

Учусук 0362 B-21

$$1) \begin{cases} x = 225 - 113K \\ y = 207 - 104K \end{cases}$$

$$2) [19; 1; 6; 1; 38]$$

$$3) x = 25491 \text{ mod } 49$$

$$4) 25 \text{ mod } 49$$

$$5) x^4 + 4x^3 - 2x^2 - x + 4$$

$$6) \emptyset$$

$$7) x = 43_5 = 23_{10}$$

$$8) \begin{cases} x = 592 - 91K \\ y = 481 - 74K \end{cases}$$

$$9) [2; 4; 4; 4; 1; 3]$$

$$10) x^2 + 2x + 1$$

$$1) 1768x - 1921y = 153$$

$$y' = -y$$

$$1768x + 1921y' = 153$$

$$\text{Hog}(1768, 1921) = 17$$

	0	1	2	3	4
1	0	1	2	3	4
2	1721	1768	153	85	68
q	-	1	11	1	1
3	0	1	-1	12	-13
4	1	0	1	-11	12
5					-23

$$a = 1768$$

$$b = -1921$$

$$c = 153$$

$$d = 17$$

$$x_1 = 25$$

$$y_1 = -y' = 23$$

$$x = 25 \cdot 9 \mp 113K = 225 - 113K$$

$$y = 23 \cdot 7 \mp 104K = 207 \mp 104K$$

$$\text{Orakem: } \begin{cases} x = 225 - 113K \\ y = 207 - 104K \end{cases}$$

$$\text{Тероверка: } \underline{1768 \cdot 225 - 1921 \cdot 207 = 153}$$

$$2) \sqrt{395} = 19 + \sqrt{395} - 19 = 19 + \frac{1}{\sqrt{395} - 19} =$$

$$= 19 + \frac{1}{\frac{\sqrt{395} + 19}{34}} = 19 + \frac{1}{\frac{34 + 4 + \sqrt{395} - 19}{34}} =$$

$$= 19 + \frac{1}{1 + \frac{1}{34}} = 19 + \frac{1}{1 + \frac{1}{\frac{34(\sqrt{395} + 15)}{5440}}} =$$

$$= 19 + \frac{1}{1 + \frac{1}{\frac{30 + 4 + \sqrt{395} - 19}{5}}} = 19 + \frac{1}{1 + \frac{1}{6 + \frac{\sqrt{395} - 15}{5}}} =$$

$$= 19 + \frac{1}{1 + \frac{1}{6 + \frac{1}{\frac{34(\sqrt{395} + 15)}{34}}}} = 19 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1 + \frac{\sqrt{395} - 19}{34}}}} =$$

$$= 19 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1 + \frac{1}{\frac{34(\sqrt{395} + 19)}{756}}}}} =$$

$$= 19 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1 + \frac{1}{1 + \frac{1}{34(\sqrt{395} + 19)}}}}} = 19 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1 + \frac{1}{38 + \frac{1}{\sqrt{395} - 19}}}}} =$$

$$\sqrt{395} = [19; 1, 6; 1, 38]$$

Пример: ~~найти наименьшее~~

Пример: $\text{см}[a_0; \dots; a_n]$

$\text{см}[2a_0] = 2a_n$, но с тем же периодом

$$19 \cdot 2 = 38 \Rightarrow \text{н.м.г.}$$

$$3) \quad x \equiv 35 \pmod{37}$$

$$x \equiv 16 \pmod{25}$$

$$x \equiv 3 \pmod{27}$$

$$x = ?$$

$$a_1 = 35 \quad m_1 = 37$$

$$a_2 = 16 \quad m_2 = 25$$

$$a_3 = 3 \quad m_3 = 27$$

$$M_0 = m_1 \cdot m_2 \cdot m_3 = 24975$$

$$M_1 = \frac{24975}{37} = 675$$

$$M_2 = \frac{24975}{25} = 999$$

$$M_3 = \frac{24975}{27} = 925$$

$$675 \cdot y_1 \equiv 35 \pmod{37} \Rightarrow 9y_1 \equiv 35 \pmod{37}$$

$$y_1 = 8$$

$$999 \cdot y_2 \equiv 16 \pmod{25} \Rightarrow 24y_2 \equiv 16 \pmod{25}$$

$$y_2 = 9$$

$$925 \cdot y_3 \equiv 3 \pmod{27} \Rightarrow 7y_3 \equiv 3 \pmod{27}$$

$$y_3 = 12$$

$$X = 8 \cdot 37 + 9 \cdot 25 + 12 \cdot 27 = \underline{25491}$$

Пример:

$$25491 \pmod{37} = 35$$

$$25491 \pmod{25} = 16$$

$$25491 \pmod{27} = 3$$

н.м.г.

$$4) \quad 23^{577}$$

$$K = 5^{77}$$

$$\varphi(99) =$$

$$K = 42^{12}$$

$$b = 5^{77}$$

$$\varphi(42) =$$

$$b = 5^{12}$$

$$77 = 12$$

$$a = 77$$

$$a = 5$$

$$b = 5^5$$

$$S_n = 10$$

$$a_i$$

$$1$$

$$0$$

$$1$$

$$2$$

$$x = 2$$

$$17$$

$$d_i$$

$$1$$

$$0$$

$$0$$

$$0$$

$$0$$

$$1$$

$$0$$

$$1$$

$$0$$

$$1$$

$$0$$

$$1$$

$$0$$

$$1$$

$$0$$

$$4) \quad 23^{5^{77}} \bmod 49$$

$$K = 5^{77}$$

$$\phi(49) = 7^2 \cdot \frac{6}{7} = 42$$

$$K = 42n + b$$

$$b = 5^{77} \bmod 42$$

$$\phi(42) = \phi(2) \cdot \phi(3) \cdot \phi(7) = 12$$

$$b = 5^{12z+d} \bmod 42$$

$$77 = 12z + d$$

$$d = 77 - 24 \cdot 3$$

$$d = 5$$

$$\begin{array}{r|l} 77 & 12 \\ - 72 & 6 \\ \hline 5 & \end{array}$$

$$b = 5^5 \bmod 42$$

$$S_n = 101_2$$

$$d: \quad c \quad c^2 \quad c^3 \cdot d \quad c^3 \cdot d \bmod K$$

$$1 \quad 1 \quad 1 \quad 5 \quad 5$$

$$0 \quad 5 \quad 25 \quad 25 \quad 25$$

$$1 \quad 25 \quad 625 \quad 3125 \quad 17$$

$$x = 23^{17} \bmod 49$$

$$17_n = 10001_2$$

$$d: \quad c \quad c^2 \quad c^3 \cdot d \quad c^3 \cdot d \bmod K$$

$$1 \quad 1 \quad 1 \quad 23 \quad 23$$

$$0 \quad 23 \quad 529 \quad 529 \quad 39$$

$$0 \quad 39 \quad 1521 \quad 1521 \quad 2$$

$$0 \quad 2 \quad 4 \quad 4 \quad 4$$

$$1 \quad 4 \quad 16 \quad 368 \quad 25$$

Answer. $x = 25 \bmod 49$

$$5) p(-3) = -38$$

$$p(2) = 42$$

$$p(-2) = -18$$

$$p(-1) = 0$$

$$p(-4) = -24$$

$$p(x) = \frac{(x-2)(x+2)(x+1)(x+4)}{(-5)(-1)(-2) \cdot 1} (-38) + \frac{(x+3)(x+2)(x+1)(x+4)}{5 \cdot 4 \cdot 3 \cdot 6} \cdot 42 +$$

$$+ \frac{(x+3)(x-2)(x+1)(x+4)}{1 \cdot (-4)(-1) \cdot 2} (-18) + \frac{(x+3)(x-2)(x+2)(x+1)}{(-1)(-6)(-2)(-3)} (-24) =$$

$$= \frac{19}{5} (x^4 + 5x^3 - 20x - 16) + \frac{7}{60} (x^4 + 10x^3 + 35x^2 + 50x + 24) - \frac{9}{4} (x^4 + 6x^3 + 11x^2 - 16x - 2)$$

$$- \frac{2}{3} (x^4 + 4x^3 - x^2 - 16x - 12) = \left(\frac{19}{5} + \frac{7}{60} - \frac{9}{4} - \frac{2}{3} \right) x^4 + \left(\frac{19}{5} + \frac{7}{60} - \frac{27}{2} + \frac{8}{3} \right) x^3 + \left(\frac{49}{12} - \frac{27}{4} + \frac{2}{3} \right) x^2 + \left(-\frac{76}{6} + \frac{19}{2} + \frac{32}{3} \right) x + \left(-\frac{304}{5} + \frac{14}{5} + 54 + 8 \right) =$$

$$= x^4 + 4x^3 - 2x^2 - x + 4$$

Омбем. $p(x) = x^4 + 4x^3 - 2x^2 - x + 4$

Проверка:

$$p(-3) = 81 - 108 - 18 + 4 = -38$$

$$p(-1) = 1 - 4 - 2 + 1 + 4 = 0$$

$$p(2) = 42$$

$$p(-2) = -18$$

$$p(-4) = -24 \quad \text{r.m.g.}$$

$$6) x^4 - 5x^3 - 6x^2 + 7x - 2 = 0$$

$$\frac{p}{q} = \frac{\pm 2; \pm 1}{\pm 1} = \pm 2; \pm 1$$

$$x_1 = 2$$

$$1 \quad -5 \quad -6 \quad 7 \quad -2$$

$$1 \quad 1 \quad -4 \quad -10 \quad -3 \quad -5$$

$$-1 \quad 1 \quad -6 \quad 0 \quad 7 \quad -9$$

$$2 \quad 1 \quad -3 \quad -12 \quad -17 \quad -36$$

$$-2 \quad 1 \quad -7 \quad 8 \quad -9 \quad 16$$

Ответ: рациональных корней нет.

$$7) 4x + 121_5 = 1003_5$$

перевод:

$$4_5 = 4_{10}$$

$$121_5 = 25 + 10 + 1 = 36$$

$$1001_5 = 125 + 3 = 128$$

$$4x = 128 - 36 = 92$$

$$x = 23_{10}$$

$$x = 43_5$$

$$\begin{array}{r} 23 \overline{) 92} \\ 4 \overline{) 4} \end{array}$$

перевод: $4x + 121_5 = 1003_5$

$$4x = 1003_5 - 121_5$$

$$\begin{array}{r} 1003 \\ - 121 \\ \hline 332_5 \end{array}$$

$$4x = 332_5 \Rightarrow x = 43_5$$

$$\begin{array}{r} 332 \overline{) 4} \\ 31 \overline{) 43} \\ \hline 22 \\ - 22 \\ \hline 0 \end{array}$$

Ответ: $x = 43_5 = 23_{10}$

$$8) x = \frac{37}{74} \pmod{91}$$

$$74x - 91y = 37$$

$$y' = -y$$

$$74x + 91y' = 37$$

$$\text{НОД}(91; 74) = 1$$

i	-1	0	1	2	3	4
a	74	74	17	6	5	1
b	-	1	4	2	1	
x	0	1	-1	5	-11	16
y'	1	0	-1	-4	+9	-13

$$a = 74$$

$$b = 91$$

$$c = 37$$

$$d = 1$$

$$x_1 = 16$$

$$y_1 = +13$$

$$x = 16 \cdot 37 - 91K = 592 - 91K$$

$$y = 13 \cdot 37 - 74K = 481 - 74K$$

$$\text{Ответ: } \begin{cases} x = 592 - 91K \\ y = 481 - 74K \end{cases}$$

$$\text{Проверка: } \underline{74 \cdot 592 - 91 \cdot 481 = 37}$$

$$g) \frac{758}{339} = 2 + \frac{80}{339} = 2 + \frac{1}{\left(\frac{339}{80}\right)} = 2 + \frac{1}{4 + \frac{19}{80}} = 2 + \frac{1}{\frac{1}{\left(\frac{80}{19}\right)}} =$$

$$= 2 + \frac{1}{4 + \frac{1}{4 + \frac{1}{18}}} = 2 + \frac{1}{4 + \frac{1}{4 + \frac{1}{19}}} = 2 + \frac{1}{4 + \frac{1}{4 + \frac{1}{\frac{4}{5}}}} =$$

$$= 2 + \frac{1}{4 + \frac{1}{4 + \frac{1}{4 + \frac{1}{1 + \frac{1}{3}}}}}$$

$$\frac{758}{330} = [2; 4, 4, 4, 1, 3]$$

2 способ: $\frac{758}{339} = 2 \cdot 339 + 80$

$$339 = 4 \cdot 80 + 19$$

$$80 = 4 \cdot 19 + 4$$

$$19 = 4 \cdot 4 + 3$$

$$y = 1.3 + 1$$

$$3 = 3 \cdot 1$$

Ans: $[2, 4, 4, 4, 1, 3]$

$$40) \frac{x^5 + x^4 + x^3 + 2x + 2}{x^3 + 2x^2 + x + 2} \in \mathbb{Z}/3\mathbb{Z}[x]$$

$$\begin{array}{r}
 x^5 + x^4 + x^3 + 2x + 2 \\
 \underline{-(x^3 + 2x^2 + x + 2)} \\
 2x^4 + x^2 + 2x \\
 \underline{-(2x^4 + x^3 + 2x^2 + x)} \\
 2x^3 + 2x^2 + x + 2 \\
 \underline{-(2x^3 + x^2 + 2x + 1)} \\
 x^2 + 2x + 1
 \end{array}$$

Проверка: $(x^3 + 2x^2 + x + 2)(x^2 + 2x + 1) + (x^2 + 2x + 1) =$
 $= x^5 + 2x^4 + 2x^3 + 2x^4 + x^3 + x^2 + x^3 + 2x^2 + 2x + 2x^2 + x + 1 + x^2 + 2x + 1 =$
 $= x^5 + x^4 + x^3 + 2x + 2, \text{ т.ч. } \text{т.ч.}$

Ответ: $x^2 + 2x + 1$