EN. 520. 680 HW1

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The average error rate is a usual way for evaluation a communication channel. It's a way of count how many time the output class agrees with the input class. Low error rate implies better transmission channel. But this method can't tell how much information was transmitted through the channel. So, another way of deriving how good is a given communication channel is to use the confusions among the class recognition. It needs to evaluates the joint entropy in the channel and compares it with the maximum possible entropy in the channel when the input and output are independent.

joint entropy: $H(X,Y) = -\sum_{j=1}^{n}\sum_{k=1}^{n} P(X_j, Y_k)\log_2 P(X_j, Y_k)$

maximum entropy: $H_{max}(X,Y) = -\sum_{j=1}^{n}\sum_{k=1}^{n} P(X_j)P(Y_k)\log_2 P(X_j)P(Y_k)$

→ total transfer in channel $I(X,Y) = H_{max}(X,Y) - H(X,Y)$

In noise-free transmission

input $P(X_1)=0.5$, $P(X_2)=0.5$, $P(Y_1)=0.5$, $P(Y_2)=0.5$

	stim 1	stim 2
resp 1	1	0
resp 2	0	1

$P(X_k, Y_k)$: 0.5 0 / 0 0.5

max $P(X_k, Y_k)$: 0.25 0.25 / 0.25 0.25

$H(X,Y) = -\sum_{j=1}^{n} P(X_j, Y_k)\log_2 P(X_j, Y_k) = 0\times 0 + 0.5\times 1 + 0\times 0 + 0.5\times 1 = 1$ bits

$H_{max}(X,Y) = 0.25\times 2 + 0.25\times 2 + 0.25\times 2 + 0.25\times 2 = 2$ bit

$I(X;Y) = 2 - 1 = 1$ bit

In Ramdom transmission:

	stim 1	stim 2
resp 1	5	5
resp 2	5	5

input: $P(X_1)=0.5$, $P(X_2)=0.5$

$P(Y_1)=0.5$ $P(Y_2)=0.5$

$P(X_j, Y_k)$: 0.25, 0.25 / 0.25 0.25

$H(X,Y) = -\sum_{j=1}^{n} P(X_j, Y_k)\log_2 P(X_j, Y_k)$

$= 0.25\times 2 + 0.25\times 2 + 0.25\times 2 + 0.25\times 2 = 2$ bit

$H_{max}(X,Y) = 2$ bit $\quad I(X;Y) = 0$ bit

If we use simple error rate to judge, the noise-free channel has no error and Ramdom transmission has 0.5 error, ~~but the fact is noise-free channel~~ as the lower error rate as good 0.5 and 0 is not so different but in actually the half-half channel is a disaster, because which it means the machine is totally guessing.