EN. 520. 665 Midterm

Qihua Gong

1. a) CaNNy is a non isotropic, LoG is isotropic

b) CaNNy is computed in terms of 1st order derivative
LoG is computed in terms of 2nd order derivative

c) Canny need the parameters of σ and two thresholds for hysteresis
LoG need σ defining the scale of Gaussian blurring

D) Canny is more likely to produce long thin contours because of non-maximum suppression which thins and hyteresis threesholding which can fill in week edge gaps.

2. rotational invariance LBP,

Let (P, R) denote a circulate neighborhood, P is a number of sampling points and R is the radius

$$(X_p, Y_q) = (x + R\cos(\frac{2\pi p}{P}), y - R\sin(\frac{2\pi p}{P}))$$

$$LBP_{P,R}(x,y) = \sum_{p=0}^{P-1} s(f(x,y) - f(x_p, y_p)) 2^p$$

$$s(z) = \begin{cases} 1, & z \geq 0 \\ 0, & z < 0 \end{cases}$$

b) Let $U_P(n,r)$ be a uniform LBP pattern, $I^{\theta\circ}(x,y)$ be the robation of image $I(x,y)$ by θ degrees, denote (x', y') as (x,y) over robate, $I^{\theta\circ}(x', y') = I(x,y)$, if there are P points in LBP, $\theta = \frac{k\,360°}{P}$, $k = 0, 1, \cdots P-1$

something rotated by P+1 steps will yield the same pattern as one rotated by 1 step, due to rotation symmetry.

$$U_P(n,r) = U_P(n, r+k \bmod P)$$

$$h_\theta(U_P(n, r+k)) = (U_P(n,r))$$

3. a) Matrix - vector

$$p(f) = \frac{\sqrt{\det B}}{\sqrt{(2\pi\sigma^2)^m}} \exp\left(\frac{(f-\mu)^T B (f-\mu)}{2\sigma^2}\right)$$

f is an $m\times 1$ vector, μ is an $m\times 1$ vector

b) Metropolis Sampling

f is initialize Randomly, S is image lattice

For $i \in S$

$f'_{i'} = f_{i'}$. for all $i' \neq i$, choose $f_i \in \mathcal{L}$ at random, $p = \min\{1, \frac{P(f')}{P(f)}\}$

Replace f by f' with probability p, generate a uniform random variable $u \in [0,1)$. if $u \leq p$ replace f by f', if $u > p$, no change

c) Gibbs Sampling.

Let f be a random cofiguration, For $i \in S$ do: compute $P_l = p(f_i = l \mid f_{N_i}) \forall l \in \mathcal{L}$

where f_{N_i} are the pixel values at neighboring sites

set f_i to l with probability p

4. a) (ii), (iv)

b) (ii)

c) (iii)

d) (iv)

e) (iii)

5. first, forward pass

$$\text{net input } h_1 = w_1 \times i_1 + w_2 \times i_2 + b_1 \times 1$$
$$= 0.15 \times 0.05 + 0.2 \times 0.1 + 0.35 \times 1 = 0.3775$$

$$\text{out } h_1 = \frac{1}{1+e^{-neth_1}} = \frac{1}{1+e^{-0.3775}} = 0.5933$$

$$\text{input } h_2 = 0.25 \times 0.05 + 0.30 \times 0.1 + 0.35 \times 1 = 0.3925$$

$$\text{out } h_2 = \frac{1}{1+e^{-0.3925}} = 0.59688$$

$$\text{input } O_1 = W_5 \times outh_1 + W_6 \times outh_2 + b_2 \times 1$$
$$= 0.4 \times 0.5933 + 0.45 \times 0.59688 + 0.6 \times 1 = 1.10591$$

$$\text{out } O_1 = \frac{1}{1+e^{-netO_1}} = \frac{1}{1+e^{-1.0596}} = 0.7514$$

$$\text{input } O_2 = 0.5 \times 0.5933 + 0.55 \times 0.59688 + 0.6 \times 1 = 1.2249$$

$$\text{Out } O_2 = \frac{1}{1+e^{-1.2249}} = 0.77293$$

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$

$$E_{O_1} = \frac{1}{2}(0.01 - 0.75136)^2 = 0.2748$$

$$E_{O_2} = \frac{1}{2}(0.99 - 0.77293)^2 = 0.02356$$

$$E_{total} = E_{O_1} + E_{O_2} = 0.27481 + 0.02356 = 0.29837$$

$$W_6^* = W_6 - n \times \left(\frac{\partial E_{total}}{\partial W_6}\right)$$

$$\frac{\partial E_{total}}{\partial W_6} = \frac{\partial E_{total}}{\partial outO_1} \times \frac{\partial OutO_1}{\partial netO_1} \times \frac{\partial netO_1}{\partial W_6}$$

$$\frac{\partial E_{total}}{\partial outO_1} = -(target\ O_1 - outO_1) = -(0.01 - 0.75136) = 0.741365$$

$$\frac{\partial outO_1}{\partial netO_1} = outO_1(1 - outO_1) = 0.75136 \times (1 - 0.751365) = 0.18682$$

$$\frac{\partial netO_1}{\partial W_6} = 1 \times out\ h_2 \times W_6^{(1-1)} + 0 + 0 = outh_2 = 0.59688$$

$$\therefore W_6^* = 0.59688 - 0.5 \times 0.741365 \times 0.18682 \times 0.59688 = 0.408666$$

$$W_2^* = W_2 - n \times \frac{\partial E_{total}}{\partial W_2}$$

$$\frac{\partial E_{total}}{\partial W_2} = \frac{\partial E_{total}}{\partial outh_2} \times \frac{\partial outh_2}{\partial neth_2} \times \frac{\partial neth_2}{\partial W_2}$$

$$\frac{\partial E_{total}}{\partial outh_2} = \frac{\partial E_{O_1}}{\partial outh_2} + \frac{\partial E_{O_2}}{\partial outh_2} = \frac{\partial E_{O_1}}{\partial netO_1} \times \frac{\partial netO_1}{\partial outh_2} + \frac{\partial E_{O_2}}{\partial netO_2} \times \frac{\partial netO_2}{\partial outh_2}$$
$$= \frac{\partial E_{O_1}}{\partial outh_2} \times \frac{\partial outh_2}{\partial netO_1} \times \frac{\partial netO_1}{\partial outh_2} + \frac{\partial E_{O_2}}{\partial outh_2} \times \frac{\partial outh_2}{\partial netO_2} \times \frac{\partial netO_2}{\partial outh_2}$$
$$= 0.041371$$

$$\frac{\partial out_{h2}}{\partial net_{h2}} = out_{h2}(1 - out_{h2}) = 0.59688 \times (1 - 0.59688) = 0.240614$$

$$\frac{\partial net_{h2}}{\partial W_2} = i_2 = 0.1$$

$$\therefore W_2^* = 0.20 - 0.5 \times 0.041371 \times 0.240614 \times 0.1 = 0.199501$$