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Real GDP Estimator with Keynesian Regressors

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1 Introduction

1.1 Overview

Real GDP is a prominent measure of output due to its simplicity, interpretability, and effectiveness in communicating the overall activity of an economy. Alongside academic settings, Real GDP has retained its relevance in politics as one of the go-to measures in public communications regarding economic growth and prosperity. As per its wide-spread adoption, the measure has accumulated a plethora of literature referencing it from a multitude of perspectives.

This paper aims to show that empirical observations of Real GDP in the U.S. economy can be modelled optimally with a **Gauss–Markov** compliant $ARX(p, q)$ specification where the exogenous contributors are theoretical descriptors of output.

1.2 Hypothesis

In the history of economic equilibria, multiple models have proposed mathematically defined relationships between Real GDP and many indicators. From the Keynesian approach, two of said indicators are:

- Price levels (CPI) through the **Aggregate Demand Curve**
- Unemployment rates through **Okun’s Law**

The formal hypothesis of this paper is that an $ARX(p, q)$ representation of Real GDP, incorporating CPI and unemployment:

$$ARX_y(p, q) = C + \sum_{i=1}^p \beta_i y_{t-i} + \sum_{j=1}^q [\phi_j CPI_{t-j} + \gamma_j u_{t-j}] + \varepsilon_t \quad (1)$$

yields OLS parameter estimates that satisfy the **Gauss–Markov** conditions for time-series estimators.

1.3 Literature Review

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2 Data

The proposed ARX model relies on two exogenous variables alongside the autoregressive target. All three variables are sourced from the Federal Reserve Bank of St. Louis database “FRED”. The implementation uses quarterly data from 1990Q1 to 2025Q1 and the following FRED series:

- Real GDP (y): “GDPC1”
- CPI (CPI): “CPIAUCSL”
- Unemployment Rate (u): “UNRATE”

NOTE: All series are seasonally adjusted by FRED.

2.1 Series Characteristics

2.1.1 Real GDP

FRED’s Real GDP series is measured in chained 2017 dollars (Bn). The series is updated quarterly, which matches the frequency of the model natively.

2.1.2 CPI

The CPI series of choice for the model records the US-wide city average CPI for all urban consumers. The series is expressed as a chain index with base period 1982-1984=100. Entries are recorded monthly, and quarter-end observations are used to align with the Real GDP series.

2.1.3 Unemployment Rate

Unemployment rate in FRED is recorded as percentages with a monthly frequency. Similar to CPI, quarter-end observations are used to match the frequencies among the series.

3 Methodology

The proposed methodology of this paper can be split into three main components:

- Data Preparation and Examination
- Model Estimation
- BLUE Evaluation

Each component is detailed in the following subsections.

3.1 Data Preparation and Examination

Data preparation does not involve any steps that are specific to a given column of the dataset. Therefore, $X_{i,j}$ notation will be used to denote any given column/observation and β_i will be used to denote any given coefficient. In addition to formatting the X matrix to match the model structure, the preparation steps emphasize transforming the raw data for interpretability while preserving its underlying distributions.

3.1.1 Dataset Structure Validation

More often than not, the performance of a non-robust, linear estimator is governed by the quality of the input variables. Ideally, we look for a dataset that's **free of multicollinearity, extreme outliers, and missing values** while being **correlated to the target**. This general construction achieves a model where all variables have uniquely identifiable contributions to the output, often leading to lower variances and better predictive power.

Autoregressive models present a challenge in this regard since variables selected to be correlated to the target (y) are also correlated to the lagged representation of it within X . Acknowledging this issue, the endogenous portion of the model will only be checked to ensure there are no singularities by perfect correlation inside X . The presence/absence of multicollinearity will be displayed through a correlation heatmap.

Furthermore, data continuity will be prioritized over outlier removal since the model itself is time-dependent. Outliers, if any, will be addressed if they are found to be damaging to the model's performance after the initial estimation.

Finally, any missing values will be handled through linear interpolation of the observations prior and after the occurrence.

3.1.2 Design Matrix Formation

X is formed by converting the model equation specified in Equation 1 into matrix form. The inner product operation specified as $ARX(p, q)$ can be written as a matrix multiplication of observations and coefficients:

$$\begin{bmatrix} 1 & X_{1,1} & \cdots & X_{1,m} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & X_{n,1} & \cdots & X_{n,m} \end{bmatrix} \times \begin{bmatrix} C \\ \beta_1 \\ \vdots \\ \beta_m \end{bmatrix} \quad (2)$$

where n is the number of observations and m is the number of features (including the intercept) dictated by the $\{p, q\}$ orders of the model.

As shown in the above equation, a closed form matrix representation of the model requires p and q to be determined. In this paper, the selection of these hyperparameters will follow the simple and idealistic approach of selecting the lowest possible orders that lead to an efficient estimator.

3.1.3 Rescaling

Rescaling the input matrix (X) allows better comparison of coefficients and ensures that $\sum X^T \varepsilon = 0$ must hold within very tight tolerances¹. A common approach to achieve this stability is standardization, which transforms each feature to have a mean of 0 and a standard deviation of 1. Standardization is performed column-wise on the input using the formula:

$$Z = \frac{X_{i,j} - \bar{X}_i}{S_i}$$

where \bar{X}_i is the mean of the i -th column, and S_i is the [unbiased] standard deviation of the i -th column.

This transformation has no effect on the error profile of the fitted model as the distribution shapes and inter-variable interactions are perfectly preserved.² However, the transformation brings all variables to the same scale and units. As a result, the **Kolmogorov–Smirnov (KS) test** which is sensitive to such differences now becomes available.

3.2 Model Estimation

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3.3 BLUE Evaluation

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¹Differences in magnitude between X columns can lead to numerical ambiguity in orthogonality checks. It is generally preferable to eliminate any differences in the order of magnitudes in X .

²The statement holds for any unpenalized estimator of the form $\hat{\beta} = \arg \min_{\beta} L(y, X\beta)$, since rescaling X can be absorbed into a reparameterization of β .