EN.520.665 HW4

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1. a) $I(x,y,t)$ = brightness at (x,y) at time t

$$I(x+\frac{dx}{dt}\delta t, y+\frac{dy}{dt}\delta t, t+\delta t) = I(x,y,t)$$

$$\frac{dI}{dt} = \frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0 \quad , \text{given } u = \frac{Wx}{Z} \quad , v = \frac{Wy}{Z}$$

$$I_x u + I_y v + I_t = 0 \quad \Rightarrow \quad I_x \frac{Wx}{Z} + I_y \frac{Wy}{Z} + I_t = 0$$

b) The optical is stationary. The basic assumption of the optical flow is that with a very short time change, the brightness in constant. The brightness should not be affect by time. Time is independ

c) for the Laplacians: $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

$$\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$$

the both Laplacians are zero, only when translates parallel to a flat object, rotates or perpendicular to the surface, travel orthogonally to surface is not.

d) How time affects, only the small movement position in the image, so that the target has a vector speed to facilitate the derivation. The specific impact should be the magnitude of the object displacement under frame sampling.

2. $E(u,v) = \int_\Omega [(I_x u + I_y v + I_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)]\,dx\,dy$

defining the Lagrangian

$$L(u(x,y), v(x,y)) \triangleq [(I_x u + I_y u + I_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)]$$

calculus of variation: $\nabla_u E = \frac{\partial L}{\partial u} - \frac{\partial}{\partial x}\frac{\partial L}{\partial u_x} - \frac{\partial}{\partial y}\frac{\partial L}{\partial u_y}$

$$= I_x^2 u + I_x I_y v - I_x I_t - \lambda \nabla^2 u$$

$$\nabla^2 u = u_{xx} + u_{yy}$$

when minimized the E, $\nabla_u E = 0$, $\nabla_v E = 0$

$$I_x^2 u + I_x I_y v = \lambda \nabla^2 u - I_x I_t$$

$$I_x I_y u + I_y^2 v = \lambda \nabla^2 v - I_y I_t$$

also can write as $\alpha = 4\lambda$

$$(I_x^2 + \alpha) u + I_x I_y v = \alpha \bar{u} - I_x I_t$$ Euler-lagrange equation.

3. $X = -U - BZ + CY$

$Y = -V - CX + AZ$

$Z = -W - AY + BX$

$(x, y) = (\frac{X}{Z}, \frac{Y}{Z})$

$(u, v) = (\dot{x}, \dot{y})$

$u = \frac{\dot{X}}{Z} - \frac{X\dot{Z}}{Z^2} = (-\frac{U}{Z} - B + Cy) - x(-\frac{W}{Z} - Ay + Bx)$

$v = \frac{\dot{Y}}{Z} - \frac{Y\dot{Z}}{Z^2} = (-\frac{V}{Z} - Cx + A) - y(-\frac{W}{Z} - Ay + Bx)$

$u = u^T + u^R, \quad v = v^T + v^R$

$u^T = (-U + xW)/Z \quad v^T = (-V + yW)/Z$

$u^R = -B + Cy + Axy - Bx^2, \quad v^R = -Cx + A + Ay^2 - Bxy$

$x = \frac{U}{W}, \quad y_0 = \frac{V}{W}$

$u^T = (x - x_0)\frac{W}{Z}, \quad v^T = (y - y_0)\frac{W}{Z}$

$\frac{Z}{W} = (x - x_0)/u^T = \frac{y - y_0}{v^T}$

$\therefore$ For any point

$u(x_0, y) = -B + Cy + Ax_0y - Bx_0^2$

$v(x, y_0) = Cx + A + Ay_0^2 - Bxy_0$

$\frac{Z}{W} = \frac{x - x_0}{u - u^R} = \frac{y - y_0}{v - v^R}$