Class - 2022/09/22

Review: the Matching Problem

1. If 4 married couple are arranged in a row, find the probability that no husband sits next to his wife

   Let A_i = couple i gets seated together

   $|S| = 8!$

   $P(A_1) = \dfrac{2*7!}{8!}$

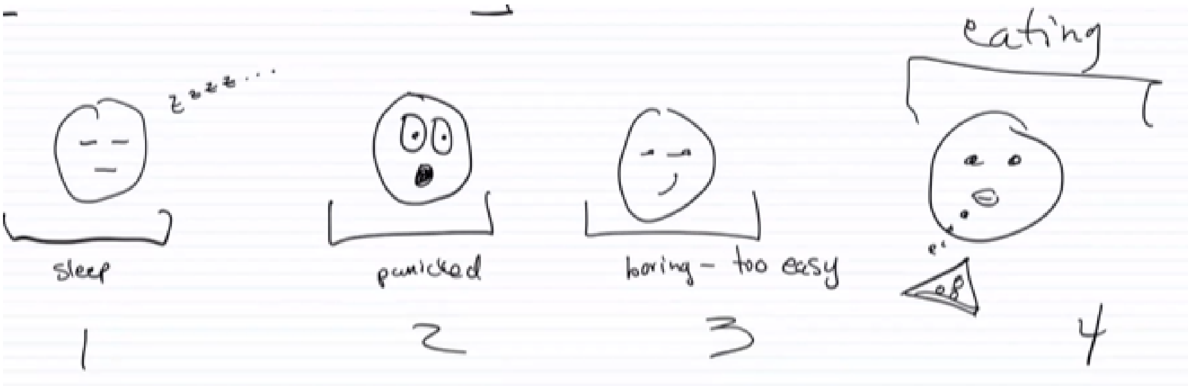
   $P(A_1A_2) = \dfrac{2*2*6!}{8!}$

   $P(A_1A_2A_3) = \dfrac{2*2*2*5!}{8!}$

   $P(A_1A_2A_3A_4) = \dfrac{2^4*4!}{8!}$

   $P(A_1 \cup A_2 \cup A_3 \cup A_4) = \binom{4}{1}\dfrac{2*7!}{8!} - \binom{4}{2}\dfrac{2*2*6!}{8!} + \binom{4}{3}\dfrac{2*2*2*5!}{8!} - \dfrac{2^4*4!}{8!}$

   $$\begin{aligned} P(\text{none}) &= 1 - P(\text{at least one match}) \\ &= 1 - P(A_1 \cup A_2 \cup A_3 \cup A_4) \end{aligned}$$



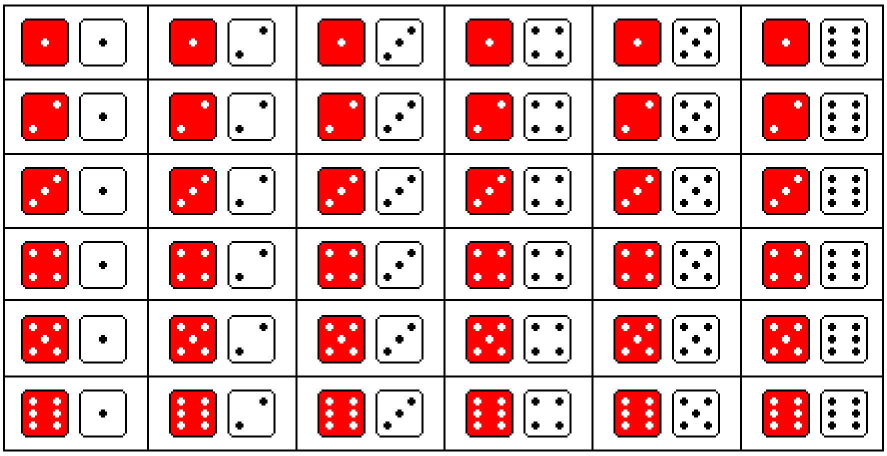
Chapter 3

Conditional Probability

Definition

- The probability of an event A given an event B
- Denoted $P(A|B)$

Example



- $A = \{\text{roll a sum of 4}\}$

  $P(A) = \frac{|A|}{|S|} = \frac{3}{36}$

- $B = \{\text{white dice is 1 or 2}\}$

  $P(B) = \frac{12}{36}$

- $P(A|B) = \frac{2}{12} \neq P(A)P(B)$
- $P(A|B) = \frac{P(AB)}{P(B)} = \frac{2/36}{12/36} = \frac{2}{12}$

Formula

$$P(A|B) = \frac{P(AB)}{P(B)}$$
$$P(B|A) = \frac{P(AB)}{P(A)}$$

$$P(AB) = P(B) \cdot P(A|B) = P(A) \cdot P(B|A)$$

Practice Problems

1. In an experiment to study the relationship of hypertension and smoking habits, the following data below are collected for 180 individuals,

	Nonsmokers	Moderate Smokers	Heavy Smokers
H	21	36	30
NH	48	26	19

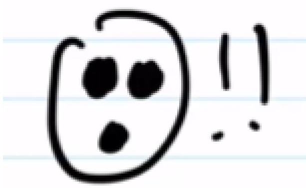
where H and NH in the table stands for *Hypertension* and *Non-hypertension* , respectively. If one of these individuals is selected at random, find the probability that the person is:

- (a) not experiencing hypertension, given that the person is a heavy smoker
- (b) a nonsmoker, given that the person is not experiencing hypertension

Answer:

- $P(H|HS) = \frac{P(H \cap HS)}{P(HS)} = \frac{30}{49}$
- $P(NS|NH) = \frac{48}{93}$

2. In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06, what is the probability that a person is diagnosed as having cancer?

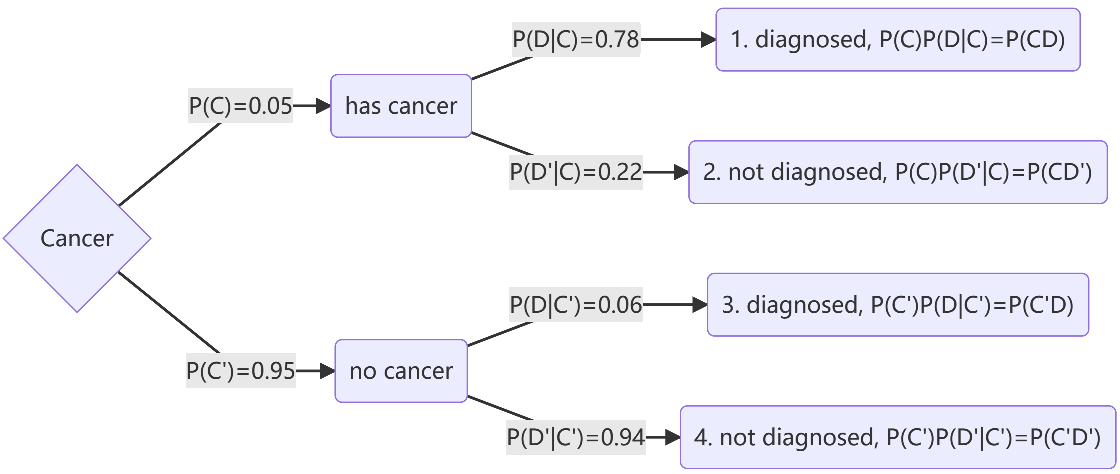


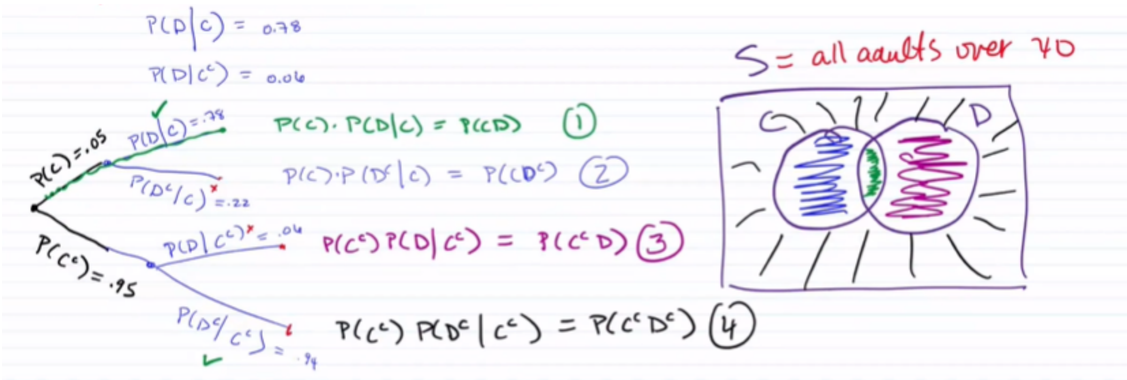
Answer:

- $C = \{\text{the person HAS cancer}\}$
  $P(C) = 0.05$
- $D = \{\text{the person is diagnosed with cancer}\}$
  $P(D) = ?$
- $P(D|C) = 0.78$

- $P(D|C^c) = 0.06$

Using a tree:





Therefore,

$$
\begin{aligned}
P(D) &= P(CD) + P(C^cD) \\
&= P(C)P(D|C) + P(C^c)P(D|C^c) \\
&= 0.05 * 0.78 + 0.95 * 0.06 \\
&= 0.096
\end{aligned}
$$

More questions:

1. What is the probability of an incorrect diagnosis?
   - $P(CD^c) + P(C^cD) = 0.05 * 0.22 + 0.95 * 0.06$
2. What is the probability you have cancer given you are diagnosed with it?
   - $P(C|D) = P(CD)/P(D) = 0.05 * 0.78/0.096 = 0.4$

3. In a small community, there is a polygamous man named Sammy. He is married to one percent of the women in the community. If you were raised to call Sammy "dad", then there is a 90 percent chance that he is your father. If you call someone else "dad", there is still a 5 percent chance that Sammy is your father. Given that Sammy is your father, what is the probability that you call him dad?

- $A =$ you call Sammy dad
- $B =$ Sammy is your dad
- $P(A) = 0.01$
- $P(B|A) = 0.9 = P(AB)P(A)$
- $P(B|A^c) = 0.05 = P(A^cB)P(A^c)$
- $P(A|B) = ?$

<u>Answer:</u>

- $P(A^c) = 0.99$
- $P(AB) = P(A)P(B|A) = 0.009$
- $P(A^cB) = P(A^c)P(B|A^c) = 0.0495$
- $P(B) = P(AB) + P(A^cB) = 0.0585$
- $P(A|B) = P(AB)/P(B) = 0.154$
- 15.4%

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)\cdot P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)}$$

B)

$$= \frac{(.01)(.9)}{(.01)(.9) + (.99)(.05)}$$

ard deck.

n that first card is not.