EN.553.436/646 Exam 1 (20 pts.)

Guidelines

syntax is insufficient.- The test is open note and open internet, but do not communicate with anyone.

- When you are ready to start, record your start time.
- Finish the exam within a 75-minute time interval.
- When you are finished, complete the honor statement and upload your IPYNB and PDF to Canvas.
- The latest you may submit the test is 1:30pm, so plan your start time accordingly.
- Some questions ask for text responses. The graders will be looking for brief, incisive remarks that indicate depth of understanding. For commenting code, a literal description of the

Honor Statement:

This work was done entirely by me within the 75-minute time limit. I have not discussed test content with any other student and I will not do so during the test period.

Signature: Qihua Gong

Start Time: 11:00

End Time: 12:00

Dataset

Simon is employed by a bureau of civil engineering. He is analyzing the Concrete Compressive Strength Dataset stored in `concrete.csv`. His goal is to assess the relation of eight attributes of concrete to the compressive strength of the concrete.

Read and run the following code cell.

In [4]:
```
import pandas as pd

data = pd.read_csv("concrete.csv")
display(data)
```

```
# Predictors: attribtues of concrete.
X = data.iloc[:,:-1]
# Target: compressive strength.
y = data.iloc[:,-1]
```

	Cement[kg/m^3]	Slag[kg/m^3]	SlyAsh[kg/m^3]	Water[kg/m^3]	Superplasticizer[kg/m^3
0	540.0	0.0	0.0	162.0	2.
1	540.0	0.0	0.0	162.0	2.
2	332.5	142.5	0.0	228.0	0.
3	332.5	142.5	0.0	228.0	0.
4	198.6	132.4	0.0	192.0	0.
...	...	...	...	...	.
1025	276.4	116.0	90.3	179.6	8.
1026	322.2	0.0	115.6	196.0	10.
1027	148.5	139.4	108.6	192.7	6.
1028	159.1	186.7	0.0	175.6	11.
1029	260.9	100.5	78.3	200.6	8.

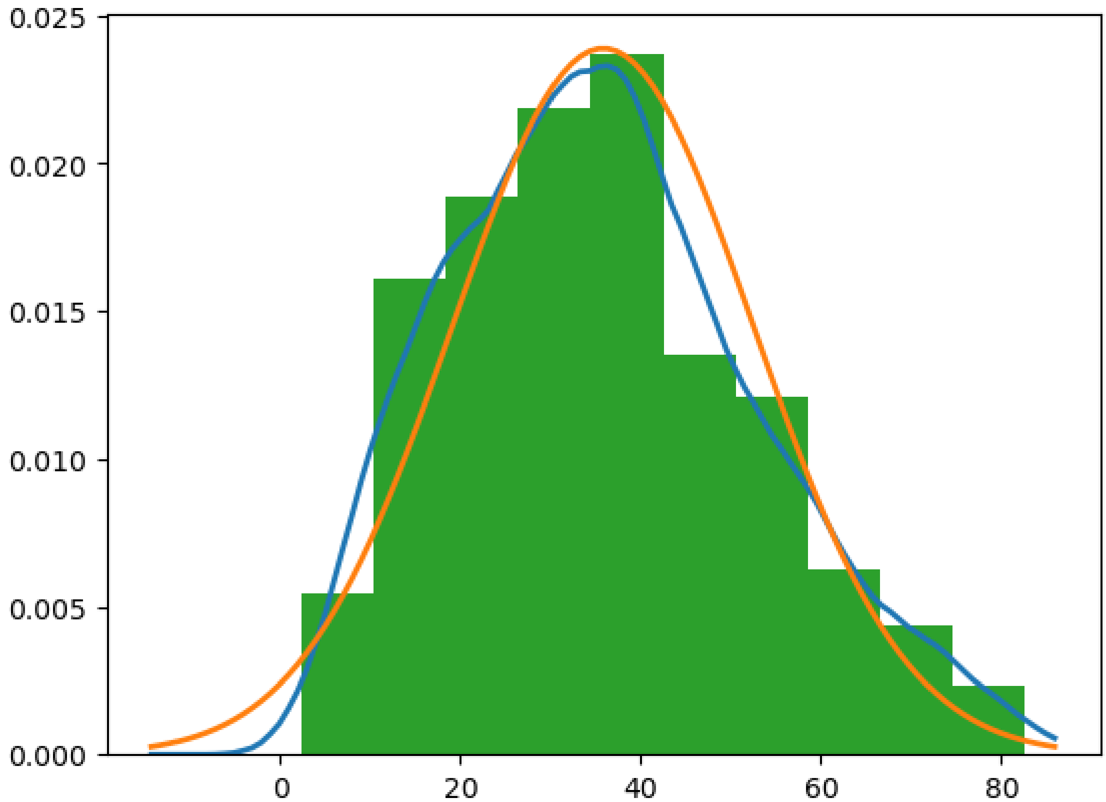
1030 rows × 9 columns

1 (4 pts.)

1. Run the following code cell.
2. Where indicated, name the method Simon is attempting to implement. (1 pt.)
3. Where indicated, write a brief comment explaining the intent of the following code. (2 pts.)
4. In the Markdown cell below, give a brief, qualitative description of the distribution of `y` based on the plot. (1 pt.)

In [5]:
```
# METHOD: Kernal Density estimation

import matplotlib.pyplot as plt
import numpy as np
from scipy import stats

# COMMENT: define the kernel density in a quadratic model
def K(u):
    return (3/4)*(1-u**2)*(np.abs(u)<=1)

# COMMENT: scale the previous function
def Kh(u, h):
    return (1/h)*K(u/h)

# COMMENT: sum the scaled kernels to calculate the Kernal Density estimation
def f(u, h):
    tmp = 0
    for y_i in y:
```

```
        tmp += Kh(u-y_i, h)
    return tmp/len(y)

# COMMENT: Do the PDF with the same mean and histogram, then compare with the k
u = np.linspace(np.mean(y)-3*np.std(y), np.mean(y)+3*np.std(y), 100)
plt.plot(u, f(u,10),linewidth=2)
plt.plot(u, stats.norm(loc=np.mean(y), scale=np.std(y)).pdf(u), linewidth=2)
plt.hist(y, density=True, bins=10)
plt.show()
```



1.3 Description: [YOUR ANSWER HERE.]

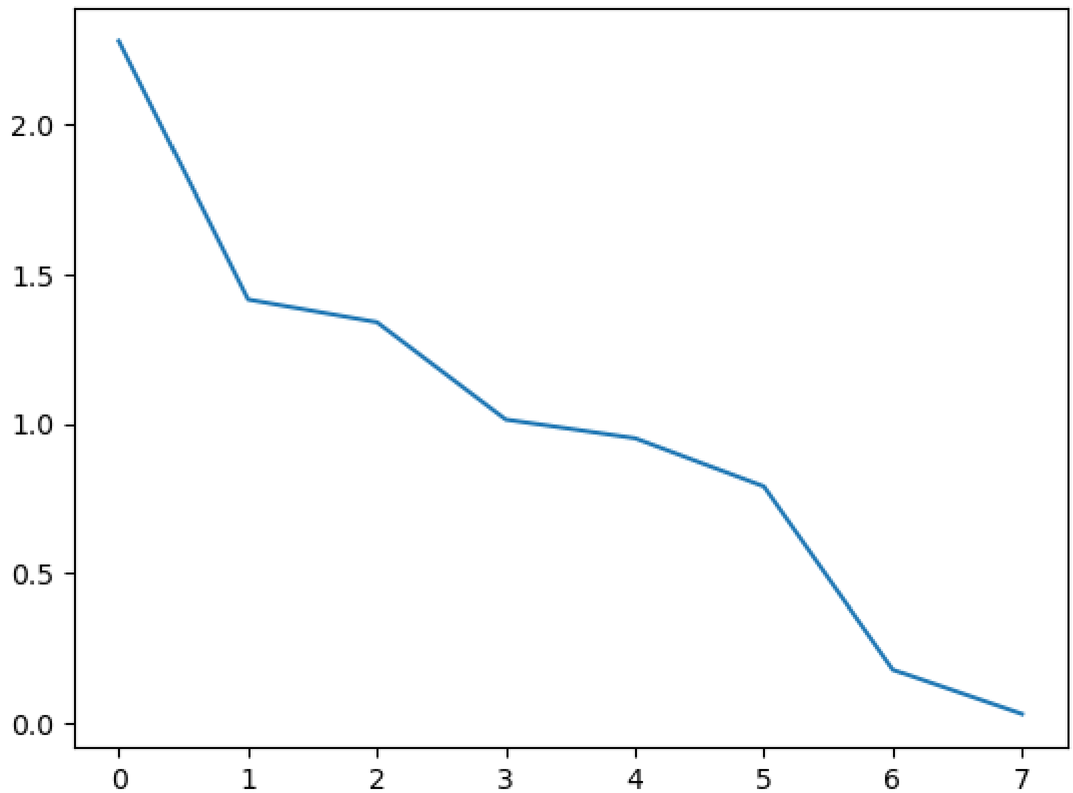
Answer: After the comparison with the normal PDF, we can see that the kernel density estimatiion is overfitting to the data

2 (8 pts.)

1. Run the following code cell.
2. Where indicated, name the method Simon is attempting to implement. (1 pt.)
3. Where indicated, write a brief comment explaining the intent of the following code. (2 pts.)
4. Simon made a comment at the bottom of the code cell. Explain that comment in the Markdown cell that follows. (3 pts.)
5. There is a mistake in the code. Fix the mistake by modifying exactly one line. Comment your correction. Run the cell to get the correct output. (2 pts.)

In [15]:
```
# METHOD: PCA

# COMMENT: compute the covariance matrix
#S = X.T @ X / (X.shape[0]-1) #modifying
S = ((X - X.mean())/X.std()).T @ ((X - X.mean())/X.std()) / (X.shape[0]-1)
# To calculate the covariance matrix, we need to calculate the mean and differe
# COMMENT: calculate the eigenvalues and eigenvectors
w, v = np.linalg.eigh(S)
# SIMON: Sorts array in descending order. It was in ascending order by default.
w = np.flip(w)
# COMMENT: print the eigenvalue of the first principal component
print(w[0]/np.sum(w))
# COMMENT: plot eigenvalues
plt.plot(w)
plt.show()

# SIMON: From the output, X is virtually one-dimensional!
```

```
0.285012420169039
```



2.3 Explanation: [YOUR ANSWER HERE.]

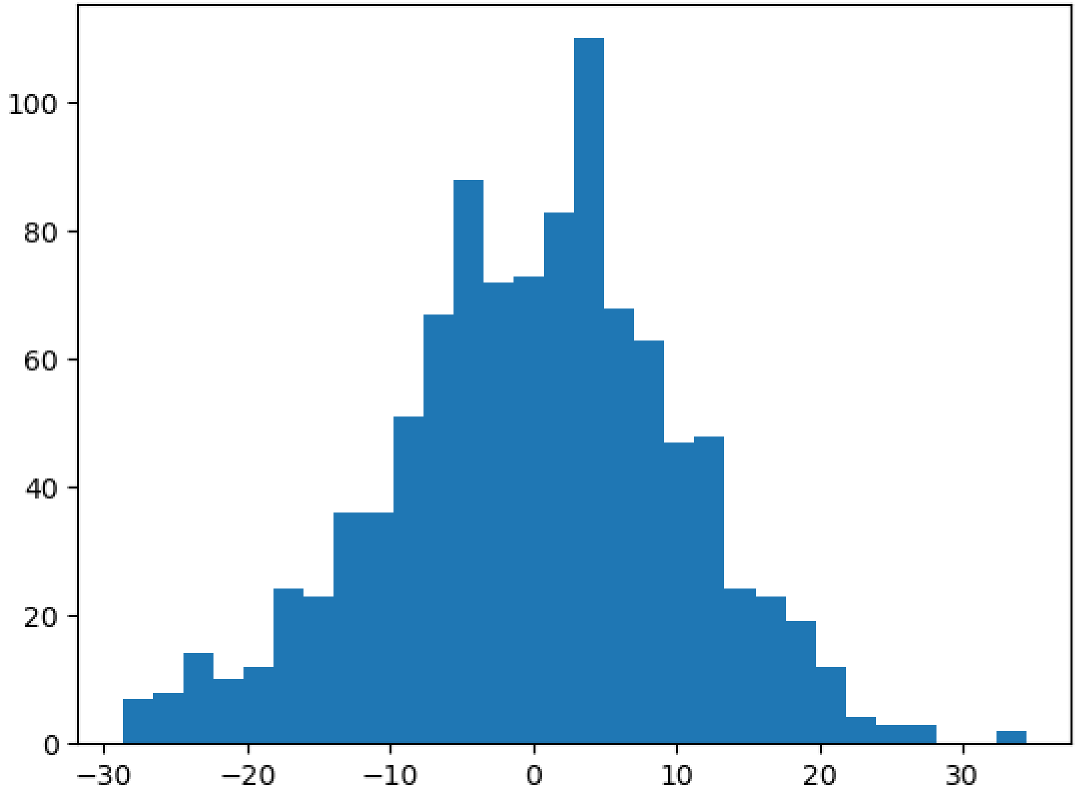
Answer: He used wrong scale to calculate the covariance matrix and the correlation matrix. So, he get a wrong plot of 0.97 PC1 and a lot of 0 in the other PCs that is why he thought only PC1 is meaningful and the X is one-dimensional. After we correct the way of calculate the covariance matrix, we can see that PCs are all meaningfull.

3 (8 pts.)

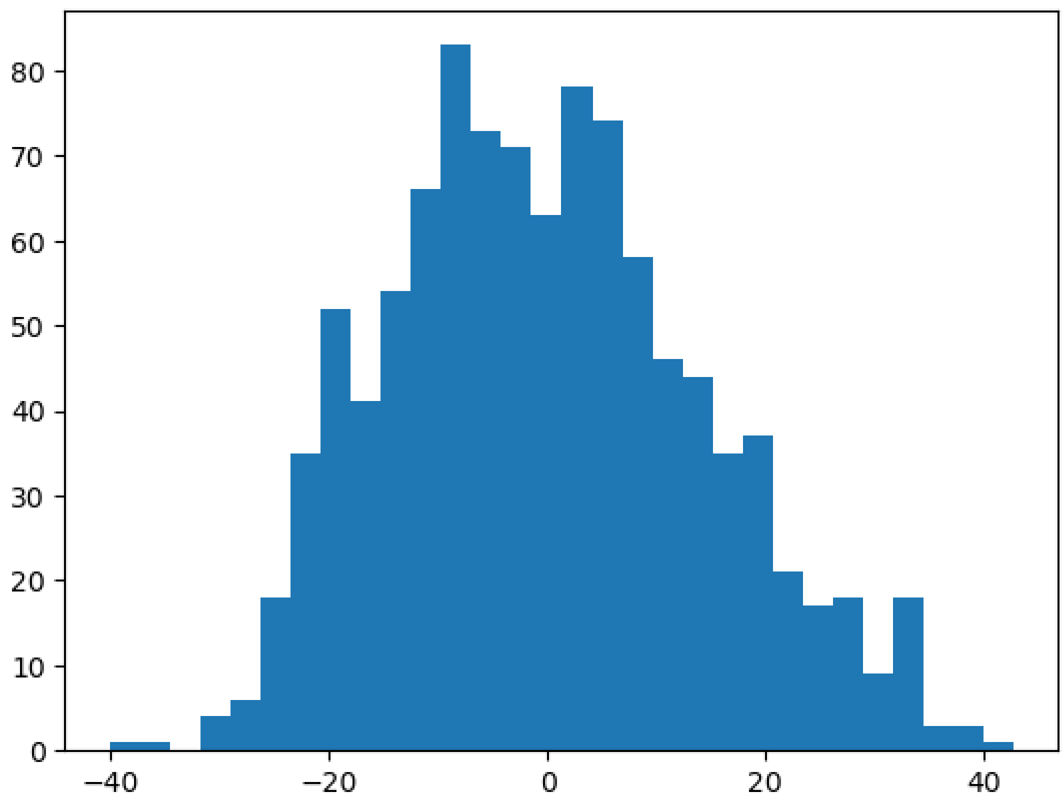
1. Read and run the following code cell.
2. Where indicated, write a brief comment explaining the intent of the following code. (4 pts.)
3. In the indicated space in `assess`, write code that will perform a linear regression to fit the coefficients of a linear model to predict `y` given `M`. Apply the model to generate predictions `y_pred` as a function of `M`. Comment your code. (4 pts.)

In [18]:
```
# METHOD: Linear Regression

from sklearn.decomposition import PCA

# COMMENT: Build matrix M
def setup(df, augment_with_quadratic=False, pca=False):
    M = df.values

    # COMMENT: Calculate the M matrix with method augment_with_quadratic
    if augment_with_quadratic:
        print("Running method augment_with_quadratic...")
        for i in range(df.shape[1]-1):
            for j in range(i+1, df.shape[1]):
                M = np.hstack((M, M[:,i:(i+1)]*M[:,j:(j+1)]))

    # COMMENT:  Calculate the M matrix PCA
    if pca:
        print("Running pca...")
        pca = PCA(n_components=2)
        M = pca.fit_transform(M)

    # COMMENT: Add a column of ones to the matrix M
    M = np.hstack((np.ones([M.shape[0], 1]), M))
    return M

# COMMENT: Define assessment function and plot
def assess(M):
    # this is placeholder to prevent error messages. you should overwrite y_pre
    #y_pred = np.zeros(y.shape) #placeholder
    # COMPLETE CODE:
    y_pred = M @ np.linalg.pinv(M.T @ M) @ M.T @ y
     #calcualte the prediction, just write them together
     # #basically from three equation: Xpinv = np.linalg.inv(X.T @ X) @ X.T, bH
    # COMMENT: Calculate the error
    e = y - y_pred
    print(np.mean(e**2))
    # COMMENT: Plot the error in a histogram
    plt.hist(e, bins=30)
    plt.show()
    return

# COMMENT: access the data by the method from four situation: raw, pca, augment
assess(setup(X))
assess(setup(X, pca=True))
assess(setup(X, augment_with_quadratic=True))
assess(setup(X, augment_with_quadratic=True, pca=True))
```

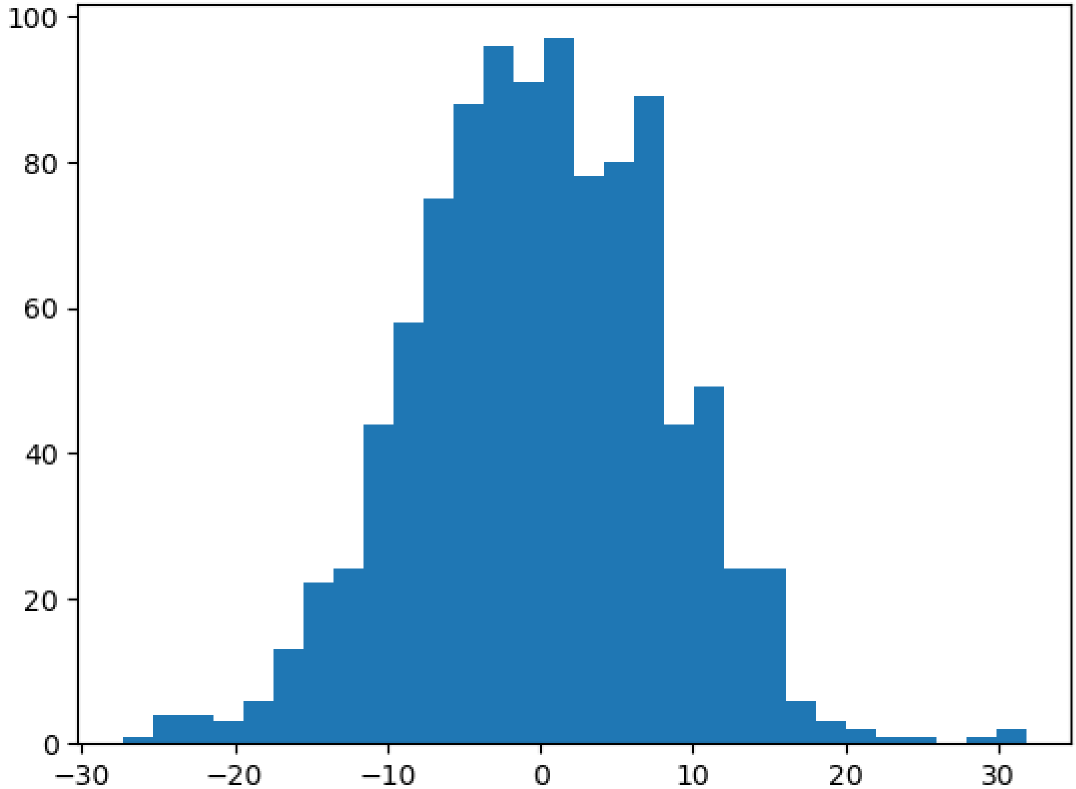
```
107.19723607482689
```




```
Running pca...
207.58044110036496
```




```
Running method augment_with_quadratic...
67.93349436115673
```




```
Running method augment_with_quadratic...
Running pca...
210.12488583372544
```