Утушкова Екатерина.

Гр. 0362. Вар. 20.

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $x = 387 + 131k$<br>$y = -585 - 198k$ | $[19, \overline{2,2,2,38}]$ | 234846 | 23 | $-x^4 - 3x^3 + 3x^2 - 5x - 3$ | нет рац. корней. |

| 7 | 8 | 9 | 10 |
|---|---|---|---|
| $x_9 = 48_9$<br>$x_{10} = 44_{10}$ | 23 | $[2;3;2;1;1;3]$ | $2x^2 + 3x + 3$ |

Вар. 20.

1. Решить диофантово ур. $2574x + 1703y = -117$

$d = НОД(2574; 1703) = 13$

| a | b | k (ост) |
|---|---|---|
| 2574 | 1703 | 871 |
| 1703 | 871 | 832 |
| 871 | 832 | 39 |
| 832 | 39 | 13 |
| 39 | 13 | 0 |
| 13 | 0 | |

$a = 2574$, $b = 1703$, $c = -117$, $d = 13$ $\Rightarrow$ $a = 198$, $b = 131$, $c = -9$, $d = 1$

$ax_1 + by_1 = c \Rightarrow 198x + 131y_1 = 9$

$ax_0 + by_0 = d$

$198x_0 + 131y_0 = 1$

$x_0 = -43 \quad y_0 = 65$

$x_1 = x_0 \cdot c/d = +43 \cdot 9 = 387$

$y_1 = y_0 \cdot c/d = -65 \cdot 9 = -585$

$x = x_1 + b/d \cdot k = 387 + 131k$

$y = y_1 - a/d \cdot k = -585 - 198k$

Проверка:

$2574 \cdot 387 - 1703 \cdot 585 = -117$ — верно.

$2574 \cdot 387$: $18018 + 20592 + 7722 = 996138$

$1703 \cdot 585$: $8515 + 13624 + 8515 = 996255$

$996255 - 996138 = 000117$

2. Цеп. др. $\sqrt{377}$

$$\sqrt{377} = 19 + \sqrt{377} - 19 = 19 + \cfrac{1}{\left(\cfrac{1(\sqrt{377}+19)}{(\sqrt{377}-19)(\sqrt{377}+19)}\right)} = 19 + \cfrac{1}{2 + \cfrac{\sqrt{377}-13}{16}} =$$

$$= 19 + \cfrac{1}{2 + \cfrac{1}{\left(\cfrac{16(\sqrt{377}+13)}{(\sqrt{377}-13)(\sqrt{377}+13)}\right)}} = 19 + \cfrac{1}{2 + \cfrac{1}{\left(\cfrac{\sqrt{377}+13}{13}\right)}} = 19 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{\sqrt{377}-13}{13}}} =$$

$$= 19 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\left(\cfrac{13(\sqrt{377}+13)}{(\sqrt{377}-13)(\sqrt{377}+13)}\right)}}} = 19 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\left(\cfrac{\sqrt{377}+13}{16}\right)}}} =$$

$$= 19 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{\sqrt{377}-19}{16}}}} = 19 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\left(\cfrac{16(\sqrt{377}+19)}{(\sqrt{377}-19)(\sqrt{377}+19)}\right)}}}} =$$

$$= 19 + \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{3}{\sqrt{377}+19}}}} = 19 + \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{38+\sqrt{377}-19}}}} =$$

$= [19, 2, 2, 2, 38]$

Проверка: $\frac{38}{19} = 2$ – верно.

№7. Реш. ур. в 9-ой с с. 2 сп.

$8x + 101 = 532$ в 9 сс.

1 сп.

$$\begin{array}{r} 532_9 \\ -\ 101_9 \\ \hline 431_9 \end{array}$$

$$\begin{array}{r|l} 431_9 & 8_9 \\ -\ 35\phantom{1} & 48 \\ \hline 71 & \\ -\ 71 & \\ \hline 0 & \end{array}$$

$X_9 = 48_9$

$X_{10} = 8 + 4 \cdot 9 = 44_{10}$

2 сп.

$8_9 = 8_{10}$

$101_9 = 1 + 1 \cdot 9^2 = 82_{10}$

$532_9 = 2 + 3 \cdot 9 + 5 \cdot 9^2 = 434_{10}$

$8x + 82 = 434$

$x = \frac{434 - 82}{8}$

$X_{10} = 44_{10}$

$X_9 = 48_9$

$$\begin{array}{r|l} 44 & 9 \\ -\ 36 & 4 \\ \hline 8 & \end{array}$$

№4. Найти ост. от дел. "$23^{9^{19}}$" на 60.

$23^{9^{19}} \bmod 60 = C$

$k = 9^{19}$

$\varphi(60) = 16$

$k = 16n + b$

$23^{16n+b} \bmod 60 = 23^{16n} \bmod 60 \cdot 23^{b} \bmod 60 = 23^{b} \bmod 60$

$k = 9^{19} = 16n + b$

$b \equiv 9^{19} \bmod 16 \equiv 9^{16+3} \bmod 16 \qquad \varphi(16) = 8$

$b \equiv 9^{2\cdot 8+3} \bmod 16 \equiv 9^{8\cdot 2} \bmod 16 \cdot 9^{3} \bmod 16 = 9^{3} \bmod 16 \equiv$

$\equiv 9 \bmod 16$

$C \equiv 23^{9} \bmod 60 \equiv 23 \bmod 60$

$23 \cdot 23 = 529 \bmod 60 \equiv 49 \bmod 60$

$49 \cdot 23 = 1127 \bmod 60 \equiv 47 \bmod 60$

$47 \cdot 23 = 1081 \bmod 60 \equiv 1 \bmod 60$

$1 \cdot 23 = 23 \bmod 60$

$23 \cdot 23 = 529 \bmod 60 \equiv 49 \bmod 60$

$49 \cdot 23 = 1127 \bmod 60 \equiv 47 \bmod 60$

$47 \cdot 23 = 1081 \bmod 60 = 1 \bmod 60$

$1 \cdot 23 = 23 \bmod 60$

№ 9 (2 сп). $\frac{140}{61}$ непр. др.

$$\frac{140}{61} = 2 + \frac{18}{61} = 2 + \frac{1}{\left(\frac{61}{18}\right)} = 2 + \frac{1}{3+\frac{7}{18}} = 2 + \frac{1}{3+\frac{1}{\left(\frac{18}{7}\right)}} = 2 + \frac{1}{3+\frac{1}{2+\frac{4}{7}}} =$$

$$= 2 + \frac{1}{3+\frac{1}{2+\frac{1}{\left(\frac{7}{4}\right)}}} = 2 + \frac{1}{3+\frac{1}{2+\frac{1}{1+\frac{3}{4}}}} = 2 + \frac{1}{3+\frac{1}{2+\frac{1}{1+\frac{1}{\left(\frac{4}{3}\right)}}}} = 2 + \frac{1}{3+\frac{1}{2+\frac{1}{1+\frac{1}{1+\frac{1}{3}}}}} =$$

$= [2; 3; 2; 1; 1; 3]$

$140 = 2 \cdot 61 + 18$

$61 = 18 \cdot 3 + 7$

$18 = 7 \cdot 2 + 4$ $\qquad [2; 3; 2; 1; 1; 3]$

$7 = 4 \cdot 1 + 3$

$4 = 3 \cdot 1 + 1$

$3 = 3 \cdot 1 + 0$

№ 3 Найти наим. натур. ч. x. (пров)

$x \equiv 4 \mod 29$

$x \equiv 8 \mod 34$

$x \equiv 6 \mod 19$

$x \equiv 6 \mod 15$

① $M = 29 \cdot 34 \cdot 19 \cdot 15 = 281010$

$M_1 = 34 \cdot 19 \cdot 15 = 9690$

$M_2 = 19 \cdot 15 \cdot 29 = 8265$

$M_3 = 15 \cdot 29 \cdot 34 = 14790$

$M_4 = 29 \cdot 34 \cdot 19 = 18734$

② $M_1 x_1 = 1 \mod m_1 \Rightarrow 9690 x_1 = 1 \mod 29$

$9690 x_1 - 29 y_1 = 1$ $\qquad 8265 x_2 - 34 y_2 = 1$

$x_1 = 22 \quad y_1 = -7351$ $\qquad x_2 = 23 \quad y_2 = -5591$

$14790 x - 19 y_3 = 1$ $\qquad 18734 x_4 - 15 y_4 = 1$

$x_3 = 12 \quad y_3 = -9341$ $\qquad x_4 = -1 \quad y_4 = 1249$

| i | x | M | m | c |
|---|---|---|---|---|
| 1 | 22 | 9690 | 29 | 4 |
| 2 | 23 | 8265 | 34 | 8 |
| 3 | 12 | 14790 | 19 | 6 |
| 4 | -1 | 18734 | 15 | 6 |

$x = (M_1 x_1 c_1 + M_2 x_2 c_2 + M_3 x_3 c_3 + M_4 x_4 c_4) \mod M = (9690 \cdot 22 \cdot 4 + 8265 \cdot 23 \cdot 8 +$

$+ 14790 \cdot 12 \cdot 6 + 18734 \cdot (-1) \cdot 6) \mod 281010 = 234846 \mod 281010$

Проверка: $234846 \bmod 29 = 4 \bmod 29$

$234846 \bmod 34 = 8 \bmod 34$

$234846 \bmod 19 = 6 \bmod 19$ $\Rightarrow$ верно

$234846 \bmod 15 = 6 \bmod 15$

№5 $p(2) = -41$; $p(1) = -9$; $p(-4) = 1$; $p(-1) = 7$; $p(-3) = 39$

p не выше 4-ой степ

$$p = \frac{(x-1)(x+4)(x+1)(x+3)}{6\cdot 3\cdot 5}(-41) + \frac{(x+2)(x+4)(x+1)(x+3)}{(-1)\cdot 5\cdot 2\cdot 4}(-9) +$$

$$+ \frac{(x-2)(x-1)(x+1)(x+3)}{(-6)(-5)(-3)(-1)}\cdot 1 + \frac{(x-2)(x-1)(x+4)(x+3)}{(-3)(-2)\cdot 3\cdot 2}\cdot 7 +$$

$$+ \frac{(x-2)(x-1)(x+4)(x+1)}{(-5)(-4)\cdot 1\cdot(-2)}\cdot 39 = -\frac{41}{90}(x-1)(x+4)(x+1)(x+3) +$$

$$+ \frac{9}{40}(x-2)(x+4)(x+1)(x+3) + \frac{1}{90}(x-2)(x-1)(x+1)(x+3) + \frac{7}{36}(x-2)\times$$

$$\times(x-1)(x+4)(x+3) - \frac{39}{40}(x-2)(x-1)(x+4)(x+1) = -\frac{41}{90}(x^4 + 7x^3 +$$

$$+11x^2 - 7x - 12) + \frac{9}{40}(x^4 + 6x^3 + 3x^2 - 26x - 24) + \frac{1}{90}(x^4 + x^3 - 7x^2 -$$

$$-x + 6) - \frac{39}{40}(x^4 + 2x^3 - 9x^2 - 2x + 8) + \frac{7}{36}(x^4 + 4x^3 - 7x^2 - 22x +$$

$$+24) = -\frac{360}{360}x^4 - \frac{1080}{360}x^3 + \frac{1080}{360}x^2 - \frac{1800}{360}x + \frac{(-1080)}{360} =$$

$$= -x^4 - 3x^3 + 3x^2 - 5x - 3$$

Проверка.

$p(2) = -16 - 8\cdot 3 + 4\cdot 3 - 10 - 3 = 41$ - верно

$p(1) = -1 - 3 + 3 - 5 - 3 = -9$ - верно.

$p(-4) = -16\cdot 16 + 3\cdot 16\cdot 4 + 16\cdot 3 + 20 - 3 = 1$ - верно.

$p(-1) = -1 + 3 + 3 + 5 - 3 = 7$ - верно.

$p(-3) = -81 + 3\cdot 27 + 27 + 15 - 3 = 39$ - верно.

№6. Н. рац. корни $x^4-5x^3-6x^2+7x-2$

Сх. Горнера. $-5+1\cdot1$

$d(1)=\{\pm1\}$ $d(-2)=\{\pm1;\pm2\}$

$\frac{p}{q}=\frac{\pm1;\pm2}{\pm1}$

| | | | | | |
|---|---|---|---|---|---|
| | 1 | -5 | -6 | 7 | -2 |
| 1 | 1 | -4 | -10 | -3 | -5 |
| -1 | 1 | -6 | 0 | 7 | -9 |
| 2 | 1 | -3 | -12 | -17 | -36 |
| -2 | 1 | -7 | 8 | -9 | 16 |

нет рациональных корней.

$x^4-5x^3-6x^2+7x-2=x(x^3-5x^2+6x+7)-2=x(x(x^2-5x+6)+$
$+7)-2=x((x-6)x(x+1)+7)-2$

Проверка:

$f(1)=1-5-6+7-2=-5\neq0$

$f(-1)=1+5-6-7-2=-9\neq0$

$f(2)=16-40-24+14-2=-36\neq0$ $\Rightarrow$ нет рациональных корней

$f(-2)=16+40-24-14-2=16\neq0$

№8. $\frac{10}{15}$ в кольце вычетов по модулю 67.

$\frac{10}{15}$ в $\mathbb{Z}_{67}$ => $10 \cdot \frac{1}{15}$ в $\mathbb{Z}_{67}$

$x = \frac{1}{15}$

$15x - 1 \,\vdots\, 67$

$15x - 67y = 1$ | => $15x + 67y' = 1$

$y' = y$

| | | | | | |
|---|---|---|---|---|---|
| r | 15 | 67 | 15 | 7 | 1 |
| q | | 0 | 4 | 2 | 7 |
| x | 1 | 0 | 1 | -4 | 9 |

$x = 9$

В $\mathbb{Z}_{67}$: $\frac{1}{15} = 9$ => $10 \cdot \frac{1}{15} = 9 \cdot 10 = 90$

$90 \bmod 67 = 23$

Ответ: 23

№10. оа. а дел. $5x^5 + 2x^4 + 4x^3 + 4x^2 + x + 4$ на $4x^3 + 6x^2 + 6x + 6$

$5x^5 + 2x^4 + 4x^3 + 4x^2 + x + 4 \;|\; 4x^3 + 6x^2 + 6x + 6$

$-\;5x^5 + 4x^4 + 4x^3 + 4x^2 \qquad\qquad\quad 3x^2 + 3x + 6$

$-\;5x^4 + 0x^3 + 0x^2 + x + 4$

$5x^4 + 4x^3 + 4x^2 + 4x$

$3x^3 + 3x^2 + 4x + 4$

$-\;3x^3 + x^2 + x + 1$

$2x^2 + 3x + 3$

Проверка. $(4x^3 + 6x^2 + 6x + 6)(3x^2 + 3x + 6) + 2x^2 + 3x + 3 =$

$= 12x^5 + 30x^4 + 60x^3 + 72x^2 + 54x + 36 + 2x^2 + 3x + 3 =$

$= 12x^5 + 30x^4 + 60x^3 + 74x^2 + 57x + 39 \pmod 7$ =>

=> $5x^5 + 2x^4 + 4x^3 + 4x^2 + x + 4$ - верно.