Bayesian Statistics and Hierarchical Bayesian Modeling for Psychological Science

Lecture 08

Lei Zhang

Social, Cognitive and Affective Neuroscience Unit (SCAN-Unit)

Department of Basic Psychological Research and Research Methods

lei.zhang@univie.ac.at
lei-zhang.net
@lei_zhang_lz

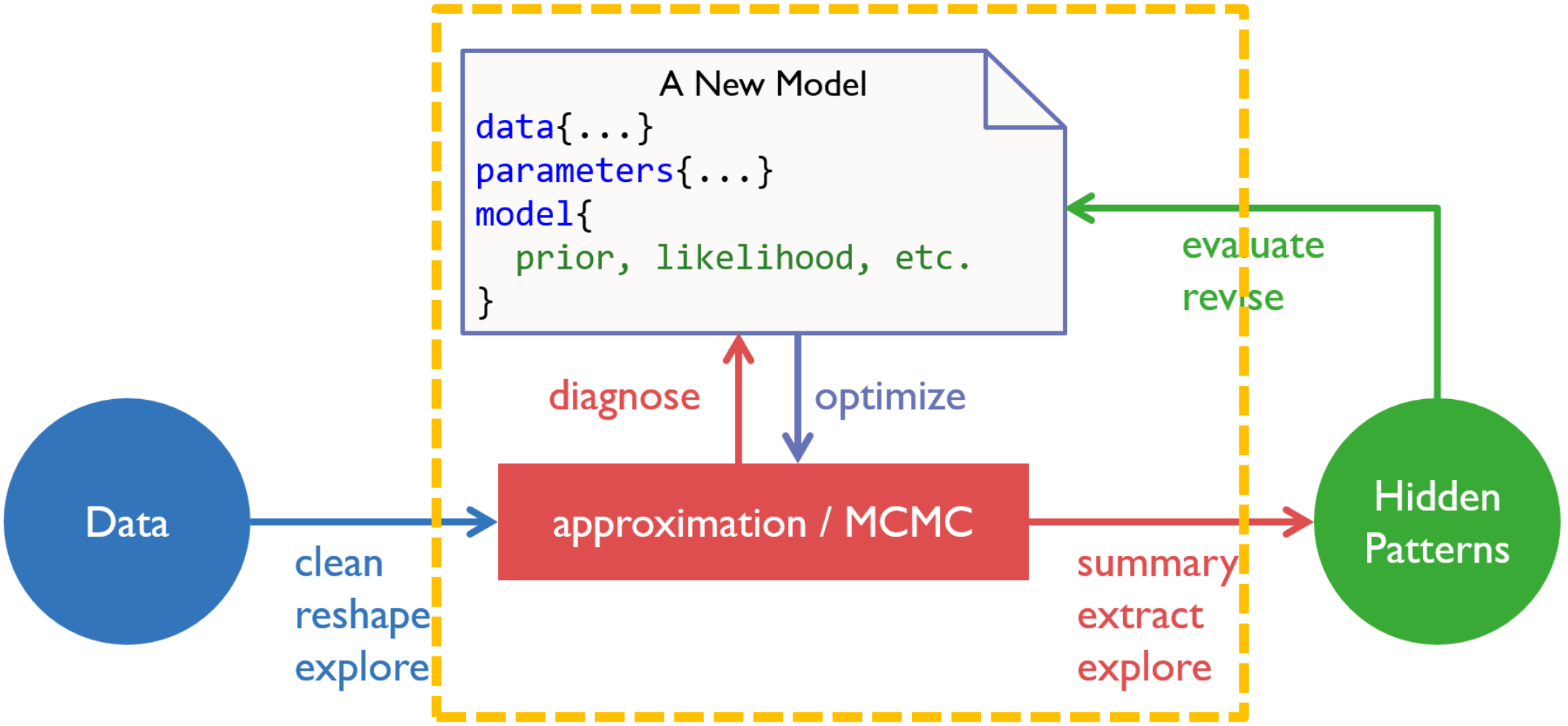
https://github.com/lei-zhang/BayesCog_Wien

universität wien
Fakultät für Psychologie

Bayesian warm-up?

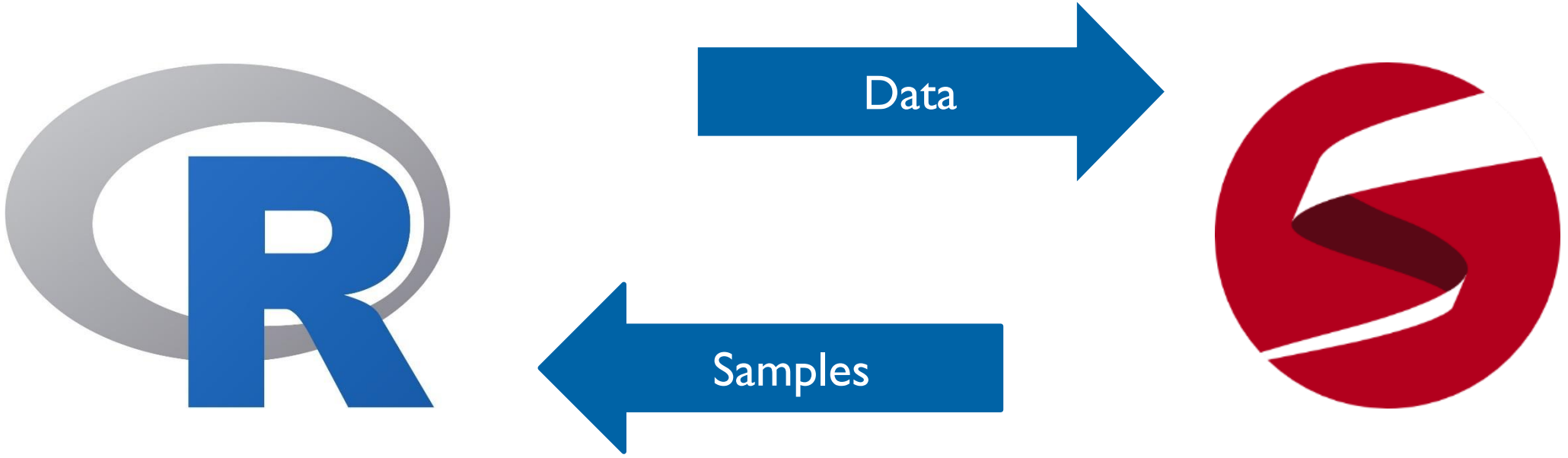
STAN PROGRAMMING LANGUAGE I

cognitive model
statistics
computing

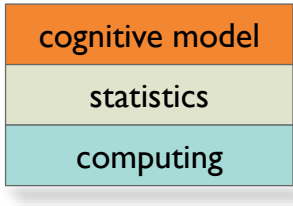


Stan and RStan

cognitive model
statistics
computing



Steps of Bayesian Modeling, with Stan



A data story	Think about how the data might arise. It can be *descriptive* or even *causal*. Write a Stan program (*.stan).
Update	Educate your model by feeding it the data. Bayesian Update: update the prior, in light of data, to produce posterior. Run Stan using RStan (PyStan, MatlabStan etc.)
Evaluate	Compare model with reality. Revise your model. Evaluate in RStan and ShinyStan.

McElreath (2016)

Steps of Using Stan

cognitive model
statistics
computing

1. Stan program read into memory
2. Source-to-source transformation into C++
3. C++ compiled and linked (takes a while)
4. Run Stan program
5. Posterior analysis / interface

```
data {
  int<lower=0> N;
  int<lower=0,upper=1> y[N];
}
parameters {
  real<lower=0,upper=1> theta;
}
model {
  y ~ bernoulli(theta);
}
```

Stan Language

model blocks

```
data {
  //... read in external data...
}

transformed data {
  //... pre-processing of data ...
}

parameters {
  //... parameters to be sampled by HMC ...
}

transformed parameters {
  //... pre-processing of parameters ...
}

model {
  //... statistical/cognitive model ...
}

generated quantities {
  //... post-processing of the model ...
}
```

cognitive model
statistics
computing

REVISIT BINOMIAL MODEL

Binomial Model

cognitive model
statistics
computing

W L W W W L W L W

$$p(w \mid N, \theta) = \binom{N}{w} \theta^{w} (1-\theta)^{N-w}$$

$$w \sim \text{Binomial}(N, \theta)$$

reads as:
w is distributed as a binomial distribution, with number of trials *N*, and success rate ϑ.

McElreath (2016)

Graphical Model Notations

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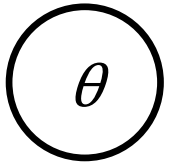
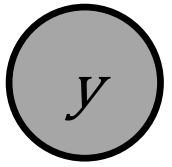
statistics
computing

	continuous	discrete
unobserved		
observed		

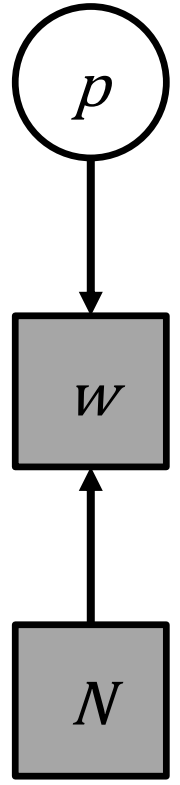
Lee & Wagenmakers (2013)

Binomial Model

cognitive model
statistics
computing

W L W W W L W L W

$$p(w \mid N, \theta) = \binom{N}{w} \theta^{w} (1-\theta)^{N-w}$$



$$\theta \sim \text{Uniform}(0, 1)$$

$$w \sim \text{Binomial}(N, \theta)$$

	continuous	discrete
unobserved	θ	δ
observed	y	N

McElreath (2016)

Binomial Model

cognitive model
statistics
computing

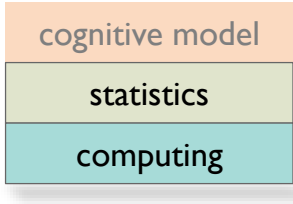
W L W W W L W L W

$$p(w \mid N, \theta) = \binom{N}{w} \theta^{w} (1-\theta)^{N-w}$$

```
data {
    int<lower=0> w;
    int<lower=0> N;
}

parameters {
    real<lower=0,upper=1> theta;
}

model {
    w ~ binomial(N, theta);
}
```

McElreath (2016)

Running Binomial Model with Stan




```
…/BayesCog/02.binomial_globe/_scripts/binomial_globe_main.R
```

```
> R.version
R version 3.5.1 (2018-07-02)

> stan_version()
[1] "2.18.0"
```

Model Summary

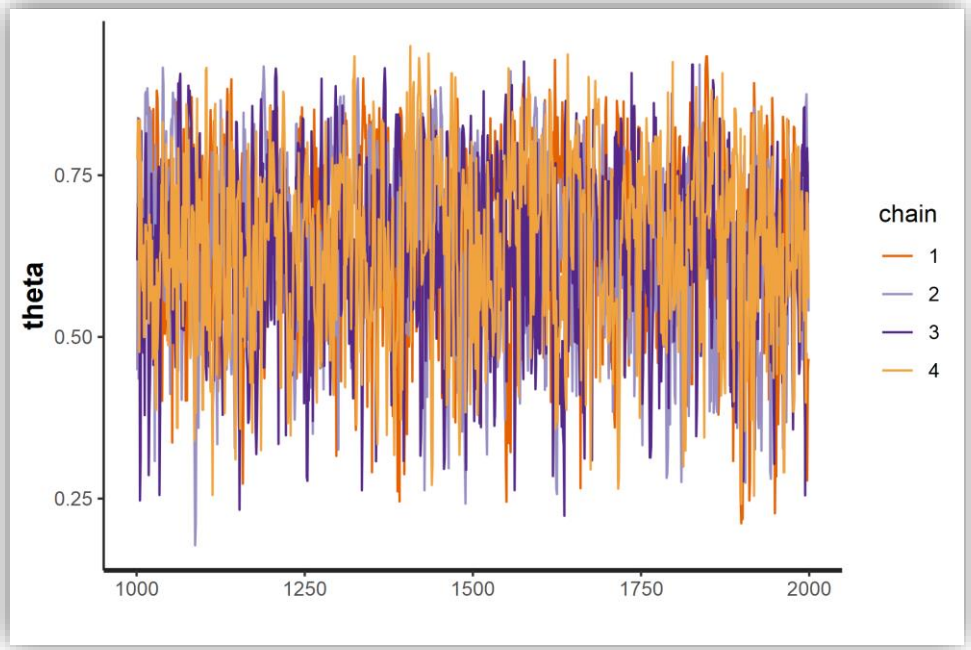
cognitive model
statistics
computing

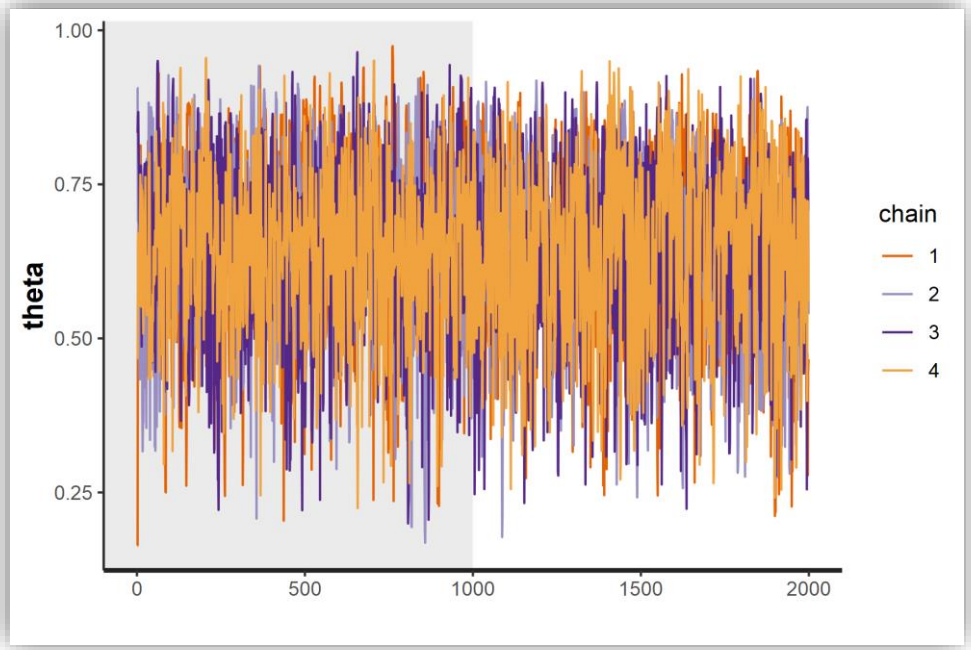
```
> print(fit_globe)
Inference for Stan model: binomial_globe_model.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

       mean se_mean   sd  2.5%   25%   50%   75% 97.5% n_eff Rhat
theta  0.64    0.00 0.14  0.35  0.54  0.65  0.74  0.87  1278    1
lp__  -7.72    0.02 0.69 -9.77 -7.89 -7.46 -7.27 -7.21  1824    1

Samples were drawn using NUTS(diag_e) at Tue Apr 09 12:44:04 2019.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

Gelman-Rubin convergence diagnostic
(Gelman & Rubin, 1992)

Diagnostics - traceplot

cognitive model
statistics
computing





Diagnostics - density

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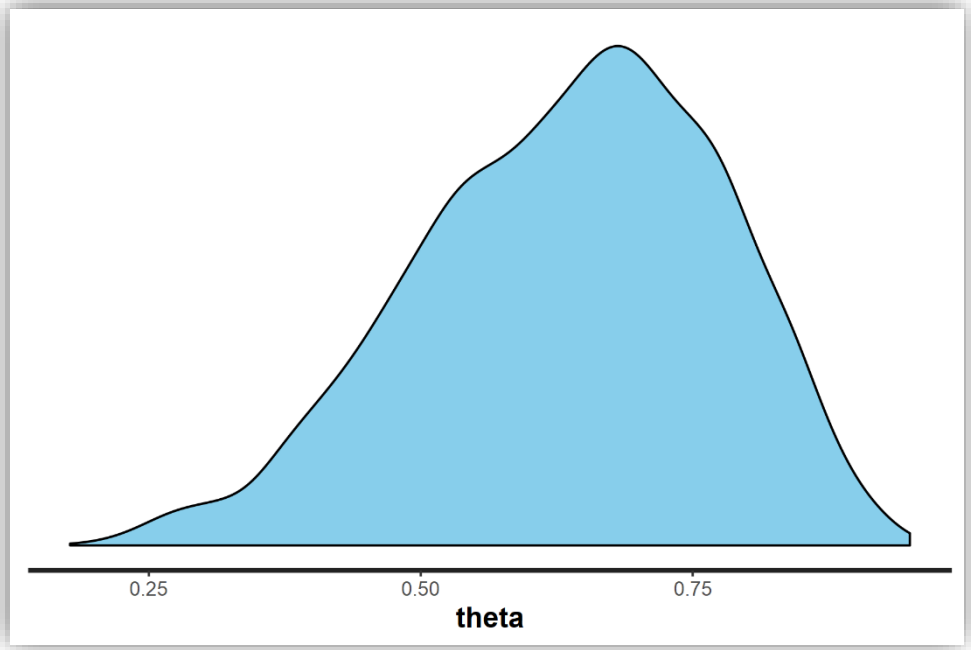
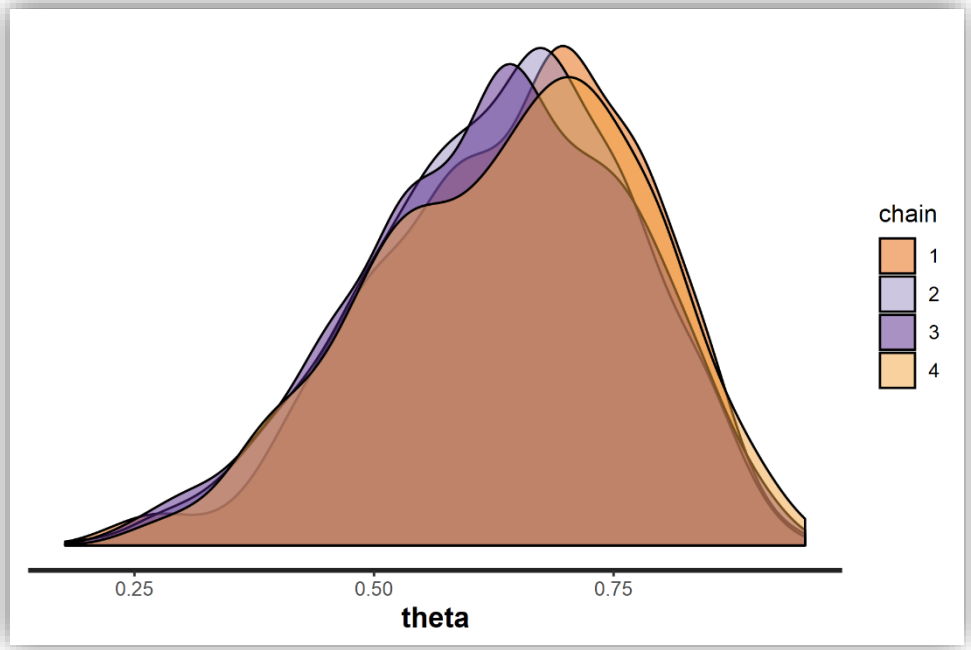
statistics
computing





Diagnostics

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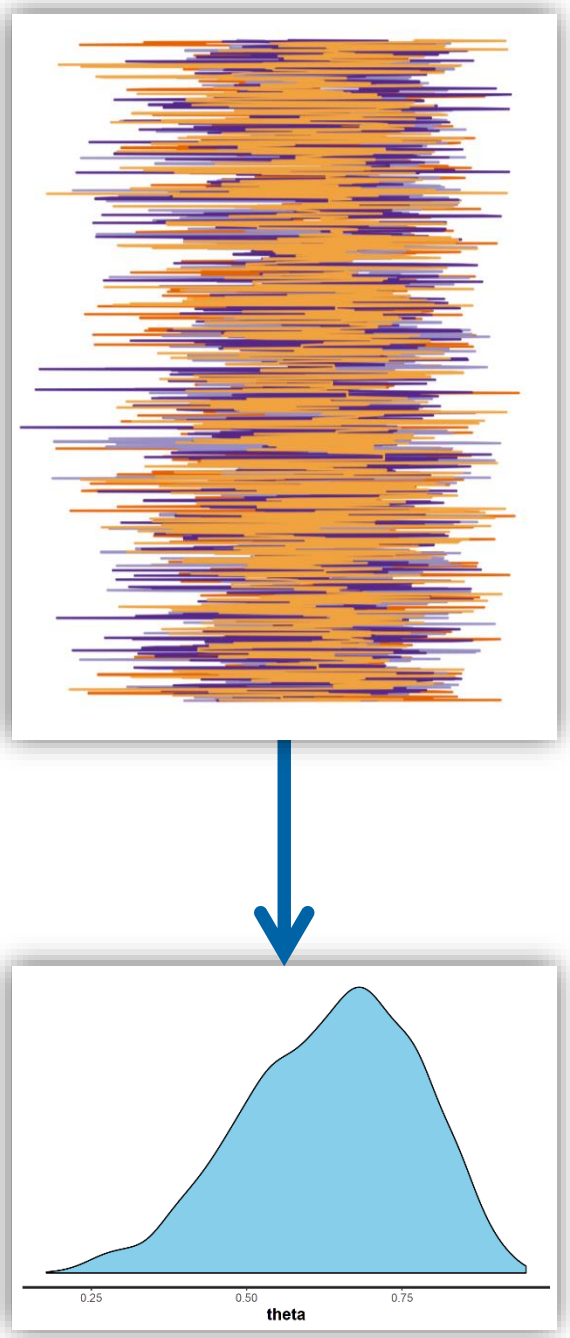
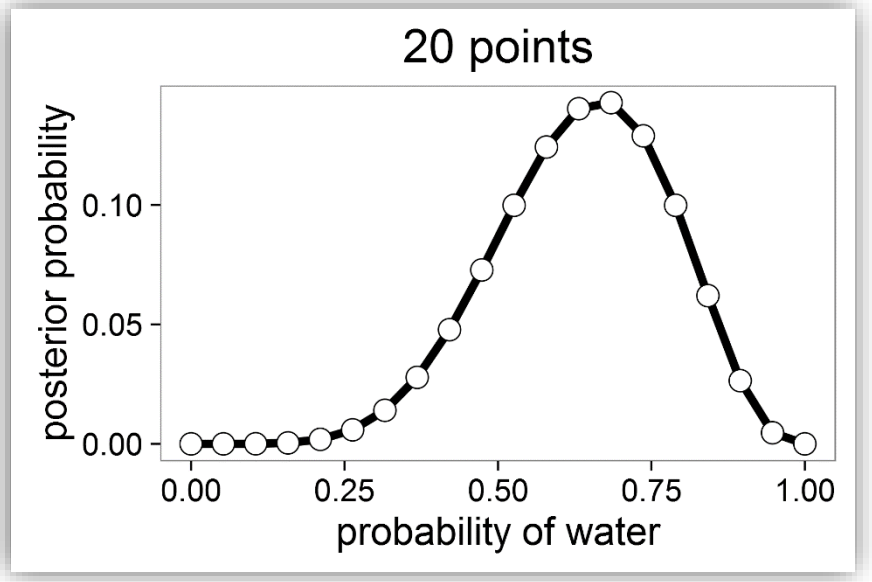
statistics
computing

MCMC



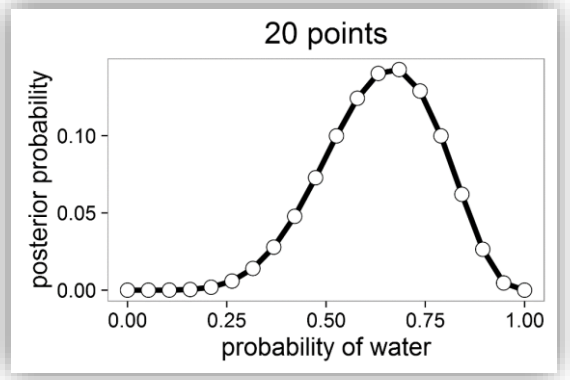
Grid Approximation

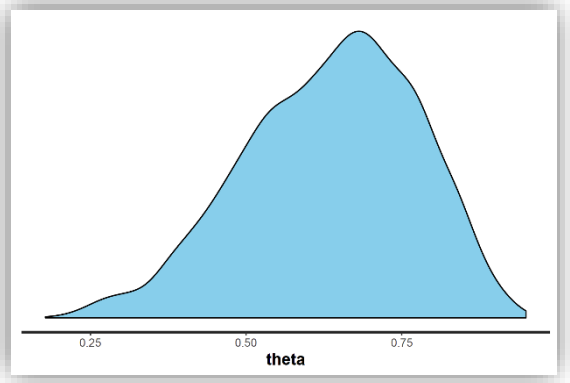


Draw a Conclusion?

cognitive model
statistics
computing

- W = 6 out of N = 9
- uncertainty (relative plausibility) of all ϑ values
- the relative plausibility of ϑ = 0.64 is the highest, but it never rules out the possibility of ϑ being other values, e.g., 0.5, 0.75
- → when ϑ = 0.5, you may still observe 6W / 9 trials





Is Anything Missing? – NO

cognitive model
statistics
computing

```
data {
    int<lower=0> w;
    int<lower=0> N;
}

parameters {
    real<lower=0,upper=1> theta;
}

model {
    theta ~ uniform(0,1);
    w ~ binomial(N, theta);
}
```

```
data {
    int<lower=0> w;
    int<lower=0> N;
}

parameters {
    real<lower=0,upper=1> theta;
}

model {
    w ~ binomial(N, theta);
}
```