EN. 520.665. HW3

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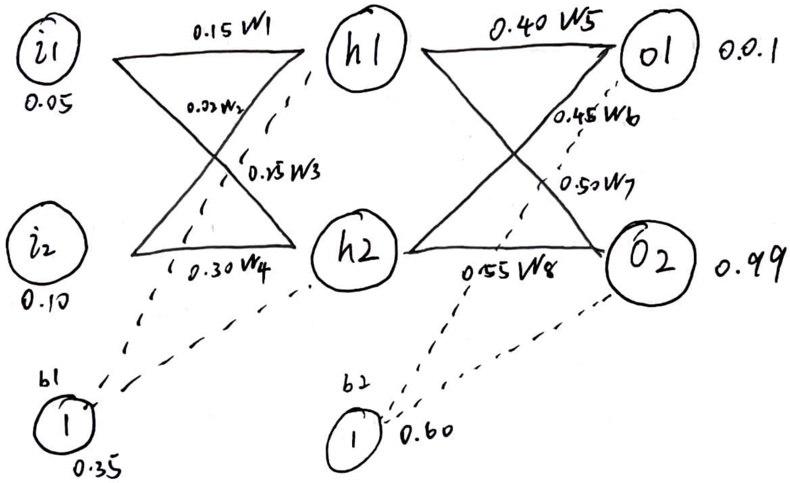
1.



first, forward pass

The net input for h_1 is calculated as the sum of the product of each weight value and input value and bias value

$$\text{net input } h_1 = w_1 \times i_1 + w_2 \times i_2 + b_1 \times 1$$
$$= 0.15 \times 0.05 + 0.2 \times 0.1 + 0.35 \times 1 = 0.3775$$

then use sigmoid function to calculate the output of h_1

$$\text{out } h_1 = \frac{1}{1+e^{-\text{net } h_1}} = \frac{1}{1+e^{-0.3775}} \approx 0.59933$$

so input $h_2 = 0.25 \times 0.05 + 0.30 \times 0.1 + 0.35 \times 1 = 0.3925$

$$\text{out } h_2 = \frac{1}{1+e^{-0.3925}} = 0.59688$$

$$\text{input } O_1 = W_5 \times \text{out } h_1 + W_6 \times \text{out } h_2 + b_2 \times 1$$
$$= 0.4 \times 0.5933 + 0.45 \times 0.59688 + 0.6 \times 1 = 1.10591$$

$$\text{out } O_1 = \frac{1}{1+e^{-\text{net } O_1}} = \frac{1}{1+e^{-1.10591}} = 0.7514$$

Same $\text{input } O_2 = 0.5 \times 0.5933 + 0.55 \times 0.59688 + 0.6 \times 1 = 1.2249$

$$\text{out } O_2 = \frac{1}{1+e^{-1.2249}} = \cancel{0.02356}\ 0.77293$$

Then calculate error $E_{total} = \sum \frac{1}{2}(\text{target} - \text{output})^2$

$$E_{O_1} = \frac{1}{2}(0.01 - 0.75136)^2 = 0.2748$$
$$E_{O_2} = \frac{1}{2}(0.99 - 0.77293)^2 = 0.02356$$
$$E_{total} = E_{O_1} + E_{O_2} = 0.27481 + 0.02356 = 0.29837$$

→ next

finally Use the back propagation to update each weight in the network.

$$\text{update } W_5^* = W_5 - n \times \left(\frac{\partial E\,total}{\partial W_5}\right)$$

the rate of change error

$$\frac{\partial E total}{\partial W_5} = \frac{\partial E total}{\partial out\,o1} \times \frac{\partial out\,o1}{\partial net\,o1} \times \frac{\partial net\,o1}{\partial W_5}$$

$$\frac{\partial E total}{\partial out\,o1} = -(target\ o1 - out\ o1) = -(0.01 - 0.75136) = 0.741365$$

$$\frac{\partial out\,o1}{\partial net\,o1} = out\ o1\ (1 - out\,o1) = 0.751365 \times (1 - 0.751365) = 0.18682$$

$$\frac{\partial net\,o1}{\partial W_5} = 1 \times out\,h1 \times W_5^{(1-1)} + 0 + 0 = out\ h1 = 0.59326$$

$$\therefore \frac{\partial E total}{\partial W_5} = 0.741365 \times 0.18682 \times 0.59326 = 0.082167$$

$$\text{update } W_5^* = 0.4 - 0.5 \times 0.082167 = 0.35892$$

as the same level backward. $W_6^* = 0.408666$

$W_7^* = 0.511301$

$W_8^* = 0.561370$

then go to next nevel layer. start with $W1$

$$W_1^* = W_1 - n \times \left(\frac{\partial E\ total}{\partial W_1}\right)$$

$$\frac{\partial E total}{\partial W_1} = \frac{\partial E total}{\partial out\,h_1} \times \frac{\partial out\,h1}{\partial net\,h_1} \times \frac{\partial net\,h1}{\partial W_1}$$

$$\frac{\partial E total}{\partial out\,h_1} = \frac{\partial E_{O_1}}{\partial out\,h_1} + \frac{\partial E_{O_2}}{\partial out\,h_1} = \frac{\partial E_{O_1}}{\partial net\,o1} \times \frac{\partial net\,o1}{\partial out\,h_1} + \frac{\partial E_{O1}}{\partial net\,o1} \times \frac{\partial net\,o1}{\partial out\,h1}$$

$$= 0.741365 \times 0.186815 + 0.1384985 \times 0.40$$

= ~~[illegible]~~

$$0.055399425 + (-0.0190491) = 0.03635$$

$$\frac{\partial out\,h1}{\partial net\,h1} = out\,h1\,(1 - out\,h1) = 0.593269\,(1 - 0.59327) = 0.2413$$

$$\frac{\partial net\,h1}{\partial W_1} = \cancel{net\,h}\ i_1 = 0.05$$

$$\therefore W_1^* = W_1 - n \times \frac{\partial E total}{\partial W_1} = 0.15 - 0.5 \times 0.03635 \times 0.2413 \times 0.05$$

$$= 0.15 - 0.5 \times 0.0004385 = 0.14978$$

as the same $W_2^* = 0.19956143$

$W_3^* = 0.24975114$

$W_4^* = 0.29950229$

addition: $W_7^* = W_7 - n \times \left(\frac{\partial E_{total}}{\partial W_7}\right)$

$$\frac{\partial E_{total}}{\partial W_7} = \frac{\partial E_{total}}{\partial out\ O_2} \times \frac{\partial out\ O_2}{\partial net\ O_2} \times \frac{\partial net\ O_2}{\partial W_7}$$

$$\frac{\partial E_{total}}{\partial out O_2} = -(target\ O_2 - out\ O_2) = -(0.99 - 0.77293) = -0.21707$$

$$\frac{\partial out O_2}{\partial net O_2} = out O_2 (1 - out O_2) = 0.77293(1-0.77293) = 0.17551$$

$$\frac{\partial net O_2}{\partial W_7} = 1 \times out\ h_1 \times W_7^{(1-1)} + 0 + 0 = out h_1 = 0.59327$$

$$W_7^* = 0.5 - 0.5 \times (-0.21707 \times 0.17551 \times 0.59327) = 0.511301$$

$$W_3^* = W_3 - n \times \frac{\partial E_{total}}{\partial W_3}$$

$$\frac{\partial E_{total}}{\partial W_3} = \frac{\partial E_{total}}{\partial out\ h_2} \times \frac{\partial out h_2}{\partial net h_2} \times \frac{\partial net h_2}{\partial W_3}$$

$$\frac{\partial E_{total}}{\partial out h_2} = \frac{\partial E_{O_1}}{\partial out h_2} + \frac{\partial E_{O_2}}{\partial out\ h_2}$$

$$= \frac{\partial E_{O1}}{\partial net\ O_2} \times \frac{\partial net O_2}{\partial out h_2} + \frac{\partial E_{O_2}}{\partial net O_2} \times \frac{\partial net\ O_2}{\partial out h_2}$$

$$= \frac{\partial E_{O1}}{\partial out h_2} \times \frac{\partial out h_2}{\partial net O_2} \times \frac{\partial net O_2}{\partial out h_2} + \frac{\partial E_{O_2}}{\partial out h_2} \times \frac{\partial out h_2}{\partial net O_2} \times \frac{\partial net\ O_2}{\partial out\ h_2}$$

$$= -0.21707 \times 0.17551 \times 0.55 + -0.21707 \times 0.17551 \times 0.55 = 0.041371$$

$$\frac{\partial out h_2}{\partial net h_2} = out h_2 (1 - out h_2) = (0.59688) \times (1 - 0.59688) = 0.240614$$

$$\frac{\partial net h_2}{\partial W_3} = i_1 = 0.05$$

$$\therefore W_3^* = 0.25 - 0.5 \times 0.041371 \times 0.240614 \times 0.05 = 0.24975114$$