Security-Constrained Unit Commitment: A Decomposition Approach Embodying Kron Reduction

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Abstract

We address the day-ahead scheduling of electricity production units throughout a network imposing N-1 security constraints, which ensures uneventful operation under any single-branch failure. For realistic electric energy systems, this optimization problem, which is mixed-integer linear or nonlinear but convex, involves millions of continuous variables, millions of constraints, and thousands of binary variables. This problem is intractable if state-of-the-art branch-and-cut solvers are used. As a solution methodology, we propose a Benders-type decomposition technique with a dynamically enriched master problem. Such master problem incorporates scheduling (binary) decisions and decisions pertaining to under-contingency operating conditions. The subproblems represent the operation of the system under no failure and single-branch failure. As the algorithm progresses, the master problem incorporates additional under-contingency operating conditions, which increases its computational burden. We use Kron reduction to compact (reducing variables and constraints) the description of the under-contingency operating conditions in the master problem without losing accuracy, which renders major computational gains. The methodology proposed allows solving, within reasonable computing times, instances intractable with state-of-the-art branch-and-cut solvers and decomposition algorithms.

Keywords: OR in energy, decomposition algorithm, network reduction, unit commitment.

1. Introduction

In power systems operations and planning, the term *system security* refers to the ability of a power system to handle a predetermined set of plausible contingencies, e.g., equipment failures. In the context of the generation scheduling problem, the solution of such problem has to be (i) the most economically efficient commitment and dispatch of generating units that can supply the demand, and (ii) able to survive such contingencies without considerable load shedding (Conejo et al., 2010). If the generation scheduling problem embodies a set of credible contingencies into its formulation is known as security-constrained unit commitment (SCUC).

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Nomenclature

Sets and indexes

$\mathcal{C}$	Contingencies.
$\mathcal{C}^{\mathrm{M},(\nu)}$	Contingencies incorporated into the master problem in iteration ν.
$\mathcal{E}$	Set of transmission lines.
$\mathcal{E}^{(\nu)}(t)$	Congested lines at iteration ν in normal operating condition at time period t.
$\mathcal{E}_c^{(\nu)}(t)$	Congested lines at iteration ν in operating condition under contingency c at time period t.
$\mathcal{G}^{\mathrm{F}}$	Dispatchable generating units (includes coal- and gas-fired units).
$\mathcal{G}^{\mathrm{N}}$	Nuclear generating units.
$\mathcal{G}^{\mathrm{W}}$	Weather-dependent generating units (includes wind, solar, and run-of-the-river hydro units).
$\mathcal{G}_i^{\mathrm{F}}$	Dispatchable generating units located at bus i.
$\mathcal{G}_i^{\mathrm{N}}$	Nuclear generating units located at bus i.
$\mathcal{G}_i^{\mathrm{W}}$	Weather-dependent generating units located at bus i.
$\mathcal{G}_r^{\mathrm{F}}$	Dispatchable generating units in reserve area r.
$\mathcal{M}^{(\nu)}(t)$	Nodes at both ends of the uncongested lines in all iterations up to iteration ν in normal operating condition at time period t.
$\mathcal{M}_c^{(\nu)}(t)$	Nodes at both ends of the uncongested lines in all iterations up to iteration ν in operation under contingency c at time period t.
$\mathcal{N}$	Buses in the electric network.
$\mathcal{N}^{(\nu)}(t)$	Nodes at both ends of the congested lines in all iterations up to iteration ν in normal operating condition at time period t.
$\mathcal{N}_c^{(\nu)}(t)$	Nodes at both ends of the congested lines in all iterations up to iteration ν in operation under contingency c at time period t.
$\mathcal{R}$	Reserve areas.
$\mathcal{T}$	Time periods, $\mathcal{T} = \{1, \ldots, T\}$.

Constants

$\boldsymbol{B}$	Nodal susceptance matrix.
$\boldsymbol{B}^c$	Nodal susceptance matrix under contingency c.
$B_{i,j}$	Element (i,j) of matrix $\boldsymbol{B}$.
c_g^{NL}	No-load cost of unit g.
c_g^1	Linear cost coefficient of unit g.
$c_g^{\mathrm{U}}/c_g^{\mathrm{D}}$	Startup/shutdown cost coefficients of unit g.
c_i^{LOL}	Value of lost load at bus i.
$D_i(t)$	Active power load of demand i in period t.
$p_g(0)$	Active power output of unit g at the beginning of the planning horizon.
$p_g^{\mathrm{N}}(t)$	Active power output of nuclear unit g in period t.
$\overline{P}_g^{\mathrm{W}}$	Maximum active power output of weather-dependent unit g.
$\overline{P}_g/\underline{P}_g$	Maximum/minimum active power output of unit g.
$R_r(t)$	Reserve required in reserve area r in period t.
$R_g^{\mathrm{D}}/R_g^{\mathrm{U}}$	Maximum ramp-down/up rate of unit g.
$\overline{S}_{i,j}$	Apparent power capacity of line (i,j).
$S_g^{\mathrm{U}}/S_g^{\mathrm{D}}$	Maximum startup/shutdown ramp rate of unit g.
$v_g(0)$	Initial commitment status of unit g (1 if online, and 0 otherwise).
κ	Threshold to determine if a transmission line is congested in the active set strategy, $\kappa \in [0,1]$.
π_c	Weight of operation costs under contingency c.

Variables

Normal operation

$D_i^{\mathrm{sh}}(t)$	Active power shed at bus i in period t.
$G_i^{\mathrm{cur}}(t)$	Generation curtailment at bus i in period t.
$p_g(t)$	Power output of unit g in period t.
$\overline{p}_g(t)$	Maximum available power in period t from unit g.
$p_g^{\mathrm{W,spill}}(t)$	Power output spilled by weather-dependent unit g in period t.
$P_i(t)$	Net power injection at bus i in period t.
$v_g(t)$	On/off status in period t of unit g: $v_g(t)$ equals 1 if unit g is on in period t, and 0 otherwise.
$y_g(t)$	Startup indicator in period t of unit g: $y_g(t)$ equals 1 if unit g starts up at the beginning of period t, and 0 otherwise.
$z_g(t)$	Shutdown indicator in period t of unit g: $z_g(t)$ equals 1 if unit g shuts down at the beginning of period t, and 0 otherwise.
$\theta_i(t)$	Voltage angle of bus i in period t.

Under-contingency operation

$D_{i,c}^{\mathrm{sh}}(t)$	Active power shed at bus i in period t under contingency c.
$G_{i,c}^{\mathrm{cur}}(t)$	Generation curtailment at bus i in period t under contingency c.
$p_{g,c}(t)$	Power output of unit g in period t under contingency c.
$p_{g,c}^{\mathrm{W,spill}}(t)$	Power output spilled by weather-dependent unit g in period t under contingency c.
$P_{i,c}(t)$	Net power injection at bus i in period t under contingency c.
$\psi_c(t)$	Approximation of the operation costs under contingency c.
$\theta_{i,c}(t)$	Voltage angle of bus i in period t under contingency c.

There exist two main formulations of the SCUC problem:

1. Preventive: The power output of the generating units does not need to change to cope with the contingencies. This formulation is conservative and generally results in higher operating costs.

2. Corrective: If a contingency occurs, the power outputs of the generating units are allowed to deviated from their scheduled values to drive the system back to a secure operating condition. This formulation is less conservative than the preventive one in terms of operating costs, but it requires more variables and constraints, which may result in computational difficulties.

We address the corrective SCUC problem. The main challenge of enforcing security pertaining to a predetermined set of contingencies into the generation scheduling problem is that for realistic power systems with a few thousand of nodes and of transmission lines, the problem becomes a large-scale one with millions of variables and constraints. The extensive form of such problem is computationally challenging or intractable even for current state-of-the-art optimization solvers.

Several solution algorithms have been proposed in the literature to address the SCUC problem (Yang et al., 2022). In the context of mathematical optimization, solution methods based on dynamic programming (Snyder et al., 1987), branch-and-bound (Shafie-Khah et al., 2011), Lagrangian relaxation (Ongsakul and Petcharaks, 2004), Benders decomposition (Liu et al., 2010; Fu et al., 2013; Wen et al., 2016; Nick et al., 2016), outer approximation (Yang et al., 2017), ordinal optimization (Wu and Shahidehpour, 2014), column-and-constraint generation (An and Zeng, 2015), and convex relaxations Quarm and Madani (2021) have been proposed. Yang et al. (2022) present a detailed review on the models and solution techniques of the SCUC problem.

One appealing decomposition technique to solve the corrective SCUC problem is Benders decomposition, which decomposes the problem into a MILP master problem that considers all the information under normal operation and several LP subproblems that check the feasibility of the solution of the master problem for all the considered contingencies. Even though Benders decomposition is one of the well-documented techniques to address this problem, it generally shows slow or very slow convergence when addressing realistic power systems mainly because the information conveyed from the subproblems to the master problem via Benders cuts is insufficient to closely approximate the contingency cost function. There are several convergence drawbacks of Benders decomposition, namely, (i) the master problem is a weak relaxation of the original problem, (ii) poor initial iterations due to the lack of second-stage information, (iii) poor bounds convergence, and (iv) computationally and increasingly expensive master problems. Such drawbacks and strategies to address them are discussed in detail in (Rahmaniani et al., 2018; Gabriel Crainic et al., 2021).

Despite of the well-documented application of Benders decomposition to solve different variants of the UC problem, little work has been reported on addressing its main drawbacks. Wu and Shahidehpour (2010) propose a method that produces stronger cuts to accelerate the convergence of Benders decomposition for the network-constrained unit commitment problem. Xiong and Jirutitijaroen (2011) propose an aggregation scheme of the multi-cut variant to speed up the computing time of the UC problem under uncertainty. Wang et al. (2013) embed variables and constraints from the scenarios into the master problem to improve the convergence of the solution of a stochastic UC problem with sub-hourly dispatch; however, the authors do not specify the criteria to add such constraints and state that the criteria are problem-dependent.

Motivated by the success of the column-and-constraint generation algorithm to address two-stage robust optimization problems, we have proposed (Constante-Flores et al., 2023) to dynamically incorporate variables and constraints from the scenarios to improve the convergence of Benders decomposition to address the week-ahead risk-constrained UC problem under uncertain generation. A recent work (Tönissen et al., 2021) use the column-and-constraint generation algorithm to address a two-stage stochastic program by (i) identifying a subset of important scenarios that are included in the master problem, and (ii) adding scenarios into the master problem iteratively and replacing the corresponding optimality cuts. Gabriel Crainic et al. (2021) also acknowledge the importance of adding primal information from the scenarios into the master problem to improve convergence of Benders decomposition. The authors identify a subset of scenarios, which remains unchanged over the evolution of the algorithm, whereas the remaining scenarios are approximated in the master problem.

In this paper, we propose a hybrid decomposition algorithm to address the SCUC problem that overcomes the aforementioned drawbacks of Benders decomposition. We decompose the problem using a Benders framework but also dynamically enrich the master problem with primal information from a selected subset of contingencies. Since this strategy increases the computational burden of the master problem, we propose to use an active set strategy combined with a network reduction technique to decrease the number of variables and constraints modeling the power flow equations and the transmission capacity limits. Additionally, we propose to suboptimally solve the master problem by imposing a relative optimality gap that is a function of the gap of the Benders decomposition and the iteration counter. This strategy allows us to reduce the solution times if the master problem does not have enough information pertaining to the considered contingencies.

Considering the above, the main contributions of this paper are summarized as follows:

1. We propose a hybrid decomposition method based on a Benders framework to address the security-constrained daily scheduling of generating units. The improvements of the proposed method with respect to other Benders-like decomposition methods rely on (i) enhancing the communication between the master problem and the subproblems by transferring primal information from a small subset of contingencies to the master problem, and (ii) reducing the size and computational complexity of the master problem by using an active set strategy on the power flow limit constraints.

2. We propose a reduced network model based on a Kron reduction technique to describe the power flow equations that can be used along with an active set strategy. The proposed reduced model (network) is able to represent the original model (network) using a significantly smaller number of variables and constraints that depends on the number of congested transmission lines, which in practice is small compared to the total number of transmission lines.

The rest of this paper is organized as follows. In Section 2, we describe the corrective security-constrained unit commitment problem, and we present the problem assumptions and its formulation. In Section 3, we describe and detail the network reduction process and the proposed hybrid decomposition technique. Next, in Section 4, we demonstrate the effectiveness of the proposed algorithm using an illustrative example and an instance of the Central Illinois 200-bus test system. Lastly, we conclude the paper with some remarks in Section 5.

2. Problem formulation

2.1. Problem statement

This paper focus on the generation scheduling problem with network contingencies also known as security-constrained unit commitment. We consider two sets of actions: (i) preventive security actions, which correspond to the generation scheduling (i.e., on/off hourly status of generating units) and their scheduled dispatch, and (ii) corrective security actions, which correspond to the rapid adjustments (for instance, within 10 minutes) in the scheduled dispatch to return the power grid to a secure operating condition once a contingency has materialized. The objective of the preventive security actions, which are ex-ante decisions, is to provide a generation scheduling able to supply the demand and with enough flexibility for executing the corrective actions (which are ex-post or recourse actions), if needed, to cope with the contingencies.

This problem can be formulated as a two-stage mixed-integer optimization problem. In the first stage, which corresponds to the normal operation without contingencies, we determine the generation scheduling and their scheduled dispatch whereas in the second stage, which corresponds to the under-contingency operation, we determine the dispatch adjustment of the generating units to bring the system back to a secure operation while shedding the minimum possible amount of demand. The adjusted dispatch have to satisfy a τ-minute (typically 10) ramping limits of the generating units and must not violate the operating range (minimum power output and capacity) of each generating unit. The first stage of this problem, includes binary and continuous variables modeling the commitment of units, and the power flows and generation dispatch under normal operation, respectively. Conversely, the second stage, includes only continuous variables to model the power flows and generation dispatch under each one of the contingencies.

2.2. Assumptions

The proposed problem formulation embodies the following assumptions.

1. We model the commitment of fossil-fuel-fired units using three binary variables.
2. The production cost of any nuclear unit is constant throughout the planning horizon; therefore, we neglect the cost of nuclear units in the objective function.
3. We assume that the no-load, start-up, and shutdown costs are per hour constant values incurred if the unit is operating, starts up or shuts down, respectively.
4. We assume that the system has to handle the contingencies in less than 10 minutes. Hence, we use the 10-minute maximum ramp rate from normal operation to under-contingency operation.
5. We assume that generating units are incapable of starting up/shutting down in less than 10 minutes. Therefore, the commitment (binary) decisions are only considered in the normal operating condition.

2.3. Model Formulation

The daily security-constrained unit commitment (SCUC) problem is formulated as follows:

$$
\begin{aligned}
\min_{\Xi} \quad & \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{F}}}\left(c_g^{\mathrm{NL}}v_g(t)+c_g^{\mathrm{U}}y_g(t)+c_g^{\mathrm{D}}z_g(t)\right)+\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{F}}}c_g^{1}p_g(t)+\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{W}}}c_g^{1}\left(p_g^{\mathrm{W}}(t)-p_g^{\mathrm{W,spill}}(t)\right)+ \\
& \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}c_i^{\mathrm{LOL}}\left(D_i^{\mathrm{sh}}(t)+G_i^{\mathrm{cur}}(t)\right)+\sum_{c\in\mathcal{C}}\pi_c\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}c_i^{\mathrm{LOL}}\left(D_{i,c}^{\mathrm{sh}}(t)+G_{i,c}^{\mathrm{cur}}(t)\right) && \text{(1a)} \\
\text{s.t.} \quad & v_g(t-1)-v_g(t)+y_g(t)-z_g(t)=0, \quad \forall g\in\mathcal{G}^{\mathrm{F}},\ t\in\mathcal{T}, && \text{(1b)} \\
& p_g(t)-p_g(t-1)\le R_g^{\mathrm{U}}v_g(t-1)+S_g^{\mathrm{U}}y_g(t), \quad \forall g\in\mathcal{G}^{\mathrm{F}},\forall t\in\mathcal{T}, && \text{(1c)} \\
& p_g(t-1)-p_g(t)\le R_g^{\mathrm{D}}\,v_g(t)+S_g^{\mathrm{D}}\,z_g(t), \quad \forall g\in\mathcal{G}^{\mathrm{F}},\forall t\in\mathcal{T} && \text{(1d)} \\
& \underline{P}_g v_g(t)\le p_g(t)\le \overline{p}_g(t)\le \overline{P}_g v_g(t), \quad \forall g\in\mathcal{G}^{\mathrm{F}},\ \forall t\in\mathcal{T}, && \text{(1e)} \\
& \overline{p}_g(t)\le p_g(t-1)+R_g^{\mathrm{U}}v_g(t-1)+S_g^{\mathrm{U}}y_g(t), \quad \forall g\in\mathcal{G}^{\mathrm{F}},\ t\in\mathcal{T}, && \text{(1f)} \\
& \overline{p}_g(t)\le \overline{P}_g\big[v_g(t)-z_g(t+1)\big]+z_g(t+1)S_g^{\mathrm{D}}, \quad \forall g\in\mathcal{G}^{\mathrm{F}},\ \forall t\in\mathcal{T}, && \text{(1g)} \\
& \sum_{g\in\mathcal{G}_r^{\mathrm{F}}}\left(\overline{p}_g(t)-p_g(t)\right)\ge R_r(t), \quad \forall r\in\mathcal{R},\ \forall t\in\mathcal{T}, && \text{(1h)} \\
& 0\le p_g^{\mathrm{W,spill}}(t)\le p_g^{\mathrm{W}}(t), \quad \forall g\in\mathcal{G}^{\mathrm{W}},\ \forall t\in\mathcal{T}, && \text{(1i)} \\
& \sum_{g\in\mathcal{G}_i^{\mathrm{F}}}p_g(t)+\sum_{g\in\mathcal{G}_i^{\mathrm{W}}}\left(p_g^{\mathrm{W}}(t)-p_g^{\mathrm{W,spill}}(t)\right)+\sum_{g\in\mathcal{G}_i^{\mathrm{N}}}p_g^{\mathrm{N}}(t)-D_i(t)+D_i^{\mathrm{sh}}(t)-G_i^{\mathrm{cur}}(t)= \\
& \quad \sum_{j\in\mathcal{N}_i}B_{i,j}\left(\theta_i(t)-\theta_j(t)\right), \quad \forall i\in\mathcal{N},\ \forall t\in\mathcal{T}, && \text{(1j)} \\
& -\overline{S}_{i,j}\le B_{i,j}\left(\theta_i(t)-\theta_j(t)\right)\le \overline{S}_{i,j}, \quad \forall (i,j)\in\mathcal{E},\ \forall t\in\mathcal{T}, && \text{(1k)} \\
& \theta_{\hat{i}}(t)=0, \quad \forall t\in\mathcal{T}, && \text{(1l)} \\
& p_{g,c}(t)-p_g(t)\le R_g^{10,\mathrm{U}}v_g(t), \quad \forall g\in\mathcal{G}^{\mathrm{F}}, \quad \forall c\in\mathcal{C},\ \forall t\in\mathcal{T}, && \text{(1m)} \\
& p_g(t)-p_{g,c}(t)\le R_g^{10,\mathrm{D}}\,v_g(t), \quad \forall g\in\mathcal{G}^{\mathrm{F}}, \quad \forall c\in\mathcal{C},\ \forall t\in\mathcal{T}, && \text{(1n)} \\
& \underline{P}_g v_g(t)\le p_{g,c}(t)\le \overline{P}_g v_g(t), \quad \forall g\in\mathcal{G}^{\mathrm{F}}, \quad \forall c\in\mathcal{C},\ \forall t\in\mathcal{T}, && \text{(1o)} \\
& 0\le p_{g,c}^{\mathrm{W,spill}}(t)\le p_g^{\mathrm{W}}(t), \quad \forall g\in\mathcal{G}^{\mathrm{W}},\ \forall c\in\mathcal{C}, \quad \forall t\in\mathcal{T}, && \text{(1p)} \\
& \sum_{g\in\mathcal{G}_i^{\mathrm{F}}}p_{g,c}(t)+\sum_{g\in\mathcal{G}_i^{\mathrm{W}}}\left(p_g^{\mathrm{W}}(t)-p_{g,c}^{\mathrm{W,spill}}(t)\right)+\sum_{g\in\mathcal{G}_i^{\mathrm{N}}}p_g^{\mathrm{N}}(t)-D_i(t)-G_{i,c}^{\mathrm{cur}}(t)+D_{i,c}^{\mathrm{sh}}(t)= \\
& \quad \sum_{j\in\mathcal{N}_i}B_{i,j}^{c}\left(\theta_{i,c}(t)-\theta_{j,c}(t)\right), \quad \forall i\in\mathcal{N},\ \forall c\in\mathcal{C},\ \forall t\in\mathcal{T}, && \text{(1q)} \\
& -\overline{S}_{i,j}\le B_{i,j}^{c}\left(\theta_{i,c}(t)-\theta_{j,c}(t)\right)\le \overline{S}_{i,j}, \quad \forall (i,j)\in\mathcal{E},\ \forall c\in\mathcal{C},\ \forall t\in\mathcal{T}, && \text{(1r)} \\
& \theta_{\hat{i},c}(t)=0, \quad \forall c\in\mathcal{C},\ \forall t\in\mathcal{T}, && \text{(1s)} \\
& v_g(t),\ y_g(t),\ z_g(t)\in\{0,1\}, \quad \forall g\in\mathcal{G}^{\mathrm{F}}, \quad \forall t\in\mathcal{T}. && \text{(1t)}
\end{aligned}
$$

where the optimization variables are the elements of the set

$$
\Xi=\left\{v_g(t),y_g(t),z_g(t),p_g(t),\overline{p}_g(t)\ \forall g\in\mathcal{G}^{\mathrm{F}}\right\}\cup\left\{P_i(t)\ \forall i\in\mathcal{N}\right\}\cup\left\{\theta_i(t)\ \forall i\in\mathcal{N}\right\}\cup\left\{p_g^{\mathrm{W,spill}}(t)\ \forall g\in\mathcal{G}^{\mathrm{W}}\right\}\cup
$$

$\left\{p_{g,c}(t)\ \forall g \in \mathcal{G}^{\mathrm{F}},\ \forall c \in \mathcal{C}\right\} \cup \left\{P_{i,c}(t)\ \forall i \in \mathcal{N},\ \forall c \in \mathcal{C}\right\} \cup \left\{\theta_{i,c}(t), D^{\mathrm{sh}}_{i,c}, G^{\mathrm{cur}}_{i,c}\ \forall i \in \mathcal{N},\ \forall c \in \mathcal{C}\right\} \cup \left\{p^{\mathrm{W,spill}}_{g,c}(t)\ \forall g \in \mathcal{G}^{\mathrm{W}},\ \forall c \in \mathcal{C}\right\}.$

The objective function (1a) includes the normal and under-contingency operation costs.

The normal operation costs comprise six components: no-load costs, startup costs, shutdown costs, production costs of fossil-fuel-fired units, production costs of weather-dependent units, and penalty costs of undercommitting and overcommitting generation, respectively.

The under-contingency operating costs of the system, $\sum_{c\in\mathcal{C}} \pi_c \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} c^{\mathrm{LOL}}_i \left(D^{\mathrm{sh}}_{i,c}(t) + G^{\mathrm{cur}}_{i,c}(t)\right)$, is due to undercommitted and overcommitted generation, respectively, of contingency c. If not enough generation is committed in time period t for the operation under contingency c, the sum of penalty terms representing the unserved demand, $D^{\mathrm{sh}}_{i,c}(t)$, is greater than 0. If the minimum committed capacity at time period t for the operation under contingency c is above the system demand, the sum of penalty terms representing the curtailed generation, $G^{\mathrm{cur}}_{i,c}(t)$, is greater than 0. The weight π_c of the operating costs under contingency c is generally equal to $\frac{1}{|\mathcal{C}|}$ and the sum of such weights has to be equal to 1. These weights can also be adjusted to consider a higher weight for a particular contingency due to, for instance, its probability of occurrence.

Constraints (1b)$-$(1l) pertain to the normal operating condition whereas constraints (1m)$-$(1t) pertain to the under-contingency operating conditions.

Constraints (1b) enforce the commitment logical coherence of fossil-fuel-fired units throughout the entire planning horizon. The commitment decisions have to satisfy the minimum up/down time limits. Constraints (1c)-(1d) enforce the ramping up/down limits of the production level of dispatchable generating units. Constraints (1e) bound the actual production of each unit. The maximum available power $\overline{p}_g(t)$ is used to meet the reserve requirements per area. Constraints (1f)-(1g) impose ramping limits on $\overline{p}_g(t)$ when a unit is online or has been started up at the beginning of period t, respectively. Constraints (1h) enforces the reserve requirements.

Weather-dependent generation is assumed to be dispatched from zero to its maximum available power. The difference between the actual dispatch and its maximum available power, known as power spillage, has to be bounded by the maximum available power. The bounds of power spillage for weather-dependent units are enforced by constraints (1i).

Constraints (1k) represent the phase-angle formulation of the per node power balance between the generation (from fuel-fired, weather-dependent, and nuclear) and demand. The active power flow through each transmission line is limited by constraints (1k). The voltage angle of the reference node is fixed by constraints (1l).

The normal operation power output is linked to the under-contingency power output of the units by constraints (1m)$-$(1n). Such constraints ensure that the system is able to handle the contingencies within τ minutes (typically 10). Constraints (1m) enforce the τ-minute maximum ramp rate limits of coal-fired and combined cycle gas turbines whereas constraints (1n) enforce such limits for gas turbines.

Constraints (1o) enforce the capacity and minimum power output of fossil-fuel-fired units during the under-contingency operating condition. The bounds of power spillage for each contingency are enforced by constraints (1p).

Constraints (1q) represent the phase-angle formulation of the per-node power balance between the generation and demand under each contingency. The active power flow through each transmission line is limited by constraints (1r) for each contingency. The voltage angle of the reference node is fixed by constraints (1s) for each contingency.

Constraints (1t) enforce the commitment variables to be binary.

3. Solution Method

The proposed solution methodology is described in detail in this section.

3.1. Motivation

We propose in the following a hybrid solution method to solve the SCUC problem (1) that exploits its decomposable structure. The SCUC problem (1) can be compactly written as follows:

$$\min_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\theta},\boldsymbol{y},\boldsymbol{\theta}_c\ \forall c \in \mathcal{C}} \quad \boldsymbol{c}_{\text{sch}}^{\top}\boldsymbol{x} + \boldsymbol{c}_{\text{dis}}^{\top}\boldsymbol{y} + \sum_{c \in \mathcal{C}} \boldsymbol{d}_c^{\top}\boldsymbol{y}_c \tag{2a}$$

$$\text{s.t.} \quad \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} \leq \boldsymbol{0}, \tag{2b}$$

$$\boldsymbol{C}\boldsymbol{y} + \boldsymbol{D}\boldsymbol{\theta} \leq \boldsymbol{0}, \tag{2c}$$

$$\boldsymbol{E}\boldsymbol{x} + \boldsymbol{F}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{y}_c \leq \boldsymbol{0}, \quad \forall c \in \mathcal{C}, \tag{2d}$$

$$\boldsymbol{C}\boldsymbol{y}_c + \boldsymbol{D}_c\boldsymbol{\theta}_c \leq \boldsymbol{0}, \quad \forall c \in \mathcal{C}, \tag{2e}$$

$$\boldsymbol{x} \in \mathbb{B}^m, \boldsymbol{y} \in \mathbb{R}^n, \boldsymbol{\theta} \in \mathbb{R}^p, \tag{2f}$$

$$\boldsymbol{y}_c \in \mathbb{R}^q, \boldsymbol{\theta}_c \in \mathbb{R}^r, \quad \forall c \in \mathcal{C}, \tag{2g}$$

where $\boldsymbol{x}$ includes the set of commitment variables, $\boldsymbol{y}$ includes the set of dispatch variables, $\boldsymbol{\theta}$ includes the network-related variables, $\boldsymbol{y}_c$ includes the set of dispatch variables for contingency c, and $\boldsymbol{\theta}_c$ includes the network-related variables for contingency c.

The objective function (2a) is comprised by the following terms:

- $\boldsymbol{c}_{\text{sch}}^{\top}\boldsymbol{x}$, the scheduling costs,
- $\boldsymbol{c}_{\text{dis}}^{\top}\boldsymbol{y}$, the normal operation dispatch costs,
- $\boldsymbol{d}_c^{\top}\boldsymbol{y}_c$, the under-contingency costs.

Constraints (2b) represent the generator-related constraints (1b)-(1i). Constraints (2c) represent the network-related constraints (1j)-(1l) in the normal operation. Constraints (2d) represent the normal and under-contingency operation coupling constraints (1m)-(1n), and the generator-related constraints (1o)-(1p). Constraints (2e) represent the network-related constraints (1q)-(1t) in under-contingency operation.

The compact form of the SCUC problem (2) allows us to identify that the commitment and dispatch variable vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, are complicating variable vectors. Fixing both variable vectors decomposes the problem into: (i) a linear programming problem that check the feasibility of the commitment and dispatch with respect to the network constraints in the normal operating condition, and (ii) as many linear subproblems as the number of contingencies that check the feasibility of the commitment and dispatch with respect to the network constraints for each one of the contingencies. This structure makes SCUC problem (2) suitable to be solved using a Benders like decomposition technique. The key challenges of using such decomposition framework are (i) the computational complexity of the master problem, and (ii) the lack of information in the master problem regarding the network feasibility of the commitment and dispatch decisions in the normal and under-contingency operating conditions.

To address the first challenge, the computational burden of the master problem, we exploit the fact that most of the inequality constraints representing the line capacity limits (2c) and (2e) are not binding at the optimal solution. Therefore, we can use an active set strategy to progressively enforce such constraints throughout the evolution of the decomposition algorithm.

In our model, we use the phase-angle formulation of the power flow equations (1j)-(1k) and (1q)-(1r). Although such formulation has a number of beneficial computational properties due to its sparsity, it is disadvantageous because it requires to model the phase angle of all the buses for all the time periods even. Therefore, such formulation is not suitable if the problem is solved using an active set strategy on the capacity limits of transmission lines. We address this drawback by using a Kron reduction technique Dörfler and Bullo (2013) that allows us to exactly model the power flow equations with a small number of variables and constraints, which are a function of the number of congested transmission lines. This network-reduction technique is described in Section 3.3.

Additionally, we tackle the computational complexity of the master problem by leveraging computational tools such as: warm starting the master problem with previous good quality solutions, and solving the master problem to suboptimality but using its bounds to maintain optimality guarantees. These strategies are detailed in Section 3.4.

To address the second challenge, the lack of information in the master problem regarding the network feasibility of the commitment and dispatch decisions in the operation under contingencies, and following the main idea behind the column-and-constraint generation algorithm, we dynamically add the variables and constraints of a small subset of contingencies to the master problem throughout the evolution of the algorithm if the Benders cuts are ineffective to keep closing the gap between the upper and lower bounds. This approach is explained in Section 3.4.

3.2. Description

The hybrid decomposition technique, illustrated in Figure 1, to address the security-constrained unit-commitment problem (1), which blends ideas from Benders decomposition and the column-and-constraint generation algorithm is presented below. First, we decompose the SCUC problem using a Benders framework into:

1. A reduced-network master problem, which is an MILP problem that includes the commitment variables, the constraints describing the normal operation, and the constraints describing the under-contingency operation of a small set of contingencies. For the normal and under-contingency opera-

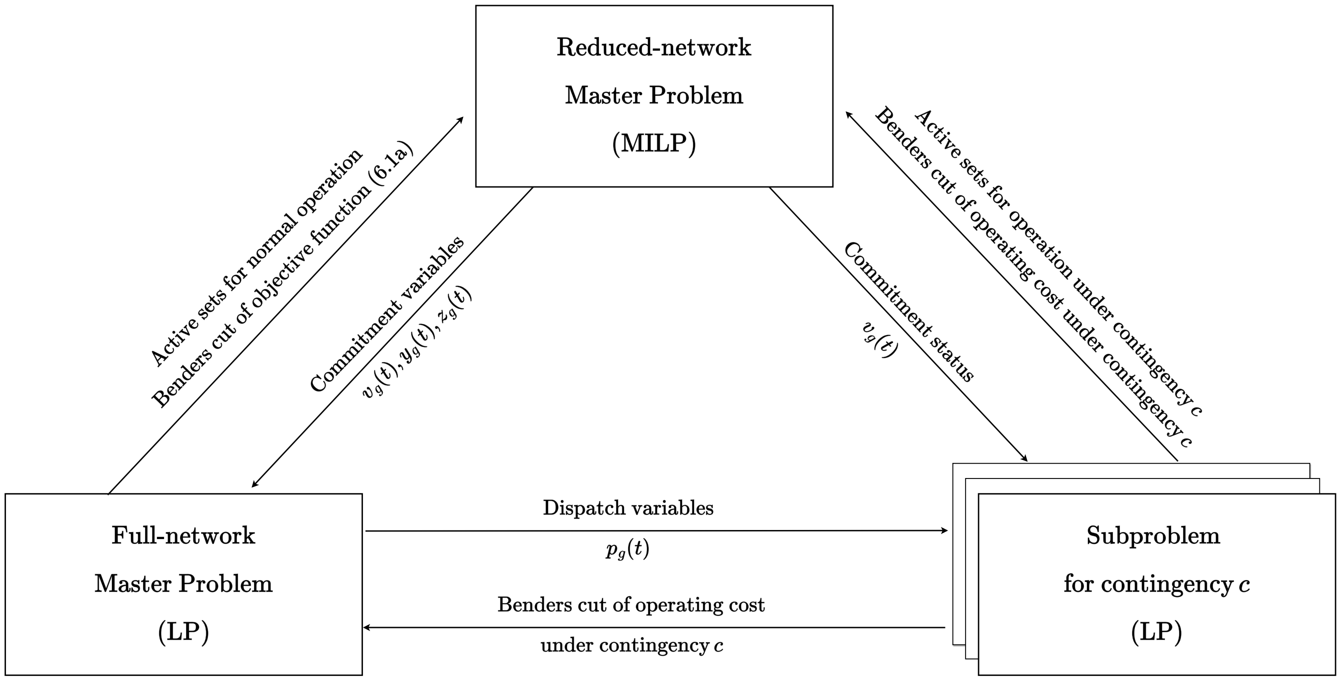


Figure 1: Illustration of the hybrid decomposition technique.

tions, we use reduced network models based on a Kron reduction technique. This problem provides the following:

(a) A lower bound of the optimal value of the objective function (1a).

(b) The commitment variables to be used in the full-network master problem and the subproblems

2. A full-network master problem, which is a linear programming problem that checks the network feasibility of the commitment variables provided by the reduced-network master problem for the normal operation and operation under the contingencies that are considered in the reduced-network master problem. This full-network master problem considers a full network representation of the power flows defined by equations (1j)-(1k) and (1q)-(1r). The solution of this problem provides the following:

   (a) A valid upper bound of the costs under normal operation, i.e.,

$$\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{F}}}\left(c_g^{\mathrm{NL}}v_g(t)+c_g^{\mathrm{U}}y_g(t)+c_g^{\mathrm{D}}z_g(t)\right)+\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{F}}}c_g^1p_g(t)+\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{W}}}c_g^1\left(p_g^{\mathrm{W}}(t)-p_g^{\mathrm{W,spill}}(t)\right)+$$
$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}c_i^{\mathrm{LOL}}\left(D_i^{\mathrm{sh}}(t)+G_i^{\mathrm{cur}}(t)\right).$$

   (b) The operation variables of the fossil-fuel-fired generating units, i.e., $p_g(t)\ \forall g\in\mathcal{G}^{\mathrm{F}}$ to be used in the subproblems.

   (c) A Benders cut, which is sent to the reduced master problem, to approximate the objective function (1a).

(d) The set of congested lines in the normal operating condition to be enforced in the reduced-network master problem in all subsequent iterations.

3. As many subproblems as the number of contingencies, which are linear programming problems that check the network feasibility of the commitment and dispatch variables provided by the reduced- and full-network master problems, respectively, for each one of the contingencies. These subproblems provide the following:

   (a) A valid upper bound of the operation cost under each contingency, i.e.,

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} c_i^{\mathrm{LOL}} \left(D_{i,c}^{\mathrm{sh}}(t) + G_{i,c}^{\mathrm{cur}}(t) \right),$$

   (b) Benders cuts, which are sent to the reduced- and full-network master problems, to approximate the operation cost under each contingency.

   (c) For the operation under each contingency, the set of congested lines whose power flow limits will be enforced, in all subsequent iterations, for each contingency considered in the reduced-network master problem.

The proposed method can be considered a hybrid technique between Benders decomposition and the column-and-constraint generation algorithm because the information between the master problems and the subproblems is transferred not only via Benders cuts but also via primal information (variables and constraints) from the normal and under-contingency operation.

The proposed solution method is as follows:

1. First, we solve the SCUC problem (1) with no contingencies (i.e., a UC problem) and identify the set of congested lines and their corresponding end nodes (under normal operation). Such nodes are used to build the initial reduced network model to be considered in the network-reduced master problem.

2. Then, we solve the reduced-network master problem. The costs of the contingencies that are not included in this master problem are approximated using Benders cuts. To improve computational efficiency, this reduced-network master problem is solved to suboptimality. We note that the lower bound of the branch-and-bound process to solve this problem is a valid lower bound of the cost of the SCUC problem (1). Additionally, this reduced-network master problem provides trial values for the commitment variables.

3. Next, we solve the full-network master problem by fixing the commitment (binary) variables and enforcing the full network power flow equations for the normal operation and operation under each contingency of the current subset of contingencies. This problem provides a Benders cut to approximate the costs under normal operation to be transferred to the reduced-network master problem, the set of congested lines and their associated nodes for the normal operation (to be used in all subsequent iterations in the reduced-network master problem) and trial values for the dispatch variables.

4. Next, we solve each one of the subproblems by fixing the commitment and dispatch variables corresponding to the normal operation. These subproblems check the network feasibility of the commitment and dispatch solutions for the operation under each contingency. Each subproblem provides a Benders cut to be incorporated into the reduced- and full-network master problems, and the set of congested lines and their associated nodes for operation under the corresponding contingency, to be used in the reduced-network master problem.

5. At every iteration and for every operating condition (normal and under contingency), we identify the set of binding transmission capacity constraints to be enforced in the next iteration of the reduced-network master problem. The set of active constraints at every iteration includes all the constraints that have been binding in previous iterations for the corresponding condition. To reduce the computational burden of the proposed reduced-network master problem, we use (i) an active set strategy on constraints representing the transmission capacity limits and (ii) a reduced network model based on Kron reduction, which significantly reduces the size of the problem.

6. An upper bound of the objective function (1a) is determined from the solution of all subproblems and the full-network master problem. The upper bound of the objective function (1a) is determined as follows:

   (a) The no-load, startup, and shutdown costs are obtained from the solution of the reduced-network master problem,

   (b) The dispatch costs (production costs and penalty costs) under normal operation are obtained from the solution of the full-network master problem,

   (c) The operation costs (penalty costs) for the operation under contingencies are obtained from the solution of the subproblems.

7. If the gap between the upper and lower bounds of (1a) is not closing for two consecutive iterations, an additional contingency is added to the reduced- and full-network master problems to provide a better approximation of the under-contingency cost functions. The additional contingency to be added into both master problems is selected based on the mismatch between the operation cost under each contingency computed at the solution of the full-network master problem and the subproblems.

8. If a convergence criterion is satisfied, the algorithm stops. Otherwise, it computes the Kron reduction matrices (for the updated set of congested lines) needed for the reduced power flow formulation and returns to Step 2.

3.3. Kron Reduction

In compact form, the power flow equations and transmission capacity limits can be expressed as follows:

$$\boldsymbol{P}^{\text{net}} = \boldsymbol{B}\boldsymbol{\theta}, \tag{3}$$

$$-\overline{\boldsymbol{S}} \leq \boldsymbol{B}_{\text{br}}\boldsymbol{\theta} \leq \overline{\boldsymbol{S}}, \tag{4}$$

where $\boldsymbol{P}^{\text{net}}$ denotes the net power injection vector, $\boldsymbol{B}$ is the nodal susceptance matrix, $\boldsymbol{\theta}$ is the voltage angle vector, $\overline{\boldsymbol{S}}$ is the transmission capacity vector, and $\boldsymbol{B}_{\text{br}}$ is the branch susceptance matrix. Equation (3) is the nodal power balance constraint and equation (4) models the transmission capacity limits. The role of equation (3) is to compute the voltage angles needed to determine the power flows through the transmission lines, which have to be within the limits enforced by (4).

In practice, most of the network constraints (4) are not binding at the optimal solution, which calls for an active set strategy. However, considering (3) and (4), we need to compute all the voltage angles even if we enforce constraints (4) only for a small number of transmission lines. This computation can be avoided using a reduced equivalent model whose nodes corresponds to the ones of the congested transmission lines. Such reduced model requires a significantly smaller number of variables and constraints to represent the power flow equations.

Network reduction techniques have been studied for decades to produce reduce order but electrically equivalent models for the network's power flows. In particular, G. Kron proposed a reduction method (Kron, 1939) based on Schur complement that has been used in a wide variety of problems in power systems, including power flow sensitivity analysis (Ward, 1949), power system monitoring (Dobson, 2012), transient stability assessment (Ishizaki et al., 2018), synchronization of generator dynamics (Dörfler and Bullo, 2010), transmission expansion planning (Ploussard et al., 2018), radial 3-phase optimal power flow (Almassalkhi et al., 2020), among others. A description of the Kron reduction, a detailed analysis of its properties, and a description of applications in power systems are reported in (Dörfler and Bullo, 2013).

In this work, to reduce the computational burden of the network-reduced master problem, we use Kron reduction to find a reduced equivalent network model within the considered active set strategy, which allows us to speed up the solution time of the network-reduced master problem.

We can express the nodal power balance constraints (3) as follows:

$$\left[\begin{array}{c} \boldsymbol{P}_r^{\text{net}} \\ \hline \boldsymbol{P}_n^{\text{net}} \end{array}\right] = \left[\begin{array}{c|c} \boldsymbol{B}_{rr} & \boldsymbol{B}_{rn} \\ \hline \boldsymbol{B}_{nr} & \boldsymbol{B}_{nn} \end{array}\right] \left[\begin{array}{c} \boldsymbol{\theta}_r \\ \hline \boldsymbol{\theta}_n \end{array}\right], \tag{5}$$

where indices r denotes the nodes at both ends of the congested lines and n denotes the remaining nodes. According to the Kron reduction, we use Gaussian elimination of the nodal susceptance matrix $\boldsymbol{B}$ with respect to the nodes indexed by n to eliminate the voltage angles $\boldsymbol{\theta}_n$ of the nodes at both ends of the uncongested lines as follows

$$\boldsymbol{P}_r^{\text{net}} + \boldsymbol{B}^{\text{ac}} \boldsymbol{P}_n^{\text{net}} = \boldsymbol{B}^{\text{red}} \boldsymbol{\theta}_r, \tag{6}$$

where the accompanying matrix $\boldsymbol{B}^{\text{ac}} = -\boldsymbol{B}_{rn}\boldsymbol{B}_{nn}^{-1} \in \mathbb{R}^{|r|\times|n|}$ and the reduced matrix $\boldsymbol{B}^{\text{red}} = \boldsymbol{B}_{rr} + \boldsymbol{B}^{\text{ac}}\boldsymbol{B}_{nr} \in \mathbb{R}^{|r|\times|r|}$.

In the proposed solution method, the congested lines and their corresponding nodes are determined using an active set strategy. Using the power balance equation of the reduced network defined by (6), instead of (3), for the normal operation condition and the operation under each of the contingencies incorporated into the network-reduced master problem allows us to drastically reduce the number of variables and equality constraints, which results in important computational gains.

3.4. Algorithm

In this section, we describe the proposed solution method and the computation of the matrices needed to find the Kron-reduction-based power flow model.

Step 0. Initialization: Initialize the iteration counter $\nu = 1$, the objective function lower bound $B_{\mathrm{L}}^{(0)} \leftarrow -\infty$, the objective function upper bound $B_{\mathrm{U}}^{(0)} \leftarrow +\infty$, the indexes of the sets of congested transmission lines in normal operation $\mathcal{E}^{(1)}(t) \leftarrow \emptyset \; \forall t \in \mathcal{T}$, and under-contingency operation $\mathcal{E}_c^{(1)}(t) \leftarrow \emptyset \; \forall t \in \mathcal{T}, \; \forall c \in \mathcal{C}$, the indexes of the sets of nodes of both ends of the congested transmission lines in normal and under-contingency operation $\mathcal{N}^{(1)}(t) \leftarrow \emptyset \; \forall t \in \mathcal{T}$ and $\mathcal{N}_c^{(1)}(t) \leftarrow \emptyset \; \forall t \in \mathcal{T}, \; \forall c \in \mathcal{C}$, respectively, and the set of contingencies initially incorporated into the master problem $\mathcal{C}^{\mathrm{M},(1)} \leftarrow \emptyset$. Also, initialize the relative optimality gap $\texttt{MIPGap}^{(1)} < 100$.

Step 1. Reduced-network master problem solution: Set the commitment (binary) variables of the coal-fired units and the combined cycle gas turbines, $v_g^{(\nu)}(t)$, $y_g^{(\nu)}(t)$, $z_g^{(\nu)}(t) \; \forall t \in \mathcal{T}, \forall g \in \mathcal{G}^{\mathrm{F}}$, and the variable representing the approximation from below of the objective function (1a), ψ, to the solution of the following network-reduced master problem:

$$
\begin{aligned}
\min_{\Xi^{\mathrm{RM}}} \quad & \psi = \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{F}}} \left(c_g^{\mathrm{NL}} v_g(t) + c_g^{\mathrm{U}} y_g(t) + c_g^{\mathrm{D}} z_g(t) \right) + \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{F}}} c_g^1 p_g(t) + \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{W}}} c_g^1 \left(p_g^{\mathrm{W}}(t) - p_g^{\mathrm{W,spill}}(t) \right) + \\
& \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} c_i^{\mathrm{LOL}} \left(D_i^{\mathrm{sh}}(t) + G_i^{\mathrm{cur}}(t) \right) + \sum_{c\in\mathcal{C}} \pi_c \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} c_i^{\mathrm{LOL}} \psi_c \qquad \text{(7a)} \\
\text{s.t.} \quad & (1\mathrm{b}) - (1\mathrm{j}), (1\mathrm{l}), (1\mathrm{t}) \\
& (1\mathrm{k}), \quad \forall (i,j) \in \mathcal{E}^{(\nu)}(t), \\
& (1\mathrm{m}) - (1\mathrm{q}), (1\mathrm{s}) \quad \forall c \in \mathcal{C}^{\mathrm{M},(\nu)}, \\
& (1\mathrm{r}), \quad \forall c \in \mathcal{C}^{\mathrm{M},(\nu)}, \; \forall (i,j) \in \mathcal{E}_c^{(\nu)}(t), \\
& \psi_c \geq 0, \quad \forall c \in \mathcal{C}, \qquad \text{(7b)} \\
& \psi_c \geq \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} c_i^{\mathrm{LOL}} \left(D_{i,c}^{\mathrm{sh}}(t) + G_{i,c}^{\mathrm{cur}}(t) \right), \quad \forall c \in \mathcal{C}^{\mathrm{M},(\nu)}, \qquad \text{(7c)} \\
& \psi_c \geq \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} c_i^{\mathrm{LOL}} \left(D_{i,c}^{\mathrm{sh},(\mu)}(t) + G_{i,c}^{\mathrm{sur},(\mu)}(t) \right) + \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{F}}} \gamma_{g,c}^{v,(\mu)} \left(v_g - v_g^{(\mu)}(t) \right) + \\
& \quad \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{F}}} \gamma_{g,c}^{\mathrm{p},(\mu)} \left(p_g - p_g^{(\mu)}(t) \right), \quad \forall c \in \mathcal{C}, \; \forall \mu = 1, \ldots, \nu - 1, \qquad \text{(7d)} \\
& \psi \geq \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{F}}} \left(c_g^{\mathrm{NL}} v_g^{(\mu)}(t) + c_g^{\mathrm{U}} y_g^{(\mu)}(t) + c_g^{\mathrm{D}} z_g^{(\mu)}(t) c_g^1 p_g^{(\mu)}(t) \right) + \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{W}}} c_g^1 \left(p_g^{\mathrm{W}}(t) - p_g^{\mathrm{W,spill},(\mu)}(t) \right) + \\
& \quad \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} c_i^{\mathrm{LOL}} \left(D_i^{\mathrm{sh},(\mu)}(t) + G_i^{\mathrm{sur},(\mu)}(t) \right) + \sum_{c\in\mathcal{C}} \pi_c \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}} c_i^{\mathrm{LOL}} \psi_c^{(\mu)} + \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{F}}} \gamma_g^{v,(\mu)} \left(v_g - v_g^{(\mu)}(t) \right) + \\
& \quad \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{F}}} \gamma_g^{y,(\mu)} \left(y_g - y_g^{(\mu)}(t) \right) + \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}^{\mathrm{F}}} \gamma_g^{z,(\mu)} \left(z_g - z_g^{(\mu)}(t) \right), \quad \forall \mu = 1, \ldots, \nu - 1, \qquad \text{(7e)}
\end{aligned}
$$

$$P_i(t) = \sum_{g \in \mathcal{G}_i^{\mathrm{F}}} p_g(t) + \sum_{g \in \mathcal{G}_i^{\mathrm{W}}} \left(p_g^{\mathrm{W}}(t) - p_g^{\mathrm{W,spill}}(t) \right) + \sum_{g \in \mathcal{G}_i^{\mathrm{N}}} p_g^{\mathrm{N}}(t) - D_i(t) + D_i^{\mathrm{sh}}(t) - G_i^{\mathrm{cur}}(t),$$
$$\forall i \in \mathcal{N},\ \forall t \in \mathcal{T}, \tag{7f}$$
$$\sum_{i \in \mathcal{N}} P_i(t) \geq 0, \quad \forall t \in \mathcal{T}, \tag{7g}$$
$$P_{\mathcal{N}_n^{(\nu)}(t)}(t) + \sum_{m=1}^{|\mathcal{M}^{(\nu)}(t)|} B_{n,m}^{\mathrm{ac}}(t) P_{\mathcal{M}_m^{(\nu)}(t)}(t) = \sum_{m=1}^{|\mathcal{N}^{(\nu)}(t)|} B_{n,m}^{\mathrm{red}}(t) \theta_{\mathcal{N}_m^{(\nu)}(t)}(t),$$
$$\forall t \in \mathcal{T},\ \forall n = 1, \ldots, |\mathcal{N}^{(\nu)}(t)|, \tag{7h}$$
$$P_{i,c}(t) = \sum_{g \in \mathcal{G}_i^{\mathrm{F}}} p_{g,c}(t) + \sum_{g \in \mathcal{G}_i^{\mathrm{N}}} p_{g,c}^{\mathrm{N}}(t) + \sum_{g \in \mathcal{G}_i^{\mathrm{W}}} \left(p_{g,c}^{\mathrm{W}}(t) - p_{g,c}^{\mathrm{W,spill}}(t) \right) - D_i(t) + D_{i,c}^{\mathrm{sh}}(t) - G_{i,c}^{\mathrm{cur}}(t),$$
$$\forall i \in \mathcal{N},\ \forall c \in \mathcal{C}^{\mathrm{M},(\nu)},\ \forall t \in \mathcal{T}, \tag{7i}$$
$$\sum_{i \in \mathcal{N}} P_{i,c}(t) \geq 0, \quad \forall c \in \mathcal{C}^{\mathrm{M},(\nu)},\ \forall t \in \mathcal{T}, \tag{7j}$$
$$P_{\mathcal{N}_{c,n}^{(\nu)}(t),c}(t) + \sum_{m=1}^{|\mathcal{M}_c^{(\nu)}(t)|} B_{n,m}^{\mathrm{ac},c}(t) P_{\mathcal{M}_{c,m}^{(\nu)}(t),c}(t) = \sum_{m=1}^{|\mathcal{N}_c^{(\nu)}(t)|} B_{n,m}^{\mathrm{red},c}(t) \theta_{\mathcal{N}_{c,m}^{(\nu)}(t),c}(t),$$
$$\forall c \in \mathcal{C}^{\mathrm{M},(\nu)},\ \forall t \in \mathcal{T},\ \forall n = 1, \ldots, |\mathcal{N}_c^{(\nu)}(t)|, \tag{7k}$$

where the optimization variables $\forall t \in \mathcal{T}$ are elements of the set

$$\begin{aligned}
\Xi^{\mathrm{RM}} = & \left\{ v_g(t), y_g(t), z_g(t), p_g(t), \overline{p}_g(t)\ \forall g \in \mathcal{G}^{\mathrm{F}} \right\} \cup \left\{ P_i(t)\ \forall i \in \mathcal{N} \right\} \cup \left\{ p_g^{\mathrm{W,spill}}(t)\ \forall g \in \mathcal{G}^{\mathrm{W}} \right\} \cup \\
& \left\{ \theta_i(t)\ \forall i \in \mathcal{N}^{(\nu)} \right\} \cup \left\{ p_{g,c}(t)\ \forall g \in \mathcal{G}^{\mathrm{F}},\ \forall c \in \mathcal{C}^{\mathrm{M},(\nu)} \right\} \cup \left\{ P_{i,c}(t)\ \forall i \in \mathcal{N},\ \forall c \in \mathcal{C}^{\mathrm{M},(\nu)} \right\} \cup \\
& \left\{ \theta_{i,c}(t), D_{i,c}^{\mathrm{sh}}, G_{i,c}^{\mathrm{cur}}\ \forall i \in \mathcal{N}_c^{(\nu)},\ \forall c \in \mathcal{C}^{\mathrm{M},(\nu)} \right\} \cup \left\{ p_{g,c}^{\mathrm{W,spill}}(t)\ \forall g \in \mathcal{G}^{\mathrm{W}},\ \forall c \in \mathcal{C}^{\mathrm{M},(\nu)} \right\} \cup \\
& \left\{ \psi_c\ \forall c \in \mathcal{C} \right\} \cup \left\{ \psi \right\}.
\end{aligned}$$

The objective function (7a) includes an estimate from below of the costs (scheduling and dispatch) for the normal operating condition and an estimate from below of the operating cost under each contingency considered in the reduced- and full-network master problem. Constraints (7b) enforce a lower bound of the operation cost under each contingency c, ψ_c. Constraints (7c) lower bound the operation costs under each contingency incorporated into the master problem (i.e., $c \in \mathcal{C}^{\mathrm{M},(\nu)}$). Constraints (7d) corresponds to the Benders cuts of operation under each contingency whereas constraints (7e) correspond to the Benders cuts of the normal operation. Constraints (7f) and (7i) are auxiliary equations to define the nodal net power balance in normal operation and under contingencies, respectively. Constraints (7g) and (7j), which are a relaxation of constraints (1f) – (1i), enforce system-wide power balance in normal and for each under-contingency operation, respectively. Constraints (7h) and (7k) pertain to the nodal power balance equations using the Kron reduction formulation in normal and for each under-contingency operation, respectively.

We note that the network-reduced master problem (7) is a relaxed version of the SCUC problem (1) because:

1. A small subset of contingencies is incorporated into the master problem, i.e., $\mathcal{C}^{\mathrm{M},(\nu)} \subseteq \mathcal{C}$.

2. The system-wide power balance constraints (7g) and (7j), which correspond to the networkless power balance equations, are a relaxation of the constraint set defined by the original power flow equations.

3. Since the subproblems, corresponding to the normal and each under-contingency operation, are convex in the normal operation decision variables, the Benders cuts (7d) and (7e) are valid approximations from below of their costs (objective function values).

4. The power flow equations defined by Kron reduction (7h) and (7k) constitute a reformulation of the phase-angle formulation. Since we enforce capacity limits for a subset of transmission lines for normal and each under-contingency operation, the formulation based on the Kron reduction is a relaxation of the constraint set defined by the original set of power flow equations.

5. When solving the relaxed master problem (7) a non-zero gap, i.e., the relative optimality gap threshold $\texttt{MIPGap}^{(\nu)}$ is set to a value greater than 0. Thus, the best lower bound provided by the branch-and-bound process is still a valid lower bound of the optimal objective function (1a) of the SCUC problem.

Step 2. Lower bound update: Set $B_{\mathrm{L}}^{(\nu)}$ to the best lower bound of the optimal value of the objective function of the reduced-network master problem (7a), i.e., $B_{\mathrm{L}}^{(\nu)} \leftarrow \psi^{(\nu)}$.

Step 3. Full-network master problem solution: Fix the commitment (binary) variables of the coal-fired units and the combined cycle gas turbines $v_g^{(\nu)}(t)$, $y_g^{(\nu)}(t)$, $z_g^{(\nu)}(t)$ $\forall t \in \mathcal{T}$, $\forall g \in \mathcal{G}^{\mathrm{F}}$ to the solution of the relaxed master problem (7).

Additionally, set the dispatch variables $p_g^{(\nu)}(t)$ $\forall t \in \mathcal{T}$, $\forall g \in \mathcal{G}^{\mathrm{F}}$, $p_g^{\mathrm{W,spill},(\nu)}(t)$ $\forall t \in \mathcal{T}$, $\forall g \in \mathcal{G}^{\mathrm{W}}$, $D_i^{\mathrm{sh},(\nu)}(t)$, $G_i^{\mathrm{sur},(\nu)}(t)$ $\forall t \in \mathcal{T}$, $\forall i \in \mathcal{N}$, and $\psi_c^{(\nu)}$ $\forall c \in \mathcal{C}$ to the solution of the following full-network master problem:

$$
\begin{aligned}
\min_{\Xi^{\mathrm{FM}}} \quad & (7\mathrm{a}) \\
\text{s.t.} \quad & (1\mathrm{b}) - (1\mathrm{l}), \\
& (1\mathrm{m}) - (1\mathrm{s}), \quad \forall c \in \mathcal{C}^{\mathrm{M},(\nu)}, \\
& v_g(t) = v_g^{(\nu)}(t), \quad \forall g \in \mathcal{G}^{\mathrm{F}},\ \forall t \in \mathcal{T}, \quad (\gamma_g^{v,(\nu)}), && (8\mathrm{a}) \\
& y_g(t) = y_g^{(\nu)}(t), \quad \forall g \in \mathcal{G}^{\mathrm{F}},\ \forall t \in \mathcal{T}, \quad (\gamma_g^{y,(\nu)}), && (8\mathrm{b}) \\
& z_g(t) = z_g^{(\nu)}(t), \quad \forall g \in \mathcal{G}^{\mathrm{F}},\ \forall t \in \mathcal{T}, \quad (\gamma_g^{z,(\nu)}), && (8\mathrm{c})
\end{aligned}
$$

where the optimization variables $\forall t \in \mathcal{T}$ are elements of the set

$$
\begin{aligned}
\Xi^{\mathrm{FM}} = & \left\{ v_g(t), y_g(t), z_g(t), p_g(t), \overline{p}_g(t)\ \forall g \in \mathcal{G}^{\mathrm{F}} \right\} \cup \left\{ P_i(t)\ \forall i \in \mathcal{N} \right\} \cup \left\{ \theta_i(t)\ \forall i \in \mathcal{N} \right\} \cup \\
& \left\{ p_g^{\mathrm{W,spill}}(t)\ \forall g \in \mathcal{G}^{\mathrm{W}} \right\} \cup \left\{ p_{g,c}(t)\ \forall g \in \mathcal{G}^{\mathrm{F}},\ \forall c \in \mathcal{C}^{\mathrm{M},(\nu)} \right\} \cup \left\{ P_{i,c}(t)\ \forall i \in \mathcal{N},\ \forall c \in \mathcal{C}^{\mathrm{M},(\nu)} \right\} \cup \\
& \left\{ \theta_{i,c}(t), D_{i,c}^{\mathrm{sh}}, G_{i,c}^{\mathrm{cur}}\ \forall i \in \mathcal{N},\ \forall c \in \mathcal{C}^{\mathrm{M},(\nu)} \right\} \cup \left\{ p_{g,c}^{\mathrm{W,spill}}(t)\ \forall g \in \mathcal{G}^{\mathrm{W}},\ \forall c \in \mathcal{C}^{\mathrm{M},(\nu)} \right\} \cup \left\{ \psi_c\ \forall c \in \mathcal{C} \right\} \cup \left\{ \psi \right\}.
\end{aligned}
$$

Step 4. Normal operation active set update: For all time periods throughout the planning horizon, set $\mathcal{E}^{(\nu+1)}(t)$ to the set of congested transmission lines whose power flow has been above $\kappa\overline{S}_{i,j}$ at the solution of the full-network master problems in any iteration up to iteration ν, where κ is a predetermined threshold between 0 and 1. Set $\mathcal{N}^{(\nu+1)}(t)$ to the set of nodes of the transmission lines in $\mathcal{E}^{(\nu+1)}(t)$.

Step 5. Solution of the subproblems: For every contingency $c \in \mathcal{C}$, fix the commitment status of the coal-fired units and the combined cycle gas turbines $v_g^{(\nu)}(t)$ $\forall t \in \mathcal{T}$, $\forall g \in \mathcal{G}^{\mathrm{F}}$ to the solution of the network-reduced master problem (7), and fix the dispatch variables $p_g^{(\nu)}(t)$ $\forall t \in \mathcal{T}$, $\forall g \in \mathcal{G}^{\mathrm{F}}$ to the solution of the full-network master problem (8). Set the slack variables $D_{i,c}^{\mathrm{sh}}(t), G_{i,c}^{\mathrm{cur}}(t)$ $\forall t \in \mathcal{T}$, $\forall i \in \mathcal{N}$ to the solution of the following subproblem:

$$\min_{\Xi_c^{\mathrm{S}}} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} D_{i,c}^{\mathrm{sh}}(t) + G_{i,c}^{\mathrm{cur}}(t) \tag{9a}$$

$$\text{s.t.} \quad (1\text{m}) - (1\text{s})$$

$$v_g(t) = v_g^{(\nu)}(t), \quad \forall g \in \mathcal{G}^{\mathrm{F}},\ \forall t \in \mathcal{T}, \quad (\gamma_{g,c}^{v,(\nu)}(t)), \tag{9b}$$

$$p_g(t) = p_g^{(\nu)}(t), \quad \forall g \in \mathcal{G}^{\mathrm{F}},\ \forall t \in \mathcal{T}, \quad (\gamma_{g,c}^{\mathrm{p},(\nu)}(t)), \tag{9c}$$

where the optimization variables for contingency $c \in \mathcal{C}$ for all $t \in \mathcal{T}$ are elements of the set

$$\Xi_c^{\mathrm{S}} = \left\{v_g(t), p_g(t),\ \forall g \in \mathcal{G}^{\mathrm{F}}\right\} \cup \left\{p_{g,c}(t)\ \forall g \in \mathcal{G}^{\mathrm{F}}\right\} \cup \left\{\theta_{i,c}(t), P_{i,c}(t), D_{i,c}^{\mathrm{sh}}, G_{i,c}^{\mathrm{cur}}\ \forall i \in \mathcal{N}\right\}.$$

The objective function (9a) corresponds a the penalty cost related to the slack variables that represent under- and over-committing generation and guarantee feasibility. Since the subproblem (9) for contingency c includes more constraints than the original problem, this subproblem provides a valid upper bound of the corresponding operating cost under contingency c.

Step 6. Contingency active set update: For all time periods throughout the planning horizon and each contingency $c \in \mathcal{C}$, set $\mathcal{E}_c^{(\nu+1)}(t)$ to the set of congested transmission lines whose power flow has been above $\kappa\overline{S}_{i,j}$ at the solution of subproblem c in any iteration up to iteration ν, where κ is a predetermined threshold between 0 and 1. Set $\mathcal{N}_c^{(\nu+1)}(t)$ to the set of nodes of the transmission lines in $\mathcal{E}_c^{(\nu+1)}(t)$.

Step 7. Upper bound update: Set $B_{\mathrm{U}}^{(\nu)}$ as follows:

$$\begin{aligned} B_{\mathrm{U}}^{(\nu)} \leftarrow & \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{F}}} \left(c_g^{\mathrm{NL}} v_g^{(\nu)}(t) + c_g^{\mathrm{U}} y_g^{(\nu)}(t) + c_g^{\mathrm{D}} z_g^{(\nu)}(t)\right) + \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{F}}} c_g^1 p_g^{(\nu)}(t) + \\ & \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{W}}} c_g^1 \left(p_g^{\mathrm{W}}(t) - p_g^{\mathrm{W,spill},(\nu)}(t)\right) + \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} c_i^{\mathrm{LOL}} \left(D_i^{\mathrm{sh},(\nu)}(t) + G_i^{\mathrm{sur},(\nu)}(t)\right) + \\ & \sum_{c\in\mathcal{C}} \pi_c \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} c_i^{\mathrm{LOL}} \left(D_{i,c}^{\mathrm{sh},(\nu)}(t) + G_{i,c}^{\mathrm{sur},(\nu)}(t)\right), \end{aligned}$$

where its constituting terms are obtained as follows:

1. The scheduling cost, i.e.,

$$\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{F}}}\Big(c_g^{\mathrm{NL}}v_g^{(\nu)}(t)+c_g^{\mathrm{U}}y_g^{(\nu)}(t)+c_g^{\mathrm{D}}z_g^{(\nu)}(t)\Big),$$

is obtained from the solution of the reduced-network master problem (7).

2. The normal operating condition cost, i.e.,

$$\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{F}}}c_g^1p_g^{(\nu)}(t)+\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{W}}}c_g^1\big(p_g^{\mathrm{W}}(t)-p_g^{\mathrm{W,spill},(\nu)}(t)\big)+\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}c_i^{\mathrm{LOL}}\big(D_i^{\mathrm{sh},(\nu)}(t)+G_i^{\mathrm{sur},(\nu)}(t)\big),$$

is obtained from the solution of the full-network master problem (8).

3. The operating cost under contingencies, i.e.,

$$\sum_{c\in\mathcal{C}}\pi_c\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}c_i^{\mathrm{LOL}}\Big(D_{i,c}^{\mathrm{sh},(\nu)}(t)+G_{i,c}^{\mathrm{sur},(\nu)}(t)\Big),$$

is obtained from the solution of all subproblems (9).

Step 8. Convergence checking: Compute the gap between the upper and lower bounds as follows:

$$\epsilon^{(\nu)}\leftarrow\frac{\min\limits_{k=1,\dots,\nu}\{\mathrm{B}_{\mathrm{U}}^{(k)}\}-\max\limits_{k=1,\dots,\nu}\{\mathrm{B}_{\mathrm{L}}^{(k)}\}}{\max\limits_{k=1,\dots,\nu}\{\mathrm{B}_{\mathrm{L}}^{(k)}\}}.$$

If $\epsilon^{(\nu)}$ is smaller than a pre-specified tolerance δ_{tol}, the algorithm has converged. Otherwise, the algorithm continues with the next step.

Step 9. Update contingencies considered into the master problem: If $\epsilon^{(\nu)}=\epsilon^{(\nu-1)}$, set the set of contingencies considered into the master problem as follows:

$$\mathcal{C}^{\mathrm{M},(\nu+1)}\leftarrow\mathcal{C}^{\mathrm{M},(\nu)}\cup\operatorname*{arg\,max}_{c\in\mathcal{C}\setminus\mathcal{C}^{\mathrm{M},(\nu)}}\left\{\left|\psi_c^{\nu}-\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}c_i^{\mathrm{LOL}}\Big(D_{i,c}^{\mathrm{sh},(\nu)}(t)+G_{i,c}^{\mathrm{sur},(\nu)}(t)\Big)\right|\right\}.$$

Step 10. Compute Kron reduction matrices: Set $\mathcal{M}^{(\nu+1)}(t)\leftarrow\mathcal{N}\setminus\mathcal{N}^{(\nu+1)}(t)\ \forall t\in\mathcal{T}$ and set $\mathcal{M}_c^{(\nu+1)}(t)\leftarrow\mathcal{N}\setminus\mathcal{N}_c^{(\nu+1)}(t)\ \forall t\in\mathcal{T},\ c\in\mathcal{C}$. For all time periods, compute the matrices $\boldsymbol{B}^{\mathrm{red}}(t)\in\mathbb{R}^{|\mathcal{N}^{(\nu+1)}(t)|\times|\mathcal{N}^{(\nu+1)}(t)|}$ and $\boldsymbol{B}^{\mathrm{ac}}(t)\in\mathbb{R}^{|\mathcal{N}^{(\nu+1)}(t)|\times|\mathcal{M}^{(\nu+1)}(t)|}$ for normal operation as follows:

$$\begin{aligned}\boldsymbol{B}^{\mathrm{ac}}(t)=&-\boldsymbol{B}_{\mathcal{N}^{(\nu+1)}(t),\mathcal{M}^{(\nu+1)}(t)}(t)\boldsymbol{B}^{-1}_{\mathcal{M}^{(\nu+1)}(t),\mathcal{M}^{(\nu+1)}(t)}(t)\\ \boldsymbol{B}^{\mathrm{red}}(t)=&\boldsymbol{B}_{\mathcal{N}^{(\nu+1)}(t),\mathcal{N}^{(\nu+1)}(t)}(t)+\boldsymbol{B}^{\mathrm{ac}}(t)\boldsymbol{B}_{\mathcal{M}^{(\nu+1)}(t),\mathcal{N}^{(\nu+1)}(t)}(t)\end{aligned}$$

Likewise, for each contingency and for all time periods, compute the matrices $\boldsymbol{B}^{\mathrm{red},c}(t)\in\mathbb{R}^{|\mathcal{N}_c^{(\nu+1)}(t)|\times|\mathcal{N}_c^{(\nu+1)}(t)|}$

and $\boldsymbol{B}^{\mathrm{ac},c}(t) \in \mathbb{R}^{|\mathcal{N}_c^{(\nu+1)}(t)| \times |\mathcal{M}_c^{(\nu+1)}(t)|}$ for operation under contingency as follows:

$$\begin{aligned}
\boldsymbol{B}^{\mathrm{ac},c}(t) =& -\boldsymbol{B}^{c}_{\mathcal{N}_c^{(\nu+1)}(t),\mathcal{M}_c^{(\nu+1)}(t)}(t)\left(\boldsymbol{B}^{c}_{\mathcal{M}_c^{(\nu+1)}(t),\mathcal{M}_c^{(\nu+1)}(t)}(t)\right)^{-1} \\
\boldsymbol{B}^{\mathrm{red},c}(t) =& \boldsymbol{B}^{c}_{\mathcal{N}_c^{(\nu+1)}(t),\mathcal{N}_c^{(\nu+1)}(t)}(t) + \boldsymbol{B}^{\mathrm{ac,c}}(t)\boldsymbol{B}^{c}_{\mathcal{M}_c^{(\nu+1)}(t),\mathcal{N}^{c,(\nu+1)}(t)}(t)
\end{aligned}$$

Once the above matrices are computed, return to Step 1.

4. Computational Experiments

In this section, we present numerical experiments to illustrate the impact of imposing security constraints as well as to validate the computational performance of the proposed solution method.

The numerical experiments have been implemented on a Windows-based laptop with an Intel Core i7 processor clocking at 2.60 GHz and 16 GB of RAM under JuMP 0.21.3 Dunning et al. (2017). We use Gurobi 9.0.3 Gurobi Optimization, LLC (2022) for the MILP and LP problems. The relative optimality gap is determined as a function of the gap between the upper and lower bounds of the Benders framework and the iteration counter as follows $\texttt{MIPGap}^{(\nu)} = \frac{\varepsilon^{(\nu)}}{2} \cdot 0.99^{\nu}$. The first term, $\frac{\varepsilon^{(\nu)}}{2}$, corresponds to half of the gap between the upper and lower bounds whereas the second term, 0.99^{ν}, is an exponentially decaying term that is a function of the iteration counted ν and prevents the `MIPGap` from stalling at a value higher than the pre-specified convergence tolerance for the algorithm.

4.1. Illustrative Example

We use the IEEE 24-bus test system to illustrate the impact of security constraints on the commitment, system costs, and unserved energy under contingencies. The IEEE 24-bus test system has 32 generating units, 1 synchronous condenser, and 38 transmission lines. In our numerical simulations, the generating units at nodes 14 and 22 are weather-dependent and are dispatched from zero to their available forecasted capacity. The remaining units need commitment variables, which render 24 units with binary variables.

As the set of contingencies, we consider the failure of all the elements in the transmission network (i.e., all transmission lines and transformers) except line 11 between nodes 7 and 8 since disconnecting such lines splits the system into two islands. We reduce the capacity of the transmission lines to be 55% of the original capacities to increase the congestion of the transmission network.

Figure 2 depicts the commitment status of the units with and without security constraints. Despite the fact that the commitment of most of the units is the same in both cases, units 8, 10, 11, and 14 are committed additional hours in the security-constrained case, which results in higher no-load costs and production costs. Even though the additional committed capacity results in a higher scheduling and production costs, it is more beneficial if any of the contingencies materialize.

Table 1 presents the cost breakdown for the generation scheduling problem with and without security constraints, SCUC and UC, respectively. Even though both problems supply the same demand, the solution of the SCUC problem shows 10% higher scheduling and dispatch costs to mitigate the unserved demand due to the contingencies. This is, the price for the security provided by the SCUC with respect to the UC.

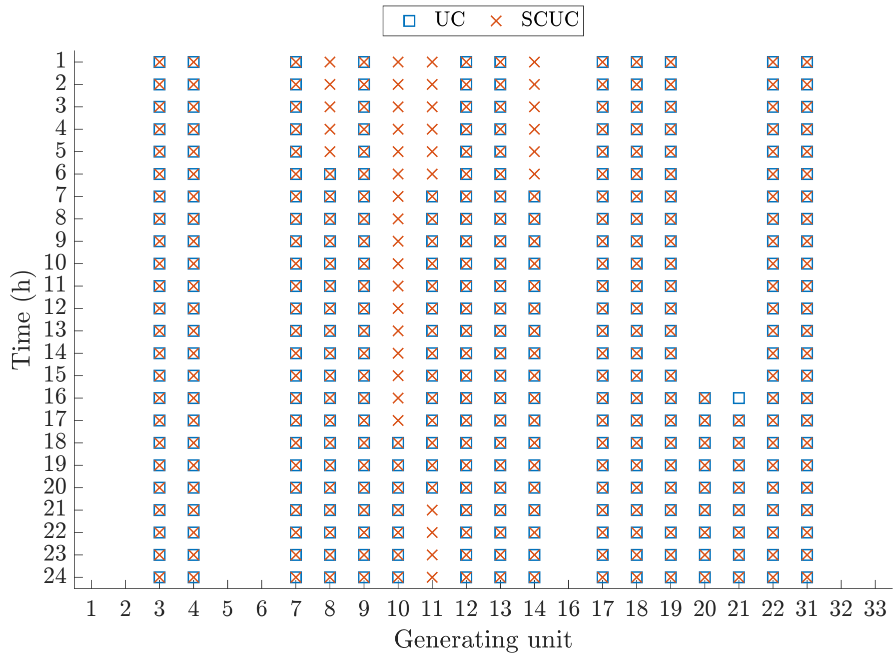


Figure 2: SCUC: Illustrative example - Comparative generation schedule with and without security constraints.

In terms of the under-contingency operating costs, the solution of the SCUC problem reduces such costs more than 4 times with respect to the solution of the UC problem.

Cost ($)	UC	SCUC
Scheduling	244,167	269,773
Fossil-fuel generation	10,430	11,189
Operation under-contingency	2,836,431	663,909

Table 1: SCUC: Illustrative example - UC vs. SCUC: Cost breakdown

In Figure 3, we present the percentage of unserved demand with respect to the total demand under each contingency for both problem, SCUC and UC. The solution of the SCUC problem reduces the unserved energy in most of the contingencies with respect to the solution of the UC problem. Note that there are several contingencies, i.e., contingencies 3, 4, 7, 14, 15, 16, 17, that even with the scheduling of the UC problem do not result in unserved demand. Hence, the UC problem is secure with respect to such subset of contingencies. Note as well that out of the 37 contingencies only a small subset of them are critical (in the sense that they result in high unserved energy for UC scheduling), e.g., contingencies 27, 18, 23, 21, and 22.

On the computational side, Figure 4 depicts the evolution of the upper and lower bound of the objective function value using the proposed solution method, which is very effective to find good quality solutions in less than 2 minutes. It is important to note that during the course of the algorithm, contingencies 27 and 18 are added to the master problem to improve the operation-cost estimate for these two contingencies. From the unserved demand perspective, the algorithm is able to identify such contingencies as the most

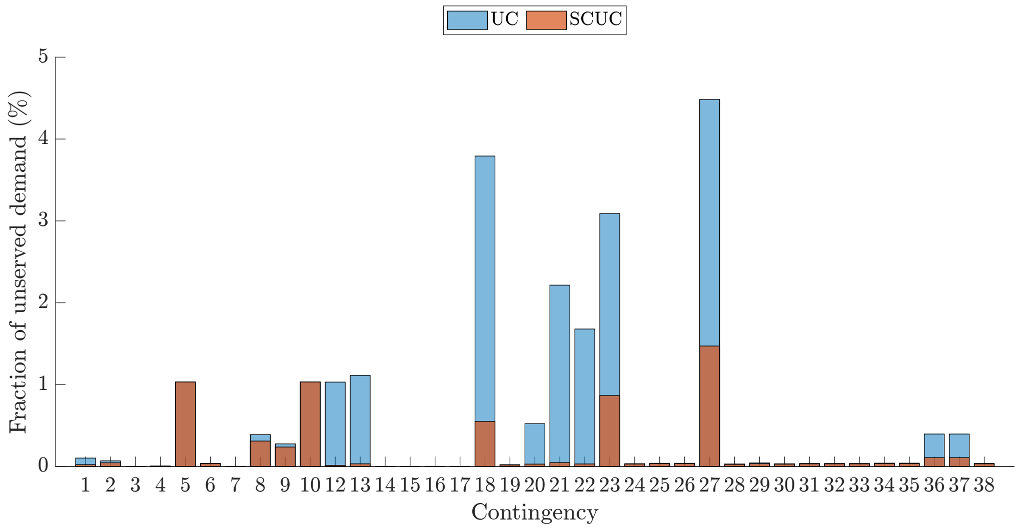


Figure 3: SCUC: Illustrative example - Comparison of load shedding under contingencies.

critical ones, which coincides with the results shown in Figure 3.

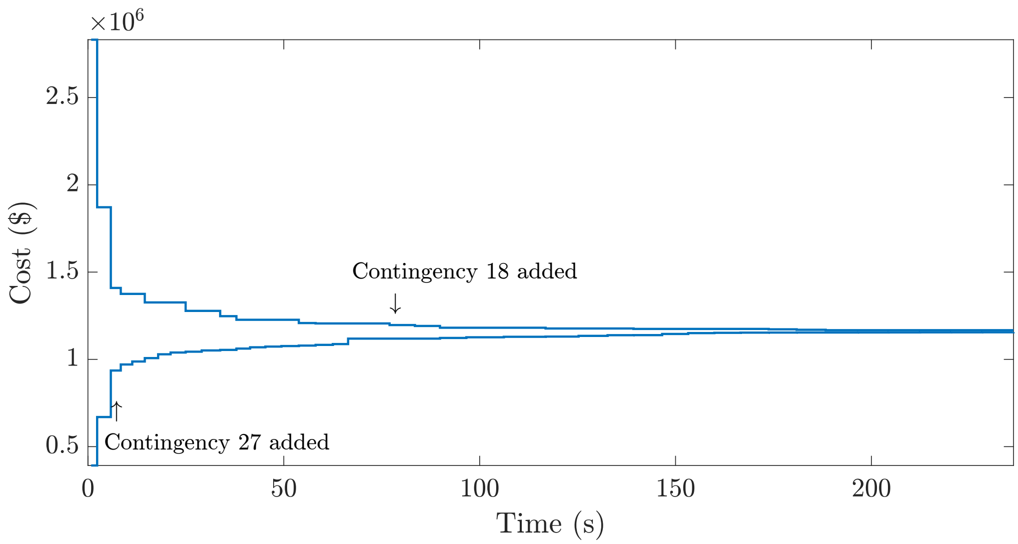


Figure 4: SCUC: Illustrative example - Evolution of the bounds of the algorithm.

4.2. Case Study

In this section, we report a series of numerical experiments using the Central Illinois 200-bus test system (Birchfield et al., 2017; Babaeinejadsarookolaee et al., 2019; Xu et al., 2017) to test the computational performance of the proposed hybrid decomposition algorithm. The Central Illinois 200-bus test system includes 200 nodes, 245 transmission lines, 108 demand loads, 25 coal-fired units, 17 gas-turbines, one nuclear plant, and six wind units. Thus, 42 units need binary variables to represent their commitment status. We consider 25 contingencies and wind spillage may happen.

4.2.1. Comparative study

We compare the computational performance of several variants of the proposed solution method with respect to the standard Benders decomposition and the solution of the extensive formulation using Gurobi Gurobi Optimization, LLC (2022). The solutions methods considered in this section are the following:

1. *Standard Benders decomposition (BD):* This approach corresponds to the multi-cut L-shaped method Birge and Louveaux (2011). The master problem enforces only constraints $(1b) - (1l)$ that pertain to the normal operating condition and the under-contingency costs are approximated from below using a Benders cut per iteration.

2. *Hybrid decomposition with full network representation (HD+FN):* This approach corresponds to the proposed hybrid decomposition framework, but the power flow equations are modeled using the full network representation.

3. *Hybrid decomposition with reduced network model (HD+RN):* This approach corresponds to the proposed hybrid decomposition framework with the power flow equations of the reduced network representation and an active set strategy pertaining to the transmission capacity limits.

4. *Extensive formulation (EF):* This corresponds to the direct solution of the extensive formulation using a MILP solver (Gurobi).

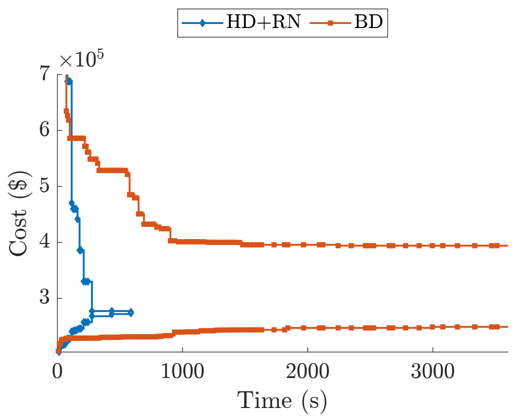
Figures 5 and 6 depict the evolution of the bounds and the gap of the aforementioned solution methods. Two stopping criteria have been chosen: (i) A time limit of 3600 seconds, and (ii) an optimality gap of 1%. For the studied instance, we can observe the following:

1. The Standard Benders decomposition shows slow convergence and is unable to close the gap beyond 33%, depicted in Fig. 5(a), within the predetermined time limit whereas all the variations of the proposed method are able to find near optimal solutions.

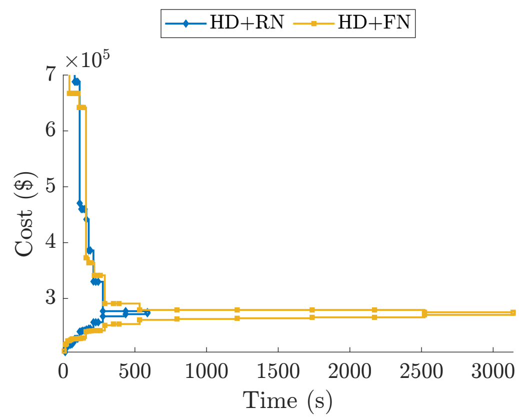
2. All the variations of the proposed hybrid decomposition outperform the solution of the extensive formulation and the standard Benders decomposition. In particular, the methods based on the reduced network model and the one with full network model and active set strategy are able to find a good quality solution ($< 1\%$ optimality gap) before Gurobi (Gurobi Optimization, LLC, 2022) is able to solve the root relaxation.

4.2.2. Impact of solving the master problem to suboptimality

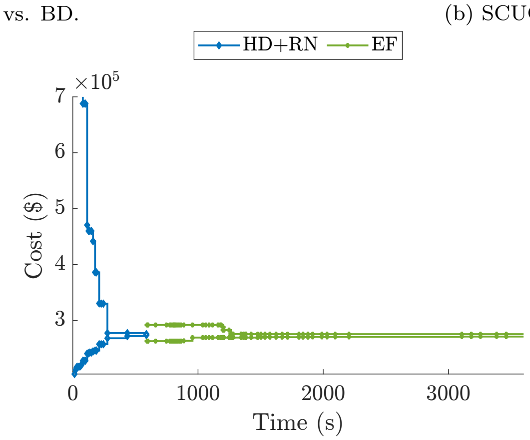
Another strategy to reduce the computational burden of the reduced-network master problem is to solve such problem to a higher relative optimality gap in the initial iterations. This might be particularly effective because at early stages the network-reduced master problem has limited to no information from the operating condition under contingencies. We compare this strategy, using a relative optimality gap that depends on the Benders optimality gap times an exponentially decaying term that depends on the iteration counter, with the strategy of solving the relaxed master problem to optimality.



(a) SCUC: HD+RN vs. BD.

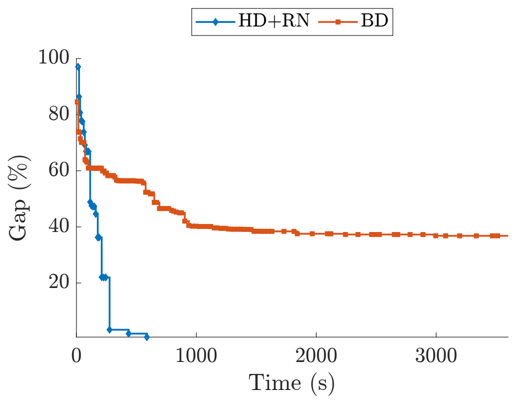


(b) SCUC: HD+RN vs. HD+FN.

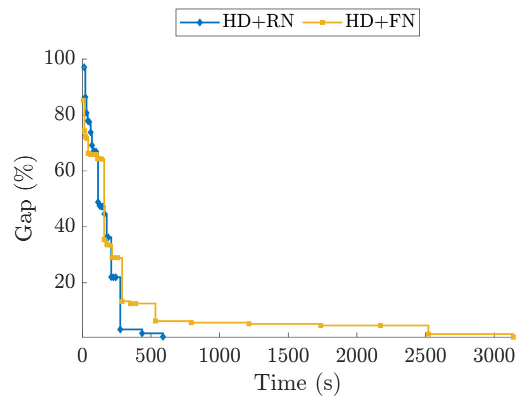


(c) SCUC: HD+RN vs. EF.

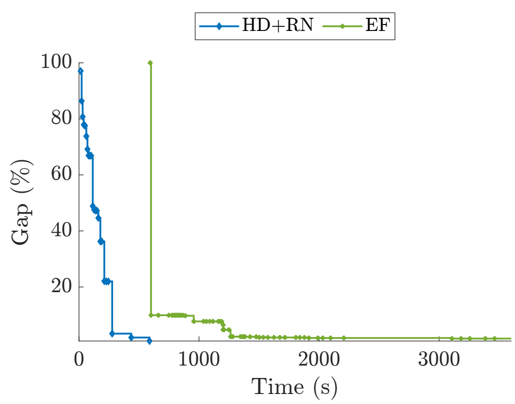
Figure 5: SCUC: Case study - Comparison of the evolution of the bounds of the algorithms.



(a) SCUC: HD+RN vs. BD.



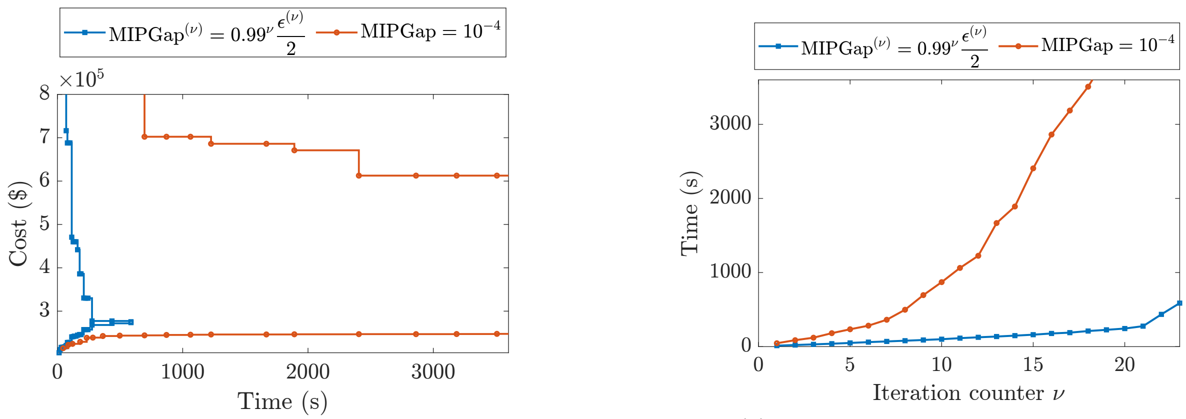
(b) SCUC: HD+RN vs. HD+FN.



(c) SCUC: HD+RN vs. EF.

Figure 6: SCUC: Case study - Comparison of the evolution of the gap of the algorithms.

Figure 7 depicts the evolution of the bounds and the solution time as a function of the time and iteration numbers, respectively. The advantage of the proposed strategy is clear as it allows to close the bounds in less computing time and a similar number of iterations. Even at the initial iterations of the algorithms, the case with the fixed optimality gap takes more time per iteration, which keeps accumulating and exacerbating the bottleneck caused by the master problem.



(a) SCUC: Evolution of bounds.

(b) SCUC: Evolution of solution times.

Figure 7: SCUC: Case study - Comparison of the bounds and solution times under both relative optimality gap strategies.

5. Conclusions

In this paper, we address the daily scheduling of electricity generation units with transmission security constraints by developing a decomposition technique that blends concepts from Benders decomposition and the column-and-constraint generation algorithm. The proposed decomposition technique overcomes some of the main drawbacks of Benders decomposition by making the master problem cognizant of information related to the system operation under certain contingencies. Additionally, we propose several strategies to reduce the solution time of the master problem using Kron reduction and leveraging computational tools.

Our numerical results show that the standard Benders decomposition is generally ineffective to address the SCUC problem since the Benders cuts convey limited contingency information. The proposed hybrid decomposition method, which conveys primal information pertaining to a small subset of contingencies into the master problem, is an effective strategy to improve the generally slow convergence of Benders-decomposition-based solution methods. Furthermore, in the studied instances, only a small subset of contingencies need to be considered into the master problems, which indicate that the information of such contingencies can not be effectively represented by Benders cuts.

The proposed Kron-based network model considered in the reduced-network master problem, which provides trial commitment decisions and constitutes the bottleneck of the algorithm, with an active set strategy significantly reduces the number of variables and constraints of such problem and speeds up its solution time. Additionally, setting the relative optimality gap for the reduced-network master problem as a function of the gap of the decomposition algorithm and the iteration number can effectively reduce the solution time. This strategy avoids unnecessary high computing times during iterations where the reduced-network master problem has not conveyed substantial information from the contingencies. In the studied instances, the proposed hybrid decomposition method outperforms commercial solvers (e.g., Gurobi) at (i) finding near optimal solutions ($< 1\%$ optimality gap) before commercial solvers compute the root relaxation of the extensive formulation, and (ii) accelerating the convergence of the bounds of the optimal objective function.

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