4.80

80. Let X be a continuous random variable with density function $f(x) = 2x$, $0 \leq x \leq 1$. Find the moment-generating function of X, $M(t)$, and verify that $E(X) = M'(0)$ and that $E(X^2) = M''(0)$.

4.91

91. Use the mgf to show that if X follows an exponential distribution, cX $(c > 0)$ does also.

5.1

1. Let $X_1, X_2, \ldots$ be a sequence of independent random variables with $E(X_i) = \mu$ and $\mathrm{Var}(X_i) = \sigma_i^2$. Show that if $n^{-2}\sum_{i=1}^{n} \sigma_i^2 \to 0$, then $\overline{X} \to \mu$ in probability.

5.10

10. A six-sided die is rolled 100 times. Using the normal approximation, find the probability that the face showing a six turns up between 15 and 20 times. Find the probability that the sum of the face values of the 100 trials is less than 300.

5.16

16. Suppose that $X_1, \dots, X_{20}$ are independent random variables with density functions

$$f(x) = 2x, \qquad 0 \leq x \leq 1$$

Let $S = X_1 + \cdots + X_{20}$. Use the central limit theorem to approximate $P(S \leq 10)$.

5.23

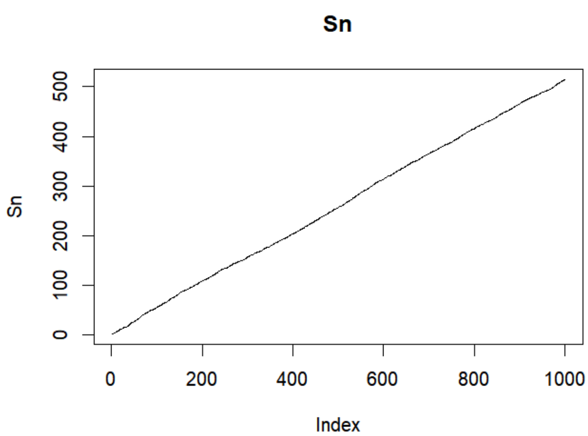
23. An irregularly shaped object of unknown area A is located in the unit square $0 \le x \le 1, 0 \le y \le 1$. Consider a random point distributed uniformly over the square; let $Z = 1$ if the point lies inside the object and $Z = 0$ otherwise. Show that $E(Z) = A$. How could A be estimated from a sequence of n independent points uniformly distributed on the square?

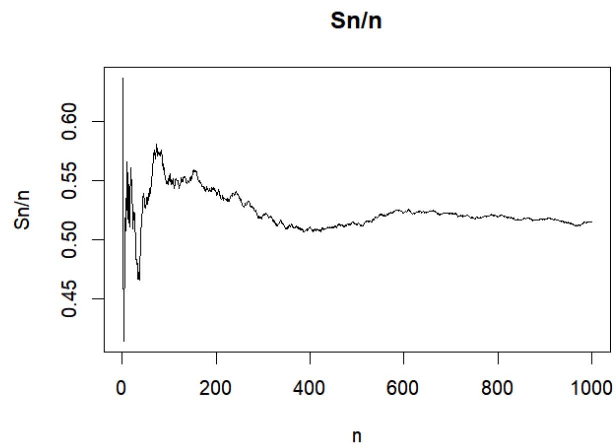
[R problem] 5.30

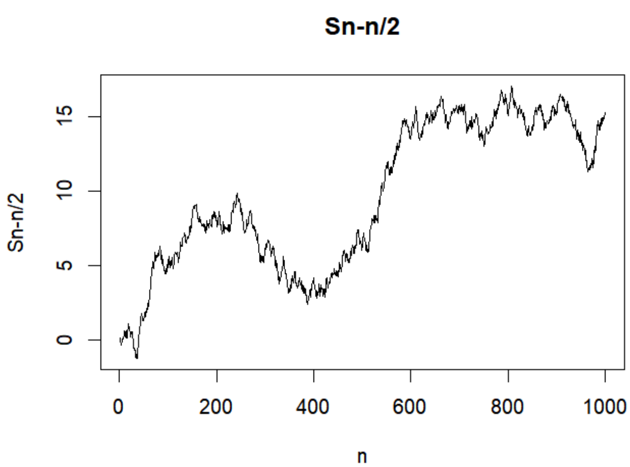
30. Generate a sequence $U_1, U_2, \ldots, U_{1000}$ of independent uniform random variables on a computer. Let $S_n = \sum_{i=1}^{n} U_i$ for $n = 1, 2, \ldots, 1000$. Plot each of the following versus n:

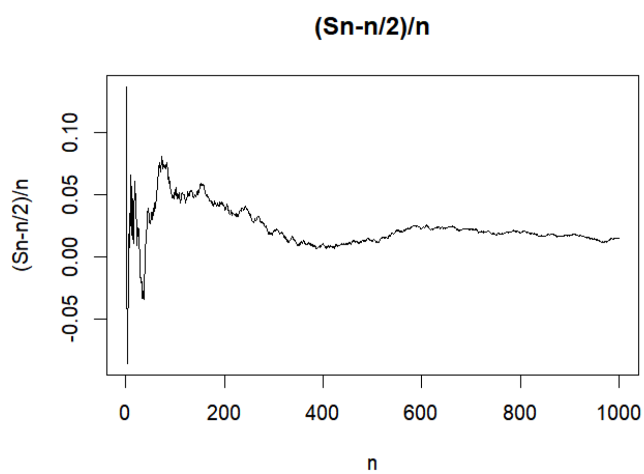
a. S_n
b. S_n/n
c. $S_n - n/2$
d. $(S_n - n/2)/n$
e. $(S_n - n/2)/\sqrt{n}$

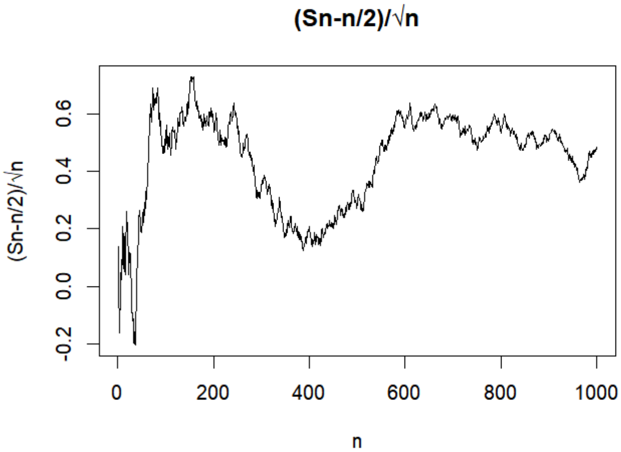
Explain the shapes of the resulting graphs using the concepts of this chapter.











(Sn-n/2)/√n
(Sn-n/2)/√n
-0.2
0.0
0.2
0.4
0.6
0
200
400
600
800
1000
n