MA212 Probability and Statistics

Chapter Two: Random Variable

Wenwu GONG & Qianqian WANG
Department of Statistics and Data Science
Email: 12031299@mail.sustech.edu.cn
12032005@mail.sustech.edu.cn

October 17, 2021

1.1 Basic concepts

Random Variable

A random variable (R.V.) is a function from Sample Space (Ω) to the real numbers. i.e., random variable $\mathrm{X} = \mathrm{X}(\omega)$ is the function of sample point ω.

- A random function $\mathrm{X}(\omega)$
- X defines on $(\Omega, \mathcal{A}, \mathrm{P})$ and maps to $(\Re, \mathcal{B}, \mathrm{P}^{\mathrm{X}})$

Discrete Random Variable

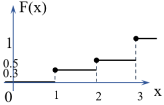
R.V. X only takes a finite or at most a countably infinite number of values. Here are two basic definitions of Discrete R.V.

- Probability Mass Function (PMF): $\mathrm{P}^{\mathrm{X}}(\mathrm{x_k}) = \mathrm{p_k} = \mathrm{P}(\mathrm{X} = \mathrm{x_k})$, A probability measure $\mathrm{p_k}$.
- Cumulative Distribution Function : $\mathrm{F}(\mathrm{x}) = \mathrm{P}(\mathrm{X} \leq \mathrm{x})$, where event $\{\mathrm{X} \leq \mathrm{x}\} = \{\omega | \mathrm{X}(\omega \leq \mathrm{x})\}$ and $-\infty < \mathrm{x} < \infty$.

1.1 Cumulative Distribution Function (Cont'd)

The Cumulative Distribution Function of r.v. X

$$\boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{P}\{\boldsymbol{X} \leq \boldsymbol{x}\}, \qquad -\infty < \boldsymbol{x} < \infty$$



The basic property of Cumulative Distribution Function

① F(x) is monotone non-decreasing function （单调不减函数）

② $0 \leq F(x) \leq 1$, and

$$F(-\infty) = \lim_{x \to -\infty} F(x) = 0 \qquad F(+\infty) = \lim_{x \to +\infty} F(x) = 1$$

③ F(x) is a Right Continuous Function（右连续函数）

$$F(x+0) = \lim_{t \to x^+} F(t) = F(x)$$

- How to use cumulative distribution function to calculate probability? $P(a < X \leq b) = P(X \leq b) - P(X \leq a)$(True difference) $= F(b) - F(a)$
- $P(X < b) = P(X \leq b) - P(X = b) = F(b^-)$, i.e., $F(b - 0)$
- For Discrete Random Variable , $P(x = c) = F(c) - F(c - 0) \neq 0$.

Continuous Random Variable

If the Cumulative Distribution Function of R.V. X can be represented as

$$F(x) = \int_{-\infty}^{x} f(t)d_t = P(X \in (-\infty, x))$$

Then, X is Continuous R.V. and the non-negative inferable function f(t) called Probability Density Function

- $\int_{-\infty}^{\infty} f(x)d_x = 1$
- For Continuous Random Variable , $P(x = c) = F(c) - F(c - \Delta x) = 0$.

1.1 Probability Density Function (Cont'd)

The property of Density Function

The property of Density Function

① $f(t) \geq 0$

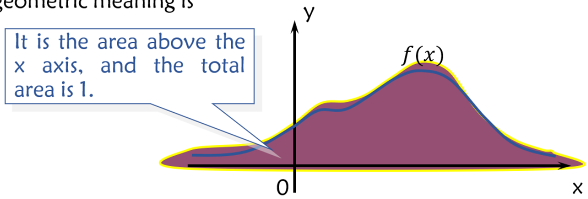
② $\int_{-\infty}^{\infty} f(t)dt = 1$

③ $\forall x_1 < x_2$ $f(x) = \lim_{\Delta x \to 0^+} \frac{F(x + \Delta x) - F(x)}{\Delta x}$

④ At the continuous point of $f(x)$

$$f(x) = F'(x)$$

The geometric meaning is



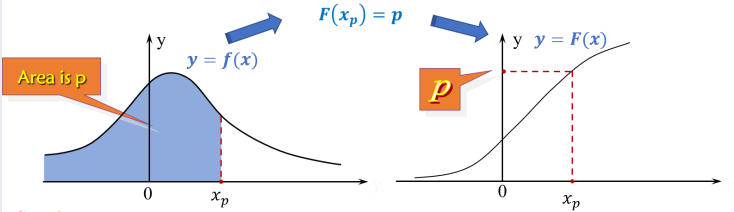
$\mathrm{P^{th}}$ Quantile

p^{th} Quantile（分位数）

Assuming $X \sim f(x)$, if $\forall 0 < p < 1$, and there is a constant x_p which follows

$$P\{X \leq x_p\} = \int_{-\infty}^{x_p} f(x)dx = p$$

Then x_p is called the **$\mathrm{p^{th}}$ Quantile** of the density function $f(x)$.



Specially,

- when p =1/2, x_p is the **median** **（中位数）** of F;
- p=1/4 and p=3/4, which corresponds to **the lower quartile** **（下四分之一分位数）** and **upper quartiles** **（上四分之一分位数）** of F.

1.2 Some important distributions of Discrete Random Variables

Binomial Distribution and Bernoulli Trail

Definition: If the frequency function of r.v. x is

$$P\{X=k\}=C_n^k p^k (1-p)^{n-k} \quad (k=0,1,2,\ldots,n)$$

Then X follows the **Binomial Distribution （二项分布）** with parameter (n, p), noted as $X \sim b(n,p)$

When n=1, b(1, p) is **(0-1)Two Points Distribution**, noted as

$$P\{X=k\}=p^k q^{1-k} \quad (k=0,1)$$

Geometric and Negative Binomial Distribution

Geometric Binomial Distribution is constructed from **independent Bernoulli Trials.**
X = k: k-1 failures followed by k_{th} success, thus

$$p(k)=P\{X=k\}=p(1-p)^{k-1} \qquad k=1,2,3,\ldots$$

Negative binomial distribution: The last trial must be success, and the remaining (r-1) successes can be assigned to the remaining (k-1) trials.

$$p(k)=P\{X=k\}=C_{k-1}^{r-1}\, p^r (1-p)^{k-r}, \qquad k=1,2,3,\ldots$$

The negative binomial distribution when r=1 is Geometric binomial distribution

Poisson Distribution

Definition: Let the value of r.v X be 0, 1, 2, …, the probability is

$$P\{X = k\} = \frac{\lambda^k}{k!} e^{-\lambda}, \qquad k = 0,1,2, \ldots$$

Then X follows the **Poisson distribution** with parameter $\lambda(\lambda > 0)$. Noted as

$$X \sim P(\lambda) \text{ or } X \sim \pi(\lambda)$$

Poisson Theorem（泊松定理）

Let $\lambda > 0$ be a constant, n is a positive integer, $\lim_{n\to\infty} np_n = \lambda$, then for any non-negative integer k,

$$\lim_{n\to\infty} C_n^k p_n^k (1 - p_n)^{n-k} = \frac{\lambda^k e^{-\lambda}}{k!}$$

Thus

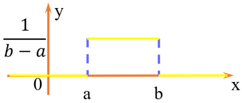
$$C_n^k p^k (1-p)^{n-k} \approx \frac{\lambda^k e^{-\lambda}}{k!} \qquad \text{(When n is very large, p is very small)}$$

1.3 Several types of Continuous Random Variables Distributions

Uniform Density

If the density function of r.v X is

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & x < a \text{ or } x > b \end{cases}$$



Then we call **X** follows the **Uniform Distribution** on a general interval (a, b), noted as $\boldsymbol{X \sim U(a,b)}$**.**

Exponential Distribution

If the **density function** of r.v. X is

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

Then, X follows the **Exponential Distribution** with parameter $\lambda > 0$, noted as $\boldsymbol{X \sim EXP(\lambda)}$**.**

1.3 Several types of Continuous Random Variables Distributions (Cont'd)

Normal distribution

If the **Density Function** of r.v. X is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \qquad -\infty < x < \infty$$

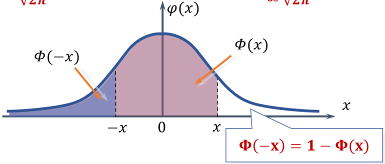
The parameters are $-\infty < \mu < \infty, \sigma > 0$, thus X follows **Normal Distribution** with parameter $(\mu,\ \sigma^2)$, noted as $X \sim N(\mu,\ \sigma^2)$

Standard Normal Distribution（标准正态分布）

When $\mu = 0,\ \sigma^2 = 1$, the normal distribution is called Standard Normal Distribution, noted as $X \sim N(0,1)$. If $X \sim N(\mu, \sigma^2)$, thus $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$

The Probability Density Function and its Distribution Function are:

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, \qquad \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$



The value of $\Phi(x)$ can be found by using the table.

The general steps

The process of finding out the Density Function of $r.v. Y = g(X)$

1. Find out the Distribution Function $F_Y(y) = P\{Y \leq y\}$ of $r.v. Y$

2. Transfer to a **probability calculation of $r.v\ X$**.
   This will use the probability of function $y = g(x)$

3. Find out $f_Y(y) = F'_Y(y)$

$$f_Y(y) = F'_Y(y) = F'_X\left(g^{-1}(y)\right)$$

You should do more exercises to understand this process and there are many problems in Mid-term and Final exams.

1.4 Functions of a Random Variable (Cont'd)

Theorem 1 (Calculation formula)

Let f(x) denote the density function of R.V. X. Y = g(x) is a monotonically function and its inverse function $h(y) = g^{-1}(y)$ is continuous derivable, then the density function of Y = g(x) is

$$f_Y(y) = \begin{cases} |h^{'}(y)|f(h(y)), & h(y) \text{ is meaningful} \\ 0, & \text{others} \end{cases}$$

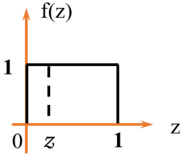
Theorem 2 (Calculation formula, more general)

Assuming the density function of R.V. X is f(x). The function g(x) is piecewise strictly monotonic between the disjoint interval $(a_1, b_1), (a_1, b_1), \dots$. Their inverse functions $h_1(y), h_2(y), \dots$ are continuous derivable. Therefore the density function of Y = g(x) is

$$f_Y(y) = \begin{cases} \sum_{i=1} |h_i^{'}(y)|f(h_i(y)), & h_1(y), h_2(y), \dots \text{ are meaningful} \\ 0, & \text{others} \end{cases}$$

Relationship between Uniform distribution and other continuous distributions

Assuming the density function of r.v X is $f(x)$, the distribution function is $F(x)$. $F(x)$ is strictly increasing between some interval I. The left end point of I is $F = 0$, the right end point is $F = 1$. I can be a bounded interval or unbounded interval. Therefore, $F^{-1}(x)$ is defined on I.



- Since $Z = F(X)$, then $Z \sim U(0,1)$.

  Proof: $P\{Z \leq z\} \ = P\{F(X) \leq z\} \ = P\{X \leq F^{-1}(z)\} \ = F(F^{-1}(z)) \ = z$

- Since $Z \sim U(0,1)$, $X = F^{-1}(z)$, then the distribution function of X is $F(x)$.

  Proof: $P\{X \leq x\} \ = P\{F^{-1}(z) \leq x\} = P\{Z \leq F(x)\} = F(x)$

2.1 Extra Exercises 1

Frequency Distribution

There are 3 boxes, the first box contains 1 white ball and 4 black balls; the second box contains 2 white balls and 3 black balls; the third box contains 3 white balls and 2 black balls. We randomly pick a box and then pick three balls from this box. Let X denote the number of white balls taken.

- The Frequency Distribution of random variable X; Full probability
- What is the probability that the number of picked white ball is not less than 2.

Cumulative Distribution

The density function of random variable X is:

$$p(x) = \begin{cases} 1 - |x|, & -1 \le x \le 1; \\ 0, & \text{others.} \end{cases}$$

Please calculate the Cumulative Distribution Function of X. probability of event

2.1 Answers1

A_i = 'the picked box is number i', $i = 1, 2, 3$

$$P(X=0) = P(A_1)P(X=0\mid A_1) + P(A_2)P(X=0\mid A_2) + P(A_3)P(X=0\mid A_3)$$

$$= \frac{1}{3}\frac{\binom{4}{3}}{\binom{5}{3}} + \frac{1}{3}\frac{\binom{3}{3}}{\binom{5}{3}} + \frac{1}{3}\times 0 = \frac{1}{6},$$

$$P(X=1) = P(A_1)P(X=1\mid A_1) + P(A_2)P(X=1\mid A_2) + P(A_3)P(X=1\mid A_3)$$

$$= \frac{1}{3}\frac{\binom{1}{1}\binom{4}{2}}{\binom{5}{3}} + \frac{1}{3}\frac{\binom{2}{1}\binom{3}{2}}{\binom{5}{3}} + \frac{1}{3}\frac{\binom{3}{1}\binom{2}{2}}{\binom{5}{3}} = \frac{1}{2},$$

$$P(X=2) = P(A_1)P(X=2\mid A_1) + P(A_2)P(X=2\mid A_2) + P(A_3)P(X=2\mid A_3)$$

$$= \frac{1}{3}\times 0 + \frac{1}{3}\frac{\binom{2}{2}\binom{3}{1}}{\binom{5}{3}} + \frac{1}{3}\frac{\binom{3}{2}\binom{2}{1}}{\binom{5}{3}} = \frac{3}{10},$$

$$P(X=3) = P(A_1)P(X=3\mid A_1) + P(A_2)P(X=3\mid A_2) + P(A_3)P(X=3\mid A_3)$$

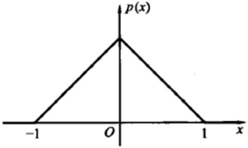
$$= \frac{1}{3}\times 0 + \frac{1}{3}\times 0 + \frac{1}{3}\frac{\binom{3}{3}}{\binom{5}{3}} = \frac{1}{30},$$

Ans : $P(X \geq 2) = P(X=2) + P(X=3) = 1/3$

2.1 Answers2

$$F(x) = \int_{-\infty}^{x} f(t)d_t$$

$$\begin{aligned}
x < -1, &\quad F(x) = \textstyle\int_{-\infty}^{x} 0d_t = 0; \\
-1 \le x < 0, &\quad F(x) = \textstyle\int_{-1}^{x} (1+t)d_t = \frac{x^2}{2} + x + \frac{1}{2}; \\
0 \le x < 1, &\quad F(x) = \textstyle\int_{-1}^{0} (1+t)d_t + \int_{0}^{x} (1-t)d_t = -\frac{x^2}{2} + x + \frac{1}{2}; \\
x > 1, &\quad F(x) = \textstyle\int_{-1}^{0} (1+t)d_t + \int_{0}^{1} (1-t)d_t = 1.
\end{aligned}$$



$$F(x) = \begin{cases} 0, & x < -1, \\ x^2/2 + x + 1/2, & -1 \leqslant x < 0, \\ -x^2/2 + x + 1/2, & 0 \leqslant x < 1, \\ 1, & x \geqslant 1. \end{cases}$$

53. A fire insurance company has high-risk, medium-risk, and low-risk clients, who have, respectively, probabilities .02, .01, and .0025 of filing claims within a given year. The proportions of the numbers of clients in the three categories are .10, .20, and .70, respectively. What proportion of the claims filed each year come from high-risk clients?

54. This problem introduces a simple meteorological model, more complicated versions of which have been proposed in the meteorological literature. Consider a sequence of days and let R_i denote the event that it rains on day i. Suppose that $P(R_i \mid R_{i-1}) = \alpha$ and $P(R_i^c \mid R_{i-1}^c) = \beta$. Suppose further that only today's weather is relevant to predicting tomorrow's; that is, $P(R_i \mid R_{i-1} \cap R_{i-2} \cap \cdots \cap R_0) = P(R_i \mid R_{i-1})$.

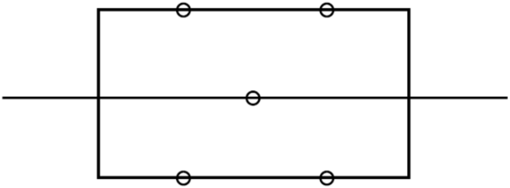
a. If the probability of rain today is p, what is the probability of rain tomorrow?
b. What is the probability of rain the day after tomorrow?
c. What is the probability of rain n days from now? What happens as n approaches infinity?

63. Suppose that the probability of living to be older than 70 is .6 and the probability of living to be older than 80 is .2. If a person reaches her 70th birthday, what is the probability that she will celebrate her 80th?

68. If A is independent of B and B is independent of C, then A is independent of C. Prove this statement or give a counterexample if it is false.

71. Show that if A, B, and C are mutually independent, then $A \cap B$ and C are independent and $A \cup B$ and C are independent.

74. What is the probability that the following system works if each unit fails independently with probability p (see Figure 1.5)?



77. A player throws darts at a target. On each trial, independently of the other trials, he hits the bull's-eye with probability .05. How many times should he throw so that his probability of hitting the bull's-eye at least once is .5?

79. Many human diseases are genetically transmitted (for example, hemophilia or Tay-Sachs disease). Here is a simple model for such a disease. The genotype *aa* is diseased and dies before it mates. The genotype *Aa* is a carrier but is not diseased. The genotype *AA* is not a carrier and is not diseased.

a. If two carriers mate, what are the probabilities that their offspring are of each of the three genotypes?

b. If the male offspring of two carriers is not diseased, what is the probability that he is a carrier?

c. Suppose that the nondiseased offspring of part (b) mates with a member of the population for whom no family history is available and who is thus assumed to have probability p of being a carrier (p is a very small number). What are the probabilities that their first offspring has the genotypes *AA*, *Aa*, and *aa*?

d. Suppose that the first offspring of part (c) is not diseased. What is the probability that the father is a carrier in light of this evidence?

1. Suppose that X is a discrete random variable with $P(X = 0) = .25$, $P(X = 1) = .125$, $P(X = 2) = .125$, and $P(X = 3) = .5$. Graph the frequency function and the cumulative distribution function of X.

15. Two teams, A and B, play a series of games. If team A has probability .4 of winning each game, is it to its advantage to play the best three out of five games or the best four out of seven? Assume the outcomes of successive games are independent.

Supplementary Questions

1. Assume that the frequency function of random variable X is

$$P(X = x) = c\left(\frac{2}{3}\right)^x, \quad x = 1, 2, 3$$

find the value of the coefficient c.

2.3 Extra Exercises 2

Possion Distribution

Suppose that X is Poisson Distribution with parameter λ. If we have $P(X = 1) = P(X = 2)$, what is $P(X = 4)$?

Uniform Distribution

Random variable ζ is uniformly distributed within $(-5, 5)$. What is the probability that the quadratic function $4x^2 + 4\zeta^2 x + 3\zeta^2 - 2 = 0$ has positive root(s)? Please write down the details.

Normal Distribution

Random variable X is normal distribution $N(\mu, \sigma^2)$. Suppose that $P(X \leq 70) = 0.5$ and $P(X \leq 60) = 0.25$, please calculate μ, σ^2.

2.3 Answer 2

Possion Distribution

$$\mathrm{P}(\mathrm{X}=1)=\mathrm{P}(\mathrm{X}=2) \Longrightarrow \lambda \exp^{\lambda}=\frac{\lambda^2}{2}\exp^{\lambda} \Rightarrow \lambda = 2$$
$$\mathrm{P}(\mathrm{X}=4)=\frac{\lambda^4}{4!}\exp^{\lambda}=0.0902$$

Uniform Distribution

$$\begin{aligned} \mathrm{f}(\zeta) &= 1/10, \zeta \in (-5,5) \\ \mathrm{P}(\zeta^2<1)+\mathrm{P}(\zeta^2>2) &= \int_{-1}^{1} 1/10 \mathrm{d}_\zeta + \int_{-5}^{-2} 1/10 \mathrm{d}_\zeta + \int_{2}^{5} 1/10 \mathrm{d}_\zeta \end{aligned}$$

Normal Distribution

$$0.25=\mathrm{P}(\mathrm{X}\le 60)=\Phi(\frac{60-70}{\sigma})=1-\Phi(10/\sigma)\ \mu=70,\ \sigma=14.81$$

31. Phone calls are received at a certain residence as a Poisson process with parameter $\lambda = 2$ per hour.

a. If Diane takes a 10-min. shower, what is the probability that the phone rings during that time?

b. How long can her shower be if she wishes the probability of receiving no phone calls to be at most .5?

33. Let $F(x) = 1 - \exp(-\alpha x^{\beta})$ for $x \geq 0$, $\alpha > 0$, $\beta > 0$, and $F(x) = 0$ for $x < 0$. Show that F is a cdf, and find the corresponding density.

40. Suppose that X has the density function $f(x) = cx^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise.

a. Find c.

b. Find the cdf.

c. What is $P(.1 \leq X < .5)$?

2. Suppose a random variable X follows the Poisson distribution. Find the value of k such that P(X = k) reaches its maximum?

2.4 Homework2

3. Assume that there are 2 defective products out of 15. Select one product each time for 3 times without replacement and let X denote the number of defective products.

   a) Compute the frequency function of X ;
   b) Find the distribution function of X and draw its graph ;
   c) Compute:

   $P\{X \le \frac{1}{2}\}, P\{1 < X \le \frac{3}{2}\}, P\{1 \le X \le \frac{3}{2}\}, P\{1 < X < 2\}.$

4. There are 2500 people of the same age and class level who bought the same life insurance from a company. The chance of each person dying within a year is 0.002. The premium is 12 RMB each year and their family will receive a settlement of 2000 RMB upon the death of an insurant. Compute

   a) The probability that the insurance company has deficit;
   b) The probability of the insurance company to gain a profit of at least 10000? 20000?

1. The density function of X is

$$f(x) = Ae^{-|x|}, \qquad -\infty < x < +\infty,$$

Compute: (1) A ;

(2) $P\{0<X<1\}$;

(3) The distribution function F(x).

Function of a Random Variable

- Random variable X is uniformly distributed within $(-1, 2)$ and Y has the relationship with X:

$$Y = \begin{cases} -1, & X < 0; \\ 1, & X \geq 0. \end{cases}$$

  What is the frequency distribution of random variable Y?

- Random variable X is uniformly distributed within $(-1, 1)$. What is the density function of $Y = \mid X \mid$?

Function of a Random Variable

$F_Y(y) = \Pr(|X| \le y)$ for different values of $y \in [0, 1]$

- By definition:

$$\begin{aligned} y < 0, &\qquad F_Y(y) = 0; \\ 0 \le y < 1, &\qquad F_Y(y) = \int_{-y}^{y} 1/2 d_x = y; \\ y \ge 1, &\qquad F_Y(y) = 1. \end{aligned}$$

- By theorem:

$$0 \le y \le 1, \; h_1(y) = -x, \; h_2(y) = x.$$

$$f_Y(y) = \begin{cases} |-1| \times 1/2 + |1| \times 1/2 = 1, & 0 \le y \le 1, \\ 0, & \text{others.} \end{cases}$$

33. Let $F(x) = 1 - \exp(-\alpha x^{\beta})$ for $x \geq 0$, $\alpha > 0$, $\beta > 0$, and $F(x) = 0$ for $x < 0$. Show that F is a cdf, and find the corresponding density.

40. Suppose that X has the density function $f(x) = cx^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise.

a. Find c. **b.** Find the cdf. **c.** What is $P(.1 \leq X < .5)$?

45. Suppose that the lifetime of an electronic component follows an exponential distribution with $\lambda = .1$.

a. Find the probability that the lifetime is less than 10.
b. Find the probability that the lifetime is between 5 and 15.
c. Find t such that the probability that the lifetime is greater than t is .01.

52. Suppose that in a certain population, individuals' heights are approximately normally distributed with parameters $\mu = 70$ and $\sigma = 3$ in.

a. What proportion of the population is over 6 ft. tall?
b. What is the distribution of heights if they are expressed in centimeters? In meters?

45. Suppose that the lifetime of an electronic component follows an exponential distribution with $\lambda = .1$.

a. Find the probability that the lifetime is less than 10.
b. Find the probability that the lifetime is between 5 and 15.
c. Find t such that the probability that the lifetime is greater than t is .01.

2. Assume that the waiting time X (in minutes) that a customer will be served at a bank window follows an exponential distribution $\exp(1/5)$. If the waiting time is longer than 10 minutes, the customer will leave. Given that the customer goes to the bank 5 times a month and let Y denote the number of times that he leaves the bank without being served. Find the distribution function of Y and $P\{Y \geq 1\}$.

52. Suppose that in a certain population, individuals' heights are approximately normally distributed with parameters $\mu = 70$ and $\sigma = 3$ in.

a. What proportion of the population is over 6 ft. tall?
b. What is the distribution of heights if they are expressed in centimeters? In meters?

53. Let X be a normal random variable with $\mu = 5$ and $\sigma = 10$. Find (a) $P(X > 10)$, (b) $P(-20 < X < 15)$, and (c) the value of x such that $P(X > x) = .05$.

56. If $X \sim N(0, \sigma^2)$, find the density of $Y = |X|$.

1. The frequency function of the r.v. X is as follow:

X	-2	-1	0	1	2
P	1/5	1/6	1/5	1/15	11/30

What is the frequency function of $Y = X^2$?

3. Let $P\{X = k\} = 0.5^k,\ k = 1,2,\dots$, and

$$Y = \begin{cases} 1, & X \text{ is an even number} \\ -1, & X \text{ is an odd number} \end{cases}.$$

Find the distribution function of Y.

59. If U is uniform on $[-1, 1]$, find the density function of U^2.

62. Show that if X has a density function f_X and $Y = aX + b$, then

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$$

3. Three players play 10 independent rounds of a game, and each player has probability $\frac{1}{3}$ of winning each round. Find the joint distribution of the numbers of games won by each of the three players.

2. The density function of r.v. X is:

$$f(x) = \begin{cases} \frac{2x}{\pi^2}, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}.$$

What is the density function of $Y = \sin(X)$?

1. The frequency function of the r.v. X is as follow:

X	-2	-1	0	1	2
P	1/5	1/6	1/5	1/15	11/30

What is the frequency function of $Y = X^2$?

2. The density function of r.v. X is:

$$f(x) = \begin{cases} \frac{2x}{\pi^2}, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}.$$

What is the density function of $Y = \sin(X)$?

3. Let $P\{X = k\} = 0.5^k,\ k = 1,2,\ldots$, and

$$Y = \begin{cases} 1, & X \text{ is an even number} \\ -1, & X \text{ is an odd number} \end{cases}.$$

Find the distribution function of Y.

4. Assume r.v. X follows the Uniform distribution $U(1,2)$. Compute the density function $f_Y(y)$ of Y where $Y = e^{2x}$.