Exercise 3.

1. 1) $\varphi(36205) = \varphi(5) \cdot \varphi(13) \cdot \varphi(557) =$

$= 4 \cdot 12 \cdot 556 = 26688$

2) $\varphi(2002) = \varphi(2) \cdot \varphi(7) \cdot \varphi(11) \cdot \varphi(13) =$

$= 1 \cdot 6 \cdot 10 \cdot 12 = 720$

3) $\varphi(13) = 12$

4) $\varphi(8) = \varphi(2^3) = 2^3\left(1 - \frac{1}{2}\right) = 8 - 4 = 4$

5) $\varphi(33) = \varphi(3) \cdot \varphi(11) = 2 \cdot 10 = 20$

2. $(13+5)^{36205^8} \bmod 33 = 18^{36205^8} \bmod 33$

$36205^8 = k \Rightarrow 18^k \bmod 33$

$\varphi(33) = \varphi(3) \cdot \varphi(11) = 2 \cdot 10 = 20$

$k = 20n + b$

$18^k \bmod 33 = 18^{20n+b} \bmod 33 = 18^b \bmod 33$

$k = 36205^8 = 20n + b$

$b = 36205^8 \bmod 20$

$36205^8 \bmod 20 = (36200+5)^8 \bmod 20 =$

$= 5^8 \bmod 20 = (20+5)^4 \bmod 20 = 5^4 \bmod 20 =$

$= (5+20)^2 \bmod 20 = 25 \bmod 20 = 5 \bmod 20 = 5$

$X = 18^5 \bmod 33$ $\quad 5_{10} = 101_2$

a_i	X	X^2	X^2a	$X^2a\,(m)$
1	1	1	18	18
0	18	324	324	27
1	27	729	13122	21

$18^5 \bmod 33 = 21 \bmod 33 = \boxed{21}$

3. $33^{36218} \bmod 2002$

$36218_{10} = 1000110101111010_2$

a_i	b	b^2	b^2a	$b^2a\,(m)$
1	1	1	33	33
0	33	1089	1089	1089
0	1089	1185921	1185921	737
0	737	543169	543169	627
1	627	393129	12973257	297
1	297	88209	2910897	1991
0	1991	3964081	3964081	121
1	121	14641	483153	671
0	671	450241	450241	1793
1	1793	3214849	106090017	33
1	33	1089	35937	1903
1	1903	3621409	119506497	1111
1	1111	1234231	40732593	1903
0	1903	3621409	3621409	1793
1	1793	3214849	106090017	33
0	33	1089	1089	1089

$33^{36218} \bmod 2002 = \boxed{1089}$