Мирошников Николай

Вариант 9.

Таблица 1. Ответы к задачам.

№	Ответ
1	$\begin{cases} x=-108+107k \\ y=112-111k \end{cases}, k\in\mathbb{Z}$
2	$[18;\overline{1,1,1,1,36}]$
3	$x=110.330$
4	85
5	$p(x)=x^4-5x^3-3x^2+5x+5$
6	Рациональных корней нет
7	$x=64_8=52_{10}$
8	32
9	$[1;2,2,1,1,6]$
10	x^2+2x+2

1.

$2553x + 2461y = -92$

$2553x + 2461y = 1$

$2553 = 2461 \cdot 1 + 92$

$2461 = 92 \cdot 26 + 69$

$92 = 69 \cdot 1 + 23$

$69 = 23 \cdot 3$

$23 = 92 - 69 \cdot 1 = 92 - (2461 - 92 \cdot 26) = 92 \cdot 27 - 2461 \cdot 1 =$
$= (2553 - 2461) \cdot 27 - 2461 = 2553 \cdot 27 - 2461 \cdot 28$

$$\begin{matrix} x = 27 \cdot (-4) = -108 \\ y = -28 \cdot (-4) = 112 \end{matrix} \Rightarrow \begin{cases} x = -108 + 107k \\ y = 112 - 111k \end{cases}, k \in \mathbb{Z}$$

Проверка:

$2553 \cdot (-108) + 2461 \cdot 112 = -275.724 + 275.632 = -92$

Ответ: $\cancel{x = -108; y = 112}$ $\begin{cases} x = -108 + 107k \\ y = 112 - 111k \end{cases}, k \in \mathbb{Z}$

(2.) $$\sqrt{346} = 18+\left(\sqrt{346}-18\right) = 18+\cfrac{1}{\left(\cfrac{1}{\sqrt{346}-18}\right)} =$$

$$= 18+\cfrac{1}{\left(\cfrac{\sqrt{346}+18}{22}\right)} = 18+\cfrac{1}{1+\left(\cfrac{\sqrt{346}-4}{22}\right)} = 18+\cfrac{1}{1+\cfrac{1}{\left(\cfrac{22}{\sqrt{346}-4}\right)}} =$$

$$= 18+\cfrac{1}{1+\cfrac{1}{\left(\cfrac{\sqrt{346}+4}{15}\right)}} = 18+\cfrac{1}{1+\cfrac{1}{1+\left(\cfrac{\sqrt{346}-11}{15}\right)}} = 18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\left(\cfrac{15}{\sqrt{346}-11}\right)}}} =$$

$$= 18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\left(\cfrac{\sqrt{346}+11}{15}\right)}}} = 18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\left(\cfrac{\sqrt{346}-4}{15}\right)}}} =$$

$$= 18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\left(\cfrac{15}{\sqrt{346}-4}\right)}}}} = 18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\left(\cfrac{\sqrt{346}+4}{22}\right)}}}} =$$

$$= 18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\left(\cfrac{\sqrt{346}-18}{22}\right)}}}} = 18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\left(\cfrac{22}{\sqrt{346}-18}\right)}}}}} =$$

$$= 18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\left(\cfrac{\sqrt{346}+18}{1}\right)}}}}} = 18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\left(\cfrac{1}{\sqrt{346}+18}\right)}}}} =$$

$$= 18 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \left(\cfrac{1}{36 + \sqrt{346} - 18}\right)}}}} \Rightarrow \sqrt{346} = [18; \overline{1, 1, 1, 1, 36}]$$

Проверка: $36 = 18 \cdot 2 \Rightarrow$ результат верный

Ответ: $\sqrt{346} = [18; \overline{1, 1, 1, 1, 36}]$

3. $\begin{cases} x \equiv 10 \bmod 28 \\ x \equiv 12 \bmod 13 \\ x \equiv 5 \bmod 25 \\ x \equiv 33 \bmod 37 \end{cases}$

	C	m
1	10	28
2	12	13
3	5	25
4	33	37

$M = 28 \cdot 13 \cdot 25 \cdot 37 = 336.700$

$M_1 = 13 \cdot 25 \cdot 37 = 12.025$

$M_2 = 28 \cdot 25 \cdot 37 = 25.900$

$M_3 = 28 \cdot 13 \cdot 37 = 13.468$

$M_4 = 28 \cdot 13 \cdot 25 = 9.100$

а) $12.025 x_1 \equiv 1 \bmod 28 \Rightarrow x_1 = 13$

r	12.025	28	13	2	1
q		429	2	6	2
x	1	0	1	-2	13

б) $25.900 x_2 \equiv 1 \bmod 13 \Rightarrow x_2 = -3 + 13 = 10$

r	25.900	13	4	1
q		1992	3	4
x	1	0	1	-3

в) $13.468 x_3 \equiv 1 \bmod 25 \Rightarrow x_3 = 7$

r	13.468	25	18	7	4	3	1
q		538	1	2	1	1	3
x	1	0	1	-1	3	-4	7

г) $9.100 x_4 \equiv 1 \bmod 37 \Rightarrow x_4 = 18$

r	9100	37	35	2	1
q		245	1	17	2
x	1	0	1	-1	18

$x \equiv (12.025 \cdot 13 \cdot 10 + 25.900 \cdot 10 \cdot 12 + 13.468 \cdot 7 \cdot 5 + 9.100 \cdot 18 \cdot 33) \bmod 336.700$

$x \equiv (1.563.250 + 3.108.000 + 471.380 + 5.405.400) \bmod 336.700$

$x \equiv (10.548.030) \bmod 336.700$

$x = 110.330$

Проверка:

1) $110.330 - 28 \cdot 3.940 = 10$

2) $110.330 - 13 \cdot 8486 = 12$

3) $110.330 - 25 \cdot 4413 = 5$

4) $110.330 - 37 \cdot 2981 = 33$

Ответ: 110.330

4. $29^{19^{83}} \bmod 88$

$k = 19^{83} \Rightarrow 29^k \bmod 88$

$\varphi(88) = \varphi(8)\cdot\varphi(11) = 4\cdot 10 = 40$

В $\mathbb{Z}_{88}$: $29^k = 29^{40n+b} = 29^{40n}\cdot 29^b = 29^b$

$k = 19^{83} = 40n + b \Rightarrow b \equiv 19^{83} \bmod 40$

$19^{83} \bmod 40$; $a^m \bmod k \Rightarrow a = 19; m = 83; k = 40$

$83_{10} = 1010011_2$

a_i	c	c^2	if(a_i==1) $c^2\cdot a$ else c^2	c^2 or $c^2\cdot a$ mod k
1	1	1	19	19
0	19	361	361	1
1	1	1	19	19
0	19	361	361	1
0	1	1	1	1
1	1	1	19	19
1	19	361	6859	19

$b \equiv 19^{83} \bmod 40$

$19 = 19^{83} \bmod 40$

$b = 18$

$29^k \bmod 88 \equiv 29^b \bmod 88 \Rightarrow k = 19^{83}$ | $29^k \bmod 88 \equiv 29^{19} \bmod 88 \Rightarrow$

$\Rightarrow 29^{19^{83}} \bmod 88 \equiv 29^{19} \bmod 88$

$29^{19} \bmod 88$, $a^m \bmod k$ $\Rightarrow$ $a = 29$, $m = 19$, $k = 88$ $19_{10} = 10011_2$

a_i	c	c^2	if(a_i=1) $c^2 a$ else c^2	c^2 or $c^2 a$ mod k
1	1	1	29	29
0	29	841	841	49
0	49	2401	2401	25
1	25	625	18125	85
1	85	~~4680~~ 7225	209.525	85

$85 \equiv 29^{19} \bmod 88$

$29^{19^{83}} \bmod 88 \equiv 29^{19} \bmod 88 \Rightarrow 85 \equiv 29^{19^{83}} \bmod 88$

Ответ: 85

5.
$$p(-2)=39$$
$$p(5)=-45$$
$$p(1)=3$$
$$p(2)=-21$$
$$p(-1)=3$$

$$p(x)=\frac{39(x-5)(x-1)(x-2)(x+1)}{(-7)(-3)(-4)(-1)}-\frac{45(x+2)(x-1)(x-2)(x+1)}{7\cdot4\cdot3\cdot6}+\frac{3(x+2)(x-5)(x-2)(x+1)}{3(-4)(-1)\cdot2}+$$

$$+\frac{21(x+2)(x-5)(x-1)(x+1)}{4\cdot3\cdot3}-\frac{3(x+2)(x-5)(x-1)(x-2)}{(-6)(-2)\cdot3}=$$

$$=\frac{234x^4-1638x^3+2106x^2+1638x-2340-45x^4+225x^2-180+63x^4-252x^3-567x^2+1008x+1260+294x^4-}{504}$$

$$\frac{-882x^3-3234x^2+882x+2940-42x^4+252x^3-42x^2-1008x+840}{}=x^4-5x^3-3x^2+5x+5 \Rightarrow$$

$$\Rightarrow p(x)=x^4-5x^3-3x^2+5x+5$$

Проверка:

x	1	-5	-3	5	5	
-2	1	-7	11	-17	39	$\Rightarrow p(-2)=39$
5	1	0	-3	-10	-45	$\Rightarrow p(5)=-45$
1	1	-4	-7	-2	3	$\Rightarrow p(1)=3$
2	1	-3	-9	-13	-21	$\Rightarrow p(2)=-21$
-1	1	-6	3	2	3	$\Rightarrow p(-1)=3$

Ответ: $x^4-5x^3-3x^2+5x+5$

⑥ $x^4-5x^3-6x^2+7x-2=0$

$$\frac{p}{q}=\frac{\pm 2,\ \pm 1}{\pm 1}$$

x	1	-5	-6	7	-2
1	1	-4	-10	-3	-5
-1	1	-6	0	7	-9
2	1	-3	-12	-17	-36
-2	1	-7	8	-9	16

Проверены все потенциальные рацион. корни

Ответ: рациональных корней нет.

7. $2X + 37 = 207$ (8 сс)

I способ:

$2_8 = 2_{10}$

$37_8 = 3 \cdot 8^1 + 7 \cdot 8^0 = 31_{10}$

$207_8 = 2 \cdot 8^2 + 0 \cdot 8^1 + 7 \cdot 8^0 = 135_{10}$

$\Rightarrow$ $2X + 37 = 207$ (8 сс) $\downarrow$ $2X + 31 = 135$ (10 сс)

$2X + 31 = 135$

$2X = 104$

$X = 52$ (10 сс) $\quad 52_{10} = \underline{6} \cdot 8^1 + \underline{4} \cdot 8^0$

$X = 64$ (8 сс)

II способ: (Вычисления ~~в~~ 8 сс)

$2X + 37 = 207$

$2X = 207 - 37$

$2X = 150$

$X = 64$ (8 сс) $\quad 64_8 = 6 \cdot 8^1 + 4 \cdot 8^0 = 52_{10}$

$X = 52$ (10 сс)

$$\begin{array}{r} \overset{\cdot}{2}\overset{8}{0}7 \\ -\ 37 \\ \hline 150 \end{array} \qquad \begin{array}{r|l} 150 & 2 \\ \cline{2-2} -14 & 64 \\ \hline 10 & \\ -10 & \\ \hline 0 & \end{array}$$

Ответ: $X = 64_8$;
$X = 52_{10}$

(8) ~~74~~ $\frac{3}{48}$ в $\mathbb{Z}_{77} \Rightarrow 3 \cdot \frac{1}{48}$ в $\mathbb{Z}_{77}$

$x = \frac{1}{48}$

$48x - 1 \vdots 91$

$\left.\begin{array}{l} 48x - 91y = 1 \\ y' = -y \end{array}\right| \Rightarrow 48x + 91y' = 1$

r	48	91	48	43	5	3	2	1
q		0	1	1	8	1	1	2
x	1	0	1	-1	2	-17	19	-36

$x = -36$, но $-36 \notin \mathbb{Z}_{77} \Rightarrow x = 77 - 36 = 41$

В $\mathbb{Z}_{77}$: $\frac{1}{48} = 41 \Rightarrow 3 \cdot \frac{1}{48} = 41 \cdot 3 = 123$

$123 \bmod 77 = 32$

Ответ: 32

9. I способ:

$$\frac{112}{79} = 1 + \frac{33}{79} = 1 + \frac{1}{\left(\frac{79}{33}\right)} = 1 + \frac{1}{2 + \left(\frac{13}{33}\right)} =$$

$$= 1 + \frac{1}{2 + \frac{1}{\left(\frac{33}{13}\right)}} = 1 + \frac{1}{2 + \frac{1}{2 + \left(\frac{7}{13}\right)}} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{\left(\frac{13}{7}\right)}}} =$$

$$= 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \left(\frac{6}{7}\right)}}} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{\left(\frac{7}{6}\right)}}}} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6}}}}} \Rightarrow$$

$$\Rightarrow \frac{112}{79} = [1; 2, 2, 1, 1, 6]$$

II способ:

$112 = 1 \cdot 79 + 33$

$79 = 2 \cdot 33 + 13$

$33 = 2 \cdot 13 + 7$

$13 = 1 \cdot 7 + 6$

$7 = 1 \cdot 6 + 1$

$6 = 6 \cdot 1$

НОД$(112, 79) = 1$

$$\frac{112}{79} = [1; 2, 2, 1, 1, 6]$$

Ответ: $[1; 2, 2, 1, 1, 6]$

(10.)

$$
\begin{array}{r|l}
5x^5+3x^4+5x^3+x^2+6x+6 & x^3+6x^2+6x+1 \\
\cline{2-2}
-\ 5x^5+2x^4+2x^3+5x^2 & 5x^2+x+4 \\
\cline{1-1}
x^4+3x^3+3x^2+6x & \\
-\ x^4+6x^3+6x^2+x & \\
\cline{1-1}
4x^3+4x^2+5x+6 & \\
-\ 4x^3+3x^2+3x+4 & \\
\cline{1-1}
x^2+2x+2 &
\end{array}
$$

$$5x^5+3x^4+5x^3+x^2+6x+6=(x^3+6x^2+6x+1)(5x^2+x+4)+x^2+\cancel{x}2x+2$$

Проверка:

$$(x^3+6x^2+6\cancel{3}x+1)(5x^2+x+4)+x^2+2x+2=5x^5+x^4+\cancel{4x^3}+2x^4+\cancel{6x^3}+\cancel{3x^2}+\cancel{2x^3}+$$
$$+\cancel{6x^2}+3x+\cancel{5x^2}+x+4+\cancel{x^2}+2x+2=5x^5+3x^4+5x^3+x^2+6x+6$$

Ответ: $\cancel{(x^3+6x^2+6x+1)(5x^2+x+4)+x^2+2x+2}$

x^2+2x+2