Семенов Егор, 0362. Вар.16. Ответы:

1. $\begin{cases} x = -2 + 111k, k \in Z \\ y = 2 - 112k, k \in Z \end{cases}$

2. $[11, \overline{3, 5, 3, 22}]$

3. 27413 ~~mod 235290~~

4. 41

5. $f(x) = x^4 + 2x^3 - 5x^2 + x - 3$

6. Рациональных корней нет.

7. 35_7

8. 12

9. $[1, 4, 2, 5, 3, 4]$

10. $5x^2 + 4x + 5$

К.ДЗ. Вариант 16.

1. $1456x + 1443y = -26$

$d = НОД(1456, 1443) = 13$

$112x + 111y = -2$

$112x_0 + 111y_0 = d$, $112x_0 + 111y_0 = 1$

$x_0 = 1 \quad y_0 = -1$

$112x_1 + 111y_1 = c$, $c = -2$

$x_1 = x_0 \cdot \frac{c}{d} = 1 \cdot -2 = -2$

$y_1 = y_0 \cdot \frac{c}{d} = -1 \cdot -2 = 2$

$x = x_1 + \frac{b}{d}k = -2 + \frac{1443}{13}k = -2 + 111k, \ k \in Z$

$y = y_1 - \frac{a}{d}k = 2 - \frac{1456}{12}k = 2 - 112k, \ k \in Z$

Проверка

при $k = 0$ $\quad 1456(-2 + 111 \cdot 0) + 1443(2 - 112 \cdot 0) = -26 \Rightarrow$

решение верно.

2. $\sqrt{128}$

$$\sqrt{128} = 11 + \sqrt{128} - 11 = 11 + \frac{1}{\left(\frac{1}{\sqrt{128} - 11}\right)} =$$

$$= 11 + \cfrac{1}{\cfrac{\sqrt{128}+11}{(\sqrt{128}-11)(\sqrt{128}+11)}} = 11 + \cfrac{1}{\cfrac{\sqrt{128}+11}{7}} = 11 + \cfrac{1}{\cfrac{3+\sqrt{128}+11-\cancel{2 \times 9}\,21}{7}} =$$

$$\cancel{11 + \cfrac{1}{3+\cfrac{\sqrt{128}+2}{3}} = 11 + \cfrac{1}{3+\cfrac{1}{\left(\cfrac{3}{\sqrt{128}+2}\right)}} = 11 + \cfrac{1}{3+\cfrac{1}{\cfrac{3(\sqrt{128}+2)}{124}}}}$$

$$= 11 + \cfrac{1}{3+\cfrac{\sqrt{128}-10}{7}} = 11 + \cfrac{1}{3+\cfrac{1}{\left(\cfrac{7}{\sqrt{128}-10}\right)}} = 11 + \cfrac{1}{3+\cfrac{1}{\left(\cfrac{7(\sqrt{128}+10)}{\cancel{28}\ 4}\right)}} =$$

$$11 + \cfrac{1}{3+\cfrac{1}{\cfrac{5+\sqrt{128}+10-20}{4}}} = 11 + \cfrac{1}{3+\cfrac{1}{5+\cfrac{\sqrt{128}-10}{4}}} = \cancel{11 + \cfrac{1}{3+\cfrac{1}{5+4(\sqrt{128}+10)}}}$$

$$= 11 + \cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{\cfrac{4}{\sqrt{128}-10}}}} = 11 + \cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{\cfrac{4(\sqrt{128}+10)}{\cancel{28}\ 7}}}} = \cancel{11 + \cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{3+\cfrac{1}{\left(\cfrac{7}{\sqrt{128}+10}\right)}}}}} =$$

$$= 11 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{\cfrac{\cancel{7}(\sqrt{128} - 10)}{\cancel{28}\,4}}}}}$$

$$= 11 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{\sqrt{128} + 10 - 21}{7}}}} = 11 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{\cfrac{7}{\sqrt{128} - 11}}}}} =$$

$$= 11 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{\cfrac{\cancel{7}(\sqrt{128} + 11)}{\cancel{7}}}}}} = 11 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{22 + \cfrac{\sqrt{128} \mp 11}{1}}}}}$$

Ответ: $[11, 3, 5, 3, \overline{22}]$

Проверка: первый элемент в два раза меньше последнего, значит, разложение верно.

3.

$x = 23 \mod 30$

$x = 1 \mod 11$

$x = 20 \mod 23$ НОД(30, 11, 23, 31) = 1 => единств. реш.

$x = 9 \mod 31$

$M = 30 \cdot 11 \cdot 23 \cdot 31 = 235290$

$M_1 = \cancel{7843}\ 11 \cdot 23 \cdot 31 = 7843$

$M_2 = 30 \cdot 23 \cdot 31 = 21390$

$M_3 = 30 \cdot 11 \cdot 31 = 10230$

$M_4 = 30 \cdot 11 \cdot 23 = 7590$

$\cancel{7843}\ M_1 x_1 = 1 \mod m \Rightarrow 7843 x_1 = 1 \mod 30$

$7843x - 30y = 1$

i	-1	0	1	2	3	4
r	7843	30	13	4	1	0
q		261	2	3	4	
x	1	0	1	-2	7	

$x_1 = 7$

$\cancel{M_2 x_2 = 11}$

$21390 x_2 = 1 \bmod 11$

$21390x - 11y = 1$

$\cancel{r\ 21}$

r	21390	11	6	5	1	0
q		1944	1	1	5	
x	1	0	1	-1	2	

$x_2 = 2$

$10230 x_3 = 1 \bmod 23$

$10230x - 23y = 1$

r	10230	23	18	5	3	2	1	0
q		444	1	3	1	1	2	
x	1	0	1	-1	4	-5	9	

$x_3 = 9$

$7590 x_4 = 1 \mod 31$

$7590x - 31 = 1$

r	7590	31	26	5	1	0
q		244	1	5	5	
x	1	0	1	-1	6	

$x_4 = 6$

$x = (7843 \cdot 7 \cdot 23 + 21390 \cdot 2 \cdot 1 + 10230 \cdot 9 \cdot 20 + 7590 \cdot 6 \cdot 9)$

$\mod 235290 = 3556763 \mod 235290 = 27413 \mod 235290$

Проверка:

$\cancel{3556763 \equiv 23 \mod 30}$

$\cancel{3556763 \equiv 1 \mod 11}$

$\cancel{3556763 \equiv 20 \mod 23}$

$\cancel{3556763 \equiv 9 \mod 31}$

$27413 < 235290$

$27413 \equiv 23 \mod 30$

$27413 \equiv 1 \mod 11$

$27413 \equiv 20 \mod 23$

$27413 \equiv 9 \mod 31$

Ответ: 27413 ~~mod 235290~~

4. $29^{7^{89}} \bmod 92$

$k = 7^{89} \Rightarrow 29^k \bmod 92$

$\varphi(92) = \varphi(2^2)\cdot\varphi(23) = 4\cdot\left(1-\frac{1}{2}\right)\cdot 22 = 44$

$k = 7^{89} = 44n + b$

$b = 7^{89} \bmod 44$

$\varphi(44) = \varphi(2^2)\cdot\varphi(11) = 2\cdot 10 = 20$

$a = 7 \quad m = 89 \quad k = 44 \quad 89 = 1011001$

d_i	c	c^2	$c^2\cdot a$	$c^2\cdot a \bmod k$
1	1	1	7	7
0	7	49	49	5
1	5	25	175	43
1	43	1849	12943	7
0	7	49	49	5
0	5	25	25	25
1	$\cancel{625}$ 25	625	4375	19

$b = 19$

$29^{44n+b} \bmod 92 = 29^{44n}\cdot 29^{b} \bmod 92 = 29^{b} \bmod 92$

$29^{19} \bmod 92$

$a = 29$ $n = 19$ $k = 92$ $19 = 10011$

a_i	c	c^2	$c^2 \cdot a$	$c^2 \cdot a \bmod k$
1	1	1	29	29
0	29	841	841	13
0	13	169	169	77
1	77	5929	171941	32
1	32	1024	209525 ~~29696~~	41

Ответ: $29^{7^{89}} \bmod 92 = 41$

7590·6·9]
3 mod235290
35290

N6.

$p(1) = -4 \qquad p(-3) = -24$

$p(-4) = 41 \qquad p(-1) = -10$

$p(2) = 11$

$$f(x) = \frac{(x+4)(x-2)(x+3)(x+1)}{5\cdot(-1)\cdot 4\cdot 2}\cdot -4 + \frac{(x-1)(x-2)(x+3)(x+1)}{-5\cdot -6\cdot -1\cdot -3}\cdot 41 +$$

$$+ \frac{(x-1)(x+4)(x+3)(x+1)}{1\cdot 6\cdot 5\cdot 3}\cdot 11 + \frac{(x-1)(x+4)(x-2)(x+1)}{-4\cdot 1\cdot -5\cdot -2}\cdot -24 +$$

$$+ \frac{(x-1)(x+4)(x-2)(x+3)}{-2\cdot 3\cdot -3\cdot 2}\cdot -10 = \frac{(x^2+2x-8)(x+3)(x+1)}{-40}\cdot -4 +$$

$$+ \frac{(x^2-3x+2)(x+3)(x+1)}{90}\cdot 41 + \frac{(x^2+3x-4)(x+3)(x+1)}{90}\cdot 11 + \frac{(x^2+3x-4)(x-2)(x+1)}{-40}$$

$$+ \frac{(x^2+3x-4)\cdot(x-2)(x+3)}{36}\cdot 10 = \frac{(x^2+2x-8)(x^2+4x+3)}{-40}\cdot -4 + \frac{(x^2-3x+2)(x^2+4x+3)}{90}$$

$$+ \frac{(x^2+3x-4)(x^2+4x+3)}{90}\cdot 11 + \frac{(x^2+3x-4)(x^2-x-2)}{-40}\cdot -24 +$$

$$+ \frac{(x^2+3x-4)(x^2+x-6)}{36}\cdot 10 = \frac{(x^4+6x^3+3x^2-26x-24)}{-40}\cdot -4 +$$

$$\cancel{} + \frac{(x^4+x^3-7x^2-x+6)}{90}\cdot 41 + \frac{(x^4+7x^3+11x^2-7x-12)}{90}\cdot 11 +$$

$$+ \frac{(x^4+2x^3-7x^2-2x+6)}{-40}\cdot -24 + \frac{(x^4+4x^3-7x^2-22x+24)}{36}\cdot -10 =$$

$$= \frac{1}{10}(x^4+6x^3+3x^2-26x-24) + \frac{41}{90}(x^4+x^3-7x^2-x+6) +$$

$$+ \frac{11}{90}(x^4+7x^3+11x^2-7x-12) + \frac{3}{5}(x^4+2x^3-7x^2-2x+8) -$$

$$- \frac{5}{18}(x^4+4x^3-7x^2-22x+24) = 0{,}1x^4 + \frac{41}{90}x^4 + \frac{11}{90}x^4 + \frac{3}{5}x^4 - \frac{5}{18}x^4 +$$

$$+ 0{,}6x^3 + \frac{41}{90}x^3 + \frac{77}{90}x^3 + \frac{6}{5}x^3 - \frac{20}{18}x^3 + 0{,}3x^2 - \frac{287}{90}x^2 + \frac{121}{90}x^2 -$$

$$- \frac{27}{5}x^2 - \frac{35}{18}x^2 - 2{,}6x - \frac{41}{90}x - \frac{77}{90}x - \frac{6}{5}x + \frac{110}{18}x - 2{,}4 + \frac{246}{90} -$$

$$- \frac{132}{90} + \frac{24}{5} - \frac{120}{18} = x^4+2x^3-5x^2+x-3$$

Проверка:

$p(1) = 1+2-5+1-3 = -4$

$p(-4) = (-4)^4 + 2\cdot(-4)^3 - 5\cdot(-4)^2 - 4 - 3 = 41$

$p(2) = 2^4 + 2\cdot 2^3 - 5\cdot 2^2 + 2 - 3 = 11$

$p(-3) = (-3)^4 + 2(-3)^3 - 5\cdot(-3)^2 - 3 - 3 = -24$

$p(-1) = (-1)^4 + 2(-1)^3 - 5(-1)^2 - 1 - 3 = -10$

6.

$x^4 - 5x^3 - 6x^2 + 7x - 2$

$p = \pm 1; \pm 2 \Rightarrow x = \pm 1; \pm 2$

$q = 1$

$x = 1$

	1	-5	-6	7	-2
1	1	-4	-10	-3	-5

~~$x = -1$~~

	~~1~~	~~-5~~	~~-6~~	~~7~~	~~-2~~
	~~1~~	~~-6~~	[illegible]	~~0~~	

$x = -1$

	1	-5	-6	7	-2
-1	1	-6	0	7	-9

$x = 2$

	1	-5	-6	7	-2
2	1	-3	-12	-17	-36

$x = -2$

	1	-5	-6	7	-2
-2	1	-7	8	-9	16

Ответ: рациональных корней нет

Проверка: ~~не равны~~ разложение по схеме ~~Гор~~ Горнера невозможно.

N10

$x^5 + x^3 + \ldots$ $\ldots x^2 + 4x + 4$

$x^3 + 4x^2 + 4x + 4$

$x^3 + 4x^2 + 4x + 4$

x^2

№7. 1 способ

$2x + 54 = 160$

$2_7 = 2_{10}$

$54_7 = \cancel{32_{10}}\ 7 \cdot 5 + 4 = 39_{10}$

$160_7 = 49 + 7 \cdot 6 = 91_{10}$

$2x + 39 = 91$

$x = \frac{91 - 39}{2} = 26$

$26_{10} = 35_7$

2 способ

$2_7 x + 54_7 = 160_7$

$x = \frac{160_7 - 54_7}{2_7} = 35_7$

$$\begin{array}{r} 160 \\ -\ 54 \\ \hline 103_7 \end{array}$$

$$\begin{array}{r|l} 103 & 2 \\ -6 & 35 \\ \hline 13 & \\ -13 & \\ \hline 0 & \end{array}$$

Ответ: 35_7

~~$f(92) \cdot f(92) = 44$~~

~~$29^{19} \bmod 44$~~ ~~$y = 29^{19} \bmod 44$~~

8) $x = 30/85 \bmod 99$

$85x = 30 \bmod 99$

$85x - 99y = 30$ $y' = -y$

$85x + 99y' = 30$

НОД$(85, 99) = 1$

r	85	99	85	14	1	0
q		0	1	6	14	
x	1	0	1	-1	7	

$x = x_3 \cdot c/d + b/d \cdot k = 7 \cdot \frac{30}{1} + \frac{99}{1}k = 210 + 99k,\ k \in Z$

$x = 210 \bmod 99 = 12.$

~~Проверка~~ Ответ: 12.

9. $\frac{824}{673}$

1 способ: $\frac{824}{673} = 1+\frac{151}{673} = 1+\frac{1}{\left(\frac{673}{151}\right)} = 1+\frac{1}{4+\frac{69}{151}} =$

$\cancel{1+\frac{1}{4+\frac{151}{69}}} = \cancel{1+\frac{1}{4+}} = 1+\frac{1}{4+\frac{1}{\left(\frac{151}{69}\right)}} = 1+\frac{1}{4+\frac{1}{2+\frac{13}{69}}} =$

$= 1+\frac{1}{4+\frac{1}{2+\frac{1}{\frac{69}{13}}}} = 1+\frac{1}{4+\frac{1}{2+\frac{1}{5+\frac{4}{13}}}} = 1+\frac{1}{4+\frac{1}{2+\frac{1}{5+\frac{1}{3+\frac{1}{4}}}}} =$

$= [1, 4, 2, 5, 3, 4]$

2 способ

$824 = 1\cdot 673 + 151$

$673 = 4\cdot 151 + 69$

$151 = 2\cdot 69 + 13$

$69 = 5\cdot 13 + 4$

$13 = 4\cdot 3 + 1$

$4 = 4\cdot 1$

$= [1, 4, 2, 5, 3, 4]$

N10 $\mathbb{Z}/7\mathbb{Z}[x]$

$$\frac{x^5+2x^4+2x^3+4x^2+6x+1}{x^3+4x^2+4x+4}$$

$$\begin{array}{l|l}
x^5+2x^4+2x^3+4x^2+6x+1 & x^3+4x^2+4x+4 \\
\underline{-\,x^5+4x^4+4x^3+4x^2} & x^2+5x+6 \\
\quad 5x^4+5x^3+0x^2+6x & \\
\quad \underline{-\,5x^4+6x^3+6x^2+6x} & \\
\qquad 6x^3+x^2+0x+1 & \\
\qquad \underline{-\,6x^3+3x^2+3x+3} & \\
\qquad\quad 5x^2+4x+5 &
\end{array}$$

Проверка: $\underline{x^5+2x^4+2x^3+4x^2+6x+1} = (x^3+4x^2+4x+4)\cdot(x^2+5x+6)+$
$+5x^2+4x+5.$

$= x^5+4x^4+4x^3+4x^2+5x^4+26x^3+26x^2+26x+6x^3+33x^2+33x+$

$33+5x^2+4x+5 = \underline{x^5+2x^4+2x^3+4x^2+6x+1}$