Жмарев 0362 Вариант 6

1. $\begin{cases} x = -4 + 106k, k \in \mathbb{Z} \\ y = -8 + 213k, k \in \mathbb{Z} \end{cases}$

2. $[17; \overline{3, 8, 3, 34}]$

3. $x = 57651 \bmod 86020$

4. $81 \bmod 98$

5. $p(x) = 2x^4 - 5x^3 - 8x^2 + 4x + 13$

6. $\varnothing$

7. 43_9

8. 52

9. $[1; 4, 1, 2, 3, 2]$

10. $x^3 + 2x^2$

Вариант 6

1.) $2769x - 1378y = -52$

$y' = -y$

$2769x + 1378y' = -52$

i	-1	0	1	2
r	2769	1378	13	0
q		2	106	
x	1	0	1	
y'	0	1	-2	

$x = x_1 \cdot \frac{c}{d} + \frac{b}{d} \cdot k = -4 + 106k, \; k \in \mathbb{Z}$

$y' = y'_1 \cdot \frac{c}{d} - \frac{a}{d} \cdot k = 8 - 213k, k \in \mathbb{Z} \Rightarrow y = -8 + 213k, \; k \in \mathbb{Z}$

Проверка: $k = 0: \; x = -4; \; y = -8; \; 2769 \cdot 4 + 1378 \cdot 8 = -52$

$k = 1: \; x = 102; \; y = 205; \; 2769 \cdot 102 - 1378 \cdot 205 = -52$

Ответ: $\begin{cases} x = -4 + 106k, \; k \in \mathbb{Z} \\ y = -8 + 213k, \; k \in \mathbb{Z} \end{cases}$

2.) $\sqrt{300} = 17 + 10\sqrt{3} - 17 = 17 + \frac{1}{\left(\frac{1}{10\sqrt{3} - 17}\right)} =$

$= 17 + \frac{1}{\left(\frac{10\sqrt{3} + 17}{11}\right)} = 17 + \frac{1}{3 + \left(\frac{10\sqrt{3} - 16}{11}\right)} = 17 + \frac{1}{3 + \frac{1}{\left(\frac{11(10\sqrt{3} + 16)}{44}\right)}} =$

$= 17 + \frac{1}{3 + \frac{1}{8 + \left(\frac{10\sqrt{3} - 16}{4}\right)}} = 17 + \frac{1}{3 + \frac{1}{8 + \frac{1}{\left(\frac{10\sqrt{3} + 16}{11}\right)}}} = 17 + \frac{1}{3 + \frac{1}{8 + \frac{1}{3 + \left(\frac{10\sqrt{3} - 17}{11}\right)}}} =$

$$= 17 + \cfrac{1}{3 + \cfrac{1}{8 + \cfrac{1}{3 + \cfrac{1}{\left(\frac{11}{10\sqrt{3} - 17}\right)}}}} = 17 + \cfrac{1}{3 + \cfrac{1}{8 + \cfrac{1}{3 + \cfrac{1}{\left(\frac{(10\sqrt{3} + 17) 11}{11}\right)}}}} =$$

$$= 17 + \cfrac{1}{3 + \cfrac{1}{8 + \cfrac{1}{3 + \cfrac{1}{34 + 10\sqrt{3} - 17}}}} = [17, \overline{3, 8, 3, 34}]$$

Проверка: $34 = 2 \cdot 17$

Ответ: $[17, \overline{3, 8, 3, 34}]$

3.) $x \equiv 11 \bmod 20$

$x \equiv 13 \bmod 23$

$x \equiv 0 \bmod 11$

$x \equiv 4 \bmod 17$

$M = 20 \cdot 23 \cdot 11 \cdot 17 = 86020$

$M_1 = M/20 = 4301$

$M_2 = M/23 = 3740$, $M_3 = M/11 = 7820$

$M_4 = M/17 = 5060$

$4301 x = 20y + 1$

$4301x - 20y = 1$

$y' = -y$

i	-1	0	1	2
r	4301	20	1	0
q		215	20	
x	1	0	1	

$x_1 = 1$

$3740x + 23y' = 1$

r	3740	23	14	9	5	4	1	0
q		162	1	1	1	1	4	
x	1	0	1	-1	2	-3	5	

$x_2 = 5$

$7820x + 11y' = 1$

r	7820	11	10	1	0
q		710	1	10	
x	1	0	1	-1	

$x_3 \equiv -1 \bmod 11 \equiv 10 \bmod 11$

$5060x + 17y' = 1$

r	5060	17	11	6	5	1	0
q		297	1	1	1	5	
x	1	0	1	-1	2	-3	

$x_4 \equiv -3 \bmod 17 \equiv 14 \bmod 17$

$x = (4301 \cdot 1 \cdot 11 + 3740 \cdot 5 \cdot 13 + 7820 \cdot 10 \cdot 0 + 5060 \cdot 14 \cdot 4) \bmod 86020 =$

$= (47311 + 243100 + 283360) \bmod 86020 =$

$= 573771 \bmod 86020 = 57651 \bmod 86020$

Перевірка: $57651 < 86020$

$57651 \equiv 11 \bmod 20$

$57651 \equiv 13 \bmod 23$

$57651 \equiv 0 \bmod 11$

$57651 \equiv 4 \bmod 17$

Відповідь: $57651 \bmod 86020$

4) $x \equiv 95^{37^{41}} \bmod 98$

$k = 37^{41}$; $\varphi(98) = \varphi(2) \cdot \varphi(7^2) = 1 \cdot 49 \cdot (1 - \frac{1}{7}) =$

$= \frac{49 \cdot 6}{7} = 42$; $k = 42n + b$; $b = 37^{41} \bmod 42$

$\varphi(42) = \varphi(2) \cdot \varphi(3) \cdot \varphi(7) = 1 \cdot 2 \cdot 6 = 12$

$b = 37^{12e+\vartheta} \mod 42$

$41 = 12e + \vartheta$; $\vartheta = 5 \Rightarrow b = 37^5 \mod 42 = 69343957$

$\mod 42 = 25 \mod 42$

$5_{10} = 101_2$

a_i	c	c^2	$c^2 \cdot a^?$	$c^2 \cdot a^? \mod k$
1	1	1	37	37
0	37	1369	1369	25
1	25	625	23125	25

$x = 95^{25} \mod 98$

$25_{10} = 11001_2$

a_i	c	c^2	$c^2 \cdot a^?$	$c^2 \cdot a^? \mod k$
1	1	1	95	95
1	95	9025	857375	71
0	71	5041	5041	43
0	43	1849	1849	85
1	85	7225	686375	81

Ombem: 81 mod 98

5.) $p(2) = -19$
$p(-1) = 8$
$p(3) = -20$
$p(-2) = 45$
$p(1) = 6$
$\deg p \geq 4$

$$P(x) = \frac{(x+1)(x-3)(x+2)(x-1)}{3 \cdot (-1) \cdot 4 \cdot 1} (-19) + \frac{(x-2)(x-3)(x+2)(x-1)}{(-3) \cdot (-4) \cdot 1 \cdot (-2)} \cdot 8 + \frac{(x-2)(x+1)(x+2)(x-1)}{1 \cdot 4 \cdot 5 \cdot 2} \cdot (-20) + \frac{(x-2)(x+1)(x-3)(x-1)}{(-4) \cdot (-1) \cdot (-5) \cdot (-3)} \cdot 45 +$$

$$+ \frac{(x-2)(x+1)(x-3)(x+2)}{(-1) \cdot 2 \cdot (-2) \cdot 3} \cdot 6 = \frac{19}{12}(x+1)(x-3)(x+2)(x-1) -$$

$-\frac{1}{3}(x-2)(x-3)(x+2)(x-1)-\frac{1}{2}(x-2)(x+1)(x+2)(x-1)+$

$+\frac{3}{4}(x-2)(x+1)(x-3)(x-1)+\frac{1}{2}(x-2)(x+1)(x-3)(x+2)=$

$=\frac{19}{12}(x^2-1)(x-3)(x+2)-\frac{1}{3}(x^2-4)(x-3)(x-1)-$

$-\frac{1}{2}(x^2-4)(x^2-1)+\frac{3}{4}(x^2-1)(x-2)(x-3)+\frac{1}{2}(x^2-4)(x+1)(x-3)=$

$=-\frac{1}{2}x^4+\frac{1}{2}x^2+2x^2-2+(x-3)\Big((x^2-1)\Big(\frac{19}{12}(x+2)+\frac{3}{4}(x-2)\Big)+$

$+(x^2-4)\Big(-\frac{1}{3}(x-1)+\frac{1}{2}(x+1)\Big)\Big)=-\frac{1}{2}x^4+\frac{5}{2}x^2-2+$

$+(x-3)\Big((x^2-1)\Big(\frac{7}{3}x+\frac{5}{3}\Big)+(x^2-4)\Big(\frac{1}{6}x+\frac{5}{6}\Big)\Big)=$

$=-\frac{1}{2}x^4+\frac{5}{2}x^2-2+(x-3)\Big(\frac{7}{3}x^3+\frac{5}{3}x^2-\frac{7}{3}x-\frac{5}{3}+\frac{1}{6}x^3+$

$+\frac{5}{6}x^2-\frac{2}{3}x-\frac{10}{3}\Big)=-\frac{1}{2}x^4+\frac{5}{2}x^2-2+(x-3)\Big(\frac{5}{2}x^3+\frac{5}{2}x^2-3x-5\Big)=$

$=-\frac{1}{2}x^4+\frac{5}{2}x^2-2+\frac{5}{2}x^4+\frac{5}{2}x^3-3x^2-5x-\frac{15}{2}x^3-\frac{15}{2}x^2+9x+15=$

$=2x^4-5x^3-8x^2+4x+13$

Проверка: $\deg p=4$

$p(2)=-19$

$p(-1)=8$

$p(3)=-20$

$p(-2)=45$

$p(1)=6$

Ответ: $p(x)=2x^4-5x^3-8x^2+4x+13$

6.) $x^4-5x^3-6x^2+7x-2$

$\frac{p}{q}=\frac{\pm2;\pm1}{\pm1}$

	1	-5	-6	7	-2
1	1	-4	-10	-3	-5
-1	1	-6	0	7	-9
2	1	-3	-12	-17	-36
-2	1	-7	8	-9	16

Проверка: $f(1)=-5$

$f(-1)=-9$

$f(2)=-36$

$f(-2)=16$

Ответ: рациональных корней нет

7.) $8_9 x + 36_9 = 423_9$

I $8_9 = 8_{10}$

$36_9 = 3 \cdot 9^1 + 6 \cdot 9^0 = 27 + 6 = 33_{10}$

$423_9 = 4 \cdot 9^2 + 2 \cdot 9^1 + 3 \cdot 9^0 = 324 + 18 + 3 = 345$

$8_{10} x + 33_{10} = 345_{10}$

$8_{10} x = 312_{10}$

$x = 39_{10} = 4 \cdot 9^1 + 3 \cdot 9^0 = 43_9$

II
$$\begin{array}{r} 423_9 \\ -\ \ 36_9 \\ \hline 376_9 \end{array}$$

$8_9 x = 376_9 \Rightarrow x = 43_9$

$$\begin{array}{r|l} 376_9 & 8_9 \\ -35_9 & \overline{43} \\ \hline 26 & \\ -\ \ \ \ & \\ \hline 0 & \end{array}$$

Отговор: 43_9

8.) $\frac{8}{25} \mod 68$

$25x \equiv 8 \mod 68$

$y' = -y$; $25x + 68y' = 8$

r	68	25	18	7	4	3	1	0
q	*	2	1	2	1	1	3	
x	0	1	-2	3	-8	11	-19	

$x = -19 \cdot 8 + 68k,\ k \in \mathbb{Z}$

$x = -152 + 68k,\ k \in \mathbb{Z}$

$-152 \equiv 52 \mod 68$

Отговор: 52

9.) $\frac{131}{108} = [1;4,1,2,3,2]$

I $131 = 1 \cdot 108 + 23$

$108 = 4 \cdot 23 + 16$

$23 = 1 \cdot 16 + 7$

$16 = 2 \cdot 7 + 2$

$7 = 3 \cdot 2 + 1$

$2 = 2 \cdot 1$

II $\frac{131}{108} = 1 + \frac{23}{108} = 1 + \frac{1}{\left(\frac{108}{23}\right)} = 1 + \frac{1}{4 + \frac{16}{23}} = 1 + \frac{1}{4 + \frac{1}{\left(\frac{23}{16}\right)}} =$

$= 1 + \frac{1}{4 + \frac{1}{1 + \frac{7}{16}}} = 1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{\left(\frac{16}{7}\right)}}} = 1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{2 + \frac{2}{7}}}} =$

$= 1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{2}}}}} = [1;4,1,2,3,2]$

10.) $\frac{x^5 + 2x^4 + x^2 + x^1}{x^3 + x^2 + x^1 + 1}$ $\quad \mathbb{Z}/3\mathbb{Z}[x]$

$$x^5+2x^4+0\cdot x^3+x^2+x \;\Big|\; \frac{x^3+x^2+x+1}{x^2+x}$$

$$-\;\underline{x^5+x^4+x^3+x^2}$$

$$x^4+2x^3+0\cdot x^2+x$$

$$-\;\underline{x^4+x^3+x^2+x}$$

$$x^3+2x^2$$

Проверка: $(x^3+x^2+x+1)(x^2+x)=x^5+x^4+x^3+x^2+x^4+x^3+$

$+x^2+x=x^5+2x^4+2x^3+2x^2+x=(x^5+2x^4+x^2+x)-$

$-(x^3+2x^2)$

Ответ: x^3+2x^2