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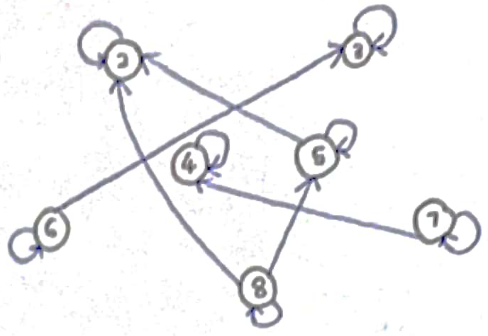
DISCRETE STRUCTURE ASSIGNMENT 2.

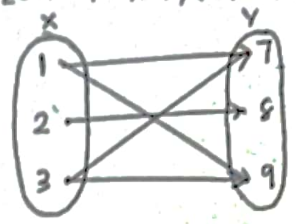
(1) $R = \{(2,2),(5,2),(8,2),(3,3),(6,3),(4,4),(7,4),(5,5),(8,5),(6,6),(7,7),(8,8)\}$



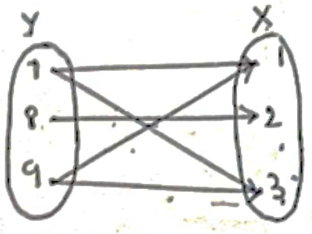
(2) (a) $R = \{(1,7),(1,9),(2,8),(3,7),(3,9)\}$

$R^{-1} = \{(7,1),(9,1),(8,2),(7,3),(9,3)\}$

(b) R =



R^{-1}:

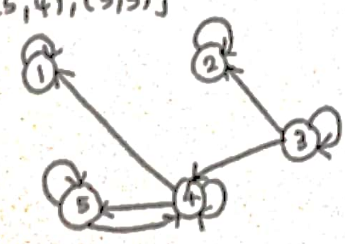


(c) R^{-1} is a function from Y to X.

(3)

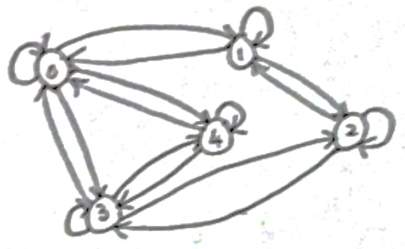
$$\begin{array}{c|c} & \begin{array}{ccccc}1 & 2 & 3 & 4 & 5\end{array} \\ \begin{array}{c}1\\2\\3\\4\\5\end{array} & \begin{bmatrix}1 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 0 & 0\\0 & 1 & 1 & 1 & 0\\1 & 0 & 0 & 1 & 1\\0 & 0 & 0 & 1 & 1\end{bmatrix}\end{array}$$

$R = \{(1,1),(2,2),(3,2),(3,3),(3,4),(4,1),(4,4),(4,5),(5,4),(5,5)\}$



	1	2	3	4	5
in	2	2	1	3	2
out	1	1	3	2	2

(4)



∴ reflexive, symmetric.

(5) $R = \{(1,3),(2,6),(3,9),(4,12)\}$

$$M_R = \begin{array}{c|c} & \begin{array}{cccc}3 & 6 & 9 & 12\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}\end{array}$$

a. Reflexive – M_R has 1's on the main diagonal

$$M_R^T = \begin{array}{c|c} & \begin{array}{cccc}3 & 6 & 9 & 12\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}\end{array}$$

b. symmetric – $M_R = M_R^T$

$$M_R = \begin{array}{c|c} & \begin{array}{cccc}3 & 6 & 9 & 12\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}\end{array} \otimes \begin{array}{c|c} & \begin{array}{cccc}3 & 6 & 9 & 12\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}\end{array}$$

$$= \begin{array}{c|c} & \begin{array}{cccc}3 & 6 & 9 & 12\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}\end{array}$$

c. transitive – $M_R \times M_R = M_R$.

$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} \otimes \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

(6)(a) $RS = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(b) $SR = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(7) Relation shows relationship between input and output while Function is a form of relation which derives one OUTPUT for each given INPUT.

(8) (i) function. One-to-one function.

(ii) function. Domain of A is X.

(iii) not function. The domain of A is not equal to X

(iv) not function. The domain of A is not equal to X.

(9) $R = \{(x,y) \mid y = x+5,\ x \text{ is } Z^+ \text{ less than } 6\}$

$x = \{1,2,3,4,5\}$

$y = \{6,7,8,9,10\}$

$R = \{(1,6),(2,7),(3,8),(4,9),(5,10)\}$

domain: $\{1,2,3,4,5\}$

range: $\{6,7,8,9,10\}$

(10) (v) $f: R \to R,\ f(x) = 1-2x$

let $f(x_1) = f(x_2)$

$1-2x_1 = 1-2x_2$

$-2x_1 = -2x_2$

$x_1 = x_2$

↳ one-to-one

let $y = 1-2x$

$x = \frac{y+1}{-2}$

$f\left(\frac{y+1}{-2}\right) = 1-2\left(\frac{y+1}{-2}\right)$

$f(y) = y \to$ onto

$f(x) = y$, Hence, f is onto

∴ f(x) is bijective because f(x) is both one-to-one and onto.

(vi). $f: R \to R,$

let $f(x_1) = f(x_2)$

$5x_1^2 - 1 = 5x_2^2 - 1$

$5x_1^2 = 5x_2^2$

$x_1^2 = x_2^2 \to x_1 = x_2 \ / \ x_1 = -x_2$

~~$x_1 = x_2 \to$ one-to-one~~ (not one to one)

let $y = 5x^2 - 1$

$5x^2 = y+1$

$x^2 = \frac{y+1}{5}$

$x = \sqrt{\frac{y+1}{5}}$

$f\left(\sqrt{\frac{y+1}{5}}\right) = 5\left(\sqrt{\frac{y+1}{5}}\right)^2 - 1$

$= y+1-1$

$= y \to$ ~~bijective~~ onto

~~∴ f(x) is bijective because f(x) is both one-to-one and onto~~ ∴ f(x) is not onetoone

f(x) is onto

(vii) $f: R \to R,\ f(x) = x^4$

let $f(x_1) = f(x_2)$

$x_1^4 = x_2^4 \to x_1 = x_2 \ / \ x_1 = -x_2$

~~$x_1 = x_2 \to$ one-to-one~~ (not one to one)

let $y = x^4$

$x = \sqrt[4]{y}$

$f(\sqrt[4]{y}) = (\sqrt[4]{y})^4$

$= y \to$ ~~one~~ onto y

∴ f(x) is not one to one

f(x) is onto

viii) $f(x_1) = f(x_2)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\cancel{x_1 x_2} - 2x_2 - 3x_1 + \cancel{6} = \cancel{x_1 x_2} - 3x_2 - 2x_1 + \cancel{6}$$

$$x_2 = x_1 \Rightarrow \text{one to one}$$

let $y = \frac{x-2}{x-3}$

$$y(x-3) = x-2$$

$$xy - 3y = x - 2$$

$$xy - x = 3y - 2$$

$$x(y-1) = 3y-2$$

$$x = \frac{3y-2}{y-1}$$

$$f(x) = \frac{\left(\frac{3y-2}{y-1}\right) - 2}{\left(\frac{3y-2}{y-1}\right) - 3}$$

$$= \frac{\left(\frac{3y-2}{y-1}\right) - \left(\frac{+2y-2}{y-1}\right)}{\left(\frac{3y-2}{y-1}\right) - \left(\frac{3y-3}{y-1}\right)}$$

$$= \frac{y}{\cancel{y-1}} \times \cancel{\frac{1}{y}}\ \frac{\cancel{y-1}}{1}$$

$$= \frac{y}{1} = y \Rightarrow \text{onto}$$

$\therefore f(x)$ is bijective because both one to one and onto.

(11) $f(g(x))$, $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1$; $g(x) = x^2 - 1$

$f(g(x)) = 3(x^2 - 1) - 1$

$= 3x^2 - 3 - 1$

$= 3x^2 - 4$

$f(g(0)) = -4$

$f(g(1)) = -1$

$f(g(2)) = 8$

$f(g(3)) = 23$

(x) $f(x) = x^2$; $g(x) = 5x - 6$

$f(g(x)) = (5x - 6)^2$

$= (5x-6)(5x-6)$

$= 25x^2 - 30x - 30x + 36$

$= 25x^2 - 60x + 36$

$f(g(0)) = 36$

$f(g(1)) = 1$

$f(g(2)) = 16$

$f(g(3)) = 81$

(xi) $f(x) = x - 1$; $g(x) = x^3 + 1$

$f(g(x)) = (x^3 + 1) - 1$

$= x^3$

$f(g(0)) = 0$

$f(g(1)) = 1$

$f(g(2)) = 8$

$f(g(3)) = 27$

(12) (xii) $a_0 = 1$, $a_1 = 6$

$a_n = 6a_{n-1} - 9a_{n-2}$

$a_2 = 6a_{2-1} - 9a_{2-2} = 6(6) - 9(1) = 27$

$a_3 = 6a_{3-1} - 9a_{3-2} = 6(27) - 9(6) = 108$

$a_4 = 6a_{4-1} - 9a_{4-2} = 6(108) - 9(27) = 405$

$a_5 = 6a_{5-1} - 9a_{5-2} = 6(405) - 9(108) = 1458$

$\therefore 1, 6, 27, 108, 405, 1458, \ldots$

(xiii) $a_0 = 2$, $a_1 = 5$, $a_2 = 15$

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

$a_3 = 6a_{3-1} - 11a_{3-2} + 6a_{3-3} = 6(15) - 11(5) + 6(2) = 47$

$a_4 = 6a_{4-1} - 11a_{4-2} + 6a_{4-3} = 6(47) - 11(15) + 6(5) = 147$

$a_5 = 6a_{5-1} - 11a_{5-2} + 6a_{5-3} = 6(147) - 11(47) + 6(15) = 455$

$a_6 = 6a_{6-1} - 11a_{6-2} + 6a_{6-3} = 6(455) - 11(147) + 6(47) = 1395$

$\therefore 2, 5, 15, 47, 147, 455, 1395, \ldots$

(xii) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$

$a_0 = 1$, $a_1 = -2$, $a_2 = -1$

$a_3 = -3a_{3-1} - 3a_{3-2} + a_{3-3} = -3(-1) - 3(-2) + 1 = 10$

$a_4 = -3a_{4-1} - 3a_{4-2} + a_{4-3} = -3(10) - 3(-1) + (-2) = -29$

$a_5 = -3a_{5-1} - 3a_{5-2} + a_{5-3} = -3(-29) - 3(10) + (-1) = 56$

$a_6 = -3a_{6-1} - 3a_{6-2} + a_{6-3} = -3(56) - 3(-29) + (10) = -71$

$\therefore 1, -2, -1, 10, -29, 56, -71, \ldots$

(13) $a_{n+1} = 5a_n - 3$; $a_1 = k$

(i) $5a_1 - 3 = k$

$5k - 3 = a_2$

$5a_2 - 3 = a_3 \Rightarrow 5(5k - 3) - 3 = a_3$

$a_3 = 25k - 18$

$5a_3 - 3 = a_4 \Rightarrow 5(25k - 18) - 3 = a_4$

$a_4 = 125k - 93$

(ii) $a_4 = 125k - 93$

$125k - 93 = 7$

$k = 0.8$.