7. Mostrar las primeras 5 iteraciones del Método de Newton para hallar el cero de $f(x) = x^2 - 6$ con $x_0 = 1$.

Sean $f(x) = x^2 - 6$, $x_0 = 1$, $f'(x) = 2x$, ent.

Iteración 1:

$f(x_0) = (1)^2 - 6 = -5$

$f'(x_0) = 2(1) = 2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{-5}{2} = \frac{7}{2} = 3.5$

Iteración 2

$f(x_1) = \left(\frac{7}{2}\right)^2 - 6 = \frac{49}{4} - 6 = \frac{25}{4} = 6.25$

$f'(x_1) = 2\left(\frac{7}{2}\right) = 7$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{7}{2} - \frac{25/4}{7} = \frac{7}{2} - \frac{25}{28} = \frac{98-25}{28} = \frac{73}{28} \approx 2.60714$

Iteración 3

$f(x_2) = \left(\frac{73}{28}\right)^2 - 6 = \frac{5329 - 4704}{784} = \frac{625}{784} \approx 0.79719$

$f'(x_2) = 2\left(\frac{73}{28}\right) = \frac{73}{14}$

$x_3 = \frac{73}{28} - \frac{625/784}{73/14} = \frac{73}{28} - \frac{8750}{57232} = \frac{10033}{4088} \approx 2.45425$

Iteración 4

$f(x_3) = \left(\frac{10033}{4088}\right)^2 - 6 = \frac{100\,661\,089 - 100270464}{16711744} = \frac{390\,625}{16\,711\,744} \approx 0.02337$

$f'(x_3) = 2\left(\frac{1033}{4088}\right) = \frac{1033}{2044}$

$x_4 = \frac{1033}{4088} - \frac{390625/16711744}{1033/2044} = \frac{1033}{4088} - \frac{798437500}{17263231552} = \frac{249079}{1206544} \approx 0.20644$

Iteración 5

$f(x_4) = \left(\frac{249079}{1206544}\right)^2 - 6 = \frac{62040348241 - 87344905436 16}{1455748423936} = \frac{-867245019375}{1455748423936} \approx -5.95738$

$f'(x_4) = 2\left(\frac{249079}{1206544}\right) = \frac{249079}{603272}$

$x_5 = \frac{249079}{1206544} - \frac{-867245019375/1455748423936}{249079/603272} = \frac{249079}{1206544} + \frac{52318463742642670 00}{3507012957387 3149 44} =$

$= \frac{1535\,582047997972422\,184\,576}{42313654416579205217935 36} \approx 0.362904$