Determinación de Series de Constantes y de Taylor

Programación en Ingeniería

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Medio de entrega:	Electrónico (Microsoft Teams)
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1. Objetivo

Determinar los valores de series de constantes y de funciones especiales para n términos mediante ciclos for, $while$ o $do-while$. El valor debe de ser calculado para cualquier valor de n. Es necesario realizar las validaciones correspondientes.

2. Formulas

2.1. Series de Constantes

- $$ln\left(2\right) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots \tag{1}$$
- $$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots \tag{2}$$
- $$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \tag{3}$$
- $$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \tag{4}$$
- $$\frac{1}{2} = \frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \frac{1}{7\times 9} + \cdots \tag{5}$$
- $$\frac{3}{4} = \frac{1}{1\times 3} + \frac{1}{2\times 4} + \frac{1}{3\times 5} + \frac{1}{4\times 6} + \cdots \tag{6}$$

2.2. Funciones exponenciales y logaritmicas

- $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \tag{7}$$

- $$xe^x = x + \frac{2x^2}{2!} + \frac{3x^3}{3!} + \frac{4x^4}{4!} + \cdots \tag{8}$$

- $$\left(x + x^2\right) e^x = x + \frac{4x^2}{2!} + \frac{9x^3}{3!} + \frac{16x^4}{4!} + \cdots \tag{9}$$

- $$\ln\left(1 + x\right) = x - \tfrac{x^2}{2} + \tfrac{x^3}{3} - \tfrac{x^4}{4} + \cdots \quad -1 < x \leq 1 \tag{10}$$

- $$\tfrac{1}{2}\ln\left(\tfrac{1+x}{1-x}\right) = x + \tfrac{x^3}{3} + \tfrac{x^5}{5} + \tfrac{x^7}{7} + \cdots \quad -1 < x < 1 \tag{11}$$

- $$\ln\left(x\right) = 2\left\{\left(\tfrac{x-1}{x+1}\right) + \tfrac{1}{3}\left(\tfrac{x-1}{x+1}\right)^3 + \tfrac{1}{5}\left(\tfrac{x-1}{x+1}\right)^5 + \cdots\right\} \quad x > 0 \tag{12}$$

- $$\ln\left(x\right) = \left(\tfrac{x-1}{x}\right) + \tfrac{1}{2}\left(\tfrac{x-1}{x}\right)^2 + \tfrac{1}{3}\left(\tfrac{x-1}{x}\right)^3 + \cdots \quad x \geq \tfrac{1}{2} \tag{13}$$

$$(1 + x)^\alpha = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha \\ 1 \end{pmatrix} x + \begin{pmatrix} \alpha \\ 2 \end{pmatrix} x^2 + \begin{pmatrix} \alpha \\ 3 \end{pmatrix} x^3 + \cdots \quad |x| < 1 \tag{14}$$

$$a^x = e^{x\ln a} = 1 + x\ln a + \frac{\left(x\ln a\right)^2}{2!} + \frac{\left(x\ln a\right)^3}{3!} + \cdots \tag{15}$$

2.3. Número de Bernoulli y de Euler

$$B_k = -\sum_{i=0}^{k-1} \begin{pmatrix} k \\ i \end{pmatrix} \frac{B_i}{k + 1 - i} \tag{16}$$

$$B_0 = 1$$

$$E_k = \frac{2^{2k+2}\left(2k\right)!}{\pi^{2k+1}}\left\{1 - \frac{1}{3^{2k+1}} + \frac{1}{5^{2k+1}} - \cdots\right\} \tag{17}$$

$$E_{2k} = i\sum_{m=1}^{2k+1}\sum_{j=0}^{m} \begin{pmatrix} m \\ j \end{pmatrix} \frac{\left(-1\right)^j\left(m - 2j\right)^{2k+1}}{2^m i^m m} \tag{18}$$

$$i^2 = -1$$

2.4. Funciones trigonométricas

$$sen\,(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \tag{19}$$

$$cos\,(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \tag{20}$$

$$tan\,(x) = x + \tfrac{x^3}{3} + \tfrac{2x^5}{15} + \tfrac{17x^7}{315} + \cdots + \tfrac{B_{2n}(-4)^n(1-4^n)}{(2n)!}x^{2n-1} + \cdots \quad |x| < \tfrac{\pi}{2} \tag{21}$$

$$sec\,(x) = 1 + \tfrac{x^2}{2} + \tfrac{5x^4}{24} + \tfrac{61x^6}{720} + \cdots + \tfrac{(-1)^n E_{2n}}{(2n)!}x^{2n} + \cdots \quad |x| < \tfrac{\pi}{2} \tag{22}$$

$$csc\,(x) = \tfrac{1}{x} + \tfrac{x}{6} + \tfrac{7x^3}{360} + \tfrac{31x^5}{15{,}120} + \cdots + \tfrac{2\left(2^{2n-1}-1\right)B_n x^{2n-1}}{(2n)!} + \cdots \quad 0 < |x| < \pi \tag{23}$$

$$sen^{-1}\,(x) = x + \tfrac{1}{2}\tfrac{x^3}{3} + \tfrac{1}{2}\tfrac{3}{4}\tfrac{x^5}{5} + \tfrac{1}{2}\tfrac{3}{4}\tfrac{5}{6}\tfrac{x^7}{7} + \cdots \quad |x| < 1 \tag{24}$$

$$cos^{-1}\,(x) = \tfrac{\pi}{2} - sen^{-1}\,(x) = \tfrac{\pi}{2} - \left(x + \tfrac{1}{2}\tfrac{x^3}{3} + \tfrac{1}{2}\tfrac{3}{4}\tfrac{x^5}{5} + \cdots\right) \quad |x| < 1 \tag{25}$$

$$tan^{-1}\,(x) = \begin{cases} x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots & |x| < 1 \\ \pm\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots & [+ \text{ si } x \geq 1, - \text{ si } x \leq -1] \end{cases} \tag{26}$$

2.5. Funciones hiperbólicas

$$sinh\,(x) = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \frac{1}{7!}x^7 + \cdots + \frac{1}{(2n+1)!}x^{2n+1} \tag{27}$$

$$cosh\,(x) = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \frac{1}{6!}x^6 + \cdots + \frac{1}{(2n)!}x^{2n} \tag{28}$$

$$tanh\,(x) = x - \tfrac{1}{3}x^3 + \tfrac{2}{15}x^5 - \tfrac{17}{315}x^7 + \cdots + \tfrac{B_{2n}4^n(4^n-1)}{(2n)!}x^{2n-1} \quad |x| < \tfrac{\pi}{2} \tag{29}$$

$$senh^{-1}\,(x) = x - \tfrac{1}{6}x^3 + \tfrac{3}{40}x^5 - \tfrac{5}{112}x^7 + \cdots + \tfrac{(-1)^n(2n)!}{4^n(n!)^2(2n+1)}x^{2n+1} \quad |x| < 1 \tag{30}$$

$$tanh^{-1}\,(x) = x + \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 + \tfrac{1}{7}x^7 + \cdots + \tfrac{1}{2n+1}x^{2n+1} \quad |x| < 1 \tag{31}$$

2.6. Series varias

$$\tfrac{\ln(1+x)}{1+x} = x - \left(1 + \tfrac{1}{2}\right) x^2 + \left(1 + \tfrac{1}{2} + \tfrac{1}{3}\right) x^3 - \cdots \quad |x| < 1 \tag{32}$$

$$e^{sen(x)} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} - \frac{x^5}{15} + \cdots \tag{33}$$

3. Procedimiento

a. Resuelva cada una de las series en la sección 2 mediante ciclos *for*, *while* y *do-while*, deben resolverse igual número de series por cada uno de los tipos de ciclo y cada serie debe ser una función, considere las validaciones necesarias de datos.

b. Realice una tabla de resultados para cada una de las series con el siguiente formato, para 8 valores diferentes de n y para 3 valores diferentes de x en el rango.

n	x	$f_n(x)$	$f_n(x) - f_{n-1}(x)$	ϵ_n
1	x_1	$f_1(x_1)$	$-$	ϵ_1
2	x_1	$f_2(x_1)$	$f_2(x_1) - f_1(x_1)$	ϵ_2
4	x_1	$f_4(x_1)$	$f_4(x_1) - f_3(x_1)$	ϵ_4
8	x_1	$f_8(x_1)$	$f_8(x_1) - f_7(x_1)$	ϵ_8
16	x_1	$f_{16}(x_1)$	$f_{16}(x_1) - f_{15}(x_1)$	ϵ_{16}
32	x_1	$f_{32}(x_1)$	$f_{32}(x_1) - f_{31}(x_1)$	ϵ_{32}
64	x_1	$f_{64}(x_1)$	$f_{64}(x_1) - f_{63}(x_1)$	ϵ_{64}
128	x_1	$f_{128}(x_1)$	$f_{128}(x_1) - f_{127}(x_1)$	ϵ_{128}
1	x_2	$f_1(x_2)$	$-$	ϵ_1
2	x_2	$f_2(x_2)$	$f_2(x_2) - f_1(x_2)$	ϵ_2
4	x_2	$f_4(x_2)$	$f_4(x_2) - f_3(x_2)$	ϵ_4
8	x_2	$f_8(x_2)$	$f_8(x_2) - f_7(x_2)$	ϵ_8
16	x_2	$f_{16}(x_2)$	$f_{16}(x_2) - f_{15}(x_2)$	ϵ_{16}
32	x_2	$f_{32}(x_2)$	$f_{32}(x_2) - f_{31}(x_2)$	ϵ_{32}
64	x_2	$f_{64}(x_2)$	$f_{64}(x_2) - f_{63}(x_2)$	ϵ_{64}
128	x_2	$f_{128}(x_2)$	$f_{128}(x_2) - f_{127}(x_2)$	ϵ_{128}
1	x_3	$f_1(x_3)$	$-$	ϵ_1
2	x_3	$f_2(x_3)$	$f_2(x_3) - f_1(x_3)$	ϵ_2
4	x_3	$f_4(x_3)$	$f_4(x_3) - f_3(x_3)$	ϵ_4
8	x_3	$f_8(x_3)$	$f_8(x_3) - f_7(x_3)$	ϵ_8
16	x_3	$f_{16}(x_3)$	$f_{16}(x_3) - f_{15}(x_3)$	ϵ_{16}
32	x_3	$f_{32}(x_3)$	$f_{32}(x_3) - f_{31}(x_3)$	ϵ_{32}
64	x_3	$f_{64}(x_3)$	$f_{64}(x_3) - f_{63}(x_3)$	ϵ_{64}
128	x_3	$f_{128}(x_3)$	$f_{128}(x_3) - f_{127}(x_3)$	ϵ_{128}