Bayesian Statistics and Hierarchical Bayesian Modeling for Psychological Science

Lecture 06

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https://github.com/lei-zhang/BayesCog_Wien

universität wien

Fakultät für Psychologie

Evaluate the review

- the ability to briefly and clearly summarize the main findings
- critical thinking
  - regarding the design
  - regarding the analyses (stats)
- appreciate the use of cognitive modeling
  - is the modeling approach appropriate to answer the research question?
  - is the interpretation sound?
  - could there be alternative models?
  - model recovery
  - posterior check etc.

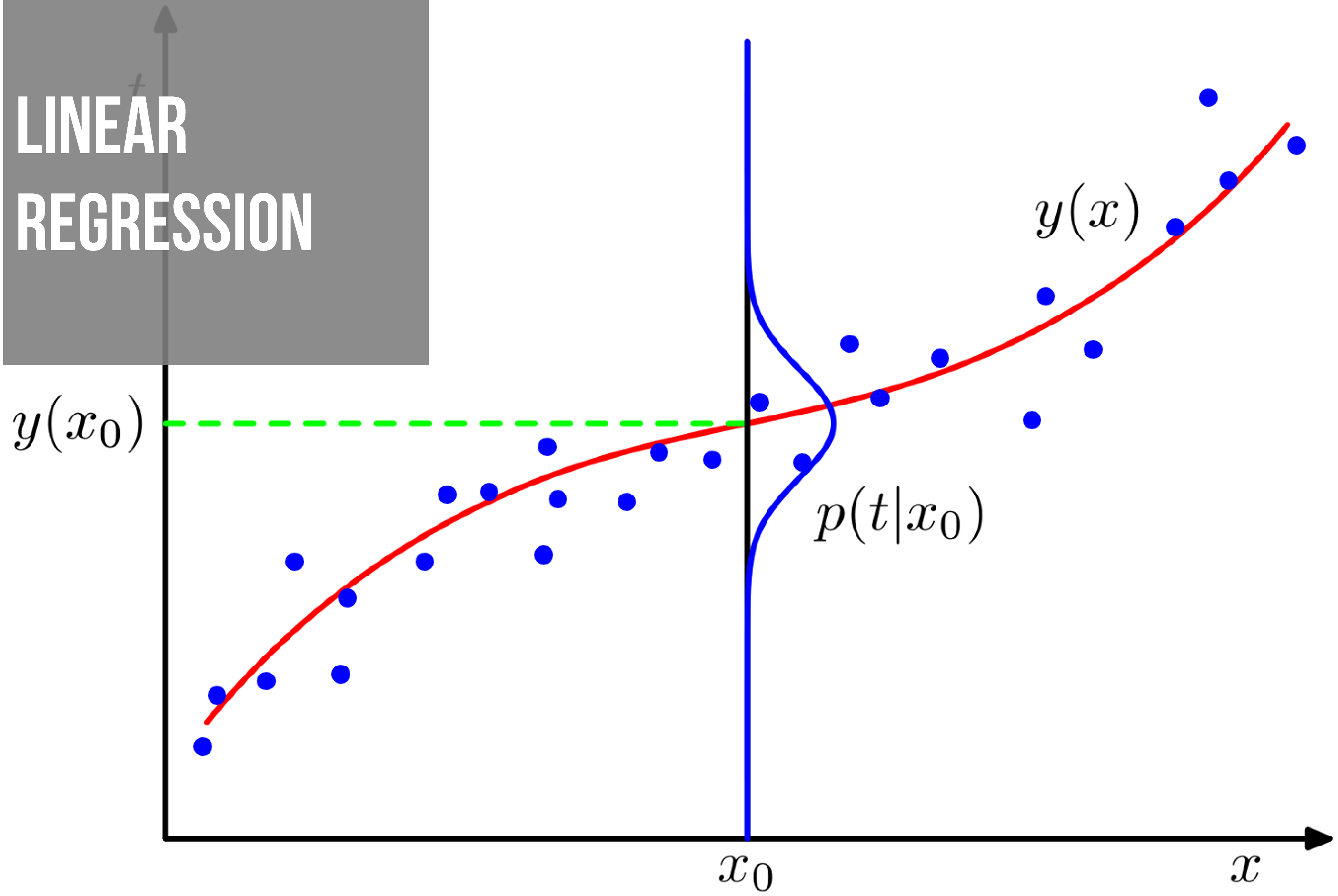
1st	2nd
70%	40%
30%	60%

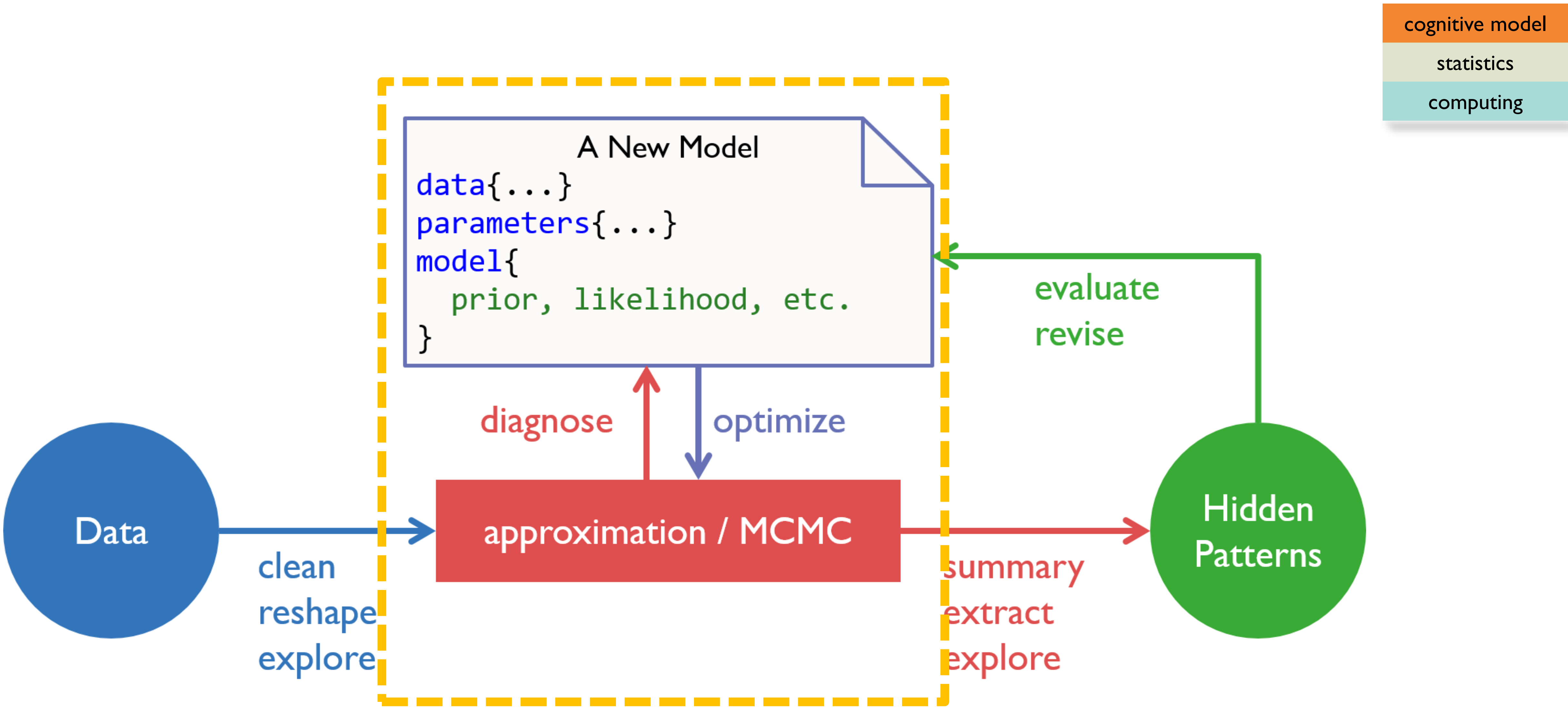
Recap

What we've learned…

- R Basics
- probability distributions
- Bayes' theorem, $p(\theta|D)$
- Binomial model
- MCMC and Stan

LINEAR REGRESSION



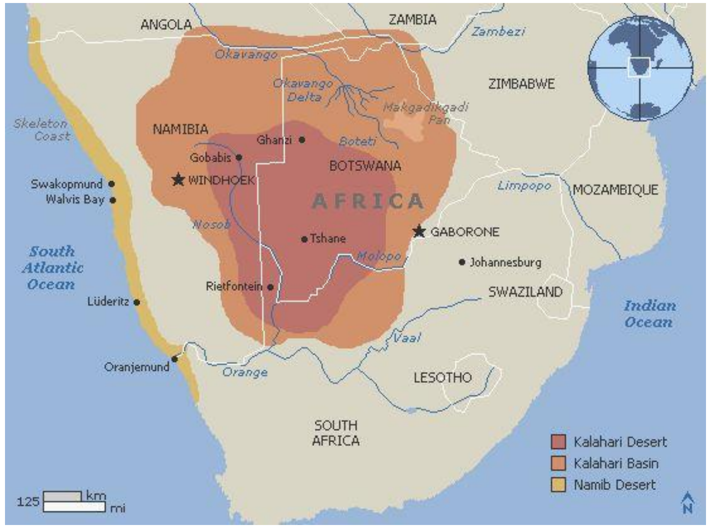

cognitive model
statistics
computing
A New Model
data{...}
parameters{...}
model{
prior, likelihood, etc.
}
evaluate
revise
diagnose
optimize
Data
clean
reshape
explore
approximation / MCMC
summary
extract
explore
Hidden Patterns

Linear Regression: height ~ weight

```
…/04.regression_height/_scripts/regression_height_main.R
```

make scatter plot and fit the model with `lm()`

```
>load('_data/height.RData')
>d <- Howell1
>d <- d[ d$age >= 18 , ]
>head(d)
height   weight age male
1 151.765 47.82561  63    1
2 139.700 36.48581  63    0
3 136.525 31.86484  65    0
4 156.845 53.04191  41    1
5 145.415 41.27687  51    0
6 163.830 62.99259  35    1
```

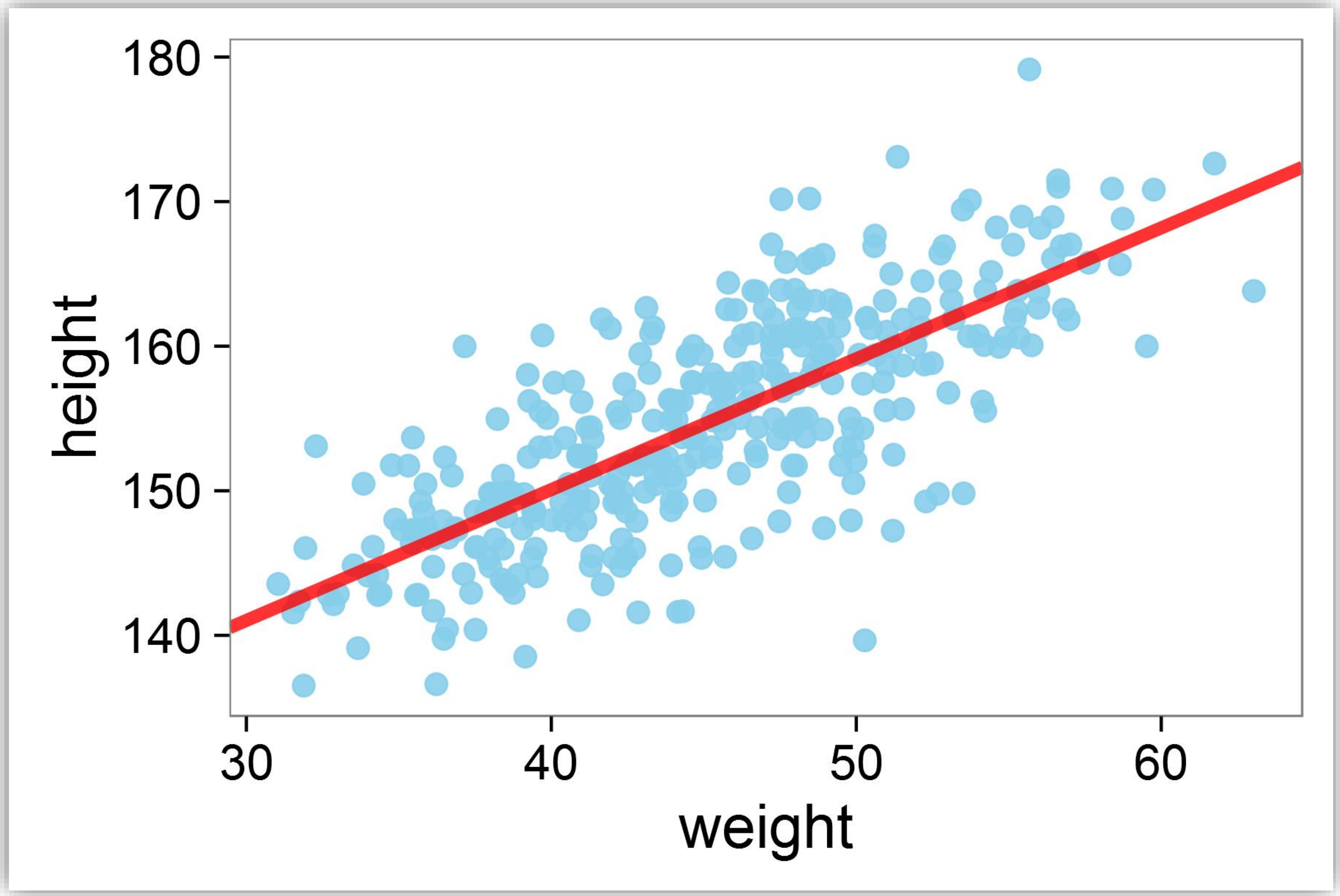


McElreath (2016)

Results with lm()


```
> L <- lm( height ~ weight, d) # estimate model by minimizing least squares errors
> summary(L)

Call:
lm(formula = height ~ weight, data = d)

Residuals:
     Min       1Q   Median       3Q      Max 
-19.7464  -2.8835   0.0222   3.1424  14.7744 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 113.87939    1.91107   59.59   <2e-16 ***
weight        0.90503    0.04205   21.52   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.086 on 350 degrees of freedom
Multiple R-squared:  0.5696, Adjusted R-squared:  0.5684 
F-statistic: 463.3 on 1 and 350 DF,  p-value: < 2.2e-16
```

height ~ weight



McElreath (2016)

Rethinking Regression Model

$$\mu_i = \alpha + \beta x_i$$

$$\cancel{y_i = \mu_i + \varepsilon}$$

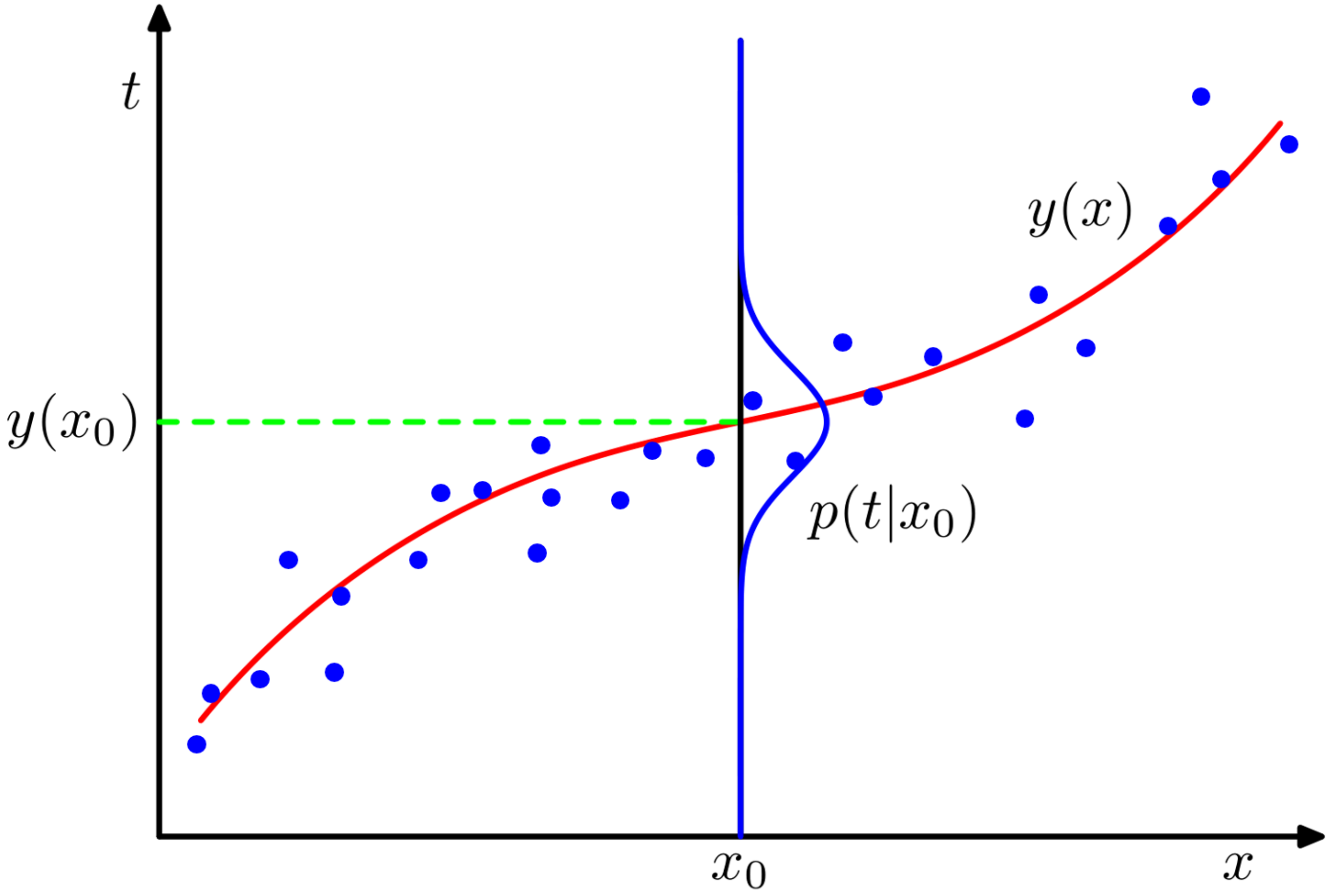
$$\cancel{\varepsilon \sim Normal(0, \sigma)}$$

$$\boxed{y_i \sim Normal(\mu_i, \sigma)}$$

Rethinking Regression Model

$$\mu_i = \alpha + \beta x_i$$

$$y_i \sim Normal(\mu_i, \sigma)$$



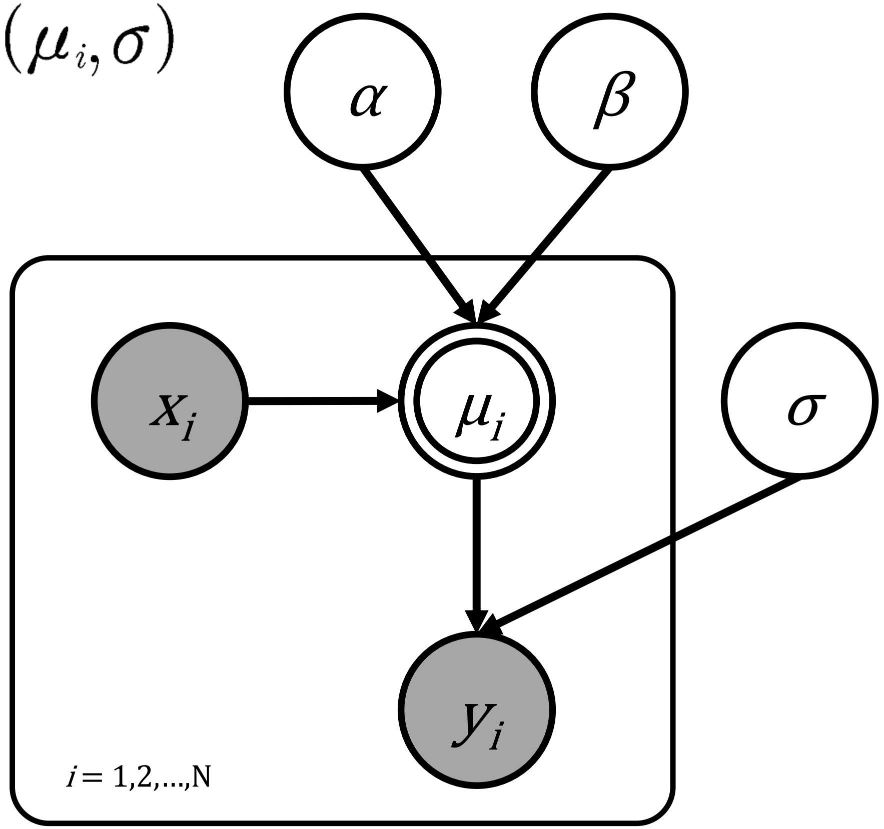
Bishop (2006)

Rethinking Regression Model

cognitive model
statistics
computing

$$\mu_i = \alpha + \beta x_i$$

$$y_i \sim Normal(\mu_i, \sigma)$$




```
model {
  vector[N] mu;
  for (i in 1:N) {
    mu[i] = alpha + beta * weight[i];
    height[i] ~ normal(mu[i], sigma);
  }
}
```

```
model {
  vector[N] mu;
  mu = alpha + beta * weight;
  height ~ normal(mu, sigma);
}
```

```
model {
  height ~ normal(alpha + beta * weight, sigma);
}
```

 deterministic variable

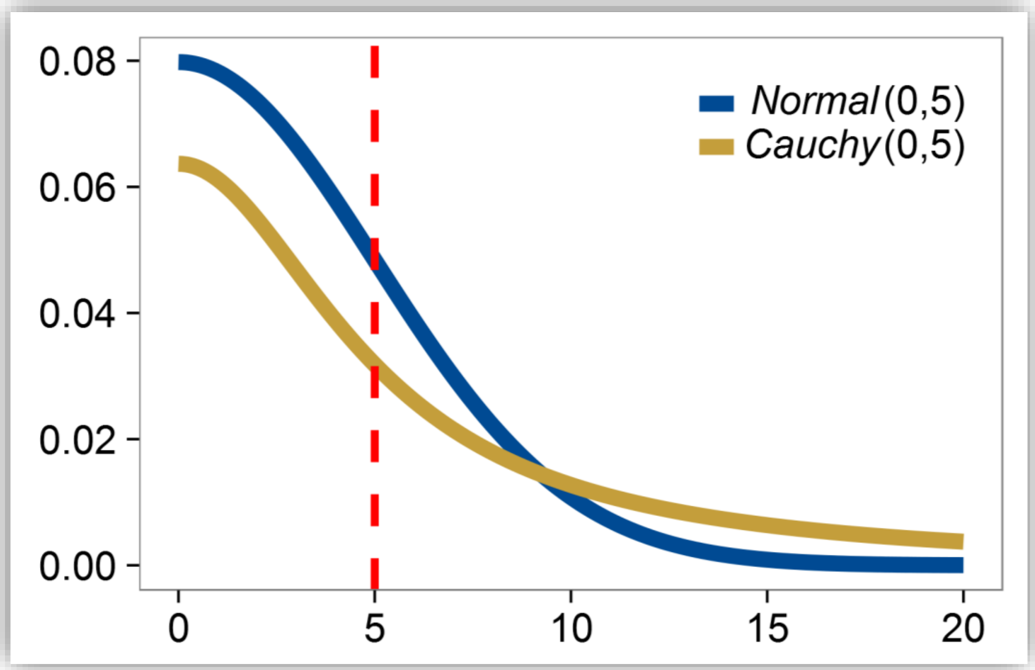
Thinking about Priors?

$$\alpha \sim Normal(170, 100) \qquad \beta \sim Normal(0,20)$$

$$\overline{\text{height}} = \alpha + \beta * \text{weight}$$

$$\sigma \sim halfCauchy(0,20)$$

$$\text{height} \sim Normal(\overline{\text{height}}, \sigma)$$

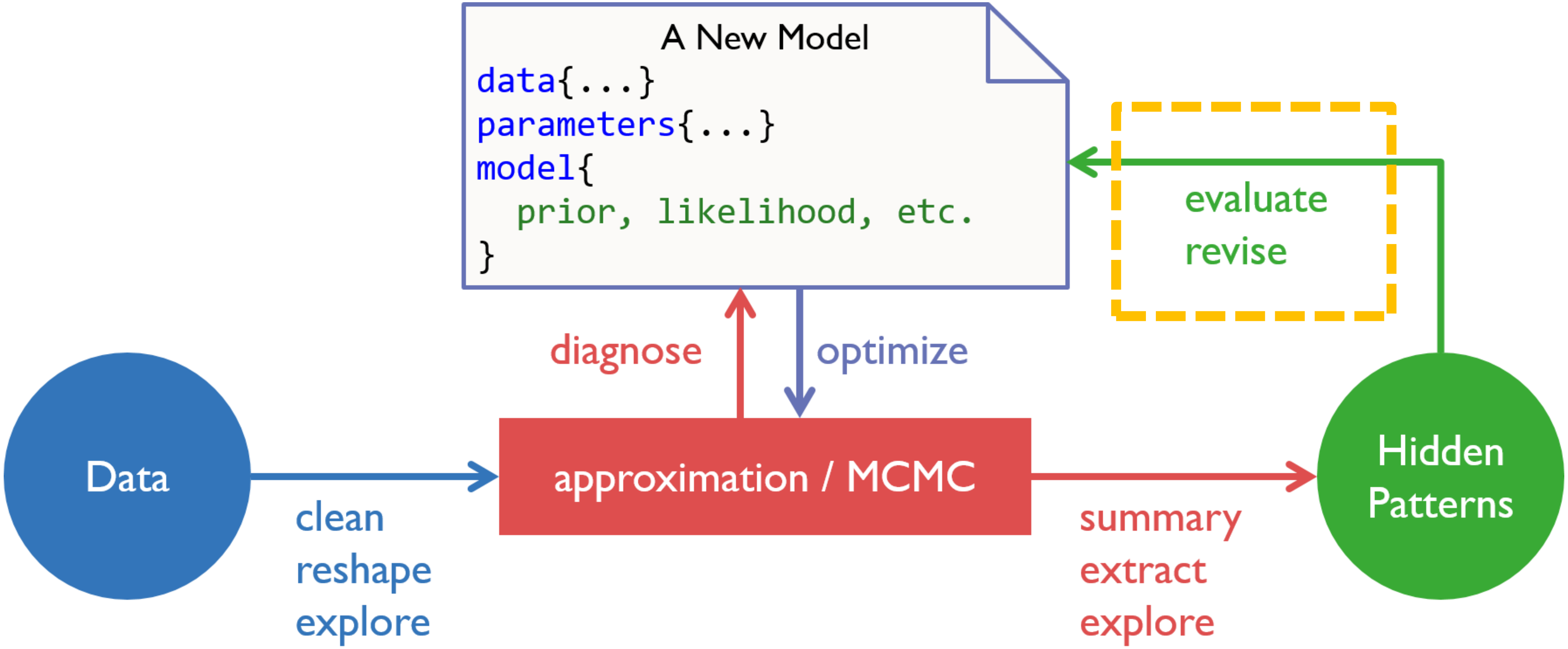


Exercise VIII


```
…/04.regression_height/_scripts/regression_height_main.R
```

TASK: estimate the model and produce the results

```
Inference for Stan model: regression_height_model.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

          mean se_mean   sd    2.5%     25%     50%     75%   97.5% n_eff Rhat
alpha   113.97    0.06 1.86  110.27  112.76  113.93  115.20  117.66   934    1
beta      0.90    0.00 0.04    0.82    0.88    0.90    0.93    0.99   922    1
sigma     5.11    0.01 0.19    4.74    4.97    5.10    5.24    5.50  1437    1
lp__   -747.61    0.04 1.23 -750.80 -748.15 -747.28 -746.72 -746.24   993    1
```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evaluate
revise
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Data
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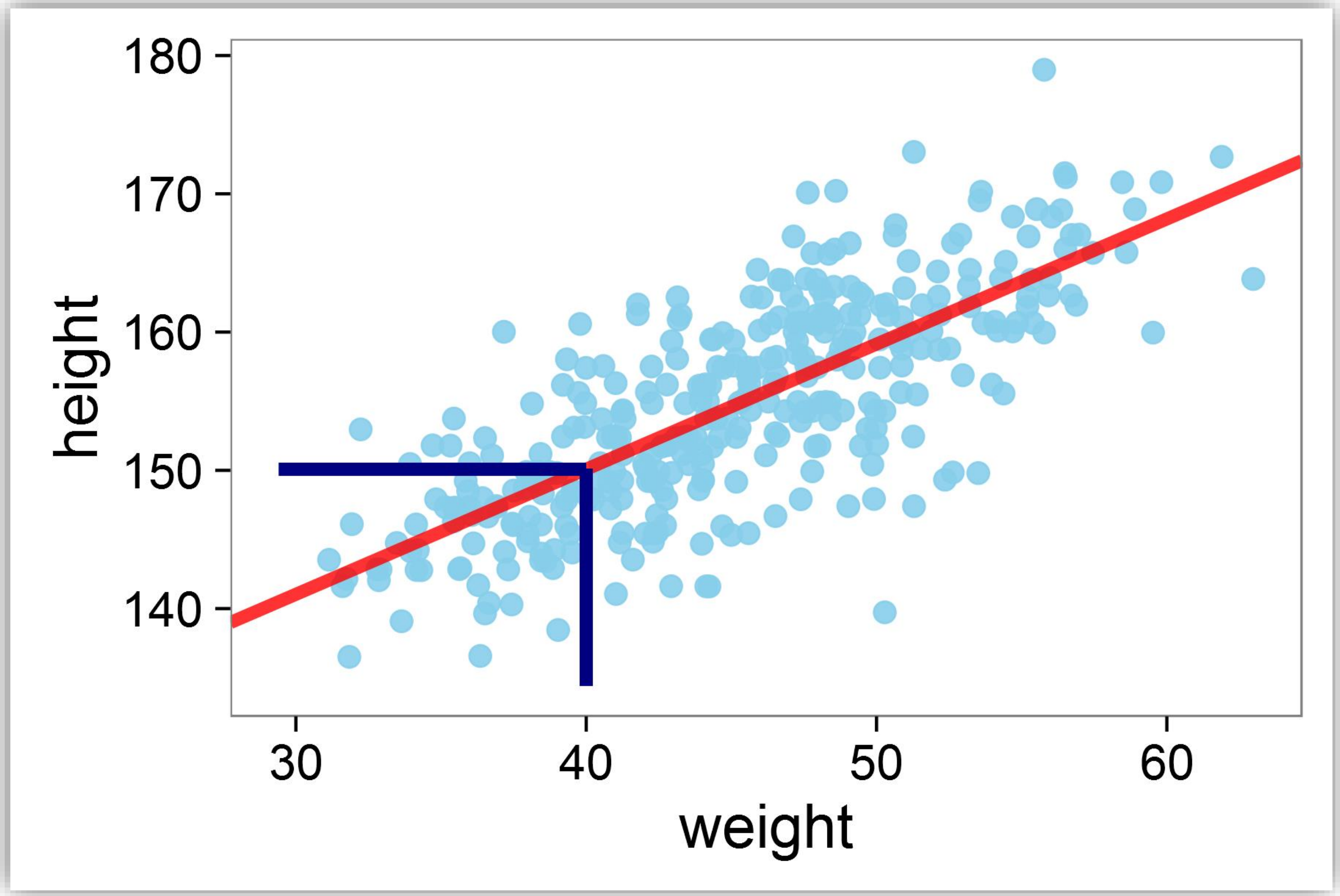
Hidden Patterns

What does the Model Predict?



$$p(y_{rep} \mid y) = \int p(y_{rep} \mid \theta)\, p(\theta \mid y)\, d(\theta)$$

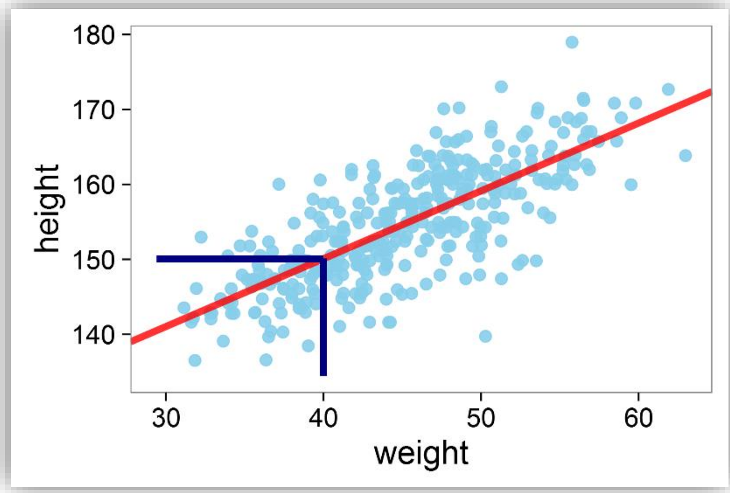
McElreath (2016)

Posterior Predictive Check (PPC)

cognitive model
statistics
computing

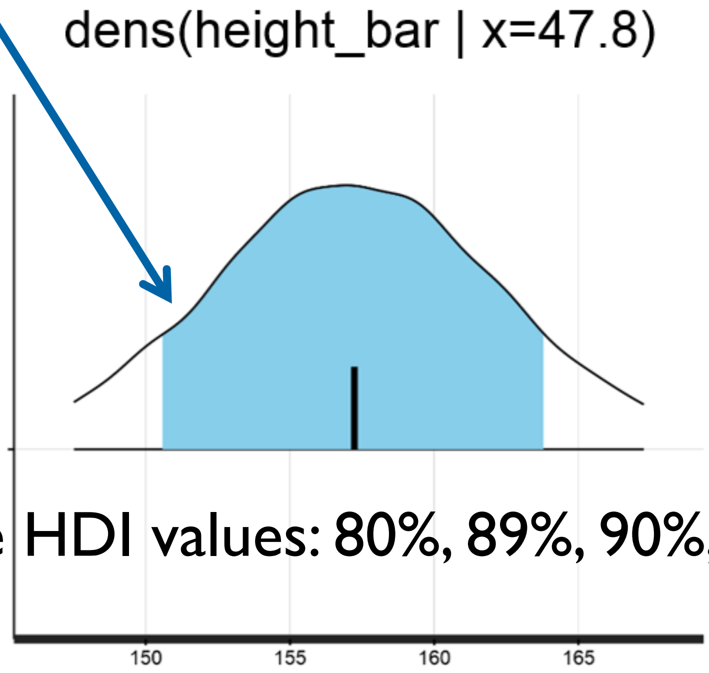
```
generated quantities {
  vector[N] height_bar;
  for (n in 1:N) {
      height_bar[n] = normal_rng(alpha + beta * weight[n], sigma);
  }
}
```

the generated quantities block runs only AFTER the sampling, and the time it costs can be essentially ignored!

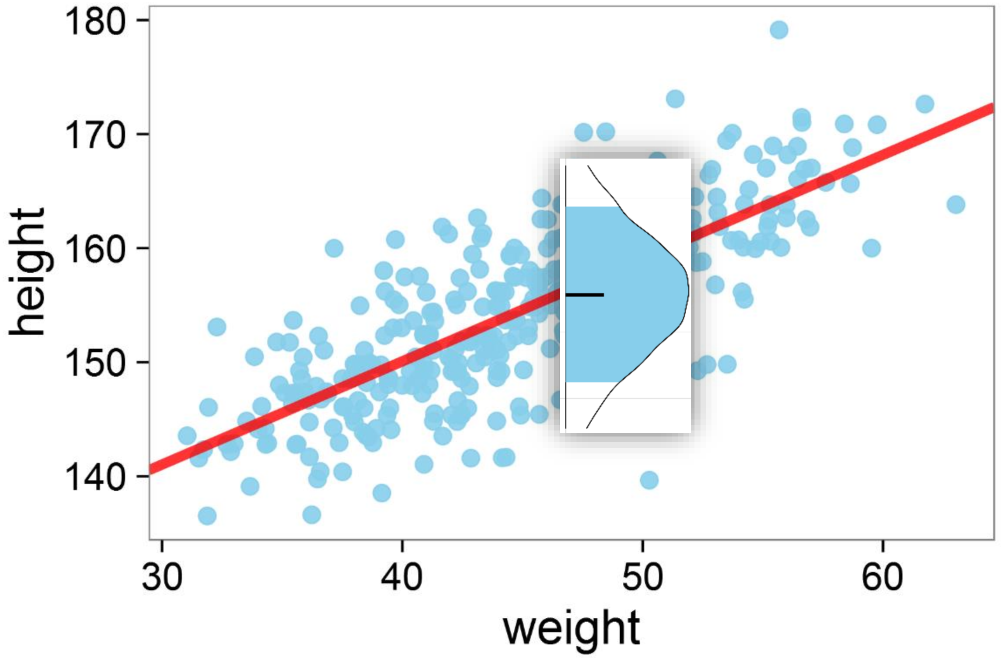


Posterior Predictive Check (PPC)

Highest density interval (HDI)

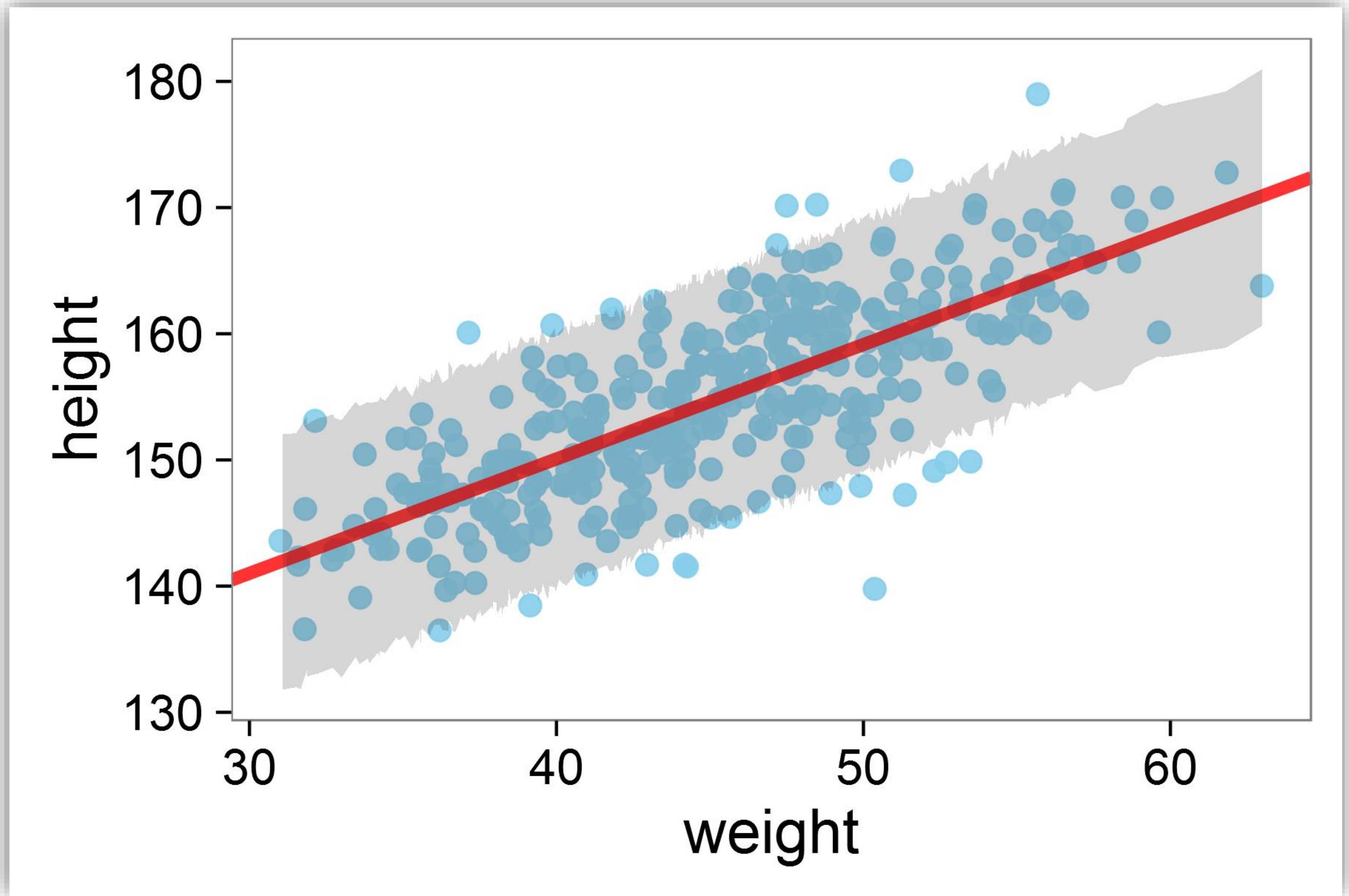


possible HDI values: 80%, 89%, 90%, 95%




```
height_bar <- extract(fit_reg_ppc, pars = 'height_bar',
                      permuted = FALSE)$height_bar
height_HDI <- apply(height_bar, 2, HDIofMCMC)
```

Posterior Predictive Check (PPC)



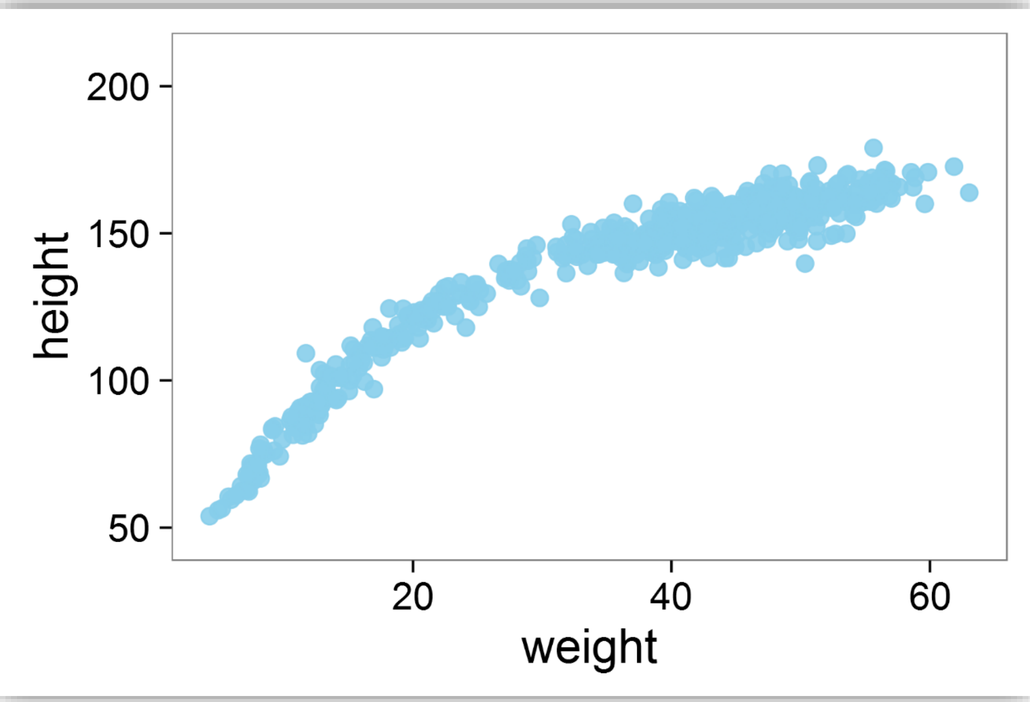
Exercise IX

cognitive model
statistics
computing

```
.../05.regression_height_poly/_scripts
/regression_height_poly_main.R
```

TASK: (1) Complete "regression_height_poly2_model.stan"

(2) produce PPC plot for both 1st order and 2nd order polynomial fit



Exercise IX – Tips

cognitive model

statistics

computing

```
> source('_scripts/regression_height_poly_main.R')

> out1 <- reg_poly(poly_order = 1)
```

$$\overline{\text{height}} = \alpha + \beta 1 * \text{weight} + \beta 2 * \text{weight}^2$$

$$\text{height} \sim Normal(\overline{\text{height}}, \sigma)$$

```
data {
  int<lower=0> N;
  vector<lower=0>[N] height;
  vector<lower=0>[N] weight;
  vector<lower=0>[N] weight_sq;
}
```

```
height ~ normal(alpha + beta1 * weight + beta2 * weight_sq, sigma);
```

Exercise IX – output2

cognitive model
statistics
computing