

0.1 Interesting functions

Weierstrass function

$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x) \quad (1)$$

Continuous but not derivative everywhere.

0.2 Differentive Equations

First Order Equation

$$\frac{dV}{dt} + \eta(t)V(t) = f(t)$$

so:

$$\frac{1}{I} \frac{d(I(t)V(t))}{dt} = \frac{dV(t)}{dt} + \frac{I'(t)}{I(t)} V(t)$$

if:

$$\frac{I'(t)}{I(t)} = \eta(t) \rightarrow I(t) = \exp\left(\int^t dt' \eta(t')\right)$$

so:

$$\frac{d(I(t)V(t))}{I dt} = f(t)$$

Floquet theory

$$\ddot{x} + \omega_0^2(1 + \mu \cos(\nu t))x = 0 \quad (2)$$

Analogy to QM(Schrodinger Equation):

$$-\frac{\hbar^2}{2m} \psi'' + U(x)\psi = E\psi$$

where $U(x) = -U_0 \cos \frac{2\pi x}{a}$. So let

$$k^2 = \frac{2mE}{\hbar^2}, u = \frac{U_0}{E}, \nu = \frac{2\pi}{a}$$

u is small, we will get

$$\psi'' + k^2 \psi = -k^2 u \cos(\nu x) \psi$$

0.3 Integrated functions

Γ function

$$\Gamma(z) \equiv \int_0^{+\infty} dt e^{-t} t^{z-1} = 2 \int_0^{+\infty} dx e^{-x^2} x^{2z-1}$$

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}$$