Force Distribution in Walking Vehicles

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This paper addresses the problem of the appropriate distribution of forces between the legs of a legged locomotion system for walking on uneven terrain. The legs of the walking machine and the terrain form closed kinematic chains. The system is statically indeterminate and an optimal solution is desired for force control of the legs. In addition, as unisense force limitations are imposed on the wrenches acting at the feet, it is important to be able to determine for any given configuration whether or not a set of valid contact forces can be found which will ensure the stability of the vehicle. Fast and efficient algorithms to solve these problems have been developed. The trade-off between computational simplicity and optimality makes it necessary to resort to suboptimal algorithms. In particular, schemes based on the Moore-Penrose Generalized Inverse, or the pseudo inverse, and linear programming were investigated. An active compliance control scheme with varying leg compliances is shown to be a suitable paradigm for control. A variation of the linear programming technique, that is well-suited to the problem of predicting instability in the vehicle, is also presented.

1 Introduction

The kinematics of a walking vehicle involves simple closed chains and multiple frictional contacts between the actively coordinated articulated legs and the passive terrain. Multifingered grippers belong to the same class devices. The kinematics and force control problems engendered by such systems have been analyzed by [1-3,7,11,22-27] with reference to multifingered grippers, and by [4,8-10,16,19,25] for walking robots. Such systems typically have a high degree of static indeterminacy [7,21-22], and an optimal distribution of force inputs is desired for control.

Another problem arises at the planning level of control. In order to predict whether or not a given (planned) configuration is stable, it is essential to determine if a valid set of foot (finger) forces can be commanded to maintain equilibrium. A foot force is valid only if the force does not tend to "pull the terrain." In other words, the foot force cannot have a positive component along the outward pointing normal to the terrain. In addition, the friction cone angle formed by the components of the foot force normal and tangential to the ground must be within a certain limit so that the foot does not slip.

In multifingered grippers, in addition to determining forces required for equilibrium, it is essential to include forces which squeeze the object to ensure that there is no slip at the points of contact [11,15]. The force distribution problem in walking vehicles seems to be different to the extent that it is not necessary to squeeze the terrain. This is because the terrain merely supports the machine and the machine need not have the ability to grip the terrain. However, as seen later, this may not be the case on uneven terrain.

Optimization is a logical choice for analysis of under-determined systems and there are several mathematical techniques that can be employed for this purpose. Linear programming has been used in [7,16], and methods based on the pseudo inverse have been described by [8,22]. While the former is unsuitable for real time computation, the latter fails to yield satisfactory solutions, unless an iterative procedure is employed [20].

It is convenient to decompose the force field into the interaction force field and the equilibrating force field [11,13]. The interaction force between any two points is the vector difference between the two contact forces along the line joining the two points [25]. Broadly speaking, the interaction force field consists of all force vectors which have a zero interaction force component. In addition, these force vectors also have a zero net resultant. The equilibrating force field consists of force vectors which maintain equilibrium with the load-wrench (and hence must have a nonzero resultant). This decomposition facilitates the development of efficient algorithms for computing the force distribution, and more importantly, allows a clear insight into the problem.

In this paper, several algorithms for force distribution are proposed and discussed. A computer simulation on a model of the Adaptive Suspension Vehicle (ASV), a six legged walking machine built at The Ohio State University [24], was used to compare these algorithms. The result of this study are briefly presented here. The details of the model and the computer simulation are described in [10].

2 Modeling and Problem Formulation

It is assumed that a desired body trajectory is available. In other words, the "synthesis problem" is not considered — it is assumed that the desired acceleration and the current velocities of the vehicle are known. The legs are assumed to be

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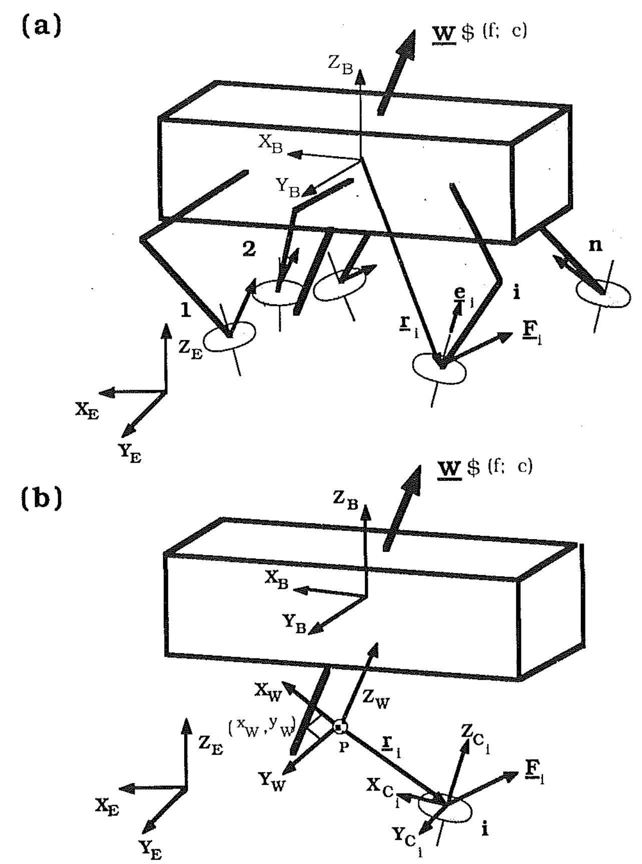


Fig. 1 Schematic of a walking vehicle (*a*) The system of forces (*b*) The coordinate systems. *n* is the number of supports, r_i is the position vector, F_i is the contact force, and e_i is the terrain normal at the i^{th} contact. w is the load wrench. E, B, W, and C_i are the coordinate systems fixed with respect to the earth, body, load wrench and the i^{th} foot, respectively

massless in comparison with the mass of the vehicle body — a realistic assumption for statically stable walking vehicles such as the ASV [24]. In addition, it is assumed that the contact interaction between the feet and the terrain can be described by a pure force through a contact center. In other words, contact moments are neglected. This model has been called point-contact by Salisbury [22].

The force (**R**) and moment (**M**) which must be applied by the feet on the body or by the terrain on the feet, are related to the given current velocities, ω (angular) and **v** (linear), and the desired accelerations by Euler's equations (all quantities are expressed in the body fixed reference frame shown in Fig. 1 (*a*)):

$$\mathbf{R} = \frac{d}{dt}(m\,\mathbf{v}) + w\mathrm{x}\; m\,\mathbf{v} - \mathbf{p}$$

$$\mathbf{M} = \frac{d}{dt}(\mathbf{H}) + \omega\mathrm{x}\,\mathbf{H} \tag{1}$$

where

$$\mathbf{p} = \mathbf{T}_E^B\,(-mg\,\hat{\mathbf{k}})$$

$\mathbf{T}_E^B$ is the homogeneous transform which transforms quantities defined in the earth fixed reference frame (**E**) to the body fixed frame (**B**), **H** is the angular momentum of the body in the body fixed frame, g is the acceleration due to gravity, and $\hat{\mathbf{k}}$ is the unit vector in the vertical direction pointing up. Now, a new coordinate system with the origin at the centroid of the centers of contact of the feet that are on the ground, and the z-axis parallel to the vector **R**, is defined. This is the X_W-Y_W-Z_W system shown in Fig. 1(*b*). From this point on all the quantities are referred to this new frame. If $\mathbf{F}_i$ are the contact forces at the contacts, whose centers are at $\mathbf{r}_i\,(x_i, y_i, z_i)$ in the new reference frame (**W**), the equations of equilibrium for the vehicle body can be written quite simply:

$$\sum_{i=1}^{n} \mathbf{F}_i = \mathbf{T}_B^W\,\mathbf{R}$$

$$\sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{F}_i) = \mathbf{T}_B^W\,\mathbf{M} \tag{2}$$

where n is the number of feet on the ground, and $\mathbf{T}_B^W$ is the transformation from **B** to **W**.
Equivalently, in matrix form,

$$\mathbf{G}\,\mathbf{q} = \mathbf{w} \tag{3}$$

where if $\mathbf{S}_{ix}$, $\mathbf{S}_{iy}$, and $\mathbf{S}_{iz}$ are the 6x1 vectors of Plucker coordinates for the zero pitch screw axes parallel to the x, y and z axes passing through the i^{th} contact point,

$$\mathbf{G} = \left[\mathbf{S}_{1x}\,\mathbf{S}_{1y}\,\mathbf{S}_{1z}\,\mathbf{S}_{2x}\,\mathbf{S}_{2y}\,\mathbf{S}_{2z}\,\ldots\,\mathbf{S}_{nx}\,\mathbf{S}_{ny}\,\mathbf{S}_{nz}\right]$$

$$\mathbf{q} = \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathrm{F}_n \end{bmatrix}$$

and

$$\mathbf{w} = \mathbf{T}_B^W \begin{bmatrix} \mathbf{R} \\ \mathbf{M} \end{bmatrix}$$

Alternatively,

$$\mathbf{G} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{I}_3 & \ldots & \mathbf{I}_3 \\ \mathbf{R}_1 & \mathbf{R}_2 & \ldots & \mathbf{R}_n \end{bmatrix}$$

where $\mathbf{I}_3$ is the 3 by 3 identity matrix and $\mathbf{R}_i$ is a skew symmetric 3 by 3 matrix:

$$\mathbf{R}_i = \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix}$$

Equation (3) represents a 6 by $3n$ system of equations which, in general, has $3n$-6 degrees of freedom. **w** may be interpreted as a wrench in screw coordinates, and hence it is called the *load wrench*. It is denoted by (**f**; **c**) in Fig. 1 in order to indicate the force and moment components of the wrench:

$$\mathbf{w} = \begin{bmatrix} \mathbf{f} \\ \mathbf{c} + \boldsymbol{\rho} \times \mathbf{f} \end{bmatrix}$$

where $\boldsymbol{\rho}$ is the position vector of any point on the wrench axis. Notice that **w** accounts for body inertial forces and thus this analysis is not a static analysis. However, it does not take the history of motion into consideration, and is, therefore, not a dynamic analysis in the true sense. Hence we use the term *quasi-static* in this paper. In general, the lines (zero pitch screws) described by the columns of **G**, do not all belong to the same linear complex, and hence the columns of **G** span the six dimensional space of all possible wrenches. In other words, the linear transformation represented by **G** is *onto* and the system of equations is consistent.

3 Analysis

Optimization. The redundancy in equation (3) can be resolved through optimization of the force field based on suitable criteria. It is necessary to ensure that the component of the foot force normal to the terrain is positive. In addition, the ratio of the tangential force component to the normal force component should be limited to within a maximum threshold value to prevent the possibility of slipping. The maximum value

Table 1 Optimization of foot contact forces

VARIABLES

$F_{1x}, F_{1y}, F_{1z}, F_{2x}, F_{2y}, F_{2z}, \ldots, F_{nx}, F_{ny}, F_{nz}$

EQUALITY CONSTRAINTS

6 Equations of Equilibrium $\rightarrow$ $\underline{G}\ \underline{q}=\underline{w}$

INEQUALITY CONSTRAINTS

$$\frac{\underline{F}_i \cdot \underline{n}_i}{|\underline{F}_i|} \geq \cos \phi_{\max}, \quad i = 1, \ldots, n$$

OBJECTIVE FUNCTION (to be minimized)

$$-\text{MINIMUM}\left[\left(\frac{\underline{F}_i \cdot \underline{n}_i}{|\underline{F}_i|}\right), \quad i = 1, \ldots, n\right]$$

or

$$\text{MAXIMUM}\ [|\underline{F}_i|, \quad i = 1, \ldots, n]$$

or

$$\sum_{i=1}^{n} |\underline{F}_i|$$

is limited to the maximum coefficient of static friction at the interface. A linear programming approach was suggested by McGhee and Orin [16,19] for the coordination of legged systems using an energy minimization objective function. However, it is believed that it is more important to consider criteria based on foot contact forces since, eventually, the traction characteristics are determined by reaction forces at the feet [10].

It is also desirable to minimize the largest contact force to prevent structural failure as well as to prevent saturation of the control system. An alternative function to consider may be the sum of the magnitudes of the contact forces. In the ASV, this is meaningful because the leakage losses in the hydraulic circuits powering the legs are proportional to the pressure differences across the actuator and are thus directly related to the contact forces. Thus it is more meaningful to consider objective functions based on foot forces than to minimize the instantaneous power consumption which does not guarantee global optimality.

This optimization exercise is summarized in Table 1. $\phi_{\max}$ is the maximum allowable friction angle. The nonlinear constraints can be approximated by linear functions (described in greater detail in a later section) so that a method like linear programming can be used.

The Pseudo-Inverse. A well-known mathematical technique traditionally used to solve underdetermined linear problems is the pseudo-inverse technique. The Moore Penrose Generalized Inverse or the pseudo inverse [18] finds the minimum norm, least-squares solution for the force vector. In this case, the transformation has been assumed to be onto and therefore the pseudo inverse of $\mathbf{G}$, $\mathbf{G}^+$ yields the minimum norm solution:

$$\mathbf{q} = \mathbf{G}^+\mathbf{w} \tag{4}$$

This minimum norm solution achieves the goal of decreasing the magnitude of the contact forces to some extent. The solution vector, $\mathbf{q}$, lies completely in the row space of $\mathbf{G}$. In other words, if $\mathbf{q}$ is expressed as $\mathbf{q} = \mathbf{q}_h + \mathbf{q}_p$, where $\mathbf{q}_h$ is a solution to the homogeneous system of equations, and $\mathbf{q}_p$ is the minimum norm solution given by equation (4),

$$\mathbf{G}\mathbf{q}_h = 0, \ \mathbf{G}\mathbf{q}_p = \mathbf{w}, \text{ and } \mathbf{q}_p = \mathbf{G}^+\mathbf{w}$$

In equation (3), the column space (and row space) of $\mathbf{G}$ is of dimension 6 (as the right inverse of $\mathbf{G}$ exists) and the null space, of dimension $3n$-6. It is possible to exploit the $3n$-6 degrees of freedom (null space components of $\mathbf{q}$) to obtain a solution that satisfies a secondary system performance goal (see [14–15]). The vector of measured (current) forces has been used as a reference vector in [9], so that

$$\mathbf{q}_d = \mathbf{G}^+\mathbf{w} + (\mathbf{I} - \mathbf{G}^+\mathbf{G})\,\mathbf{q}_a$$

where $\mathbf{q}_a$ is the vector of actual (measured) forces. This tends to smooth out discontinuities in forces caused by leg phase transitions (between protraction and retraction). However, this does not address the problem of improving traction. In fact, it is difficult to use this formulation to optimize contact conditions [10].

Decomposition of the Force Field. The $3n$ force components acting at the n contact points constitute a vector field which may be called a force field. This force field can be decomposed [Waldron 86] into two force fields:

(*a*) the equilibrating force field
(*b*) the interaction force field

The *interaction force* between any two contacts is the component of the vector difference between the contact forces, along the line joining the contact points. If i and j denote the two points, the interaction force $\mathbf{F}_{ij}$ is defined by:

$$\mathbf{F}_{ij} = (\mathbf{F}_i - \mathbf{F}_j).\ (\mathbf{r}_i - \mathbf{r}_j), \ \forall\ i, j = 1, \ldots, n, \ i \neq j.$$

These forces are similar to the scalar internal forces which describe the pinch between two fingers in a gripper [22]. There is a subtle difference between interaction forces and internal forces, which is only of academic interest and is explained elsewhere [13].

The *equilibrating forces* are forces required to maintain the body in equilibrium against the load wrench, and *do not* have any interaction force components. Thus these forces must have a nonzero resultant. The interaction force field consists of forces which must have a zero net resultant. It includes force components which squeeze the body (in the case of multifingered grippers) or the terrain (in the case of walking vehicles).

The central idea behind this decomposition is that the vector of desired foot forces belong to the equilibrating force field. This is so that the feet do not "fight each other." Thus all sets of interaction forces must equal zero. This is illustrated by examples for two and three contact point cases in Fig. 2. Mathematically, this condition can be expressed by:

$$(\mathbf{F}_i - \mathbf{F}_j).\ (\mathbf{r}_i - \mathbf{r}_j) = 0, \ \forall\ i, j = 1, \ldots, n, \ i \neq j \tag{5}$$

This is the *zero interaction force condition* [25].

It is possible to establish a correspondence with this decomposition, which was derived from the mechanics of locomotion, and a geometrical technique of resolving redundancies — the pseudo inverse. The following theorem which is proved in [13] is presented here without proof:

Theorem. If a body is subjected to multiple frictional contacts modeled by point contact, and if the system of zero pitch contact wrenches span a six-dimensional space, the Moore-Penrose generalized inverse (or the pseudo inverse) solution to the equilibrium equations yields a solution vector which lies in the equilibrating force field and has no interaction force components.

In other words, the pseudo inverse of **G** is a right inverse that yields a minimum norm solution, and this solution belongs to the equilibrating force field. It has also been shown [13] that this force field is a *helicoidal vector field* (see [5] for the definition of a helicoidal field), and is mathematically isomorphic to the velocity field of a rigid body. These results enable simple analytical expressions for the equilibrating force solution and closed form analytical formulae for equilibrating force vectors have been derived in [13]. However, an alternative scheme (which is derived from the decomposition) for computing the force distribution is pursued here. The scheme presented here is more efficient though it does sacrifice to a lesser extent the optimal characteristics of a minimum norm solution.

4 Force Distribution by Decomposition

Equilibrating Forces. The system of equations for the equilibrating force solution is further decomposed into two subsystems. The equilibrating force field is now partitioned into a set of forces which are parallel to the axis of the load wrench (or Z_W) and into another set of forces which lie on planes perpendicular to this axis.

The equilibrium equations involving the forces on the X_W-Y_W plane can be written in reference frame **W**:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 \\ -y_1 & x_1 & -y_2 & x_2 & \dots & -y_n & x_n \end{bmatrix} \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ \vdots \\ F_{nx} \\ F_{ny} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix} \tag{6}$$

This is an under-determined set of three equations with $2n$ unknowns which can be inconsistent if and only if the projections of the n contact points on the X_W-Y_W plane are concurrent. This can never happen in an actual situation. The hypothesis that the equilibrating forces cannot have any interaction force components yields nC_2 equality conditions of the form of equation (5) of which, it can be shown, only $2n$-3 are linearly independent. Combining these equations with (6), we have a $(nC_2 + 3)$x$2n$ system of equations in which $2n$ are linearly independent. Fortunately, an analytical inversion is possible and simple expressions (which are easily verified by substitution in (5-6)) are obtained:

$$F_{ix} = \frac{c(-y_i)}{nI}$$
$$F_{iy} = \frac{c(x_i)}{nI} \tag{7}$$

where

$$I = \sum_{i=1}^{n} \frac{(x_i^2 + y_i^2)}{n}$$

Having found the forces, F_{ix} and F_{iy}, if (x_w, y_w) are the coordinates of the point of intersection of the load wrench with the X_W-Y_W plane, the three remaining equations of equilibrium can be written as:

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ (x_1 - x_w) & (x_2 - x_w) & \dots & (x_n - x_w) \\ (y_1 - y_w) & (y_2 - y_w) & \dots & (y_n - y_w) \end{bmatrix} \begin{bmatrix} F_{1z} \\ F_{2z} \\ \vdots \\ F_{nz} \end{bmatrix} = \begin{bmatrix} f \\ \sum z_i F_{ix} \\ \sum z_i F_{iy} \end{bmatrix} \tag{8}$$

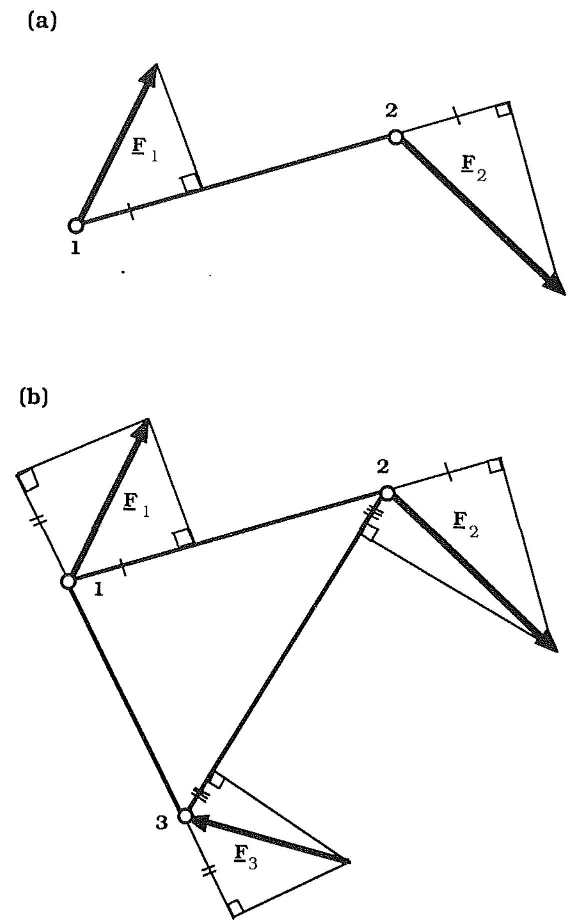


Fig. 2 The zero interaction force condition for (*a*) Two and (*b*) Three contacts (F_i is the contact force at the i^{th} contact)

This constitutes a 3xn system of equations which is undetermined when $n > 3$, or when n is equal to 3 and the support points are colinear. The later condition is dynamically unstable and can be safely excluded from this analysis.

In this case, the zero interaction force hypothesis over-constrains the problem and cannot be applied. Moreover the argument about the feet "fighting each other" would seem to more applicable to the forces on the X_w-Y_w plane (which would be coincidental with the horizontal plane in the absence of inertial forces). As an alternative, it is assumed that the feet have equal compliances along the Z_W direction. This is reasonable if the legs are assumed to be identical in terms of the structure and the control circuits, which is definitely true for the ASV. This leads to a planar force distribution [10]:

$$F_{iz} = A + B\,(x_i - x_w) + C\,(y_i - y_w) \tag{9}$$

in which the constants can be determined directly by substituting (9) into equation (8). This is also true for the special case in which n equals 3 and (8) represents 3 equations in 3 unknowns.

It may be shown that the solution obtained for each of the subproblems (6), (8), is in fact identical to the pseudo inverse solution for the two subproblems. However, the force distribution thus obtained differs from the pseudo inverse solution for equation (3), reported in [13]. We nevertheless loosely refer to this solution as the equilibrating force solution. Though equations (7) and (9) yield a force distribution which would equilibrate the load wrench, the solution does not lie in the equilibrating force field. This makes the method suboptimal (if we accept the minimum norm solution to be optimal) but its implementation is computationally more efficient and faster because of the closed form and simple nature of the expressions in (7) and (9).

Interaction Forces. In the previous subsection, we discussed methods to find appropriate equilibrating forces. It is often essential to superimpose a set of bias forces or interaction forces which will ensure that the friction angle constraints are met. This is similar to the idea of using null space components to satisfy a secondary objective function. In simple cases where the number of contacts or supports is restricted to 2 or 3, it is easy to describe the interaction force field. It is possible to obtain analytical expressions for the forces, if a solution exists [10] (Also see [1,6,17] for analysis of similar problems with reference to multifingered grippers.) However, when the number of contacts is greater than three, it is somewhat more difficult to visualize the interaction force field. Some preliminary studies in this context are reported in [13].

Linear programming may be used to determine the interaction forces. This questions the motivation for force decomposition, since linear programming could have been used for the original problem prior to the decomposition as discussed earlier. However, this method was initially pursued with two objectives. Firstly, it was possible that linear programming could offer a simpler method for obtaining the interaction forces only (as opposed to using it to compute the total force field). This was found later not to be the case. Secondly, linear programming is a simple and very reliable technique and it was felt that a solution for the interaction forces would yield some insight into the nature of the interaction force field. The application of this technique is briefly described in the next section.

5 Linear Programming Solution

A foot coordinate system ($\mathbf{C}_i$) is defined for each of the supports with the z-axis along the normal to the terrain pointing outward and the origin at the center of contact. If $\mathbf{e}_i$ is the normal for the i^{th} contact point in the earth-fixed coordinate system, and $\mathbf{R}^E_{Ci}$ is the rotation transformation which relates the foot coordinate system to the earth-fixed coordinate system,

$$\mathbf{R}^E_{Ci} = \begin{bmatrix} \dfrac{e_z}{s} & \dfrac{-e_x\, e_y}{t} & e_x \\ 0 & \dfrac{e_x^2\, e_z^2}{t} & e_y \\ \dfrac{-e_x}{s} & \dfrac{-e_y\, e_z}{t} & e_z \end{bmatrix},$$

$$s = \sqrt{e_x{}^2 + e_z{}^2},\ t = \sqrt{(e_y s)^2 + (e_x e_z)^4}$$

The force components $\mathbf{F}_i$ can be transformed into the foot coordinate systems, to yield the components ζ_i:

$$\zeta_i = \mathbf{R}^{Ci}_E\, \mathbf{R}^E_B\, \mathbf{R}^B_W\, \mathbf{F}_i$$

The inequality constraints in Table 1 can be easily linearized by substituting the friction cone with a "friction pyramid" [16]. The pyramid may be defined to lie entirely within the cone and to provide a conservative estimate. If μ is the maximum coefficient of friction, and μ_{eff}, the effective coefficient of friction is defined as $\mu/\sqrt{2}$, each of the four bounding planes of the pyramid may be represented as:

$$\begin{aligned} \xi_{ix} &\le \mu_{eff}\xi_{iz} \\ -\xi_{ix} &\le \mu_{eff}\xi_{iz} \\ \xi_{iy} &\le \mu_{eff}\xi_{iz} \\ -\xi_{iy} &\le \mu_{eff}\xi_{iz} \end{aligned} \tag{10}$$

There are a variety of objective functions that can be used some of which are listed in Table 1. The objective functions based on infinity norms add some complexity to the linear programming formulation. Extra variables must be introduced which increases the number of unknown and linear equations. Details of a method for such functions can be found in [10]. An alternative which is not listed in Table 1 is to maximize the smallest distance from the side-constraints, the friction angle inequality constraints. This can be done by considering $4n$ slack variables (one for each of the inequalities in (10) for each of the n footholds) and maximizing the smallest of the variables. Again, more information is available in [10].

6 Variable Compliance Method

Another way of resolving the static indeterminacy is by using the principle of geometric compatibility, which is extensively used for passive structures encountered in solid mechanics problems. If the body is assumed to be rigid (very low compliance compared to the legs), and the legs are assumed to be linearly elastic, possessing stiffnesses, k_{ix}, k_{iy}, and k_{iz} in the three orthogonal directions, then the force displacement relations are:

$$F_{ix} = k_{ix}\,\delta_{ix},\ F_{iy} = k_{iy}\,\delta_{iy},\ \text{and}\ F_{iz} = k_{iz}\,\delta_{iz}$$

where $\boldsymbol{\delta}_i$ is the vector displacement at the i^{th} contact point with components δ_{ix}, δ_{iy}, and δ_{iz}. If the walking platform has a small linear displacement, $\boldsymbol{\Delta}$ (components Δ_x, Δ_y, and Δ_z), and angular displacements in the form of small rotations, $\partial\Theta_x$, $\partial\Theta_y$, and $\partial\Theta_z$ about the x, y, and z axes, respectively, ($\partial\boldsymbol{\Theta}$ being a vector representation of the angular displacements) then, for all contacts:

$$\boldsymbol{\delta}_i = \boldsymbol{\Delta} + \partial\boldsymbol{\Theta} \times \mathbf{r}_i \tag{11}$$

These are the geometric compatibility conditions. Combining these conditions with the force-displacement relationships, the following equations can be written for each of the n legs:

$$\begin{bmatrix} F_{ix} \\ F_{iy} \\ F_{iz} \end{bmatrix} = \begin{bmatrix} k_{ix} & 0 & 0 & 0 & z_i k_{ix} & -y_i k_{ix} \\ 0 & k_{iy} & 0 & -z_i k_{iy} & 0 & x_i k_{iy} \\ 0 & 0 & k_{iz} & y_i k_{iz} & -x_i k_{iz} & 0 \end{bmatrix} \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \\ \partial\Theta_x \\ \partial\Theta_y \\ \partial\Theta_z \end{bmatrix} \tag{12}$$

This is a very important simplification since the $3n$ unknown force components have been expressed as functions of only six unknown quantities. Futher, as the equilibrium equations provide six conditions, in general, a unique solution for the six force displacements can be obtained by substitution in equation (3). However, the compliances (stiffnesses) of the legs still remain unknown. The stiffness is a function not only of the structure of the leg and the drives actuating it, but also a function of the simulated electronic compliance. The important point here is that these stiffnesses can be controlled.

Ideally, the compliances of the legs should be chosen to match or mismatch the terrain impedance with the leg impedance. A mismatch of leg impedance with the terrain impedance would minimize the power transfer through the leg to the terrain and in turn reduce energy spent in doing wasteful work in deforming the terrain. Alternatively, a strategy of varying compliances can be used in a scheme of hybrid control allocated by legs in which the compliances are selected to minimize dynamic force interactions between the legs [9]. In this case, in the absence of information about the terrain impedance or the exact control laws, a heuristic choice of compliances is made.

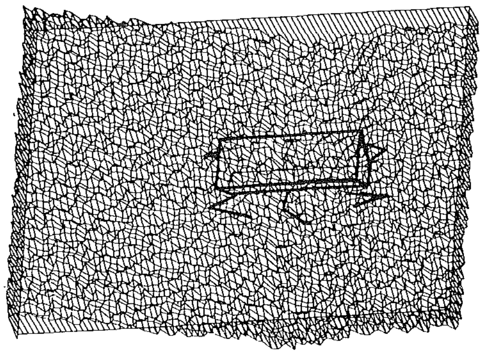


Fig. 3 Wire frame simulation model of the ASV and the rough terrain

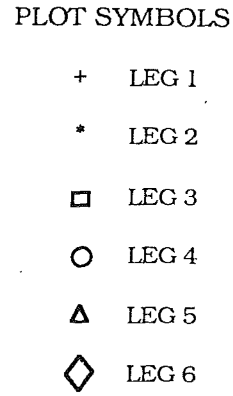
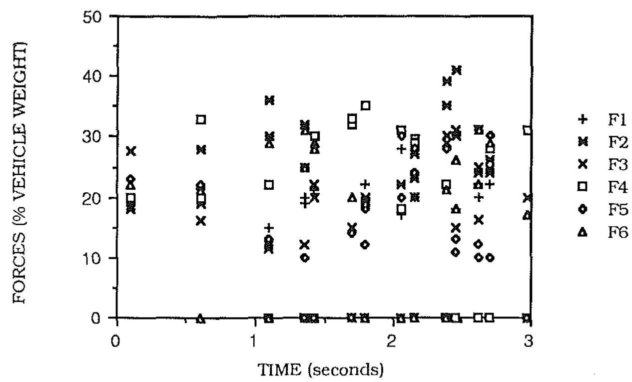


Fig. 4 Force distribution for the ASV. ϵ is the ratio of the tangential component to the normal component of the foot force.



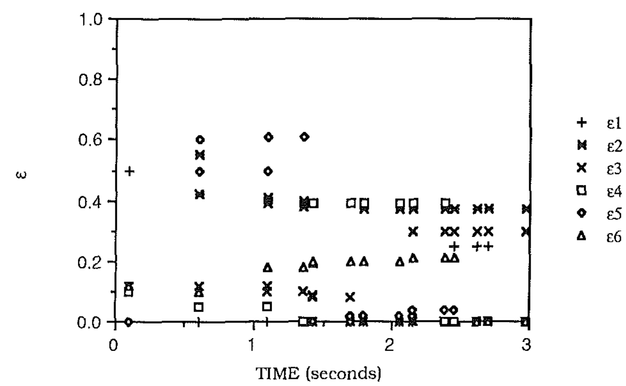


Fig. 4(*a*) Decomposition of the force field (equilibrating forces only)

Since the dominant component of the load wrench is the vehicle weight, the pitch of the wrench axis is likely to be close to zero. If the normals to the terrain surface at the contacts are parallel to the load-wrench, the z-component of contact force will have a zero friction angle. In such a situation, the friction angle associated with the net contact forces will be small. Thus, characterizing available footholds based on the angle between the terrain surface normals at the footholds and the wrench axis is appropriate. A soft (compliant) leg would be preferred for poor footholds and a stiff leg for good footholds.

The larger the angle between the wrench axis and the foothold normal, the lower is the desired stiffness. If k_{ix} and k_{iy} are assumed to be equal, the forces on the X_W-Y_W plane are as before (equation (7)). In the z-direction, then,

$$\begin{bmatrix} k_1 & k_2 & \ldots & k_n \\ k_1 y_1 & k_2 y_2 & \ldots & k_n y_n \\ -k_1 x_1 & -k_2 x_2 & \ldots & -k_n x_n \end{bmatrix} \begin{bmatrix} 1 & y_1 & -x_1 \\ 1 & y_2 & -x_2 \\ \vdots & \vdots & \vdots \\ 1 & y_n & -x_n \end{bmatrix} \begin{bmatrix} \Delta_z \\ \partial\Theta_x \\ \partial\Theta_y \end{bmatrix}$$

$$= \begin{bmatrix} f \\ f\,y_w + \sum z_i F_{iy} \\ -f\,x_w - \sum z_i F_{ix} \end{bmatrix} \tag{13}$$

or,

$$\begin{bmatrix} \sum k_i & \sum k_i y_i & -\sum k_i x_i \\ \sum k_i y_i & \sum k_i y_i^2 & -\sum k_i x_i y_i \\ -\sum k_i x_i & -\sum k_i x_i y_i & \sum k_i x_i^2 \end{bmatrix} \begin{bmatrix} \Delta z \\ \partial\Theta x \\ \partial\Theta y \end{bmatrix}$$

$$= \begin{bmatrix} f \\ f\,y_w + \sum z_i F_{iy} \\ -f\,x_w - \sum z_i F_{ix} \end{bmatrix}$$

The quantities Δ_z, $\partial\Theta_y$, and $\partial\Theta_x$ can be easily found by analytical inversion, and F_{iz} has a distribution:

$$F_{iz} = k_i\,(\Delta_z + y_i\,\partial\Theta_x \text{-} x_i\,\partial\Theta_y) \tag{14}$$

Analytical expressions for the constants in equation (14) can be obtained from [10]. The use of different compliances for different legs in a direction along the load wrench axis modifies the equilibrating forces obtained through equation (9). It produces a nonuniform distribution of forces as it redistributes the equilibrating forces so that the legs with poorer footholds take a smaller share of the load. It also provides ratios of compliances for the legs which can be used in active compliance control schemes as suggested in [9].

7 Comparative Study of Different Methods

Computer Simulation and Organization of Results

A computer simulation of several methods was performed using a model of the ASV. A 3 second interval of vehicle motion was considered in which the vehicle starts from rest at the origin (reference frame **E**) and reaches a point (7.45, 2.7, 0.28) (all dimensions are in feet), where the linear velocity is (3.18, 1.17, 0.26) feet/sec. A computer simulated uneven terrain that was generated by random numbers (shown in Fig. 3) was used. The vehicle weight is 7000 lb_m. The maximum allowable coefficient of friction was arbitrarily chosen to be 0.4 for all legs. More details regarding the model and the simulation can be obtained from [10].

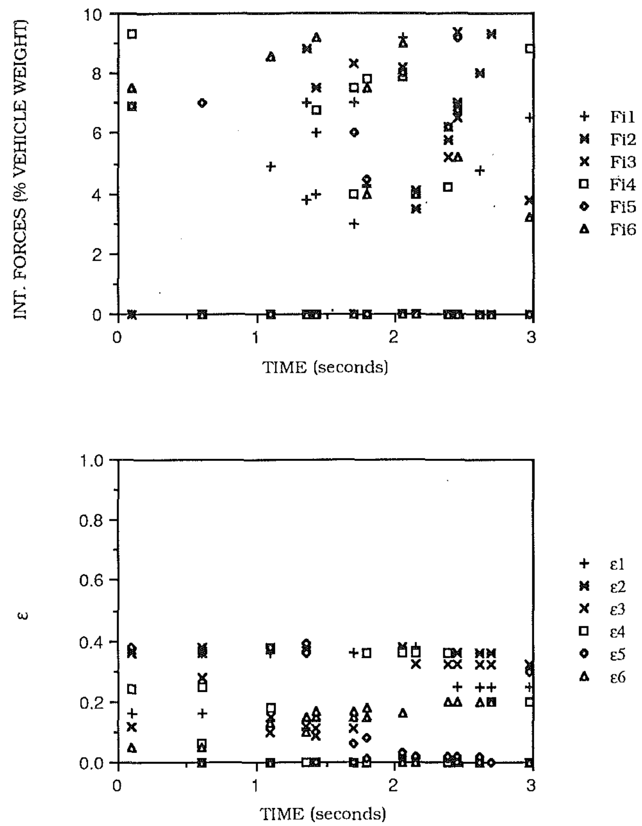


Fig. 4(*b*) Decomposition of the force field (interaction forces by linear programming)

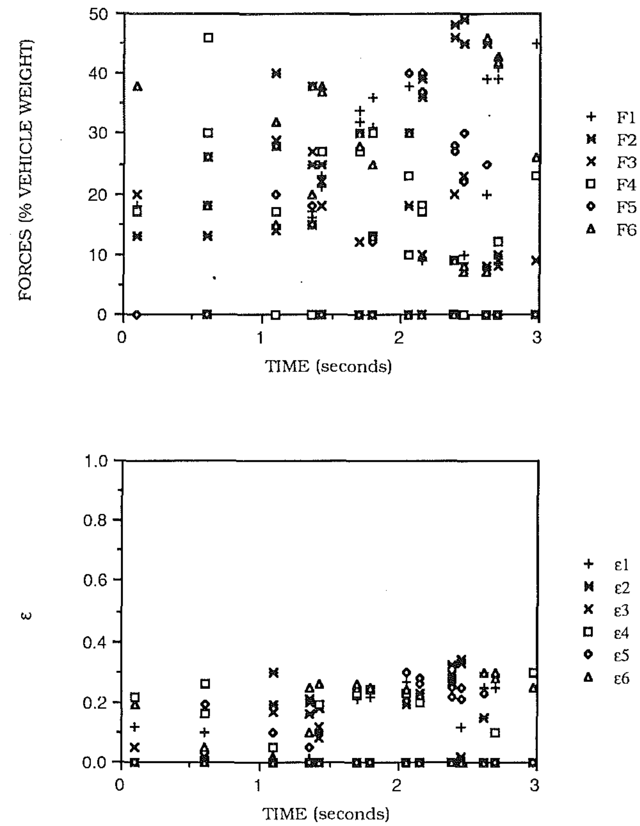


Fig. 4(*d*) Linear programming (maximizing smallest distance from frictional constraints)

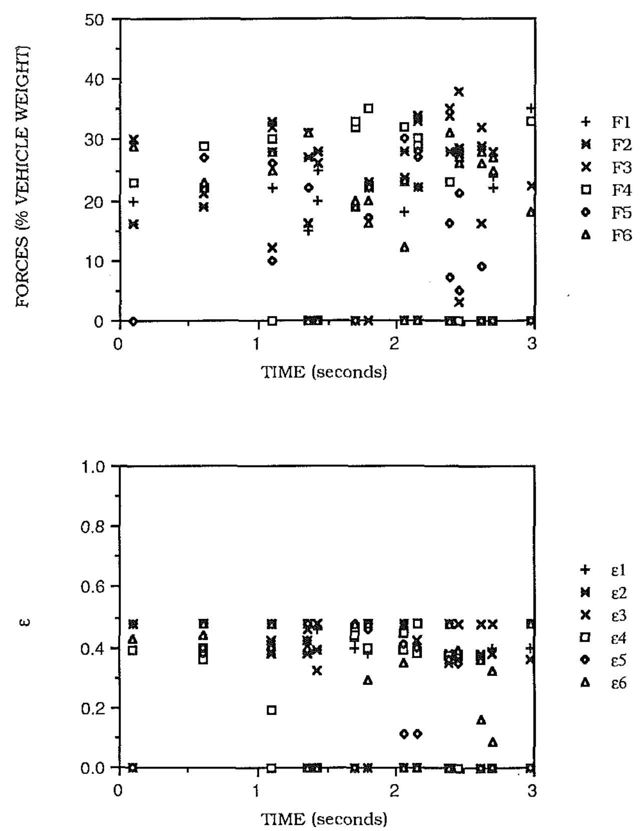


Fig. 4(*c*) Linear programming (minimization of the largest foot component)

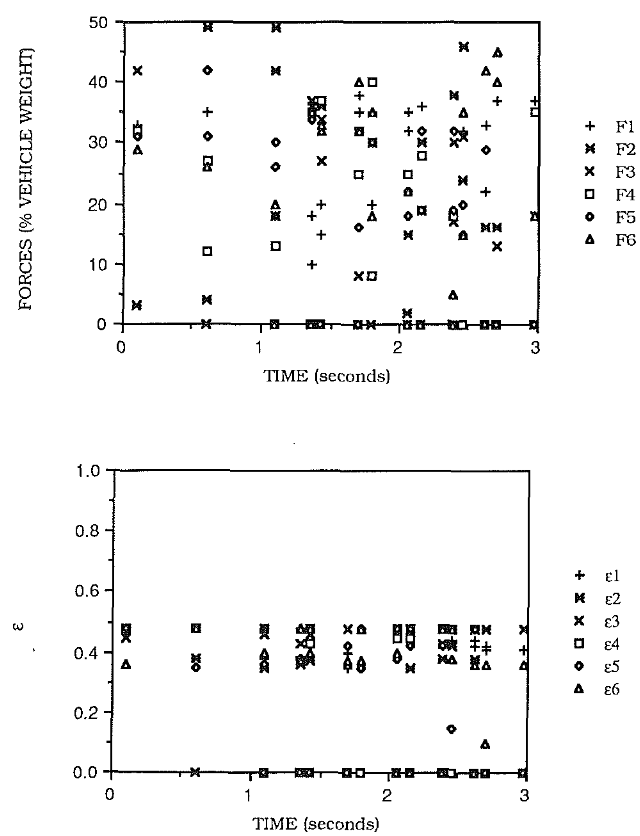


Fig. 4(*e*) Linear programming— Phase I, simplex method

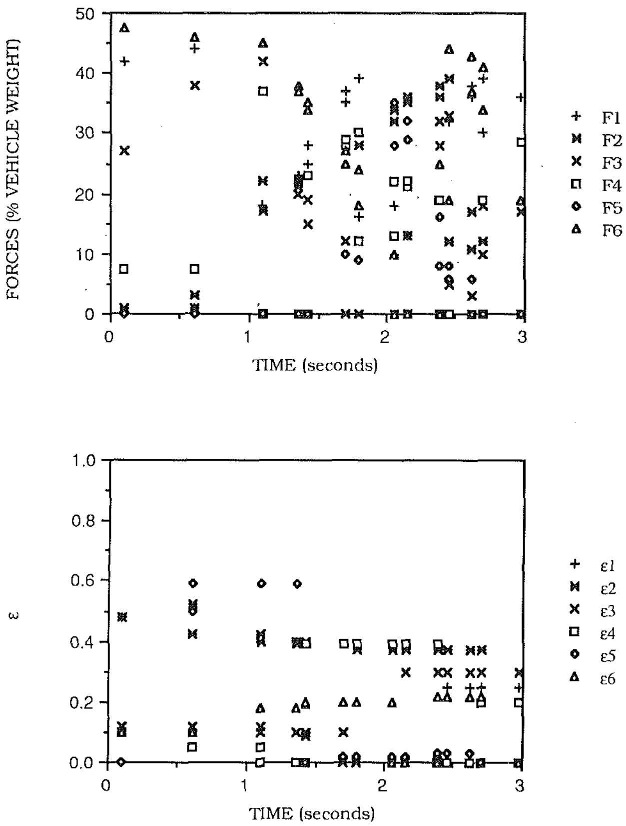


Fig. 4(*f*) Variable compliance method

Table 2 Computational times for proposed schemes

All figures are in seconds. t_4 and t_5 are the times required for the computations with 4 and 5 support legs respectively on the VAX 8500. The average time is the time required for one set of computations in the test case.

SCHEME	TIME REQUIRED		AVERAGE TIME
	t_4	t_5	
(a), Force Decomposition (No Int. Forces)	0.016	0.020	0.018
(b), Force Decomposition (Int. Forces by Linear Prog.)	3.021	5.332	3.704
(c), Linear Prog. (Minimizing Max. Force Comp.)	0.647	1.130	0.794
(d), Linear Prog. (Max. Distance from Side Constr.)	1.773	3.455	2.227
(e), Linear Prog. (Phase I, Simplex)	0.390	0.657	0.462
(f), Variable Compliance	0.028	0.037	0.029

Figure 4 shows the force distribution resulting from different algorithms. Only the peak values of the forces and the friction angles are particularly meaningful. It should be pointed out that the force distribution has been determined only at instances just before and after a leg has been lifted or placed. A linear interpolation between these instants yields the force distribution at intermediate time instants, though, in practice, a real time algorithm would be used to compute the forces at *all* these instants.

As the loading is not likely to change drastically, and more importantly, the location of the support points (and hence the normals to the terrain at the supports) remain the same, the approximate solutions obtained by linear interpolation is expected to suffice for planning purposes. In any event, a study of the force distribution at instants just before stepping and lifting events should be adequate to reveal any problems with vehicle instability (see Fig. 4).

A comparison of the computational times for the different schemes are reported in Table 2. These data are the execution times on the VAX 8500 obtained by averaging the run time over several runs. In the 3 second interval of vehicle motion considered, a variable duty factor was used, but the number of support legs ranged between 4 and 5. This should explain the format in Table 2.

Results

The following options were evaluated using the computer simulation:

(*a*) Decomposition of the force field
- Equilibrating forces only (equations (7) and (9))
- No (additional) interaction force components

(*b*) Decomposition of the force field
- Equilibrating forces only (equations (7) and (9))
- Interaction force components by linear programming

(*c*) Linear Programming
- Minimization of the largest force component in the foot coordinate system

(*d*) Linear programming
- Maximizing the smallest distance from the side-constraints

(*e*) Linear programming
- Phase I: Simplex method

(*f*) Variable compliance method
- Equations (8) and (12, 13)

The equilibrating force solution (Method (*a*)) is shown in Fig. 4(*a*). The peak load is 40 percent of the peak vehicle weight (on leg 2). However, as discussed earlier, the friction angles can become very high, since the algorithm does not take into account the variation of the terrain. The high friction angles for the legs 2, 4, and 5 are undesirable — the largest ratio of tangential to normal forces (ϵ) is almost 0.6. It should be recalled that the equilibrating forces do include some interaction force components because of the decomposition of equation (3) into equations (6) and (8). However, the appearance of these components is incidental as opposed to Method (*b*) in which an interaction force field is deliberately superposed on the equilibrating force field.

The problem of finding interaction forces in a computationally efficient manner proved to be quite intractable. However the nature of the interaction force field and the feasiblity of the decomposition scheme had to be investigated. The Simplex method (see, for example, [23] for description) was used to solve a linear programming formulation of the problem of determining the interaction forces. The maximum interaction force component was used as an objective function for minimization (see Fig. 4(*b*)). By comparing Figures 4(*a*) and 4(*b*), it may be seen that the interaction forces are small in magnitude compared to the equilibrating forces (less than 20 percent of the maximum equilibrating force) but they bias the force distribution so that the friction angle constraints are easily met. From the point of view of traction, this is a suitable solution. The high computational time is undesirable, but should not come as a surprise. Clearly, this is not a practical method in terms of computational time, but it demonstrates that the decomposition scheme works effectively even for highly exaggerated rough terrain conditions.

Methods (c) and (d) are simple linear programming exercises. It is evident that minimizing the largest force component (Method (c), Fig. 4(c)) results in smaller foot forces (peak force less than 35 percent of the vehicle weight), but the solutions lie along the side-constraints. In other words, the friction angles, though acceptable, are close to the maximum allowable values. An attempt to maximize the smallest distance from the side-constraints (Fig. 4(d)) reduces the frictional angles significantly (the maximum value of ϵ is about 0.36 compared to 0.48 in Fig. 4(c)) but the magnitudes of the foot forces are high (the peak value is about 50 percent of the vehicle weight). The trade-off between the two desired characteristics for locomotion is clearly reflected in this study. It should also be noted that the number of dummy (slack) variables is much larger in Method (d) and the almost threefold increase in computational time in Table 2 is indicative of that.

The Simplex Method has two phases of computations. In Phase I, a valid (or feasible) solution is sought without considering the objective function (or the optimality of the solution), and Phase II involves the generation of an optimal solution using the feasible solution from Phase I as a starting value. In this problem, the Phase I solution would be *any* solution in which the stipulated conditions on the friction angles were met. Such a solution is shown in Fig. 4(e). The advantage is clearly the reduction in computational time by almost a factor of 2 over the other linear programming methods (see Table 2). The nonoptimality of the solution is not a factor when the stability of a stance or configuration needs to be verified. (The vehicle is stable if and only if a feasible (with non-negative contact wrenches satisfying the frictional constraints) solution can be found to the force distribution problem in equation (4).) The computational burden is nevertheless heavy and is a major deterrent. However, it is possible that the parallelism in computations for Phase I can be exploited to yield better results in terms of computational time. Thus it may be a viable option at the planning level to determine whether or not a planned configuration is stable.

The variable compliance method (Method (f)) shown in Fig. 4(f) is a derivative of Method (a). (Both methods yield identical results on even terrain.) The frictional angles are almost identical for the most part even on uneven terrain. However, the redistribution of equilibrating forces is evident from a comparison of Figs. 4(a) and 4(f). For example, the load taken by legs 2 and 5 is reduced because of the large friction angles; legs 1 and 6 support a larger fraction of the load. The computational load compared to Method (a) is only 30 percent more, and the advantages outweigh this increase. It needs about 25 milliseconds for 5 support legs (for a main frame computer such as the VAX 8500) with a completely serial processing environment and holds definite promise for use in a real time system.

8 Concluding Remarks

Different ways of formulating and solving the force distribution problem for statically stable walking vehicles have been proposed and studied. A comparative study of several algorithms for application on uneven terrain using digital simulation is described. The algorithms described here have two applications: (*1*) Evaluating and predicting stability in rough terrain for use by a guidance system; (*2*) Generating force set-points for force control schemes, and desired compliances in active compliance control schemes.

The concept of decomposing the force field into an equilibrating force field and an interaction force field is particularly useful. The equilibrating forces are designed to maintain equilibrium with an external load wrench — they are required to satisfy the minimum norm condition. The interaction forces are bias forces that are superposed on the equilibrating forces in order to optimize the contact conditions. The equilibrating forces can be efficiently computed if a small deviation from the definition is permitted. An efficient algorithm to compute interaction forces in real-time is yet to be found, though some useful theoretical results are presented in [13]. However, varying the compliances of the legs to enable legs with better footholds to support a greater fraction of the load yields a satisfactory equilibrating force solution which obviates the need for superposition of interaction forces.

Another useful approach is the well-known technique of linear programming. The application of this method to optimize the contact conditions for legged locomotion is reported here for the first time. An implementation of the Phase I of the Simplex Method is believed to be appropriate for verifying if a given vehicle configuration is stable.

9 Acknowledgments

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