8. Sea $f(x) = x^2 - 6$ con $x_0 = 3$ y $x_1 = 2$, hallar x_4 para

a) El método de Secante.

b) El método de Newton.

a)

Sean $f(x) = x^2 - 6$, $x_0 = 3$, $x_1 = 2$, entonces

$$f(x_0) = (3)^2 - 6 = 3$$

$$f(x_1) = (2)^2 - 6 = -2$$

$$x_2 = x_1 - f(x_1)\frac{x_1 - x_0}{f(x_1) - f(x_0)} = 2 - (-2)\frac{2-3}{-2-(3)} = 2 + 2\frac{-1}{-5} = 2 + \frac{2}{5} = \frac{12}{5} = 2.4$$

$$f(x_2) = \left(\frac{12}{5}\right)^2 - 6 = -\frac{6}{25}$$

$$x_3 = \frac{12}{5} - \left(-\frac{6}{25}\right)\frac{(12/5) - 2}{(-6/25) - (-2)} = \frac{12}{5} + \frac{6}{25}\cdot\frac{2/5}{44/25} = \frac{12}{5} + \frac{3}{55} = \frac{27}{11}$$

$$f(x_3) = 2\left(\frac{27}{11}\right) = \frac{54}{11}$$

$$x_4 = \frac{27}{11} - \frac{54}{11}\cdot\frac{(27/11) - (12/5)}{(54/11) - (-6/25)} = \frac{27}{11} - \frac{54}{11}\cdot\frac{\frac{3}{55}}{\frac{1416}{275}} = \frac{27}{11} - \frac{135}{2596} = \frac{567}{236} \approx 2.40254$$

b)

Sean $f(x) = x^2 - 6$, $x_0 = 3$, entonces $f'(x) = 2x$ (Ignoremos el x_1 dado)

$$f(x_0) = 3^2 - 6 = 3, \quad f'(x_0) = 2(3) = 6$$

$$x_1 = 3 - \frac{3}{6} = \frac{5}{2}$$

$$f(x_1) = \frac{25}{4} - 6 = \frac{1}{4}, \quad f'(x_1) = 5$$

$$x_2 = \frac{5}{2} - \frac{1/4}{5} = \frac{5}{2} - \frac{1}{20} = \frac{49}{20}$$

$$f(x_2) = \frac{2401}{400} - 6 = \frac{1}{400}, \quad f'(x_2) = \frac{49}{10}$$

$$x_3 = \frac{49}{20} - \frac{1/400}{49/10} = \frac{49}{20} - \frac{1}{1960} = \frac{4801}{1960}$$

$$f(x_3) = \left(\frac{4801}{1960}\right)^2 - 6 = \frac{23049601}{3841600} - 6 = \frac{1}{3841600}$$

$$f'(x_3) = 2\,\frac{4801}{1960} = \frac{4801}{980}$$

$$x_4 = \frac{4801}{1960} - \frac{1/3841600}{4801/980} = \frac{4801}{1960} - \frac{1}{18819920} = \frac{46099201}{18819920} \approx 2.44948$$