Minimum Energy Force Distribution for a Walking Robot

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Here, the foot force distribution for a six-legged walking machine is resolved for minimum energy consumption over a full cycle for regular wave gaits. Some insects utilize horizontal foot forces to reduce muscle forces and energy consumption [R. J. Full, R. Blickhan, and L. H. Ting, J. Exp. Biology 158 (1991), 369–390]. Foot force distribution for minimum energy consumption when applied to the walking machine, also supports this observation. In this study, geometric work loss for a walking machine with articulated legs is minimized by controlling interaction forces at the foot-ground interface. Minimum energy foot forces are studied for various duty factors, lateral offsets, link proportions, and friction between foot and ground.

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1. INTRODUCTION

The superior mobility characteristics of legged animals compared to those of wheeled or tracked vehicles for offroad locomotion motivated the development of artificial walking machines. However, legged machines have hardly begun to be used for practical applications. One of the major reasons behind this slow progress is their poor energy efficiency. An industrial manipulator can perform its function even if its energy efficiency is low, because the controller, power source, and actuators can be separated from the manipulator itself. In contrast, an autonomous walking vehicle cannot function satisfactorily with low energy efficiency, due to the fact that it has to carry all driving and control units in addition to body and payload. Long duration missions such as planetary explorations are subject to resource constraints. The minimization of energy consumption plays a key role in the design of walking machines. A reduction in energy consumption results in machines which can not only travel more, but also require smaller actuators (as size and weight of onboard energy storage device reduces) that typically yield a reduction in vehicle weight and cost.

Several avenues of research were pursued in the past to realize energy efficient walking vehicles. Some of these are: (1) design of energy efficient artificial leg structure,[2,3] (2) the employment of energy storage devices to recover energy from one step to the next,[4,5] and (3) finding an optimal manner of walking which reduces energy consumption.[6,7] In this study, we concentrate on the third aspect.

For a statically stable walker, at any instant, at least three legs should be on the ground. If a three-dimensional ground reaction force vector on each grounded foot is considered, the foot force distribution problem becomes indeterminate. It means, there exist many solutions which can satisfy the force-moment balance criteria. Here, we focus on determining the foot force distribution for minimum energy consumption of the legged vehicle.

The motivation behind this work is the study made by Full, Blickhan, and Ting[1] on the foot force pattern and energy consumption characteristics of cockroaches. They measured the ground reaction forces for different legs of the cockroach during walking to find out why animals travelling at a constant average speed generate significant horizontal ground reaction forces and observed that the ground reaction force vectors minimize joint moments and muscle forces by directing reaction forces toward the coxal joint (where the leg is connected to the body of the insect). We also study the role of duty factor, link proportions of the leg, and friction at the foot-ground interface on power consumption of the walking vehicle.

In the following section, different modes of transportation are compared based on the cost of transportation. Section 3 deals with the design aspects of walking machine legs. The foot force distribution problem and different approaches for resolving forces are discussed in Section 4. In Section 5 the foot force distribution problem is formulated for minimum energy consumption. Simulation results and their analysis are given in Section 6.

2. MODE OF TRANSPORTATION AND COST

Biological systems have three major modes of transportation: swimming, flying, and running. To compare the cost in different modes, a suitable basis is necessary. Gabrielli and Von Karman[8] introduced *specific resistance* as an energy requirement per unit body weight to move unit distance and compared various means of locomotion from an engineering point of view. As per their study, merchant ships have the least specific resistance, jet fighters have the maximum, and natural legged locomotion falls in between. In a different study confined to animals, Schmidt-Nielsen[9] found that for a given body weight, energy cost of swimming is the least and flying is a far cheaper way to move to a distant point than running. In an effort to compare selected land vehicles with the legged machines of today, Gregorio, Ahmadi, and Buehler[10] plotted specific resistance against speed in log–log scale. It was prominent that legged machines appeared at a position far from the Gabrielli–Von Karman line (a practical limit to vehicle performance based on 1950 data) in spite of the fact that the weight of the power source is not included in walking vehicle mass while calculating specific resistance. This means, there is enormous scope to improve the energy efficiency of artificial walking machines. In this study, we also use the specific resistance as an index for comparison. However, we refer to it as *specific energy consumption* hereafter.

3. IMPORTANCE OF LEG DESIGN

The leg geometry is a crucial aspect of walking machine design since it strongly influences the efficiency of the vehicle. Since all aspects of walking are ultimately governed by the physical limitations of the leg, it is important to select a leg that has more range of motion and that will not impose constraints on gait selection. Synthesis of a walking machine configuration begins with the identification of a suitable leg type. A variety of leg geometries can provide the motion required by a terrain adaptive walker so that it can adapt its foot placements to the terrain and can move its body on a stable trajectory relatively independent of terrain details. A survey of the literature shows that there are a number of different leg designs currently employed for walking robots. Hirose and Umetani[2] used a three-dimensional pantograph and later Song et al.[3] used a two-dimensional pantograph leg mechanism. These legs can avoid geometric work loss (energy loss when two actuators work against each other) by locking one actuator while the other is being used. Pantograph leg mechanisms have been shown to be very effective in walking machine prototypes. However, these designs include linear motions which are bulky and less reliable compared to rotary motions and could result in locking of joints. Bares and Whittaker[11] used spatially decoupled orthogonal legs revolute-prismatic-prismatic (RPP) to avoid geometric work loss. This type of leg is specially advantageous in extreme terrain because the foot can be placed into ditches and holes and near walls without concern for shank rocking collisions. Due to this property, orthogonal legs have superior mobility characteristics compared to legs based on pantograph mechanisms. However, actuator locking problems are also present in orthogonal legs.

Here, we considered a three-dimensional articulated mechanical leg structure. Animals using legs for terrestrial locomotion show excellent performance on rough terrain over a wide range of speed and body size. All employ open chain based legs for generation of propulsive forces. Because these legs are energy inefficient compared to the type of legs mentioned above, energy minimization is important in this case.

4. THE FORCE DISTRIBUTION OF WALKING MACHINE

The control of walking machines involves the solution of the force distribution problem. For these machines, at any instant, at least 3 feet should be on the ground. For each foot, there will be three unknown force components, if rotational torques at the foot are neglected. Thus, the force system involves $3m$ unknown force components when m feet are on the ground. Since only six static equilibrium equations are available to determine these force components, and $m \geq 3$ for static equilibrium, the system is always statically indeterminate. The degree of indeterminacy increases with m.

Many researchers tried to resolve the foot force distribution problem for different objectives. Klein and Chung[12] and Klein and Kittivatcharapong[13] used the homogeneous solution to minimize the discontinuities in commanded forces when the leg phase alternates between transfer and support. Waldron and Kumar decomposed the foot force field into the interaction force field and the equilibrating force field, and obtained the foot forces for zero interaction force based on planar vertical force distribution.[14,15] Gao and Song[16] used the stiffness matrix method to get an exact solution from the viewpoint of structural mechanics. Huang and Waldron[17] used planar force distribution to find the relationship between payload and speed of the walking machine. However, none of these studies addressed the energy consumption aspects of the walker.

So far, only Orin and Oh[6,7] addressed the problem from the energy consumption point of view. They tried to resolve the foot force distribution for minimum energy consumption and load balance (minimization of the maximum normal tip reaction force) between several legs. They simplified the friction cone constraint to eliminate the associated nonlinearity by inscribing a pyramid within the desired friction cone. Because of this simplification, the solution was conservative. Their simulation was restricted to joint torque computations of hexapod legs using tripod gait only.

5. OPTIMUM FOOT FORCE DISTRIBUTION FOR MINIMUM POWER CONSUMPTION

5.1. The Walking Machine Model

A simplified diagram of a six-legged machine is shown in Figure 1. Each leg has three joints powered by independent actuators. The first joint is for the swivel of the leg about the vertical axis passing through the hip. The other two axes (hip and knee) are horizontal and parallel to each other. The vehicle walks along a straight path on horizontal ground and during walking, it maintains fixed body height. The center of gravity (CG) is located at the geometric center of the vehicle body. Let the reference

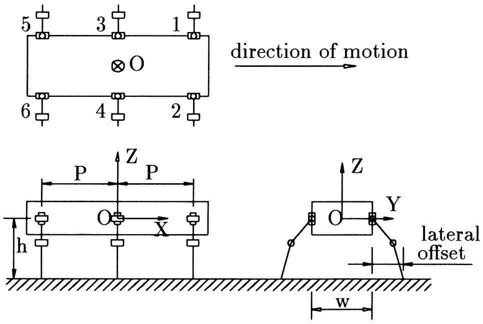

Figure 1. Simplified six-legged machine.

frame be a vehicle body-fixed frame defined as having its origin at the CG, the x axis along the longitudinal axis of symmetry (positive forward), the z axis aligned vertically (positive up), and the y axis being the remaining one forming a right-handed orthogonal set. Leg number is assigned as 1, 3, 5 on the left side and 2, 4, 6 on the right side starting from the front to the rear.

Suppose that the contact surfaces between the feet and the terrain are small enough to be considered as points. Then, the interaction forces between them can be modelled as point forces. At any instant, for m grounded legs, let the ground reaction force at foot i be denoted by $\mathbf{F}_i = (F_{xi}, F_{yi}, F_{zi})^T$ and let the foot location be denoted by $\mathbf{r}_i = (x_i, y_i, z_i)^T$. Let us assume that the vehicle weight is lumped into the center of gravity and the legs are assumed to be massless in comparison with the mass of the vehicle body.

Due to the optimal stability characteristic of wave gait, it is considered here for analysis. It is widely accepted for use in walking machines. Wave gait is a regular, periodic, and symmetric gait in which the placing of the foot runs from the rear leg to the front leg as a wave along either side of the body. For a hexapod, the gait can be expressed analytically in the gait formula[18] as

$$\phi_1 = 0$$
$$\phi_3 = \beta$$
$$\phi_5 = 2 * \beta - 1$$
$$\phi_2 = \phi_1 + 0.5$$
$$\phi_4 = \phi_3 + 0.5$$
$$\phi_6 = \phi_5 + 0.5$$

for $0.5 \leq \beta \leq 1.0$, where β is the duty factor and is defined as the fraction of the cycle time for which a leg remains on the ground. Gait can also be represented graphically using a gait diagram as illustrated in Figure 2.

5.2. Problem Formulation

Here, the problem of force distribution for minimum energy consumption is presented mathematically. The formulation includes: the force and moment balance equations, the constraints and the energy consumption (objective function). At any instant, considering the static equilibrium of the vehicle,

$$\sum_{i=1}^{m} \mathbf{F}_i + \mathbf{R} = 0 \tag{1}$$

$$\sum_{i=1}^{m} (\mathbf{r}_i \times \mathbf{F}_i) + \mathbf{T} = 0 \tag{2}$$

where $\mathbf{R}$ and $\mathbf{T}$ are the gross external force and the moment applied on the vehicle. $\mathbf{R}$, $\mathbf{T}$, and $\mathbf{F}_i$ are three-dimensional vectors. If we assume that there are no external forces or moments other than gravity, $\mathbf{R}$ reduces to $(0, 0, -W)^T$ and $\mathbf{T}$ reduces to null.

An important inequality constraint is associated with limiting the angle between the resultant foot force and the normal to the ground at the point of contact, to avoid foot slippage. This is limited by the coefficient of static friction at the interface. Then,

$$\left(F_{xi}^2 + F_{yi}^2\right)^{1/2} \leq \mu F_{zi}$$

for $i = 1, 2, \ldots, m$. Here, μ is the coefficient of static friction between foot and ground. This is a nonlinear set of constraints. Because these are also nonsmooth at $F_{xi} = F_{yi} = 0$, we replace these by equiva-

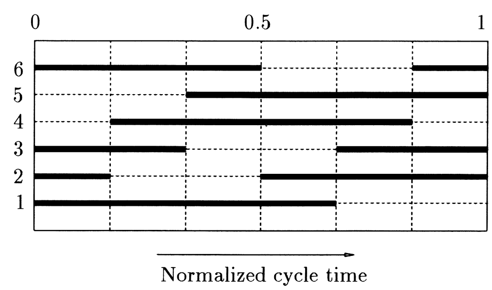

Figure 2. Gait diagram for wave gait ($\beta = 2/3$).

lent smooth constraints,

$$\left(F_{xi}^2 + F_{yi}^2\right) \leq \mu^2 F_{zi}^2 \tag{3}$$

An additional requirement is that since the feet of a walking machine cannot grasp the ground, normal forces cannot be allowed to become negative. It means,

$$F_{zi} \geq 0 \tag{4}$$

The objective function is the energy consumed. It is assumed that the trajectory of the system is prescribed. Since energy is the integral of power consumption over time, minimization of energy results if the power is minimized at each point of the trajectory. To calculate instantaneous power consumption of the vehicle, it is necessary to consider all the actuators responsible for the vehicle motion. The vehicle has six legs and each leg has three joints. Legs which are in transfer phase do not support any load and joint power requirements for these legs are considered nil. Let, at any instant, p_{ik} be the power requirement at the kth joint of the ith supporting leg. Then,

$$p_{ik} = \tau_{ik} \dot{\theta}_{ik} \tag{5}$$

where, τ_{ik} and $\dot{\theta}_{ik}$ are torque and angular speed, respectively, at the kth joint of the ith supporting leg. Since the joint which executes negative work ($p_{ik} \leq 0$), does not regenerate power, the total instantaneous power consumption can be written as

$$P_{\text{inst}} = \sum_{i,k} \max(p_{ik}, 0)$$

for $i = 1, \ldots, m$ and $k = 1, 2, 3$. Thus, the problem is to *minimize* P_{inst} subject to the constraints mentioned in Eqs. (1)–(4). The objective function is nonsmooth. To develop a formulation with smooth functions, we introduce $3m$ new variables u_{ik}. These are bounded by p_{ik} as

$$u_{ik} \geq p_{ik}$$
$$u_{ik} \geq 0$$

for each joint. Thus, the problem is reduced to

$$\min \sum_{i,k} u_{ik}$$

6. RESULTS AND DISCUSSION

The optimization problem formulated here is solved using a sequential quadratic programming based non linear programming (NLP) solver. Wave gaits with different duty factors are considered. The physical values of the parameters used for simulations are: pitch (linear distance between two adjacent legs of the same side of the vehicle) $P = 1.0$, leg stroke $R = 1.0$, length of leg segments $L_1 = 0.5$, $L_2 = 0.5$, body elevation $h = 0.6$, and track width $w = 1.2$. All these values are expressed as a fraction of $(L_1 + L_2)$. The coefficient of friction at the foot-ground interface was $\mu = 0.3$. The optimal foot force distribution for $\beta = 2/3$ is shown in Figure 3 over a gait cycle. The foot force components as a fraction of the vehicle weight are plotted against the normalized cycle time at an interval of 0.02. The three components f_x, f_y, f_z are shown by a continuous line, a dashed line, and a center line, respectively. It is observed that, the foot force pattern for the opposite legs are exactly the same (opposite in sign, in case of component along the y direction) but are out of phase by 180°. This is because these are mounted in

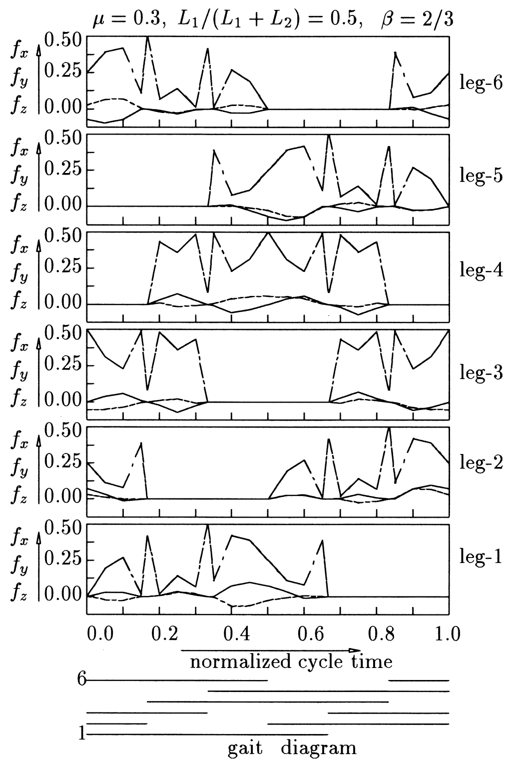

Figure 3. Foot force distribution for minimum power consumption ($\beta = 2/3$, lateral offset = 0.1).

the same body segment and in the gait cycle are out of phase by a half cycle. Discontinuities in vertical force variation of a foot with time occur when another foot is placed on or lifted from the ground.

The variation of instantaneous power consumption throughout a locomotion cycle of the vehicle is shown in Figure 4(a) and (b) for duty factors 2/3 and 3/4, respectively. It is observed that at most points of the cycle, the power consumption at duty factor = 2/3 is more compared to power consumption at duty factor = 3/4. The highest peaks in Figure 4(a) represent a four-legged support state of the machine. Power value becomes minimum when the machine is supported by five legs including one of the middle legs (either leg No. 3 or leg No. 4) at the middle of its stroke. When for a five-legged support state, one of the extreme (either front or hind) legs is at the midstroke, the intermediate peaks arise. In Figure 4(b), ($\beta = 3/4$), highest peaks arise during four-legged support states and none of the supporting legs are at their midstrokes. However, when two out of the four supporting legs are close to their midstrokes flat minima in the power curve occur. Minima also occur at six-legged support states. For five-legged support states with one of the middle legs close to midstroke, intermediate flats occur. So, power consumption characteristics improve when legs are at their midstroke. It also improves when more number of legs are on the ground.

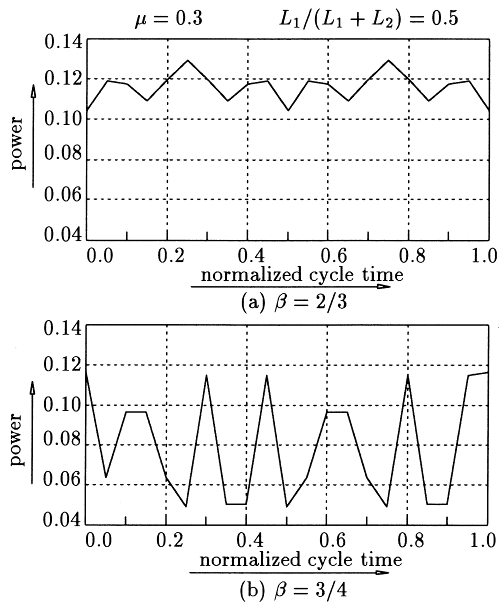

Figure 4. Variation of power consumption with normalized cycle time (lateral offset = 0.1).

The total energy consumption of the vehicle over a complete cycle is calculated from the instantaneous power consumption data to determine the specific energy consumption (E_{sp}). The variation of specific energy consumption with the duty factor is shown by dotted lines in Figure 5. At this point, the three-dimensional force distribution model can be compared with the planar vertical force distribution model. Keeping all the parameters unchanged, using the planar vertical force distribution[17] and the zero horizontal foot forces, variation of specific energy consumption with the duty factor is shown by continuous lines in Figure 5. It is clear that the horizontal foot force components reduce energy consumption significantly. For $\beta = 0.8$, the specific energy consumption is reduced by as high as 57% of that using planar foot force distribution.

To study the significance of foot force direction on specific energy consumption of the vehicle, the values of E_{sp} are computed for different lateral offset. Lateral offset is the shortest distance between vertical projection of the hip on the ground and the corresponding track (Fig. 1). It is expressed as a fraction of $(L_1 + L_2)$. As shown in Figure 6, the specific energy consumption gradually decreases with a lateral offset and reaches the minimum at offset = 0.45. If the offset is increased further, specific energy consumption increases sharply. To explain this behavior, it is necessary to choose any particular instant of the gait and to check the direction of the ground reaction force vector (for minimum power consumption) for different values of lateral offset. For example, at normalized cycle time = 0.2, for $\beta = 2/3$, (Fig. 2) four legs are on the ground. Up to offset = 0.2, only one foot force vector (leg 4) is directed toward the knee. As the offset is increased, the force vector at leg 3 also passes

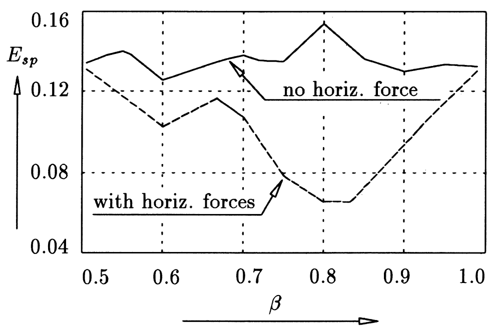

Figure 5. Variation of specific energy consumption (E_{sp}) with duty factor ($\mu = 0.3$, link ratio = 0.5, lateral offset = 0.1).

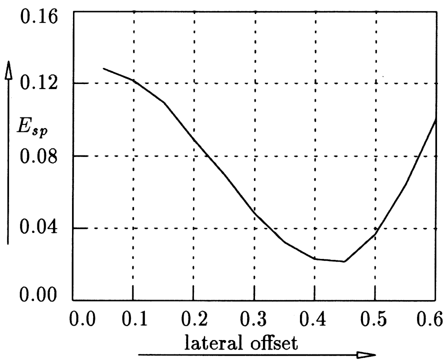

Figure 6. Variation of specific energy consumption with lateral offset ($\mu = 0.3$, link ratio = 0.5, $\beta = 2/3$).

through its knee. At offset = 0.45, foot forces for legs 1, 3, and 6 pass through their knees and the foot force for leg 4 passes through its hip. The instantaneous power consumption at this condition becomes minimum. Similar situations occur at any other instant of the gait cycle. Beyond offset = 0.45, less number of force vectors are directed either to hip or to knee and instantaneous power consumption increases with offset. Thus, for minimum energy foot force distribution, the ground reaction force vectors pass close to the hip or the knee of the corresponding legs. It supports the observations by Full, Blickhan, and Ting[1] for biological systems.

The above studies were performed for a coefficient of friction $\mu = 0.3$. To study the influence of coefficient of friction at foot-ground interface on E_{sp}, optimizations are done for different values of μ, keeping all other parameters unchanged. The variation is shown in Figure 7. Initially, for small values of μ the specific energy consumption is large. It reduces gradually with an increase in the value of the coefficient of friction. This is because an increase in the value of the coefficient of friction means relaxation of the friction cone constraint and less number of active friction constraints at any instant. Beyond 0.3, μ does not have any more appreciable influence.

So far, we considered that both the links of the leg are equal in length. To study the variation in specific energy consumption with link proportion, specific energy consumption is determined for different link proportion $L_1/(L_1 + L_2)$ keeping the total link length (i.e., $L_1 + L_2$) unchanged. The variation is shown in Figure 8. It is observed that, specific energy consumption is minimum for link proportion = 0.25. It increases considerably at both the extremes. This is because, in these situations, the leg has to operate near the boundary of its workspace.

7. CONCLUSION

In this study, the foot force distribution for a six legged walking machine is resolved for minimum energy consumption. It is observed that the energy consumption characteristics can be improved significantly by controlling the foot force components. The minimum energy foot force distribution obtained supports the observations by Full, Blickhan, and Ting[1] on biological systems that the reaction forces are directed to reduce the joint torque requirements. A study on instantaneous power consumption shows that power demands are less when any of the grounded legs are at the middle of their stroke and also when more number of legs are on the ground.

Specific energy consumption is found to be very sensitive to lateral offset. Also, it reduces with an increase in the coefficient of friction at the foot contact point on the ground. The link proportion also influences specific energy consumption significantly.

As long as the number of legs on the ground and the foot coordinates of all the grounded legs are

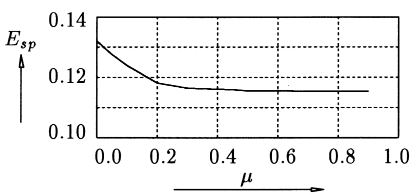

Figure 7. Variation of specific energy consumption with coefficient of friction ($\beta = 2/3$, link ratio = 0.5, lateral offset = 0.1).

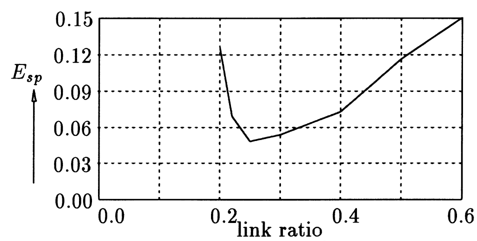

Figure 8. Variation of specific energy consumption with link proportion ($\beta = 2/3$, $\mu = 0.3$, lateral offset = 0.1).

known, the scheme can be used for any type of gait. Some modifications are necessary to analyze vehicle motions with body inclination. The optimal foot forces cannot be obtained in real time. However, it is useful to investigate them since they provide insights that can be used to evaluate the effectiveness of the scheme. Further, in some situations, look up tables can be generated using this technique off-line. The study is also useful for finding limits of system performance.

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