Risk Evaluation Based on Variable Fuzzy Sets and Information Diffusion Method

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Outline

Background

- Many cities suffer extreme **natural hazards** frequently.
- Natural hazards cause **devastating economic and social losses**.
- Many papers build models to assess natural hazards' **risk evaluation**.
- An effective way to reduce the **negative impacts** and guide relevant decision makers.
- Global climate changeable, modernization requires more **effective emergency plans**.

Literature Reviews

- Qualitative techniques
  - Geographic information systems [1,2], Remote sensing assessment technology [3,4], etc.
- Quantitative methods
  - Bayesian belief network model [5], Regression model [6,7], Probabilistic model [8,9,10], etc.

Literature Reviews

- Qualitative techniques
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- Quantitative methods
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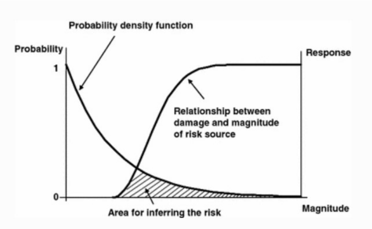
- **Improved methods for quantifying risk**
  - Neural network model [11,13]
  - Fuzzy model to deal with incomplete data sets [14,15,16]
  - Dynamic risk [17,18,19]
  - Others [20,21]

Existing Problems of Risk Evaluation

A general methodology is lacking for assessing the **multiple hazards dynamic risk** when sample sets are **incomplete**

- How to assess **multiple hazards risk**?
- How to deal with the fuzziness related to **multiple hazards indicator**?
- How to assess **dynamic risk** when sample sets are incomplete?

Research Aims and Objectives



$$Risk = \sum_{j=1}^{J} p(x; u_j) \cdot f(x; u_j, v).$$

- **Comprehensive evaluation model to deal with multiple hazards indicator**
  - Variable fuzzy sets (VFS)
  - Information entropy method (IEM)

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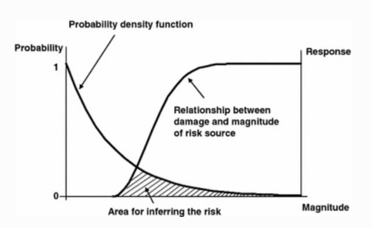

$$Risk = \sum_{j=1}^{J} p(x; u_j) \cdot f(x; u_j, v).$$

- **Comprehensive evaluation model to deal with multiple hazards indicator**
  - Variable fuzzy sets (VFS)
  - Information entropy method (IEM)
- **Dynamic probability risk model to address incomplete data and dynamic risk**
  - Comprehensive multiple hazards level
  - Information diffusion method (IDM)

Outline

Variable fuzzy set theory

- The concept of variable fuzzy set is proposed by Chen [22]
- For fuzzy set U and random element $u \in U$, mapping D

$$u \in U \longmapsto D(u) \in [-1, 1]. \quad (1)$$

- Define **relative membership degree functions** (RMDFs) $\mu_A(u)$ and $\mu_A^c(u)$

$$\mu_A(u) + \mu_A^c(u) = 1 \qquad D_A(u) = \mu_A(u) - \mu_A^c(u), \quad (2)$$

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$$\mu_A(u) = [1 + D_A(u)]/2. \quad (3)$$

Variable fuzzy set theory

- For any sample point x which is coordinated with fuzzy set $u \in U$

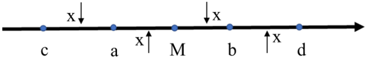

Figure: The linear relationship between different position point x, M and zone [a, b], [c, d]

- Ratio can be used to represent the mapping $D(u, x)$

Variable fuzzy set theory

For $u \in U$, RMDFs can be expressed by **Eq. 4**

$$\begin{cases} \mu_A(u) = 0.5[1 + \left(\frac{x-b}{M-b}\right)] & x \in [M, b] \\ \mu_A(u) = 0.5[1 - \left(\frac{x-b}{d-b}\right)] & x \in [b, d], \end{cases} \tag{4}$$

where parameter M_{rl} [23] is given by Eq. 5

$$M_{rl} = \frac{L-l}{L-1} a_{rl} + \frac{l-1}{L-1} b_{rl}. \tag{5}$$

Normal diffusion function

Let $X = \{x_i | i = 1, 2, \ldots, n\}$ be **one dimension sample sets**, $V = \{v_j | j = 1, 2, \ldots, J\}$ be the **universal field** [24].

$$\mu_{(1)}(x_i, v_j) = exp[-\frac{(x_i - v_j)^2}{2h^2}], \quad x_i \in X, v_j \in V. \tag{6}$$

$$h = \begin{cases} 0.6841(b-a), & for \quad n=5; \\ 0.5404(b-a), & for \quad n=6; \\ 0.4482(b-a), & for \quad n=7; \\ 0.3839(b-a), & for \quad n=8; \\ 2.6581(b-a)/(n-1), & for \quad n \geq 9. \end{cases}$$

$$where \quad b = \max_{1 \leq i \leq n}\{x_i\}, \quad a = \min_{1 \leq i \leq n}\{x_i\}.$$

Figure: Normal diffusion coefficient

Principle of information diffusion

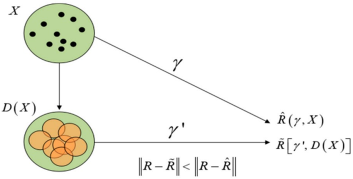

Figure: The principle of information diffusion method

Let $X = \{x_i | i = 1, 2, \ldots, n\}$ be a given sample which can be used to estimate the relationship R on universe V, γ is a reasonable operator. Using it to deal with X directly, we can obtain an estimator for R, denoted as $\hat{R}(\gamma, X)$.

Principle of information diffusion

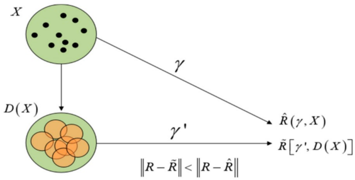

Figure: Principle of information diffusion method

If and only if X is incomplete, there must exist a **reasonable diffusion function** $\mu(x_i, v)$ and a corresponding operator γ' which can transform X into the *fuzzy sample set* $D(X)$. It leads to a diffusion estimator $R'(\gamma', D(x))$ [24].

$$\|R - R'\| < \|R - \hat{R}\|.$$

Corollary: Normal Information Diffusion Estimator

Suppose $\mu(x_i, v_j)$ is given by Eq. 6, let

$$q_j = \sum_{i=1}^{n} \mu(x_i, v_j) \quad and \quad Q = (q_j) \quad j = 1, 2, \ldots, J. \tag{7}$$

Discrete **probability density function** [25] is given by Eq. 8

$$p_j = q_j / H, \quad H = \sum_{j=1}^{J} q_j \quad j = 1, 2, \ldots, J. \tag{8}$$

Corollary: Max-min inference for R_f

Define the matrix $Q = (q_{jkl})$, where $q_{jkl} = \sum_{i=1}^{n} \mu(x_i; u_j, v_k, o_l)$, let

$$\begin{cases} s_l = \max\limits_{1 \le j \le J} q_{jkl}, \quad l = 1,\ 2, \cdots,\ L. \\ \mu_l(u_j, v_k) = \frac{q_{jkl}}{s_l}, \quad j = 1,\ 2, \cdots,\ J \quad k = 1, 2, \cdots, K. \end{cases} \tag{9}$$

The **fuzzy relation** R_f model is denoted as Eq.10

$$r_{jk} = \mu_l(u_j, v_k) \quad and \quad R_f = (r_{jkl})_{J \times K \times L}. \tag{10}$$

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Corollary: Max-min inference for R_f

For two dimension fuzzy output B, Eq.11,

$$\mu_B(o_l) = \max_{\substack{u_j \in U \\ v_k \in V}} \{\min \mu_A(u_j, v_k), r_{jkl}\}, \quad o_l \in O, \tag{11}$$

can make **more accurate inference for R_f model** [25] when the sample sets are **incomplete**.

Outline

Calculation of relative membership degree

- When random point x locates in the lowest grade [26]

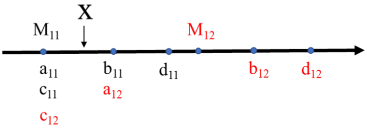

Figure: Position between random point x with parameters

$$\begin{cases} \mu_A(u)_1 = [\mu_A(u)_{11} \quad \mu_A(u)_{12} \quad 0 \quad \cdots \quad 0] \\ \mu_A(u)_{11} + \mu_A(u)_{12} = 1 \\ 0.5 \le \mu_A(u)_{11} \le 1 \\ 0 \le \mu_A(u)_{12} \le 0.5 \end{cases} \tag{12}$$

Indicator Weights

- For each element in $U = (u_{rl})$, where u_{rl} is the measured value from lth class to rth indicator

$$f_{rl} = u_{rl} / \sum_{l=1}^{L} u_{rl} \qquad h_r = \frac{-1}{lnL} \cdot \sum_{l=1}^{L} (f_{rl} ln f_{rl}). \tag{13}$$

- The **entropy coefficient** [27] of indicators can be defined as

$$\omega_r = (1 - h_r)/(R - \sum_{r=1}^{R} h_r). \tag{14}$$

VFS-IEM to evaluate the comprehensive degree value

1. The weights ω_r of indicators.
2. Relative membership degree matrix:

$$\mu_A(u) = (\mu_A(u)_{rl}) \tag{15}$$

3. Comprehensive degree value of each sample:

$$\nu_A(u)_l = \frac{-1}{1 + \left(\frac{\sum_{r=1}^{R}[\omega_r(1-\mu_A(u)_{rl})]^{\alpha}}{\sum_{r=1}^{R}[\omega_r \mu_A(u)_{rl}]^{\alpha}}\right)^{\frac{\beta}{\alpha}}}. \tag{16}$$

4. **Multiple hazards level**:

$$H = (1 \quad 2 \ldots L) \cdot (\nu_A^o(u)_l)^T. \tag{17}$$

VFS-IEM to evaluate the comprehensive degree value

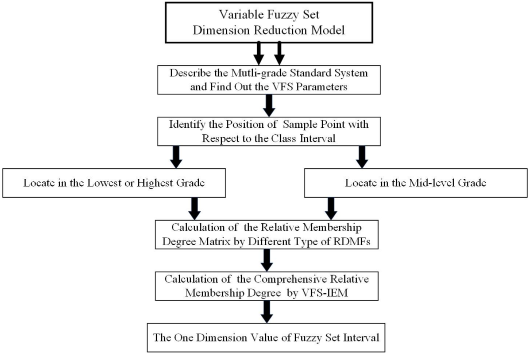

Figure: Flow chart of the VFS-IEM comprehensive evaluation model

Conditional probability distribution

Define 2-dimensional diffusion function of X on U and V as:

$$\mu(x_i; u_j, v_k) = exp[-\frac{(x_i - u_j)^2}{2{h_u}^2} - \frac{(x_i - v_k)^2}{2{h_v}^2}]. \quad (18)$$

Discrete joint probability distribution p_{jk} function and **conditional probability distribution** function are as follows:

$$p_{jk} = \frac{\sum_{i=1}^{n} \mu(x_i; u_j, v_k)}{\sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{i=1}^{n} \mu(x_i; u_j, v_k)}, \quad (19)$$

$$p(x; u_j) = p_{v|u_j}(v|u_j) = \frac{p_{jk}}{\sum_{k=1}^{K} p_{jk}}, j = 1, 2, \cdots J. \quad (20)$$

Vulnerability curve

For a fuzzy input A with membership function $\mu(x_i; u_j, v_k)$ and fuzzy relation R_f model, the inference of output B is given by:

$$\mu_B(o_l) = \max_{\substack{u_j \in U \\ v_k \in V}} \{\min \mu_A(u_j), r_{jkl} \quad \min \mu_A(v_k), r_{jkl}\}, \tag{21}$$

By the center-of-gravity method [14], the **discrete vulnerability curve** is given by Eq. 22.

$$f(x; u_j, v_k) = \frac{\sum_{l=1}^{L} \mu_B(o_l) \cdot o_l}{\sum_{l=1}^{L} \mu_B(o_l)}. \tag{22}$$

Expected value risk

Multiple hazards dynamic risk can be quantified as the **expected value** of conditional probability distribution and vulnerability curve:

$$Risk_{v_k} = \sum_{j=1}^{J} p(x; u_j) \cdot f(x; u_j, v_k), \tag{23}$$

where k denotes dynamic risk of different months.

VFS-IDM Dynamic Probability Risk Model

1. To eliminate the fuzziness of multiple hazards data sets by using VFS-IEM model,
2. Construct the information matrix of each sample by using normal information diffusion function,
3. To change the information matrix into probability matrix or fuzzy relationship matrix,
4. Get the result of conditional probability distribution and vulnerability surface based on Eqs. 20 and 22,
5. Calculate the expected value of dynamic risk by Eq. 23.

Outline

Recap

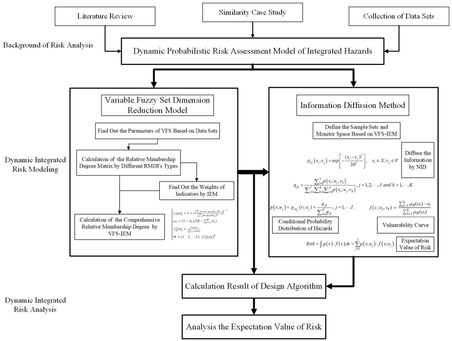

Figure: The VFS-IDM dynamic probability risk assessment model

Highlights

In this paper, the proposed model highlights:

- The calculation of relative membership degree can be classified into three types.
- VFS-IEM model can improve the estimation accuracy of hazards level degree.
- Information diffusion method can be used to solve the low accuracy of risk evaluation.
- The proposed VFS-IDM model can assess the dynamic risk of multiple hazards.

Future Work

- Case study will be the focus and there is a application by using the proposed model
- More accurate method of weight calculation
- The change of vulnerability curve in the internal attributes
- New Algorithms to solve multiple hazards level

Thank you

Thank you for listening!

Q&A

Questions?

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