EN. 520. 650. HW3

Qihua Gong

1. from question:

F	P	T
friendly	0.75	Day
unfriendly	0.25	night

set the table $\Rightarrow$

F	T	P
friendly	Day	$0.9 \times 0.75 = 0.675$
unfriendly	Day	0
friendly	night	$0.75 \times 0.1 = 0.075$
unfriendly	night	0.25

$\because P(a|b) = \frac{P(a,b)}{P(b)}$

$$P(F=f \mid T=n) = \frac{P(F=f, T=n)}{P(T=n)} = \frac{0.075}{0.25+0.075} = 0.23$$

2. draw the table

D	S	P
disease	positive	0.99
disease	negative	0.01
no disease	positive	0.01
no disease	negative	0.99

$P(\text{disease}) = 0.0001$

$P(\text{no disease}) = 0.9999$

$$P(D=d \mid S=P) = \frac{0.0001 \times 0.99}{0.0001 \times 0.99 + 0.9999 \times 0.01} \approx 0.0099 = 0.99\%$$

$\therefore$ It's really a rare disease and the chance you get this disease is 0.99%

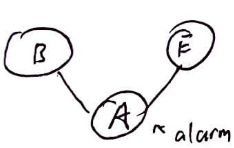
3. a) If no evidence, the Burglary and Earthquake is independent.

For numerical: $P(B,E) = P(B \mid \text{parents}(B))\, P(E \mid \text{parents}(E))$

$= P(B)\,P(E)$

$\therefore$ they are independent.

For topological: list the table



B	P(B)
t	0.001
f	0.999

E	P(E)
t	0.002
f	0.998

$\Rightarrow$

B	E	A	P(A|B, E)
t	t	t	0.95
t	t	f	0.05
t	f	t	0.94
t	f	f	0.06
f	t	t	0.29
f	t	f	0.71
f	f	t	0.001
f	f	f	0.999

we can see that burglary is non-decendant of earthquake, so they are independent in topological semantics

3. b) when Alarm is true, they are not independent.

to check independent or not, we need to verify if $P(B,E|A) = P(B|A)P(E|A)$ holds

$$P(A) = P(A|B,E)P(B)P(E) + P(A|B,-E)P(B)P(-E) + P(A|-B,E)P(-B)P(E) + P(A|-B,-E)P(-B)P(-E)$$

$$= 0.0000019 + 0.000938 + 0.000579 + 0.000989 = 0.00251$$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.95\times0.001\times0.002 + 0.94\times0.001\times0.998}{0.00251} = 0.37451$$

$$P(E|A) = \frac{P(A,E)}{P(A)} = \frac{P(A,E|B)P(B) + P(A,E|-B)P(-B)}{P(A)}$$

$$= \frac{0.95\times0.001\times0.002 + 0.29\times0.999\times0.002}{0.00251} = 0.2316$$

$$P(B,E|A) = \frac{P(A|B,E)P(B,E)}{P(A)} = \frac{P(A|B,E)P(B)P(E)}{P(A)} = \frac{0.95\times0.001\times0.002}{0.00251} = 0.000757$$

$$P(B|A)P(E|A) = 0.0867 \neq P(B,E|A)$$

So they are not independent

4.

O_1	O_2	O_3
Red	Blue	Yellow

$$\alpha_1(1) = \pi_1 b_1(O_1) = 0.3\times0.5 = 0.15$$
$$\alpha_1(2) = \pi_2 b_2(O_1) = 0.4\times0.1 = 0.04$$
$$\alpha_1(3) = \pi_3 b_3(O_1) = 0.3\times0.4 = 0.12$$
$$\alpha_2(1) = \sum_{i=1}^{3} \alpha_1(i) a_{i1} b_1(O_2) = (0.15\times0.4 + 0.04\times0.2 + 0.12\times0.3)\times0.2 = 0.0208$$
$$a_2(2) = \sum_{i=1}^{3} a_1(i) a_{i2} b_2(O_2) = (0.15\times0.3 + 0.04\times0.7 + 0.12\times0.2)\times0.2 = 0.0194$$
$$a_2(3) = \sum_{i=1}^{3} a_1(i) a_{i3} b_3(O_2) = (0.15\times0.3 + 0.04\times0.1 + 0.12\times0.5)\times0.2 = 0.0218$$
$$a_3(1) = \sum_{i=1}^{3} a_2(i) a_{i1} b_1(O_3) = (0.02\times0.4 + 0.02\times0.2 + 0.02\times0.3)\times0.1 = 1.8\times10^{-3}$$
$$a_3(2) = 0.02\times(0.3+0.7+0.2)\times0.1 = 2.4\times10^{-3}$$
$$a_3(3) = 0.02\times(0.3+0.1+0.5)\times0.3 = 5.4\times10^{-3}$$

$$P(O|\lambda) = 9.6\times10^{-3}$$

$\delta_1(1) = \pi_1 b_1(O_1) = 0.3\times0.5 = 0.15$ $\quad \psi_1(1) = 0$

$\delta_1(2) = 0.04$ $\quad \psi_1(2) = 0$

$\delta_1(3) = 0.12$ $\quad \psi_1(3) = 0$

$\delta_2(1) = \max[\delta_1(i) a_{i1}] b_1(O_2)$ $\quad \psi_2(1) = 1$

$\delta_2(2) = \max[\delta_1(i) a_{i2}] b_2(O_2)$ $\quad \psi_2(2) = 1$

$\delta_2(3) = \max[\delta_1(i) a_{i3}] b_2(O_3)$ $\quad \psi_2(3) = 3$

same in $\delta_3(1)$ $\quad \psi_3(1) = 1$, $\psi_3(2) = 2$ $\quad \psi_3(3) = 3$

$\therefore q_3 = 2$

$q_2 = \psi_2(q_3^*) = \psi_2(2) = 1$

$q_1 = \psi_1(q_2^*) = \psi_1(1) = 0$