2) $\sqrt{215} = 14 + \frac{1}{\alpha_1}$

$$\alpha_1 = \frac{1}{\sqrt{215} - 14} = \frac{\sqrt{215} + 14}{19} = 1 + \frac{\sqrt{215} - 5}{19} = 1 + \frac{1}{\alpha_2}$$

$$\alpha_2 = \frac{19}{\sqrt{215} - 5} = \frac{\sqrt{215} + 5}{10} = 1 + \frac{\sqrt{215} - 5}{10} = 1 + \frac{1}{\alpha_3}$$

$$\alpha_3 = \frac{10}{\sqrt{215} - 5} = \frac{\sqrt{215} + 5}{19} = 1 + \frac{\sqrt{215} - 14}{19} = 1 + \frac{1}{\alpha_4}$$

$$\alpha_4 = \frac{19}{\sqrt{215} - 14} = \sqrt{215} + 14 = 28 + \frac{1}{\alpha_5}$$

$$\alpha_5 = \frac{1}{\sqrt{215} - 14} = \alpha_1$$

$$\sqrt{215} = [14; (1, 1, 1, 28)]$$

3) $\begin{cases} x \equiv 31 \bmod 33 \\ x \equiv 25 \bmod 32 \\ x \equiv 28 \bmod 29 \\ x \equiv 0 \bmod 23 \end{cases}$ используем китайский теорем

$x \equiv b_1 \pmod{n_1}$

$x \equiv b_2 \pmod{n_2}$

$x \equiv b_3 \pmod{n_3}$

$x \equiv b_4 \pmod{n_4}$

$N = n_1 n_2 n_3 n_4$

$N_i = \frac{N}{n_i}$

$N_1 = n_2 n_3 n_4$

$N_2 = n_1 n_3 n_4$

$N_3 = n_1 n_2 n_4$

$N_4 = n_1 n_2 n_3$

$N = 33 \times 32 \times 29 \times 23$

$=$

b_i	N_i	x_i	$b_i N_i x_i$
31	21344	-14	-9263296
25	22011	13	7153575
28	24288	2	1360128
0	30624	2	0

$21344 x_1 \equiv 1 \bmod 33$: $(33(647) + 7)$

$22011 x_2 \equiv 1 \bmod 32$: $(32(628) + 5)$

$24288 x_3 \equiv 1 \bmod 29$: $(29(837) + 15)$

$30624 x_4 \equiv 1 \bmod 23$: $(23(1332) + 12)$

$7x_1 \equiv 1 \bmod 33$

$5x_2 \equiv 1 \bmod 32$

$15x_3 \equiv 1 \bmod 29$

$12x_4 \equiv 1 \bmod 23$

$7x_1 \equiv 1 \bmod 33$

$7x_1 - 1 = 33k$

$7x_1 - 33k = 1$

$33 = 4(7) + 5$

$7 = 1(5) + 2$

$5 = 2(2) + 1$

$1 = 5 - 2(2)$

$1 = 5 - 2[7 - 1(5)]$

$1 = 3(5) - 2(7)$

$1 = 3[33 - 4(7)] - 2(7)$

$1 = 33(3) + 7(-14)$

$\underline{x_1 = -14}$

$5x_2 \equiv 1 \bmod 32$

$5x_2 - 1 = 32k$

$5x_2 - 32k = 1$

$32 = 6(5) + 2$

$5 = 2(2) + 1$

$1 = 5 - 2(2)$

$1 = 5 - 2[32 - 6(5)]$

$1 = 13(5) - 2(32)$

$1 = 5(13) + 32(-2)$

$\underline{x_2 = 13}$

$15x_3 \equiv 1 \bmod 29$

$15x_3 - 1 = 29k$

$15x_3 - 29k = 1$

$29 = 1(15) + 14$

$15 = 1(14) + 1$

$1 = 15 - 1(14)$

$1 = 15 - 1[29 - 1(15)]$

$1 = 2(15) - 1(29)$

$1 = 15(2) + 29(-1)$

$\underline{x_3 = 2}$

$12x_4 \equiv 1 \bmod 23$

$12x_4 - 1 = 23k$

$12x_4 - 23k = 1$

$23 = 1(12) + 11$

$12 = 1(11) + 1$

$1 = 12 - 1(11)$

$1 = 12 - 1[23 - 1(12)]$

$1 = 2(12) - 1(23)$

$1 = 12(2) + 23(-1)$

$\underline{x_4 = 2}$

$x = \sum_i b_i N_i x_i$

$= -9263296 + 7153575 + 1360128$

$= \underline{-749593 \bmod 704352}$

4) $38^{29^{77}} \mod 55$

1) $(38, 55) = 1$

$38^{\varphi(55)} \equiv 1 \mod 55$

$\varphi(55) = 55(1 - \frac{1}{5})(1 - \frac{1}{11})$

$= 40$

$38^{40} \equiv 1 \mod 55$

2) $29^{77} \equiv x \mod 40$

$(29, 40) = 1$

$29^{\varphi(40)} \equiv 1 \mod 40$

$\varphi(40) = 40(1 - 1/2)(1 - \frac{1}{5})$

$= 16$

$29^{16} \equiv 1 \mod 40$

$29^{77} \equiv 29^{16 \cdot 4} \cdot 29^{13} \mod 40 = 29^{13} \mod 40$

$13 = 1101_2$

$29^2 = 841 \equiv 1 \mod 40$

$1^2 29 = 29 \equiv 29 \mod 40$ $\qquad$ $\underline{x = 29}$

3) $29^{77} \equiv 29 \mod 40$

$38^{29^{77}} \equiv 38^{29} \mod 40$

$29 = 11101_2$

$38^2 \cdot 38 = 54\,872 \equiv 32 \mod 40$

Ответ 38.

18-03-2021

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дискретная математика "ИДЗ"

1) $917x + 868y = -14$

$ax + by = c$

НОД(917, 868)

$917 = 1(868) + 49$

$868 = 17(49) + 35$

$49 = 1(35) + 14$

$35 = 2(14) + 7$

$14 = 2(7) + 0$

НОД(917, 868) = 7

$7 = 35 - 2(14)$

$7 = 35 - 2(49 - 1(35))$

$7 = [868 - 17(49)] - 2[49 - 1(35)]$

$7 = 868 - 19(49) + 2(35)$

$7 = 868 - 19[917 - 1(868) +$

$7 = 35 - 2(14)$

$7 = [35 - 2(49 - 1(35)]$

$7 = 3(35) - 2(49)$

$7 = 3[(868 - 17(49)] - 2(49)]$

$7 = 3(868) - 51(49) - 2(49)]$

$7 = 3(868) - 53(49)$

$7 = 3(868) - 53(917 - 1(868)]$

$7 = 868(56) + 917(-53)$

$7 = 917(-53) + 868(56)$

$-14 = 917(742) + 868(-784)$

$x_0 = 742 \quad y_0 = -784$

$x = x_0 + bn, \ n \in \mathbb{Z}$

$y = y_0 - a_n, \ n \in \mathbb{Z}$

$x = 742 + 868n$

$y = -784 - 917n$

6) Найдём рациональных корни.

$12x^4 - 8x^3 - 9x^2 - 7x - 3$

$-3 : \pm\{1, 3\}$

$12 : \pm\{1, 2, 3, 4, 6, 12\}$

рациональные корни $\dfrac{\pm(1,3)}{\pm(1,2,3,4,6,12)} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, \pm \frac{1}{6}, \pm \frac{1}{12}$

$\pm 3, \pm \frac{3}{2}, \pm 1, \pm \frac{3}{4}, \pm \frac{1}{2}, \pm \frac{1}{4}$

	12	−8	−9	−7	−3
1	12	4	−5	−12	−15 ≠ 0
−1	12	−20	11	−18	15 ≠ 0
3/2	12	10	6	2	0

$(x - 3/2)(12x^3 + 10x^2 + 6x + 2)$

$12x^3 + 10x^2 + 6x + 2$

$2 : \pm\{1, 2\}$

$12 : \pm\{1, 2, 3, 4, 6, 12\}$

$\dfrac{\pm\{1,2\}}{\pm\{1,2,3,4,6,12\}} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, \pm \frac{1}{6}, \pm \frac{1}{12}$

$\pm 2, \pm 1, \pm \frac{2}{3}, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$

	12	10	6	2
1	12	22	28	30 ≠ 0
−1	12	−2	8	−6 ≠ 0
1/2	12	16	14	9 ≠ 0
−1/2	12	4	4	0

$(x + 1/2)(12x^2 + 4x + 4)$

$(12x^2 + 4x + 4) \Rightarrow$ Нет рациональных корни в R.

$= \underline{(x - 3/2)(x + 1/2)(12x^2 + 4x + 4)}$

7) $6x_9 + 40_9 = 286_9$

$$\begin{array}{r} 286_9 \\ -\ 40_9 \\ \hline 246_9 \end{array}$$

$$\begin{array}{r|l} 246_9 & 6_9 \\ -20 & 37 \\ \hline 046_9 & \\ 46_9 & \\ \hline -- & \end{array}$$

$(37)_9$

$(37_9) = 7 \cdot 9^0 + 3 \cdot 9^1$
$= 7 + 27$
$= 34_{10}$

$\underline{x = 37_9\ ;\ 34_{10}}$

8) вычислите $8/29$ по модулю 45

$29x \equiv 8 \bmod 45$

НОД$(29, 45) = 1$ $\Rightarrow$ существует единственное решение

$29x - 45y = 8$
$45 = 1(29) + 16$
$29 = 1(16) + 13$
$16 = 1(13) + 3$
$13 = 4(3) + 1$

$1 = 13 - 4(3)$
$1 = 13 - 4[16 - 1(13)]$
$1 = 5(13) - 4(16)$
$1 = 5[29 - 1(16)] - 4(16)$
$1 = 5(29) - 9(16)$
$1 = 5(29) - 9[45 - 1(29)]$
$1 = 14(29) - 9(45)$
$1 = 29(14) - 45(9)$

$x_0 = x = 14 \cdot 8 = 112 \equiv \underline{22 \bmod 45}$

9) $\frac{829}{196} = 4 + \cancel{\frac{1}{\frac{196}{633}}}$ $\quad 4 + \frac{1}{\frac{633}{196}} = 4 + \frac{1}{3+\frac{196}{45}} =$

$$4 + \cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{\frac{45}{16}}}} = 4 + \cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{16/13}}}} =$$

$$= 4 + \cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{\frac{13}{3}}}}}} = 4 + \cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{4+1/3}}}}}$$

Ответ: $[4, 3, 4, 2, 1, 4, 3]$

10). $2x^5+2x^3+2$ на x^3+x^2+x+2 в кольце $\mathbb{Z}/3\mathbb{Z}[x]$

$$\begin{array}{r|l}
2x^5+2x^3+2 & x^3+x^2+x+2 \\
\hline
-\;2x^5+2x^4+2x^3+4x^2 & 2x^2-2x+2. \\
\hline
-2x^4-4x^2+2 & \\
-\;-2x^4-2x^3-2x^2-4x & \\
\hline
2x^3-2x^2+4x+2 & \\
-\;2x^3+2x^2+2x+4 & \\
\hline
-4x^2+2x-2. &
\end{array}$$

5) $y_1 = 30 \qquad x_1 = 2$

$y_2 = -40 \qquad x_2 = -3$

$y_3 = -22 \qquad x_3 = -2$

$y_4 = -24 \qquad x_4 = -4$

$y_5 = -6 \qquad x_5 = -1$

$$P_4(x) = \sum_i y_i Q_i$$

$$= (30)\frac{(x+3)(x+2)(x+4)(x+1)}{(2+3)(2+2)(2+4)(2+1)} + (-40)\frac{(x-2)(x+2)(x+4)(x+1)}{(-3-2)(-3+2)(-3+4)(-3+1)} +$$

$$(-22)\frac{(x-2)(x+3)(x+4)(x+1)}{(-2-2)(-2+3)(-2+4)(-2+1)} + (-24)\frac{(x-2)(x+3)(x+2)(x+1)}{(-4-2)(-4+3)(-4+2)(-4+1)}$$

$$+ (-6)\frac{(x-2)(x+3)(x+2)(x+4)}{(-1-2)(-1+3)(-1+2)(-1+4)}$$

$$= \frac{1}{12}(x^4+10x^3+35x^2+50x+24) + 4(x^4+5x^3-20x-16) - \frac{11}{4}(x^3+6x^3+3x^2-26x-24)$$

$$\cancel{4} \; -2/3\,(x^4+4x^3-x^2-16x-12) + \frac{1}{3}(x^4+7x^3+8x^2-28x-48)$$

$$= \begin{cases} x^4\left(\frac{1}{12} + 4 - \frac{11}{4} - 2/3 + 1/3\right) = x^4 \\ x^3\left(\frac{10}{12} + 20 - 33/2 - 8/3 + 7/3\right) = 4x^3 \\ x^2\left(35/12 - 33/4 + 2/3 + 8/3\right) = -2x^2 \\ x\left(50/12 - 20x + 286/4 + 32/3 - 28/3\right) = 57x \\ 24 - 64 + 66 + 8 - 16 = 18 \end{cases}$$

$$= \underline{x^4 + 4x^3 - 2x^2 + 57x + 18}$$