

Кислицина Юлия

Вариант 7

Таблица 1. Ответы к задачам

| N  | Ответ  |
|----|--|
| 1  | $\begin{cases} x = 207 + 105k \\ y' = -270 - 137k \end{cases}, k \in \mathbb{Z}$ |
| 2  | $[11, 5, 1, 1, 5, 22]$   |
| 3  | $x \equiv 96960 \pmod{234600}$   |
| 4  | $c \equiv 11 \pmod{49}$  |
| 5  | $P(x) = x^4 + 3x^3 - 5x^2 - 3x + 1$  |
| 6  | $x_1 = -2; x_2 = 3; x_3 = -\frac{1}{2}; x_4 = \frac{3}{2}$                       |
| 7  | $x = 43_6 = 27$  |
| 8  | $27 \pmod{75}$   |
| 9  | $[2, 1, 2, 4, 4, 4]$   |
| 10 | $3x^2 + 3x + 2$  |

# Задание 1 $2603x - 1995y = 171$

1) Пусть  $y' = -y \Rightarrow 2603x + 1995y' = 171$

2) НОД(2603, 1995) = 19  $\rightarrow 137x + 105y' = 9$

$$2603 = 1 \cdot 1995 + 608$$

$$1995 = 3 \cdot 608 + 171$$

$$608 = 3 \cdot 171 + 95$$

$$171 = 1 \cdot 95 + 76$$

$$95 = 1 \cdot 76 + 19$$

$$76 = 4 \cdot 19$$

3)

|   |     |     |    |    |     |     |     |   |
|---|-----|-----|----|----|-----|-----|-----|---|
| i | -1  | 0   | 1  | 2  | 3   | 4   | 5   | 6 |
| r | 137 | 105 | 32 | 9  | 5   | 4   | 1   | 0 |
| q |     | 1   | 3  | 3  | 1   | 1   | 4   |   |
| x | 1   | 0   | 1  | -3 | 10  | -13 | 23  |   |
| y | 0   | 1   | -1 | 4  | -13 | 17  | -30 |   |

$$x_0 = 23; y'_0 = -30$$

$$x_1 = x_0 \cdot \frac{c}{d} = 23 \cdot 9 = 207$$

$$y'_1 = y'_0 \cdot \frac{c}{d} = -30 \cdot 9 = -270$$

4)  $\begin{cases} x = 207 + 105k \\ y' = -270 - 137k \end{cases}, k \in \mathbb{Z}$

Ответ:  $\begin{cases} x = 207 + 105k \\ y' = -270 - 137k \end{cases}, k \in \mathbb{Z}$

## Проверка:

Пусть  $k = 1$ , т.е.  $x = 312; y' = -407 \Rightarrow 2603 \cdot 312 + 1995(-407) = 812136 - 811965 = 171$

$$2603(207 + 105k) + 1995(-270 - 137k) = 171$$

$$538821 + 273315k + (-538650 - 273315k) = 171$$

$$171 = 171 \Rightarrow \text{левая часть равна правой} \Rightarrow \text{ответ верный}$$

## Задание 2

$$\begin{aligned} \sqrt{125} &= 11 + \sqrt{125} - 11 = 11 + \frac{1}{\left(\frac{1}{\sqrt{125} - 11}\right)} = 11 + \frac{1}{5 + \frac{\sqrt{125} - 9}{4}} = 11 + \frac{1}{5 + \frac{1}{\left(\frac{4}{\sqrt{125} - 9}\right)}} \\ &= 11 + \frac{1}{5 + \frac{1}{\left(\frac{\sqrt{125} + 9}{11}\right)}} = 11 + \frac{1}{5 + \frac{1}{1 + \frac{\sqrt{125} - 2}{11}}} = 11 + \frac{1}{5 + \frac{1}{1 + \frac{1}{\left(\frac{11(\sqrt{125} + 2)}{125 - 11}\right)}}} \\ &= 11 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\left(\frac{11(\sqrt{125} + 9)}{444}\right)}}}} = 11 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{\sqrt{125} - 11}{4}}}}}} \\ &= 11 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{1}{\left(\frac{4(\sqrt{125} + 11)}{4}\right)}}}}} = 11 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{1}{22 + (\sqrt{125} - 11)}}}}} = [11, 5, 1, 1, 5, 22] \end{aligned}$$

Проверка:  $22 = 2 \cdot 11 \Rightarrow$  все верно  
[a, ..., 2a]

Ответ:  $\sqrt{125} = [11, 5, 1, 1, 5, 22]$

Задача 3  $x \equiv 10 \pmod{25}$ ;  $x \equiv 0 \pmod{24}$ ;  $x \equiv 9 \pmod{17}$ ;  $x \equiv 15 \pmod{23}$

1)  $M = 25 \cdot 24 \cdot 17 \cdot 23 = 234600$

$M_1 = 9384$

$M_3 = 13800$

$M_2 = 9775$

$M_4 = 10200$

2)  $9384x_1 - 25y = 1$

$9384x_1 + 25y' = 1$

$x_1 = -11$

| r | 9384 | 25  | 9 | 7  | 2 | 1   |
|---|------|-----|---|----|---|-----|
| q |      | 375 | 2 | 1  | 3 | 2   |
| x | 1    | 0   | 1 | -2 | 3 | -11 |

$9775x_2 - 24y = 1$

$9775x_2 + 24y' = 1$

$x_2 = 7$

| r | 9775 | 24  | 7 | 3  | 1 |  |
|---|------|-----|---|----|---|--|
| q |      | 407 | 3 | 2  | 3 |  |
| x | 1    | 0   | 1 | -3 | 7 |  |

$13800x_3 - 17y = 1$

$13800x_3 + 17y' = 1$

$x_3 = 4$

| r | 13800 | 17  | 13 | 4  | 1 |
|---|-------|-----|----|----|---|
| q |       | 811 | 1  | 3  | 4 |
| x | 1     | 0   | 1  | -1 | 4 |

$10200x_4 - 23y = 1$

$10200x_4 + 23y' = 1$

$x_4 = -2$

| r | 10200 | 23  | 11 | 1  |
|---|-------|-----|----|----|
| q |       | 443 | 2  |    |
| x | 1     | 0   | 1  | -2 |

3)  $x \equiv (9384 \cdot (-11) \cdot 10 + 9775 \cdot 7 \cdot 0 + 13800 \cdot 4 \cdot 9 + 10200 \cdot (-2) \cdot 15) \pmod{234600}$

$x \equiv (-1032240 + 496800 - 306000) \pmod{234600}$

$x \equiv -841440 \pmod{234600}$

$-841440 + 234600 \cdot 4 = 96960 \Rightarrow x \equiv 96960 \pmod{234600}$

Ответ:  $x \equiv 96960 \pmod{234600}$

Проверка:

$96960 : 25 = 3878 \text{ (ост. 10)}, \text{ т.е. } 96960 \equiv 10 \pmod{25}$

$96960 : 24 = 4040 \text{ (ост. 0)}, \text{ т.е. } 96960 \equiv 0 \pmod{24}$

$96960 : 17 = 5703 \text{ (ост. 9)}, \text{ т.е. } 96960 \equiv 9 \pmod{17}$

$96960 : 23 = 4215 \text{ (ост. 15)}, \text{ т.е. } 96960 \equiv 15 \pmod{23}$

Задача 4  $C \equiv 4^{25^{55}} \pmod{49}$

1)  $k = 25^{55} \Rightarrow C \equiv 4^k \pmod{49}$

2)  $\varphi(49) = 42 \Rightarrow k = 25^{55} = 42 \cdot n + b \Rightarrow b \equiv 25^{55} \pmod{42}$

3)  $\varphi(42) = \varphi(7) \varphi(3) \varphi(2) = 12 \Rightarrow b \equiv 25^{12 \cdot 4 + 7} \pmod{42} \equiv 25^7 \pmod{42}$

$m = 7 = 111_2$

$b \equiv 25 \pmod{42}$

| $a_i$ | C  | $C^2$ | $C^2 \cdot a_i$ | $C \equiv C^2 \cdot a_i \pmod{k}$ |
|-------|----|-------|-----------------|-----------------------------------|
| 1     | 1  | 1     | 25              | 25                                |
| 1     | 25 | 625   | 15625           | 1                                 |
| 1     | 1  | 1     | 25              | 25                                |

4)  $C \equiv 4^{25} \pmod{49}$

$m = 25 = 11001_2$

$C \equiv 11 \pmod{49}$

| $a_i$ | C  | $C^2$ | $C^2 \cdot a_i$ | $C \equiv C^2 \cdot a_i \pmod{k}$ |
|-------|----|-------|-----------------|-----------------------------------|
| 1     | 1  | 1     | 4               | 4                                 |
| 1     | 4  | 16    | 64              | 15                                |
| 0     | 15 | 225   | 225             | 29                                |
| 0     | 29 | 841   | 841             | 8                                 |
| 1     | 8  | 64    | 256             | 11                                |

Ответ:  $C \equiv 11 \pmod{49}$



Задание 5  $P(-3) = -35$ ;  $P(-2) = -21$ ;  $P(-1) = -3$ ;  $P(1) = -3$ ;  $P(2) = 15$

$$P(x) = \frac{(x+2)(x+1)(x-1)(x-2)}{(-1)(-2)(-4)(-5)}(-35) + \frac{(x+3)(x+1)(x-1)(x-2)}{1(-1)(-3)(-4)}(-21) + \frac{(x+3)(x+2)(x-1)(x-2)}{2 \cdot 1 \cdot (-2)(-3)}(-3) + \frac{(x+3)(x+2)(x+1)(x-2)}{4 \cdot 3 \cdot 2 \cdot (-1)}(-3) + \frac{(x+3)(x+2)(x+1)(x-1)}{5 \cdot 4 \cdot 3 \cdot 1}15 = -\frac{7}{8}(x^4 - 5x^2 + 4) + \frac{7}{4} \cdot (x^4 + x^3 - 7x^2 - x + 6) - \frac{1}{4}(x^4 + 2x^3 - 7x^2 - 8x + 12) + \frac{1}{8}(x^4 + 4x^3 - x^2 - 16x - 12) + \frac{1}{4}(x^4 + 5x^3 + 5x^2 - 5x - 6) = x^4 \left( -\frac{7}{8} + \frac{7}{4} - \frac{1}{4} + \frac{1}{8} + \frac{1}{4} \right) + x^3 \left( \frac{7}{4} + \frac{5}{4} - \frac{2}{4} + \frac{5}{4} \right) + x^2 \left( \frac{35}{8} - \frac{49}{4} + \frac{7}{4} - \frac{1}{8} + \frac{5}{4} \right) + x \left( -\frac{7}{4} + \frac{8}{4} - \frac{16}{8} - \frac{5}{4} \right) + \left( -\frac{28}{8} + \frac{42}{4} - \frac{12}{4} - \frac{12}{8} - \frac{6}{4} \right) = x^4 + 3x^3 - 5x^2 - 3x + 1$$

Проверка:

$$P(-3) = (-3)^4 + 3(-3)^3 - 5(-3)^2 - 3(-3) + 1 = 81 - 81 - 45 + 9 + 1 = -35$$

$$P(-2) = (-2)^4 + 3(-2)^3 - 5(-2)^2 - 3(-2) + 1 = 16 - 24 - 20 + 6 + 1 = -21$$

$$P(-1) = (-1)^4 + 3(-1)^3 - 5(-1)^2 - 3(-1) + 1 = 1 - 3 - 5 + 3 + 1 = -3$$

$$P(1) = 1^4 + 3 \cdot 1^3 - 5 \cdot 1^2 - 3 \cdot 1 + 1 = 1 + 3 - 5 - 3 + 1 = -3$$

$$P(2) = 2^4 + 3 \cdot 2^3 - 5 \cdot 2^2 - 3 \cdot 2 + 1 = 16 + 24 - 20 - 6 + 1 = 15$$

Ответ:  $P(x) = x^4 + 3x^3 - 5x^2 - 3x + 1$

Задание 6  $4x^4 - 8x^3 - 23x^2 + 27x + 18 = 0$

$$\frac{p}{q} = \frac{\pm 18; \pm 9; \pm 6; \pm 3; \pm 2; \pm 1}{\pm 4; \pm 2; \pm 1}$$

| x    | 4 | -8  | -23 | 27  | 18  |
|------|---|-----|-----|-----|-----|
| 1    | 4 | -4  | -27 | 0   | 18  |
| -1   | 4 | -12 | -11 | 38  | -20 |
| 2    | 4 | 0   | -23 | -19 | -20 |
| -2   | 4 | -16 | 9   | 9   | 0   |
| 3    | 4 | -4  | -3  | 0   |     |
| 1/2  | 4 | -2  | -4  |     |     |
| -1/2 | 4 | -6  | 0   |     |     |
| 3/2  | 4 | 0   |     |     |     |

$$(4x^3 - 16x^2 + 9x + 9)(x+2)$$

$$(4x^2 - 4x - 3)(x+2)(x-3)$$

$$(4x-6)(x+2)(x-3)\left(x+\frac{1}{2}\right)$$

$$4x^4 - 8x^3 - 23x^2 + 27x + 18 = 4(x+2)(x-3)\left(x+\frac{1}{2}\right)\left(x-\frac{3}{2}\right)$$

Проверка:

$$x = -2 : 4(-2)^4 - 8(-2)^3 - 23(-2)^2 + 27(-2) + 18 = 0$$

$$x = 3 : 4(3)^4 - 8(3)^3 - 23(3)^2 + 27(3) + 18 = 0$$

$$x = -\frac{1}{2} : 4\left(-\frac{1}{2}\right)^4 - 8\left(-\frac{1}{2}\right)^3 - 23\left(-\frac{1}{2}\right)^2 + 27\left(-\frac{1}{2}\right) + 18 = 0$$

$$x = \frac{3}{2} : 4\left(\frac{3}{2}\right)^4 - 8\left(\frac{3}{2}\right)^3 - 23\left(\frac{3}{2}\right)^2 + 27\left(\frac{3}{2}\right) + 18 = 0$$

Ответ:  $x_1 = -2$ ;  $x_2 = 3$ ;  $x_3 = -\frac{1}{2}$ ;  $x_4 = \frac{3}{2}$

Задача 7  $4x + 223_6 = 523_6$

1 способ:

$$4x = 523_6 - 223_6$$

$$4x = 300_6$$

$$x = 43_6$$

$$\begin{array}{r|l} 300 & 4 \\ -24 & \\ \hline 20 & 43 \\ -20 & \\ \hline 0 & \end{array}$$

2 способ:

$$4_6 = 4$$

$$223_6 = 2 \cdot 36 + 12 + 3 = 87$$

$$523_6 = 5 \cdot 36 + 12 + 3 = 195$$

$$4x = 195 - 87$$

$$4x = 108$$

$$x = 27 = 43_6$$

Ответ:  $x = 43_6 = 27$

Задача 8  $x \equiv \frac{6}{53} \pmod{75} \Rightarrow 53x \equiv 6 \pmod{75}$

$$53x - 75y = 6$$

$$53x + 75y' = 6$$

НОД(53, 75) = 1  $\Rightarrow 53x_0 + 75y_0' = 1$

$$75 = 1 \cdot 53 + 22$$

$$53 = 2 \cdot 22 + 9$$

$$22 = 2 \cdot 9 + 4$$

$$9 = 2 \cdot 4 + 1$$

$$4 = 4 \cdot 1$$

| r | 53 | 75 | 53 | 22 | 9 | 4  | 1  |
|---|----|----|----|----|---|----|----|
| q |    | 0  | 1  | 2  | 2 | 2  |    |
| x | 1  | 0  | 1  | -1 | 3 | -7 | 17 |

$$x_1 = 17 \cdot 6 = 102$$

$$x = 102 + 75k, k \in \mathbb{Z}$$

$$x \equiv 102 \pmod{75} \equiv 27 \pmod{75}$$

Ответ:  $\frac{6}{53} \pmod{75} \equiv 27 \pmod{75}$

Задача 9

$$1) \frac{627}{233} = 2 + \frac{161}{233} = 2 + \frac{1}{\left(\frac{233}{161}\right)} = 2 + \frac{1}{1 + \frac{72}{161}} = 2 + \frac{1}{1 + \frac{1}{\left(\frac{161}{72}\right)}} = 2 + \frac{1}{1 + \frac{1}{2 + \frac{17}{72}}} =$$

$$= 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\left(\frac{72}{17}\right)}}} = 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4 + \frac{4}{17}}}} = 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4 + \frac{1}{\left(\frac{17}{4}\right)}}}} = 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4 + \frac{1}{4 + \frac{1}{4}}}}} =$$

$$= [2, 1, 2, 4, 4, 4]$$

$$2) 627 = 2 \cdot 233 + 161$$

$$233 = 1 \cdot 161 + 72$$

$$161 = 2 \cdot 72 + 17$$

$$72 = 4 \cdot 17 + 4$$

$$17 = 4 \cdot 4 + 1$$

$$4 = 4 \cdot 1$$

$$\frac{627}{233} = [2, 1, 2, 4, 4, 4]$$

Ответ:  $\frac{627}{233} = [2, 1, 2, 4, 4, 4]$

Задача 10

$$\frac{x^5 + 2x^4 + 4x^3 + x^2 + 4x + 4}{3x^3 + 2x^2 + 4x + 4}$$

В поле 2/5

$$\begin{array}{r|l} x^5 + 2x^4 + 4x^3 + x^2 + 4x + 4 & 3x^3 + 2x^2 + 4x + 4 \\ \hline x^5 + 4x^4 + 3x^3 + 3x^2 & 2x^2 + x + 3 \\ \hline -3x^4 + x^3 + 3x^2 + 4x & \\ -3x^4 + 2x^3 + 4x^2 + 4x & \\ \hline 4x^3 + 4x^2 + 0x + 4 & \\ -4x^3 + x^2 + 2x + 2 & \\ \hline 3x^2 + 3x + 2 & \end{array}$$

Проверка:  $(3x^3 + 2x^2 + 4x + 4)(2x^2 + x + 3) + 3x^2 + 3x + 2 = x^5 + \underline{3x^4} + \underline{4x^3} + \underline{4x^4} + \underline{2x^3} + \underline{x^2} +$   
 $+ \underline{3x^3} + \underline{4x^2} + \underline{2x} + \underline{3x^2} + \underline{4x} + 2 + \underline{3x^2} + \underline{3x} + 2 = x^5 + 2x^4 + 4x^3 + \cancel{x^2} + 4x + 4 - \text{Верно}$

Ответ:  $3x^2 + 3x + 2$