The Time Value of Money

Outline

- Interest, present and future value
  - Future value
  - Interest: simple vs. compound
  - Discount rates and present values
- Multiple cash flows
- Level cash flows: perpetuities and annuities
- Effective annual interest rates
- Inflation and the time value of money

Time Value of Money

Money has a time value. This can be expressed in multiple ways:

A dollar today held in savings will grow.

A dollar received in a year is not worth as much as a dollar received today.

Future Values

Future Value: Amount to which an investment will grow after earning interest.

Let r = annual interest rate

Let t = # of years

Simple Interest

$$\boxed{FV_{Simple} = \text{Initial investment} \times (1 + r \times t)}$$

Compound Interest

$$\boxed{FV_{Compound} = \text{Initial investment} \times (1 + r)^t}$$

Simple Interest: Example

Interest earned at a rate of 7% for five years on a principal balance of $100.

Example - Simple Interest

	Today			Future Years		
		1	*2*	*3*	*4*	*5*
Interest Earned		***7***	***7***	***7***	***7***	***7***
Value	*100*	***107***	***114***	***121***	***128***	***135***

Value at the end of Year 5: $135

Compound Interest: Example

Interest earned at a rate of 7% for five years on the previous year's balance.

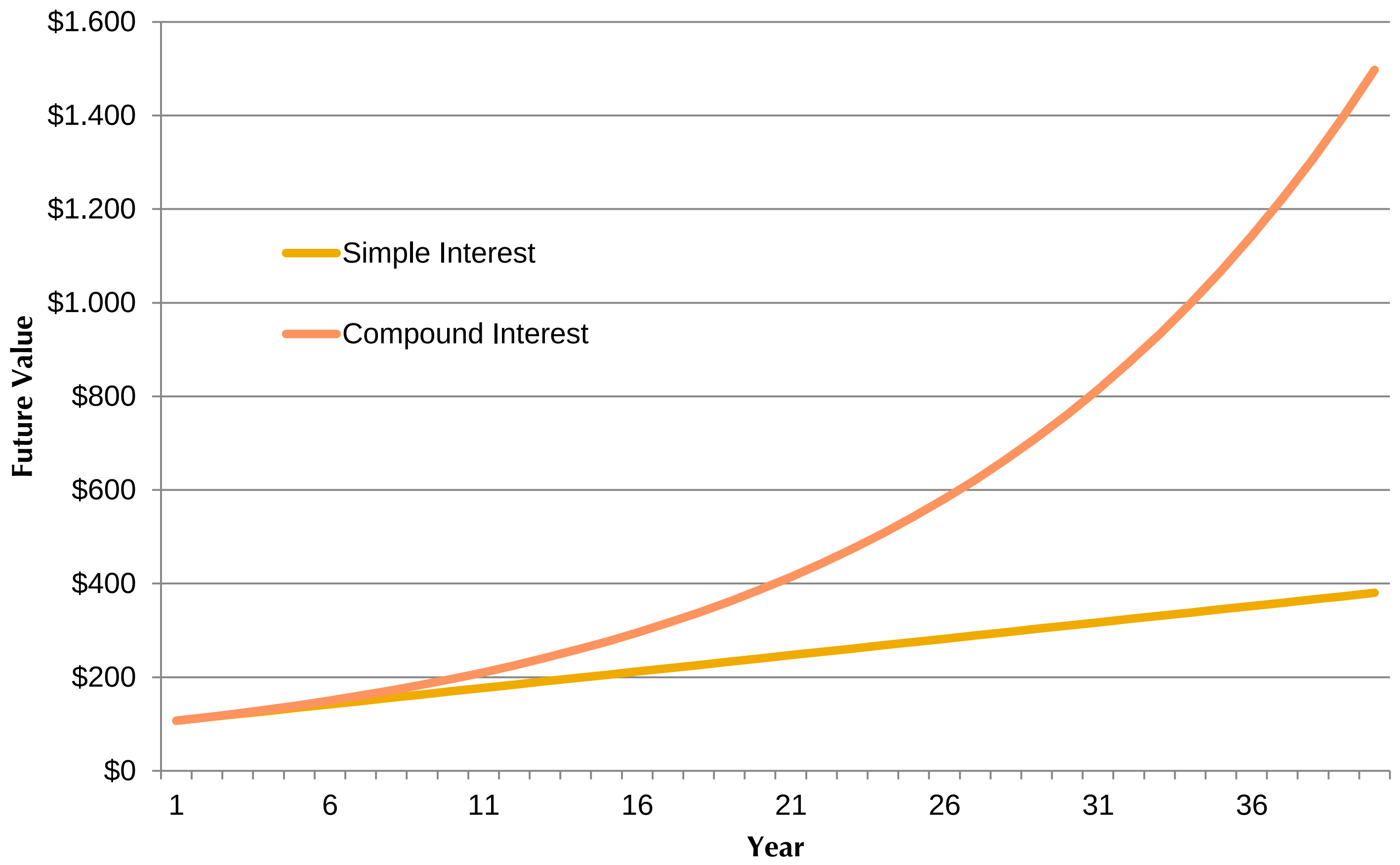
Example - Compound Interest

	Today	Future Years				
		1	2	3	4	5
Interest Earned		7	7.49	8.01	8.58	9.18
Value	100	107	114.49	122.50	131.08	140.26

Value at the end of Year 5 = $140.26

The Power of Compounding

Interest earned at a rate of 7% for the first forty years on the $100 invested using simple and compound interest.



Present Value

What is it? Value today of a future cash flow

- ✓ Discount Rate: r
- ✓ Discount Factor: $DF = \frac{1}{(1+r)^t}$
- ✓ Present Value: $PV = FV \times \frac{1}{(1+r)^t}$

Recall: t = number of years

Present Value: Example

Always ahead of the game, Tommy, at 8 years old, believes he will need \$100,000 to pay for college. If he can invest at a rate of 7% per year, how much money should he ask his rich Uncle GQ to give him?

$$FV = \$100,000 \qquad t = 10\ yrs \qquad r = 7\%$$

$$PV = FV \times \frac{1}{(1+r)^t} = \$100,000 \times \frac{1}{(1.07)^{10}} \approx \$50,835$$

Note: Ignore inflation/taxes

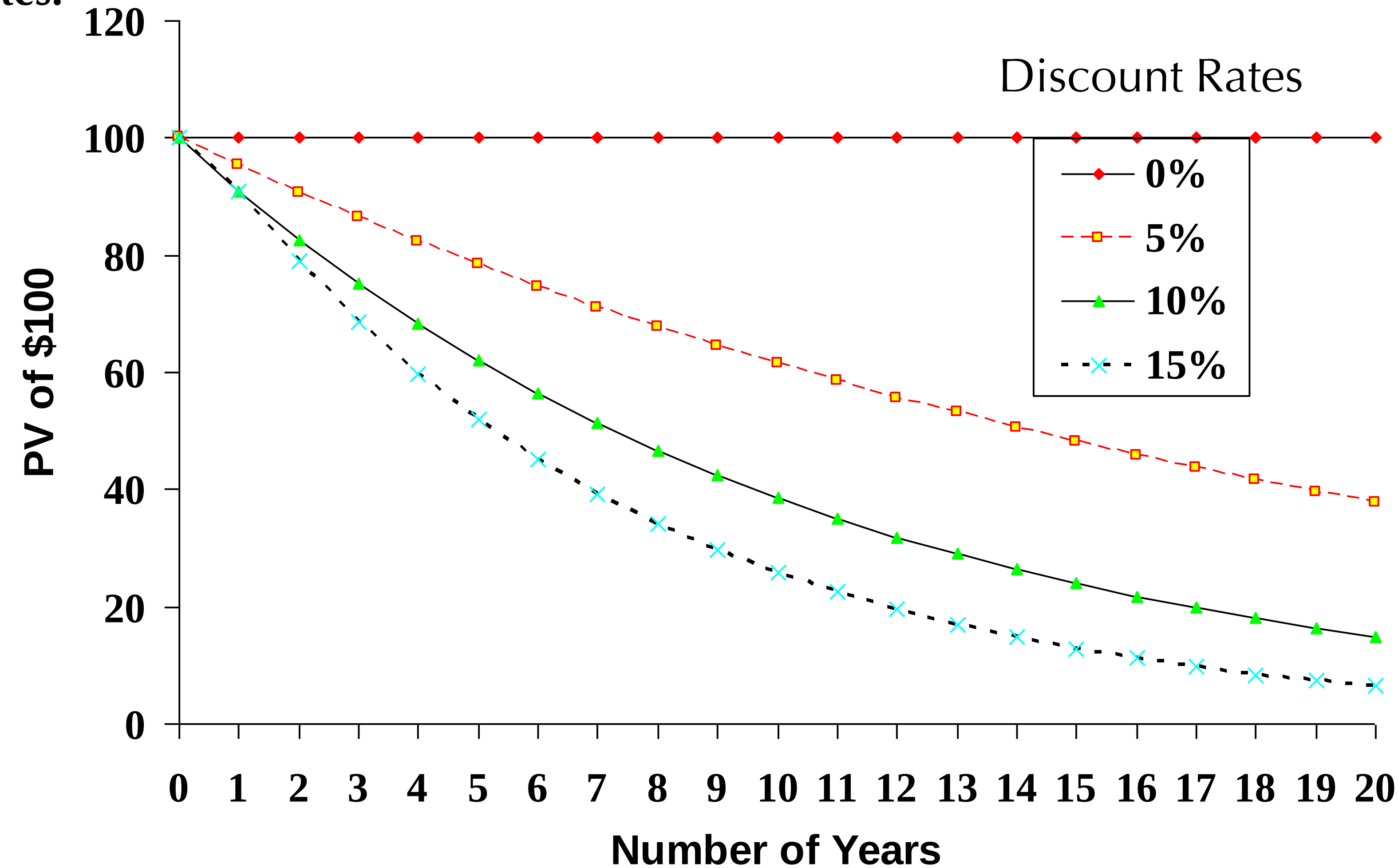
Time value of money:(applications)

The PV formula has many applications. Given any variables in the equation, you can solve for the remaining variable.

$$PV = FV \times \frac{1}{(1+r)^t}$$

Present Values: Changing Discount Rates

The present value of $100 to be received in 1 to 20 years at varying discount rates:



PV of Multiple Cash Flows

The present value of multiple cash flows can be calculated:

$Denote:$

$C_1 =$ The cash flow in year 1

$C_2 =$ The cash flow in year 2

$C_t =$ The cash flow in year t (with any number of cash flows in between)

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_t}{(1+r)^t}$$

Multiple cash flows: Example

Your auto dealer gives you the choice to pay $15,500 cash now or make three payments: $8,000 now and $4,000 at the end of the following two years. If your cost of money (discount rate) is 8%, which do you prefer?

$$\text{Initial Payment*} \quad 8{,}000.00$$

$$PV\ of\ C_1 = \frac{4{,}000}{(1+.08)^1} = 3{,}703.70$$

$$PV\ of\ C_2 = \frac{4{,}000}{(1+.08)^2} = 3{,}429.36$$

$$\text{Total PV} = \$15{,}133.06$$

*

Perpetuities

What are they? A stream of level cash payments that never ends.

Let C = Yearly Cash Payment

PV of Perpetuity:

$$\boxed{PV = \frac{C}{r}}$$

Recall: r = the discount rate

Perpetuities: Example

In order to create an endowment, which pays $185,000 per year forever, how much money must be set aside today if the rate of interest is 8%?

$$PV = \frac{185,000}{.08} = \$2,312,500$$

What if the first payment won't be received until 3 years from today?

$$PV = \frac{2,312,500}{(1+.08)^2} = \$1,982,596$$

Annuities

What are they? Annuities are equally-spaced, level streams of cash flows lasting for a limited period of time.

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

$$FV = PMT\left[\frac{(1+r)^t - 1}{r}\right]$$

The terms within the brackets are collectively called the "annuity factor."

PMT=yearly cash payment
t=number of years the payment is received

Present value of annuities: Example

You are purchasing a home and are scheduled to make 30 annual installments of \$10,000 per year. Given an interest rate of 5%, what is the price you are paying for the house (i.e. what is the present value)?

$$PV = \$10,000\left[\frac{1}{.05} - \frac{1}{.05(1+.05)^{30}}\right]$$

$$PV = \$153,724.51$$

Future value of annuities: Example

You plan to save $4,000 every year for 20 years and then retire. Given a 10% rate of interest, how much will you have saved by the time you retire?

$$FV = 4{,}000\left[\frac{1.10^{20} - 1}{.10}\right] = 229{,}100$$

Annuity due

What is it? An annuity whose payment is to be made immediately, rather than at the end of the period.

How does it differ from an ordinary annuity?

$$\boxed{PV_{Annuity\ Due} = PV_{Annuity} \times (1 + r)}$$

How does the future value differ from an ordinary annuity?

$$\boxed{FV_{Annuity\ Due} = FV_{Annuity} \times (1 + r)}$$

Recall: r = the discount rate

Annuities due: Example

$$\boxed{FV_{AD} = FV_{Annuity} \times (1+r)}$$

<u>*Example*</u>*: Suppose you invest $429.59 annually at the beginning of each year at 10% interest. After 50 years, how much would your investment be worth?*

$$FV_{AD} = FV_{Annuity} \times (1+r)$$

$$FV_{AD} = (\$500{,}000) \times (1.10)$$

$$FV_{AD} = \$550{,}000$$

Compounding frequency: EAR and APR

- Quoted, or nominal rate called **annual percentage rate (APR)**
- Rate that incorporates compounding called **effective annual rate (EAR)**
- Relationship between APR and EAR:

$$EAR = \left(1 + \frac{APR}{m}\right)^m - 1$$

m=compound frequency

Used in situations that do not use yearly time periods

- *Semiannual* bond payments
- *Quarterly* stock dividends
- Consumer loans – *monthly* payments

EAR vs. APR Example

- Assumptions:
  - Borrow $100 today
  - 12% annual interest rate
  - APR: Loan compounds *annually*; you pay 12.00%
  - EARS: Loan compounds *monthly;* you pay 12.68%
- Formula to convert APR to EAR:

$$EAR = \left(1 + \frac{0.12}{12}\right)^{12} - 1$$

Inflation and real interest

Exact calculation:

$$\boxed{1 + \text{real interest rate} = \frac{1+\text{nominal interest rate}}{1+\text{inflation rate}}}$$

Approximation:

$$\boxed{\text{Real interest rate} \approx \text{nominal interest rate} - \text{inflation rate}}$$

Inflation: Example

If the nominal interest rate on your interest-bearing savings account is 2.0% and the inflation rate is 3.0%, what is the real interest rate?

$$1 + \text{real interest rate} = \frac{1+.02}{1+.03}$$

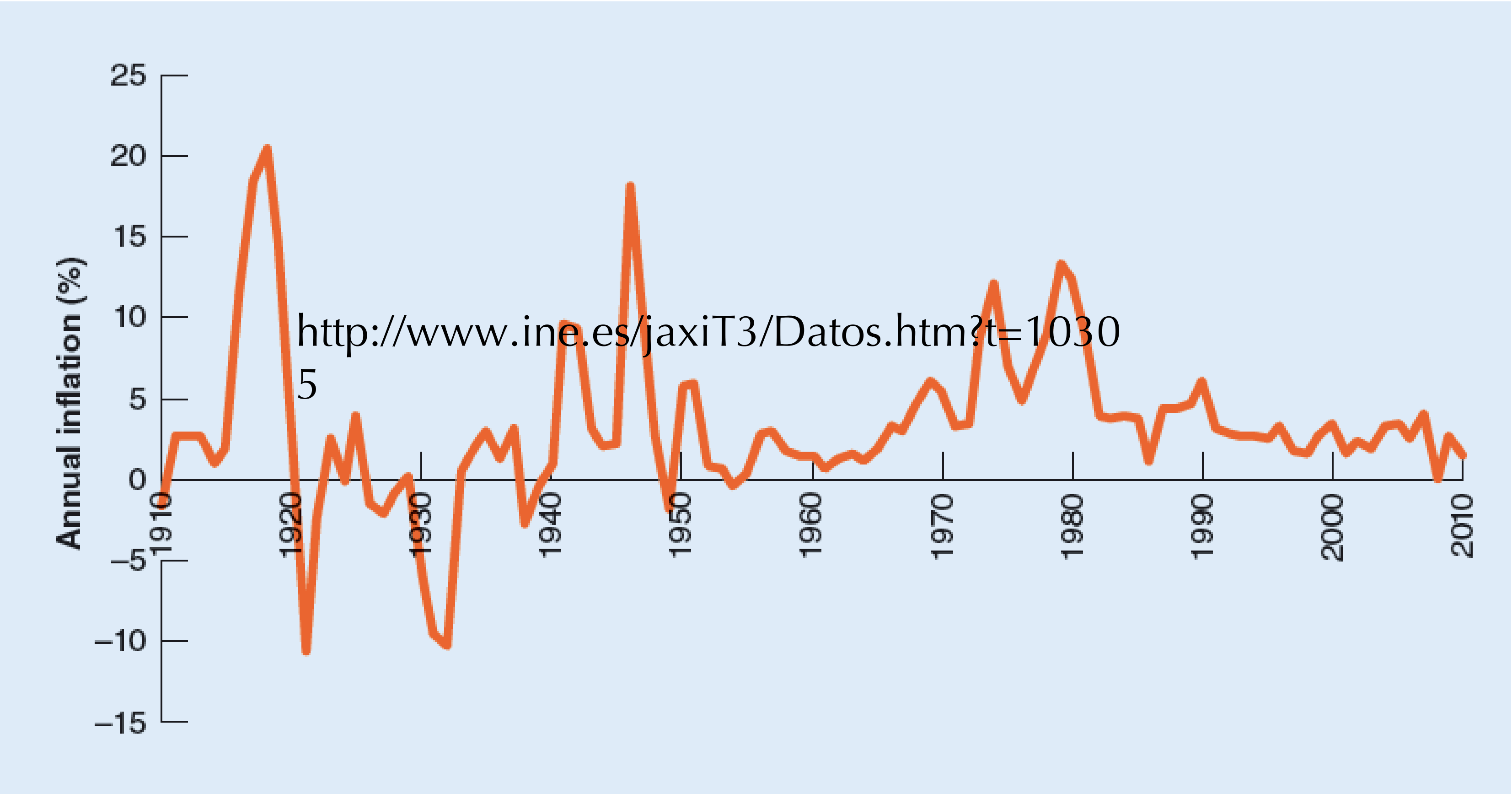
$$1 + \text{real interest rate} = 0.9903$$

$$\text{real interest rate} = -.0097 \text{ or } -.97\%$$

$$\text{Approximation} = .02-.03 = -.01 = -1\%$$

Appendix A: Inflation

Annual U.S. Inflation Rates from 1900 - 2010



Source: Bureau of Labor Statistics.

Annual Spain Inflation Rates from 1962 - 2015