

0	9	81	81	410
	11	121	828	
1	40	1600	196800	17
1	17	289	19652	50

Exercise 3

1.

$$\phi(36216) = \cancel{2^3} \cdot \cancel{3^2} \cdot \cancel{503} = 2^3 \left(1 - \frac{1}{2}\right) \cdot 3^2 \left(1 - \frac{1}{3}\right) \cdot 503 = 4 \cdot 6 \cdot 503 = 12048$$

$$\phi(2002) = \cancel{2} \cdot \cancel{7} \cdot \cancel{11} \cdot \cancel{13} = 1 \cdot 6 \cdot 10 \cdot 12 = 720$$

$$\phi(14) = \phi(2) \cdot \phi(7) = 1 \cdot 6 = 6 \quad \phi(4) = 2^2 \cdot \left(1 - \frac{1}{2}\right) = 2$$

$$\phi(63) = \cancel{3^2} \cdot \cancel{7} = 3^2 \left(1 - \frac{1}{3}\right) \cdot 6 = 36$$

$$2. (d+5)^{k^m} \bmod x = 19 \bmod 63$$

$$k = 16^4 \Rightarrow 19^k \bmod 63$$

$$\phi(63) = 36 \quad k = 16^4 = 36n + 6$$

$$8 = 16 \text{ mod } 36$$

$$\phi(36) = \phi(2^2) \cdot \phi(3^2) = 2 \cdot 6 = 12$$

$$\cancel{8=16} \quad d=16 \quad m=4 \quad k=36$$

$$4 = 100$$

d_i	C	C^2	$C^2 \cdot d$	$C^2 \cdot d \text{ mod } t$
1	1	1	16	16
0	16	256	256	4
0	4	16	16	16

$$b=16$$

$$19^{36n+b} \text{ mod } 63 = 19^{36n} \cdot 19^b \text{ mod } 63 = 19^b \text{ mod } 63$$

$$19^{16} \text{ mod } 63$$

$$d=19 \quad m=16 \quad k=63$$

d_i	C	C^2	$C^2 \cdot d$	$C^2 \cdot d \text{ mod } t$
1	1	1	19	19
0	19	361 196	361 196	46
0	46	2116 49	2116 49	37
0	37	1369 2401	4369 2401	46
0	46	49 2116	49 2116	37

$$\text{Answer: } 19^{16} \text{ mod } 63 = \boxed{37}$$