Bayesian Statistics and Hierarchical Bayesian Modeling for Psychological Science

Lecture 08

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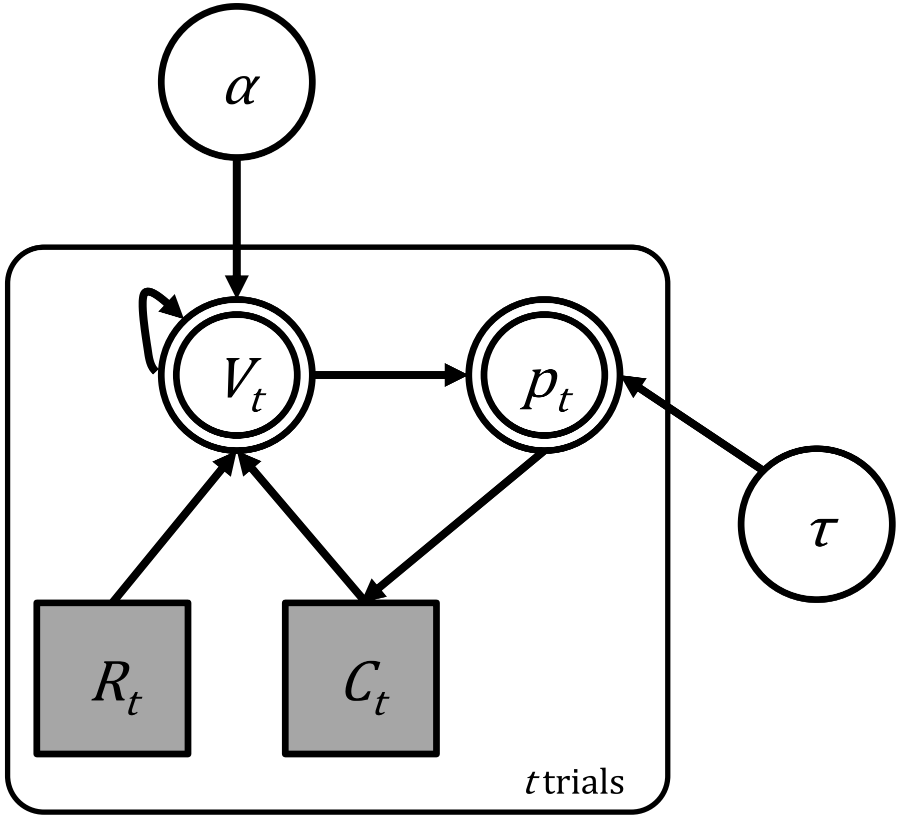
https://github.com/lei-zhang/BayesCog_Wien

universität wien

Fakultät für Psychologie

RL – Implementation



$$\alpha \sim Uniform(0,1)$$

$$\tau \sim Uniform(0,3)$$

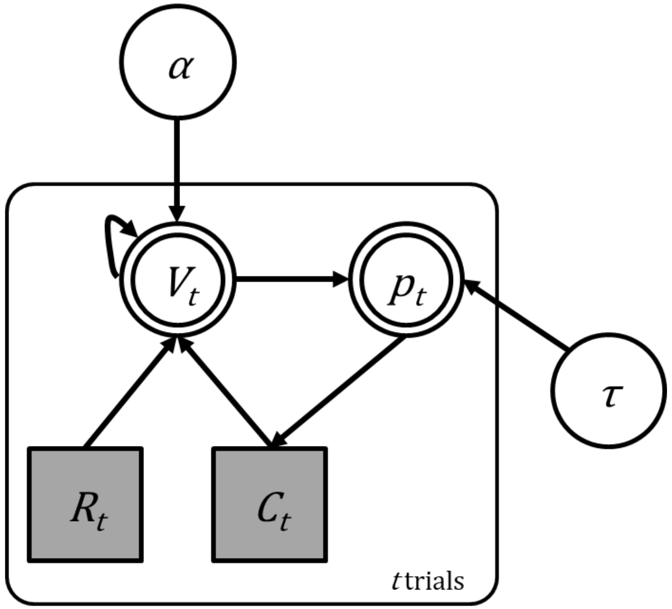
$$p_t(C=A) = \frac{1}{1+e^{\tau(V_t(B)-V_t(A))}}$$

$$V_{t+1}^{c} = V_t^C + \alpha\left(R_t - V_t^C\right)$$

deterministic variable

RL – Implementation

cognitive model
statistics
computing



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$$p_t(C=A) = \frac{1}{1+e^{\tau(V_t(B)-V_t(A))}}$$

$$V_{t+1}^{c} = V_t^{C} + \alpha\left(R_t - V_t^{C}\right)$$

```
transformed data {
  vector[2] initV;
  initV = rep_vector(0.0, 2);
}

model {
  vector[2] v[nTrials+1];
  real pe[nTrials];

  v[1] = initV;

  for (t in 1:nTrials) {
    choice[t] ~ categorical_logit( tau * v[t] );

    pe[t] = reward[t] - v[t,choice[t]];

    v[t+1] = v[t];
    v[t+1, choice[t]] = v[t, choice[t]] + lr * pe[t];
  }
}
```

RL – Implementation

cognitive model
statistics
computing

```
model {
  vector[2] v[nTrials+1];
  real pe[nTrials];

  v[1] = initV;

  for (t in 1:nTrials) {
    choice[t] ~ categorical_logit( tau * v[t] );
    pe[t] = reward[t] - v[t,choice[t]];

    v[t+1] = v[t];
    v[t+1, choice[t]] = v[t, choice[t]] + lr * pe[t];
  }
}
```

```
model {
  vector[2] v;
  real pe;

  v = initV;

  for (t in 1:nTrials) {
    choice[t] ~ categorical_logit( tau * v );
    pe = reward[t] - v[choice[t]];

    v[choice[t]] = v[choice[t]] + lr * pe;
  }
}
```

RL – Fitting with Stan

cognitive model
statistics
computing

```
…/06.reinforcement_learning/_scripts/reinforcement_learning_single_parm_main.R
```

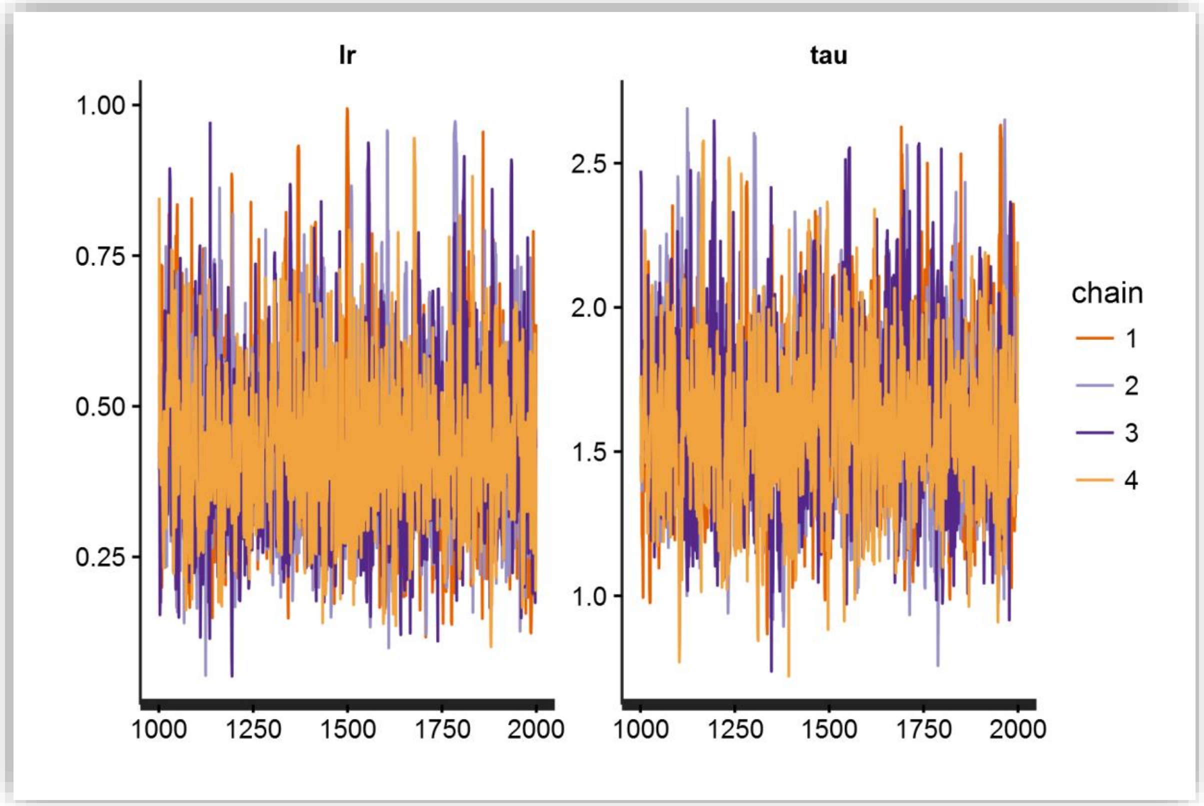
TASK: fit the model for single participants

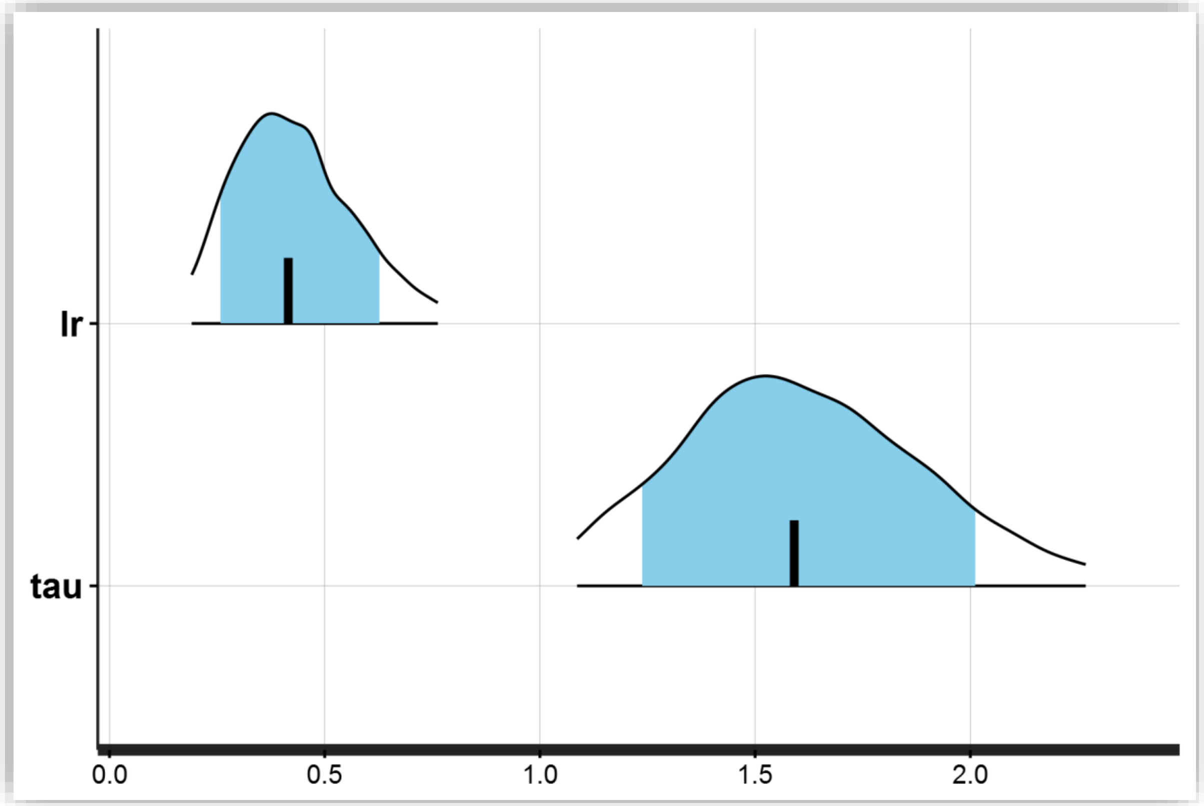
```
> source('_scripts/reinforcement_learning_single_parm_main.R’) # a function

> fit_rl1 <- run_rl_sp(multiSubj = FALSE)
```

```
> load('_data/rl_sp_ss.RData')
> head(rl_ss)
     [,1] [,2]
[1,]    2   -1
[2,]    1    1
[3,]    1    1
[4,]    1    1
[5,]    2   -1
[6,]    1    1
```

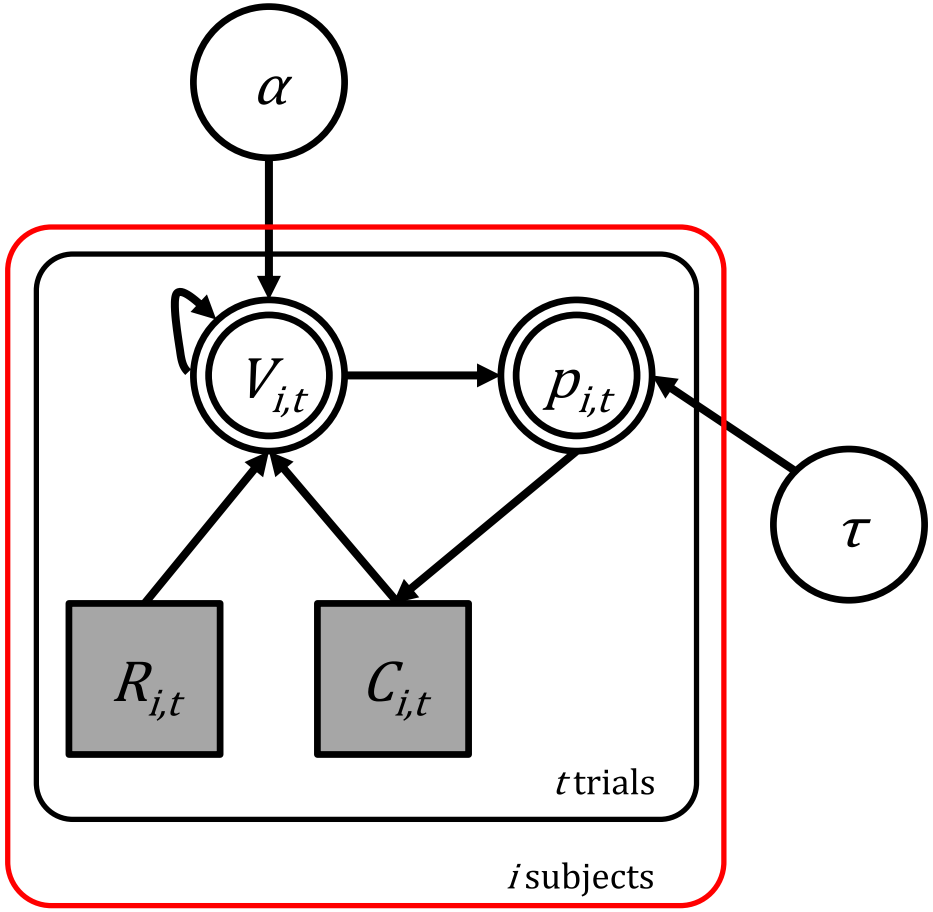
RL – MCMC Output

cognitive model
statistics
computing





Fitting Multiple Participants as ONE




```
model {
  for (s in 1:nSubjects) {
    vector[2] v;
    real pe;
    v = initV;

    for (t in 1:nTrials) {
      choice[s,t] ~ categorical_logit( tau * v );
      pe = reward[s,t] - v[choice[s,t]];
      v[choice[s,t]] = v[choice[s,t]] + lr * pe;
    }
  }
}
```

Exercise X

cognitive model
statistics
computing

```
…/06.reinforcement_learning/_scripts/reinforcement_learning_single_parm_main.R
```

TASK:

(1) complete the model (Tip: the for-loop)

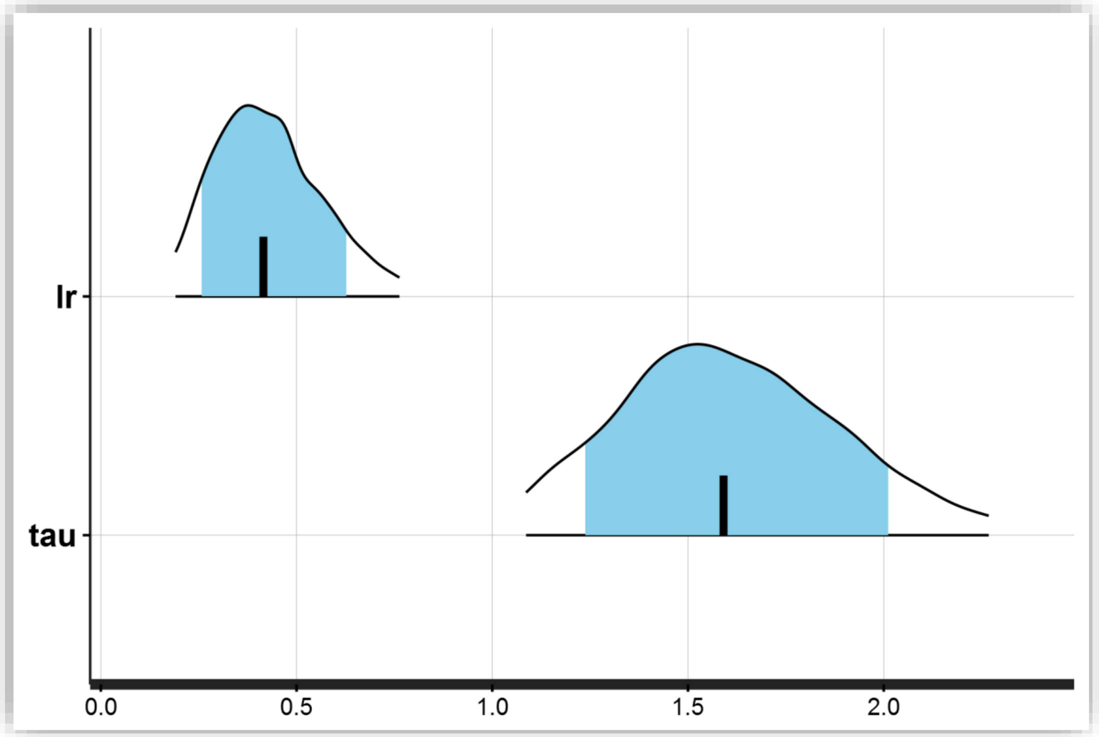
(2) fit the model for multiple participants (assuming same parameters)

```
> source('_scripts/reinforcement_learning_single_parm_main.R')

> fit_rl2 <- run_rl_sp(multiSubj = TRUE)
```

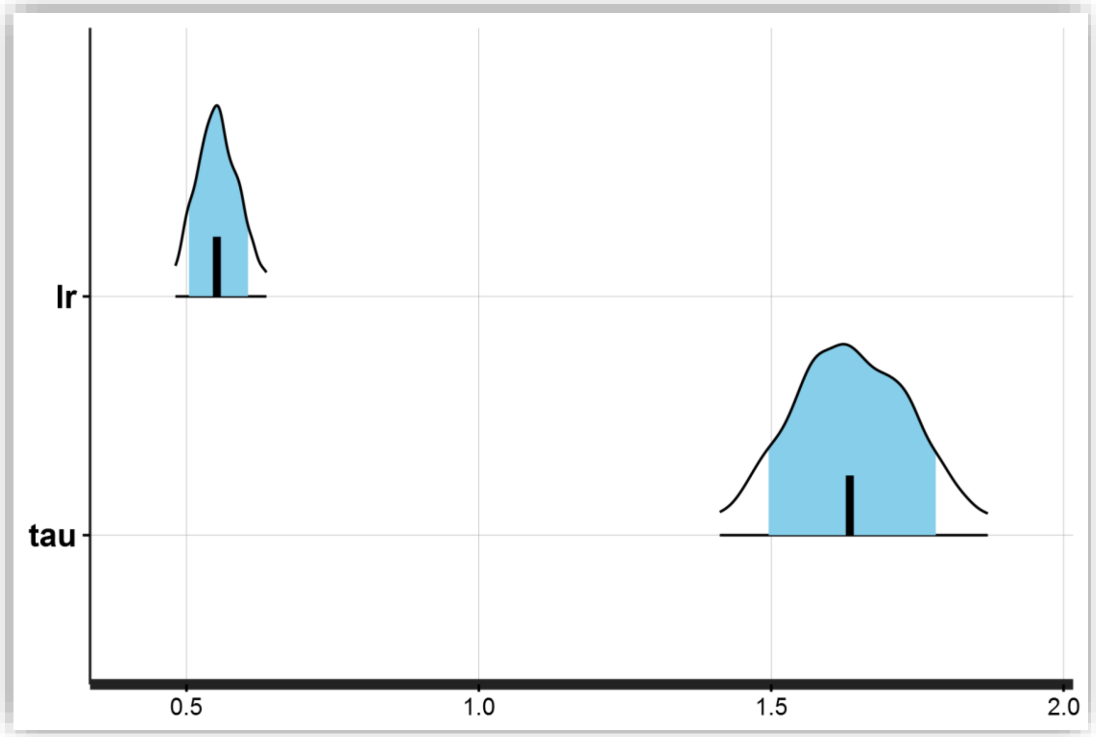
Exercise X

cognitive model
statistics
computing

N = 1

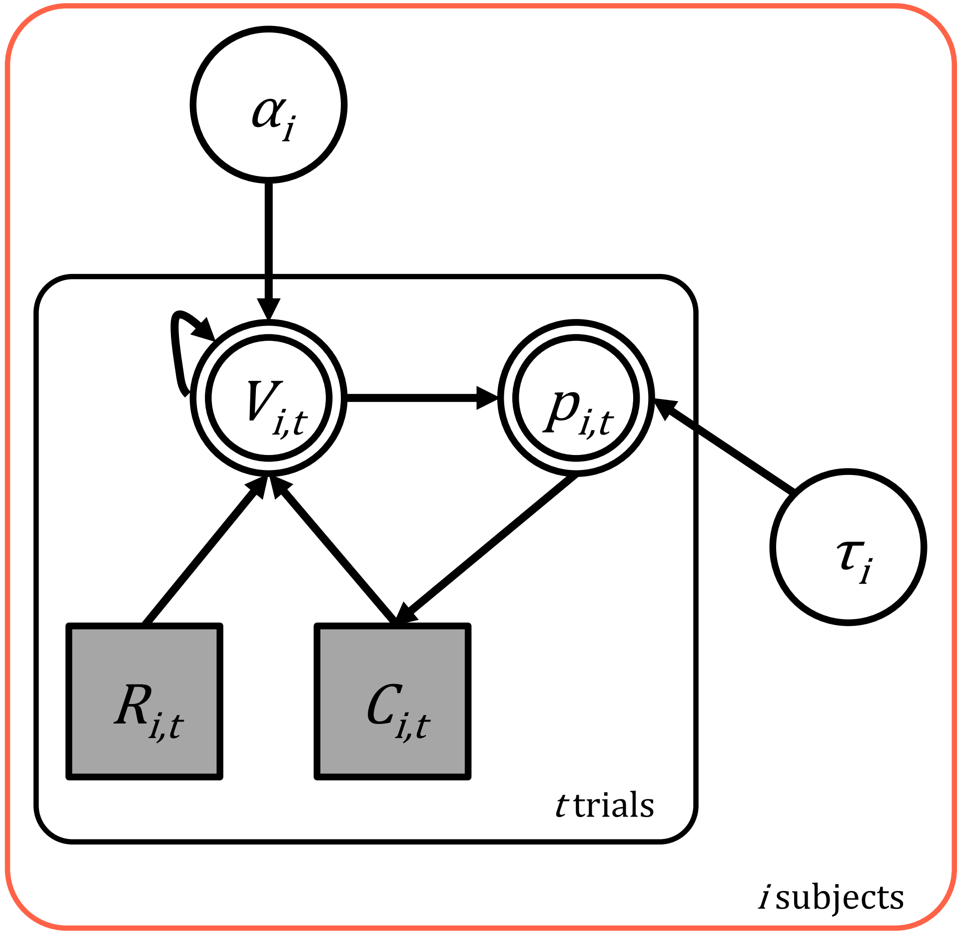


N = 10



Fitting Multiple Participants Independently

cognitive model
statistics
computing



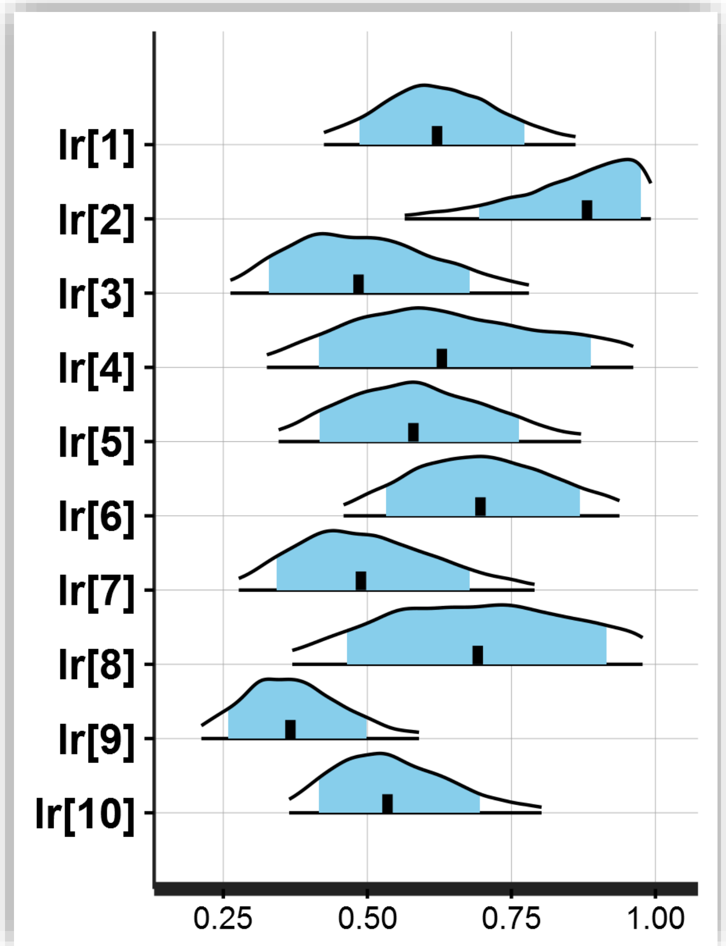

```
model {
  for (s in 1:nSubjects) {
    vector[2] v;
    real pe;
    v = initV;

    for (t in 1:nTrials) {
      choice[s,t] ~ categorical_logit( tau[s] * v );
      pe = reward[s,t] - v[choice[s,t]];
      v[choice[s,t]] = v[choice[s,t]] + lr[s] * pe;
    }
  }
}
```

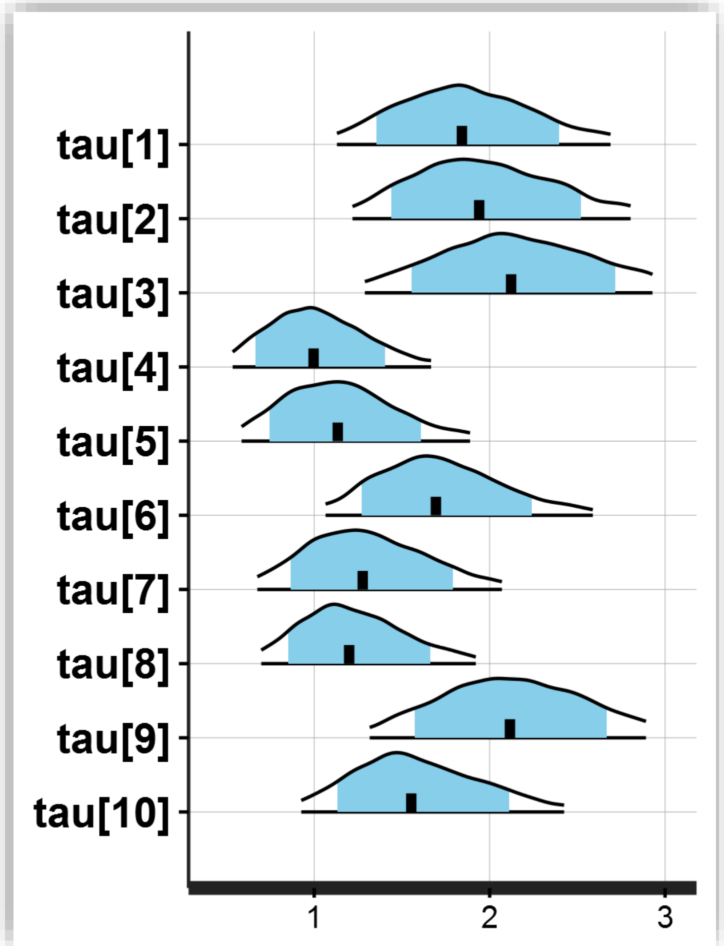
Individual Fitting

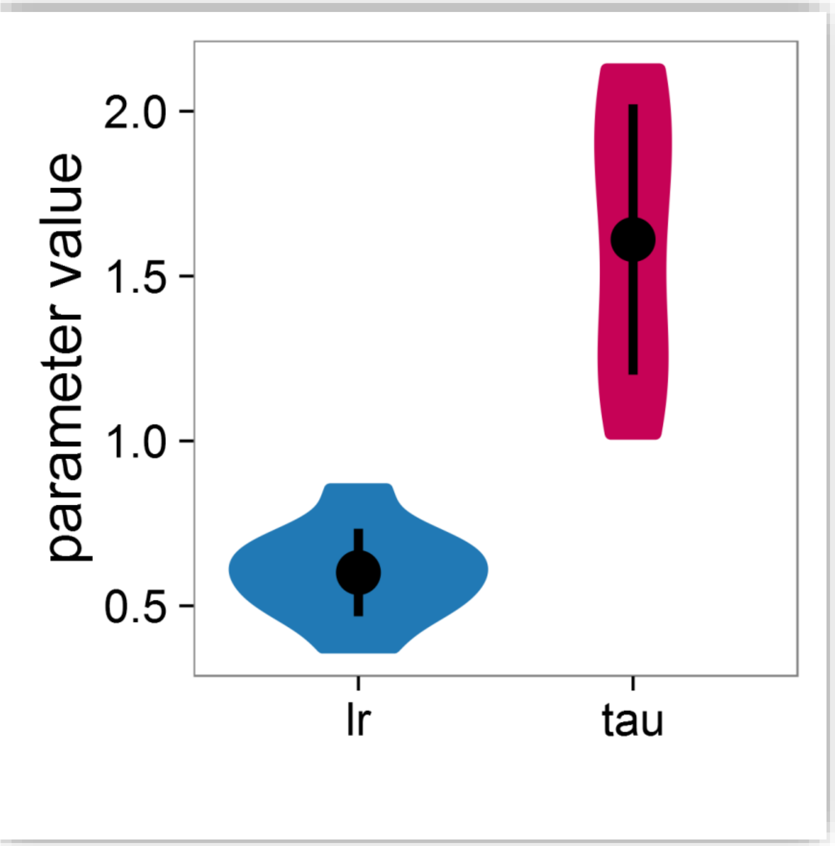
cognitive model

statistics

computing

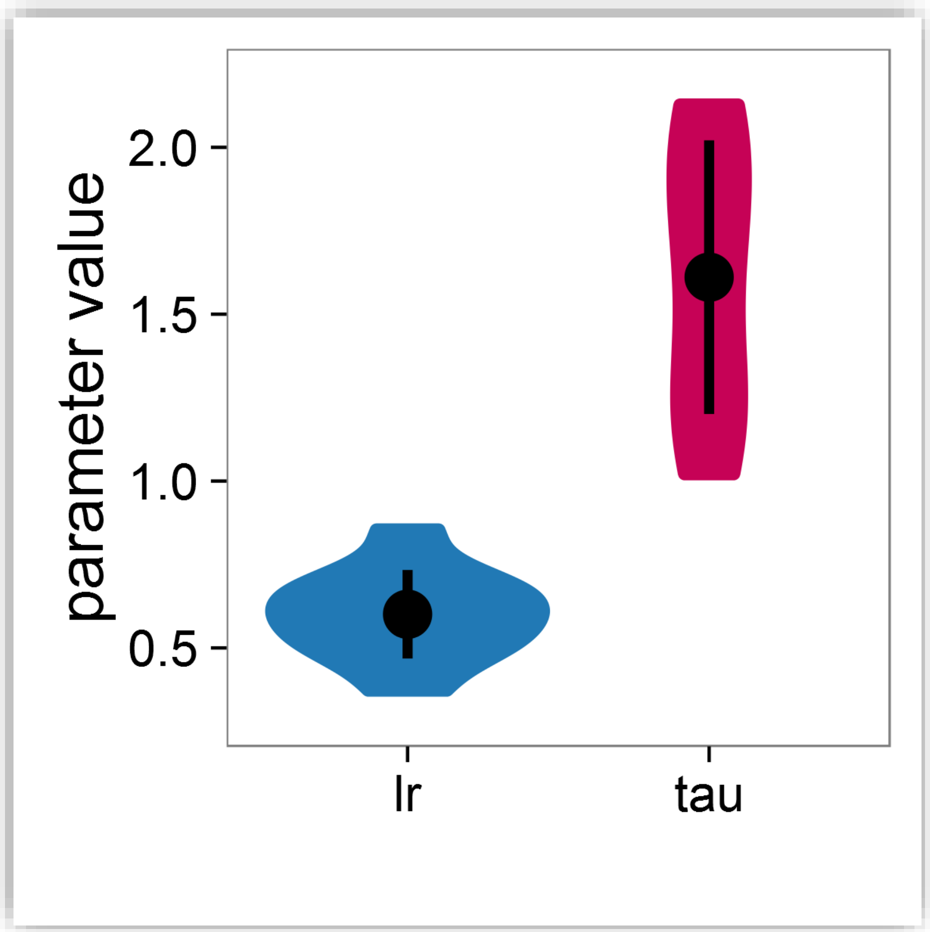




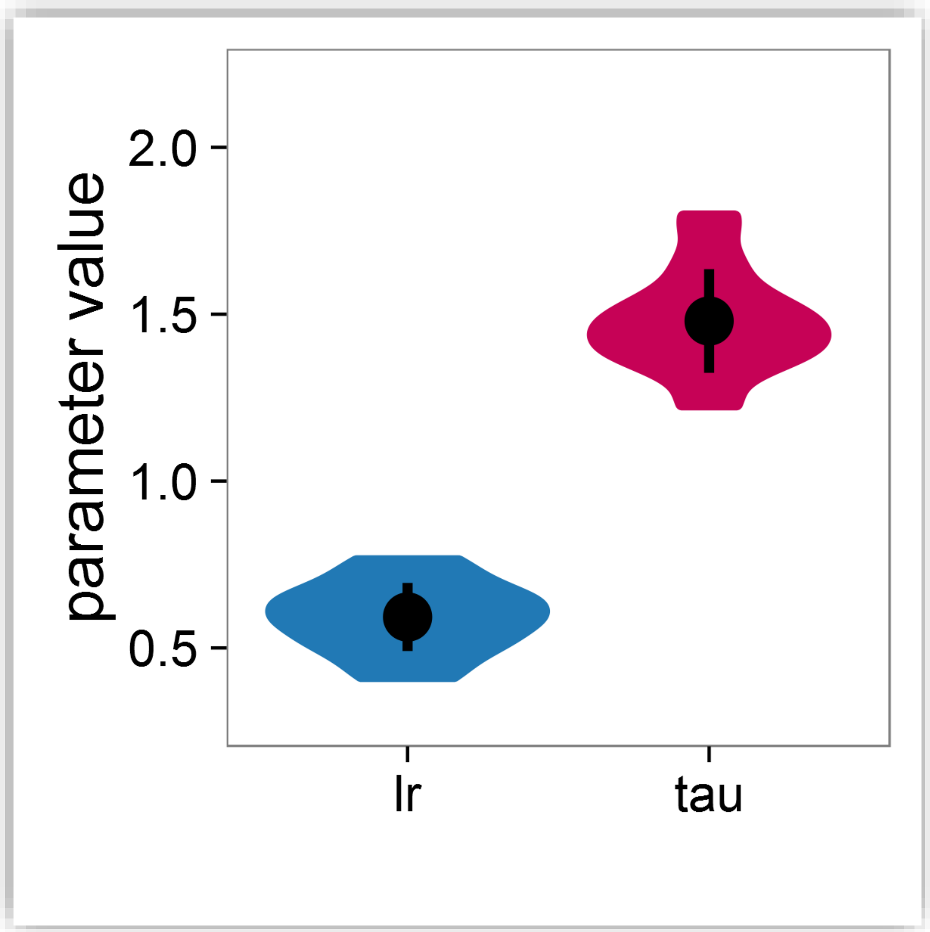


Comparing with True Parameters

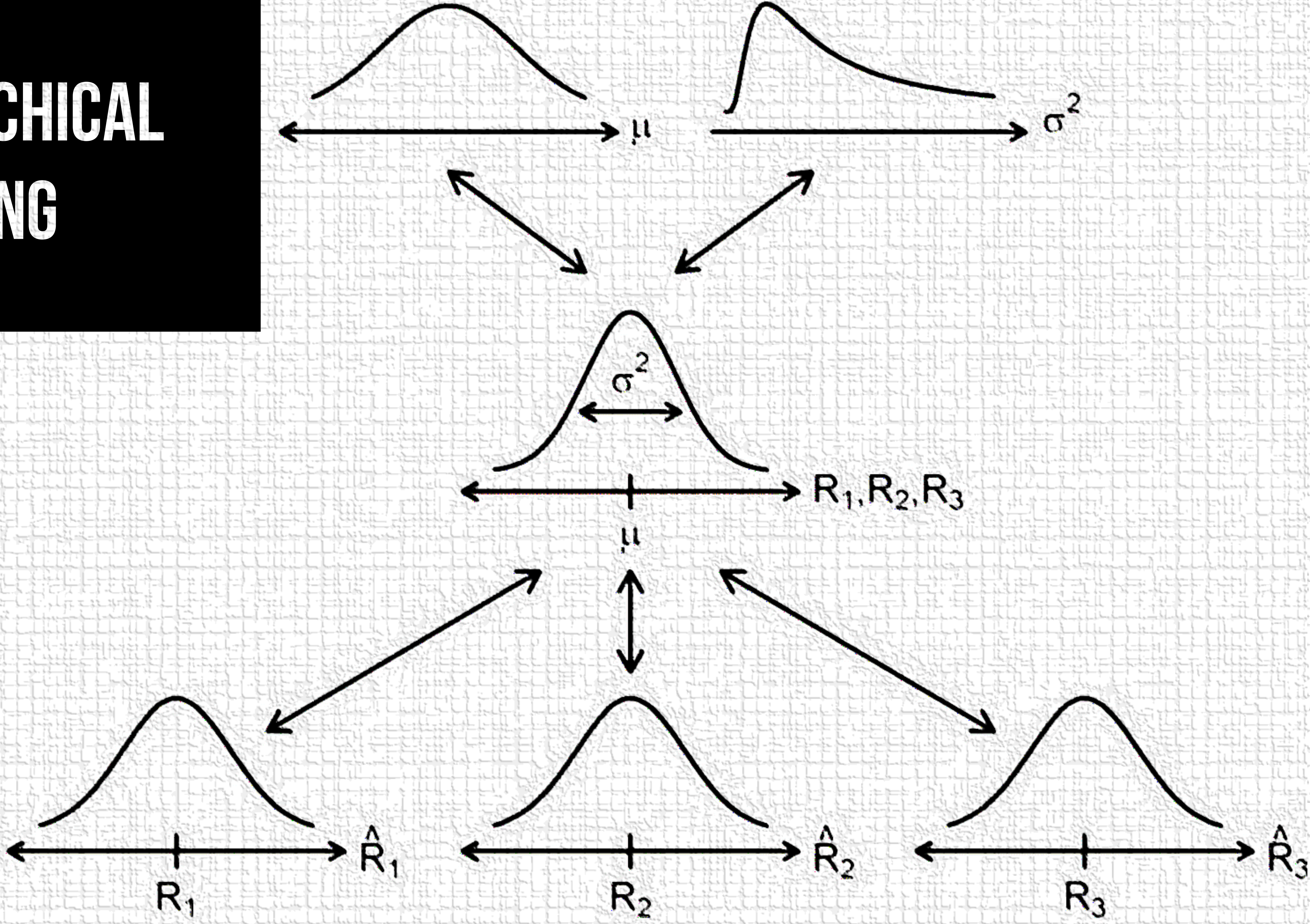
Posterior Means

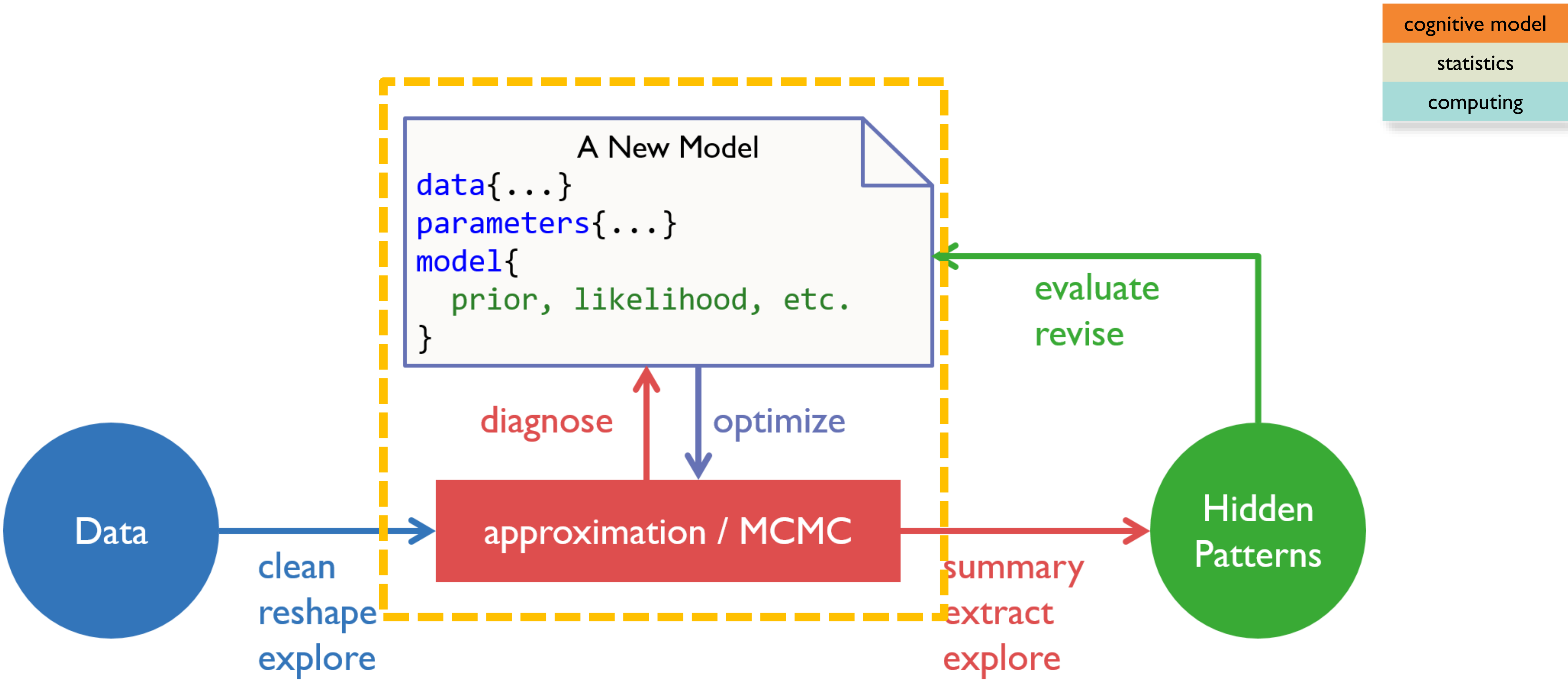


True Parameters



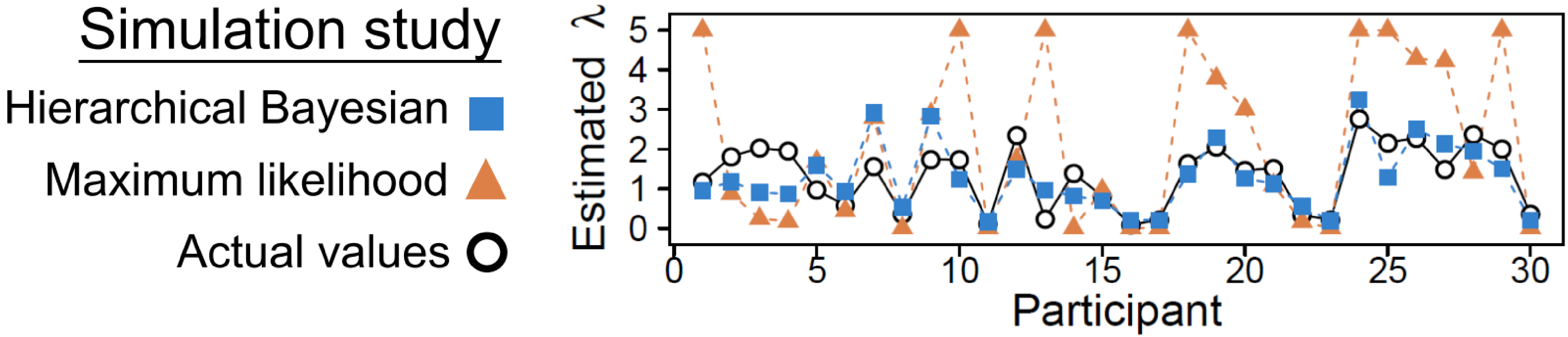
HIERARCHICAL MODELING




cognitive model
statistics
computing
A New Model
data{...}
parameters{...}
model{
prior, likelihood, etc.
}
evaluate
revise
diagnose
optimize
Data
approximation / MCMC
Hidden Patterns
clean
reshape
explore
summary
extract
explore

Why Hierarchical Bayesian Cognitive Modeling?

cognitive model
statistics
computing



Ahn et al. (2011)

Why Hierarchical Bayesian Cognitive Modeling?

cognitive model
statistics
computing

Fixed effects

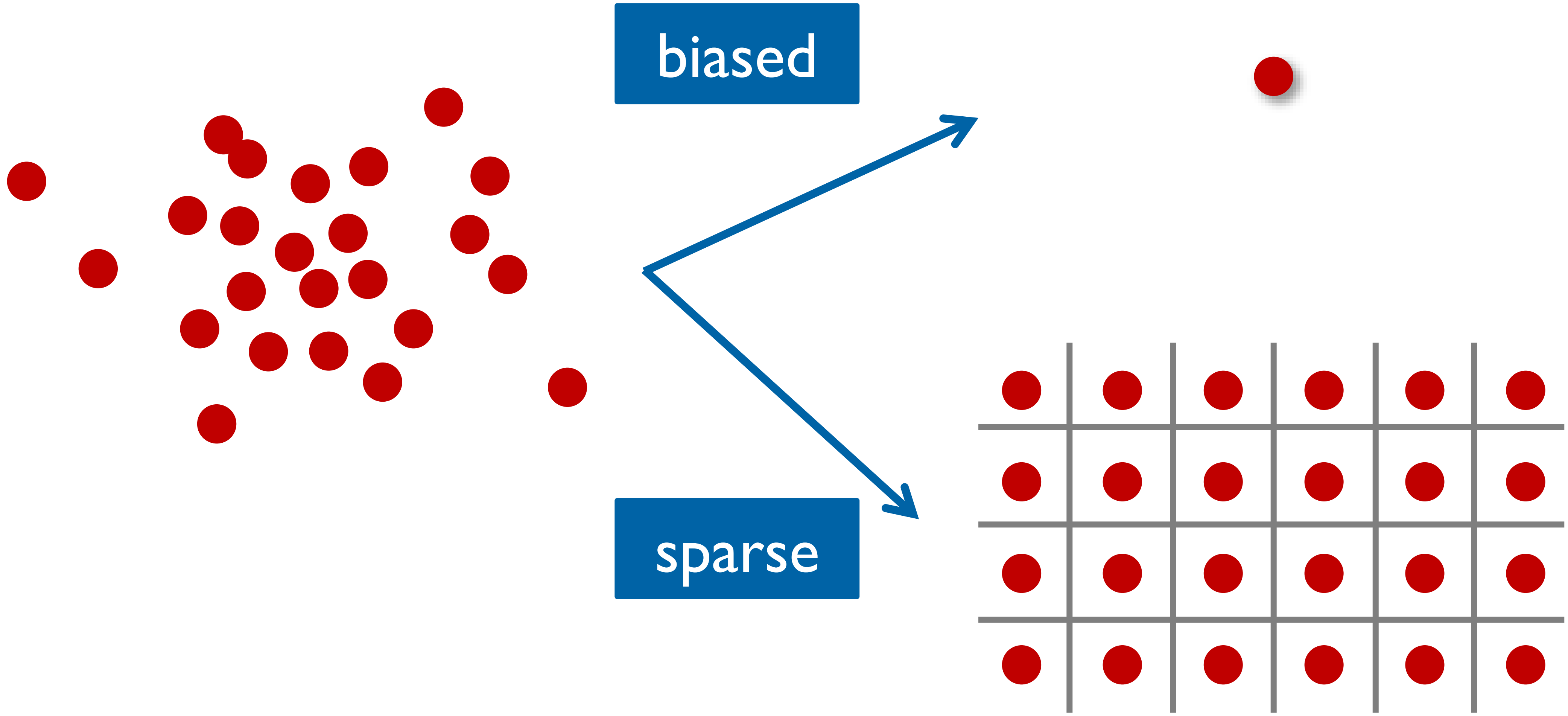
- all subjects are fitted with the same set of parameters
- worse model fit than "random effects"

Random effects

- each subject is fitted independently of the others
- best model fit for each subject
- parameter estimates can be noisy

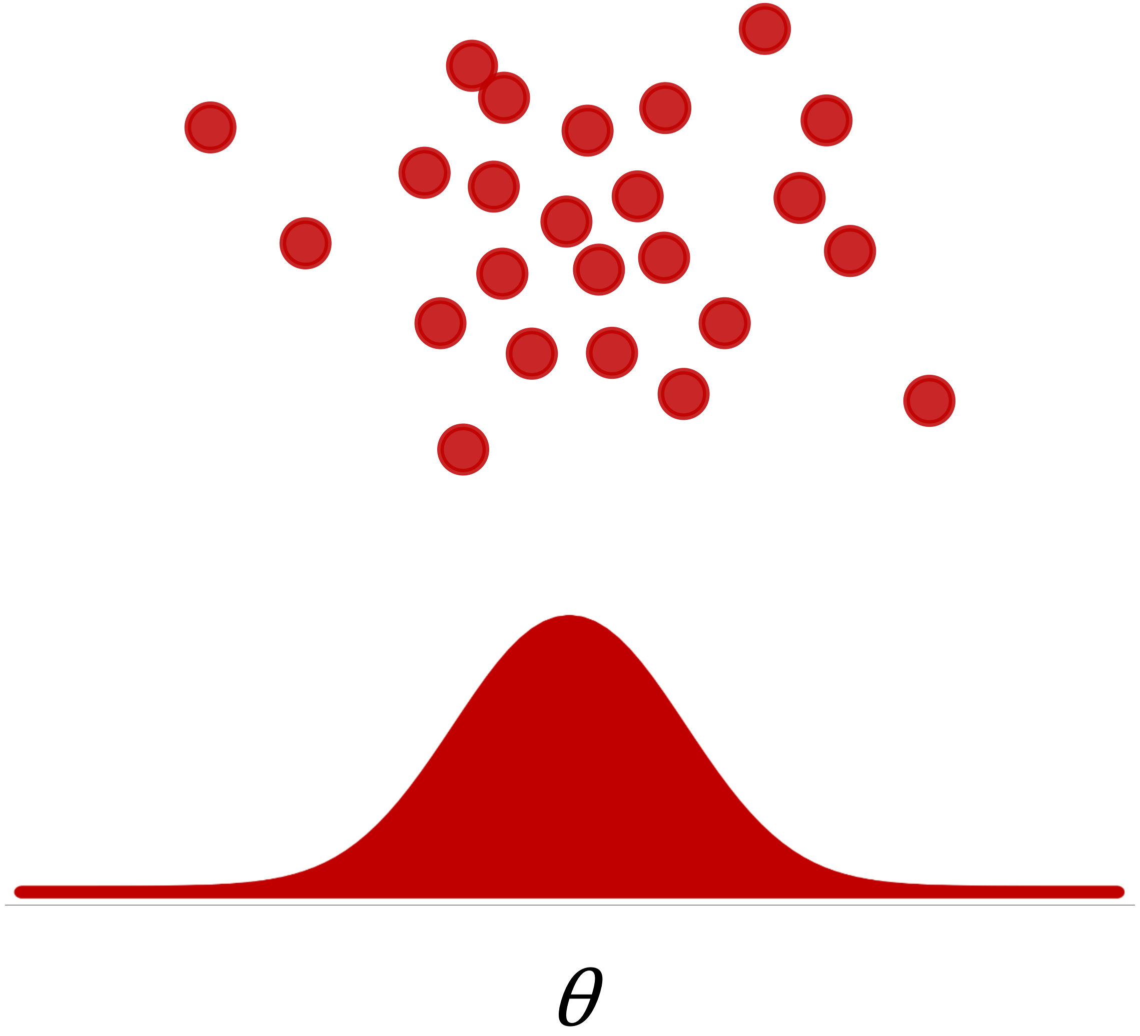
Adapted from Jan Gläscher's workshop

Fitting Multiple Participants



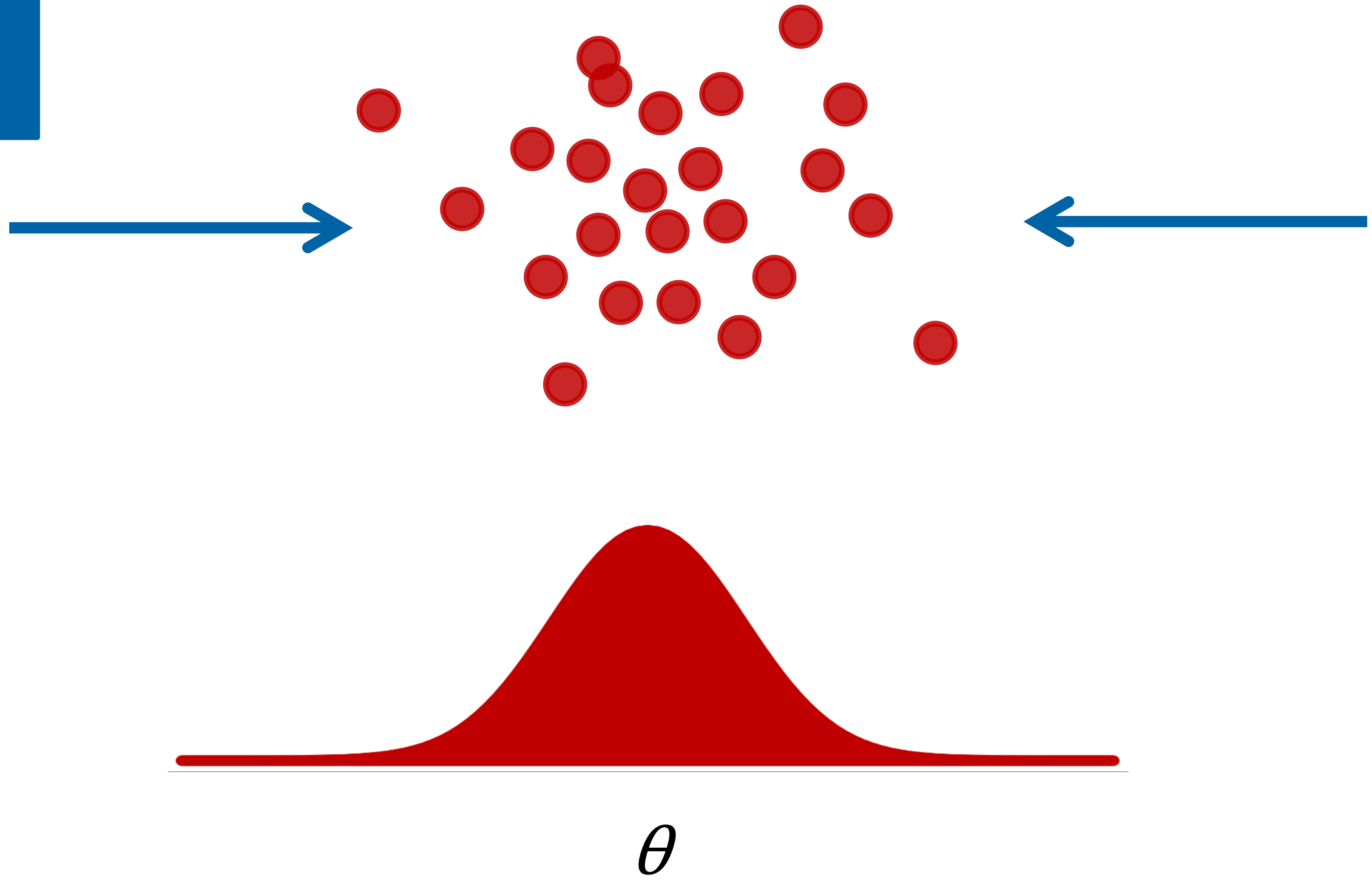
Fitting Multiple Participants

cognitive model
statistics
computing



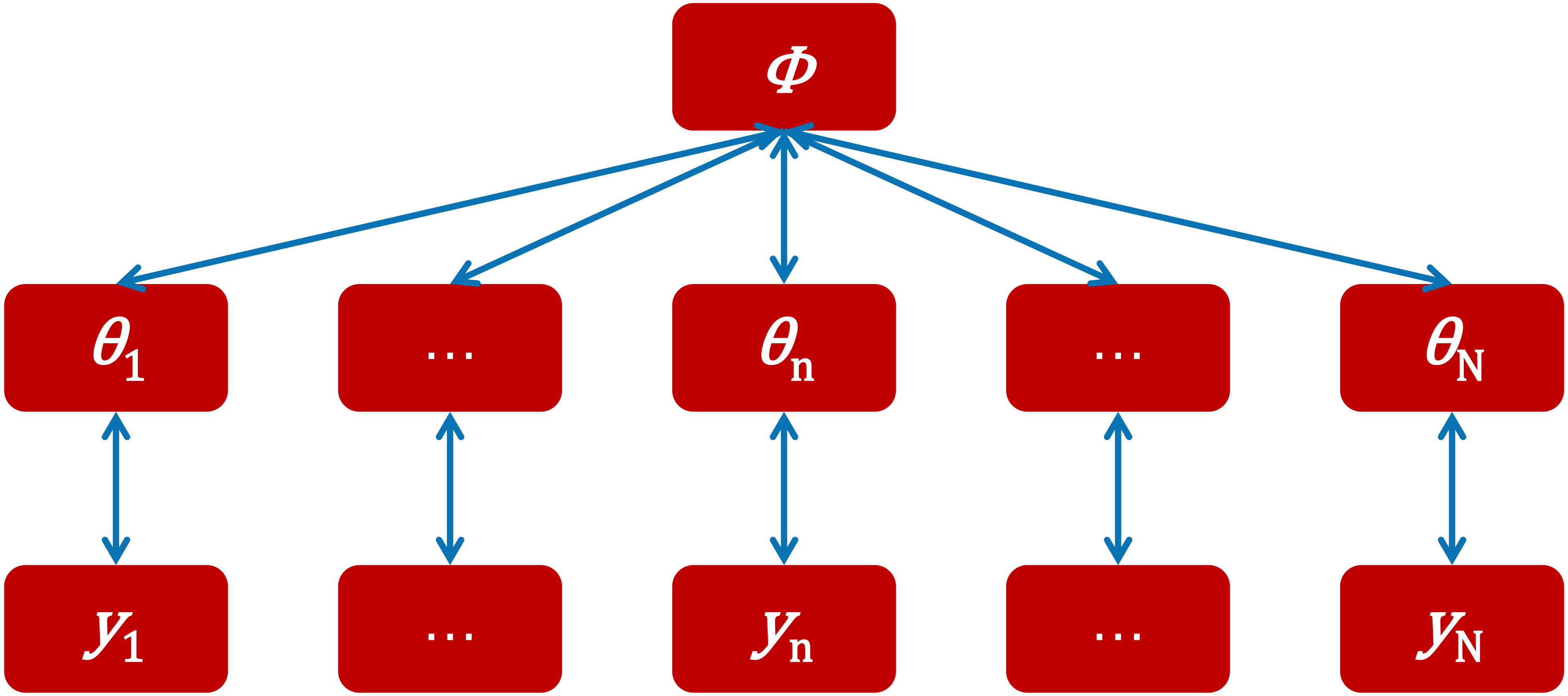
Fitting Multiple Participants

shrinkage effect



Hierarchical Structure

cognitive model
statistics
computing



Hierarchical Structure

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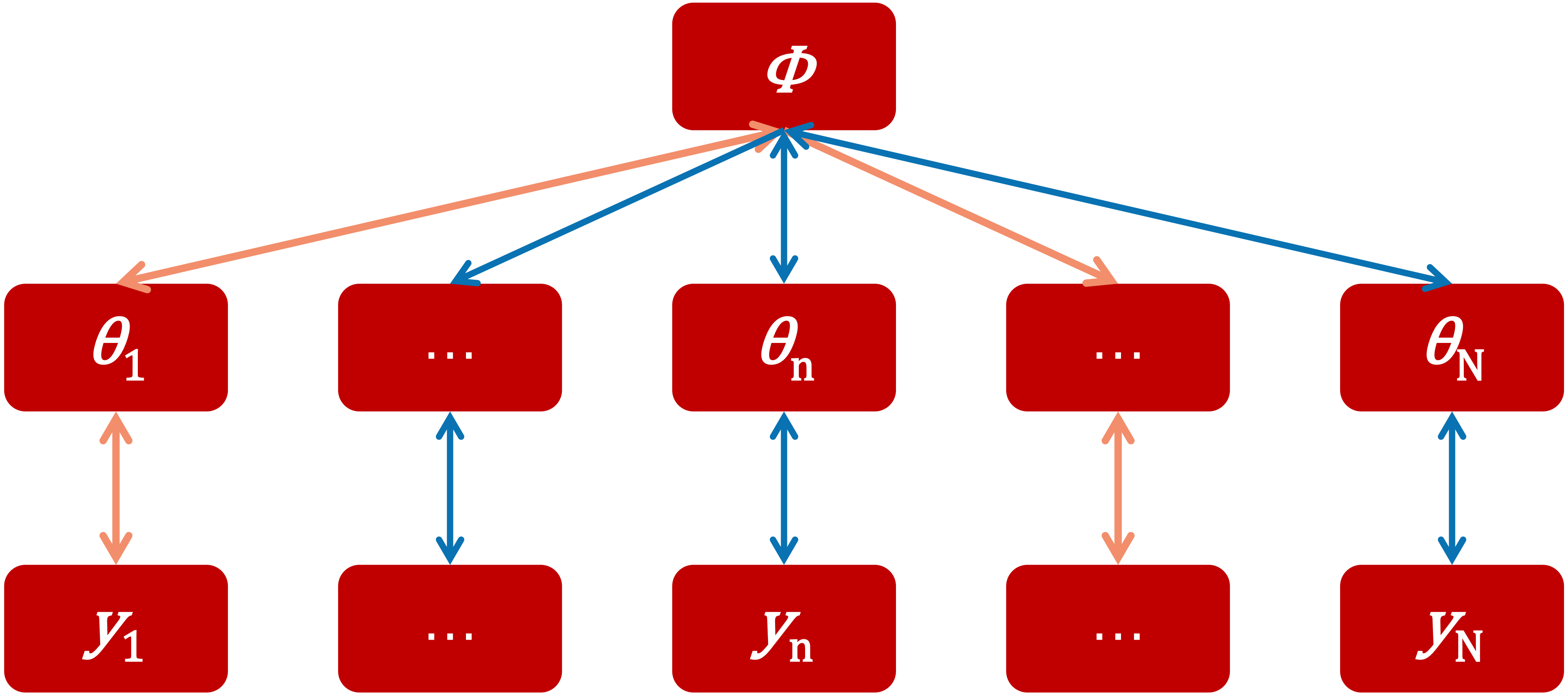
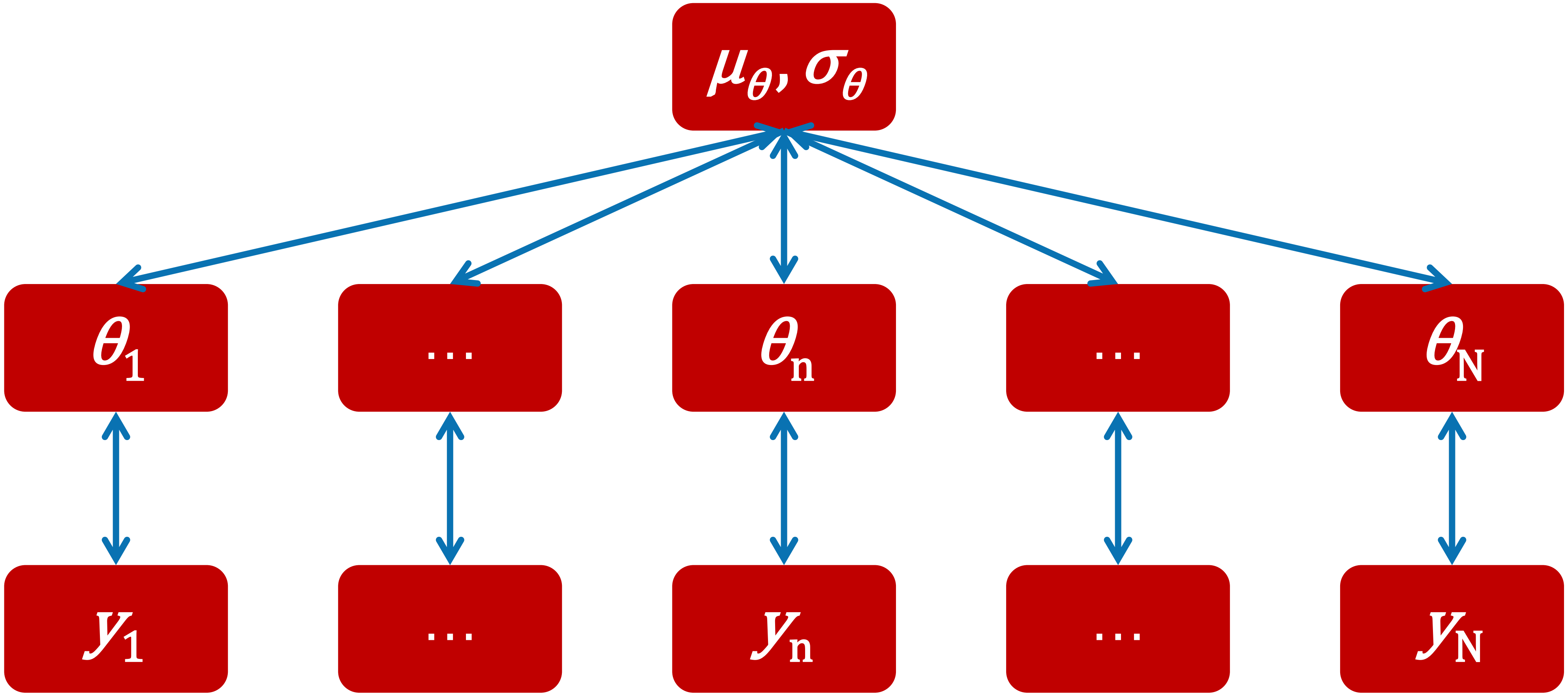
statistics
computing

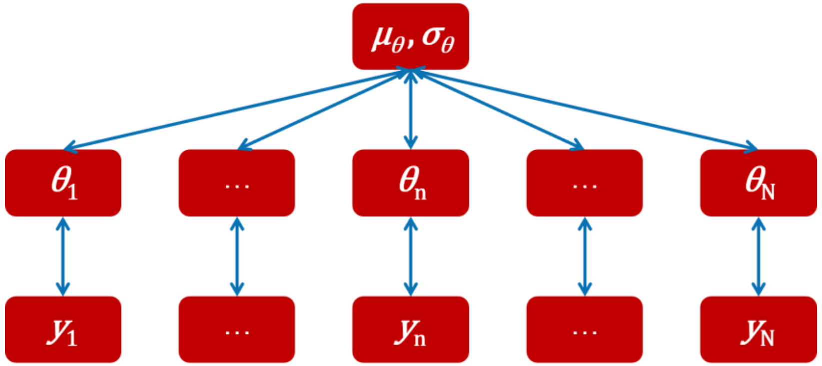


Hierarchical Structure

cognitive model
statistics
computing



Hierarchical Structure



$$P(\Theta,\Phi \mid D) = \frac{P(D \mid \Theta,\Phi)P(\Theta,\Phi)}{P(D)} \propto P(D \mid \Theta)P(\Theta \mid \Phi)P(\Phi)$$