Hoja Nº ....

Nombre: ....

GUÍA TP Nº 1

(1) a- CONDICIONES INICIALES:

$y(0)=0 \quad ; \quad y'(0)=1$

1) $\frac{d^2y}{dt} + 3\frac{dy}{dt} + 2y = 0$

$(D^2+3D+2).y(t)=0$

$D^2+3D+2=0$

$\frac{-3 \pm \sqrt{9-4.2.1}}{2.1} = \frac{-3 \pm 1}{2} \to D_1=-1, \; D_2=-2$

$D_1=-1 \wedge D_2=-2$

RAÍCES REALES Y DISTINTAS $\Rightarrow y(t) = C_1.e^{D_1 t} + C_2.e^{D_2 t}$

$y(t) = C_1.e^{-t} + C_2.e^{-2t}$

POR CONDICIÓN INICIAL: $y(0)=0=C_1.e^{-0}+C_2.e^{-2.0}$

$y(0) = C_1 + C_2 = 0 \Rightarrow C_1 = -C_2$

$y'(t) = -C_1.e^{-t} - 2C_2.e^{-2t}$

$y'(0) = 1 = -C_1.e^{-0} - 2C_2.e^{-2.0}$

$y'(0) = -C_1 - 2C_2 = 1$

$C_1 = -C_2 \Rightarrow C_2 - 2C_2 = 1$

$-C_2 = 1 \to C_2 = -1$

$\boxed{y(t) = e^{-t} - e^{-2t}}$

$C_1 = 1$

2) $\frac{d^2y}{dt} - \frac{dy}{dt} = 0$

$(D^2-D).y(t)=0$

$D^2-D=0$

$\frac{1 \pm \sqrt{1-4.1.0}}{2} = \frac{1 \pm 1}{2} \to D_1=1, \; D_2=0$

$D_1=1 \wedge D_2=0$

$y(t) = C_1.e^{t} + C_2.e^{0} = C_1.e^{t} + C_2$

$y(0) = 0 = C_1 \cdot e^0 + C_2 \Rightarrow C_1 + C_2 = 0$

$C_1 = -C_2$

$y'(t) = C_1 \cdot e^t$

$y'(0) = 1 = C_1 \cdot e^0 = C_1 \Rightarrow \underline{C_1 = 1}$

$\underline{C_2 = -1}$

$\boxed{y(t) = e^t - 1}$

3) $9 \frac{d^2y}{dt} - \frac{dy}{dt} = 0$

$(9D^2 - D) \cdot y(t) = 0$

$9D^2 - D = 0$

$\frac{1 \pm \sqrt{1 - 4 \cdot 0 \cdot 9}}{2 \cdot 9} = \frac{1 \pm 1}{18}$ $\rightarrow D_1 = \frac{1}{9}$, $\rightarrow D_2 = 0$

$y(t) = C_1 \cdot e^{1/9 t} + C_2 \cdot e^0$

$y(t) = C_1 \cdot e^{1/9 t} + C_2$

$y(0) = 0 = C_1 e^0 + C_2 \Rightarrow C_1 + C_2 = 0$

$C_1 = -C_2$

$y'(t) = \frac{1}{9} C_1 \cdot e^{1/9 \cdot t}$

$y'(0) = 1 = \frac{1}{9} \cdot C_1 \cdot e^{1/9\, 0} = \frac{1}{9} C_1$

$\underline{C_1 = 9} \rightarrow \underline{C_2 = -9}$

$\boxed{y(t) = 9e^{1/9 t} - 9}$

4) $\frac{d^2y}{dt} - 8\frac{dy}{dt} + 15y = 0$

$D^2 - 8D + 15 = 0$

$\frac{8 \pm \sqrt{64 - 4 \cdot 15 \cdot 1}}{2} = \frac{8 \pm 2}{2}$ $\rightarrow D_1 = 5$, $\rightarrow D_2 = 3$

$(D^2 - 8D + 15)\, y(t) = 0$

$y(t) = C_1 \cdot e^{5t} + C_2 e^{3t}$

$y(0) = 0 = C_1 e^{5 \cdot 0} + C_2 e^{3 \cdot 0} \Rightarrow C_1 + C_2 = 0$

$C_1 = -C_2$

$$y'(t) = 5C_1 e^{5t} + 3C_2 e^{3t}$$

$$y'(0) = 1 = 5C_1 e^{5\cdot 0} + 3C_2 e^{3\cdot 0}$$

$$1 = 5C_1 + 3C_2$$

$$1 = -5C_2 + 3C_2$$

$$1 = -2C_2 \Rightarrow \underline{\underline{C_2 = -1/2}}$$

$$\underline{C_1 = 1/2}$$

$$\boxed{y(t) = \frac{1}{2} e^{5t} - \frac{1}{2} e^{3t}}$$

b- <u>Condiciones iniciales:</u>

$$y(0) = 1 \quad ; \quad y'(0) = 0$$

1) $2 \cdot \frac{d^2y}{dt} + 3\frac{dy}{dt} + y = 4 - e^t$

$$2D^2 + 3D + 1 = 0 \quad \begin{matrix} \to D_1 = -1/2 \\ \to D_2 = -1 \end{matrix}$$

$$y_B = A + Be^t \qquad y_A = C_1 e^{-1/2 t} + C_2 e^{-t}$$

$$y_B' = Be^t$$

$$y_B'' = Be^t$$

$$2Be^t + 3Be^t + A + Be^t = 4 - e^t$$

$$A = 4 \quad ; \quad 2Be^t + 3Be^t + Be^t = -e^t$$

$$2B + 3B + B = -1 \longrightarrow B = -1/6$$

$$y_B = 4 - \frac{1}{6}e^t$$

$$y(t) = y_A + y_B \longrightarrow y(t) = C_1 e^{-1/2 t} + C_2 e^{-t} + 4 - \frac{1}{6}e^t$$

$$CI \Rightarrow y(0) = C_1 + C_2 + 4 - \frac{1}{6} = 1$$

$$C_1 + C_2 = \frac{-17}{6} \longrightarrow C_1 = \frac{-17}{6} - C_2$$

$$y'(t) = -\frac{1}{2} C_1 e^{-1/2 t} - C_2 e^{-t} - \frac{1}{6} e^t$$

$$y'(0) = -\frac{1}{2} C_1 - C_2 - \frac{1}{6} = 0$$

$$-\frac{1}{2}C_1 - C_2 = \frac{1}{6} \longrightarrow C = \left(\frac{1}{6} + C_2\right) \cdot -2 = -\frac{1}{3} - 2C_2$$

$$-\frac{1}{3} - 2C_2 = \frac{-17}{6} - C_2 \Rightarrow C_2 = \frac{5}{2} \longrightarrow C_1 = \frac{-16}{3}$$

$$\boxed{y(t) = \frac{-16}{3} e^{-1/2 t} + \frac{5}{2} e^{-t} + 4 - \frac{1}{6} e^t}$$

2) $\frac{d^2y}{dt} + 9\frac{dy}{dt} = 2t^2+4t+7$

$D^2+9D=0 \rightarrow D_1=0$
$\rightarrow D_2=-9$

$y_B = \underbrace{B_2 t^3 + B_1 t^2 + B_0 t}$

$y_A = C_1 \cdot e^0 + C_2 \cdot e^{-9t}$

SE DESPEJARO EN $t \Rightarrow t \cdot (B_2t^2+B_1t+B_0)$

$\underline{y_A = C_1 + C_2\, e^{-9t}}$

$y_B' = 3B_2t^2 + 2B_1t + B_0$

$y_B'' = 6B_2t + 2B_1$

$6B_2t + 2B_1 + 9\cdot(3B_2t^2+2B_1t+B_0) = 2t^2+4t+7$

$6B_2t + 2B_1 + 27B_2t^2 + 18B_1t + 9B_0 = 2t^2+4t+7$

$2B_1 + 9B_0 = 7$; $6B_2t + 18B_1t = 4t$; $27B_2t^2 = 2t^2$

$2\cdot\frac{16}{81} + 9B_0 = 7$ $\qquad$ $6\cdot\frac{2}{27} + 18B_1 = 4$ $\qquad$ $B_2 = 2/27$

$9B_0 = 7 - \frac{32}{81}$ $\qquad$ $18B_1 = 4 - \frac{12}{27}$

$B_0 = \frac{535}{81} \cdot \frac{1}{9}$ $\qquad$ $B_1 = \frac{32}{9} \cdot \frac{1}{18}$

$B_1 = \frac{16}{81}$

$B_0 = \frac{535}{729}$

$y_B = \frac{2}{27}\cdot t^3 + \frac{16}{81}\cdot t^2 + \frac{535}{729}t$

$y(t) = y_A + y_B = C_1 + C_2\,e^{-9t} + \frac{2}{27}t^3 + \frac{16}{81}t^2 + \frac{535}{729}t$

CI $\Rightarrow$ $y(0) = C_1 + C_2 = 1$

$C_1 = 1 - C_2$

$y'(t) = -9C_2\,e^{-9t} + \frac{6}{27}t^2 + \frac{32}{81}t + \frac{535}{729}$

$y'(0) = -9C_2 + \frac{535}{729} = 0$

$C_2 = -\frac{535}{729}\cdot\left(-\frac{1}{9}\right) = \frac{535}{6561} \longrightarrow C_1 = \frac{6026}{6561}$

$$\boxed{y(t) = \frac{6026}{6561} + \frac{535}{6561}e^{-9t} + \frac{2}{27}t^3 + \frac{16}{81}t^2 + \frac{535}{729}t}$$

3) $\frac{d^2y}{dt}+2\frac{dy}{dt}+y=te^t-e^t$

$D^2+2D+1=0 \rightarrow D_1=D_2=-1$

$y_A=C_1.e^{-t}+C_2te^{-t}$

$y_B=B_0te^t+B_1e^t$

$y_B'=B_0e^t+B_0te^t+B_1e^t$

$y_B''=B_0e^t+B_0e^t+B_0te^t+B_1e^t = 2.B_0e^t+B_0te^t+B_1e^t$

$2B_0e^t+B_0te^t+B_1e^t+2.(B_0e^t+B_0te^t+B_1e^t)+B_0te^t+B_1e^t=te^t-e^t$

$2B_0e^t+2B_0te^t+2B_1e^t+2B_0e^t+2B_0te^t+2B_1e^t=te^t-e^t$

$4B_0e^t+4B_0te^t+4B_1e^t=te^t-e^t$

$4B_0te^t=te^t \rightarrow 4B_0=1$

$B_0=1/4$

$4B_0e^t+4B_1e^t=-e^t$

$4.\frac{1}{4}+4B_1=-1$

$4B_1=-2 \rightarrow B_1=-1/2$

$y_B=\frac{1}{4}te^t-\frac{1}{2}e^t$

$y(t)=C_1e^{-t}+C_2te^{-t}+\frac{1}{4}te^t-\frac{1}{2}e^t$

$CI \rightarrow \quad y(0)=C_1-\frac{1}{2}=1$

$C_1=\frac{3}{2}$

$y'(t)=C_1e^{-t}+C_2.e^{-t}+C_2te^{-t}+\frac{1}{4}.e^t+\frac{1}{4}t.e^t-\frac{1}{2}e^t$

$y'(t)=C_1e^{-t}+C_2e^{-t}+C_2te^{-t}-\frac{1}{4}e^t+\frac{1}{4}t.e^t$

$y'(0)=-C_1+C_2-\frac{1}{4}=0$

$C_2=\frac{1}{4}+\frac{3}{2}=\frac{7}{4}$

$$\boxed{y(t)=\frac{3}{2}e^{-t}+\frac{7}{4}te^{-t}+\frac{1}{4}te^t-\frac{1}{2}e^t}$$

4) $\frac{d^2y}{dt}+3\frac{dy}{dt}=2\,sen(t)+cos(t)$

$D^2+3D=0 \rightarrow D_1=0,\ D_2=-3$

$y_n=C_1e^0+C_2e^{-3t}$

$y_n=C_1+C_2e^{-t}$

$y_B=B_0\,sen(t)+B_1.cos(t)$

$y_B'=B_0\,cos(t)-B_1\,sen(t)$

$y_B''=-B_0.sen(t)-B_1.cos(t)$

$-B_0 \cdot \mathrm{Sen}(t) - B_1 \cos(t) + 3 B_0 \cos(t) - 3 B_1 \cdot \mathrm{Sen}(t) = 2 \cdot \mathrm{sen}(t) + \cos(t)$

$-B_0 \cdot \mathrm{Sen}(t) - 3B_1 \cdot \mathrm{sen}(t) = 2\,\mathrm{sen}(t)$ $\qquad$ $-B_1 \cdot \cos(t) + 3 B_0 \cos(t) = \cos(t)$

$-B_0 - 3B_1 = 2$ $\qquad$ $-B_1 + 3B_0 = 1$

$B_0 = -2 - 3B_1$ $\qquad$ $B_0 = \frac{1}{3} + \frac{1}{3} B_1$

$$-2 - 3B_1 = \frac{1}{3} + \frac{1}{3} B_1$$

$$-2 - \frac{1}{3} = \frac{1}{3} B_1 + 3B_1$$

$$-\frac{7}{3} = \frac{10}{3} B_1 \Rightarrow B_1 = \frac{-7}{10}$$

$$B_0 = -2 - 3 \cdot \left(\frac{-7}{10}\right) \Rightarrow B_0 = \frac{1}{10}$$

$$Y_B = \frac{1}{10} \mathrm{sen}(t) - \frac{7}{10} \cdot \cos(t)$$

$$Y(t) = c_1 + c_2 e^{-3t} + \frac{1}{10} \mathrm{sen}(t) - \frac{7}{10} \cos(t)$$

$CI \Rightarrow$ $Y(0) = c_1 + c_2 - \frac{7}{10} = 1$

$$c_1 + c_2 = 1 + \frac{7}{10}$$

$$c_1 = \frac{17}{10} - c_2$$

$$Y'(t) = -3c_2 e^{-3t} + \frac{1}{10} \cos(t) + \frac{7}{10} \cdot \mathrm{sen}(t)$$

$$Y'(0) = -3c_2 + \frac{1}{10} = 0$$

$$c_2 = \frac{1}{30} \Rightarrow c_1 = \frac{17}{10} - \frac{1}{30} = \frac{5}{3}$$

$$\boxed{Y(t) = \frac{5}{3} + \frac{1}{30} e^{3t} + \frac{1}{10} \mathrm{sen}(t) - \frac{7}{10} \cos(t)}$$

5) $5 \frac{d^2Y}{dt} + 15 \frac{dY}{dt} - 25Y = 5e^{-4t} + 15e^{3t}$

$5D^2 + 15D - 25Y = 0$

$5 \cdot (D^2 + 3D - 5Y) = 0$

$D^2 + 3D - 5Y = 0$

$D = \frac{-3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-5)}}{2} = \frac{-3 \pm \sqrt{29}}{2} \rightarrow D_1 = 1{,}19 \rightarrow D_2 = -4{,}1$

$Y_A = c_1 \cdot e^{1{,}19t} + c_2\, e^{-4{,}19t}$

$Y_B = B_0 e^{-4t} + B_1 e^{3t}$

$Y_B' = -4 B_0 \cdot e^{-4t} + 3 B_1 e^{3t}$

$Y_B'' = 16 B_0\, e^{-4t} + 9 B_1 e^{3t}$

$80 B_0 e^{-4t} + 45 B_1 e^{3t} - 60 B_0 e^{-4t} + 45 B_1 e^{3t} - 25 B_0 e^{-4t} - 25 B_1 e^{3t} = 5e^{-4t} + 15e^{3t}$

$-5B_0 e^{-4t} + 65 B_1 e^{3t} = 5e^{-4t} + 15e^{3t}$

$-5B_0 = 5 \longrightarrow B_0 = -1$

$65 B_1 = 15 \longrightarrow B_1 = \frac{3}{13}$

$$Y_B = -e^{-4t} + \frac{3}{13} e^{3t}$$

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$$Y(t) = C_1 e^{1,19t} + C_2 e^{-4,19t} - e^{-4t} + \frac{3}{13} e^{3t}$$

$$CI \Rightarrow Y(0) = C_1 + C_2 - 1 + \frac{3}{13} = 1$$

$$C_1 + C_2 = \frac{23}{13} \rightarrow C_1 = \frac{23}{13} - C_2$$

$$Y'(t) = 1,19\,C_1 e^{1,19t} - 4,19\,C_2 e^{-4,19t} + 4e^{-4t} + \frac{9}{13} e^{3t}$$

$$Y'(0) = 1,19\,C_1 - 4,19\,C_2 + 4 + \frac{9}{13} = 0$$

$$1,19\,C_1 = \frac{-61}{13} + 4,19\,C_2$$

$$\frac{23}{13} - C_2 = \frac{-61}{13} + 4,19\,C_2$$

$$\frac{84}{13} = 5,19\,C_2 \Rightarrow C_2 = 1,24$$

$$C_1 = 0,52$$

$$\boxed{Y(t) = 0,52\, e^{1,19t} + 1,24\, e^{-4,19t} - e^{-4t} + \frac{3}{13} e^{3t}}$$

6) $\frac{d^2y}{dt} + 2\frac{dy}{dt} + 10y = 3t^2$

$$D = \alpha \pm \beta i$$

$$D^2 + 2D + 10 = 0$$

$$D = \frac{-2 \pm \sqrt{4 - 4 \cdot 10}}{2} = \frac{-2 \pm 6i}{2} \begin{cases} D_1 = -1 + 3i \\ D_2 = -1 \end{cases}$$

$$y_B = B_0 t^2 + B_1 t + B_2$$

$$y_B' = 2B_0 t + B_1$$

$$y_B'' = 2B_0$$

$$y_h = C_1 e^{\alpha x} \cdot \cos(\beta x) + C_2 e^{\alpha x} \operatorname{sen}(\beta x)$$

$$y_h = C_1 e^{-t} \cdot \cos(3t) + C_2 e^{-t} \operatorname{sen}(3t)$$

$$2B_0 + 4B_0 t + 2B_1 + 10B_0 t^2 + 10B_1 t + 10B_2 = 3t^2$$

$$2B_0 + 2B_1 + 10B_2 = 0$$

$$4B_0 t + 10B_1 t = 0$$

$$10B_0 t^2 = 3t^2 \rightarrow \underline{B_0 = \frac{3}{10}}$$

$$t\,4 \cdot \frac{3}{10} + 10B_1 t = 0$$

$$10B_1 t = -\frac{6}{5} t$$

$$B_1 = -\frac{6}{5} \cdot \frac{1}{10} = \frac{-3}{25}$$

$$2 \cdot \frac{3}{10} + 2\left(\frac{-3}{25}\right) + 10B_2 = 0$$

$$10B_2 = \frac{-3}{5} + \frac{6}{25}$$

$$B_2 = \frac{-9}{25} \cdot \frac{1}{10} = \frac{-9}{250}$$

$$y_B = \frac{3}{10} t^2 - \frac{3}{25} t - \frac{9}{250}$$

$$Y(t) = C_1 e^{-t} \cos(3t) + C_2 e^{-t} \operatorname{sen}(3t) + \frac{3}{10} t^2 - \frac{3}{25} t - \frac{9}{250}$$

$$CI \Rightarrow Y(0) = C_1 - \frac{9}{250} = 1$$

$$C_1 = 1 + \frac{9}{250}$$

$$C_1 = \frac{259}{250}$$

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$Y(t) = -c_1 e^{-t}\cos(3t) - 3c_1 e^{-t}\,\text{sen}(3t) + 3c_2 e^{-t}\cos(3t) - c_2 e^{-t}\,\text{sen}(3t) + \frac{3}{5}t - \frac{3}{25}$

$Y'(0) = -c_1 + 3c_2 - \frac{3}{25} = 0$

$3c_2 = \frac{3}{25} + c_1 = \frac{3}{25} + \frac{259}{250} = \frac{289}{250} \Rightarrow c_2 = \frac{289}{250}\cdot\frac{1}{3}$

$c_2 = \frac{289}{750}$

$$\boxed{Y(t) = \frac{259}{250}e^{-t}\cos(3t) + \frac{289}{750}e^{-t}\,\text{sen}(3t) + \frac{3}{10}t^2 - \frac{3t}{25} - \frac{9}{250}}$$

7) $\frac{d^2Y}{dt} = \cos(3t)$

$D^2 = 0 \rightarrow D_1 = D_2 = 0$

$Y_A = c_1 e^{0t} + c_2 t e^{0t}$

$Y_A = c_1 + c_2 t$

$Y_B = B_0\cos(3t) + B_1\,\text{sen}(3t)$

$Y_B' = -3B_0\,\text{sen}(3t) + 3B_1\cos(3t)$

$Y_B'' = -9B_0\cos(3t) - 9B_1\,\text{sen}(3t)$

$-9B_0\cos(3t) - 9B_1\,\text{sen}(3t) = \cos(3t)$

$-9B_0\cos(3t) = \cos(3t)$

$-9B_0 = 1 \rightarrow B_0 = -\frac{1}{9}$

$B_1 = 0$

$Y_B = -\frac{1}{9}\cos(3t)$

$Y(t) = c_1 + c_2 t - \frac{1}{9}\cos(3t)$

$c_1 \Rightarrow Y(0) = c_1 - \frac{1}{9} = 1$

$c_1 = \frac{10}{9}$

$Y'(t) = c_2 + \frac{1}{3}\,\text{sen}(3t)$

$Y'(0) = c_2 = 0$

$$\boxed{Y(t) = \frac{10}{9} - \frac{1}{9}\cos(3t)}$$

② 1) $F(t) = 2e^{-t}\cos(10t) - t^4 + 6e^{-(t-10)}$

$\mathcal{L}\{U\} = \mathcal{L}\{2e^{-t}\cos(10t)\} - \mathcal{L}\{t^4\} + \mathcal{L}\{6e^{-(t-10)}\}$

$\mathcal{L}\{U\} = 2\cdot\mathcal{L}\{e^{-t}\cdot\cos(10t)\} - \frac{4!}{s^5} + 6\cdot\mathcal{L}\{e^{-(t-10)}\}$ $\qquad e^{t-10} = e^{t}\, e^{-10}$

$\mathcal{L}\{U\} = 2\cdot\frac{s-1}{(s-1)^2+10^2} - \frac{24}{s^5} + 6\,\mathcal{L}\{e^{-t}\}\cdot e^{10}$

$$\boxed{\mathcal{L}\{U\} = \frac{2(s+1)}{(s+1)^2+100} - \frac{24}{s^5} + \frac{6e^{10}}{s+1}}$$

2) $F(t) = sen(wt)$

$\mathcal{L}\{U\} = \int_0^B e^{-st}\cdot U(t)\,dt = \lim_{B\to\infty}\int_0^B e^{-st}\, U(t)\,dt$

$= \lim_{B\to\infty}\int_0^B e^{-st}\cdot sen(wt)\,dt = \int_0^\infty e^{-st}\, sen(wt)\,dt$ $\qquad U = sen(wt)$

$du = \frac{1}{w}\cos(wt)\cdot dt$

$= -\frac{1}{s}e^{-st}\cdot sen(wt)\Big|_0^\infty + \frac{w}{s}\int_0^\infty e^{-st}\cdot\cos(wt)\,dt$ $\qquad dv = e^{-st}$

$L \to 0$

$= \frac{w}{s}\int_0^\infty e^{-st}\cdot\cos(wt)\cdot dt = \left[-\frac{1}{s}e^{-st}\cdot\cos(wt)\Big|_0^\infty - \frac{w}{s}\int_0^\infty e^{-st}\cdot sen(wt)\,dt\right]\cdot\frac{w}{s}$ $\qquad V = -\frac{1}{s}e^{-st}$

$= \frac{w}{s^2} - \frac{w^2}{s^2}\int_0^\infty e^{-st}\, sen(wt)\,dt$

$\int_0^\infty e^{-st}\, sen(wt)\,dt + \frac{w^2}{s^2}\int_0^\infty e^{-st}\cdot sen(wt)\,dt = \frac{w}{s^2}$

$\left[1+\frac{w^2}{s^2}\right]\cdot\int_0^\infty e^{-st}\cdot sen(wt)\,dt = \frac{w}{s^2}$

$\int_0^\infty e^{-st}\cdot sen(wt)\cdot dt = \frac{w}{s^2+w^2}$

$$\boxed{\mathcal{L}\{sen(wt)\} = \frac{w}{s^2+w^2}}$$

3) $y''+3y'+2y=5$ $\qquad y'(0)=2 \quad ; \quad y(0)=1$

$\mathcal{L}\{y''+3y'+2y\} = \mathcal{L}\{5\}$

$\mathcal{L}\{y''\}+3\mathcal{L}\{y'\}+2\mathcal{L}\{y\} = \frac{5}{s}$

$\mathcal{L}\{y\}(s) = Y(s)$

$\mathcal{L}\{y'\}(s) = sY(s) - y(0) = sY(s) - 1$

$\mathcal{L}\{y''\}(s) = s^2Y(s) - sY(0) - y'(0) = s^2Y(s) - s - 2$

$$(s^2Y(s)-3)+3\cdot(sY(s)-1)+2Y(s)=\frac{5}{s}$$

$$s^2Y(s)-s-2+3sY(s)-3+2Y(s)=\frac{5}{s}$$

$$Y(s)\cdot(s^2+3s+2)=\frac{5}{s}+5+s$$

$$\boxed{Y(s)=\frac{5+5s+s^2}{s\cdot(s^2+3s+2)}}$$

4) $Y'''+5Y''+8Y'=0$ $\qquad Y''(0)=0\ ;\ Y'(0)=0\ ;\ Y(0)=5$

$$\mathcal{L}\{Y'''+5Y''+8Y'\}=\mathcal{L}\{0\}$$

$$\mathcal{L}\{Y'''\}+5\mathcal{L}\{Y''\}+8\mathcal{L}\{Y'\}=0$$

$$\mathcal{L}\{Y'\}(s)=s\cdot Y(s)-Y(0)=sY(s)-5$$

$$\mathcal{L}\{Y''\}(s)=s^2Y(s)-s\cdot Y(0)-Y'(0)=s^2Y(s)-s\cdot 5-0=s^2Y(s)-5s$$

$$\mathcal{L}\{Y'''\}(s)=s^3Y(s)-s^2Y(0)-sY'(0)-Y''(0)=s^3Y(s)-5s^2-s\cdot 0-0=s^3Y(s)-5s^2$$

$$s^3Y(s)-5s^2+5\cdot(s^2Y(s)-5s)+8\,(sY(s)-5)=0$$

$$s^3Y(s)-5s^2+5s^2Y(s)-25s+8sY(s)-40=0$$

$$Y(s)\,(s^3+5s^2+8s)=5s^2+25s+40$$

$$Y(s)=\frac{5\cdot(s^2+5s+8)}{s\cdot(s^2+5s+8)}\implies\boxed{Y(s)=\frac{5}{s}}$$

5) $F(s)=\dfrac{-(s^2+s+1)}{s^2\cdot(s+1)\cdot(s+2)}$

$$F(s)=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+1}+\frac{D}{s+2}=\frac{A\cdot(s+1)\cdot(s+2)+B\cdot s\cdot(s+1)(s+2)+C\,s^2\cdot(s+2)+D\,s^2\cdot(s+1)}{s^2\cdot(s+1)\cdot(s+2)}$$

$$-s^2-s-1=A\,(s+1)\cdot(s+2)+B\,s\,(s+1)(s+2)+C\,s^2(s+2)+D\,s^2\cdot(s+1)$$

Si $s=0\implies -1=2A\to A=-\frac{1}{2}$

Si $s=-1\implies -(1)^2-(-1)-1=C\cdot(-1)^2\cdot(-1+2)$

$$-1=C$$

Si $s=-2\implies -(-2)^2-(-2)-1=D\cdot(-2)^2(-2+1)$

$$-3=-4D\to D=\frac{3}{4}$$

Si $s=1\implies -1^2-1-1=-\frac{1}{2}(1+1)\cdot(1+2)+B\cdot 1(1+1)(1+2)-1^2\cdot(1+2)+\frac{3}{4}\cdot 1^2(1+1)$

$$-3=-3+6B-3+\frac{3}{2}$$

$$6B=\frac{3}{2}\to B=\frac{1}{4}$$

$F(s)=\dfrac{-1}{2s^2}+\dfrac{1}{4s}-\dfrac{1}{s+1}+\dfrac{3}{4(s+2)}\ ;\quad \mathcal{L}^{-1}\{F(s)\}=-\dfrac{1}{2}t\cdot e^{0t}+\dfrac{1}{4}e^{0}-e^{-t}+\dfrac{3}{4}e^{-2t}$

$$\boxed{\mathcal{L}^{-1}\{F(s)\}=-\frac{1}{2}t+\frac{1}{4}-e^{-t}+\frac{3}{4}e^{-2t}}$$

6) $F(s) = \dfrac{s^2+9s+19}{(s+1)(s+2)(s+3)}$

$$= \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} = \frac{A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)}{(s+1)(s+2)(s+3)}$$

$$s^2+9s+19 = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$

Si $s=-2 \Longrightarrow 5 = -B \longrightarrow B=-5$

Si $s=-3 \Longrightarrow 1 = 2C \longrightarrow C=\frac{1}{2}$

Si $s=-1 \Longrightarrow 11 = A2 \longrightarrow A=\frac{11}{2}$

$$F(s) = \frac{11}{2}\cdot\frac{1}{s+1} - \frac{5}{s+2} + \frac{1}{2}\cdot\frac{1}{s+3}$$

$$\boxed{\mathcal{L}^{-1}\{F(s)\} = \frac{11}{2}e^{-t} - 5e^{-2t} + \frac{1}{2}e^{-3t}}$$

7) $F(s) = \dfrac{2\cdot(s+1)}{s(s+3)(s+5)^2}$

$$= \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+5)^2} + \frac{D}{s+5} = \frac{A\cdot(s+3)\cdot(s+5)^2 + Bs(s+5)^2 + Cs(s+3) + Ds\cdot(s+3)(s+}{s\cdot(s+3)(s+5)^2}$$

$$2(s+1) = A(s+3)(s+5)^2 + Bs(s+5)^2 + Cs(s+3) + Ds(s+3)(s+5)$$

Si $s=0 \Longrightarrow 2 = 75A \longrightarrow A=\frac{2}{75}$

Si $s=-3 \Longrightarrow -4 = -12B \longrightarrow B=\frac{1}{3}$

Si $s=-5 \Longrightarrow -8 = 10C \longrightarrow C=-\frac{4}{5}$

Si $s=1 \Longrightarrow 4 = \frac{96}{25} + 12 - \frac{16}{5} + 24D \longrightarrow D=\frac{-9}{25}$

$$F(s) = \frac{2}{75}\cdot\frac{1}{s} + \frac{1}{3}\cdot\frac{1}{s+3} - \frac{4}{5}\cdot\frac{1}{(s+5)^2} - \frac{9}{25}\,\frac{1}{s+5}$$

$$\boxed{\mathcal{L}^{-1}\{F(s)\} = \frac{2}{75} + \frac{1}{3}e^{-3t} - \frac{4}{5}te^{-5t} - \frac{9}{25}e^{-5t}}$$

8) $F(s) = \dfrac{1}{s\cdot(s+1)^3(s+2)}$

$$= \frac{A}{s} + \frac{B}{(s+1)^3} + \frac{C}{(s+1)^2} + \frac{D}{s+1} + \frac{E}{s+2} = \frac{A(s+1)^3(s+2) + Bs(s+2) + Cs(s+1)(s+2) + Ds(s+1)^2(s+2) + E}{s(s+1)^3(s+2)}$$

$$1 = A(s+1)^3(s+2) + Bs(s+2) + Cs(s+1)(s+2) + Ds(s+1)^2(s+2) + Es(s+1)^3$$

SI $s=0 \Longrightarrow 1=A2 \longrightarrow A=1/2$

SI $s=-1 \Longrightarrow 1=-B \longrightarrow B=-1$

SI $s=-2 \Longrightarrow 1=2E \longrightarrow E=1/2$

PARA C Y D $\longrightarrow$ DERIVO EN AMBOS MIEMBROS

$$0=3A(s+1)^2(s+2)+A(s+1)^3+B(s+2)+Bs+Cs(s+1)+Cs(s+2)+C(s+1)(s+2)+2Ds(s+1)(s+2)$$
$$+Ds(s+1)^2+D(s+1)^2(s+2)+E(s+1)^3+3Es(s+1)^2$$

SI $s=-1 \longrightarrow 0=B(-1+2)+B(-1)+C(-1)(-1+2)$

$0=-C \longrightarrow C=0$

SI $s=-2 \longrightarrow 0=A(-2+1)^3+B(-2)+C(-2)(-2+1)+D(-2)(-2+1)^2+E(-2+1)^3+3E(-2)(-2+1)^2$

$0=-\frac{1}{2}+2-\frac{1}{2}-3-2D \longrightarrow D=1$

$$F(s)=\frac{1}{2}\cdot\frac{1}{s}-\frac{1}{(s+1)^3}+0\cdot\frac{1}{(s+1)^2}-\frac{1}{s+1}+\frac{1}{2}\cdot\frac{1}{s+2}$$

$$\boxed{\mathcal{L}^{-1}\{F(s)\}=\frac{1}{2}-e^{-t}t^2-e^{-t}+\frac{1}{2}e^{-2t}}$$

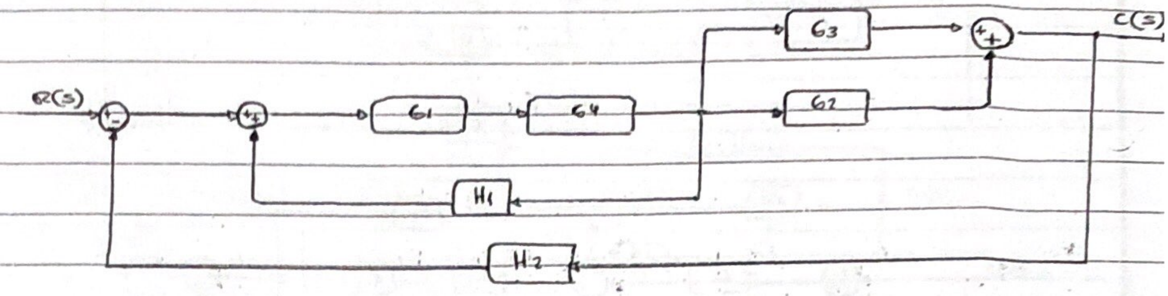
③ $\frac{d}{dt}\left(x_2(t)\cdot x_1(t)\right)+x_3(t)\cdot 0.3$

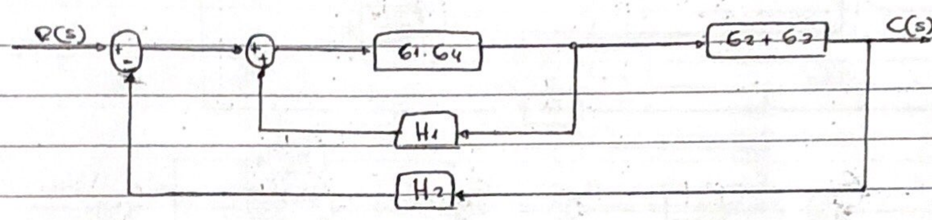
$$\frac{d}{dt}\left(\operatorname{sen}(t)\cdot\cos(2t)\right)+0{,}3t^2=\cos(t)\cdot\cos(2t)+\operatorname{sen}(t)\cdot\left(-2\operatorname{sen}(2t)\right)+0{,}3t^2$$

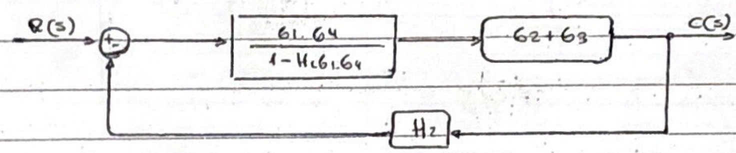
$$\boxed{\text{SALIDA SISTEMA}=\cos(t)\cdot\cos(2t)-2\operatorname{sen}(t)\cdot\operatorname{sen}(2t)+0{,}3t^2}$$

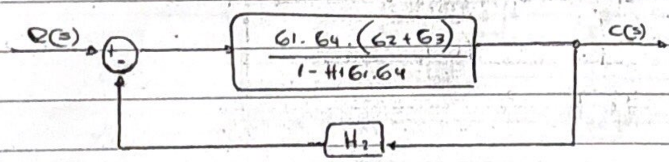
Guía TP N° 2

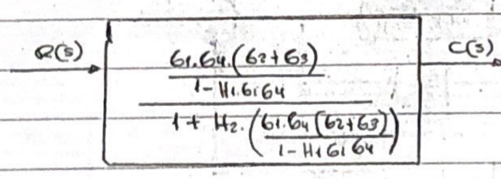
1

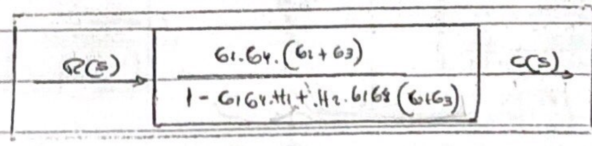

R(s)
G3
G1
G4
G2
H1
H2
C(s)


R(s)
G1.G4
G2+G3
H1
H2
C(s)

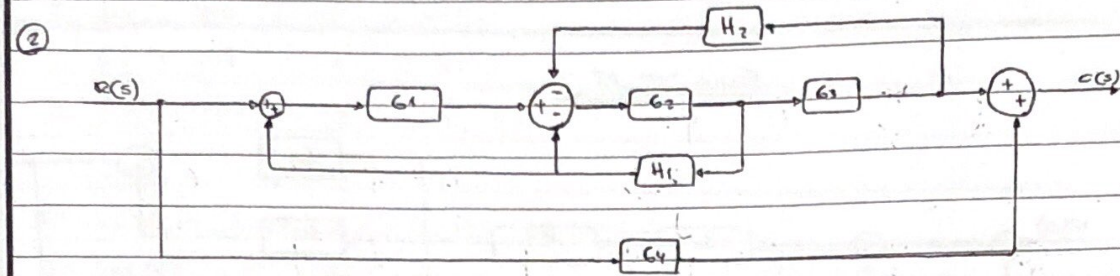

R(s)
G1.G4 / 1-H1G1.G4
G2+G3
H2
C(s)

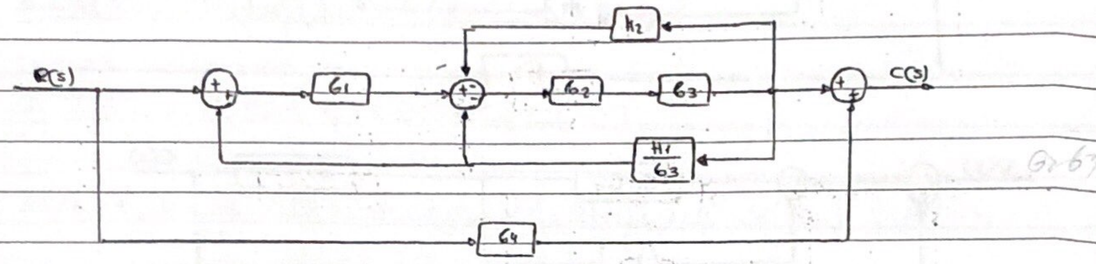

R(s)
G1. G4 .(G2+G3) / 1-H1G1.G4
H2
C(s)

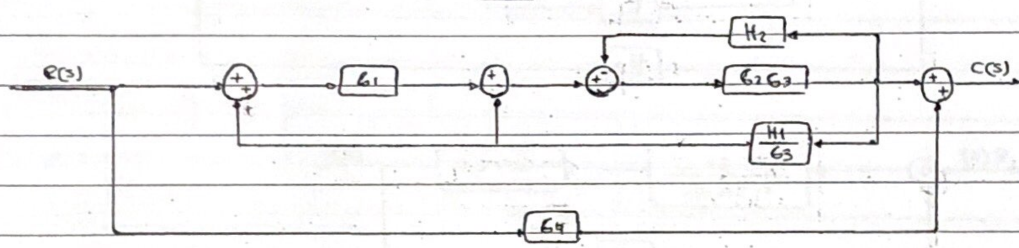

R(s)
G1.G4.(G2+G3) / 1-H1.G1G4
1+ H2.(G1.G4(G2+G3) / 1-H1G1G4)
C(s)

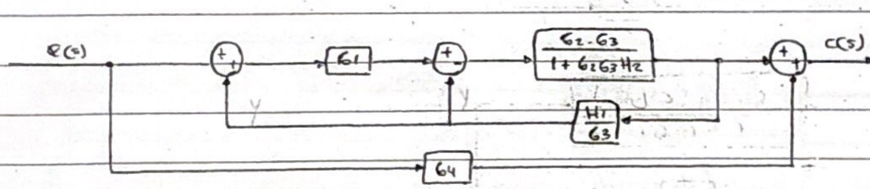

R(s)
G1.G4.(G2+G3)
1- G1G4.H1+ H2.G1G4(G2G3)
C(s)

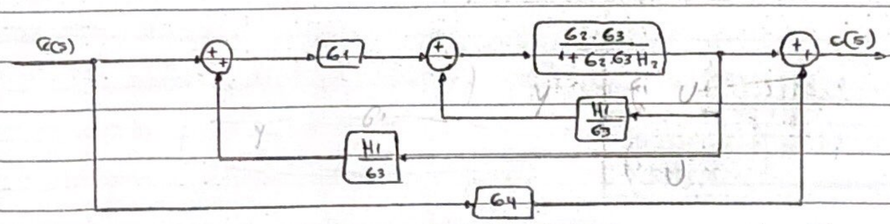
②

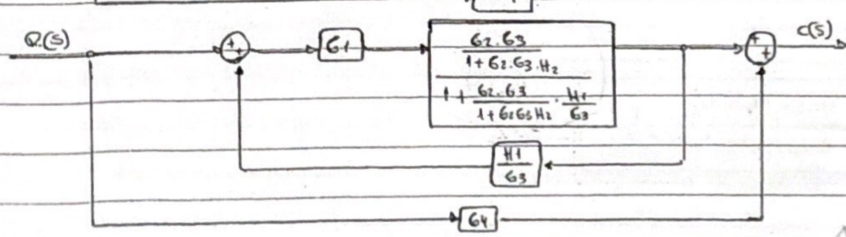








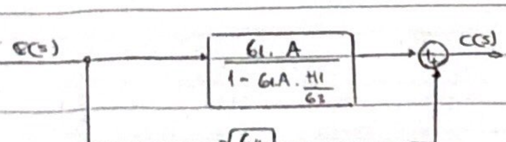


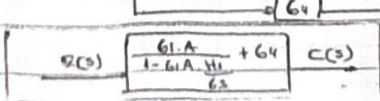


$$A = \frac{\frac{G_2 G_3}{1+G_2G_3H_2}}{1+\frac{G_2G_3}{1+G_2G_3H_2}\cdot\frac{H_1}{G_3}}$$

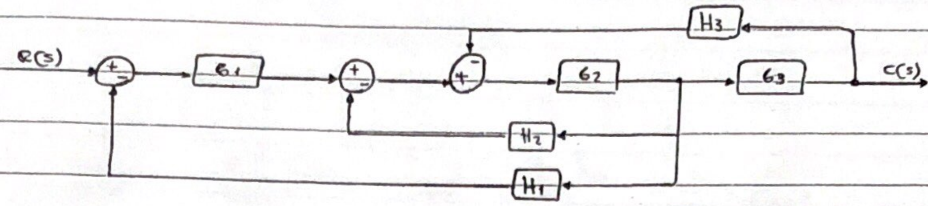
$A = 0,4$

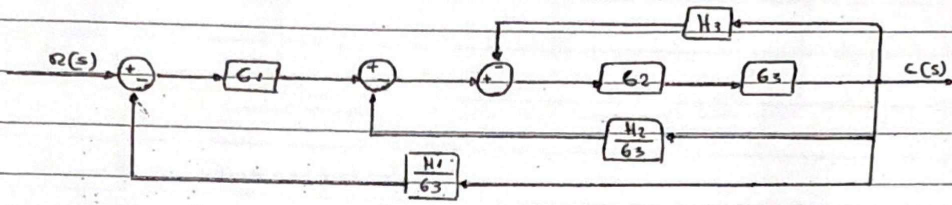
$$A = \frac{2\cdot 3}{1+12} \qquad 1+\frac{2}{1+12} \qquad \frac{6/13}{1+2/13}$$

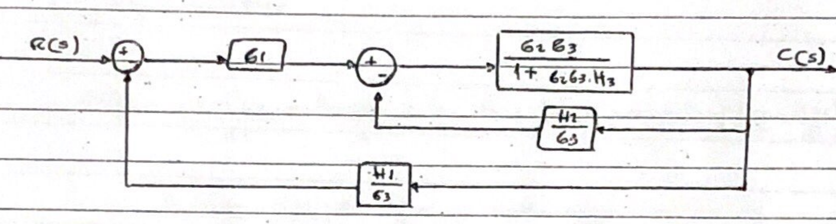


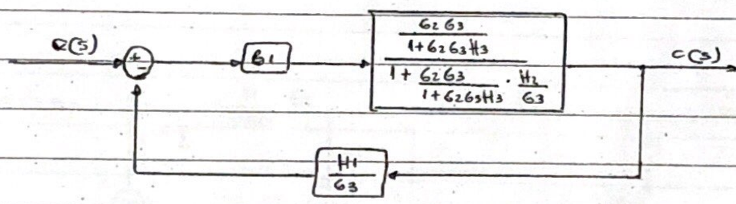


3

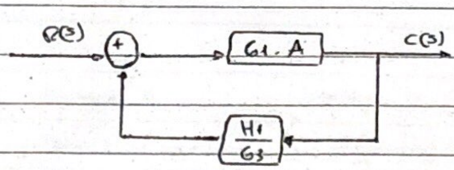


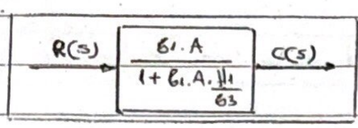






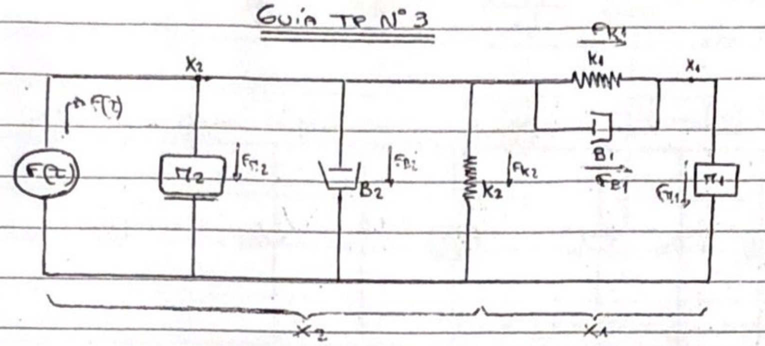
$$A = \frac{\frac{G_2 G_3}{1+G_2G_3H_3}}{1+\frac{G_2 G_3}{1+G_2G_3H_3}\cdot\frac{H_2}{G_3}}$$





Guía TP N° 3

(1) 1.1) a-



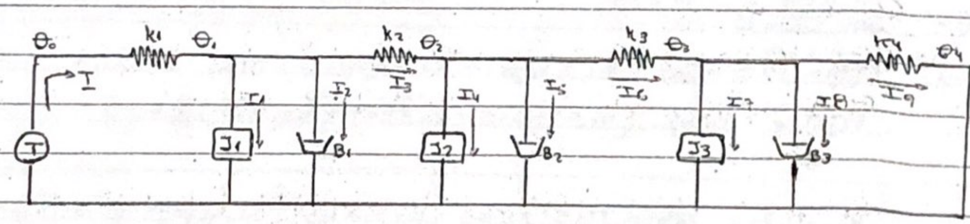
b- $F(t) = F_{M2} + F_{B2} + F_{K2} + F_{K1} + F_{M1}$

$F_{K1} + F_{B1} = F_{M1}$

$$F(t) = M_2 \cdot \frac{d^2x_2}{dt^2} + B_2 \cdot \frac{dx_2}{dt} + k_2 \cdot x_2 + K_1 (x_2 - x_1) + B_1 \cdot \left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right)$$

$$B_1 \left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right) + k_1 (x_2 - x_1) = M_1 \cdot \frac{d^2x_1}{dt^2}$$

1.2) a-



b-

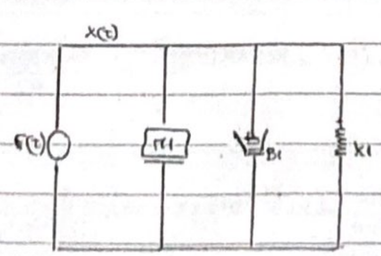
$T(t) = I \quad \Longrightarrow \quad T(t) = k_1 \cdot (\theta_0 - \theta_1)$

$I = I_1 + I_2 + I_3 \qquad k_1 (\theta_0 - \theta_1) = J_1 \cdot \frac{d^2\theta_1}{dt^2} + B_1 \cdot \frac{d\theta_1}{dt} + k_2 \cdot (\theta_1 - \theta_2)$

$I_3 = I_4 + I_5 + I_6 \longrightarrow k_2 (\theta_1 - \theta_2) = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \cdot \frac{d\theta_2}{dt} + k_3 (\theta_2 - \theta_3)$

$I_6 = I_7 + I_8 + I_9 \Longrightarrow k_3 (\theta_2 - \theta_3) = J_3 \cdot \frac{d^2\theta_3}{dt^2} + B_3 \cdot \frac{d\theta_3}{dt} + k_4 \cdot (\theta_3 - \theta_4)$

1.3) a-

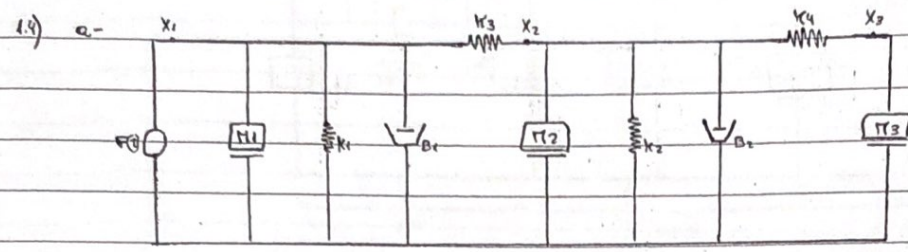


húsares

b- $F(t)\cdot l_1 = f'(t)\cdot l_2 \Longrightarrow \frac{F(t)\cdot l_1}{l_2} = M_1\cdot\frac{d^2x}{dt^2} + B_1\cdot\frac{dx}{dt} + K_1\cdot x$

$f'(t) = F_M + F_B + F_K$

1.4) a-



b- $F(t) = F_{M1} + F_{B1} + F_{K1} + F_{K3} \longrightarrow F(t) = M_1\cdot\frac{d^2x_1}{dt^2} + B_1\cdot\frac{dx_1}{dt} + K_1\cdot x_1 + K_3\cdot(x_2-x_1)$

$F_{K3} = F_{M2} + F_{B2} + F_{K2} + F_{K4} \longrightarrow K_3(x_2-x_1) = M_2\cdot\frac{d^2x_2}{dt^2} + B_2\cdot\frac{dx_2}{dt} + K_2\cdot x_2 + K_4\cdot(x_3-x_2)$

$F_{K4} = F_{M3} \Longrightarrow K_4(x_3-x_2) = M_3\cdot\frac{d^2x_3}{dt^2}$

c- Con CI = 0

$F(s) = M_1S^2X_1(s) + B_1SX_1(s) + K_1\cdot X_1(s) + K_3X_2(s) - K_3\cdot X_1(s)$

$F(s) = X_1(s)\cdot(M_1S^2 + B_1S + K_1 - K_3) + K_3\cdot X_2(s)$

$K_3\cdot X_2(s) - K_3\cdot X_1(s) = M_2S^2X_2(s) + B_2SX_2(s) + K_2X_2(s) + K_4\cdot X_3(s) - K_4\cdot X_2(s)$

$-K_3X_1(s) = X_2(s)\cdot(M_2S^2 + B_2S + K_2 - K_4) + K_4X_3(s)$

$X_1(s) = -\frac{X_2(s)\cdot(M_2S^2 + B_2S + K_2 - K_4) + K_4X_3(s)}{K_3}$

$K_4X_3(s) - K_4X_2(s) = M_3S^2X_3(s)$

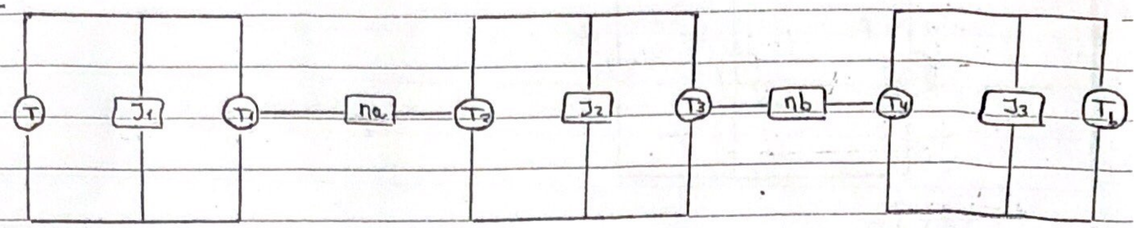
$X_2(s) = \frac{-M_3S^2X_3(s) + K_4X_3(s)}{K_4} = -\frac{X_3(s)\cdot(M_3S^2 + K_4)}{K_4}$

$F(s) = -\frac{X_2(s)\cdot(M_2S^2 + B_1S + K_2 - K_4 - K_3) + K_4\cdot X_3(s)}{K_3}\cdot(M_1S^2 + B_1S + K_1 - K_3) + X_2(s)\cdot K_3$

$F(s) = X_3(s)\left[\frac{\frac{(M_3S^2+K_4)}{K_4}(M_2S^2 + B_1S + K_2 - K_4 - K_3) + K_4}{K_3}\right]\cdot(M_1S^2 + B_1S + K_1 - K_3) - \frac{X_3(s)(M_3S^2+K_4)\cdot K_3}{K_4}$

$$\boxed{F(s) = X_3(s)\cdot\left[\frac{\frac{M_3S^2+K_4}{K_4}\cdot(M_2S^2 + B_1S + K_2 - K_4 - K_3) + K_4}{K_3}\cdot(M_1S^2 + B_1S + K_1 - K_4) - \frac{(M_3S^2+K_4)\cdot K_3}{K_4}\right]}$$

1.5) a-



b-

$$T = J_1 \frac{d^2\theta_1}{dt^2} + T_1 \qquad n_a = \frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_1}{N_2} \longrightarrow T_1 = \frac{T_2}{n_a}$$

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + T_3 \qquad n_b = \frac{T_4}{T_3} = \frac{\theta_2}{\theta_3} = \frac{N_3}{N_4} \longrightarrow T_3 = \frac{T_4}{n_b}$$

$$T_4 = J_3 \frac{d^2\theta_3}{dt^2} + T_L$$

c-

$$T = J_1 \frac{d^2\theta_1}{dt^2} + \frac{T_2}{n_a} = J_1 \frac{d^2\theta_1}{dt^2} + \frac{J_2}{n_a}\frac{d^2\theta_2}{dt^2} + \frac{T_3}{n_a} = J_1 \frac{d^2\theta_1}{dt^2} + \frac{J_2}{n_a}\frac{d^2\theta_2}{dt^2} + \frac{T_4}{n_a n_b}$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + \frac{J_2}{n_a}\frac{d^2\theta_2}{dt^2} + \frac{J_3}{n_a n_b}\frac{d^2\theta_3}{dt^2} + \frac{T_L}{n_a n_b}$$

$$n_a = \frac{\theta_1}{\theta_2} \longrightarrow \theta_2 = \frac{\theta_1}{n_a}$$

$$n_b = \frac{\theta_2}{\theta_3} \longrightarrow \theta_3 = \frac{\theta_2}{n_b} = \frac{\theta_1}{n_a n_b}$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + \frac{J_2}{n_a^2}\frac{d^2\theta_1}{dt^2} + \frac{J_3}{n_a^2 n_b^2}\frac{d^2\theta_1}{dt^2} + \frac{T_L}{n_a n_b}$$

$$\boxed{T = \underbrace{\left(J_1 + \frac{J_2}{n_a^2} + \frac{J_3}{n_a^2 n_b^2}\right)}_{\text{J EQUIVALENTE}} \cdot \frac{d^2\theta_1}{dt^2} + \frac{T_L}{n_a n_b}}$$

d- SI $N_1 = 10$, $N_2 = 50$, $N_3 = 10$ Y $N_4 = 20$

$n_a = \frac{1}{5}$

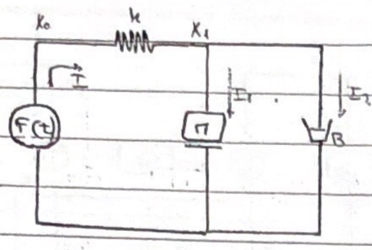
EN CAMBIO SI $N_2 = 20$ Y $N_4 = 40$ ENTONCES $n_a = \frac{1}{2}$

LA ACELERACIÓN ANGULAR ES $\alpha = \frac{d^2\theta_1}{dt^2}$

MIENTRAS MÁS CHICO SEA Na, J EQUIVALENTE VA A SER MAYOR POR LO QUE α TAMBIEN VA A SER MAYOR.

ENTONCES EL SISTEMA QUE MAYOR ACELERACIÓN ANGULAR VA A TENER ES EL QUE TIENE $n_a = \frac{1}{5}$

② 2.1)



$I = I_1 + I_2$

$F(t) = k.(x_0 - x_1)$ (I)

$k(x_0 - x_1) = M.\frac{d^2x_1}{dt^2} + B\frac{dx_1}{dt}$

$k.\,x_0(t) = M\frac{d^2x_1}{dt^2} + B.\frac{dx_1}{dt} + k x_1 \longrightarrow x_0(t) = \frac{M}{k}\frac{d^2x_1}{dt^2} + \frac{B}{k}\frac{dx_1}{dt} + x_1$

$X_0(s) = \frac{M}{k}s^2 X_1(s) + \frac{B}{k}s\,X_1(s) + X_1(s)$

$\boxed{X_0(s) = X_1(s).\left(\frac{M}{k}s^2 + \frac{B}{k}s + 1\right)}$ ⟶ MODELO QUE DESCRIBE COMPORTAMIENTO DEL SISTEMA

FUNCIÓN DE TRANSFERENCIA ⟶ $\frac{X_1(s)}{X_0(s)} = \frac{1}{\frac{M}{k}s^2 + \frac{B}{k}s + 1}$ $C(s)$, $R(s)$

$\frac{C(s)}{R(s)}$

SE NORMALIZA MULTIPLICANDO POR $\frac{k}{M}$ CADA TERMINO ⟶ $\frac{X_1(s)}{X_0(s)} = \frac{\frac{k}{M}}{s^2 + \frac{B}{M}s + \frac{k}{M}}$

FORMA NORMAL: $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ F.T = $\frac{\text{Salida}}{\text{Entrada}}$

2ζ

F.N = $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\omega_n^2 = \frac{k}{m}$; $2\zeta\omega_n = \frac{B}{m}$

$m = 20\,kg$

$k = 150\,N/m$

$B = 90\,\frac{N.seg}{m}$

$F(t)$ = ESCALÓN UNITARIO

$\omega_n^2 = \frac{k}{m} = \frac{150\,\frac{kg.m}{seg^2}.\frac{1}{m}}{20\,kg} = 7{,}5\,\frac{1}{seg^2}$

$2\zeta\omega_n = \frac{B}{m} = \frac{90\,\frac{kg.m}{seg}.\frac{seg}{m}}{20\,kg} = 4{,}5$

FRECUENCIA NATURAL NO AMORTIGUADA ⟶ $\omega_n = \sqrt{7{,}5\,\frac{1}{seg^2}} = 2{,}73\,\frac{1}{seg} \Longrightarrow \boxed{\omega_n = 2{,}73\,\frac{1}{seg}}$

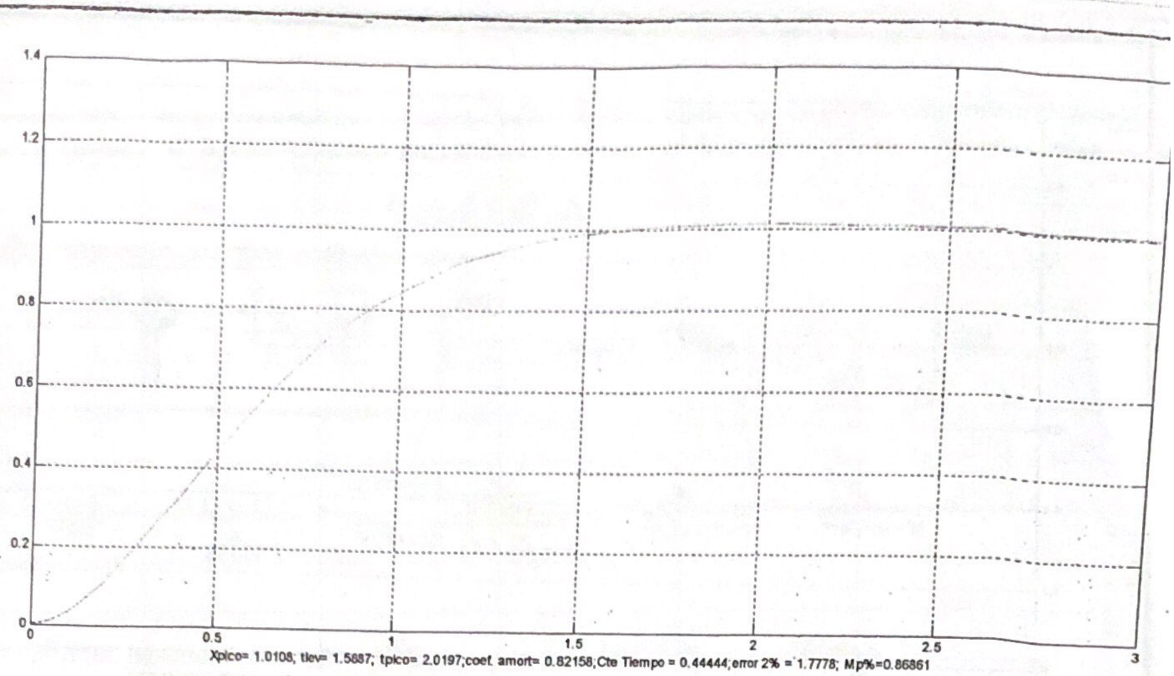
COEFICIENTE DE AMORTIGUAMIENTO ⟶ $\zeta = \frac{4{,}5}{2\omega_n} = \frac{4{,}5}{2.\,2{,}73} = 0{,}82 \Longrightarrow \boxed{\zeta = 0{,}82}$

FRECUENCIA NATURAL AMORTIGUADA ⟶ $\omega_d = \omega_n\sqrt{1-\zeta^2} = 2{,}73\sqrt{1-0{,}82^2} = 1{,}56 \Longrightarrow \boxed{\omega_d = 1{,}56}$

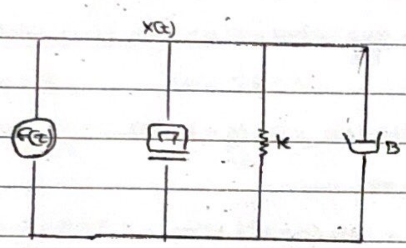
TIEMPO DE SOBREPICO ⟶ $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{1{,}56\,\frac{1}{seg}} = \Longrightarrow \boxed{T_p = 2{,}01\,seg}$

VALOR DE SOBREPICO ⟶ $X_p = 1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 1 + e^{-\frac{0{,}82\pi}{\sqrt{1-0{,}82^2}}} = 1 + 0{,}011 \Longrightarrow \boxed{X_p = 1{,}011}$

$= \frac{4}{\ldots} = \frac{4}{\ldots} = 1{,}78 \longrightarrow \boxed{T_s = 1{,}78}$



2.2)



$$F(t) = M\frac{d^2x}{dt} + B\frac{dx}{dt} + Kx$$

$$F(s) = M s^2 X(s) + B s \cdot X(s) + K \cdot X(s)$$

$$F(s) = X(s) \cdot (M s^2 + Bs + K)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2+Bs+K} = \frac{\frac{1}{M}}{s^2+\frac{B}{M}s+\frac{K}{M}}$$

MULTIPLICO Y DIVIDO POR $K \longrightarrow = \frac{1}{K} \cdot \left(\frac{\frac{K}{M}}{s^2+\frac{B}{M}s+\frac{K}{M}} \right)$

$$\omega_n^2 = \frac{K}{m} \quad ; \quad 2\zeta\omega_n = \frac{B}{M}$$

$M = 5\,kg \quad , \quad K = 1250\,\frac{N}{m} \quad , \quad B = 30\,\frac{N.seg}{m}$

$$\omega_n^2 = \frac{1250\,\frac{kg.m}{seg^2}\,\frac{1}{m}}{5\,kg} = 250\,\frac{1}{seg^2} \Longrightarrow \boxed{\omega_n = 15{,}81\,\frac{1}{seg}}$$

$$2\zeta\omega_n = \frac{30}{5} = 6 \longrightarrow \zeta = \frac{6}{2\omega_n} = \frac{6}{2 \cdot 15{,}81} = 0{,}1897 \Longrightarrow \boxed{\zeta = 0{,}1897}$$

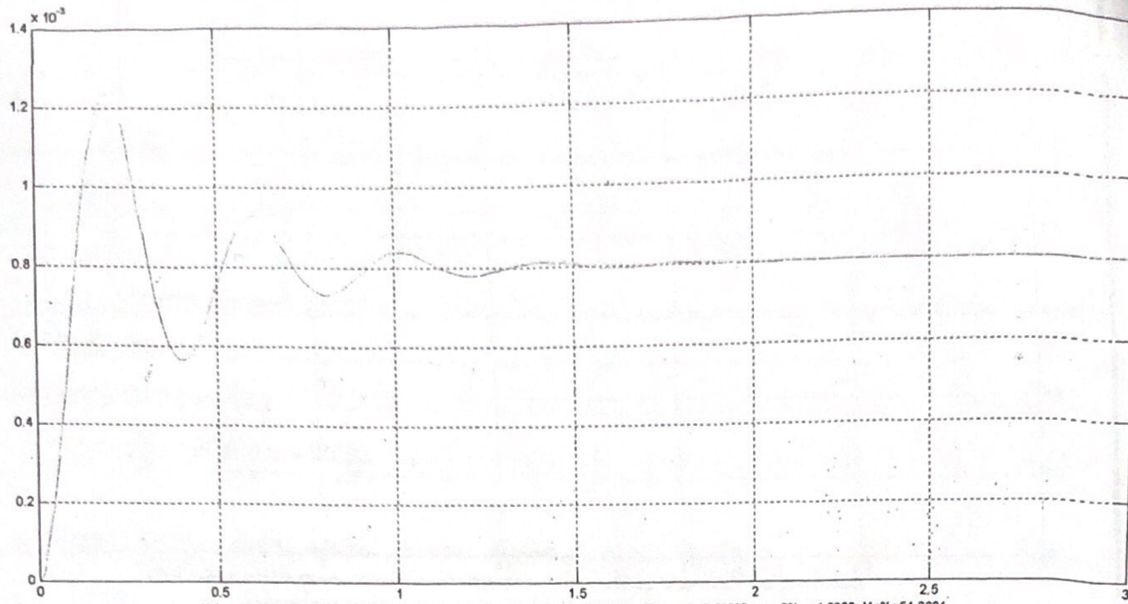
$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 15{,}81 \cdot \sqrt{1-0{,}1897^2} = 15{,}52 \Longrightarrow \boxed{\omega_d = 15{,}52}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{15{,}52} = 0{,}2024 \Longrightarrow \boxed{t_p = 0{,}2024}$$

M_p = VALOR PICO CUANDO TIENDE A 1 $= e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{-\frac{0{,}189\pi}{\sqrt{1-0{,}189^2}}} = 0{,}562$

$$X_p = \frac{F(t)}{K} + \frac{M_p}{K} = \frac{1}{1250} + \frac{0{,}562}{1250} = 0{,}001235 \Longrightarrow \boxed{X_p = 0{,}001235}$$

$$T_s = 4\tau = \frac{4}{\zeta\omega_n} = \frac{4}{0{,}189 \cdot 15{,}52} = 1{,}36 \Longrightarrow \boxed{T_s = 1{,}36}$$



Xpico= 0.0012347; tlev= 0.10856; tpico= 0.20677;coef. amort= 0.18974;Cte Tiempo = 0.33333;error 2% = 1.3333; Mp%=54.3091

2.3) EN $t=0 \rightarrow P = K \cdot X(0) \rightarrow X(0) = \frac{P}{K} = \frac{100}{82} = 1{,}21$

CI: $X(0) = 1{,}21$; $X'(0) = 0$

EN $t>0 \rightarrow M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = 0$; $M=8$, $B=10$, $K=82$

ECUACIÓN CARACTERÍSTICA: $8D^2+10D+82=0$

$$D_{1,2} = \frac{-10 \pm \sqrt{100-4.8.82}}{16} = \frac{-10 \pm 50{,}23\,i}{16} = -0{,}625 \pm 3{,}13\,i$$

$$x(t) = C_1 \cdot e^{-0{,}625t} \cdot \cos(3{,}13t) + C_2\, e^{-0{,}625t} \cdot \operatorname{sen}(3{,}13t)$$

$$x(0) = 1{,}21 = C_1$$

$$x'(t) = -0{,}625\, C_1 \cdot e^{-0{,}625t} \cdot \cos(3{,}13t) + \left(-3{,}13\, C_1\, e^{-0{,}625t} \cdot \operatorname{sen}(3{,}13t)\right) - 0{,}625\, C_2\, e^{-0{,}625t} \operatorname{sen}(3{,}13t) + 3{,}13\, C_2\, e^{-0{,}625t} \cdot \cos(3{,}13t$$

$$x'(0) = -0{,}625\, C_1 + 3{,}13\, C_2 = 0 \Longrightarrow C_2 = 0{,}2416$$

$$\boxed{x(t) = 1{,}21\, e^{-0{,}625t} \cdot \cos(3{,}13t) + 0{,}2416 \cdot e^{-0{,}625t} \cdot \operatorname{sen}(3{,}13t)}$$

$$8D^2+10D+82=0 \longrightarrow D^2 + \frac{5}{4}D + 10{,}25 = 0$$

$$w_n^2 = 10{,}25 \Longrightarrow \boxed{w_n = 3{,}20}$$

$$2\zeta w_n = \frac{5}{4} \longrightarrow \zeta = \frac{5}{4.2.w_n} = \frac{5}{8.3{,}20} = 0{,}1952 \Longrightarrow \boxed{\zeta = 0{,}1925}$$

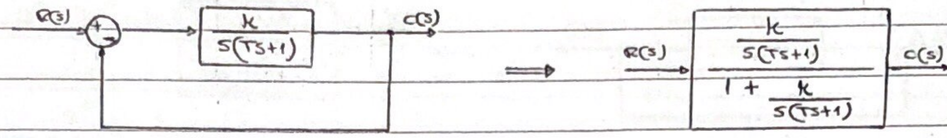
$$w_d = w_n \cdot \sqrt{1-\zeta^2} = 3{,}20\,\sqrt{1-0{,}19^2} = 3{,}13 \Longrightarrow \boxed{w_d = 3{,}13}$$

$$T_s\big|_{2\%} = 4\tau = \frac{4}{\zeta w_n} = \frac{4}{0{,}19.3{,}20} = 6{,}5 \longrightarrow \boxed{T_s = 6{,}5}$$

LA ALTURA PICO ES 1,21 QUE SE DA EN $t=0$

Guía TP N° 4

① Función de transferencia



$$\frac{C(s)}{R(s)} = \frac{\frac{k}{s.(Ts+1)}}{1+\frac{k}{s(Ts+1)}} = \frac{s(Ts+1).\frac{k}{s(Ts+1)}}{s(Ts+1).\left(1+\frac{k}{s(Ts+1)}\right)} = \frac{k}{s(Ts+1)+k} = \frac{k}{Ts^2+s+k}$$

Forma normalizada $\longrightarrow \dfrac{\frac{k}{T}}{s^2+\frac{s}{T}+\frac{k}{T}} \Longrightarrow w_n^2 = \dfrac{k}{T} \;;\; 2\zeta w_n = \dfrac{1}{T}$

Análisis curva de respuesta

$t_p = 3 \quad ; \quad X_p = 25,4\,\%$

$t_p = \dfrac{\pi}{w_d} \longrightarrow w_d = \dfrac{\pi}{t_p} = \dfrac{\pi}{3} = 1,047$

Si $X_p = 25,4\,\% \longrightarrow \zeta \cong 0,4$

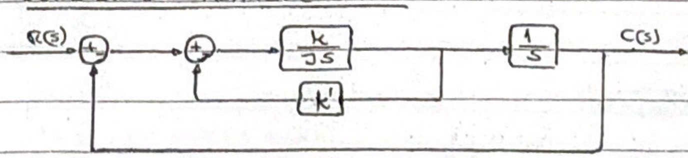
$w_d = w_n.\sqrt{1-\zeta^2} \longrightarrow w_n = \dfrac{w_d}{\sqrt{1-\zeta^2}} = \dfrac{1,047}{\sqrt{1-0,4^2}} = 1,1423$

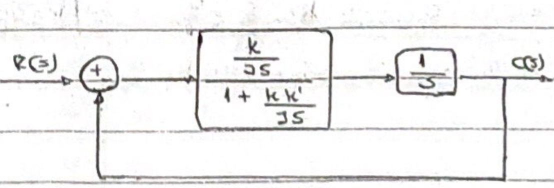
$\dfrac{k}{T} = w_n^2 = 1,1423^2 = 1,30$

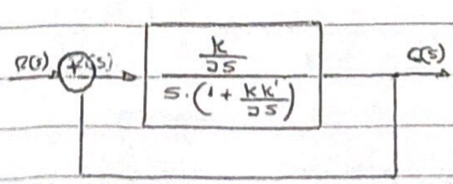
$\dfrac{1}{T} = 2\zeta w_n = 2 \cdot 0,4 \cdot 1,1423 = 0,913 \longrightarrow T = \dfrac{1}{0,913} \Longrightarrow \boxed{T = 1,094}$

$k = 1,30 \cdot T = 1,30 \cdot 1,094 \Longrightarrow \boxed{k = 0,42}$

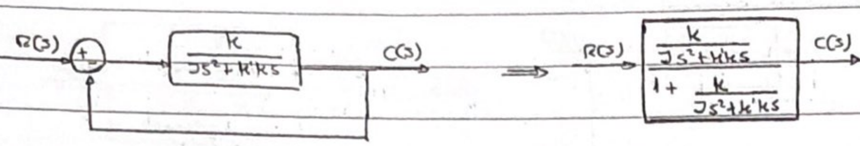
② Función de transferencia







$$\frac{\frac{K}{Js}}{\left(1+\frac{KK'}{Js}\right)s} = \frac{\frac{K}{Js}}{s+\frac{KK'}{J}} = \frac{\frac{J}{K}\cdot\frac{K}{Js}}{\frac{J}{K}\left(s+\frac{KK'}{J}\right)} = \frac{\frac{1}{s}}{\frac{Js+K'}{K}} = \frac{\frac{1}{s}}{\frac{Js+K'K}{K}} = \frac{K}{s(Js+K'K)} = \frac{K}{Js^2+K'Ks}$$



$$\frac{C(s)}{R(s)} = \frac{\frac{K}{Js^2+KK's}}{1+\frac{K}{Js^2+KK's}} = \frac{(Js^2+KK's)\cdot\frac{K}{(Js^2+KK's)}}{(Js^2+KK's)\cdot\left(1+\frac{K}{Js^2+KK's}\right)} = \frac{K}{Js^2+KK's+K}$$

FORMA NORMALIZADA $\longrightarrow \dfrac{\frac{K}{J}}{s^2+\frac{KK'}{J}s+\frac{K}{J}} \Longrightarrow \omega_n^2=\frac{K}{J}$; $2\zeta\omega_n = \frac{KK'}{J}$

Si $J=1 \longrightarrow \omega_n^2=K$; $2\zeta\omega_n = K\cdot K'$

$t_p=2$; $X_p=25\% \longrightarrow \zeta=0{,}4$

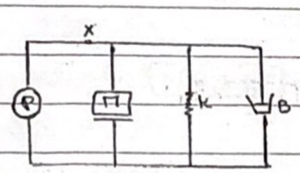
$t_p=\frac{\pi}{\omega_d} \longrightarrow \omega_d=\frac{\pi}{t_p}=\frac{\pi}{2}=1{,}57$

$\omega_d=\omega_n\sqrt{1-\zeta^2} \longrightarrow \omega_n=\frac{\omega_d}{\sqrt{1-\zeta^2}}=\frac{1{,}57}{\sqrt{1-0{,}4^2}}=1{,}71$

$K=\omega_n^2=1{,}71^2=2{,}93 \Longrightarrow \boxed{K=2{,}93}$

$K'K=2\zeta\omega_n=2\cdot0{,}4\cdot1{,}71=1{,}36 \longrightarrow K'=\frac{1{,}36}{K}=\frac{1{,}36}{2{,}93} \longrightarrow \boxed{K'=0{,}46}$

③



$P = M\frac{d^2x}{dt^2}+B\frac{dx}{dt}+K\cdot x$

$P = Ms^2X(s)+B\,s\,X(s)+KX(s)$

$\frac{X(s)}{P}=\frac{1}{Ms^2+Bs+K}=\frac{1}{K}\left(\frac{\frac{K}{M}}{s^2+\frac{B}{M}s+\frac{K}{M}}\right) \longrightarrow \omega_n^2=\frac{K}{M}$; $2\zeta\omega_n=\frac{B}{M}$

$t_p=2$

$t_p=\frac{\pi}{\omega_d} \longrightarrow \omega_d=\frac{\pi}{t_p}=\frac{\pi}{2}=1{,}57$

$X_{ESTABILIZACIÓN}=0{,}1 \longrightarrow X(\infty)=0{,}1$

Si $t\to\infty \Longrightarrow P=KX(\infty)$

$X(\infty)=\frac{P}{K}=\frac{2}{K}=0{,}1 \longrightarrow K=\frac{2}{0{,}1}=20 \Longrightarrow \boxed{K=20}$

$M_p=\frac{X(t_p)-X(\infty)}{X(\infty)}\cdot100=\frac{0{,}1095-0{,}1}{0{,}1}\cdot100 \longrightarrow M_p=9{,}5\% \Longrightarrow \zeta\cong0{,}6$

$\omega_n=\frac{\omega_d}{\sqrt{1-\zeta^2}}=\frac{1{,}57}{\sqrt{1-0{,}6^2}}=1{,}96$

$M=\frac{K}{\omega_n^2}=\frac{20}{1{,}96^2} \Longrightarrow \boxed{M=5{,}19}$

$B=2\zeta\omega_n\cdot M=2\cdot0{,}6\cdot1{,}96\cdot5{,}19 \Longrightarrow \boxed{B=12{,}21}$