Physcis

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Contents

Chapter 1

Math

1.1 Formulas

1.1.1 Fourier transform

$$\delta^3(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\vec{x}}$$

1.1.2 Integral

$$I = \int_{-\infty}^{\infty} dk \frac{e^{ikr} - e^{-ikr}}{k}$$

Note that the integrand does not blow up as $k \to 0$, so

$$I = \lim_{\delta \to 0} \left[\int_{-\infty}^{\infty} dk \frac{e^{ikr} - e^{-ikr}}{k + i\delta} \right]$$

So we have pole at $k = -i\delta$. For e^{ikr} we must close the contour up to get exponential decay at large k. This misses the pole, so this term gives zero. So

$$I = \int_{-\infty}^{\infty} dk \frac{-e^{-ikr}}{k + i\delta} = -(2\pi i)(-e^{-\delta r}) = 2\pi i e^{-i\delta r}$$

1.2 Functions

1.2.1 Interesting functions

Weierstrass function

$$f(x) = \sum_{n=0}^{\infty} a^n cos(b^n \pi x) \tag{1.1}$$

Continuous but not derivative everywhree.

1.2.2 Differentive Equations

First Order Equation

$$\frac{dV}{dt} + \eta(t)V(t) = f(t)$$

so:

$$\frac{1}{I}\frac{d(I(t)V(t))}{dt} = \frac{dV(t)}{dt} + \frac{I'(t)}{I(t)}V(t)$$

if:

$$\frac{I'(t)}{I(t)} = \eta(t) \to I(t) = exp(\int^t dt'\eta(t'))$$

so:

$$\frac{d(I(t)V(t))}{Idt} = f(t)$$

Floquet theory

$$\ddot{x} + \omega_0^2(1 + \mu cos(\nu t))x = 0 \tag{1.2}$$

Analogy to QM(Schrodinger Equation):

$$-\frac{\hbar^2}{2m}\psi'' + U(x)\psi = E\psi$$

where $U(x) = -U_0 cos\frac{2\pi x}{a}$. So let

$$k^2 = \frac{2mE}{\hbar^2}, u = \frac{U_0}{E}, \nu = \frac{2\pi}{a}$$

u is small, we will get

$$\psi'' + k^2\psi = -k^2 u cos(\nu x)\psi$$

1.2.3 Integrated functions

Γ function

$$\Gamma(z) \equiv \int_0^{+\infty} dt e^{-t} t^{z-1} = 2 \int_0^{+\infty} dx e^{-x^2} x^{2z-1}$$

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{sin \pi z}$$

Chapter 2

Classical Mechanics

Chapter 3

Electramegnetic Mechanics

3.1 Maxwell Equation

$$\begin{cases} \nabla \cdot E = \dfrac{\rho}{\epsilon} \\ \nabla \times B = \dfrac{1}{c}\left(J + J_D + J_{ind}\right) \\ \nabla \cdot B = 0 \\ \nabla \times E = -\dfrac{1}{c}\partial_t B \end{cases} \longrightarrow \begin{cases} \nabla \cdot D = \rho \\ \nabla \times H = \dfrac{1}{c}J + \dfrac{1}{c}\partial_t D \\ \nabla \cdot B = 0 \\ \nabla \times E = -\dfrac{1}{c}\partial_t B \end{cases} \tag{3.1}$$

where

$$J_D = \epsilon\partial_t E; \quad J_{induced} = c\chi_m^B \nabla \times B$$

Note that the Maxwell equation is relationship about **E** and **B**, the $\vec{D}$ and $\vec{H}$ is introduced to help reduce the equations to a more concised form.

To solve it, using iterative method (in power of $1/c$)

i) 0th order:

$$\nabla \cdot E^{(0)} = \rho \qquad \nabla \times B^{(0)} = 0$$
$$\nabla \times E^{(0)} = 0 \qquad \nabla \cdot B^{(0)} = 0$$

ii) 1th order:

$$\nabla \cdot E^{(1)} = 0 \qquad \nabla \times B^{(1)} = \frac{j}{c} + \frac{1}{c}\partial_t E^{(0)}$$
$$\nabla \times E^{(1)} = 0 \qquad \nabla \cdot B^{(1)} = 0$$

iii) 2th order:

$$\nabla \cdot E^{(2)} = 0 \qquad \nabla \times B^{(2)} = 0$$
$$\nabla \times E^{(2)} = -\frac{1}{c}\partial_t B^{(1)} \qquad \nabla \cdot B^{(2)} = 0$$

iv) 3th order ...

$$\nabla \cdot E^{(3)} = 0 \qquad \nabla \times B^{(3)} = \frac{1}{c}\partial_t E^{(3)}$$
$$\nabla \times E^{(3)} = -\frac{1}{c}\partial_t B^{(1)} \qquad \nabla \cdot B^{(3)} = 0$$

So in the quasi-static approximation, we get

$$\begin{aligned} E &= E^{(0)} + E^{(2)} + \dots \\ B &= B^{(1)} + B^{(3)} + \dots \end{aligned}$$

3.1.1 Boundary conditions

$$\begin{cases} \vec{e}_n \cdot (\vec{D}_2 - \vec{D}_1) = \sigma \\ \vec{e}_n \times (\vec{H}_2 - \vec{H}_1) = \dfrac{\vec{\alpha}}{c} \\ \vec{e}_n \cdot (\vec{B}_2 - \vec{B}_1) = 0 \\ \vec{e}_n \times (\vec{E}_2 - \vec{E}_1) = 0 \end{cases} \tag{3.2}$$

Note that here σ and $\vec{\alpha}$ are **free** charge and **free** current. The induced charges and current make trivial contribution due to infinitesimal integral volume (or area).

3.1.2 Plain wave

If $J = \rho = 0$, then

$$\begin{aligned} \nabla \times (\nabla \times E) &= \nabla(\nabla \cdot E) - \nabla^2 E \\ &= -\frac{1}{c}\partial_t(\nabla \times B) = -\frac{1}{c^2} \cdot \epsilon\mu \cdot \partial_t^2 E \end{aligned}$$

let

$$n^2 = \epsilon\mu$$

So

$$\nabla^2 E - \frac{n^2}{c^2}\partial_t^2 E = 0$$

Because $E = E_0 e^{-i\omega t}$, so we get

$$\nabla^2 E + \frac{n^2}{c^2}\omega^2 E = 0$$

$$E = E_0 e^{i(kx-\omega t)}, \quad k^2 = n^2\omega^2/c^2$$

let $B = B_0 e^{i(kx-\omega t)}$

$$\nabla \times E = -\frac{1}{c}\partial_t B = \frac{i\omega}{c} B$$

$$\vec{B} = \frac{c}{i\omega}\nabla \times E = \frac{c}{\omega}\vec{k} \times \vec{E}$$

3.1.3 Metal

$$\begin{aligned}\nabla \times H &= \frac{1}{c}(J + \partial_t D)\\ &= \frac{1}{c}(\sigma E - i\omega\epsilon E)\\ &= \frac{\sigma - i\omega\epsilon}{c} E\end{aligned}$$

So

$$\nabla \times (\nabla \times E) = -\nabla^2 E = -\frac{1}{c}\nabla \times \partial_t B = -\frac{1}{c}\partial_t(\nabla \times \mu H) = -\mu\frac{\sigma - i\omega\epsilon}{c^2}(-i\omega)E = k^2 E$$

where,

$$k^2 = \frac{\omega^2\mu(\epsilon + i\sigma/\omega)}{c^2} = \frac{\omega^2\mu\hat{\epsilon}}{c^2}$$

Helmholtz eqn:

$$(\nabla^2 + \frac{\omega^2\mu\hat{\epsilon}}{c^2})E = 0$$

3.2 EM in relativity

$$F_{\mu\nu} \equiv \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3.3}$$

The Maxwell Eqn:

$$\begin{aligned}\partial_\mu F_{\mu\nu} = \Box A_\nu - \partial_\nu(\partial_m u A_\mu) = \Box A_\mu = 0,\\ \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0\end{aligned} \tag{3.4}$$

Chapter 4

Quantum Mechanics

4.1 Rotation

For spin-1/2,

$$D(R_{\hat{n}}(\phi)) = e^{-i\frac{\phi}{\hbar}\hat{n}\cdot\vec{S}} = e^{-i\frac{\phi}{2}\hat{n}\cdot\vec{\sigma}} = cos(\frac{\phi}{2}) - i\hat{n}\cdot\vec{\sigma} sin(\frac{\phi}{2}) = \begin{pmatrix} \cos\left(\frac{\phi}{2}\right) - in_z\sin\left(\frac{\phi}{2}\right) & (-in_x - n_y)\sin\left(\frac{\phi}{2}\right) \\ (-in_x + n_y)\sin\left(\frac{\phi}{2}\right) & \cos\left(\frac{\phi}{2}\right) + in_z\sin\left(\frac{\phi}{2}\right) \end{pmatrix}$$

Chapter 5

QFT

- What's a field, classical field is a spatial distribution, quantum field is a analogy to classical one, but make up of creation and destruction operator.
- How to construct Lagrangian from a field ← Lorantz invariant.
- EOM
- Neother theorem. Conserved current and charge
- Symmetry. What's each group? corresponding representation. How to embed particles into lorentz group and unitary group?

5.1 Motivation

particle number is not conserved. The creation and destruction of particls, which is possible due to the most famous eqn. of special relativity $E = mc^2$. *Lorentz invariance* guides the definition of particle.

Why QFT: $\boxed{\text{quantum mechanics plus Poincaré symmetry}}$.

$\boxed{\text{Quantum field theory is just quantum mechanics with an infinite number of harmonic oscillators}}$

The Quamtum Mechanics can describe a system with a fixed number of particles in terms of a many-body wave function. The Relativistic Quantum Field Theory with creation and annihilation operators was developed in order to include processes in which the number of particles is not conserved, and to describe the conversion of mass into energy and vice versa. A consequence of relativity is that the number of particles isn't fixed, ??? though the converse is false: particle production can happen without relativity.

Construct $\mathcal{L}$ from field ϕ and its derivative under the rule of Lorentz Invariant. How to incorporate symmetry ?

QFT is the quantum mechanics of *extensive degrees of freedom* $\langle x|\phi\rangle = \phi(x)$ is a function of space, the wavefunction. This looks like a field. It is not what we mean by field in QFT. meaningless phrases like "second quantization" may conspire to try to confuse you.

It is not a coincidence that the harmonic oscillator plays an important role. After all, electromagenetic waves oscillate harmonically.

There are two common ways to quantize a field theory:

- canonical quantization
- Feynman path integral
- Ohter alternatives: perturbation theory

5.2 Convention

In relativity, the symmetry refer to invariance after the transformation of *coordinate system*. So the rotation is:

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{5.1}$$

or in other way:

$$R^T \mathbb{1} R = \mathbb{1}$$

5.2.1 4D time-space

$dx^\mu \equiv (dt, d\vec{x})^\mu$
$ds^2 = dt^2 - d\vec{x} \cdot d\vec{x} = \eta_{\mu\nu} dx^\mu dx^\nu$ with

$$\eta^{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}_{\mu\nu} \tag{5.2}$$

Rotation and boost in 4D time-space around x axes is:

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \cos\theta_x & \sin\theta_x \\ & & -\sin\theta_x & \cos\theta_x \end{pmatrix}, \begin{pmatrix} \cosh\beta_x & \sinh\beta_x & & \\ \sinh\beta_x & \cosh\beta_x & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

5.2.2 quantization

$$[a_k, a_p^\dagger] = (2\pi)^3\delta^3(\vec{p}-\vec{k}), a_p^\dagger |0\rangle = \frac{1}{\sqrt{2\omega_p}} |\vec{p}\rangle$$

Function derivatives $\frac{\delta\phi(x)}{\delta\phi(y)} = \delta(x-y)$,

$$\frac{\partial(\partial_\alpha A_\alpha)^2}{\partial(\partial_\mu A_\nu)} = 2(\partial_\alpha A_\alpha)\frac{\partial(\partial_\beta A_\gamma)}{\partial(\partial_\mu A_\nu)} g_{\beta\gamma} = 2(\partial_\alpha A_\alpha) g_{\beta\gamma} g_{\gamma\nu} g_{\beta\gamma} = 2\partial_\nu(\partial_\alpha A_\alpha)$$

notation ϕ and π for scalar fields, ψ, ξ, χ for fermions, A_μ, J_μ, V_μ for vectors and h_μ, T_μ for tensors.

Kinetic term Anything with just two fields of the same or different type can be called a kinetic term. Kinetic terms tell you about the free (non-interacting) behavior. Though, sometimes it is useful to think of a *mass term* such as $m^2\phi^2$, as an interaction rather than a kinetic term.

Boundary conditions we will always assume that our fields vanish on asymptotic boundaries. so we can integrate by part:

$$A\partial_\mu B = -(\partial_\mu A)B$$

We define quantum fields as integrals over creation and annihilation operators for each momentum: (Why define it this way ???)

$$\phi_0(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{\omega_p}} (a_p e^{i\vec{p}\vec{x}} + a_p^\dagger e^{-i\vec{p}\vec{x}}) \tag{5.3}$$

$$|x\rangle = \phi_0(\vec{x})|0\rangle$$

There is no physical content in the above equation. It is just a definition. The physical content is in the algebra of a_p and $a_p^\dagger$ and in the Hamiltonian H_0. Nevertheless, we will see that collections of a_p and $a_p^\dagger$ in the form of Eq.5.3 are very useful in quantum field theory.

Following this defition, we can get:

$$\pi(\vec{x}) \equiv \partial_t\phi(\vec{x})|_{t=0} = -i\int \frac{d^3p}{(2\pi)^3}\sqrt{\frac{\omega_p}{2}} (a_p e^{i\vec{p}\vec{x}} - a_p^\dagger e^{-i\vec{p}\vec{x}})$$

$\pi[\phi, \dot{\phi}]$ can also be implicityly defined as:

$$\frac{\partial\mathcal{H}[\phi,\pi]}{\partial\pi} = \dot{\phi}$$

5.2.3 Hamiltonian & Lagrangian

Why do we restrict to Lagrangians of the form $\mathcal{L}[\phi, \partial_\mu \phi]$? First of all, this is the form that all "classical" Lagrangians had. If only first derivatives are involved, boundary conditions can be specified by initial positions and velocities only, in accordance with Newton's laws. In the quantum theory, if kinetic terms have too many derivatives, for example $\mathcal{L} = \phi \Box^2 \phi$, there will generally be disastrous consequences. For example, there may be states with negative energy or negative norm, permitting the vacuum to decay. But interactions with multiple derivatives may occur. Actually, they must occur due to quantum effects in all but the simplest renormalizable field theories; for example, they are generic in all effective field theories.

Hamiltonian and *Lagrangian* density: (How to connect field ϕ with $\mathcal{L}$ or $\mathcal{H}$???).

$$\mathcal{L}[\phi, \dot{\phi}] = \pi[\phi, \dot{\phi}]\dot{\phi} - \mathcal{H}[\phi, \pi[\phi, \dot{\phi}]]$$

Or inversely:

$$\mathcal{H}[\phi, \pi] = \pi\dot{\phi}[\phi, \pi] - \mathcal{L}[\phi, \dot{\phi}[\phi, \pi]], \frac{\partial \mathcal{L}[\phi, \dot{\phi}]}{\partial \dot{\phi}} = \pi$$

The Halmiltonian corresponds to a conserved quantity - the total energy of the system - while the Lagrangian does not. The problem with Halmiltonian is that they are not Lorentz invariant. It is the 0 component of a Lorentz vector: $P^\mu = (H, \vec{P})$. And $\mathcal{H}$ is the 00 compnent of a Lorentz tensor, the energy-momentum tensor $\mathcal{T}_{\mu\nu}$. Halmiltonians are great for non-relativistic systems, but for relativistic systems we will almost exclusively use Lagrangians.

Time evolution is generated by a hamiltonian H. $i\hbar\partial_t |\phi\rangle = H |\phi\rangle$

5.2.4 Norther's theorem

If there is such a symmetry that depends on some parameter α that can be taken small (continuous), than we find:

$$0 = \frac{\delta \mathcal{L}}{\delta \alpha} = \sum_n \left\{ \left[\frac{\partial \mathcal{L}}{\partial \phi_n} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \right] \frac{\delta \phi_n}{\delta \alpha} + \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha} \right] \right\}$$

If the EOM is satisfied, then it reduces to $\partial \mu J_\mu = 0$, where

$$J_\mu = \sum_n \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha} \tag{5.4}$$

A vector field J_μ that satisfies $\partial_\mu J_\mu = 0$ is called *conserved current*. The total charge Q, defined as:

$$Q = \int d^3x J_0$$

, satisfies

$$\partial_t Q = \int d^3x \partial_t J_0 = \int d^3x \vec{\nabla} \cdot \vec{J} = 0$$

Neother's theorem: If a Lagrangian has a continuous symmetry then there exists a current associate with that symmetry that is conserved when the equations of motion are satisfied.

- continuous
- the current is conserved *on-shell*, that is, when the EOM are satisfied. (The field dies out at asymptotic boundary).
- It works for *global symmetries*, parametrized by number α, not only for *local(gauge) symmetries* parametrized by functions $\alpha(x)$.

5.2.5 Energy-Momentum Tensor

global space-time translation $\rightarrow$ energy-momentum tensor.

$$\phi(x) \rightarrow \phi(x+\xi) = \phi(x) + \xi^\nu \partial_\nu \phi(x) + \cdots$$

With infinitesimal change:

$$\frac{\delta\phi}{\delta\xi^\nu} = \partial_\nu\phi, \frac{\delta\mathcal{L}}{\delta\xi^\nu} = \partial_\nu\mathcal{L},$$

So:

$$\delta S = \int d^4x \delta\mathcal{L} = \xi^\nu \int d^4x \partial_\nu \mathcal{L} = 0$$

which leads to:

$$\partial_\nu\mathcal{L} = \frac{\delta\mathcal{L}[\phi_n, \partial_\mu\phi_n]}{\delta\xi^\nu} = \partial_\mu\left(\sum_n \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_n)}\frac{\delta\phi_n}{\delta\xi^\nu}\right)$$

Or equivalently

$$\partial_\mu\left(\sum_n \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_n)}\partial_\nu\phi_n - g_{\mu\nu}\mathcal{L}\right) = 0$$

The four symmetries have produced four Noether current, one for each ν:

$$\mathcal{T}_{\mu\nu} = \sum_n \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \partial_\nu \phi_n - g_{\mu\nu}\mathcal{L} \tag{5.5}$$

The corresponding conserved current:

$$Q_\nu = \int d^3x \mathcal{T}_{0\nu}$$

Electron has an inherent two-valuedness called spin, while a photon has an inherent two-valuedness called polarization.

5.3 Symmetry

5.3.1 Lorentz invariance

the symmetry group associated with *special relativity.*
For scalar field, the physical content of Lorentz invariance is that nature has a symmetry under which scalar fields do not transform. Take, for example, the temperature of a fluid, which can vary from point to point. If we change reference frames, the labels for the points change, but the temperature at each point stays the same.

As for vector field, the difference is that the compnents of a vector field at the point x transform into each other as well.

The simplest Lorentz-invariant operator that we can write down involving derivatives is the d'Alembertian:

$$\Box = \partial_\mu^2 = \partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2$$

Objects such as $v^2 = V_\mu V^\mu, \phi, 1, \partial_\mu V^\mu$ are *Lorentz invariant*, meaning they do not depend on the Lorentz frame at all. While objects like: $V_\mu, F_\mu \nu, \partial_\mu, x_\mu$ are *Lorentz covariant*, meaning they do change in different frames, but precisely as the Lorentz transformation dictates.

5.3.2 Unitary

the probability should add up to *1.*

5.3.3 Poincaré Group

The group of translations and Lorentz transformation is called the **Poincaré group**, ISO(1,3) (the isometry group of Minkowski space).

5.4 Field

5.4.1 Scalar Field

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\vec{\nabla}\phi \cdot \vec{\nabla}\phi - \frac{1}{2}m^2\phi^2 = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2)$$

EOM:

$$-\partial_t^2\phi + \nabla^2\phi - m^2\phi = (\partial_\mu\partial^\mu + m^2)\phi = 0$$

$$\boldsymbol{\phi}(\vec{x}) = \int \frac{d^d k}{(2\pi)^d}\sqrt{\frac{\hbar}{2\omega_k}}(e^{i\vec{k}\cdot\vec{x}}a_k + e^{-i\vec{k}\vec{x}}a_k^\dagger)$$

$$\boldsymbol{\pi}(\vec{x}) = \frac{\partial\mathcal{L}}{\partial_\mu\phi} = \frac{1}{i}\int \frac{d^d k}{(2\pi)^d}\sqrt{\frac{\hbar\omega_k}{2}}(e^{i\vec{k}\cdot\vec{x}}a_k - e^{-i\vec{k}\vec{x}}a_k^\dagger)$$

$$[\phi(\vec{x}), \pi(\vec{x'})] = i\hbar\delta^d(\vec{x} - \vec{x'})$$

$$H = \sum_n(p_n\dot{q}_n) = \int dx(\pi(x)\dot{q}(x) - \mathcal{L})$$

5.4.2 Klein-Gordon Field

For a massive scalar (spin 0) and neutral (charge 0) field:

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu\phi)(\partial^\mu\phi) - m^2\phi^2]$$

The Euler-Lagrange formula:

$$(\Box + m^2)\phi = 0$$

This is the Klein-Gordon equation. It was quantized by Pauli and Weisskopf in 1934. The Klein-Gordon equation was historically rejected as a fundamental quantum equation because it predicted negative probability density.

5.4.3 Dirac Field

Dirac was looking for an equation linear in E or in ∂_t. For a massive spinor (spin 1/2) field the Lagrangian density is:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi; \quad \bar{\psi} = \psi^*\gamma^0 \quad \text{Dirac adjoint}$$

The 4×4 Dirac matrices $\gamma^\mu(\mu = 0, 1, 2, 3)$ satisfy the Clifford algebra:

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$$

The corresponding EOM is the Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0; \quad i(\partial_\mu \bar{\psi})\gamma^\mu + m\bar{\psi} = 0$$

The γ matrices:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

Where σ is the Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5.6}$$

Quantization of the Dirac field is achieved by replacing the spinors by field operators and using the Jordand and Wigner quantization rules. Heisenberg's EOM for the field operator $\hat{\psi}(\vec{x}, t)$ reads:

$$i\partial_t \hat{\psi}(\vec{x}, t) = [\hat{\psi}(\vec{x}, t), \hat{H}]$$

There are both positive and negative eigenvalues in the energy spectrum. The later are problematic in view of Einsteins energy of a particle at rest $E = mc^2$. Diracs way out of the negative energy catastrophe was to postulate a Fermi sea of antiparticles. This genial assumption was not taken seriously until the positron was discovered in 1932 by Anderson.

5.4.4 Maxwell Field

$$\tag{5.7}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - j_\mu A^\mu = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - j_\mu A^\mu = -\frac{1}{2}(\partial_\mu A_\nu) + \frac{1}{2}(\partial_\mu A_\mu)^2 - j_\mu A^\mu = (E^2 - B^2)/2 - \phi V + \vec{j}\vec{A}$$

where J_μ is the external current:

$$J_\mu(x) = \begin{cases} J_0(x) = \rho(x) \\ J_i(x) = v_i(x) \end{cases}$$

Field strength tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

with components

$$F^{0i} = \partial^0 A^i - \partial^i A^0 = -E^i, \quad F^{ij} = \partial^i A^j - \partial^j A^i = -\varepsilon^{ijk} B^k$$

EOM $\frac{\partial \mathcal{L}}{\partial A_\nu} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = 0$

$$-J_\nu - \partial_\mu(-\partial_\mu A_\nu) - \partial_\nu(\partial_\mu A_\mu) = 0$$

which gives

$$\partial_\mu F_{\mu\nu} = J_\nu$$

Lorentz gauge: $\partial_\mu A_\mu = 0$

$$J_\nu = \partial_\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) = \Box A_\nu - \partial_\nu(\partial_\mu A_\mu) = \Box A_\nu$$

so

$$A_\nu(x) = \frac{1}{\Box} J_\nu(x)$$

propagator

$$\Pi_A = \frac{1}{\Box}$$

Note that the propagator has nothing to do with the source. In fact it is entirely determined by the kinetic terms for a field.

5.4.5 Proca (Massive Vector Boson) Field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu - j_\mu A^\mu$$

EOM:

$$\Box A^\nu - \partial^\mu(\partial_\mu A^\mu) + m^2 A^\nu = j^\nu$$

Chapter 6

Examples

6.1 Dimensional analysis

6.1.1 Black body radiation

$$I(\omega) \equiv \frac{1}{V}\frac{d}{d\omega}E(\omega)$$

This has units of $[Energy] \times [time] \times [distance]^{-3}$ that can be constructed out of $\omega, k_B T$ and c. So easily:

$$I(\omega) = const \times c^{-3} w^2 k_B T$$

6.2 First Principle

6.2.1 Special relativity field

$$\text{special relativity scalar field}\phi \xrightarrow{simplest possible\, Lorentz invariant} \Box\phi = 0$$

One solution is:

$$\phi(x) = a_p(t)e^{i\vec{p}\cdot\vec{x}}$$

So we get:

$$(\partial_t^2 + \vec{p}\cdot\vec{p})a_p(t) = 0$$

This is exactly the equation of motion of harmonic oscillator. General solution:

$$\phi(x,t) = \int \frac{d^3p}{(2\pi)^3}[a_p(t)e^{i\vec{p}\cdot\vec{x}} + a_p^*(t)e^{-i\vec{p}\cdot\vec{x}}]$$

Chapter 7

doubts

7.1 Doubts

Some doubt about physics

7.1.1 Classical Physics

Newton's Second Law
: Why it is relationship between acceleration and force, rather than velocity and force?

Action
: $S = \int Ldt$, why L, but not other quantities, e.g. H.

Lagrangian
: What's the intuitive physical meaning of $L = E_k - E_p$

7.1.2 Statistical Physics

Enthalpy, Free energy, Gibbs Free energy
: What do H,F,G stand for? Physical meaning?

Entropy
: How do we know the entropy defined in $\Delta\Gamma = \frac{d\Gamma}{dE}\Delta E = e^S$ is the same one as we defined in $dE = TdS - PdV$?

7.2 Notation

7.2.1 Electrodynamics

The reason a conductor puts boundary conditions on the EM field is that the electrons move around to compensate for an applied field. But there is a limit on hwo fast the electrons can move. The resulting cutoff frequency is called the *plasma frequency*

Maxwell Eqn

$$\left.\begin{aligned}\overrightarrow{\nabla}\times(\overrightarrow{\nabla}\varphi)=0\\ \overrightarrow{\nabla}\cdot(\overrightarrow{\nabla}\times\overrightarrow{A})=0\end{aligned}\right\}\Rightarrow\quad\left.\begin{aligned}\overrightarrow{\nabla}\cdot\overrightarrow{E}&=\rho\\ \overrightarrow{\nabla}\times\overrightarrow{B}&=\frac{1}{c}\overrightarrow{j}+\frac{1}{c}\partial_t\overrightarrow{E}\\ \overrightarrow{\nabla}\cdot\overrightarrow{B}&=0\\ -\overrightarrow{\nabla}\times\overrightarrow{E}&=\frac{1}{c}\partial_t\overrightarrow{B}\end{aligned}\right\}\tag{7.1}$$

With $\overrightarrow{E}$ and $\overrightarrow{B}$ still connected with each other, there is one **DOF**.

7.2.2 Statistical Physics

Partisian func

When count from particle (one particle can occupy how many states), remember the $\frac{1}{N!}$ factor! When count from state (how many particles in one state), don't need $\frac{1}{N!}$.

7.2.3 Quantum Mechanics

Baker-Campbell-Hausdorff

if $[A,B]=\mathbb{C}$, then:

$$e^Ae^B=e^Be^Ae^{[A,B]},e^Ae^B=e^{A+B+\frac{1}{2}[A,B]},e^{A+B}=e^{B+A}$$

Angular Momentum

Rep. of J in the Hilbert Space are discrete (finite) $\Leftarrow$ subspace.

$$Y_l^m(\theta,\phi)=\langle\theta,\phi|l,m\rangle=\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(cos\theta)e^{im\phi}$$

Harmonic Oscillator

What's the physical meaning of the energy eigenstate $|\phi\rangle$, what's its relationship to $|x\rangle$ state?

Chapter 8

Terminology

analyticity

bilinear In QFT, a bilinear term means it has exactly two fields. Such as:

$$\mathcal{L}_K \supset \frac{1}{2}\phi\Box\phi, \frac{1}{4}F_{\mu\nu}^2, \frac{1}{2}m^2\phi^2, \frac{1}{2}\phi_1\Box\phi_2, \phi_1\partial_\mu A_\mu, \ldots$$

Boltzmann distribution $n_i = Ne^{-\beta E_i}$

causality

cluster decomposition principle

covariant vectors vectors with lower indices

contravariant vectors vectors with upper indices

equipartion theorem a body in thermal equilibrium should have energy equally distributed among all possible modes, (mode is a seperation of phase space), which means all modes have the same energy.

first principle

Fermi's golden rule $\Gamma \sim |\mathcal{M}|^{\in\delta(\mathcal{E}_{\{}-\mathcal{E}_{\rangle})}$

Legendre transformation

lightlike $V^\mu V_\mu = 0$

little group The representation of the full Poincaré group is induced by a representation of the subgorup of the Poincaré group that holds p^μ fixed, called the *little group*

locality

Lorentz group this is the generalization of the rotation group to include both rotations and boosts.

quantize promote x and p as operators and impose the canonical commutation relations: $[x, p] = i$

quantum process time evolution of an open quantum system ???

representation A set of objects that mix under a transformation group is called a representation of the group.

second quantization canonical quantization of relativistic fields, $H_0 = \int \frac{d^3p}{(2\pi)^3}\omega_p(a_p^{\deg}a_p + \frac{1}{2})$. First quantization refer to the discrete mode, for example, of a particle in a box. Second quantization refers to the integer numbers of excitations of each of these modes. There are two features in second quantization:

1. We have many quantum mechanical systems - one for each $\vec{p}$ - all at the same time.
2. We interpret the nth excitation of the $\vec{p}$ harmonic oscillator as having n *particles*.

S-matrix

spacelike $V^\mu V_\mu < 0$

SO(n) the group of nD rotations ($det(R) = 1$)

timelike $V^\mu V_\mu > 0$

Table 8.1: Frequently used constants

Constant	S.I.	Gauss
h	6.626×10^{-34}	