Optimal Force Distribution for the Legs of an Hexapod Robot

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Abstract— **In this paper we present direct and inverse kinematic models and a methodology for optimal force distribution calculation for the legs of an hexapod robot. This approach is used in the case of real time hexapod force control. The force distribution problem is formulated in terms of a nonlinear programming problem under equality and inequality constraints. Then, according to [1], the friction constraints are transformed from nonlinear inequalities into a combination of linear equalities and linear inequalities. The original nonlinear constrained programming problem is then transformed into a quadratic optimization problem. Some simulation results are given and perspectives on hexapod control are discussed.**

Keywords : hexapod Robot, Optimal Force Distribution, Quadratic Programming, Friction Constraints.

I. Introduction

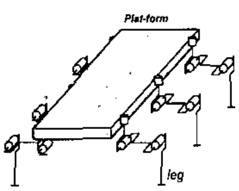

Fig. 1. view of the hexapod

Hexapode robots, as part of legged vehicles, can be used in work spaces with rough terrain, e.g. map building on an uneven ground, hazardous tasks like land mine searching and removing, volcano data collection, etc. Legged vehicles force control need calculation of real-time force distribution on the robot's legs. Due to the existence of three actuated joints in each leg, the hexapod robot has redundant actuation leading to more active joints (18) than the robot platform degree-of-freedom (6 dof). Thus, when formulating the force distribution problem, we find fewer force moment balancing equations than unknown variables. So, the solution of these equations is not unique. Moreover, some physical constraints, that concern the contact nature, friction, etc , must be taken into account in the calculation of force distribution. In addition, joints torque saturation must also be considered. Thus the Force Distribution Problem (FDP) can be formulated as a nonlinear constrained programming problem under nonlinear equality and inequality constraints. Several approaches have been proposed for solving such a problem [1]-[21], dealing with the following four principal methods :

1) Linear-Programming (LP) Method [2], [3]
2) Compact-Dual LP (CDLP) Method [4],
3) Quadratic Programming (QP) Method [5], [6]
4) Analytical Method [7], [8]

A comparative study for the four cited methods can be found in [1]. Some researchers proposed the optimal force distribution scheme of multiple cooperating robots by combining the Dual Method with the QP [9].

In FDP solving, according to some criteria, physical aspects of the robot crawling have to be considered. This concerns the following points:

1) preventing legs from slipping
2) avoiding discontinuities of the foot forces.
3) avoiding impact by making the foot forces of the swing leg increase smoothly from zero, after contact between leg and ground arises.

In this paper we propose an approach to solve the problem of real-time force distribution for an hexapod robot, based on the approach proposed in [1] for a quadruped robot.

This approach consists of the combination of the QP Method with the reduction technique of problem size. The robot crawling is divided into 3 phases. The first phase, only 3 legs are supporting the robot, for instance legs $1-2-3$, leading to a force distribution problem with 9 unknown variables. Then, in the second phase, all the six legs are supporting the robot leading to a force distribution problem with 18 unknown variables. In order to reduce the problem complexity, we consider that the contact forces on the legs 1-2-3 can be deduced from the first phase by introducing a continuous, decreasing function that varies from 1 to 0. Thus, the problem dimension, in the second phase, can be reduced from 18 to 9. The third phase is similar to the first one, with the legs 4-5-6 supporting the robot. In the three phases, the force-distribution problem is the same and solved with the same algorithm. The main idea concerns the transformation of the original nonlinear constrained problem into a linear one, by reducing the problem size and transforming the nonlinear constraints into a linear ones, respecting some physical considerations.

The rest of the paper is organized as follows. The direct and inverse geometrical models of the hexapod are presented in section 2. Section 3 concerns the force distribution problem. Problem reduction and optimal solution are presented in section 4. Before presenting some remarks and perspectives,

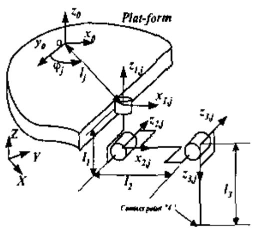

Fig. 2. geometrical model

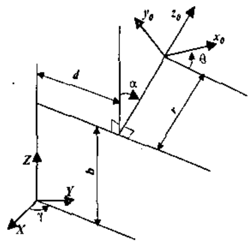

Fig. 3. geometrical parameters

simulation results are given in section 5.

II. Geometrical Modelling

Before presenting the direct and inverse geometrical model, let us consider the hexapod architecture. As the hexapod legs are identical, only one leg modelling is considred, the leg j architecture is given in figure (2). Every leg "j" j=1,...,6 is fixed at the plat-form by a revolute joint located at l_j distance from the center of gravity of the plat-form (the body). The angle ϕ_j represents the orientation of the coordinate frame $(x_{1,j}, y_{1,j}, z_{1,j})$ fixed at the first articulation of the leg and the coordinate fixed coordinate frame of the body (x_0, y_0, z_0). A walking robot is considerated as an arborescent robot conteining some closed loops. So to study this kind of robots we use the method defined by Khalil and Kleinfinger [10].

The transformation matrix from ith joint's attached coordinate frame to the (i-1)th joint's attached coordinate frame is given by figure (3):

$$^{i-1}\boldsymbol{T}_i = R(Z,\gamma)T(Z,b)R(X,\alpha)T(X,d)R(Z,\theta)T(Z,r) \tag{1}$$

The table (1) describes the transformation from the world ground coordinate frame (X, Y, Z) to the coordinate frame at the contact point "4" of each leg. The transformation providing

frame	α	d	θ	r	b	γ
plat-form	α	d	θ	r	h	β
liaison"1"	0	l_j	$\theta_{1,j}$	0	0	ϕ_j
liaison"2"	$-\pi/2$	0	$\theta_{2,j}$	0	$-l1$	0
liaison"3"	0	l_2	$\theta_{3,j}$	0	0	0
contact Pt"4"	0	l_3	$\theta_{4,j}$	0	0	0

TABLE I

GEOMETRICAL PARAMETERS

the exact position of the contact point"4" of any leg in the absolute coordinate frame fixed at the ground is given by :

$$^{R}\boldsymbol{T}_4 = {}^{R}\boldsymbol{T}_0^0\boldsymbol{T}_1^1\boldsymbol{T}_2^2\boldsymbol{T}_3^3\boldsymbol{T}_4 \tag{2}$$

When the position and the orientation of the last coordinate frame fixed to the end of each leg "j" are known, we apply the method proposed by Paul [11]. It provides the values of the joints' coordinates $\theta_{i,j}$ $(i = 1,2,3)$ $(j = 1,..,6)$ as follow:

$$\begin{cases} \theta_{1,j} = \arctan(S1, C1) \\ \theta_{2,j} = \arctan(S2, C2) \\ \theta_{3,j} = \arctan(S3, C3) \end{cases} \tag{3}$$

with:

$$\begin{cases} S1 = S\phi X_0 - C\phi Y_0 \\ C1 = S\phi Y_0 - C\phi X_0 - l_j \\ X_0 = (e-f)P_{x,j} + (e+f)P_{y,j} + gP_{z,j} - gh - dS\theta \end{cases}$$

$$\begin{cases} X_0 = (e-f)P_{x,j} + (e+f)P_{y,j} + gP_{z,j} \\ \quad -gh - dS\theta \\ Y_0 = (-e-f)P_{x,j} + (-e+f)P_{y,j} + gP_{z,j} \\ \quad -gh + dS\theta \\ e = C\beta C\theta, \quad f = S\beta C\alpha S\theta, \quad g = S\alpha S\theta \end{cases}$$

$$\begin{cases} S2 = \frac{XZ + Y\sqrt{X^2+Y^2-Z^2}}{X^2+Y^2} \\ C2 = \frac{XZ - Y\sqrt{X^2+Y^2-Z^2}}{X^2+Y^2} \end{cases} \quad \begin{cases} X = 2Z_2X_1 \\ Y = -2Z_2Y_1 \\ Z = W^2 - X_1^2 \\ \quad -Y_1^2 - Z_1^2 \end{cases}$$

$$\begin{cases} Z_2 = l_2 \\ W = -l_3 \\ X_1 = S\beta S\alpha P_{x,j} - C\beta S\alpha P_{y,j} + C\alpha P_{z,j} \\ \quad -hC\alpha - r + l_1 \\ Y_1 = C(\phi + \theta_{j,1})(X_0) + S(\phi + \theta_{j,1})(Y_0 - lC\theta_{j,1}) \end{cases}$$

$$\begin{cases} S3 = \frac{X_1C2 + Y_1S2}{W} \\ C3 = \frac{X_1S2 - Y_1S2 + Z_2}{W} \end{cases}$$

NB: S*=sin(*); C*=cos(*); $(P_{x,j}, P_{y,j}, P_{z,j})$, are the coordinates of the point "4" expressed in (x_0, y_0, z_0).

III. Force Distribution Problem

A. Problem formulation

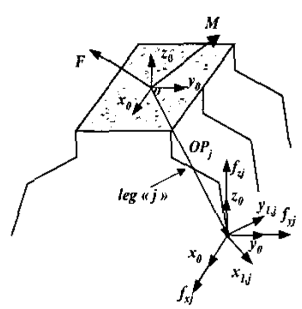

Fig. 4. Orientation of coordinate frame

The force system acting on a hexapod robot is shown in figure (4). For simplicity, only the force components on the foot are presented here. In the general case, rotational torques at the feet are neglected. Let (x_0, y_0, z_0) be the robot fixed body coordinate frame in which the body is located in the (x_0, y_0) plane and $(x_{1,j}, y_{1,j}, z_{1,j})$ denote the coordinate frame fixed at the foot "j", in which the leg j lies in the $(x_{1,j}, z_{1,j})$ plane and its z axis is normal to the support surface of the foot which is assumed to be parallel to the (x_0, y_0, z_0) plane. $\boldsymbol{F} = [F_X F_Y F_Z]^T$ and $\boldsymbol{M} = [M_X M_Y M_Z]^T$ denote respectively the robot body force vector and moment vector, which results from the gravity and the external force acting on the robot body. Define $f_{x,j}, f_{y,j}$, and $f_{z,j}$ as the components of the force acting on the supporting foot "j" in the directions of x_0, $y_0\, and\, z_0$, respectively. The number of supporting feet, n, can vary between 3 and 6 for an hexapod robot. The robot's quasi-static force/moment equation can be written as

$$\begin{cases} \sum_{j=1}^{n} \boldsymbol{f}_j = \boldsymbol{F} \\ \sum_{j=1}^{n} \boldsymbol{OP}_j \wedge \boldsymbol{f}_j = \boldsymbol{M} \end{cases} \tag{4}$$

where $\boldsymbol{OP}_j$ is the position vector joining contact point of the leg "j" and gravity center of the body. The general matrix form of this equation can be written as :

$$\boldsymbol{A}\,\boldsymbol{G} = \boldsymbol{W} \tag{5}$$

with :

$$\begin{cases} \boldsymbol{G} = [\boldsymbol{f}_1^T \boldsymbol{f}_2^T \cdots \boldsymbol{f}_n^T]^T & \in \Re^{3n} \\ \boldsymbol{f}_j^T = [f_{x,j}\, f_{y,j}\, f_{z,j}]^T & \in \Re^3 \\ \boldsymbol{W} = [\boldsymbol{F}^T \boldsymbol{M}^T]^T & \in \Re^6 \end{cases}$$

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{I}_3 & \cdots & \cdots & \boldsymbol{I}_3 \\ \boldsymbol{B}_1 & \cdots & \cdots & \boldsymbol{B}_n \end{pmatrix} \quad \in \Re^{6\times 3n}$$

$$\boldsymbol{B}_j \equiv \widehat{\boldsymbol{OP}}_j \equiv \begin{pmatrix} 0 & -P_{z,j} & P_{y,j} \\ P_{z,j} & 0 & -P_{x,j} \\ -P_{y,j} & P_{x,j} & 0 \end{pmatrix} \quad \in \Re^{3\times 3}$$

where $\boldsymbol{I}_3$ is the identity matrix and $\mathbf{G}$ is the foot force vector, corresponding to three ($\boldsymbol{G} \in \Re^9$) or six ($\boldsymbol{G} \in \Re^{18}$) supporting legs. $\boldsymbol{A}$ is a coefficient matrix which is a function of the positions of the supporting feet, and $\boldsymbol{B}_j$ is a skew symmetric matrix consisting of $(P_{x,j}, P_{y,j}, P_{z,j})$, which is the position coordinate of the supporting foot "j" in (x_0, y_0, z_0). $\boldsymbol{W}$ is a total body force/moment vector. It is clear that (5) is an underdetermined system and its solution is not unique. In other words, the feet forces have many solutions according to the equilibrium equation. However, the feet forces must meet the needs for the following physical constraints, otherwise they become invalid :

1) Supporting feet should not slip when the robot walks on the ground. It results in the following constraint:

$$\sqrt{f_{x,j}^2 + f_{y,j}^2} \le \mu\, f_{z,j} \tag{6}$$

where μ is the static coefficient of friction of the ground

2) Since the feet forces are generated from the corresponding actuators of joints, the physical limits of the joint torques must be taken into account. It follows that :

$$-\tau_{jmax} \le \boldsymbol{J}_j^{T\,j} \boldsymbol{A}_{0j} \begin{pmatrix} f_{x,j} \\ f_{y,j} \\ f_{z,j} \end{pmatrix} \le \tau_{jmax} \tag{7}$$

for $(j = 1, ..., n)$, where $\boldsymbol{J}_j \in \Re^{3\times 3}$ is the Jacobian of the leg "j", $\tau_{jmax} \in \Re^{3\times 1}$ is the maximum joint torque vector of the leg "j", and $\boldsymbol{A}_{0j} \in \Re^{3\times 3}$ is the orientation matrix of $(x_{1,j}, y_{1,j}, z_{1,j})$ with respect to (x_0, y_0, z_0).

3) In order to have definite contact with the ground, there must exist a $f_{z,j}$ such that :

$$f_{z,j} \ge 0 \tag{8}$$

In the following, we propose an approach for problem size reduction, linearisation and solving for the hexapod case. Clearly, it is difficult to solve such a nonlinear programming problem for real-time feet force distribution with complex constraints.

B. Problem Size Reduction

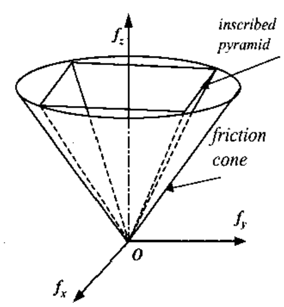

Fig. 5. conservative inscribed pyramid

The equation (6) is a formulation of the friction cone figure(5). In order to overcome the non linearities induced by the following equations, most researches substitute this friction cone by the inscribed pyramid [3], [12], [5]. Thus, the nonlinear friction constraints are approximately expressed by the following linear inequalities :

$$f_{x,j} \ge \acute{\mu}\, f_{z,j}, \quad f_{y,j} \ge \acute{\mu}\, f_{z,j}, \quad j = 1, ..., n \tag{9}$$

where $\acute{\mu} = \frac{\sqrt{2}\mu}{2}$ is for the inscribed pyramid. Thus, the initial non linear constrained programming problem, substituting the non linear constraint Eq(6) by the linear one of Eq(9), becomes a linear programming problem [3], [4], and [5]. The possibility of slipping can be minimized, by optimizing the ratio of tangential to normal forces at the feet. In [13], the authors have shown that, for multi-legged robots, all ratios (at the feet) are equal to the global ratio. This leads Liu and Wen [7] to find the relationship between the feet forces and transform the initial friction constraints from the nonlinear inequalities into a set of linear equalities. Let us define the global ratio by the ratio of the tangential to normal forces at the robot body. The advantage of the existing methodes lies in the fact that part of component of the feet forces satisfy the global ratio relation ship and lets the other components satisfy the linear inequality constraints as Eq (9). For example, defining $f_{x,j}$ $(j = 1;;n)$ and $f_{y,j}$ $(j = 1;;n)$, for a feet j Chen et al [1], show that :

$$f_{x,j} = k_{xz} f_{z,j} \quad , (i = 1, ..., n) \tag{10}$$

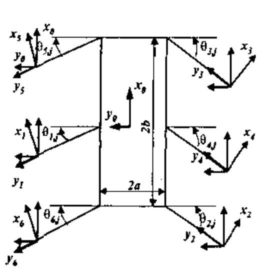

Fig. 6. bottom View of the hexapod

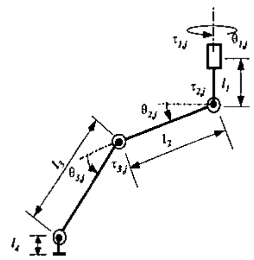

Fig. 7. basic mechanism of the leg

$$f_{y,j} \leq \mu^{\star} f_{z,j} \quad ,(i=1,...,n) \tag{11}$$

where $k_{xz} = \frac{F_X}{F_Z}$ is the global ratio of forces at the robot body in direction of x_0 and z_0. $\mu^{\star}$ is the given coefficient for friction constraints. According to Eq(6), we have $\mu^{\star} = \sqrt{\mu^2 - k_{xz}^2}$. Finally, the force distribution problem is transformed into a linear one by replacing Eq (6) with Eqs (10) and (11).

C. Problem transformation and Continuous solution

In modelling the hexapod robot walking behavior, we consider that three legs support the robot at a time. So, discontinuity avoidance of the feet forces must be taken into account. The consideration should be twofold: the first is to assure that the foot force of the swing leg continuously transits while the leg moves from free to placement on the ground; the second is to make the feet forces increase smoothly from zero, in order to avoid impact resulting from the placement of the swing leg. The hexapod robot crawling steps are as follow :

- $[t_1; t_2]$ denotes the time period from the time t_1 at which the legs (4,5,6) (the swing leg) is lifted, to the time t_2 at which the legs (4,5,6) is placed.
- $[t_2; t_3]$ denotes the time period from the time t_2 at which the leg (4,5,6) is placed, to the time t_3 at which the legs (1,2,3) (next swing leg) is lifted .
- $[t_3; t_4]$ denotes the time period from the time t_3 at which the legs (1,2,3) (the swing leg) is lifted, to the time t_4 at which the legs (1,2,3) is placed.

In the time period $(t_1; t_2)$ $n = 3$, so $\boldsymbol{G}$ and $\boldsymbol{A}$ become a vector of 9×1 and a matrix of 6×9, respectively. Equation (5) contains nine unknown variables with six equations. By adding the Eq (10) to the Eq (5) we obtain nine equations.

$$\overline{\boldsymbol{A}}\,\boldsymbol{G} = \overline{\boldsymbol{W}} \tag{12}$$

with :

$$\overline{\boldsymbol{A}} = \begin{pmatrix} & \boldsymbol{I}_3 & & & \boldsymbol{I}_3 & & & \boldsymbol{I}_3 & \\ & \boldsymbol{B}_1 & & & \boldsymbol{B}_2 & & & \boldsymbol{B}_3 & \\ 1 & 0 & -k_{xz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -k_{xz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -k_{xz} \end{pmatrix}$$

$$\boldsymbol{G} = \begin{pmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \\ \boldsymbol{f}_3 \end{pmatrix} \qquad \overline{\boldsymbol{W}} = \begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{M} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using some rows combination of the matrix $\overline{\boldsymbol{A}}$, Eq (12) can be written as :

$$\widehat{\boldsymbol{A}}\,\boldsymbol{G} = \widehat{\boldsymbol{W}} \tag{13}$$

With: $\widehat{\boldsymbol{A}} =$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -k_{xz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -k_{xz} & 0 & 0 & 0 \\ -P_{y,1} & P_{x,1} & 0 & -P_{y,2} & P_{x,2} & 0 & 1 & 0 & -k_{xz} \\ P_{z,1} & 0 & -P_{x,1} & P_{z,2} & 0 & -P_{x,2} & -P_{y,3} & P_{x,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{z,3} & 0 & -P_{x,3} \\ 0 & -P_{z,1} & P_{y,1} & 0 & -P_{z,2} & P_{y,2} & 0 & -P_{z,3} & P_{y,3} \end{pmatrix} \quad \widehat{\boldsymbol{W}} = \begin{pmatrix} F_X \\ F_Y \\ 0 \\ 0 \\ M_Z \\ M_Y \\ 0 \\ M_X \end{pmatrix}$$

where $\widehat{\boldsymbol{A}} \in \Re^{8\times 9}$ is the resulting matrix of $\overline{\boldsymbol{A}}$ after combination. $\boldsymbol{G} \in \Re^{8}$ is the foot force vector. $\widehat{\boldsymbol{W}} \in \Re^{8}$ is the resulting vector of $\boldsymbol{W}$ after combination. Thus, the force distribution problem is subjected to the inequality constraints expressed by (7), (8) (10). In the time period $(t_2; t_3)$, $\boldsymbol{G}$ is a 18 dimensional vector, and $\boldsymbol{A}$ is a matrix of 6×18. For the sake of continuity of solution, the foot forces of the legs (1,2,3) denoted by $\boldsymbol{f}_j = [f_{x,j}\, f_{y,j}\, f_{z,j}]^T$ should be changed smoothly from $\boldsymbol{f}_j(t_2)$ to $\boldsymbol{0}$ (j=1,2,3). Therefore, the foot force of the legs f_j $(j = 1,2,3)$ in the time period $(t_2; t_3)$ can be expressed as:

$$\boldsymbol{f}_j = \delta(t)\boldsymbol{f}_j(t_2) \quad (j = 1,2,3) \tag{14}$$

where $\delta(t)$ is any desired continuous scalar function varying from $\delta(t_2) = 0$ *to* $\delta(t_3) = 1$. So, the initial problem with 18 unknow variables $[\boldsymbol{f}_1, \boldsymbol{f}_2, ..., \boldsymbol{f}_6] \in \Re^{1\times 18}$ is reduced to 9. Thus, $\boldsymbol{f}_j$ (j=1,2,3) are known in this time period and Eq (6) can be rewritten as.

$$\begin{pmatrix} \boldsymbol{I}_3 & \boldsymbol{I}_3 & \boldsymbol{I}_3 \\ \boldsymbol{B}_4 & \boldsymbol{B}_5 & \boldsymbol{B}_6 \end{pmatrix} \begin{pmatrix} \boldsymbol{f}_4 \\ \boldsymbol{f}_5 \\ \boldsymbol{f}_6 \end{pmatrix} = \begin{pmatrix} F_X \\ F_Y \\ F_Z \\ M_X \\ M_Y \\ M_Z \end{pmatrix} - \begin{pmatrix} \boldsymbol{I}_3 & \boldsymbol{I}_3 & \boldsymbol{I}_3 \\ \boldsymbol{B}_1 & \boldsymbol{B}_2 & \boldsymbol{B}_3 \end{pmatrix} \begin{pmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \\ \boldsymbol{f}_3 \end{pmatrix}$$

(15)

The general form of Eq (15) :

$$\widetilde{A}G = \widetilde{W} \tag{16}$$

with :

$$\begin{cases} \widetilde{W} = W - A_i V \\ V = [f_1 f_2 f_3]^T \end{cases}$$

As in the first period (t_1, t_2) by combining Eq (16) with (10) we obtain eight independent equations with nine unknown variables, similar to those in Eq (13), with $\widehat{A}$ the resulting matrix of $\widetilde{A}$ after combination, and $\widehat{W}$ the resulting vector of $\widetilde{W}$ after combination.

IV. Quadratic Problem Formulation and Solution

The solution to the inverse dynamic equations of a hexapod robot is not unique, but it can be chosen in an optimal manner by minimizing some objective function. The approach taken here is to minimize the sum of the weighted torque of the robot, which results in the following objective function [6], [9] :

$$f_G = p^T G + \frac{G^T Q G}{2} \tag{17}$$

with:

$$p^T = [\widehat{\tau}_1^T] J_1^T \,.......\, , \widehat{\tau}_n^T J_n^T] \quad \in \Re^{3n}$$

$$Q = \begin{pmatrix} J_1 q_1 J_1^T & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & J_n q_n J_n^T \end{pmatrix} \quad \in \Re^{3n \times 3n}$$

where $\widehat{\tau}_j$ is the joint torque vector due to the weight and inertia of the leg "j", $\mathbf{J}_j$ is the Jacobian of the leg "j", and $\mathbf{q}_j$ is a positive definite diagonal weighting matrix of the leg j. This objective function is strictly convex. Because the time for obtaining a solution does not depend on an initial guess, a quadratic programming is superior to linear programming in both speed and quality of the obtained solution [6]. The general linear-quadratic programming problem of the force distribution for the legs of a hexapod robot is stated by :

$$p^T G + \frac{G^T Q G}{2} \tag{18}$$

$$\widehat{A}\widehat{G} = \widehat{W} \tag{19}$$

$$B\widehat{G} \le C \tag{20}$$

where $\mathbf{G} \in \Re^9$ is a vector of the design variables (the feet forces). It should be pointed out that, Eq. (19) denotes Eq (13), and Eq(20) is the resulting inequality constraints for the combination of Eq (7), Eq (8) and (11) where

$$B = [B_1^T \, B_2^T \, B_3^T \, B_4^T]^T \quad \in \Re^{9 \times 24}$$

$$C = [\tau_{1max} \, \tau_{2max} \, \tau_{3max} \, - \tau_{1max} \, - \tau_{2max} \, - \tau_{3max} \, 0\,0\,0\,0\,0\,0]^T \quad \in \Re^{24}$$

with

$$B_1 = \begin{bmatrix} J_1^T R_1 & 0 & 0 \\ 0 & J_2^T R_2 & 0 \\ 0 & 0 & J_3^T R_3 \end{bmatrix} \quad \in \Re^{9 \times 9}$$

$$B_2 = \begin{bmatrix} -J_1^T R_1 & 0 & 0 \\ 0 & -J_2^T R_2 & 0 \\ 0 & 0 & -J_3^T R_3 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 0 & 1 & -\mu^\star & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\mu^\star & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\mu^\star \end{bmatrix}$$

In Eq (19), we have eight linear independant equations with nine unknown variables. By using Gauss algorithm, this equation is transformed as follow :

$$[I_8 \quad \widehat{A}_r] \begin{bmatrix} \widehat{G}_b \\ \widehat{G}_r \end{bmatrix} = \widehat{W}_r, \tag{21}$$

where $I_8 \in \Re^{8 \times 8}$ identity matrix, $\widehat{A}_r \in \Re^8$ is the remaining column of the matrix $\widehat{A}$ after transformation. $\widehat{\mathbf{G}}_b \in \Re^8$ is the partial vector of G. $\widehat{G}_r \in \Re$ is the unknown element of $\widehat{G}$ which denotes the design variable. $\widehat{W}_r \in \Re^8$ is the resulting vector of $\widehat{W}$ after transformation. Equation(21) may be rewritten by the following form

$$I_8 \widehat{G}_b + \widehat{A}_r \widehat{G}_r - \widehat{W}_r = 0, \tag{22}$$

Which yields to

$$\widehat{G}_b = \widehat{W}_r - \widehat{A}_r \widehat{G}_r. \tag{23}$$

Finally, it results in

$$G = \begin{bmatrix} \widehat{G}_b \\ \widehat{G}_r \end{bmatrix} = \begin{bmatrix} \widehat{W}_r \\ 0 \end{bmatrix} + \begin{bmatrix} -\widehat{A}_r \\ 1 \end{bmatrix} \widehat{G}_r. \tag{24}$$

Now let $\widehat{G}_0 = [\widehat{W}_r^T \quad 0]^T \in \Re^8$ *and* $N = [-\widehat{A}_r^T \quad 1]^T \in \Re^9$, then Eq (24) becomes

$$G = \widehat{G}_0 + N\widehat{G}_r. \tag{25}$$

Substituting Eq (25) into Eqs (18) and (20), the linear quadratic programming problem can be expressed by :

$$minimize \quad f(\widehat{G}_r), \tag{26}$$

$$subject\,to \quad BN\widehat{G}_r \le C - B\widehat{G}_0. \tag{27}$$

where

$$\begin{aligned} f(\widehat{G}_r) = & \, p^T \widehat{G}_0 + \tfrac{1}{2} \widehat{G}_0^T Q \widehat{G}_0 + p^T N \widehat{G}_r \\ & + \tfrac{1}{2} \widehat{G}_0^T Q N \widehat{G}_r + \tfrac{1}{2} \widehat{G}_r^T N^T Q \widehat{G}_0 \\ & + \tfrac{1}{2} \widehat{G}_r^T N^T Q N \widehat{G}_r \end{aligned}$$

Since $\widehat{G}_r$ is a single variable denoted by x, the optimal force distribution can be written as :

$$minimize \quad a_0x^2 + a_1x + a_2 \qquad subject\,to \quad x \in [b_1\, b_2] \tag{28}$$

With

$$a_0 = \tfrac{1}{2}N^TQN$$
$$a_1 = p^TN + \tfrac{1}{2}\widehat{G}_0^TQN + \tfrac{1}{2}N^TQ\widehat{G}_0$$
$$a_2 = p^T\widehat{G}_0 + \tfrac{1}{2}\widehat{G}_0^TQ\widehat{G}_0$$

Where $[b_1\, b_2]$ denotes the bound resulted from Eq(27). Since it is clear that $a_0 \geq 0$ because of the positive-definite matrix Q, There must be an optimal solution for the force distribution problem.

V. SIMULATIONS

The basic mechanism, size and parameters of Hexapod robot are shown in Figure (6) and (7), where a = 0.25 [m], b = 0.61 [m], $l_1 = 0.05$ [m], $l_2 = 0.20$ [m], $l_3 = 0.30$ [m] and $l_4 \simeq 0$ [m]. There are tree actuated joints $\theta_{1,j}$, $\theta_{2,j}$, $and \theta_{3,j}$ in the leg "j", whose torques are denoted as $\tau_{1,j}\, \tau_{2,j}\, and\, \tau_{3,j}$, for (j=1,...,6), respectively. Assume the weight of leg is ignorable, then the Jacobian of the leg i can be expressed by.

$$J_j = \sigma[J_{j1} \quad J_{j1} \quad J_{j1}] \tag{29}$$

for (j=1,...,6), where

$$J_{j1} = \begin{bmatrix} 0 \\ -l_3C(\theta_{2,j}+\theta_{3,j}) + l_2C(\theta_{2,j}) \\ 0 \end{bmatrix}$$
$$J_{j2} = \begin{bmatrix} -l_3S(\theta_{2,j}+\theta_{3,j}) - l_2S(\theta_{2,j})] \\ 0 \\ -l_3C(\theta_{2,j}+\theta_{3,j}) - l_2C(\theta_{2,j})] \end{bmatrix}$$
$$J_{j3} = \begin{bmatrix} l_3S(\theta_{2,j}+\theta_{3,j}) \\ 0 \\ l_3C(\theta_{2,j}+\theta_{3,j}) \end{bmatrix}$$

and $\sigma = -1$ for (i=1,5,6) $\sigma = 1$ for (i=2,3,4). From Figure (6) the orientation matrix of $(x_{1,j}, y_{1,j}, z_{1,j})$ with respect to the frame (x_0, y_0, z_0) can be obtained by,

$$A_{0j} = \begin{pmatrix} Cos\,\theta_{1,j} & Sin\,\theta_{1,j} & 0 \\ -Sin\,\theta_{1,j} & Cos\,\theta_{1,j} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{30}$$

A. Simulation results

In order to show the effectivenesse of proposed approch, some simulations were conducted under Matlab. The simulation's parameters are given in table (2). We consider that the hexapod robot is crawling in circular trajectory, with a radius of 3.5 [m], on an uneven ground, in the X-Y plane. The z axis trajectory is given by :

$$h(t) = l_1 + l_3 + 2sin(\pi\, t)$$

Furthermore, the force tensor acting at the body center is given by:

Fx = -5, Fy = 10, Fz = -250 [N], Mx = 3 , My = 1 , Mz = 2 [Nm].

For the objective function Eq (17), the weighting matrix are choosen as follow : $p = 0$ and $Q = I$ (the identity matrix).

The figure 8 shows the circular trajectory of the hexapod in the X-Y plane, with a sinusoidal trajectory imposed in the Z-axis. The associated joints coordinates are obtained by using the direct and inverse geometric model (Eq (2)and Eq (3)). In the figures 9-11, the force distribution of the hexapod legs are given. We can show that, this distribution validate the following force equilbrium equation :

$$\Sigma fx_j = Fx, \quad ,\Sigma fy_j = Fy, \quad \Sigma fz_j = Fz$$

The crawling steps (section 3.3) imposed to the hexapod is ensured by the obtained force distribution figure 9-11. Elsewhere, the z-force components fz_j are never negative, respecting the contact constraint. We can also show that the constraint Eq (6) are always satisfiyed as shown in figure 12 for leg(j) ,j=1,2,3 where the curve $fx_j^2 + fy_j^2$ is always under the curve $\mu^2 fz_j^2$

Parameter	Value	Designation
$T = (t_4 - t_1)$	2s	time fraction of agait cycle
tp	0.2s	sampling time
$t_2 - t_1$	9*tp	period when the legs (1,2,3) are supporting the body
$t_3 - t_2$	2*tp	intermediate period
$t_4 - t_3$	9*tp	period when the legs (4,5,6) are supporting the body
$\delta(t)$	$\frac{t}{2tp} - \frac{T-2tp}{4tp}$	linear scalar function
μ	0.05	static coefficient of friction
τ_j	$[10\,10\,10]^T$	maximum joint torque vector

TABLE II

SIMULATION PARAMETERS

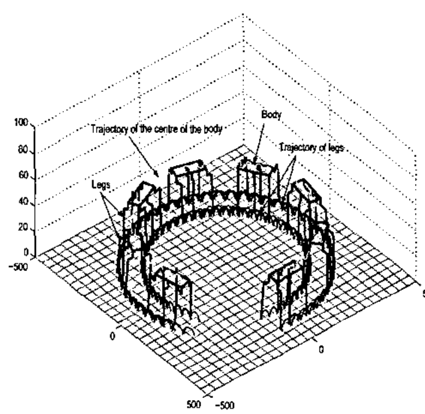

Fig. 8. View of the hexapod crawling

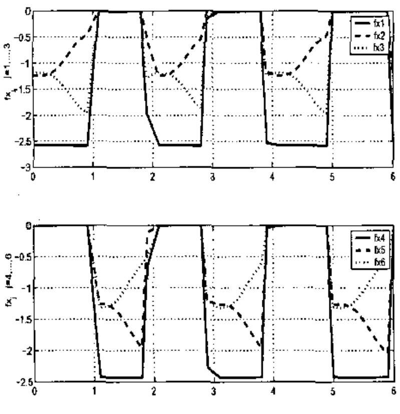

Fig. 9. Forces $fx_j \quad j = 1,..6$

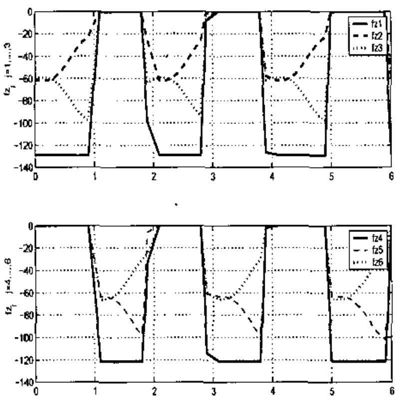

Fig. 10. Forces $fz_j \quad j = 1,..6$

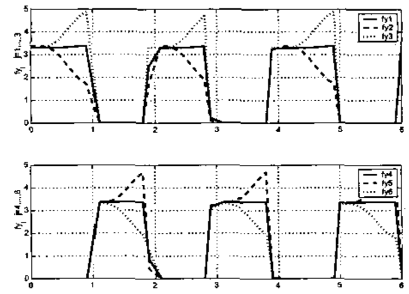

Fig. 11. Forces $fy_j \quad j = 1,..6$

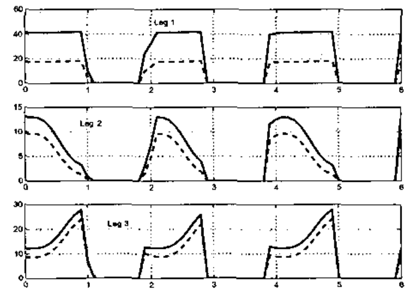

Fig. 12. Constraint $fx_j^2 + fy_j^2 \leq \mu^2 fz_j^2$ satisfaction

VI. Conclusion

In this paper, the authors have presented the force distribution problem is the case of an hexapod robot. First, the hexapod inverse and direct geometric models where presented. Then, the real time force distribution problem where formulated in terms of non linear programming problem. After problem size reduction and transformation, the initial problem is solved in terms of quadratic programming problem. Simulations results where presented. Actually, we are working on the generalisation of the approach proposed by Fijany et al [14] to the hexapod robot case. This approach leads to a 0 (log N) algorithm on N processors for dynamic simulation and control. Finaly, a new learning approach is under developpment for real-time operationnal space control of the hexapod robot.

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