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FINANCIAL AND COST ANALYSIS

Time Value of Money

Outline

Interest and future value
- Future value
- Interest: simple vs. compound

Present value
- Discount rates and present values
- Multiple cash flows
- Level cash flows: perpetuities and annuities
- Annuities due

Effective annual interest rates

Inflation and the time value of money

PV **Present Value**

FV_t **Future Value**: Amount to which an investment will grow after earning interest

$$FV_1 = PV \cdot (1+r) = 100 \cdot (1+7\%) = 100 \cdot (1+0.07) = 107$$

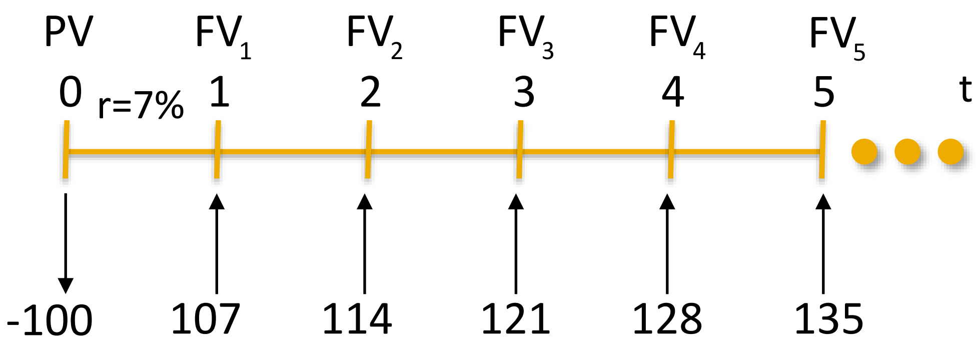
$$FV_2 = PV \cdot (1+r \cdot 2) = 100 \cdot (1+0.07 \cdot 2) = 114$$

$$FV_3 = PV \cdot (1+r \cdot 3) = 100 \cdot (1+0.07 \cdot 3) = 121$$

$$FV_4 = PV \cdot (1+r \cdot 4) = 100 \cdot (1+0.07 \cdot 4) = 128$$

$$FV_5 = PV \cdot (1+r \cdot 5) = 100 \cdot (1+0.07 \cdot 5) = 135$$

Let **r** = annual interest rate
Let **t** = number of periods



Simple Interest

$$\boxed{FV_{Simple} = \text{Initial investment} \times (1 + r \times t)}$$

PV **Present Value**

FV_t **Future Value**: Amount to which an investment will grow after earning interest

$$FV_1 = PV \cdot (1+r) = 100 \cdot (1+7\%) = 100 \cdot (1+0.07) = 107$$

$$FV_2 = 107 \cdot (1+0.07) = 100 \cdot (1+0.07)^2 = 114.49$$

$$FV_3 = 114.49 \cdot (1+0.07) = 100 \cdot (1+0.07)^3 = 122.50$$

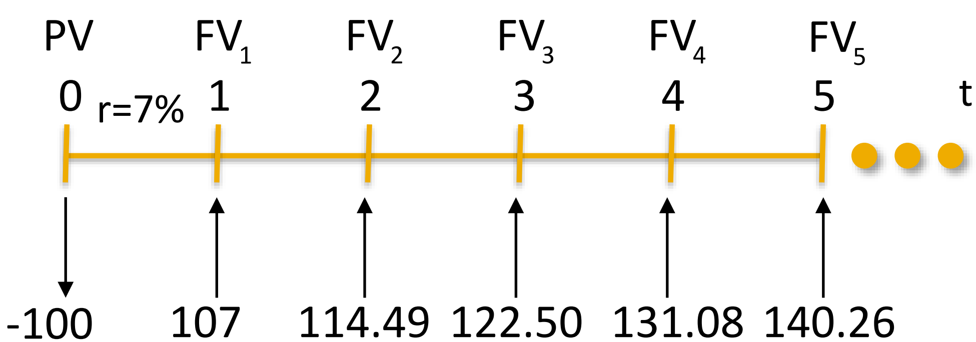
$$FV_4 = 122.50 \cdot (1+0.07) = 100 \cdot (1+0.07)^4 = 131.08$$

$$FV_5 = 131.08 \cdot (1+0.07) = 100 \cdot (1+0.07)^5 = 140.26$$

$$FV_5 = PV \cdot (1+r)^5$$

Let **r** = annual interest rate

Let **t** = number of periods



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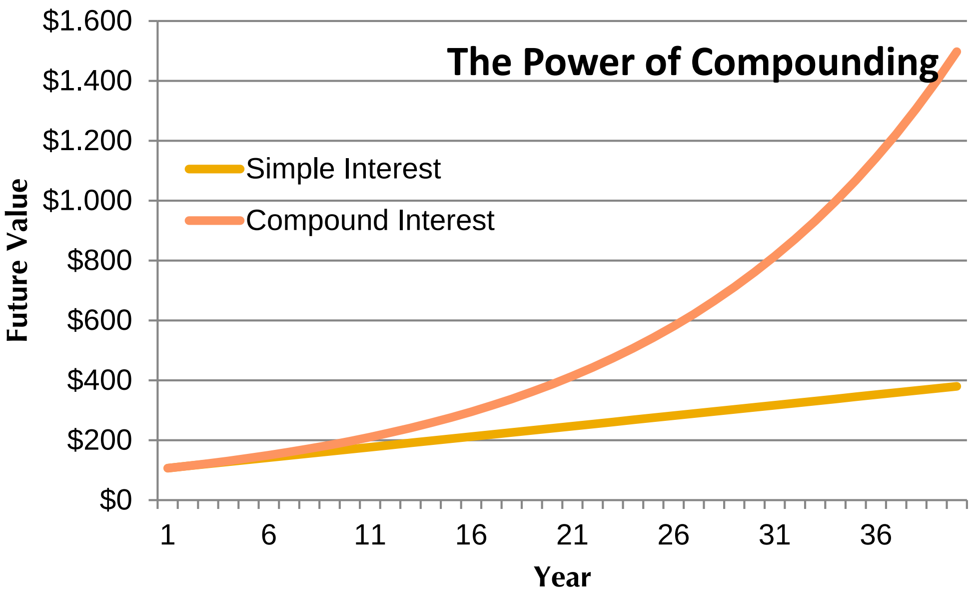
$$FV_{Simple} = \text{Initial investment} \times (1 + r \times t)$$

Compound Interest

$$FV_{Compound} = \text{Initial investment} \times (1 + r)^t$$

Interest earned at a rate of 7% for the first forty years on the $100 invested using simple and compound interest



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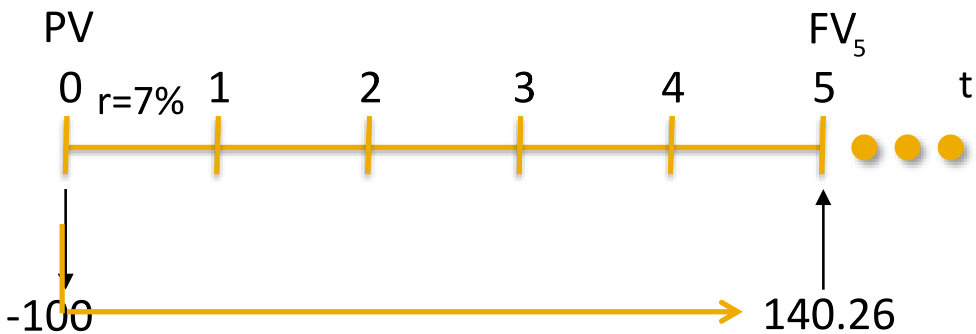
Let **t** = number of periods



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$$FV_{Simple} = \text{Initial investment} \times (1 + r \times t)$$

Compound Interest

$$FV_{Compound} = \text{Initial investment} \times (1 + r)^t$$

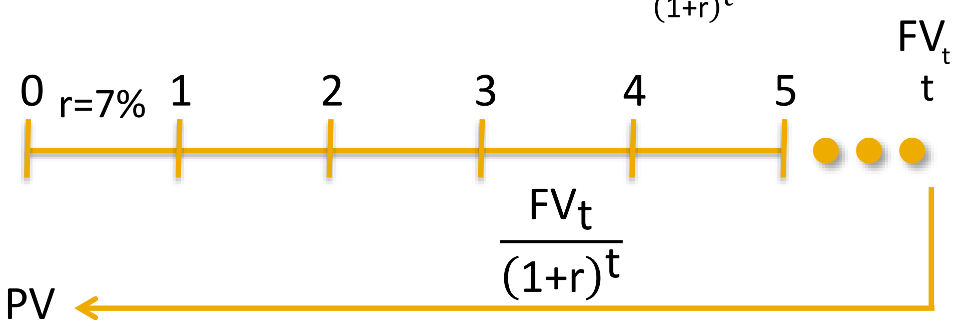
PV **Present Value**: Value today of a future cash flow

$$\frac{FV_t}{(1+r)^t} = PV$$

The PV formula has many applications. Given any variables in the equation, you can solve for the remaining variable

Let **r** = annual interest rate = **discount rate**
Let **t** = number of periods

discount factor $= \frac{1}{(1+r)^t}$



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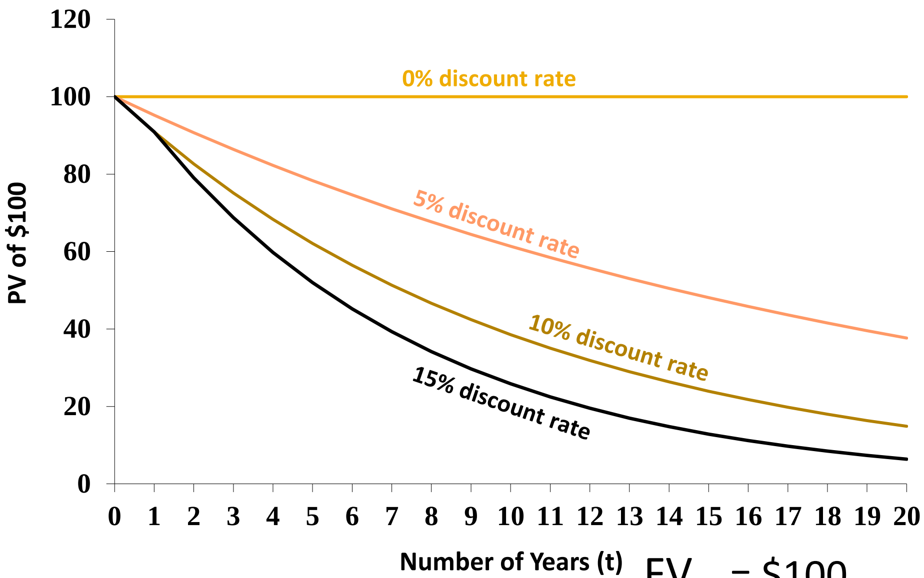
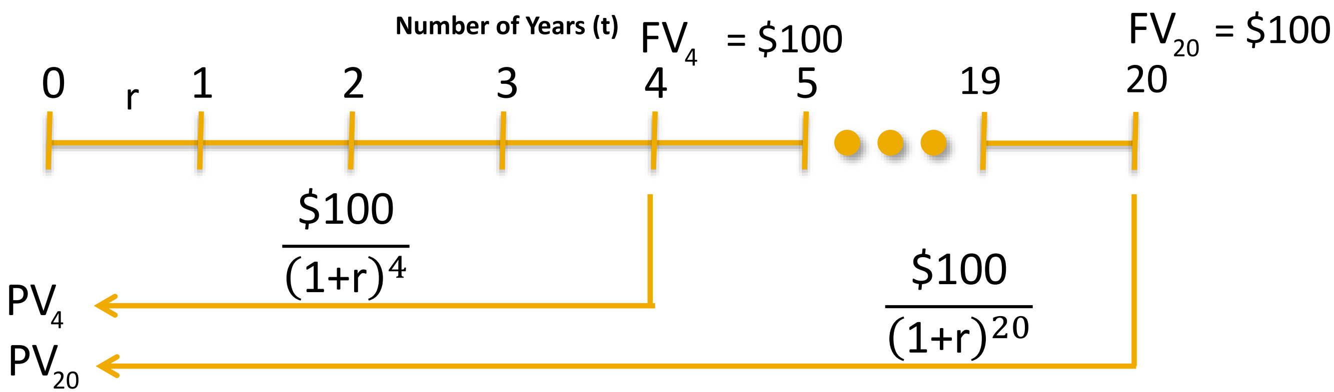
PV **Present Value**: Value today of a future cash flow

Changing Discount Rates The present value of $100 to be received in 1 to 20 years at varying discount rates:





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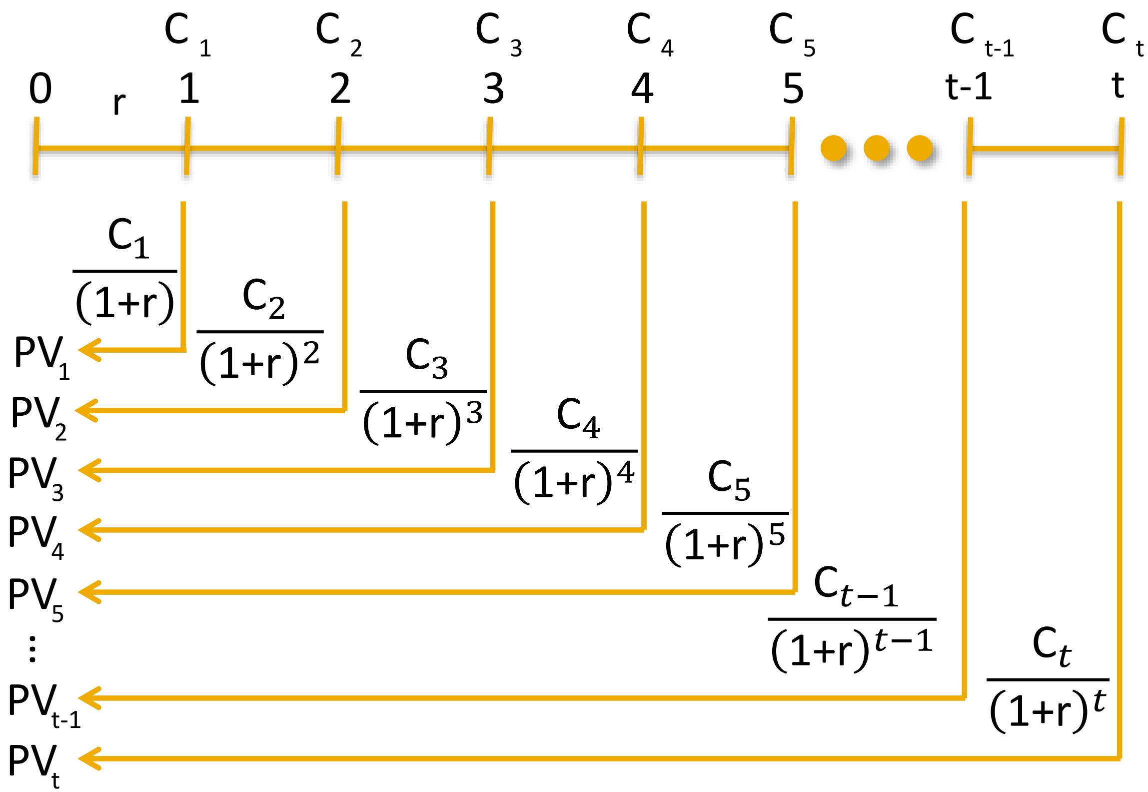

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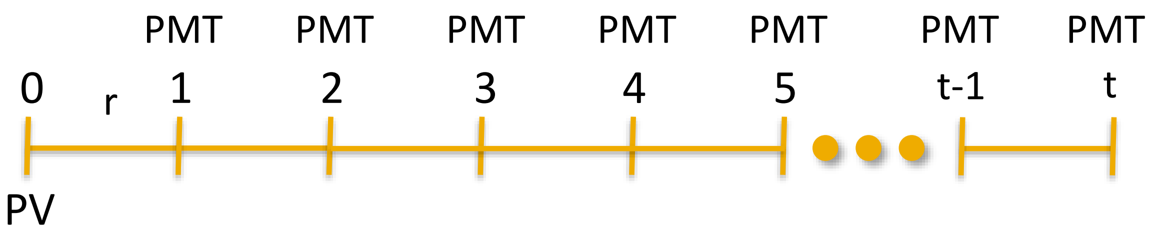
PV of Multiple Cash Flows: The present value of multiple cash flows can be calculated:

$$PV=PV_1+PV_2+\cdots+PV_t=\frac{C_1}{(1+r)}+\frac{C_2}{(1+r)^2}+\cdots+\frac{C_t}{(1+r)^t}=\sum_{i=1}^{t}\frac{C_i}{(1+r)^i}$$

where

C_1 is the cash flow in year 1
C_2 is the cash flow in year 2
C_t is the cash flow in year t (with any number of cash flows in between)



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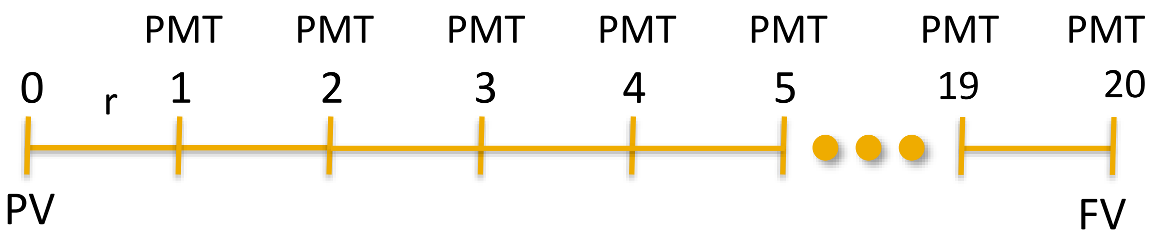
Annuities What are they? Annuities are equally-spaced, level streams of cash flows lasting for a limited period of time

PMT = yearly cash payment
t = number of years the payment is received

$$PV = PMT \cdot \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right]$$

$$FV = PMT \cdot \left[\frac{(1+r)^t - 1}{r} \right]$$

The terms within the brackets are collectively called the "annuity factor"



Annuities What are they? Annuities are equally-spaced, level streams of cash flows lasting for a limited period of time

Annuity due What is it? An annuity whose payment is to be made immediately, rather than at the end of the period

How does it differ from an ordinary annuity?

$$PV_{Annuity\ Due} = PV_{Annuity} \cdot (1+r)$$

How does the future value differ from an ordinary annuity?

$$FV_{Annuity\ Due} = FV_{Annuity} \cdot (1+r)$$

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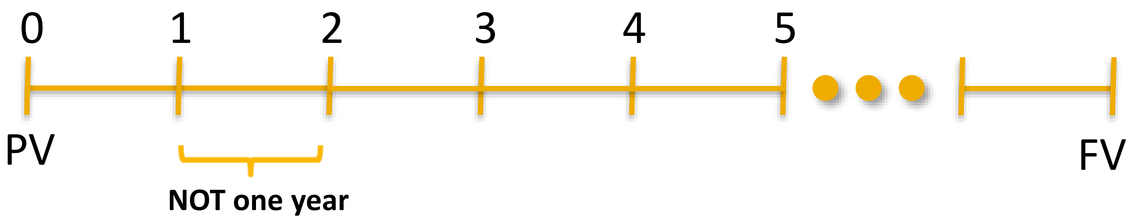

Used in situations that do not use yearly time periods

Semiannual bond payments
Quarterly stock dividends
Consumer loans – monthly payments
⋮

Effect of Compounding Frequency

Assumptions

$100 deposit today
12% annual interest rate
Bank compounds interest at six months instead of end of year
Interest is earned on interest
$\$112.36 = \$106 \cdot (1+0.06)$

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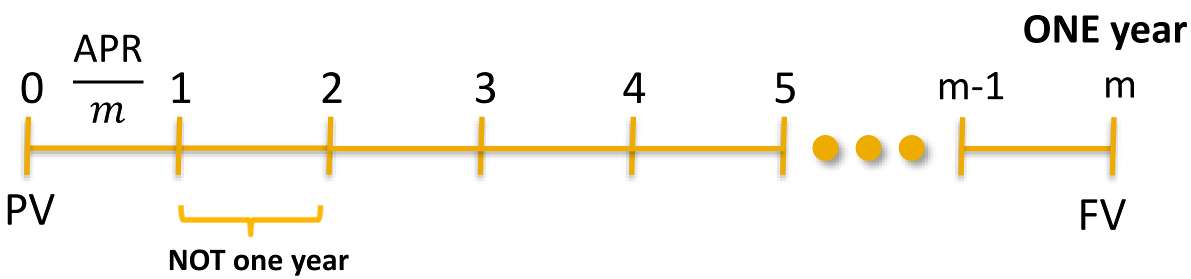

Assumptions

Borrow \$100 today
12% annual interest rate
APR: Loan compounds *annually*; you pay 12.00%
EARS: Loan compounds *monthly*; you pay 12.68%

$$\text{EAR} = \left(1+\frac{0.12}{12}\right)^{12} - 1$$

APR Quoted, or nominal rate called **annual percentage rate**

EAR Rate that incorporates compounding called **effective annual rate**

Relationship between APR and EAR:

$$\cancel{PV} \cdot (1+\text{EAR})^1 = \cancel{PV} \cdot \left(1+\frac{\text{APR}}{m}\right)^m \qquad \text{EAR} = \left(1+\frac{\text{APR}}{m}\right)^m - 1$$

m = compound frequency

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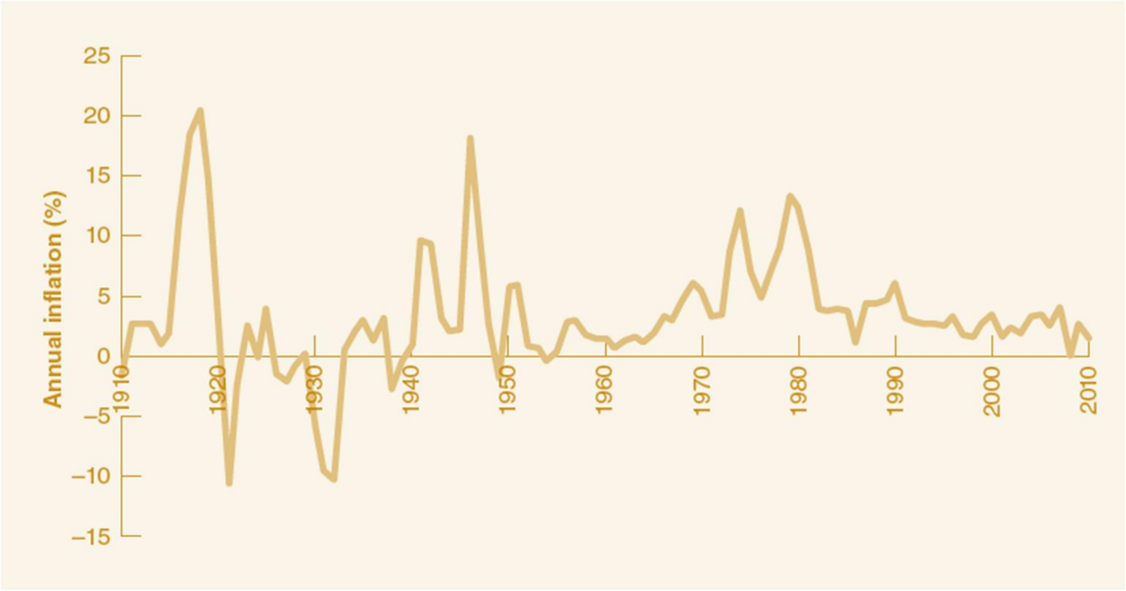
Annual U.S. Inflation Rates from 1900 - 2010



Source: Bureau of Labor Statistics.