Вариант 2

1/

$$1853x - 1836y = -51$$

$$1853 = 1(1836) + 17$$

$$1836 = 108(17) + 0$$

$$17 = 1853 - 1(1836)$$

$$x_0 = -3 \quad y_0 = 3$$

$$\begin{cases} x = x_0 + b_n \\ y = y_0 - b_n \end{cases} \Rightarrow \begin{cases} x = -3 - 1836n \\ y = 3 - 1853n \end{cases}$$

2/

$$\sqrt{321} = 17 + \frac{1}{\alpha_1}$$

$$\alpha_1 = \frac{1}{\sqrt{321} - 17} = \frac{\sqrt{321} + 17}{32} = 1 + \frac{\sqrt{321} - 15}{32} = 1 + \frac{1}{\alpha_2}$$

$$\alpha_2 = \frac{32}{\sqrt{321} - 15} = \frac{32(\sqrt{321} + 15)}{96} = \frac{\sqrt{321} + 15}{3} = 10 + \frac{\sqrt{321} - 15}{3} = 10 + \frac{1}{\alpha_3}$$

$$\alpha_3 = \frac{3}{\sqrt{321} - 15} = \frac{3(\sqrt{321} + 15)}{96} = \frac{\sqrt{321} + 15}{32} = 1 + \frac{\sqrt{321} - 17}{32} = 1 + \frac{1}{\alpha_4}$$

$$\alpha_4 = \frac{32}{\sqrt{321} - 17} = \frac{32(\sqrt{321} + 17)}{32} = \sqrt{321} + 17 = 34 + \frac{1}{\alpha_5}$$

$$\alpha_5 = \frac{1}{\sqrt{321} - 17} = \alpha_1$$

$$\sqrt{321} = [17; (1, 10, 1, 34)]$$

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3) $x = 6 \mod 34$ $\quad N = 517\ 650$

$x = 6 \mod 25$

$x = 3 \mod 29$

$x = 14 \mod 21$

b_i	$N_i = N/x_i$	x_i	$b_i n_i x_i$
6	15225	4	365 400
6	20706	-4	-496 944
3	17850	2	107 100
14	24650	2	690 200

$15225\, x_1 = 1 \mod 34$; $15225 = 34(447) + 27$

$20706\, x_2 = 1 \mod 25$; $20706 = 25(828) + 6$

$17850\, x_3 = 1 \mod 29$; $17850 = 29(615) + 15$

$24650\, x_4 = 1 \mod 21$; $24650 = 21(1173) + 17$

$$\begin{cases} 27 x_1 = 1 \mod 34 \\ 6 x_2 = 1 \mod 25 \\ 15 x_3 = 1 \mod 29 \\ 17 x_4 = 1 \mod 21 \end{cases}$$

- $27 x_1 = 1 \mod 34 \implies 27x - 34y = 1$

$34 = 1(27) + 7$

$27 = 3 \times 7 + 6$

$6 = 6 \times 1 = 0$

$\text{pgcd}(27; 34) = 1 \iff$ $6 = 27 - 3(7)$

$6 = 27 - 3[34 - 27]$

$6 = 4(27) - 3(24)$

$x_1 = 4$

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4) $57^{25^{41}} \pmod{98} \equiv ?$ $\quad$ $\operatorname{pgcd}(57, 98) = 1 \Rightarrow 57^{\varphi(98)} \equiv 1 \pmod{98}$

$\varphi(98) = \varphi(2\times 7^2) = \varphi(2)\cdot\varphi(7^2) = 1\times(49-7) = 42$

$\varphi(98) = 42 \qquad 57^{42} \equiv 1 \pmod{98}$

$\cancel{25^{41} (\bmod 98)}$. $\quad$ $\operatorname{pgcd}(25, 42) = 1 \qquad 25^{\varphi(42)} \equiv 1 \pmod{42}$

$\varphi(42) = \varphi(2\times 7\times 3) = 1\times 2\times 6 = 12$

$25^{12} \equiv 1 \pmod{42}$

$25^{41} = (25^{12})^3 \cdot 25^5$

$25^2 = 625 \underset{42}{\equiv} 37 \quad ; \quad 25^3 = 25\times 25^2 = 37\times 25 \underset{42}{\equiv} 1$

$\cancel{25^6 = (25^2)^3 = (1)^3 \underset{42}{\equiv} 1}$ $\qquad 25^5 = 25^2 \cdot 25^3 = 1\times 37$

$\cancel{25^{42} = (1)^3}.$ $\qquad = 37 \pmod{42}$

$57^{25^{41}} \pmod{98} = 57^{37} \pmod{98}$

$57^2 \equiv 3249 \underset{98}{\equiv} 15$

$57^4 = (57^2)^2 \underset{98}{\equiv} 29$

$57^6 = 57^4 \times 57^2 = 29\times 15 = 433 \underset{98}{\equiv} 43$

$57^{37} = ((57)^6)^6 \cdot 57 = (43)^6 \cdot 57 \underset{98}{\equiv} 15$ $\qquad$ om lem : $57^{25^{41}} \pmod{98} = 15$

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10) $2x^5+x^3+2x^2+x+2 \;\big|\; x^3+x^2+x+1$

$-\;2x^5+2x^4+2x^3+2x^2 \qquad\qquad 2x^2+x+1$

x^4+2x^3+x+2

$-\;x^4+x^3+x^2+x$

x^3+2x^2+2

$-\;x^3+x^2+x+1$

x^2+2x+1

$$2x^5+x^3+2x^2+x+2=(x^3+x^2+x+1)(2x^2+x+1)+(x^2+2x+1)$$

6) $x^4-5x^3-6x^2+7x-2$

	1	-5	-6	7	-2
1	1	-4	-10	-3	-5 ≠0
-1	1	-6	0	7	-9 ≠0
2	1	-3	-12	-17	-36 ≠0
-2	1	-7	8	-9	16 ≠0

нет рационнальные корни

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7) $7x_9 + 50_9 = 646_9$

$7x = 646 - 50$

$7x = 586$

$$\begin{array}{r|l} 586 & 7 \\ \cline{2-2} -54 & 76 \\ \cline{1-1} 46 & \\ -46 & \\ \cline{1-1} 00 & \end{array}$$

$(76)_9 \longrightarrow (?)_{10}$

$7 \times 9^1 + 6 \times 9^0$

$(69)_{10}$

$\{(76)_9 ; (69)_{10}\}$

8) $32/49 \mod 65$

$\text{Mod}(49, 32) = 1 \Rightarrow 49x - 65y = 32$

$65 = 1(49) + 16$

$49 = 3(16) + 1$

$1 = 49 - 3(16)$

$1 = 49 - 3(65 - 1(49))$

$1 = 4(49) - 3(65)$

$x = X_0 = 4 \times 3 = 128 \mod 65$

$63 \mod 65$

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9) $$\frac{447}{202} = 2 + \frac{1}{202/43}$$

$$= 2 + \cfrac{1}{4 + \cfrac{1}{43/30}}$$

$$= 2 + \cfrac{1}{4 + \cfrac{1}{1 + 13/30}}$$

$$= 2 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{30/13}}}$$

$$= 2 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + 4/13}}}$$

$$= 2 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{13/4}}}}$$

$$= 2 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4}}}}}$$

ОТВЕТ: [2, 4, 1, 2, 3, 4]

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