0.1 Dirac Equation

current

Show that:

$$\overline{u}_f \gamma^\mu \gamma_i = \frac{1}{2m} \overline{u}_f \left((p_f + p_i)^\mu + i\sigma^{\mu\nu}(p_f - p_i)_\nu \right) u_i$$

0.1.1 Conplex Conjugate

0.1.2 γ^5

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$$

0.2 Slashed

- $$\not{p}\gamma^\nu = p_\mu \gamma^\mu \gamma^\nu = p_\mu (2\eta^{\mu\nu} - \gamma^\nu\gamma^\mu) = 2p^\nu - \gamma^\nu \not{p}$$
  $$\Downarrow$$
  $$\not{p}\gamma^\nu + \gamma^\nu \not{p} = 2p^\nu \cdot \mathcal{I}$$

- $$\not{p}\not{q} = p_\mu q_\nu \gamma^\mu\gamma^\nu = 2pq - \not{q}\not{p}$$
  $$\Downarrow$$
  $$\not{p}\not{q} + \not{q}\not{p} = 2pq \cdot \mathcal{I}$$

- $$\gamma^\mu \not{p} \gamma_\mu = \gamma^\mu\gamma^\nu p_\nu \gamma_\mu = (2\eta^{\mu\nu} - \gamma^\nu\gamma^\mu) p_\nu \gamma_\mu = 2\not{p} - \not{p}\gamma^\mu\gamma_\mu = -2\not{p}$$

- $$\gamma^\mu \not{p} \gamma^\nu \gamma_\mu = (2p^\mu - \not{p}\gamma^\mu)\gamma^\nu\gamma_\mu = 2\gamma^\nu\not{p} + 2\not{p}\gamma^\nu = 2p^\nu \cdot \mathcal{I}$$

- $$\gamma^\mu \not{p}\not{q}\gamma_\mu = (2p^\mu - \not{p}\gamma^\mu)(2q_\mu - \gamma_\mu \not{q}) = 4pq - 2\not{p}\not{q} - 2\not{p}\not{q} + 4\not{p}\not{q} = 4pq \cdot \mathcal{I}$$

- $$\begin{aligned}
\gamma^\mu \not{p}\gamma_\nu\not{q}\gamma_\mu &= (2p^\mu - \not{p}\gamma^\mu)\gamma_\nu(2q_\mu - \gamma_\mu\not{q}) = 4pq\gamma_\nu - 2\gamma_\nu\not{p}\not{q} - 2\not{p}\not{q}\gamma_\nu + \not{p}\gamma^\mu\gamma_\nu\gamma_\mu\not{q} \\
&= 4pq\gamma_\nu - 2\gamma_\nu\not{p}\not{q} - 2\not{p}\not{q}\gamma_\nu - 2\not{p}\gamma_\nu\not{q} \\
&= 2\gamma_\nu(2pq - \not{p}\not{q}) - 2\not{p}(\not{q}\gamma_\nu + \gamma_\nu\not{q}) \\
&= \gamma_\nu 2\not{q}\not{p} - 4\not{p}q_\nu \\
&= 2(\gamma_\nu\not{q} - 2q_\nu)\not{p} \\
&= -2\not{q}\gamma_\nu\not{p}
\end{aligned}$$

0.3 Trace