EN. 520.665. HW5

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1. We can design a least square fitting of Two 3-D point set.

for the basic two point sets $\{p_i\}$ and $\{p_i'\}$, $i=1,2,\cdots,N$, p_i and p_i' are consider as a 3x1 column matrix, and they are the point set we need.

$$p_i' = Rp_i + T + N_i$$

here R is a 3x3 rotation matrix and T is a translation vector 3x1 column matrix, and N_i is a noise vector.

here we want to fint R and T to minimize

$$\Sigma^2 = \sum_{i=1}^{N} \| p_i' - (Rp_i + T) \|^2$$

and it will based on a singular value decomposition SVD of 3x3 matrix.

$$p' = p'' \Rightarrow \begin{cases} p' \triangleq \frac{1}{N}\sum_{i=1}^{N} p_i' \\ p'' \triangleq \frac{1}{N}\sum_{i=1}^{N} p_i'' = \hat{R}p + \hat{T} \\ p \triangleq \frac{1}{N}\sum_{i=1}^{N} p_i \end{cases}$$

$$q_i \triangleq p_i - p$$

$$q_i' \triangleq p_i' - p'$$

$$\Sigma^2 = \sum_{i=1}^{N} \| q_i - Rq_i \|^2$$ then we will find the translation

$$\hat{T} = p' - \hat{R}p$$

after that use SVD Algorithm for finding $\hat{R}$

1. Form $\{p_i\}$, $\{p_i'\}$ calculate p, p'; and then $\{q_i\}$, $\{q_i'\}$
2. calculate the 3x3 matrix

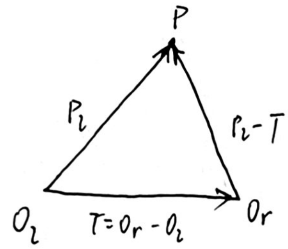
$$H \triangleq \sum_{i=1}^{N} q_i q_i'^t$$

3. Find the SVD of H, $H = U\Lambda V'$
4. calculate $X = VU'$.
5. calculate, $\det(X)$ and determinant X

   if $\det(X) = +1$, then X is a rotation which is the desired solution, $\hat{R} = X$

   if $\det(X) = -1$, then X is a reflection, and the algorithm fails.

2. a) for points in one image and epipolar lines in the others



$$(P_l - T)^T (T \times P_l) = 0$$

using $P_r = R(P_l - T)$

$$(R^T P_r)^T T \times P_l = 0$$

$$(R^T P_r)^T S P_l = 0 \quad \text{or} \quad P_r^T R S P_l = 0 \quad \text{where } S = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix}$$

$$P_r^T E P_l = 0 \quad \text{or} \quad P_r^T E P_l = 0$$

where $E = RS$ is essential matrix

and the equation $P_r^T E P_l = 0$ defines a mapping between points and epipolar lines

b) if the edges ordered the same way, we will have pixel coordinates $\bar{p}_l$ and $\bar{p}_r$

$$\bar{p}_l = M_l p_l \quad , \quad \bar{p}_r = M_r p_r$$

$$\bar{p}_r^T F \bar{p}_l = 0$$

where $F = (M_r^{-1})^T E M_l^{-1} = (M_r^{-1})^T R S M_l^{-1}$ for the fundamental matrix

3. $f = \frac{2p}{1+\sqrt{1+p^2+q^2}}$, $g = \frac{2q}{1+\sqrt{1+p^2+q^2}}$

then for the image irradiance equation

$$E(x,y) = R(f(x,y), g(x,y))$$

for the brightness error

$$\iint_\Omega (E(x,y) - R(f(x,y), g(x,y)))^2 \, dx dy$$

$$p = \frac{4f}{4-f^2-g^2} \quad \text{and} \quad q = \frac{4g}{4-f^2-g^2}$$

then the euler equation

$$\frac{f_y(4+f^2-g^2) - g_x(4-f^2+g^2) + 2(g_y - f_x)fg}{(4-f^2-g^2)^2} = 0$$

$$\begin{cases} (E-R)R_f + \lambda \nabla^2 f = 0 \\ (E-R)R_g + \lambda \nabla^2 g = 0 \end{cases} \Rightarrow \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

also in the depth: $\delta z = p\delta x + q\delta y$

$$z(x,y) = z(x_0, y_0) + \int_c (p dx + q dy)$$

$$\nabla^2 z = p_x + q_y$$

$$(z_x, z_y) \cdot n = (p, q) \cdot n$$

$$n = \left(-\frac{dy}{ds}, \frac{dx}{ds}\right)$$

$$z_{ij}^{k+1} = \bar{z}_{ij}^{k} - \frac{\epsilon}{4}(h_{ij} + v_{ij})$$

where $\bar{z}_{ij} = \frac{1}{4}(z_{i+1,j} + z_{i-1,j} + z_{i,j+1} + z_{i,j-1})$

$h_{ij} = \frac{1}{2}(p_{i+1,j} - p_{i-1,j})$ and $v_{ij} = \frac{1}{2}(q_{i,j+1} - q_{i,j-1})$

$\frac{1}{2\epsilon}(z_{i+1,j} - z_{i-1,j}) = p_{ij}$ and $\frac{1}{2\epsilon}(z_{i,j+1} - z_{i,j+1}) = q_{ij}$

4. $\because$ $cut(A,B) = \sum_{u\in A, u\in B} w(u,v)$

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

for the given

$cut(A,B) = \sum_{u\in A, v\in B} w(u,v)$

$$Ncut = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

$asso(X,V) = \sum_{u\in x, t\in V} w(u,t)$

~~At~~ $cut(A,B) = 2+2+2 = 6$

$assov(A,V) = 3+2+2+2 = 9$

$assov(B,V) = 3+2+2+2 = 9$

$\therefore Ncut = \frac{6}{9} + \frac{6}{9} = \frac{4}{3} = 1.33$

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