A Polynomial Time Approximation Scheme for the Maximal Overlap of Two Independent Erdös-Rényi Graphs

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Based on a joint work with Jian Ding(PKU) and Hang Du(PKU)

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- **Random Graph Matching** is an extensively studied topic in recent years, which lies in the intersection of **probability, statistics and computer science**.

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- The motivations and prototypes of the problem come from various applied fields.

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- Sample two independent Erdös-Rényi graphs $G_1(n, p)$ and $G_2(n, p)$, where $p = n^{-\alpha}$.
- Find a **bijection** π between the two vertex set such that the number of **common edges** under π is maximized. Let $C(\pi)$ be the number of common edges under π. Formally,

$$C(\pi) := \sum_{i,j=1}^{n} G^{(1)}_{i,j} G^{(2)}_{\pi(i),\pi(j)}.$$

where $G^{(i)}$ is the adjacency matrix for G_i.

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- $\mathbb{P}$ the joint law of G_1 and G_2
- What is the typical behavior of $\mathrm{C}(\pi^*)$ under different α?
- A union bound on $\mathrm{C}(\pi^*)$ yields an **upper bound**,

$$\mathbb{P}\left[\mathrm{C}(\pi^*) > \gamma(n)\right] \le n!\mathbb{P}[\mathbf{B} > \gamma(n)] \text{ where } \mathbf{B} \sim \mathbf{Bin}\left(\binom{n}{2}, p^2\right).$$

But is this $\gamma(n)$ the right asymptotic?

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- For the critical case $\alpha = 1/2$, things become more subtle.

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Theorem (Ding-Du-G.' 22+)

For any constant $\epsilon > 0$, there exists $C = C(\epsilon)$ together with an $O(n^C)$-time algorithm, which takes G_1, G_2 as input and outputs some π^*, such that

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- This proves that the asymptotic for $\mathsf{C}(\pi^*)$ is typically $\frac{n}{2\alpha-1}$.

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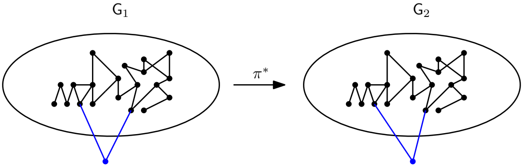

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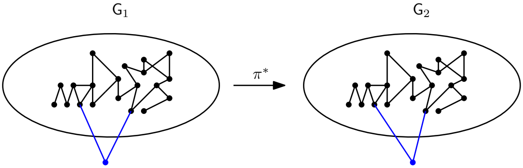

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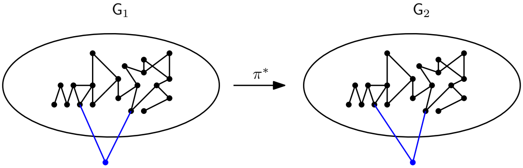

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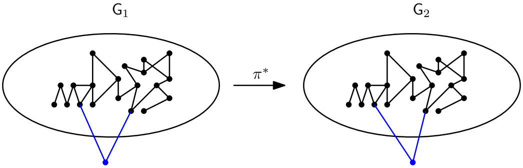

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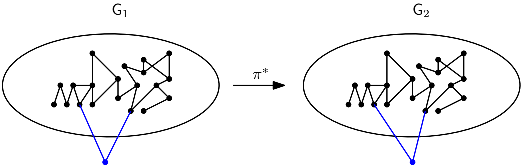
- For each pair (u, u') in $N_{u_k}(M_k)$, we try to find a $v \in \mathsf{G}_2$ s.t. $(\pi^*(u), v)$, $(\pi^*(u'), v) \in \mathsf{G}_2$.



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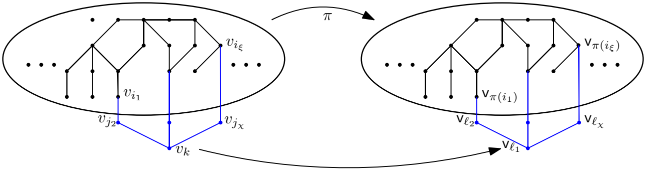

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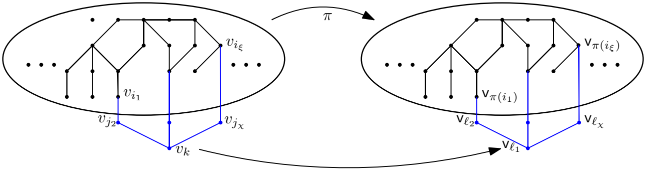

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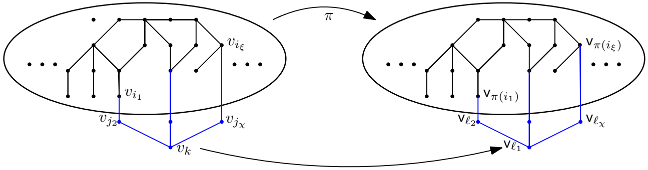

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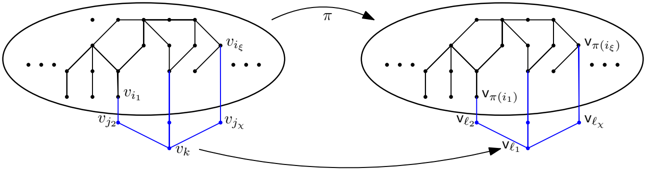

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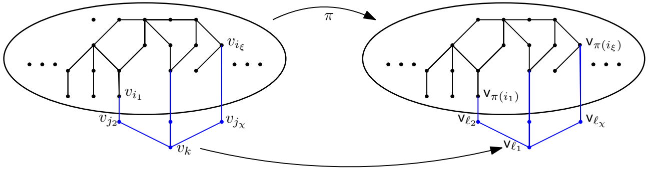
- If success, we add this tree to M_k(blue).



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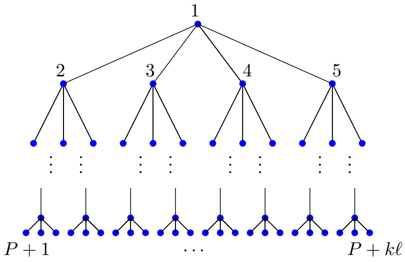

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- Our goal is to show that in most$(n-o(n))$ steps, we can find a tuple in CAND_k which matches successfully.

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- It's fair to say most of our work is to compute the moments, but it is not as obvious as it may seem.

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- If the previous conditioning(positive and negative) does not have devastating effects, then our idea works. Given this,

Proposition

For $1 \leq k \leq (1 - \varepsilon/3)n$, *for any "good" realization* $\mathcal{F}_{k-0.5}$ *and any tuple in* CAND_k, *it holds uniformly for all* k *that,*

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- The claim $\mathbb{P}[X_k > 0|\mathcal{F}_{k-0.5}] \geq 1-o(1)$ follows.

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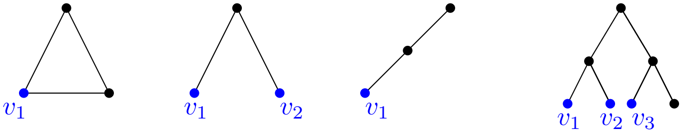

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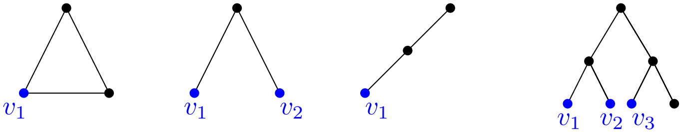

- The expectation may not be able to reflect the actual enumerations. It depends on the structure. e.g. For the second structure above, we have,

$$\max_{v_1, v_2} \mathrm{Ext}(v_1, v_2, \mathrm{Structure}) = O(1) \neq np^2 = o(1)$$

while for the third structure the expectation reflects the structures.

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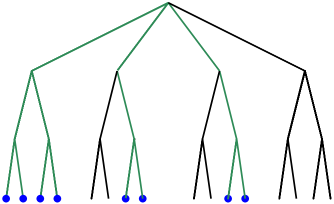

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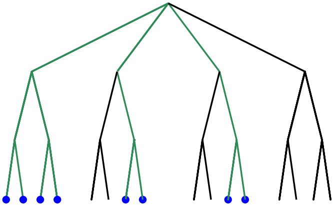

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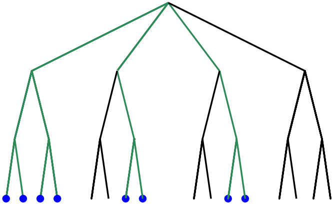

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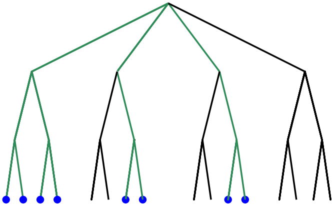
- We now focus on counting the extensions of a tree-like structure **T** given some leaves $v_1, v_2, \cdots, v_k$.
- We take a tree for example to give you a flavor of this method.



- We first count the green tree spanned by the fixed vertices. To compute this enumeration, we **decompose** the tree(green) into dense parts and sparse parts.
- We then count the enumeration of the black parts according to its expectation.
- This finishes the counting of tree extension. To learn more about the extensions(of other shapes), see Spencer's paper in the reference.

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- Based on the belief of the **information computation gap**, our original goal is to establish some negative results on the performance of efficient algorithm. But later things turned out to evolve in an unexpected way.
- Our result contributes an example for which approximation algorithms were discovered for random instances whereas the worst-case of the problem is known as NP-hard.

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- For the case $0 < \alpha < 1/2$, determine the second order of the maximal overlap.

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Thanks!