Овсянникова

Вариант 10

Ответы

1. $\begin{cases} x = 108 + 109k \\ y = -106 - 107k \end{cases}$
2. $[11; \overline{1, 10, 1, 22}]$
3. 113929
4. 9 mod 56
5. $x^4 - 5x^3 + 2x^2 - 2x + 5$
6. $\varnothing$
7. 62
8. 64 mod 67
9. $[3; 1, 2, 4, 4, 4]$
10. $2x^2 + 2x + 1$

① $1391x + 1417y = 26$

$a = 1391,\ b = 1417,\ c = 26$

i	-1	0	1	2	3
r	1417	1391	26	13	0
q		1	53	2	
x	0	1	-1	54	
y	1	0	1	-53	

$x_1 = 54,\ y_1 = -53$

$$\begin{cases} x = x_1 \cdot \frac{c}{d} + \frac{b}{d}k = 108 + 109k \\ y = y_1 \cdot \frac{c}{d} - \frac{a}{d}k = -106 - 107k \end{cases},\ k \in \mathbb{Z}$$

Ответ: $\begin{cases} x = 108 + 109k \\ y = -106 - 107k \end{cases}$

Проверка: при $k = 1$ $\begin{cases} x = 217 \\ y = -213 \end{cases}$

$1391 \cdot 217 + 1417 \cdot (-213) = 26$

$26 = 26 \implies$ верно

2 $\sqrt{142} = 11 + \left(\sqrt{142} - 11\right) =$

$$= 11 + \frac{1}{\left(\frac{1}{\sqrt{142}-11}\right)} = 11 + \frac{1}{1+\frac{\sqrt{142}-10}{21}} =$$

$$= 11 + \frac{1}{1+\frac{1}{\left(\frac{21}{\sqrt{142}-10}\right)}} = 11 + \frac{1}{1+\frac{1}{10+\frac{\sqrt{142}-10}{2}}} =$$

$$= 11 + \frac{1}{1+\frac{1}{10+\frac{1}{\left(\frac{2}{\sqrt{142}-10}\right)}}} =$$

$$= 11 + \frac{1}{1+\frac{1}{10+\frac{1}{1+\frac{\sqrt{142}-11}{21}}}} =$$

$$= 11 + \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{\left(\cfrac{21}{\sqrt{142} - 11}\right)}}}} =$$

$$= 11 + \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{22 + (\sqrt{142} - 11)}}}} =$$

$$= [11; \overline{1, 10, 1, 22}]$$

Ответ: $[11; \overline{1, 10, 1, 22}]$

Проверка:

$22 = 2 \cdot 11 \Rightarrow$ верно

(3) $x \equiv 9 \mod 16$
$x \equiv 12 \mod 17$
$x \equiv 4 \mod 15$
$x \equiv 6 \mod 37$

$M = 16 \cdot 17 \cdot 15 \cdot 37 = 150\,960$
$M_1 = 17 \cdot 15 \cdot 37 = 9435$
$M_2 = 16 \cdot 15 \cdot 37 = 8880$
$M_3 = 16 \cdot 17 \cdot 37 = 10064$
$M_4 = 16 \cdot 17 \cdot 15 = 4080$

1) $9435 x_1 - 16 y = 1$ $\qquad y' = -y$

i	-1	0	1	2	3	4
r	9435	16	11	5	1	0
q		589	1	2	5	
x	1	0	1	-1	3	

$x_1 = 3$

2) $8880 x_2 - 17 y = 1$ $\qquad y' = -y$

i	-1	0	1	2	3	4
r	8880	17	6	5	1	0
q		522	2	1	5	
x	1	0	1	-2	3	

$x_2 = 3$

3) $10064 x_3 - 15y = 1 \quad y' = -y$

i	−1	0	1	2	3	4
r	10064	15	14	1	0	
q		670	1	14		
x	1	0	1	−1		

$x_3 = -1$

4) $4080 x_4 - 37y = 1 \quad y' = -y$

i	−1	0	1	2	3	4	5
r	4080	37	10	7	3	1	0
q		110	3	1	2	3	
x	1	0	1	−3	4	−11	

$x_4 = -11$

$x = (9435 \cdot 3 \cdot 9 + 8880 \cdot 3 \cdot 12 + 10064 \cdot (-1) \cdot 4 +$
$+ 4080 \cdot (-11) \cdot 6) \bmod 150960 =$
$= 264889 \bmod 150960 = 113929$

Ответ: 113929

Проверка:

113 929 : 16 = 7120 (ост 9)

113 929 : 17 = 6701 (ост 12)

113 929 : 15 = 7595 (ост 4)

113 929 : 37 = 3079 (ост 6)

4

$25^{17^{37}} \bmod 56 \equiv C$

$k = 17^{37}$

$\varphi(56) = 24$

$k = 24n + b$

$b = 17^{37} \bmod 24$

$37_{10} = 100\,101_2$

a_i	c	c^2	c^2a	$c^2a \bmod k$
1	1	1	17	17
0	17	289	289	1
0	1	1	1	1
1	1	1	17	17
0	17	289	289	1
1	1	1	17	17

$25^{24n+6} \bmod 56 = 25^{24n} \cdot 25^{6} \bmod 56 =$

$= 25^{6} \bmod 56$

$25^{17} \bmod 56 \equiv C$

$17_{10} = 10001_2$

a_i	C	C_2	C^2a	$C^2a \bmod k$
1	1	1	25	25
0	25	625	625	9
0	9	81	81	25
0	25	625	625	9
1	9	81	2025	9

$C \equiv 9 \bmod 56$

Ответ: $9 \bmod 56$

5 $\deg p(x) \leq 4$

i	
0	$p(-1) = 15$
1	$p(2) = -15$
2	$p(5) = 45$
3	$p(4) = -35$
4	$p(1) = 1$

$$p(x)=\frac{15(x-2)(x-5)(x-4)(x-1)}{(-1-2)(-1-5)(-1-4)(-1-1)}-$$

$$-\frac{15(x+1)(x-5)(x-4)(x-1)}{(2+1)(2-5)(2-4)(2-1)}+$$

$$+\frac{45(x+1)(x-2)(x-4)(x-1)}{(5+1)(5-2)(5-4)(5-1)}-$$

$$-\frac{35(x+1)(x-2)(x-5)(x-1)}{(4+1)(4-2)(4-5)(4-1)}+$$

$$+\frac{(x+1)(x-2)(x-5)(x-4)}{(1+1)(1-2)(1-5)(1-4)}=$$

$$=\frac{1}{12}(x^4-12x^3+49x^2-78x+40)-$$

$$-\frac{1}{6}(5x^4-45x^3+95x^2+45x-100)+$$

$$+\frac{1}{8}(5x^4-30x^3+35x^2+30x-40)+$$

$$+\frac{1}{6}(7x^4-49x^3+63x^2+49x-70)-$$

$$-\frac{1}{24}(x^4-10x^3+27x^2-2x-40)=$$

$$= x^4\left(\frac{1}{12} - \frac{5}{6} + \frac{5}{8} + \frac{7}{6} - \frac{1}{24}\right) +$$

$$+ x^3\left(-1 + \frac{45}{6} - \frac{30}{8} - \frac{49}{6} + \frac{10}{24}\right) +$$

$$+ x^2\left(\frac{49}{12} - \frac{95}{6} + \frac{35}{8} + \frac{63}{6} - \frac{27}{24}\right) +$$

$$+ x\left(-\frac{78}{12} - \frac{45}{6} + \frac{30}{8} + \frac{49}{6} + \frac{2}{24}\right) +$$

$$+ \left(\frac{40}{12} + \frac{100}{6} - \frac{40}{8} - \frac{70}{6} + \frac{10}{24}\right) =$$

$$= x^4 - 5x^3 + 2x^2 - 2x + 5$$

Ответ: $p(x) = x^4 - 5x^3 + 2x^2 - 2x + 5$

Проверка:

$$\left.\begin{aligned} p(-1) &= 1+5+2+2+5 = 15 \\ p(2) &= 16-40+8-4+5 = -15 \\ p(5) &= 625-625+50-10+5 = 45 \\ p(4) &= 256-320+32-8+5 = -35 \\ p(1) &= 1-5+2-2+5 = 1 \end{aligned}\right\} \Rightarrow \text{верно}$$

⑥ $x^4 - 5x^3 - 6x^2 + 7x - 2 = 0$

$$\frac{p}{q} = \frac{\pm 2, \pm 1}{\pm 1} = \pm 2$$

Ответ: рациональных корней не имеется, т.к. коэффициент при старшем члене равен 1.

⑦ $4x + 70 = 460$ (8 с.с.)

I способ

$4x = 460 - 70$

$4x = 370$

$x = \frac{370}{4}$

$x = 62_{10}$

$$\begin{array}{r} 460 \\ -\ 70 \\ \hline 370 \end{array} \qquad \begin{array}{r|l} 370 & 4 \\ \cline{2-2} -34 & 76 \\ \hline 30 & \\ -30 & \\ \hline 0 & \end{array}$$

$76_8 = 7 \cdot 8 + 6 = 62_{10}$

II способ

$4x + 70 = 460$

$4_8 = 4_{10}$

$70_8 = 7 \cdot 8 = 56_{10}$

$460_8 = 4 \cdot 8^2 + 6 \cdot 8 = 304_{10}$

$4x + 56 = 304$

$4x = 248$

$x = 62$

Ответ: $x = 62$

(8) $x = \frac{18}{61} \bmod 67$

$61x = 18 \bmod 67$

$61x - 67y = 18$ $\quad y' = -y$

$61x + 67y' = 18$

i	-1	0	1	2	3
r	67	61	6	1	0
q		1	10	6	
x	0	1	-1	11	

$\underline{1}\ \ 61x + 67y' = 1$

$x_1 = 11 \cdot 18 = 198$

$x = 198 \bmod 67 = 64 \bmod 67$

Ответ: $x = 64 \bmod 67$

I способ

9. $$\frac{860}{233} = 3 + \frac{161}{233} = 3 + \frac{1}{\left(\frac{233}{161}\right)} =$$

$$= 3 + \frac{1}{1 + \frac{72}{161}} = 3 + \frac{1}{1 + \frac{1}{\left(\frac{161}{72}\right)}} =$$

$$= 3 + \frac{1}{1 + \frac{1}{2 + \frac{17}{72}}} = 3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\left(\frac{72}{17}\right)}}} =$$

$$= 3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4 + \frac{4}{17}}}} = 3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4 + \frac{1}{\left(\frac{17}{4}\right)}}}} =$$

$$= 3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4 + \frac{1}{4 + \frac{1}{4}}}}} =$$

$$= 3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4}}}}} = [3; 1, 2, 4, 4, 4]$$

II способ

$\frac{860}{233}$

$$860 = 233 \cdot 3 + 161$$
$$233 = 161 \cdot 1 + 72$$
$$161 = 72 \cdot 2 + 17$$
$$72 = 17 \cdot 4 + 4$$
$$17 = 4 \cdot 4 + 1$$
$$4 = 1 \cdot 4 + 0$$

$\Rightarrow$

$$\Rightarrow \frac{860}{233} = [3; 1, 2, 4, 4, 4]$$

Ответ: $[3; 1, 2, 4, 4, 4]$

(10) $\frac{2x^5}{2x^3+2x^2+x+1}$ в кольце $\frac{\mathbb{Z}}{3\mathbb{Z}}[x]$

$$\begin{array}{r|l}
2x^5+0x^4+0x^3+0x^2+0x+0 & 2x^3+2x^2+x+1 \\
-\underline{2x^5+2x^4+x^3+x^2} & \overline{x^2+2x+2} \\
x^4+2x^3+2x^2+0x & \\
-\underline{x^4+x^3+2x^2+2x} & \\
x^3+0x^2+x+0 & \\
-\underline{x^3+x^2+2x+2} & \\
2x^2+2x+1 &
\end{array}$$

Проверка:

$(2x^3+2x^2+x+1)(x^2+2x+2)+2x^2+2x+1 =$

$= 2x^5+0x^4+0x^3+0x^2+0x+0 = 2x^5$

$\Rightarrow$ верно

Ответ: $2x^2+2x+1$