma_p^2\right)\Delta t + \sigma_p Z_p(t)\sqrt{\Delta t} + CPP(\Delta t)$$

  where,
  $CPP(\Delta t) = \sum_{f=1}^{F} \beta_f \sum_{i=1}^{N_f(\Delta t)} \log(J_{f,i}) + \sum_{k=1}^{N_s(\Delta t)} \log(J_{s,k})$

- $v_t (volatility^2)$ follows a mean-reverting Stochastic Differential Equation :

$$dv_t = \alpha(\mu - v_t)d_t + \sigma_v \sqrt{v_t} dW_t$$

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Implement the answer of the Assignment 1

The payoff structure of the portfolio

- One zero coupon bond paying 10000 in 10 years
- 10 years European call option on UK Equity with strike price of 7000
- 10 years European put option on UK Equity with strike price of 5000

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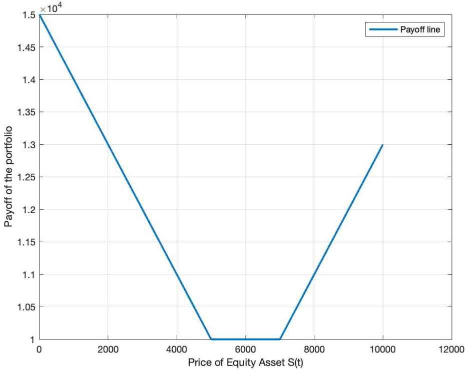
The payoff structure of the portfolio

$$Payoff = 10000 + (5000 - S)_{S \leq 5000} + (S - 7000)_{S \geq 7000}$$



Implement the answer of the Assignment 1

Calculate the market consistent value of the portfolio

- In terms of martingale X_t, it follows :

$$X_0 = E(X_t | F_0)$$

- The market consistent value of portfolio is :

$$P_0 = E(\frac{P_T}{B(T)} | X_0 = 5000) = 12066.67$$

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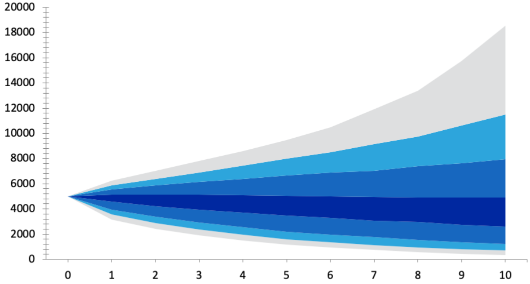
Estimate the 95% confidence interval for the value of the portfolio

- The graph of price of Equity Asset :



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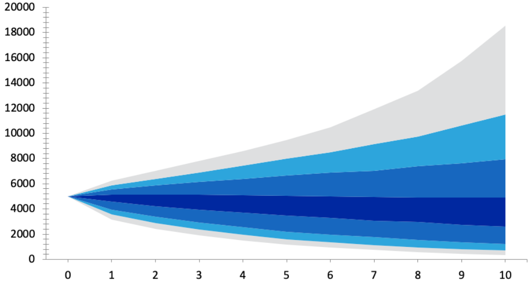

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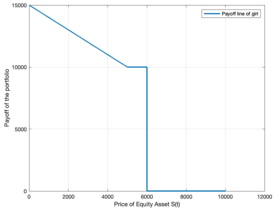
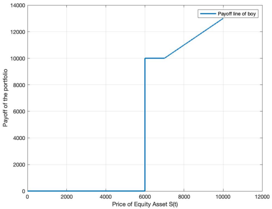
Payoff structure of each child

- The graph payoff structure of each child :





- The market consistent value :
  boy :3989.569.
  girl :8077.096.

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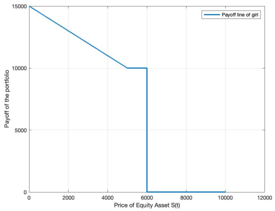

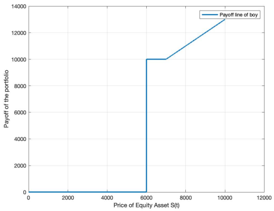

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- Waste computing resource.
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Assignment 2 : Value at Risk

- Main difference between Real World and Market Consistent
- Value at Risk theory
- Implement the answer of the Assignment 2 part 1

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Real World Model

- Two Economics : GBP and EUR
- Two Equities : E-GBP and E-EUR
- Two Govt Bonds(annually) : GBP-GOV-30ys-0 and EUR-GOV-30ys-0
- Two Risky Bonds : GBP-BBB-30ys-2 (semiannually) and EUR-AAA-30ys-5 (annually)
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Difference between RW and MC

Parent equity asset

- Parent Equity Asset :
  Parent equity assets are addable models. Correlation of these assets is done via the equity asset factor loadings.

Difference between RW and MC

Difference

- Difference between Parent Equity Asset and Parent Equity Asset Correlation Model :
  Depend on whether they are correlated using an equity asset factor model or the correlation matrix.
  The fundamental difference between the two types, is that the former splits risk into systematic risk driven by factor loadings, and specific risk driven by the equity asset volatility model. Whereas the correlated version of the equity assets does not make this distinction, and all risk is driven by the equity asset volatility model.

VaR Theory

- From Financial Risky Theory course, VaR is a measure of the risk of loss for investments, i.e. 0.995 VaR is 0.5 means means that there is a 0.05 probability that the portfolio will increase in value by less than 0.5.
- The definition of Upper-Quantiles(1) :
  For $\alpha \in (0, 1)$, the number $q^{\alpha}(X) = \inf\{x : \alpha < F_X(x)\}$, is called the upper-$\alpha$-quantile of X.
- The definition of VaR(1) :
  For $\alpha \in (0, 1)$, we define the VaR of X, at confidence level 1-α, as : $\text{VaR}^{\alpha}(X) = -q^{\alpha}(X) = -\inf\{x : \alpha < F_X(x)\}$.

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VaR Theory

- In usual case :
  We now consider an investment where at time zero we buy x shares of risky assets and y units of the risk-free asset, and we use $V_{(x;y)}(t)$to denote the value of the portfolio at time t, with $X_{(x;y)}$ to denote the value of the portfolio.
  Thus, VaR is(1) :
  $\mathrm{VaR}^{\alpha}(X_{(x;y)})=V_{(x;y)}(0)\text{-}x\mathrm{e}^{-rT}q^{\alpha}(\mathrm{S(T)})\text{-}y$
- In my case, I decide to analyse VaR from Macro way, i.e. from the 0.5 percentage of lower bound in Total Return Index plot.

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- In my case, I decide to analyse VaR from Macro way, i.e. from the 0.5 percentage of lower bound in Total Return Index plot.

Implement the answer of the Assignment 2 part 1

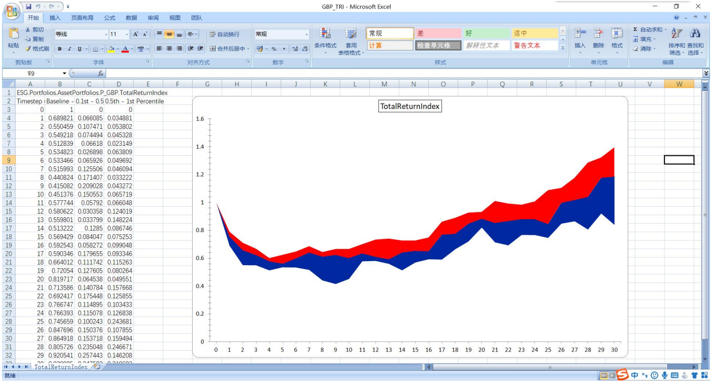
VaR change

- We observe the changes of VaR during these conditions :
  1. Assets' weights in the portfolio : (equity :gov bond :risky bond)
  (based) 0.45 :0.1 :0.45 ; 0.25 :0.5 :0.25 ;
  2. Different random number of seed :
  (based) 1 ; 100 ; 10000 ;
  3. Different numbers of trails :
  100 ; (based) 1000 ; 10000 ;
  4. Different Economic :
  GBP ; EUR.

Implement the answer of the Assignment 2 Part 1

Conclusion

- The graph of 0.45 :0.1 :0.45 is :

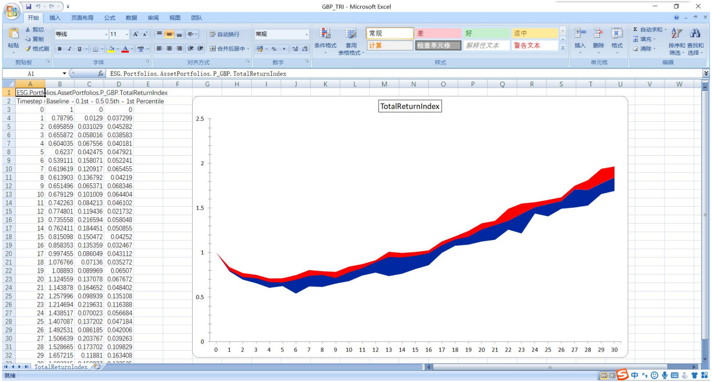


	A	B	C	D
1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep (	Baseline - 0.1st - 0.5	0.5th - 1st Percentile	
3	0	1	0	0
4	1	0.689821	0.066085	0.034881
5	2	0.550459	0.107471	0.053802
6	3	0.549218	0.074494	0.045328
7	4	0.512839	0.06618	0.023149
8	5	0.534823	0.026898	0.063809
9	6	0.533466	0.065926	0.049692
10	7	0.515993	0.125506	0.046094
11	8	0.440824	0.171407	0.033222
12	9	0.415082	0.209028	0.043272
13	10	0.451376	0.150553	0.065719
14	11	0.577744	0.05792	0.066048
15	12	0.580622	0.030358	0.124019
16	13	0.559801	0.033799	0.148224
17	14	0.513222	0.1285	0.086746
18	15	0.569429	0.084047	0.075253
19	16	0.592543	0.058272	0.099048
20	17	0.590346	0.179655	0.093346
21	18	0.664012	0.111742	0.115263
22	19	0.72054	0.127605	0.080264
23	20	0.819717	0.064538	0.049551
24	21	0.713586	0.140784	0.157668
25	22	0.692417	0.175448	0.125855
26	23	0.766747	0.114895	0.103433
27	24	0.766393	0.115078	0.126838
28	25	0.745659	0.100243	0.243681
29	26	0.847696	0.150376	0.107855
30	27	0.864918	0.153718	0.159494
31	28	0.805726	0.235048	0.246671
32	29	0.920541	0.257443	0.146208

Implement the answer of the Assignment 2 Part 1

Conclusion

- Change to 0.25 :0.5 :0.25 is :



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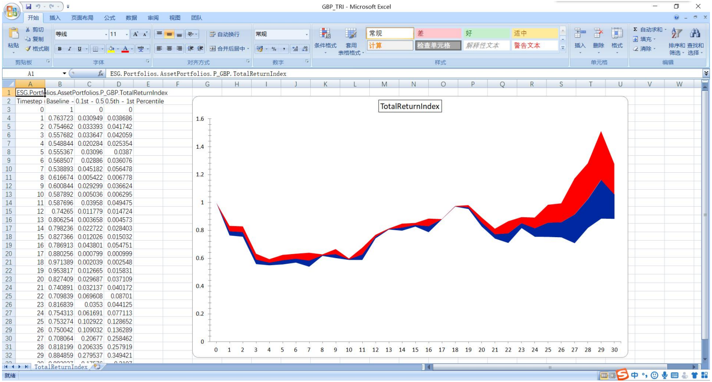
3	0	1	0	0
4	1	0.78795	0.0129	0.037299
5	2	0.695859	0.031029	0.045282
6	3	0.655872	0.058016	0.038583
7	4	0.604035	0.067556	0.040181
8	5	0.6237	0.042475	0.047921
9	6	0.539111	0.158071	0.052241
10	7	0.619619	0.120917	0.065455
11	8	0.613903	0.136792	0.04219
12	9	0.651496	0.065371	0.068346
13	10	0.679129	0.101009	0.064404
14	11	0.742263	0.084213	0.046102
15	12	0.774801	0.119436	0.021732
16	13	0.735558	0.216594	0.058048
17	14	0.762411	0.184451	0.050855
18	15	0.815098	0.150472	0.04252
19	16	0.858353	0.135359	0.032467
20	17	0.997455	0.086049	0.043112
21	18	1.076766	0.07136	0.035272
22	19	1.08893	0.089969	0.06507
23	20	1.124559	0.137078	0.067672
24	21	1.143878	0.164652	0.048402
25	22	1.257996	0.098939	0.135108
26	23	1.214694	0.219631	0.116388
27	24	1.438517	0.070023	0.056684
28	25	1.407087	0.137202	0.047184
29	26	1.492531	0.086185	0.042006
30	27	1.506639	0.203767	0.039263
31	28	1.528665	0.173702	0.109829
32	29	1.657215	0.11881	0.163408

Implement the answer of the Assignment 2 Part 1

Conclusion

- with 100 trails is :



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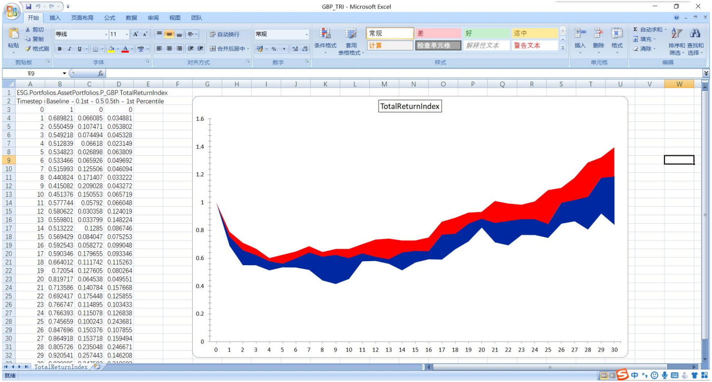

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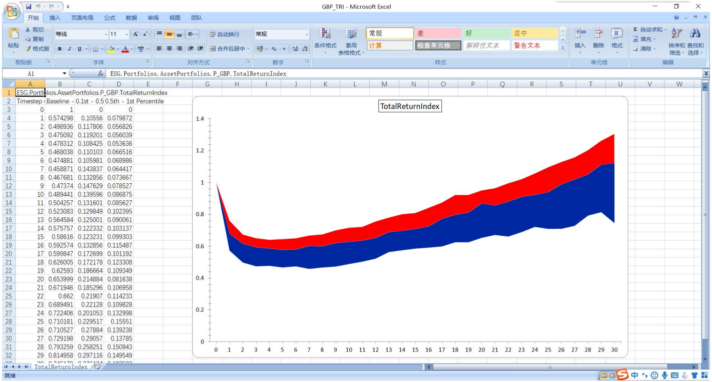
Implement the answer of the Assignment 2 Part 1

Conclusion

- with 10000 trails is :



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Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
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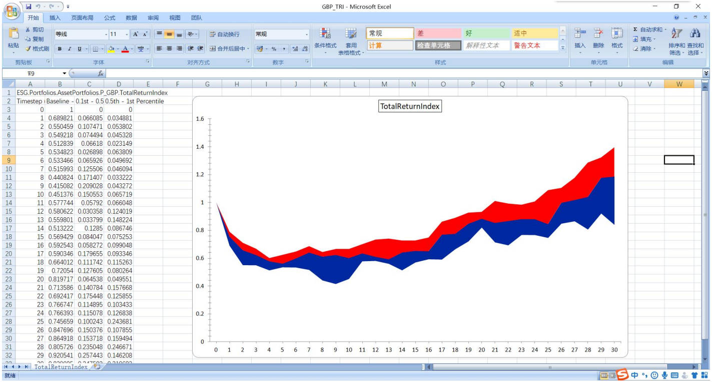
2	0.498936	0.117806	0.056826
3	0.475092	0.119201	0.056039
4	0.478312	0.108425	0.053636
5	0.468038	0.110103	0.066516
6	0.474881	0.105981	0.068986
7	0.458871	0.143837	0.064417
8	0.467681	0.132856	0.073667
9	0.47374	0.147629	0.078527
10	0.489441	0.139596	0.086875
11	0.504257	0.131601	0.085627
12	0.523083	0.129849	0.102395
13	0.564584	0.125001	0.090061
14	0.575757	0.122332	0.103137
15	0.58616	0.123231	0.099303
16	0.592574	0.132856	0.115487
17	0.599847	0.172699	0.101192
18	0.626005	0.172178	0.123308
19	0.62593	0.186664	0.109349
20	0.653999	0.214884	0.081638
21	0.671946	0.185296	0.106958
22	0.662	0.21907	0.114233
23	0.689491	0.22128	0.109828
24	0.722406	0.201053	0.132998
25	0.710181	0.229517	0.15551
26	0.710527	0.27884	0.139238
27	0.729198	0.29057	0.13785
28	0.793259	0.258251	0.150943
29	0.814958	0.297116	0.149549

Implement the answer of the Assignment 2 Part 1

Conclusion

- with 1 random seed is :

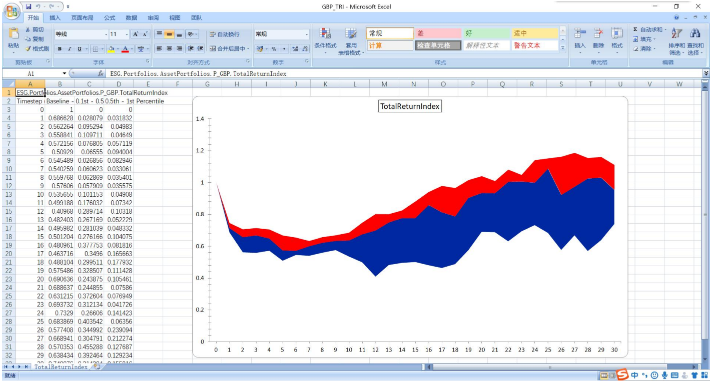


	A	B	C	D
1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep (	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.689821	0.066085	0.034881
5	2	0.550459	0.107471	0.053802
6	3	0.549218	0.074494	0.045328
7	4	0.512839	0.06618	0.023149
8	5	0.534823	0.026898	0.063809
9	6	0.533466	0.065926	0.049692
10	7	0.515993	0.125506	0.046094
11	8	0.440824	0.171407	0.033222
12	9	0.415082	0.209028	0.043272
13	10	0.451376	0.150553	0.065719
14	11	0.577744	0.05792	0.066048
15	12	0.580622	0.030358	0.124019
16	13	0.559801	0.033799	0.148224
17	14	0.513222	0.1285	0.086746
18	15	0.569429	0.084047	0.075253
19	16	0.592543	0.058272	0.099048
20	17	0.590346	0.179655	0.093346
21	18	0.664012	0.111742	0.115263
22	19	0.72054	0.127605	0.080264
23	20	0.819717	0.064538	0.049551
24	21	0.713586	0.140784	0.157668
25	22	0.692417	0.175448	0.125855
26	23	0.766747	0.114895	0.103433
27	24	0.766393	0.115078	0.126838
28	25	0.745659	0.100243	0.243681
29	26	0.847696	0.150376	0.107855
30	27	0.864918	0.153718	0.159494
31	28	0.805726	0.235048	0.246671
32	29	0.920541	0.257443	0.146208

Implement the answer of the Assignment 2 Part 1

Conclusion

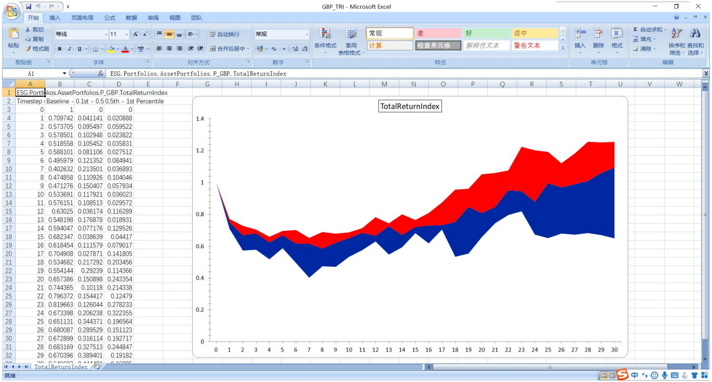
- with 100 random seed is :



Implement the answer of the Assignment 2 Part 1

Conclusion

- with 10000 random seed is :



ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
0	1	0	0
1	0.709742	0.041141	0.020888
2	0.573705	0.095497	0.059522
3	0.578501	0.102948	0.023822
4	0.518558	0.105452	0.035831
5	0.588101	0.081106	0.027512
6	0.495979	0.121352	0.084941
7	0.402632	0.213501	0.036893
8	0.474858	0.110926	0.104046
9	0.471276	0.150407	0.057934
10	0.533691	0.117921	0.036023
11	0.576151	0.108513	0.029572
12	0.63025	0.036174	0.116289
13	0.548198	0.176878	0.018931
14	0.594047	0.077176	0.129526
15	0.682347	0.038639	0.04417
16	0.618454	0.111579	0.079017
17	0.704908	0.027871	0.141805
18	0.534682	0.217292	0.203456
19	0.554144	0.29239	0.114366
20	0.657386	0.150898	0.243354
21	0.744365	0.10118	0.214338
22	0.796372	0.154417	0.12479
23	0.819663	0.126044	0.278233
24	0.673398	0.206238	0.322355
25	0.651131	0.344371	0.196564
26	0.680087	0.289529	0.151123
27	0.672899	0.316114	0.192717
28	0.683169	0.327513	0.244847
29	0.670396	0.389401	0.19182

Implement the answer of the Assignment 2 Part 1

Conclusion

- What could we conclude are :
  1. Along with time, the 0.995 VaR changes from negative to positive, which means that if you want this portfolio to earn money, it is better to hold for a long time ;
  2. Changing higher the risk-free asset means you could gain much more, but still you need to hold for a long time ;

Implement the answer of the Assignment 2 Part 1

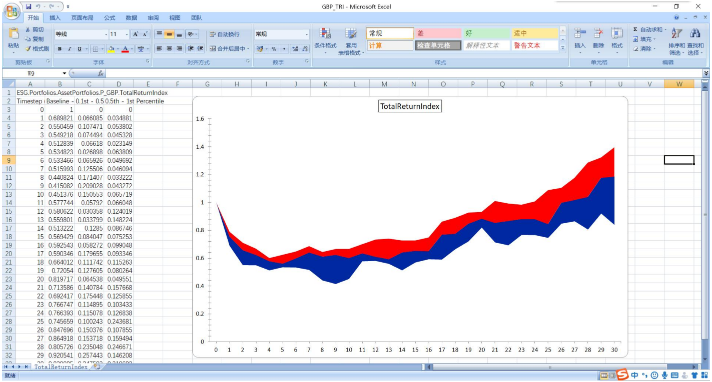
Conclusion

- What could we conclude are :
  3. More trails makes the plot more smoothly, which means during a certain range of value, it could get the point as large as possible, i.e. could simulate more suitable situations ;
  4. More random seeds makes the plot more fluctuating because the seed is an initial number of the pseudorandom number generator which is in fact fully deterministic. Larger numbers somehow returns a sequence of numbers that looks more random.

Implement the answer of the Assignment 2 Part 1

Conclusion

- Similarly, show the EUR plot in the same order :



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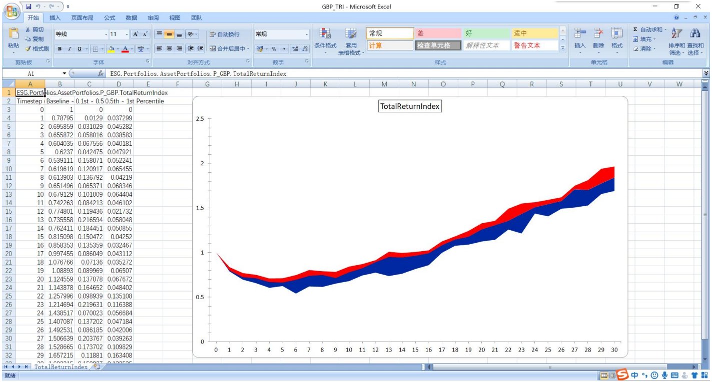
Implement the answer of the Assignment 2 Part 1

Conclusion

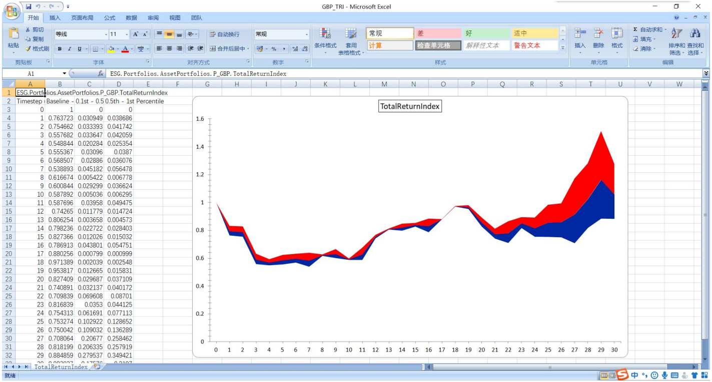
- Change to 0.25 :0.5 :0.25 is :



Implement the answer of the Assignment 2 Part 1

Conclusion

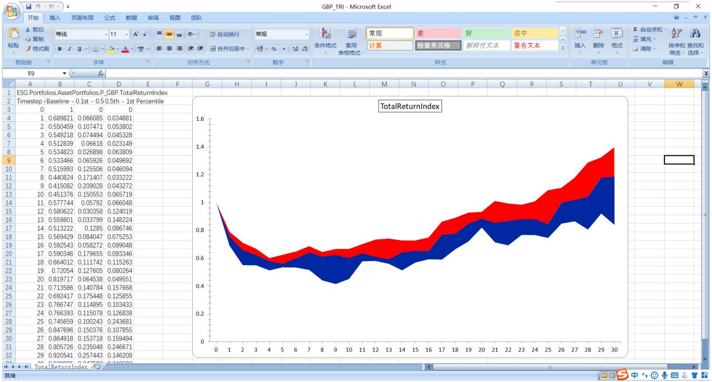
- With 100 trails is :



Implement the answer of the Assignment 2 Part 1

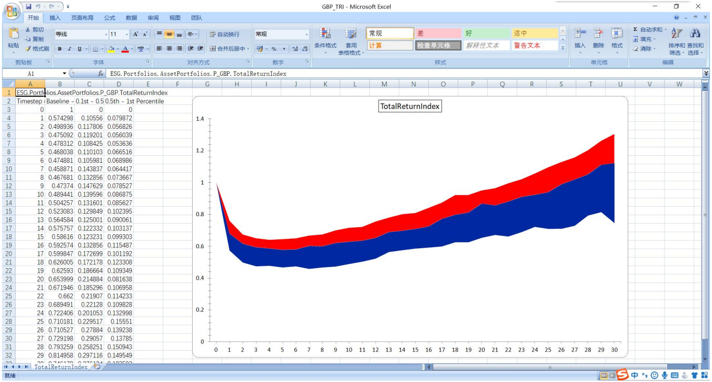
Conclusion

- With 1000 trails is :



Implement the answer of the Assignment 2 Part 1
Conclusion

- With 10000 trails is :



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1	ESG.Portfolios.AssetPortfolios.P_GBP.TotalReturnIndex			
2	Timestep	Baseline -	0.1st - 0.5	0.5th - 1st Percentile
3	0	1	0	0
4	1	0.574298	0.10556	0.079872
5	2	0.498936	0.117806	0.056826
6	3	0.475092	0.119201	0.056039
7	4	0.478312	0.108425	0.053636
8	5	0.468038	0.110103	0.066516
9	6	0.474881	0.105981	0.068986
10	7	0.458871	0.143837	0.064417
11	8	0.467681	0.132856	0.073667
12	9	0.47374	0.147629	0.078527
13	10	0.489441	0.139596	0.086875
14	11	0.504257	0.131601	0.085627
15	12	0.523083	0.129849	0.102395
16	13	0.564584	0.125001	0.090061
17	14	0.575757	0.122332	0.103137
18	15	0.58616	0.123231	0.099303
19	16	0.592574	0.132856	0.115487
20	17	0.599847	0.172699	0.101192
21	18	0.626005	0.172178	0.123308
22	19	0.62593	0.186664	0.109349
23	20	0.653999	0.214884	0.081638
24	21	0.671946	0.185296	0.106958
25	22	0.662	0.21907	0.114233
26	23	0.689491	0.22128	0.109828
27	24	0.722406	0.201053	0.132998
28	25	0.710181	0.229517	0.15551
29	26	0.710527	0.27884	0.139238
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Implement the answer of the Assignment 2 Part 1

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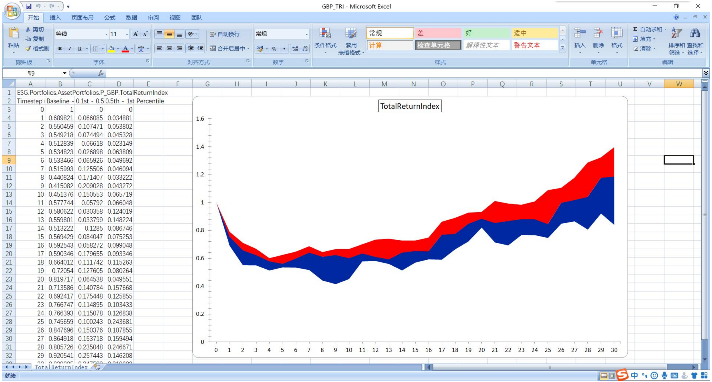

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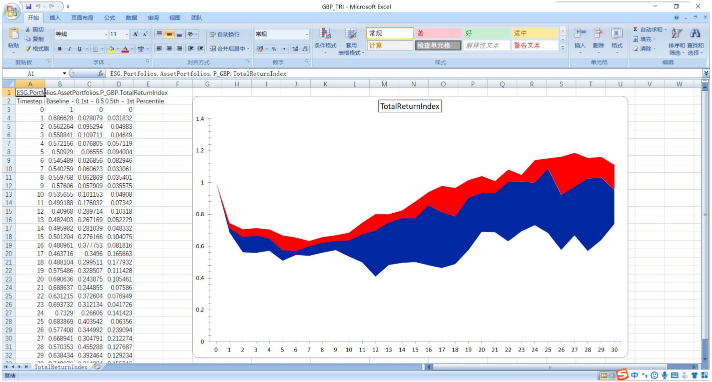
Implement the answer of the Assignment 2 Part 1

Conclusion

- With 100 random seeds is :



Implement the answer of the Assignment 2 Part 1

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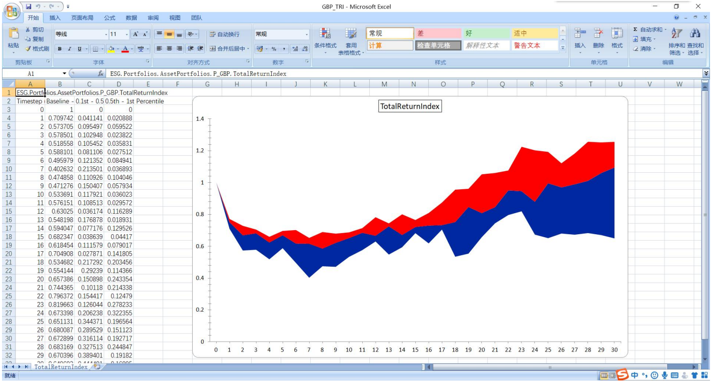

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Implement the answer of the Assignment 2 Part 1

Conclusion

- What could we conclude from EUR and GBP are :
  No marked difference. For each part compared in these two Economies, same trend means these two somehow have the similar market with each other, which could not be difficult to image from history.

Assignment 3 : Correlation Analysis

Correlation Analysis

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- Why we need **CORRELATION** ?
- Correlation in use
  Mr.Pearson

$$\rho = \frac{E(V_1, V_2) - E(V_1)E(V_2)}{SD(V_1)SD(V_2)}$$

correlation $\neq$ *dependence*

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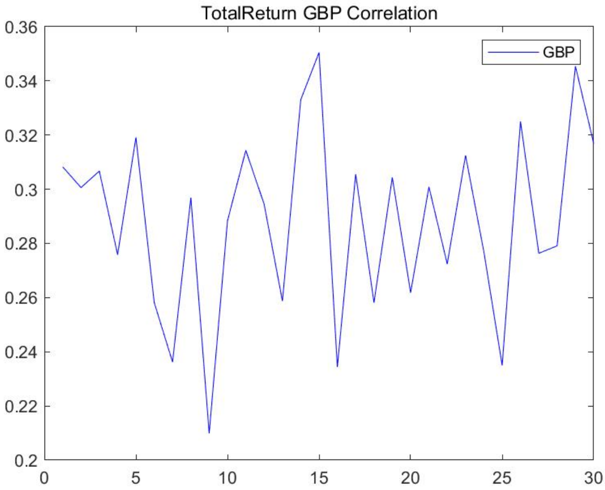

Vary Over Time – GARCH

- Log Return : Stochastic Process
  Equity asset : $\{X_t\}$; Risky Bonds :$\{Y_t\}$ where $t \in \{1, ..., 30\}$
- Reshaped Data : $X'_t = \frac{X_{t+1} - X_t}{X_t}$,$Y'_t = \frac{Y_{t+1} - Y_t}{Y_t}$
- Covariance rate of $\{X_t\}$,$\{Y_t\}$=Covariance of $\{X'_t\}$,$\{Y'_t\}$
- $Cov_n = EX_nY_n - E(X_n)E(Y_n)$
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Vary Over Time – GARCH

GARCH models can also be used for updating covariance rate estimates and forecasting the future level of covariance rates.For example,the GARCH(1,1) model for updating a covariance rate between X and Y is

- $COV_n = \omega + \alpha X_{n-1} Y_{n-1} + \beta COV_{n-1}$
- $Cor_n = \frac{COV_n}{SD(X_{n-1})SD(Y_{n-1})}$
- where $\{X_t\}$,$\{Y_t\}$ donate log return
- Fit the model with our data, and we got $\alpha and \beta : \alpha = -0.1839; \beta = 34.37.$

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- Because we focus on the majority of generated data, rather than extreme value(tail value)
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Tail Dependence

- Define :
  $$TailDependence(P : x, y) = \frac{JointTailEventCount(P;x,y)}{\sqrt{SingleTailEventCount(p;x)SingleTailEventCount(p;y)}}$$
- where p$\in (0, 100), donateauser - specifiedpercentage$
- SingleTailEventCount(p ;y)= $\{n : X_n \leqslant y(p)\}$
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- SingleTailEventCount(p ;y)= $\{n : X_n \leqslant y(p)\}$
- JointTailEventCount(P ;x,y) = $\{n : X_n \leqslant x(p) \quad and \quad Y_n \leqslant y(P)\}$

Tail Dependence

- Define :
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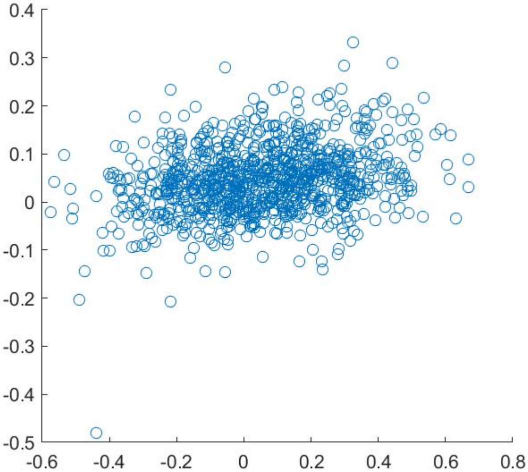
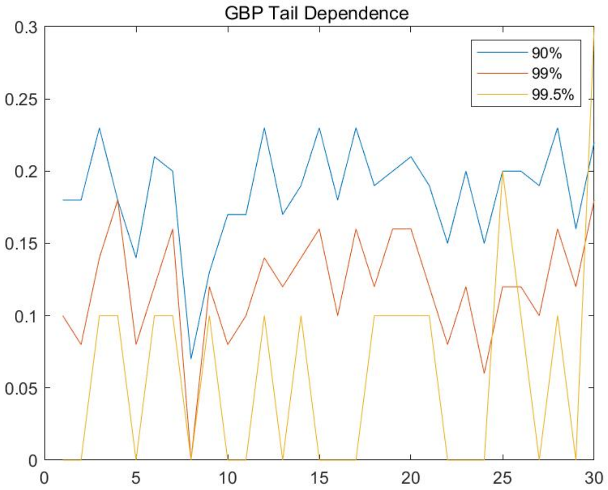
Tail Dependence

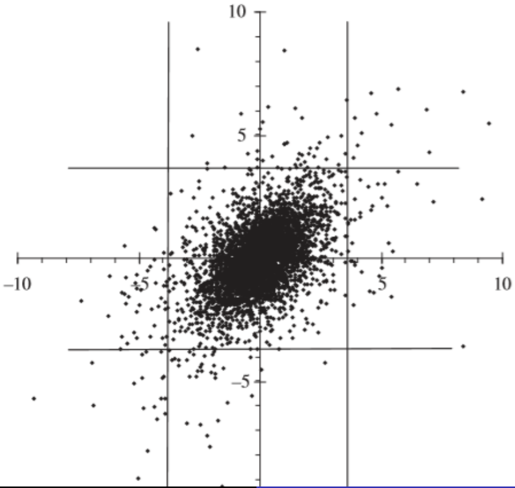


Tail Dependence



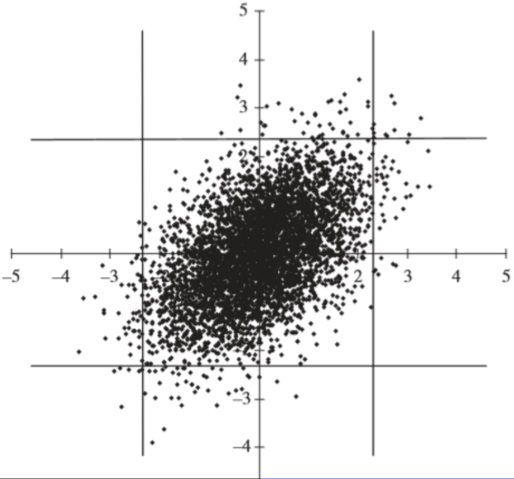
Copula

Student's t-distribution with mark tail dependence



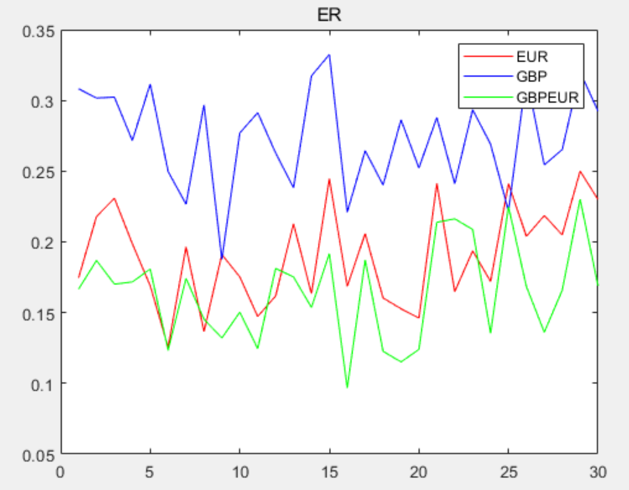
Copula

Gaussian distribution with slight tail dependence



ExcessReturn Anaysis

ExcessReturn Correlation



Conclusion

- We discuss the method to price the portfolio via Monte Carlo Simulation
- We use VaR to measure the risk of portfolio in real world
- We identify dependence between two assets via correlation, notice the weakness for model's plausibility, and suggest with some other tools, including copula, GARCH.

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Thanks

Thanks