Examples - Functional Data Analysis

Introduction to Functional Data Analysis

Example 1.1. Consider the dataset growth in library fda. Create a graph which shows observed discrete growth values for each person. Do the observed growth curves differ significantly in subpopulations of boys and girls?

Example 1.2. Consider the dataset CanadianWeather in library fda. Plot the daily average temperature values of the different weather stations.

Example 1.3. El Nino and La Nina are disruptions in the ocean-atmosphere system in the Tropical Pacific that cause changes in sea surface temperatures (SST's) and have important consequences for global weather and climate. El Nino is characterized by unusually warm SST's while La Nina has unusually cool SST's. Data for monthly SST (in degrees C) in various regions of the south Pacific ocean from 1950 to 2013 are available in the file ninoSST.csv. Using the NINO3 column, treat data for each calendar year as a functional observation.

The years of 1965, 1972, 1982, and 1997 were pronounced El Nino years. Emphasize these curves in the plot by making them thick red. Emphasize the pronounced La Nina years of 1955, 1973, 1975, 1988, and 1999 with thick blue.

Representing Functional Data

Example 2.1. Show that B-spline of order 3 with the knots at $0, 1, 2, 3$ has the form

$$B_3(t) = \begin{cases} \frac{t^2}{2}, & 0 \leq t \leq 1, \\ \frac{-2t^2+6t-3}{2}, & 1 \leq t \leq 2, \\ \frac{(3-t)^2}{2}, & 2 \leq t \leq 3. \end{cases}$$

Example 2.2. Construct B-spline basis of order 3 with the knots at $0, 1, 2, 3$ with R.

Example 2.3. Convert the growth data to functional objects by smoothing the data with B-spline basis, where $K = 12$, order 6, and knots with equal intervals. Do the smoothing separately for boys and girls, and use the least squares method to estimate coefficient vectors $\mathbf{c}_i$. Calculate also fitted values $\hat{\mathbf{y}}_i = \mathbf{H}\mathbf{y}_i$, and also consider calculation of the SSE value.

Example 2.4. Use smooth.basis function to convert the growth data to functional objects by smoothing the data with B-spline basis, where $K = 12$, order 6 and knots with equal intervals. Also consider first and second derivatives of the smooth curves.

Example 2.5. Convert the growth data to functional objects by smoothing the data with B-spline basis, where the order of the B-spline is 6, and knots are in observed point values. Do the smoothing separately for boys and girls, and use the penalized least squares method to estimate coefficient vectors $\mathbf{c}_i$ with roughness penalty being the square of the second derivative $[D^2 x_i(t)]^2$.