Егориков О362

Вариант 25

Номер	Ответ
1	$\begin{cases} x = 212 + 107k \\ y = -208 - 105k \end{cases}$
2	[11, 4, 2, 4, 22]
3	90946 mod 205530
4	19
5	$p(x) = x^4 - 2x^3 - 4x^2 - 4x - 5$
6	корней нет
7	36_7
8	14
9	[1, 1, 2, 5, 1, 4]
10	$2x^5 + 2x^3 + x^2 + 2x$

№1.

$1365x + 1391y = 52$

$1391 = 1365 \cdot 1 + 26$

$1365 = 26 \cdot 52 + 13$

$26 = 13 \cdot 2$

НОД$(1365, 1391) = 13$

$13(105x + 107y) = 52$

$105x + 107y = 4$

i	-1	0	1	2
r	107	105	2	1
q		1	52	
x	0	1	-1	53
y	1	0	1	1

$x_0 = 53$

$y_0 = 1 \equiv -52 \bmod$

$y_0 = -52$

$c = 4;\ d = 1$

$x_1 = x_0 \cdot \frac{c}{d} = 53 \cdot 4 = 212$

$y_1 = y_0 \cdot \frac{c}{d} = -52 \cdot 4 = -208$

$$\left[\begin{array}{l} x = 212 + 107k \\ y = -208 - 105k \end{array}\right.$$

Проверка: При $k = 1$: $\left\{\begin{array}{l} x = 319 \\ y = -313 \end{array}\right.$

$1365 \cdot 319 - 1391 \cdot 313 = 52$

Ответ: $\left[\begin{array}{l} x = 212 + 107k \\ y = -208 - 105k \end{array}\right.$

№2.

$$\sqrt{126} = 11 + (\sqrt{126} - 11) = 11 + \cfrac{1}{\left(\cfrac{1}{\sqrt{126}-11}\right)} = 11 + \cfrac{1}{\cfrac{\sqrt{126}+11}{5}} = 11 + \cfrac{1}{4 + \cfrac{\sqrt{126}-9}{5}} =$$

$$= 11 + \cfrac{1}{4 + \cfrac{1}{\left(\cfrac{5}{\sqrt{126}-9}\right)}} = 11 + \cfrac{1}{4 + \cfrac{1}{\left(\cfrac{5(\sqrt{126}+9)}{45}\right)}} = 11 + \cfrac{1}{4 + \cfrac{1}{\left(\cfrac{\sqrt{126}+9}{9}\right)}} =$$

$$= 11 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{\sqrt{126}-9}{9}}} = 11 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{45}{9(\sqrt{126}+9)}}} = 11 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{\left(\cfrac{\sqrt{126}+9}{5}\right)}}} =$$

$$= 11 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{\sqrt{126}-11}{5}}}} = 11 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{5}{5(\sqrt{126}+11)}}}} =$$

$$= 11 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{22 + \sqrt{126} - 11}}}} = [11, \overline{4, 2, 4, 22}]$$

$22 = 11 \cdot 2$

Т.к. первое число ответа в два раза меньше последнего, то ответ был получен верный. Ответ: $[11, \overline{4, 2, 4, 22}]$

№ 3.

$x \equiv 23 \bmod 31 \quad x \equiv 16 \bmod 30 \quad x \equiv 11 \bmod 13 \quad x \equiv 13 \bmod 17$

① $M = 31 \cdot 30 \cdot 13 \cdot 17 = 205530$

$M_1 = 30 \cdot 13 \cdot 17 = 6630$

$M_2 = 31 \cdot 13 \cdot 17 = 6851$

$M_3 = 31 \cdot 30 \cdot 17 = 15810$

$M_4 = 31 \cdot 30 \cdot 13 = 12090$

$31 \times 30 = 930$

930×13: $2790 + 930 = 12090$

12090×17: $84630 + 12090 = 205530$

② Поиск x_1, x_2, x_3, x_4:

1) $M_1 x_1 = 1 \bmod m_1$

$6630 x_1 - 31y = 1$

$6630 = 31 \cdot 213 + 27$

$31 = 27 \cdot 1 + 4$

$27 = 4 \cdot 6 + 3$

$4 = 3 \cdot 1 + 1$

$3 = 1 \cdot 3$

2) $M_2 x_2 = 1 \bmod m_2$

$6851 x_2 - 30y = 1$

$6851 = 30 \cdot 228 + 11$

$30 = 11 \cdot 2 + 8$

$11 = 8 \cdot 1 + 3$

$8 = 3 \cdot 2 + 2$

$3 = 2 \cdot 1 + 1$

$2 = 1 \cdot 2$

3) $M_3 x_3 = 1 \bmod m_3$

$15810 x_3 - 13y = 1$

$15810 = 13 \cdot 1216 + 2$

$13 = 2 \cdot 6 + 1$

$2 = 1 \cdot 2$

4) $M_4 x_4 = 1 \bmod m_4$

$12090 x_4 - 17y = 1$

$12090 = 17 \cdot 711 + 3$

$17 = 3 \cdot 5 + 2$

$3 = 2 \cdot 1 + 1$

$2 = 1 \cdot 2$

i	−1	0	1	2	3	4
r	6630	31	27	4	3	1
q		213	1	6	1	
x	1	0	1	−1	7	−8

$x_1 = -8 \Longrightarrow x_1 = -8 \bmod 31 = 23 \bmod 31$

$x_1 = 23$

i	−1	0	1	2	3	4	5
r	6851	30	11	8	3	2	1
q		228	2	1	2	1	
x	1	0	1	−2	3	−8	11

$x_2 = 11$

i	−1	0	1	2
r	15810	13	2	1
q		1216	6	
x	1	0	1	−6

$x_3 = -6 \Longrightarrow x_3 = -6 \bmod 13 = 7 \bmod 13$

$x_3 = 7$

i	−1	0	1	2	3
r	12090	17	3	2	1
q		711	5	1	
x	1	0	1	−5	6

$x_4 = 6$

$x = (6630 \cdot 23 \cdot 23 + 6851 \cdot 11 \cdot 16 + 15810 \cdot 7 \cdot 11 + 12090 \cdot 6 \cdot 13) \bmod 205530 =$
$= (3507270 + 1205776 + 1217370 + 943020) \bmod 205530 = 6873436 \bmod$
$\bmod 205530 = 90946 \bmod 205530$

Проверка:

1) $90946 : 31 = 2933$; $90946 - 62 = 289$; $289 - 279 = 104$; $104 - 93 = 116$; $116 - 93 = 23$

2) $90946 : 30 = 3031$; $90 - 90$; $94 - 90 = 46$; $46 - 30 = 16$

3) $90946 : 13 = 6995$; $90 - 78$; $129 - 117 = 124$; $124 - 117 = 76$; $76 - 65 = 11$

4) $90946 : 17 = 5349$; $90 - 85$; $59 - 51 = 84$; $84 - 68 = 166$; $166 - 153 = 13$

Ответ: $x = 90946 \bmod 205530$

№4.

$67^{5^{35}} \bmod 78 \quad k=5^{35} \Rightarrow 67^k \bmod 78$

$\varphi(78)=\varphi(2)\cdot\varphi(3)\cdot\varphi(13)=2\left(1-\frac{1}{2}\right)\cdot 3\left(1-\frac{1}{3}\right)\cdot 13\left(1-\frac{1}{13}\right)=2\cdot\frac{1}{2}\cdot 3\cdot\frac{2}{3}\cdot$
$\cdot 13\cdot\frac{12}{13}=1\cdot 2\cdot 12=24$

$k=24n+b$

$67^k \bmod 78=67^{24n+b} \bmod 78=67^b \bmod 78$

$k=5^{35}=24n+b$

$b=5^{35} \bmod 24$

$5^{35} \bmod 24=(3120+5)^{30} \bmod 24=(3120+5)^{25} \bmod 24=(3120+5)^{20} \bmod 24=$
$=(3120+5)^{15} \bmod 24=(3120+5)^{10} \bmod 24=(3120+5)^{5} \bmod 24=(3120+5) \bmod 24=$
$=5$

$x=67^5 \bmod 78 \quad 5_{10}=101_2 \quad a=67$

a_i	x	x^2	$x^2\cdot a^{a_i}$	$x^2\cdot a^{a_i} \bmod 78$
1	1	1	67	67
0	67	4489	4489	43
1	43	1849	123883	19

$x=67^5 \bmod 78=19$

Ответ: 19

№5.

$p(-1)=-2;\ p(-2)=19;\ p(3)=-26;\ p(1)=-14;\ p(4)=43$

$\frac{(x+2)(x-3)(x-1)(x-4)}{1\cdot(-4)\cdot(-2)\cdot(-5)}\cdot(-2)+\frac{(x+1)(x-3)(x-1)(x-4)}{(-1)\cdot(-5)\cdot(-3)\cdot(-6)}\cdot 19+\frac{(x+1)(x+2)(x-1)(x-4)}{4\cdot 5\cdot 2\cdot(-1)}\cdot$
$\cdot(-26)+\frac{(x+1)(x+2)(x-3)(x-4)}{2\cdot 3\cdot(-2)\cdot(-3)}\cdot(-14)+\frac{(x+1)(x+2)(x-3)(x-1)}{5\cdot 6\cdot 1\cdot 3}\cdot 43=$
$=\frac{1}{20}(x^2+2x-3x-6)(x^2-x-4x+4)+\frac{19}{90}(x^2-1)(x^2-3x-4x+12)+\frac{13}{20}(x^2-1)\cdot$
$\cdot(x^2+2x-4x-8)-\frac{7}{18}(x^2+x+2x+2)(x^2-3x-4x+12)+\frac{43}{90}(x^2-1)(x^2+2x-3x-6)=$
$=\frac{1}{20}(x^2-x-6)(x^2-5x+4)+\frac{19}{90}(x^2-1)(x^2-7x+12)+\frac{13}{20}(x^2-1)(x^2-2x-8)-$
$-\frac{7}{18}(x^2+3x+2)(x^2-7x+12)+\frac{43}{90}(x^2-1)(x^2-x-6)=\frac{1}{20}(x^4-x^3-6x^2-5x^3+5x^2+$
$+30x+4x^2-4x-24)+\frac{19}{90}(x^4-x^2-7x^3+7x+12x^2-12)+\frac{13}{20}(x^4-x^2-2x^3+2x-8x^2+$
$+8)-\frac{7}{18}(x^4+3x^3+2x^2-7x^3-21x^2-14x+12x^2+36x+24)+\frac{43}{90}(x^4-x^2-x^3+x-6x^2+6)=$

$$= \frac{1}{20}(x^4-6x^3+3x^2+26x-24)+\frac{19}{90}(x^4-7x^3+11x^2+7x-12)+\frac{13}{20}(x^4-2x^3-9x^2+2x+8)-$$

$$-\frac{7}{18}(x^4-4x^3-7x^2+22x+24)+\frac{43}{90}(x^4-x^3-7x^2+x+6)=x^4\left(\frac{1}{20}+\frac{19}{90}+\frac{13}{20}-\frac{7}{18}+\right.$$

$$\left.+\frac{43}{90}\right)+x^3\left(-\frac{6}{20}-\frac{19\cdot7}{90}-\frac{26}{20}+\frac{28}{18}-\frac{43}{90}\right)+x^2\left(\frac{3}{20}+\frac{19\cdot11}{90}-\frac{13\cdot9}{20}+\frac{49}{18}-\right.$$

$$\left.-\frac{43\cdot7}{90}\right)+x\left(\frac{26}{20}+\frac{19\cdot7}{90}+\frac{13\cdot2}{20}-\frac{7\cdot22}{18}+\frac{43}{90}\right)+\left(-\frac{24}{20}-\frac{19\cdot12}{90}+\frac{13\cdot8}{20}-\right.$$

$$\left.-\frac{7\cdot24}{18}+\frac{43\cdot6}{90}\right)=x^4\left(\frac{9+38+117-70+86}{180}\right)+x^3\left(\frac{-54-266-234+280-86}{180}\right)$$

$$+x^2\left(\frac{27+418-1053+490-602}{180}\right)+x\left(\frac{234+266+234-1540+86}{180}\right)+$$

$$+\left(\frac{-216-456+936-1680+516}{180}\right)=\frac{180}{180}x^4-\frac{360}{180}x^3-\frac{720}{180}x^2-\frac{720}{180}x-$$

$$-\frac{900}{180}=x^4-2x^3-4x^2-4x-5$$

Проверка: $p(-1)=(-1)^4-2\cdot(-1)^3-4\cdot(-1)^2-4\cdot(-1)-5=1+2-4+4-5=-2$

$p(-2)=(-2)^4-2\cdot(-2)^3-4\cdot(-2)^2-4\cdot(-2)-5=16+16-16+8-5=19$

$p(3)=3^4-2\cdot3^3-4\cdot3^2-4\cdot3-5=81-54-36-12-5=-26$

$p(1)=1^4-2\cdot1^3-4\cdot1^2-4\cdot1-5=1-2-4-4-5=-14$

$p(4)=4^4-2\cdot4^3-4\cdot4^2-4\cdot4-5=256-128-64-16-5=43$

Ответ: $p(x)=x^4-2x^3-4x^2-4x-5$

$x^4-5x^3-6x^2+7x-2=0$ $\boxed{№6}$ $\frac{p}{q}=\frac{\pm2;\ \pm1}{\pm1}$

x	1	−5	−6	7	−2
1	1	−4	−10	−3	−5
−1	1	−6	0	7	−9
2	1	−3	−12	−17	−36
−2	1	−7	8	−9	16

$f(1)=-5$ $f(2)=-36$

$f(-1)=-9$ $f(-2)=16$

Ответ: рациональные корни отсутствуют.

№7

$3_7 x + 30_7 = 204_7$

1 способ:

$3_7 x = 204_7 - 30_7$

$x = \frac{144}{3}$

$x = 36_7$

$36_7 = 6 \cdot 7^0 + 3 \cdot 7^1 = 6 + 21 = 27_{10}$

$$\begin{array}{r} 204 \\ -\ \ 30 \\ \hline 144_7 \end{array}$$

$$\begin{array}{r|l} 144 & 3 \\ -12 & 36_7 \\ \hline 24 & \\ 24 & \\ \hline 0 & \end{array}$$

2 способ:

$3_7 = 3_{10}$ $\quad 30_7 = 0 \cdot 7^0 + 3 \cdot 7^1 = 21_{10}$ $\quad 204_7 = 4 \cdot 7^0 + 0 \cdot 7^1 + 2 \cdot 7^2 = 4 + 98 = 102_{10}$

$3x + 21 = 102$

$x = \frac{102 - 21}{3}$

$x = 27_{10} = 36_7$

$$\begin{array}{r|l} 27 & 7 \\ 21 & 3 \;|\; 7 \\ \hline 6 & 0 \;|\; 0 \\ & 3 \end{array}$$

Ответ: 36_7

№8

$x = \frac{30}{51} \mod 91 \Longrightarrow 51x = 30 \mod 91$

$51x - 91y = 30$

$51x + 91y' = 30$

НОД$(51, 91) = 1 \Longrightarrow 51x + 91y' = 1$

i	-1	0	1	2	3	4	5	6
r	51	91	40	11	7	4	3	1
q		1	1	3	1	1	1	
x	1	0	1	-1	4	-5	9	-14
y	0	1	-1	0	-7	9	-16	25

$91 = 51 \cdot 1 + 40$

$51 = 40 \cdot 1 + 11$

$40 = 11 \cdot 3 + 7$

$11 = 7 \cdot 1 + 4$

$7 = 4 \cdot 1 + 3$

$4 = 3 \cdot 1 + 1$

$3 = 1 \cdot 3$

$x_0 = -14;\ y'_0 = 25$

$ax + by = c$

$ax_0 + by_0 = 1$

$x = 51 \cdot (-14) + 91n,\ n \in \mathbb{Z}$

$x = -714 + 91 \cdot 8 = -714 + 728 = 14$

$x = -714 \mod 91 \Longrightarrow -714 \equiv 14 \mod 91$

Ответ: 14

№9.

1 способ:

$$\frac{155}{92}=1+\frac{63}{92}=1+\frac{1}{\left(\frac{92}{63}\right)}=1+\frac{1}{1+\frac{29}{63}}=1+\frac{1}{1+\frac{1}{\left(\frac{63}{29}\right)}}=1+\frac{1}{1+\frac{1}{2+\frac{5}{29}}}=$$

$$=1+\frac{1}{1+\frac{1}{2+\frac{1}{\left(\frac{29}{5}\right)}}}=1+\frac{1}{1+\frac{1}{2+\frac{1}{5+\frac{4}{5}}}}=1+\frac{1}{1+\frac{1}{2+\frac{1}{5+\frac{1}{\left(\frac{5}{4}\right)}}}}=$$

$$=1+\frac{1}{1+\frac{1}{2+\frac{1}{5+\frac{1}{1+\frac{1}{4}}}}}=[1,1,2,5,1,4]$$

2 способ:

$$\begin{aligned}155&=92\cdot1+63\\92&=63\cdot1+29\\63&=29\cdot2+5\\29&=5\cdot5+4\\5&=4\cdot1+1\\4&=1\cdot4\end{aligned}\Longrightarrow[1,1,2,5,1,4]$$

Ответ: $[1,1,2,5,1,4]$

№10.

$$\frac{2x^5+2x^3+x^2+2x}{2x^3+x^2+2x+2}$$ в кольце $\frac{\mathbb{Z}}{3\mathbb{Z}}[x]$

$$\begin{array}{l|l}
2x^5+0\cdot x^4+2x^3+x^2+2x & 2x^3+x^2+2x+2 \\
\cline{2-2}
-\;2x^5+1\cdot x^4+2x^3+\cancel{2x^2} & x^2+x+1 \\
\cline{1-1}
2x^4+0\cdot x^3+2x^2+2x & \\
-\;2x^4+1\cdot x^3+2x^2+2x & \\
\cline{1-1}
2x^3+0\cdot x^2+0x+0 & \\
-\;2x^3+x^2+2x+2 & \\
\cline{1-1}
2x^2+x+1 &
\end{array}$$

$-1\equiv2\bmod3\Longrightarrow-1x^4\equiv2x^4$

$-1\equiv2\bmod3\Longrightarrow-x^2\equiv2x^2$

$-1\equiv2\bmod3\Longrightarrow-x^3\equiv2x^3$

$-2\equiv1\bmod3\Longrightarrow\cancel{-2x}\equiv1\cdot x$

Проверка: $(2x^3+x^2+2x+2)(x^2+x+1)+2x^2+x+1=2x^5+x^4+2x^3+2x^2+2x^4+x^3+2x^2+2x+$
$+2x^3+x^2+2x+2+2x^2+x+1=2x^5+3x^4+5x^3+7x^2+5x+3=2x^5+2x^3+x^2+2x$