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## Direct Kinematic Analysis of a Hexapod Spider-like Mobile Robot

Mostafa Ghayour and Amir Zareei

Mechanical Engineering Department Islamic Azad University, Khomeini-shahr Branch Isfahan, Iran

Ghayour@iaukhsh.ac.ir, Amir.Zareei@iaukhsh.ac.ir

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**Abstract.** In this paper, an appropriate mechanism for a hexapod spider-like mobile robot is introduced. Then regarding the motion of this kind of robot which is inspired from insects, direct kinematics of position and velocity of the centre of gravity (C.G.) of the body and noncontact legs are analysed. By planning and supposing a specific time variation for each joint variable, location and velocity of the C.G. of the robot platform and angular velocity of the body are obtained and the results are shown and analysed.

### Introduction

Hexapod walking mobile robots are so important in researchers' sight because of their competence like walking in unstructured terrain, going forward with many kinds of gaits to adopt different speeds and loads and because the redundant limb exists, hexapod robot could continue its work even if a limb were lost. These advantages make it competent for some autonomous and high reliability works, such as field scouting, underwater searching, and space exploring.

Saha et al. 2005 studied inverse and forward kinematics of a dodekapod with 12 DOF (degree of freedom). A hierarchical method to solve the kinematics was proposed.[1] Jianhau et al. 2006 designed a hexapod with 42 DOF which can realize omni-directional movement. The tripod gait of hexapod simulated and according to the model kinematic angular displacement, angular velocity and angular acceleration of each joint were analysed.[2] Daniali et al. 2008 proposed homotopy continuation method to solve the forward and inverse kinematic problems of an offset 3-UPU translational parallel manipulator.[3] Chablat et al. 2008 derives the inverse and the forward kinematic equation of a serial-parallel 5-axis machine tool. The parallel module consists of moving platform that is connected to a fixed base by three non-identical legs.[4] Huang et al. 2009 analysed the direct kinematics of Stewart platform which consist of two regular hexagons platforms. A concise algebraic elimination method was used to solve closed-form forward kinematics in this paper [5].

Spider-like walking mobile robot is consisted from six limbs that are axially located around circular platform as shown in Fig. 1. Each leg has two parts with three DOF that two degree of freedoms are located on connection point of femur and platform ( $\theta_1, \theta_2$  shown in Fig. 2) and the other DOF is placed on the joint between the femur and tibia. ( $\theta_3$  shown in Fig. 2).



Figure 1. Hexapod spider-like mobile robot [6]

Walking in every direction without need to turn, ability to carry more loads because of the axisymmetric distribution of the legs around the platform are the advantages of this kind of robot.

### Direct Kinematics

Direct kinematics for robots with parallel structure means “finding the location of the robot platform versus joint variables”. In mentioned spider-like mobile robot through walking, legs are divided into two groups including legs which are in contact with the ground and those are defined as contact limbs and limbs that have no contact with the ground and are moving through the space while robot is walking and are defined as noncontact legs. For the purpose of forward kinematic analysis first by using the joint variables related to contact legs, location and velocity of C.G. of body are determined, then direct kinematics of noncontact legs are calculated.

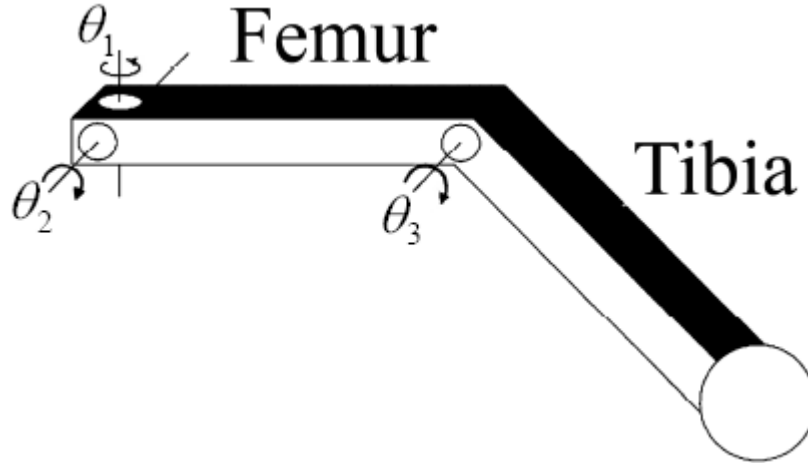


Figure 2. Schematic view a leg of the robot with its degree of freedoms

Limbs are numbered from one to six and the connection points of six legs of the robot to the platform are called  $B_1$  to  $B_6$ . End points of legs that contact with the ground surface are called  $A_1$  to  $A_6$ . The fixed reference frame has its origin at the centre of the platform on the ground, X-axis direction is direct to  $A_1A_3$  vector and Z-axis is normal to the ground surface and the Y-axis of the coordinate system is specified by the right hand rule. Similarly the moveable frame P has its origin at centre of platform, with its x-axis direct to  $B_1B_3$  vector and z-axis is normal to the platform plane. Other moveable frames have their origin at the  $B_i$ , with x-axes direct to radial direction of platform and z-axes is normal to the platform plane. Other coordinate frames systems are similar to  $B_i$  coordinate frames and installed at points  $A_i$ . The only difference between these two kinds of coordinate frame is the direction of their z-axes. The z-axes of  $B_i$  coordinate frames are normal to platform plane whereas z-axes of  $A_i$  coordinate frames are normal to the ground surface. In addition to mentioned frames, on each leg and by origin of actuators, standing coordinate systems that their z-axis are direct to the rotation of link and their x-axis are direct to common normal between two links. Installed coordinate frames systems are shown in Fig. 3.

**Position Direct Kinematics of Robot Platform.** In order to study the direct kinematics of the robot at first by using the joint variables of contact limbs, position and orientation of robot platform based on fixed frame are determined.

By attention to Fig. 4 and knowing the  $\overrightarrow{OA_i}$  vectors (end point of contact legs) can write:

$$\overrightarrow{r_{B_i}} = \overrightarrow{r_{A_i}} + \overrightarrow{r_{M_i/A_i}} + \overrightarrow{r_{B_i/M_i}} \quad (1)$$

That in (1)  $\overrightarrow{r_{B_i}}$ ,  $\overrightarrow{r_{A_i}}$  represent position vector of  $B_i$ ,  $A_i$  in base coordinate frame respectively and  $\overrightarrow{r_{M_i/A_i}}$  is position vector of  $M_i$  versus  $A_i$  and similarly  $\overrightarrow{r_{B_i/M_i}}$  is position vector of  $B_i$  versus  $M_i$ .

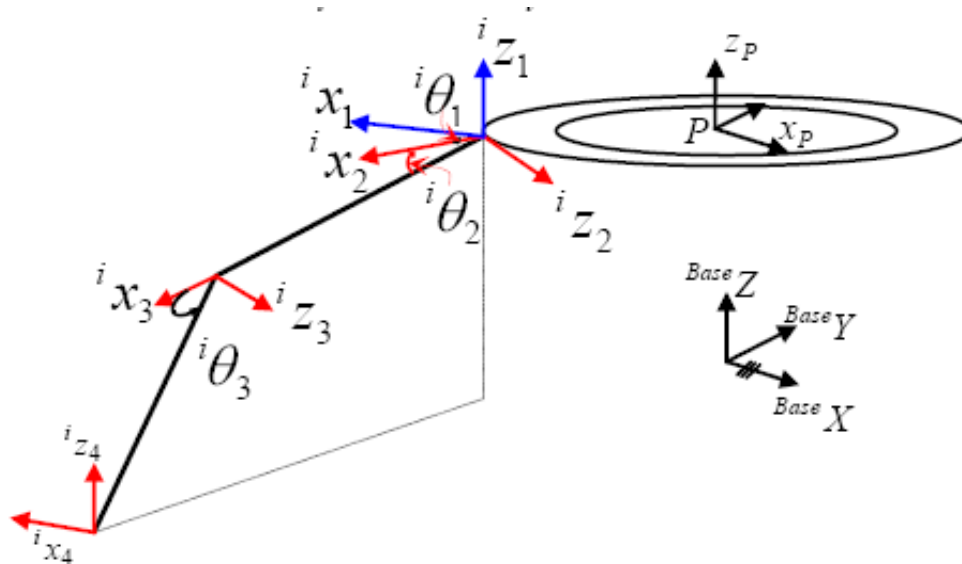


Figure 3. Specifying coordinate frames of the robot

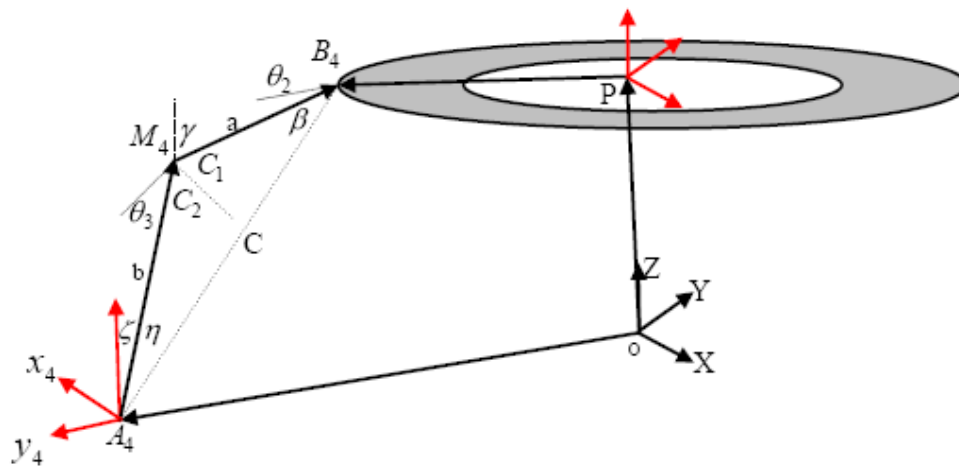


Figure 4. Parameters of the robot

Suppose that legs 1, 3 and 5 are contact and limbs 2, 4 and 6 are noncontact legs. According to relation (1) the location of points  $B_i$  versus fixed coordinate are determined and as direction of x-axis of P-coordinate system is direct to  $\overrightarrow{B_3B_1}$  vector can determine the direction of x-axis unit vector:

$$\vec{e}_x = \frac{\overrightarrow{B_3B_1}}{\|\overrightarrow{B_3B_1}\|} \quad (2)$$

From other side can determine the direction of unit vector of  $\overrightarrow{B_3B_5}$  vector:

$$\vec{e}_m = \frac{\overrightarrow{B_3B_5}}{\|\overrightarrow{B_3B_5}\|} \quad (3)$$

By having vectors  $\vec{e}_x$ ,  $\vec{e}_m$  and by using the cross product characterization of two mentioned vectors, the direction of unit vector, normal to the platform plane,  $\vec{e}_z$ , determined:

$$\vec{e}_z = \vec{e}_m \times \vec{e}_x \quad (4)$$

By specifying of unit vectors,  $\vec{e}_x$  and  $\vec{e}_z$  from P-coordinate system can specify the y-axis direction of this coordinate system:

$$\vec{e}_y = \vec{e}_z \times \vec{e}_x \quad (5)$$

By knowing the unit vectors  $\vec{e}_x$ ,  $\vec{e}_y$  and  $\vec{e}_z$ , rotation matrix of platform versus fixed coordinate is determined as follow:

$${}^P_B R = [\vec{e}_x \quad \vec{e}_y \quad \vec{e}_z] \quad (6)$$

For specifying the origin of P-coordinate system in fixed coordinate frame can determine the circle equation of the platform by using from coordinates of  $B_1$ ,  $B_3$  and  $B_5$ :

$$(x_{B_1} - x_p)^2 + (y_{B_1} - y_p)^2 + (z_{B_1} - z_p)^2 = r^2 \quad (7)$$

$$(x_{B_3} - x_p)^2 + (y_{B_3} - y_p)^2 + (z_{B_3} - z_p)^2 = r^2 \quad (8)$$

$$(x_{B_5} - x_p)^2 + (y_{B_5} - y_p)^2 + (z_{B_5} - z_p)^2 = r^2 \quad (9)$$

That in relations (7) to (10) “r” indicates radius of the platform. By solving the above system of non linear equation ((7) to (9)) coordinate of the C.G. of body is determined.

**Direct Kinematics of Platform Velocity.** Direct kinematic of velocity in parallel robots means to determine the velocity and angular velocity of robot platform by using the position and velocity of joint variables. In order to specify the direct kinematics of platform velocity can use (10):

$$\vec{OA_i} + \vec{A_iM_i} + \vec{M_iB_i} + \vec{B_iP} = \vec{OP} \quad (10)$$

In (10)  $\vec{OA_i}$  indicates a vector that was drawn from fix coordinate origin to point “A” from leg No. i. By differentiating form (10) the relation between velocity of joint variables and platform velocity specified as follow:

$$\vec{V}_p = \vec{\omega_i}^{B \rightarrow Tib} \times \vec{A_iM_i} + \vec{\omega_i}^{B \rightarrow Fem} \times \vec{M_iB_i} + \vec{\omega}^{B \rightarrow P} \times \vec{B_iP} \quad (11)$$

In above equation  $\vec{\omega_i}^{B \rightarrow Fem}$ ,  $\vec{\omega_i}^{B \rightarrow Tib}$  indicate the absolute angular velocity of femur and tibia of limb No. i respectively. (11) can be used for each three contact limbs. By using this equation for two legs and elimination of  $\vec{V}_p$  from these two equations can drive  $\vec{\omega}^{B \rightarrow P}$  that shows the absolute angular velocity of platform. By using from  $\vec{\omega}^{B \rightarrow P}$  in (11) can obtain  $\vec{V}_p$ . Regarding to Fig. 4  $\vec{\omega_i}^{1 \rightarrow Fem}$ ,  $\vec{\omega_i}^{1 \rightarrow Tib}$  determine as follow:

$$\vec{\omega_i}^{1 \rightarrow Tib} = \dot{\theta}_1 \vec{i}k_1 + \dot{\zeta} \vec{i}k_2 \quad (12)$$

$$\vec{\omega_i}^{1 \rightarrow Fem} = \dot{\theta}_1 \vec{i}k_1 + \dot{\gamma} \vec{i}k_3 \quad (13)$$

Regarding to Fig. 4 and because two parts of leg are in a plane we can write:

$$\zeta = \frac{\pi}{2} - \theta_2 - \theta_3 \longrightarrow \dot{\zeta} = -(\dot{\theta}_2 + \dot{\theta}_3) \quad (14)$$

$$\gamma = \frac{\pi}{2} - \theta_2 \longrightarrow \dot{\gamma} = -\dot{\theta}_2 \quad (15)$$

That in (12) and (13)  $\vec{i}k_1$  indicates the unit vector direct to z-axis of first coordinate frame of limb No. i. As shown in Fig.(4) the relation between the unit vectors of different coordinate frames of each leg is determined as follow:

$$\vec{i}k_3 = \vec{i}k_2 \quad (16)$$

$$\vec{i}k_2 = -\sin(\theta_1) \vec{i}i_1 + \cos(\theta_1) \vec{i}j_1 \quad (17)$$

$$\vec{i}j_4 = -\vec{i}k_3 \quad (18)$$

By using (12) to (18)  $\vec{\omega_i}^{1 \rightarrow Tib}$ ,  $\vec{\omega_i}^{1 \rightarrow Fem}$  are defined as follow:

$$\vec{\omega_i}^{1 \rightarrow Tib} = \dot{\theta}_1 \vec{i}k_1 - (\dot{\theta}_2 + \dot{\theta}_3) \left( -S(\theta_1) \vec{i}i_1 + C(\theta_1) \vec{i}j_1 \right) \quad (19)$$

$$\vec{\omega_i}^{1 \rightarrow Fem} = \dot{\theta}_1 \vec{i}k_1 - \dot{\theta}_2 \left( -S(\theta_1) \vec{i}i_1 + C(\theta_1) \vec{i}j_1 \right) \quad (20)$$

In (19), (20) C means Cos and S shows Sin.

$$\vec{\omega_i}^{B \rightarrow Fem} = {}^P_B R {}^1P_i R \vec{\omega}^{1 \rightarrow Fem} \quad (21)$$

$${}^B \rightarrow {}^{Fem} \omega_i = {}^1 R_i {}^1 \rightarrow {}^{Fem} \omega_i \quad (22)$$

And similarly we can write:

$${}^B \rightarrow {}^{Tib} \omega_i = {}^1 R_i {}^1 \rightarrow {}^{Tib} \omega_i \quad (23)$$

In above equations  ${}^P R$  is rotation matrix of platform relative to fix coordinate frame and  ${}^P R_i$  is rotation matrix of first coordinate frame of limb No.i relative to P-coordinate frame system. Regarding to installing figure of coordinate frames on platform  ${}^P R_i$  is determined as follow:

$${}^P R_i = \begin{bmatrix} \cos[(i-1)\frac{\pi}{3} + \frac{\pi}{6}] & -\sin[(i-1)\frac{\pi}{3} + \frac{\pi}{6}] & 0 \\ \sin[(i-1)\frac{\pi}{3} + \frac{\pi}{6}] & \cos[(i-1)\frac{\pi}{3} + \frac{\pi}{6}] & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (24)$$

That 'i' in above equation is the number of limb.

### Direct kinematics of Noncontact Legs

Direct kinematics of position for a noncontact limb likes direct kinematics for a serial robot. As shown in Fig. 4 can write:

$$\overrightarrow{OA_i} = \overrightarrow{OP} + \overrightarrow{PB_i} + \overrightarrow{B_iM_i} + \overrightarrow{M_iA_i} \quad (25)$$

$$\overrightarrow{PB_i} = {}^P R {}^P \overrightarrow{PB_i} \quad (26)$$

$$\overrightarrow{B_iM_i} = {}^P R {}^P R_i {}^1 \overrightarrow{B_iM_i} \quad (27)$$

$$\overrightarrow{M_iA_i} = {}^P R {}^P R_i {}^1 \overrightarrow{M_iA_i} \quad (28)$$

In above equations  $\overrightarrow{PB_i}$ ,  $\overrightarrow{B_iM_i}$  and  $\overrightarrow{M_iA_i}$  demonstrate these vectors in base coordinate frame and  ${}^P \overrightarrow{PB_i}$  indicates this vector in P-coordinate.  ${}^1 \overrightarrow{B_iM_i}$  and  ${}^1 \overrightarrow{M_iA_i}$  shows these vectors in the first coordinate frame belong to limb No.i.  ${}^P \overrightarrow{PB_i}$  is specified as follow:

$${}^P \overrightarrow{PB_i} = \begin{Bmatrix} r \cos(\frac{\pi}{6} + (i-1)\frac{\pi}{3}) \\ r \sin(\frac{\pi}{6} + (i-1)\frac{\pi}{3}) \\ 0 \end{Bmatrix} \quad (29)$$

**Direct Kinematics of Noncontact Legs Velocity.** In order to specify the velocity of end point of noncontact legs should differentiate from (25) that its result is as bellow:

$$\overrightarrow{V_{A_i}} = \overrightarrow{V_P} + {}^B \rightarrow {}^P \omega \times \overrightarrow{PB_i} + {}^B \rightarrow {}^{Fem} \omega_i \times \overrightarrow{B_iM_i} + {}^B \rightarrow {}^{Tib} \omega_i \times \overrightarrow{M_iA_i} \quad (30)$$

By using  $\overrightarrow{V_P}$ ,  ${}^B \rightarrow {}^P \omega$  were derived from (11) and regarding to Fig. 4  ${}^B \rightarrow {}^{Tib} \omega_i$ ,  ${}^B \rightarrow {}^{Fem} \omega_i$  are specified as follow:

$${}^B \rightarrow {}^{Fem} \omega_i = \dot{\theta}_1 {}^i \overrightarrow{k_1} + \dot{\theta}_2 {}^i \overrightarrow{k_2} + {}^B \rightarrow {}^P \omega \quad (31)$$

$${}^B \rightarrow {}^{Tib} \omega_i = \dot{\theta}_3 {}^i \overrightarrow{k_3} + \dot{\theta}_1 {}^i \overrightarrow{k_1} + \dot{\theta}_2 {}^i \overrightarrow{k_2} + {}^B \rightarrow {}^P \omega \quad (32)$$

By using (30) to (32) the velocity of end point of noncontact legs are specified.

### Simulation

In this section a numerical example is taken to demonstrate the forward kinematics algorithm of hexapod mobile robot proposed in this paper. The limbs part length and the outer diameter of the platform are consumed 150 mm and suppose that legs 1,3 and 5 are contact and the other limbs are noncontact. The initial and final value of joint variables of contact and one of noncontact legs are reputed as follow:

$$\begin{cases} {}^1\theta_1(0)={}^3\theta_1(0)={}^5\theta_1(0)=0 \\ {}^1\theta_1(1\text{Sec})={}^3\theta_1(1)={}^5\theta_1(1)=\frac{\pi}{18} \end{cases} \quad (33)$$

$$\begin{cases} {}^1\theta_2(0)={}^3\theta_2(0)={}^5\theta_2(0)=\frac{4\pi}{9} \\ {}^1\theta_2(1\text{Sec})=\frac{\pi}{3}, {}^3\theta_2(1)=\frac{7\pi}{18}, {}^5\theta_2(1)=\frac{13\pi}{36} \end{cases} \quad (34)$$

$$\begin{cases} {}^1\theta_3(0)={}^3\theta_3(0)={}^5\theta_3(0)=\frac{\pi}{18} \\ {}^1\theta_3(1\text{Sec})=\frac{5\pi}{36}, {}^3\theta_3(1)=\frac{\pi}{6}, {}^5\theta_3(1)=\frac{\pi}{12} \end{cases} \quad (35)$$

$$\begin{cases} {}^2\theta_1(0)={}^0\theta_2(0)=\frac{\pi}{36}, {}^2\theta_3(0)=\frac{4\pi}{9} \\ {}^2\theta_1(1\text{Sec})=\frac{\pi}{18}, {}^2\theta_2(1)=0, {}^2\theta_3(1)=\frac{\pi}{3} \end{cases} \quad (36)$$

In above equations  ${}^1\theta_1$ ,  ${}^3\theta_1$  and  ${}^5\theta_1$  represent  $\theta_1$  of first, third and fifth limb and similarly  ${}^1\theta_2$ ,  ${}^3\theta_2$  and  ${}^5\theta_2$  are  $\theta_2$  of mentioned limbs.

Joint variables with initial and final velocity and acceleration of zero arrive to final quantity by polynomial form. Finally time variation of the position and velocity of platform and one of the noncontact legs are described in Fig. 5 to Fig. 12.

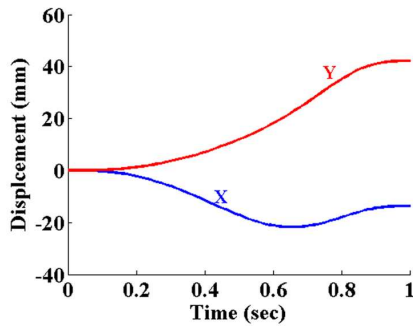


Figure 5. Displacement of C.G. of robot in direction of x and y-axis relative to fixed coordinate system

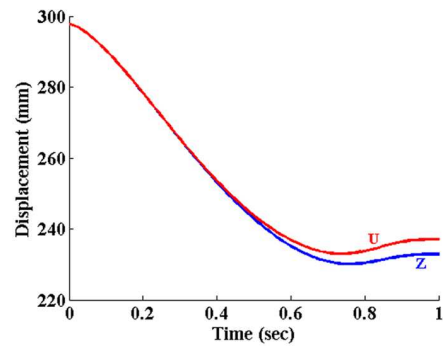


Figure 6. Displacement of C.G. of robot in direction of z-axis relative to fixed coordinate frame system (Z) and displacement norm (U)

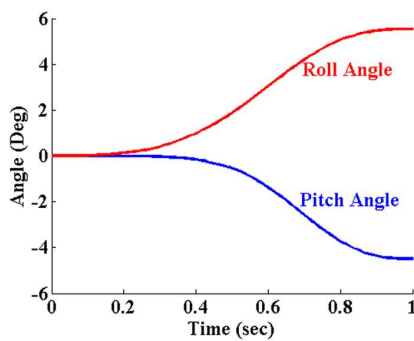


Figure 7. Roll and pitch angle variation of the platform

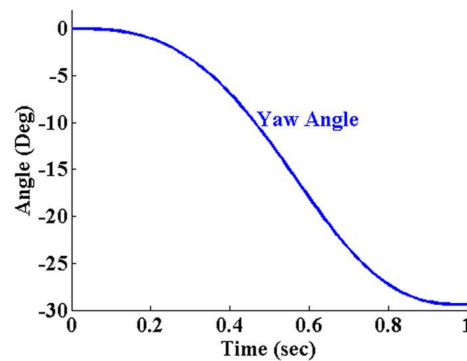


Figure 8. Yaw angle variation of the platform

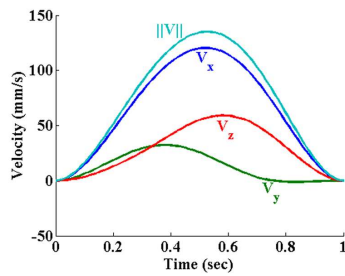


Figure 9. Velocity components and velocity norm of C.G. of body relative to fixed coordinate system

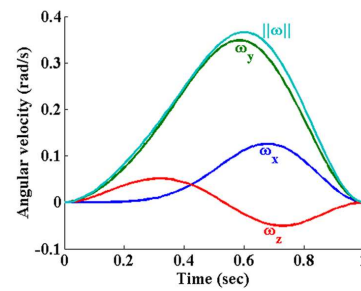


Figure 10. Angular velocity components and angular velocity norm of the gravity centre of the platform

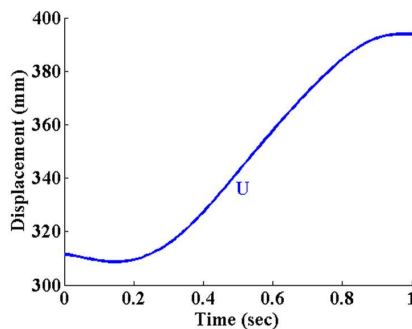


Figure 11. Displacement norm of the end point of second leg (noncontact leg)

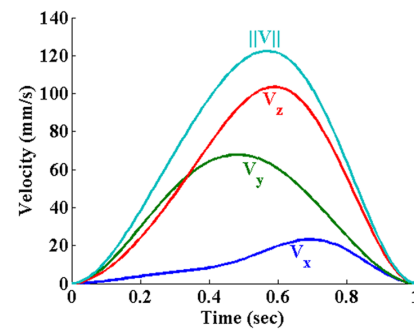


Figure 12. Velocity norm and velocity components of the end point of second leg (noncontact limb)

## Result

In this article a proper mechanism was designed for a hexapod spider-like mobile robot. Then regarding the motion of this kind of robot direct kinematics of position and velocity of C.G. of the body and noncontact limbs were calculated. In further a numerical example was taken to demonstrate the forward kinematics algorithm of hexapod mobile robot. For this numerical example a polynomial form with zero initial and final value of velocity and acceleration was supposed as time variation of joint variables and consequently the position and velocity of the C.G. of platform and angular velocity of the platform were calculated and shown in Fig. 5 to Fig. 10. In order to study the direct kinematics of noncontact legs a polynomial form with zero initial and final value of velocity and acceleration were specified for joint variables of a noncontact limb then displacement and velocity of the tip of this leg were demonstrated and shown in Fig. 11 and fig. 12. Fig. 9 described the velocity of C.G. of the platform and Fig. 10 showed the angular velocity of the body. The amount of displacement and velocity of C.G. of the platform and the end point of the noncontact foot and angular velocity of the body were in a sensible range with logical variation and smooth behaviour. Fig. 9 and Fig. 10 has shown that platform was moved with initial and final velocity and angular velocity of zero value which according to the format of joint variables motion this is a good validation for the results.



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