Implementing

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- Two elements: Hubs , non-Hubs , 10 nodes

- key Cases:
  i) Hub cost
  ii) Transportation costs:
    - Collection ($\chi = 3$) : non-hub → hub
    - Transfer ($\alpha = 1$) : hub → hub
    - Distribution ($\delta = 2$) : hub → non-hub

- Decide: where to open hubs.

  $Y_k = 1$ if node k is a hub, else 0

- Assign non-hubs → hubs

Collection: $X_{ik} = 1$ if non-hub i —Parcel→ hub k, else 0

Distribution: $z_{ij} = 1$ if hub l —Parcel→ non-hub j

Transfer: t_{kl} = Parcel shipped from hub k → hub l

- Total Cost = $\sum(f_k \cdot y_k)$ (leaving) Hub cost
  + $\chi \cdot \sum(c_{ik} \cdot q_i \cdot x_{ik})$ Collection
  + $\alpha \cdot \sum(c_{kl} \cdot t_{kl})$ Transfer $k \to l$
  + $\delta \cdot \sum\sum(c_{ij} \cdot d_j \cdot z_{ij})$ (arriving)

- Constraint: (Collection Assignment.)
- Each non-hub i assigns to one hub for collection: $\sum_{k=1}^{10} x_{ik}$ for all i $(x_{i1} + x_{i2} + \cdots + x_{i(10)} = 1)$
- Distribution Assignment: hub assigns to one non-hubs

  For each non-hub j: $\sum_{l=1}^{10} z_{lj} = 1$ for all j

- New objective Function

$$\text{Cost} = \underbrace{\sum_{k \in N} f_k y_k}_{\text{Hub Cost}} + \underbrace{\chi \sum_{i \in N} \sum_{k \in N} c_{ik} q_i x_{ik}}_{\text{Collection Cost}} + \underbrace{\alpha \sum_{k \in N} \sum_{l \in N} c_{kl} t_{kl}}_{\text{Transfer Cost}} + \underbrace{\delta \sum_{l \in N} \sum_{j \in N} c_{lj} d_j z_{lj}}_{\text{Distribution Cost}}$$

Assignment Constraints:

- Each non-hub assigns to one hub for collection:

$$\sum_{k \in N} x_{ik} = 1 \quad \forall i \in N$$

- Each non-hub assigns to one hub for distribution:

$$\sum_{l \in N} z_{lj} = 1 \quad \forall j \in N$$

- Hub Activation:

  Non-hubs can only assign to open hubs:

$$x_{ik} \le y_k \quad \forall i,k \in N$$

$$z_{lj} \le y_l \quad \forall l,j \in N$$

3. Flow conservation:

  outflow from hub k = Parcels received from non-hubs

$$\sum_{l \in N} t_{kl} = \sum_{i \in N} q_i x_{ik} \quad \forall k \in N$$

  Inflow to hub l: Parcels sent to non-hubs

$$\sum_{l \in N} t_{kl} = \sum_{i \in N} q_i x_{ik} \quad \forall k \in N$$

Inflow to hub l: parcels sent to non-hubs.

$$\sum_{k \in N} t_{kl} = \sum_{j \in V} d_j z_{lj} \quad \forall l \in V$$

$y_k, x_{ik}, z_{lj} \in \{0,1\}, \quad t_{kl} \geq 0 \quad \forall i,j,k,l \in N$

When is $t_{kl} > 0$:

only if: $\exists i$ with $x_{ik}=1$ (non-hub → hub)

$\exists j$ with $z_{lj}=1$ (hub → non-hub)

and $k \neq l$

Variable	Type	Description.
y_k	Binary	$=1$ if node k is a hub
x_{ik}	Binary	$=1$ if non-hub $i \rightarrow$ hub k (collection)
z_{lj}	Binary	$=1$ if hub $l \rightarrow$ non hub j (distribution)
t_{kl}	continuous	Flow from hub $k \rightarrow$ hub l