17 вариант

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1	Частное реш. $x_1 = -144$ $y_1 = -261$	общее реш $x = -144 - 123k$ $y = -261 - 223k$
2	$[17; \overline{3, 8, 3, 34}]$	
3	45131	
4	63	
5	$-x^4 - 4x^3 - x^2 + x + 3$	
6	рациональных корней нет	
7	35_{10} или 43_8	
8	21	
9	[4, 2, 5, 1, 3, 2]	
10	$3x^2 + x + 4$	

① $1561x - 861y = -63 \Rightarrow \begin{array}{l} a = 1561 \\ b = -861 \\ c = -63 \end{array}$

Пусть $y' = -y \Rightarrow 1561x + 861y' = -63 \Rightarrow \begin{array}{l} a' = 1561 \\ b' = 861 \\ c' = -63 \end{array}$

	-1	0	1	2	3	4	5	6
r	1561	861	700	161	56	49	7	0
q		1	1	4	2	1	7	
x	1	0	1	-1	5	-11	16	
y	0	1	-1	2	-9	20	-29	

$d = НОД(a, b) = 7$

$x_0 = 16 \Rightarrow x_1 = x_0 \cdot \frac{c}{d} = 16 \cdot \frac{-63}{7} = -144$

$y'_0 = -29 \Rightarrow y'_1 = y'_0 \cdot \frac{c'}{d} = -29 \cdot \frac{-63}{7} = 261$

$\left.\begin{array}{l} y'_1 = 261 \\ y'_1 = -y_1 \end{array}\right| \Rightarrow y_1 = -261$

Тогда $\begin{array}{l} x_1 = -144 \\ y_1 = -261 \end{array}$ – частное решение

$x = x_1 + \frac{b}{d}k = -144 + \frac{-861}{7}k = -144 - 123k$

$y = y_1 - \frac{a}{d}k = -261 - \frac{1561}{7}k = -261 - 223k$ – общее решение

Проверка.

Подставлю $x_1 = -144$; $y_1 = -261$ в исходное уравнение

$1561 \cdot (-144) - 861 \cdot (-261) = -224784 + 224721 = -63 \Rightarrow$
$\Rightarrow x_1, y_1$ верны

Найду ещё пару корней с помощью общего решения и проверю их с помощью подстановки.

$x_2 = -144 - 123(-1) = -21$
$y_2 = -261 - 223(-1) = -38$

$1561 \cdot (-21) - 861 \cdot (-38) = -32781 + 32718 = -63 \Rightarrow x_2, y_2$ верны

Ответ: частное решение: $x_1 = -144$
$y_1 = -261$
общее решение: $x = -144 - 123k$
$y = -261 - 223k$

1561×144:
6244
6244
1561
224784

861×261:
861
5166
1722
224721

861×38:
6888
2583
32718

1561×21:
1561
3122
32781

② $$\sqrt{300} = 17 + (\sqrt{300}-17) = 17 + \frac{1}{\left(\frac{1}{\sqrt{300}-17}\right)} = 17 + \frac{1}{\left(\frac{\sqrt{300}+17}{(\sqrt{300}-17)(\sqrt{300}+17)}\right)} =$$

$$= 17 + \frac{1}{\left(\frac{\sqrt{300}+17}{11}\right)} = 17 + \frac{1}{\left(\frac{17+(\sqrt{300}-17)+\cancel{17}}{11}\right)} = 17 + \frac{1}{\left(\frac{34+(\sqrt{300}-17)}{11}\right)} =$$

$$= 17 + \frac{1}{3+\frac{\sqrt{300}-16}{11}} = 17 + \frac{1}{3+\frac{1}{\left(\frac{11}{\sqrt{300}-16}\right)}} = 17 + \frac{1}{3+\frac{1}{\left(\frac{(\sqrt{300}+16)\cdot 11}{(\sqrt{300}-16)(\sqrt{300}+16)}\right)}} =$$

$$= 17 + \frac{1}{3+\frac{1}{\left(\frac{(\sqrt{300}+16)\cancel{11}}{\cancel{44}\ 4}\right)}} = 17 + \frac{1}{3+\frac{1}{\left(\frac{17+(\sqrt{300}-17)+16}{4}\right)}} = 17 + \frac{1}{3+\frac{1}{\left(\frac{33+(\sqrt{300}-17)}{4}\right)}} =$$

$$= 17 + \frac{1}{3+\frac{1}{8+\frac{\sqrt{300}-16}{4}}} = 17 + \frac{1}{3+\frac{1}{8+\frac{1}{\left(\frac{4}{\sqrt{300}-16}\right)}}} = 17 + \frac{1}{3+\frac{1}{8+\frac{1}{\left(\frac{4(\sqrt{300}+16)}{(\sqrt{300}-16)(\sqrt{300}+16)}\right)}}} =$$

$$= 17 + \frac{1}{3+\frac{1}{8+\frac{1}{\left(\frac{\cancel{4}(\sqrt{300}+16)}{\cancel{44}\ 11}\right)}}} = 17 + \frac{1}{3+\frac{1}{8+\frac{1}{\left(\frac{17+(\sqrt{300}-17)+16}{11}\right)}}} = 17 + \frac{1}{3+\frac{1}{8+\frac{1}{\left(\frac{33+(\sqrt{300}-17)}{11}\right)}}} =$$

$$= 17 + \frac{1}{3+\frac{1}{8+\frac{1}{3+\frac{\sqrt{300}-17}{11}}}} = 17 + \frac{1}{3+\frac{1}{8+\frac{1}{3+\frac{1}{\left(\frac{11}{\sqrt{300}-17}\right)}}}} =$$

$$= 17 + \frac{1}{3+\frac{1}{8+\frac{1}{3+\frac{1}{\left(\frac{(\sqrt{300}+17)\cdot 11}{(\sqrt{300}-17)(\sqrt{300}+17)}\right)}}}} = 17 + \frac{1}{3+\frac{1}{8+\frac{1}{3+\frac{1}{\left(\frac{\cancel{11}(\sqrt{300}+17)}{\cancel{11}}\right)}}}} =$$

$$= 17 + \frac{1}{3+\frac{1}{8+\frac{1}{3+\frac{1}{17+(\sqrt{300}-17)+17}}}} = 17 + \frac{1}{3+\frac{1}{8+\frac{1}{3+\frac{1}{34+(\sqrt{300}-17)}}}}$$

В результате: $\sqrt{300} = [17; \overline{3, 8, 3, 34}]$

Проверка: $34 = 17\cdot 2 \Rightarrow$ $\Rightarrow$ результат верен

Ответ: $\sqrt{300} = [17; \overline{3, 8, 3, 34}]$

3

$x \equiv 1 \bmod 10$

$x \equiv 8 \bmod 13$

$x \equiv 28 \bmod 37$

$x \equiv 13 \bmod 17$

	c	m
1	1	10
2	8	13
3	28	37
4	13	17

$M = 13 \cdot 10 \cdot 37 \cdot 17 = 81770$

$M_1 = 13 \cdot 37 \cdot 17 = 8177$

$M_2 = 10 \cdot 37 \cdot 17 = 6290$

$M_3 = 10 \cdot 13 \cdot 17 = 2210$

$M_4 = 10 \cdot 13 \cdot 37 = 4810$

а) $8177 x_1 \equiv 1 \bmod 10 \Rightarrow x_1 = 3$

r	8177	10	7	3	1
q		817	1	2	3
x	1	0	1	-1	3

б) $6290 x_2 \equiv 1 \bmod 13 \Rightarrow x_2 = 6$

r	6290	13	11	2	1
q		483	1	5	2
x	1	0	1	-1	6

в) $2210 x_3 \equiv 1 \bmod 37 \Rightarrow x_3 = 11$

r	2210	37	27	10	7	3	1
q		59	1	2	1	2	3
x	1	0	1	-1	3	-4	11

г) $4810 x_4 \equiv 1 \bmod 17$

r	4810	17	16	1
q		282	1	16
x	1	0	1	-1

$-1 \bmod 17 \equiv 16 \bmod 17$

$x \equiv (8177 \cdot 3 \cdot 1 + 6290 \cdot 6 \cdot 8 + 2210 \cdot 11 \cdot 28 + 4810 \cdot 16 \cdot 13) \bmod 81770$

$x \equiv (24531 + 301920 + 680680 + 1000480) \bmod 81770$

$x \equiv 2007611 \bmod 81770$

$x = 45131$

Проверка:

$45131 = 4513 \cdot 10 + 1 \Rightarrow 45131 \equiv 1 \bmod 10$

$45131 = 3471 \cdot 13 + 8 \Rightarrow 45131 \equiv 8 \bmod 13$

$45131 = 1219 \cdot 37 + 28 \Rightarrow 45131 \equiv 28 \bmod 37$

$45131 = 2654 \cdot 17 + 13 \Rightarrow 45131 \equiv 13 \bmod 17$

$\Rightarrow$ 45131 – искомое число

Ответ: 45131.

④ $19^{13^{77}} \mod 86$

$k = 13^{77} \Rightarrow 19^k \mod 86$

$\varphi(86) = \varphi(2)\cdot\varphi(43) = 1\cdot 42 = 42$

в $\mathbb{Z}_{86}$: $19^k = 19^{42n+b} = 19^{42n}\cdot 19^b = 19^b$

$k = 13^{77} = 42n + b \Rightarrow b \equiv 13^{77} \mod 42$

$13^{77} \mod 42$; $a^m \mod k \Rightarrow a = 13;\ m = 77;\ k = 42$

77	38	19	9	4	2	1
1	0	1	1	0	0	1

$77_{10} = 1001101_2$

a_i	c	c^2	if(a_i==1) c^2*a else c^2	c^2 или c^2*a mod k
1	1	1	13	13
0	13	169	169	1
0	1	1	1	1
1	1	1	13	13
1	13	169	2197	13
0	13	169	169	1
1	1	1	13	$\boxed{13}$

121×19: $1089 + 121 = 2299$

65×65: $325 + 390 = 4225$

361×19: $3249 + 361 = 6859$

169×13: $507 + 169 = 2197$

$b \equiv 13^{77} \mod 42$

$13 \equiv 13^{77} \mod 42$

$b = 13$

$19^k \mod 86 \equiv 19^b \mod 86 \Rightarrow$

$\Rightarrow 19^k \mod 86 \equiv 19^{13} \mod 86$, $k = 13^{77}$ $\Rightarrow$

$\Rightarrow 19^{13^{77}} \mod 86 \equiv 19^{13} \mod 86$

$19^{13} \mod 86$, $a^m \mod k$ $\Rightarrow$ $a = 19$, $m = 13$, $k = 86$

13	6	3	1
1	0	1	1

$13_{10} = 1101_2$

a_i	c	c^2	if(a_i==1) c^2*a else c^2	c^2 или c^2*a mod k
1	1	1	19	19
1	19	361	6859	65
0	65	4225	4225	11
1	11	121	2299	$\boxed{63}$

$63 \equiv 19^{13} \mod 86$

$19^{13^{77}} \mod 86 \equiv 19^{13} \mod 86$ $\Rightarrow 63 \equiv 19^{13^{77}} \mod 86$

Ответ: 63

(5)

$p(2) = -47$
$p(-3) = 18$
$p(-2) = 13$
$p(-4) = -17$
$p(-1) = 4$

$$P(x) = \frac{(x+3)(x+2)(x+4)(x+1)}{4\cdot5\cdot6\cdot3}\cdot(-47) + \frac{(x-2)(x+2)(x+4)(x+1)}{(-5)\cdot(-1)\cdot1\cdot(-2)}\cdot18 + \frac{(x-2)(x+3)(x+4)(x+1)}{(-4)\cdot1\cdot2\cdot(-1)}\cdot13 + \frac{(x-2)(x+3)(x+2)(x+1)}{(-6)\cdot(-1)\cdot(-2)(-3)}(-17) + \frac{(x-2)(x+3)(x+4)(x+2)}{(-3)\cdot\cancel{2}\cdot1\cdot3}\cdot\cancel{4}^{2} =$$

$$= -\frac{47}{360}(x+3)(x+2)(x+4)(x+1) - \frac{18}{10}(x^2-4)(x+4)(x+1) + \frac{13}{8}(x-2)(x+3)(x+4)(x+1) - \frac{17}{36}(x^2-4)(x+3)(x+1) - \frac{2}{9}(x^2-4)(x+3)(x+4) =$$

$$= \overset{x^2+4x+3}{(x+1)(x+3)}(x+4)\left(\frac{13}{8}(x-2) - \frac{47}{360}(x+2)\right) - (x^2-4)\left(\frac{18}{10}(x+4)(x+1) + \frac{17}{36}(x+3)(x+1) + \frac{2}{9}(x+3)(x+4)\right) = (x^3+4x^2+4x^2+16x+3x+12)\left(\frac{13^{(45}}{8}x - \frac{13}{4} - \frac{47}{360}x - \frac{47}{180}\right) -$$

$$- (x^2-4)\left(\frac{18}{10}(x^2+5x+4) + \frac{17}{36}(x^2+4x+3) + \frac{2}{9}(x^2+7x+12)\right) =$$

$$= (x^3+8x^2+19x+12)\left(\frac{585-47}{360}x - \frac{585+47}{180}\right) - (x^2-4)\left(\frac{18^{(18}}{10}x^2 + 9^{(9}x + \frac{36^{(12}}{5} + \frac{17^{(5}}{36}x^2 + \frac{17}{9}x + \frac{17^{(5}}{12} + \frac{2^{(20}}{9}x^2 + \frac{14}{9}x + \frac{8^{(20}}{3}\right) = (x^3+8x^2+19x+12)\left(\frac{269}{180}x - \frac{158}{45}\right) -$$

$$- (x^2-4)\left(\frac{324+85+40}{180}x^2 + \frac{81+17+14}{9}x + \frac{432+85+160}{60}\right) =$$

$$= \frac{269}{180}x^4 - \frac{158}{45}x^3 + \frac{538}{45}x^3 - \frac{1264^{(4}}{45}x^2 + \frac{5111}{180}x^2 - \frac{3002}{45}x + \frac{269^{(3}}{15}x - \frac{632}{15} - (x^2-4)\left(\frac{449}{180}x^2 + \frac{112}{9}x + \frac{677}{60}\right) =$$

$$= \frac{269}{180}x^4 + \frac{380}{45}x^3 + \frac{55}{180}x^2 - \frac{2195}{45}x - \frac{632}{15} - \left(\frac{449}{180}x^4 + \frac{112}{9}x^3 + \frac{677}{60}x^2 - \frac{449}{45}x^2 - \frac{448}{9}x - \frac{677}{15}\right) = \frac{269}{180}x^4 + \frac{380}{45}x^3 + \frac{55}{180}x^2 - \frac{2195}{45}x - \frac{632}{15} -$$

$$- \frac{449}{180}x^4 - \frac{\overset{560}{112^{(5}}}{9}x^3 - \frac{\overset{2031}{677^{(3}}}{60}x^2 + \frac{\overset{1796}{449^{(4}}}{45}x^2 + \frac{\overset{2240}{448^{(5}}}{9}x + \frac{677}{15} =$$

$$= \left(\frac{269}{180} - \frac{449}{180}\right)x^4 + \left(\frac{380}{45} - \frac{560}{45}\right)x^3 + \left(\frac{55-2031+1796}{180}\right)x^2 + \left(\frac{2240}{45} - \frac{2195}{45}\right)x + \frac{677}{15} - \frac{632}{15} = -x^4 - 4x^3 - x^2 + x + 3 \Rightarrow$$

$P(x) = -x^4 - 4x^3 - x^2 + x + 3$

Проверка:

x	-1	-4	-1	1	3	
2	-1	-6	-13	-25	-47	$\Rightarrow p(2) = -47$
-3	-1	-1	2	-5	18	$\Rightarrow p(-3) = 18$
-2	-1	-2	3	-5	13	$\Rightarrow p(-2) = 13$
-4	-1	0	-1	5	-17	$\Rightarrow p(-4) = -17$
-1	-1	-3	2	-1	4	$\Rightarrow p(-1) = 4$

Ответ: $-x^4 - 4x^3 - x^2 + x + 3$.

⑥ $x^4-5x^3-6x^2+7x-2=0$

$\frac{p}{q}=\frac{\pm2;\ \pm1}{\pm1}$

x	1	−5	−6	7	−2		Проверка:
1	1	−4	−10	−3	−5	$\Rightarrow f(1)=-5$	$f(1)=1-5-6+7-2=-5$
−1	1	−6	0	7	−9	$\Rightarrow f(-1)=-9$	$f(-1)=\cancel{1+5}\ \cancel{-6}-7-2=-9$
2	1	−3	−12	−17	−36	$\Rightarrow f(2)=-36$	$f(2)=16-40-24+14-2=-36$
−2	1	−7	8	−9	16	$\Rightarrow f(-2)=16$	$f(-2)=16+40-24-14-2=16$

Проверены все потенциально возможные рациональные корни.

Ответ: рациональных корней нет.

⑦ $4x+77=313\ (8\ cc)$

I способ:

$4_8=4_{10}$

$\overset{1}{7}\overset{0}{7}_8=7\cdot8^1+7\cdot8^0=63_{10}$

$\overset{2}{3}\overset{1}{1}\overset{0}{3}_8=3\cdot8^2+1\cdot8^1+3\cdot8^0=192+8+3=203_{10}$

$\Rightarrow$ $4x+77=313\ (8cc)$ $\downarrow$ $4x+63=203\ (10\ cc)$

$4x+63=203$
$4x=140$
$x=35\ (10\ cc)$
$x=43\ (8\ cc)$

$35\,|\,4$
$3\,|\,4$

$35_{10}=43_8$

II способ: Все вычисления в 8 сс.

$4x+77=313$
$4x=313-77$
$4x=214$
$x=43\ (8\ cc)$
$x=35\ (10\ cc)$

$$\begin{array}{r} 313 \\ -\ 77 \\ \hline 214 \end{array}$$

$$\begin{array}{r|l} 214 & 4 \\ \cline{2-2} -20 & 43 \\ \hline 14 & \\ -14 & \\ \hline 0 & \end{array}$$

$\overset{1}{4}\overset{0}{3}_8=4\cdot8^1+3\cdot8^0=35_{10}$

Ответ: $x=35_{10}$;
$x=43_8$.

⑧ $\frac{7}{48}$ в $\mathbb{Z}_{91}$ => $7 \cdot \frac{1}{48}$ в $\mathbb{Z}_{91}$

$x = \frac{1}{48}$

$48x - 1 \vdots 91$

$\left.\begin{matrix} 48x - 91y = 1 \\ y' = -y \end{matrix}\right| \Rightarrow 48x + 91y' = 1$

r	48	91	48	43	5	3	2	1
q		0	1	1	8	1	1	2
x	1	0	1	-1	2	-17	19	-36

$x = -36$, но $-36 \notin \mathbb{Z}_{91}$ $\Rightarrow x = 91 - 36 = 55$

В $\mathbb{Z}_{91}$: $\frac{1}{48} = 55 \Rightarrow 7 \cdot \frac{1}{48} = 55 \cdot 7 = 385$

$385 \bmod 91 = 21$

Ответ: 21.

⑨ $\frac{504}{113}$

I способ:

$504 = 4 \cdot 113 + 52$

$113 = 2 \cdot 52 + 9$

$52 = 5 \cdot 9 + 7$

$9 = 1 \cdot 7 + 2$

$7 = 3 \cdot 2 + 1$

$2 = 2 \cdot 1$

НОД$(504, 113) = 1$

$\frac{504}{113} = [4; 2; 5; 1, 3; 2]$

II способ:

$\frac{504}{113} = 4 + \frac{52}{113} = 4 + \frac{1}{\left(\frac{113}{52}\right)} =$

$= 4 + \frac{1}{2 + \frac{9}{52}} = 4 + \frac{1}{2 + \frac{1}{\left(\frac{52}{9}\right)}} =$

$= 4 + \frac{1}{2 + \frac{1}{5 + \frac{7}{9}}} = 4 + \frac{1}{2 + \frac{1}{5 + \frac{1}{\left(\frac{9}{7}\right)}}} =$

$= 4 + \frac{1}{2 + \frac{1}{5 + \frac{1}{1 + \frac{2}{7}}}} = 4 + \frac{1}{2 + \frac{1}{5 + \frac{1}{1 + \frac{1}{\left(\frac{7}{2}\right)}}}} =$

$= 4 + \frac{1}{2 + \frac{1}{5 + \frac{1}{1 + \frac{1}{3 + \frac{1}{2}}}}} = [4, 2, 5, 1, 3, 2]$

Ответ: $[4; 2, 5, 1, 3, 2]$

(10)

$$
\begin{array}{l|l}
-\;\cancel{x^5} + 0\cdot x^4 + x^3 + 3x^2 + 3x + 3 & x^3 + 3x^2 + 4x + 4 \\
\quad \cancel{x^5} + 3x^4 + 4x^3 + 4x^2 & \hline x^2 + 2x + 1 \\
\hline
\qquad -\;\cancel{2x^4} + 2x^3 + 4x^2 + \cancel{3x} + 3 & \\
\qquad\quad \cancel{2x^4} + x^3 + 3x^2 + \cancel{3x} & \\
\hline
\qquad\qquad -\;\cancel{x^3} + x^2 + 0\,x + 3 & \\
\qquad\qquad\quad \cancel{x^3} + 3x^2 + 4x + 4 & \\
\hline
\qquad\qquad\qquad 3x^2 + x + 4 &
\end{array}
$$

$$x^5 + x^3 + 3x^2 + 3x + 3 = (x^3 + 3x^2 + 4x + 4)(x^2 + 2x + 1) + 3x^2 + x + 4$$

Проверка: $(x^2 + 2x + 1)(x^3 + 3x^2 + 4x + 4) + 3x^2 + x + 4 = x^5 \cancel{+3x^4} \cancel{+4x^3} \cancel{+4x^2} \cancel{+2x^4} +$
$\cancel{+x^3} \cancel{+3x^2} + 3x + x^3 \cancel{+3x^2} + 4x + 4 + 3x^2 + x + 4 = x^5 + x^3 + 3x^2 + 3x + 3$

Ответ: $3x^2 + x + 4$