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1) $\begin{cases} x = -6 - 118 \cdot k \\ y = -6 - 119 \cdot k \end{cases}$

2) $[13, 3, \, 13, 26]$

3) ~~эквивалентных корней нет~~ $x = 136896 \mod M$

4) 3

5) $\frac{17}{15}x^4 - \frac{89}{30}x^3 - \frac{128}{15}x^2 + \frac{100}{30}x - \frac{47}{5}$

6) эквивалентных корней нет

7)

8) $x = 24$

9) 4

10) $6x^3 + 5x^2 + 4$

Задание №1

$1547x - 1534y = -78$

$a = 1547$

$b = -1534$

$c = -78$ $d = HOD = 13$

$119x - 118y = -6$

$a = 119$

$b = -118$

$c = -6$

$d = 13$

$\boxed{\dfrac{119}{118}} = 1 + \dfrac{x}{118} = 1 + 0 = \boxed{\dfrac{1}{1}}$ — y_1 и x_1

$x + 4y$

$\dfrac{y_1}{x_1} = \dfrac{\pm l}{\pm p}$

$\begin{cases} x_1 = 1 \\ y_1 = 1 \end{cases}$ $| \cdot -6$

$\begin{cases} x_0 = -6 \\ y_0 = -6 \end{cases}$ $\begin{cases} x = x_0 + b \cdot k \\ y = y_0 + a \cdot k \end{cases}$ $\begin{cases} x = -6 - 118 \cdot k \\ y = -6 - 119 \cdot k \end{cases}$

$$\sqrt{176} = 13 + (\sqrt{176} - 13) = 13 + \cfrac{1}{\frac{1}{\sqrt{176}-13}} = 13 + \cfrac{1}{\frac{\sqrt{176}+13}{\sqrt{176}+13}}$$

$$+ 13 + \cfrac{1}{(\sqrt{176}+13)} = 13 + \cfrac{1}{\frac{3+\sqrt{176}-13 \cdot 21}{7}} = 13 + \cfrac{1}{\frac{3+\sqrt{176}-13}{7}}$$

$$= 13 + \cfrac{1}{3 + \cfrac{1}{\frac{7}{\sqrt{176}-8}}} = 13 + 3 + \cfrac{1}{\left(\frac{\sqrt{176}-1}{\sqrt{176}-64}\right)} = 13 + \cfrac{1}{3 + \cfrac{1}{\frac{\sqrt{176}+8}{18}}}$$

$$= 13 + \cfrac{1}{3 + \cfrac{1}{\left(\frac{7}{176-8}\right)}} = 13 + \cfrac{1}{3 + \cfrac{1}{\left(\frac{\sqrt{176}+8-16}{176-64}\right)}}$$

$$= 13 + \cfrac{1}{3 + \cfrac{1}{1 + \frac{\sqrt{176}+8}{10}}} = 13 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{\frac{\sqrt{176}+8}{8}}}}$$

$$= 13 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3 + \frac{\sqrt{176}-13}{7}}}} = 13 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3\left(1 + \frac{1}{\sqrt{176}-13}\right)}}}$$

$$= 13 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{16 + (\sqrt{176}-13)}}}}$$

$$[13, 3, 1, 3, 16]$$

Задание №3

$x = 5 \mod 33$ $M = 33 \cdot 16 \cdot 17 \cdot 31 = 278256$

$x = 4 \mod 16$ $M_1 = 16 \cdot 17 \cdot 31 = 8432$

$x = 9 \mod 17$ $M_2 = 33 \cdot 17 \cdot 31 = 17391$

$x = 13 \mod 31$ $M_3 = 33 \cdot 16 \cdot 31 = 16368$

$M_4 = 33 \cdot 16 \cdot 17 = 8976$

$8432x - 33y = 1$ $x = 2$

$17391x - 16y = 1$ $x = -1$

$16368x - 17y = 1$ $x = -6$

$8976x - 31y = 1$ $x = 11$

$x = (2 \cdot 8432 \cdot 5 + (-1) \cdot 17391 \cdot 4 + (-6) \cdot 16368 \cdot 9 + 11 \cdot 8976 \cdot 13) =$
$= 914452 \mod 278256$

$$X = 136196 \mod M$$

Задание № 6

$x^4 - 5x^3 - 6x^2 + 7x - 2$ $1 -5 -6 7 -2$

$p = \pm 2; \pm 1$ $1 \quad 1 -4 -10 -3 -5$

$q = \pm 1$ $-1 \quad 1 -6 \; 0 \; 7 -9$

$2 \quad 1 -3 -12 -17 -36$

$-2 \quad 1 -7 \; 8 -9 \; 16$

рациональных корней нет

Задание №4

$17^{17^{150}} \mod 47$

$k = 17^{150}$

$\varphi(47) = 46$

$\left.\begin{array}{l} k = 46n + b \\ k = 17^{150} \end{array}\right\} \Rightarrow b = 17^{150} \mod 46$

~~$x = 170^{150} \mod 46$~~

~~$150_{10} = 10010110$~~

$\varphi(46) = \varphi(2) \cdot \varphi(23) = 1 \cdot 22 = 22$

$17^{22 \cdot 6 + 19} \mod 46 = 17^{19} \mod 46$

$b = 17^{19} \mod 46 \qquad a = 17$

$19 = 10011$

a_i	b	b^2	$b^2 a$	$b^2 a (M)$
1	1	1	17	17
0	17	289	289	13
0	13	168	169	31
1	31	968	16337	7
1	7	49	833	5

$x = 14^5 \mod 47$

$5 = 101$

1	1	1	14	~~14~~ 14
0	14	196	196	~~54~~ ~~4~~ 8
1	~~34~~	~~156~~ ~~16~~	~~65~~	~~38~~ ~~14~~ 38
	8	64	896	(3)

Задание №5

$$p(-2)=-28 \qquad y_1=-28 \qquad x_1=-2$$
$$x(-1)=3 \qquad y_2=3 \qquad x_2=-1$$
$$p(4)=-27 \qquad y_3=-27 \qquad x_3=4$$
$$p(-2)=39 \qquad y_4=39 \qquad x_4=-2$$
$$p(3)=-41 \qquad y_5=-41 \qquad x_5=3$$

$$P_4(x)=\ (-28)\frac{(x-(-1))(x-4)(x-(-2))(x-3)}{(-2-(-1))(-2-4)(-2-(-2))(-2-3)}+$$

$$+3\frac{(x-(-2))(x-4)(x-(-2))(x-3)}{(-1-(-2))(-1-4)(-1-(-2))(-1-3)}+6\frac{(x-(-2))(x-(-1))(x-(-2))(x-3)}{(4-(-2))(4-(-1))(4-(-2))(4-3)}\to$$

$$+39\frac{(x-(-2))(x-(-1))(x-4)(x-3)}{(-2-(-2))(-2-(-1))(-2-4)(-2-3)}+(-41)\frac{(x-(-2))(x-(-1))(x-4)(x-(-2))}{(3-(-2))(3-(-1))(3-4)(3-(-2))}=$$

$$=(-28)\frac{(x+1)(x-4)(x+2)(x-3)}{(-1)\ (-6)}(-28)\frac{(x+1)(x-4)(x+2)(x-3)}{(3)\ (-2)\ (4)\ (-1)}+$$

$$+3\frac{(x-3)(x-4)(x+2)(x-3)}{(-3)\ (-5)\ (1)\ (-4)}+6\frac{(x-3)(x+4)(x+2)(x-3)}{(2)\ (5)\ (6)\ (-1)}+$$

$$+39\frac{(x-2)(x+4)(x-4)(x-3)}{(-4)\ (-3)\ (-6)\ (-5)}+(-41)\frac{(x-2)(x+1)(x-4)(x+2)}{(1)\ (4)\ (-1)\ (5)}=$$

$$=-\frac{7}{8}(x+1)(x-4)(x+2)(x-3)+\left(-\frac{1}{20}\right)(x-2)(x-4)(x+2)(x-3)+\left(-\frac{1}{10}\right)(x-2)(x+1)(x+2)(x-3)+$$

$$+\frac{13}{120}(x-2)(x+1)(x-4)(x-3)+\frac{41}{20}(x-2)(x+1)(x-4)(x+2)=$$

$$=-\frac{105}{120}(x^4-4x^3-7x^2+22x+24)+\left(-\frac{6}{120}\right)(x^4-7x^3+8x^2+28x-48)+\left(-\frac{12}{120}\right)(x^4-2x^3-7x^2+8x+12)+$$

$$+\frac{13}{120}(x^4-8x^3+17x^2+2x-24)+\left(\frac{246}{120}\right)(x^4-3x^3-8x^2+22x+16)=\frac{17}{15}x^4-\frac{79}{30}x^3-\frac{122}{15}x^2+\frac{100}{50}x+\cdots$$

Задание №8

$x = \dfrac{47}{78} \bmod 91$

$78x = 47 \bmod 91$

$78x - 91y = 47$

$78x + 91y = 1$

$x = 16 \qquad y = -13$

$x = 47 \cdot 16 + 91n \quad n \in \mathbb{Z}$

$x = 752 \bmod 91$

$x = 24$

Задача №9

$$\frac{261}{151} = 1\frac{80}{151} \qquad [1;2;3;1;4;4]$$

$$\frac{151}{80} = 2\frac{21}{80}$$

$$\frac{80}{21} = 3\frac{17}{21}$$

$$\frac{21}{17} = 1\frac{4}{17}$$

$$\frac{17}{4} = 4\frac{1}{4}$$

$$\frac{4}{1} = 4$$

Задача №10

$$\frac{4x^5+2x^4+2x^3+3x+4}{5x^3+5x^2+x+3} \qquad x \neq x$$

$$
\begin{array}{r|l}
4x^5+2x^4+2x^3+0\cdot x^2+3x+4 & 5x^3+5x^2+x+3 \\
\underline{4x^5+4x^4+5x^3+1\cdot x^2} & \overline{5x^2+x} \\
5x^4+4x^3+6x^2+3x & \\
\underline{5x^4+5x^3+x^2+3x} & \\
6x^3+5x^2 \qquad +4 &
\end{array}
$$