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Tripod Gaits Planning and Kinematics Analysis of a Hexapod Robot

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Abstract—**This paper proposes a scheme for the design, gaits planning and kinematics control of a hexapod robot. The robot is symmetrical structure with six identical legs. Each leg consists of three revolute joints which are actuated by position-controlled servos. The paper focuses on two crucial problems for multi-legged robot control, gaits planning and kinematics solving. Based on tripod gaits, a new turn gait which has static stability is proposed. Considering turn gait, there are two important parameters called the reachable area of each leg and maximum rotation angle of robot body. The two parameters are determined by the initial state and mechanical structure of hexapod robot. Then, an analytical inverse kinematics solution is discussed to realize kinematics control of the robot. The maximum rotation angles are solved and the obstacle avoidance experiment proves that the turn gait is feasible and the hexapod robot can walk rapidly and agilely on even terrain.**

I. INTRODUCTION

RESEARCHES on multi-legged walking robots which are created by imitating limb structure and motion control of insects or arthropod animals have extended for many decades[1]. Among many multi-legged robots, hexapod robot is one of the most typical robots. Hexapod robot has the advantage (compared with a quadruped robot) that it can establish a static equilibrium easily while moving and can successfully walk in unstructured terrain. Besides, it can go forward with many kinds of gaits to adapt different speeds and loads. And because the redundant limb exists, hexapod robot could continue its work even if limb is lost. These advantages make it competent for some autonomous and high-reliability works, such as field scouting, underwater searching, and space exploring. However, the control of those 6 legs becomes a complex task, due to the number of variables that must be monitored. There is large body of work devoted to the control of hexapod robot which contains gait planning and kinematics control [2, 3].

Gait takes an important role in the control of walking machine. The gait synthesis might be based on kinematics model of the robot and walking rules that are well known from insect walking [4]. There are two main types of gaits adopted in walking machines – periodic gait and non-periodic gait which is also called free gait [4, 5]. The free gait increases the adaptability of the walking machine because it can move on the uneven terrain. However, the free gait is hard to be realized in the real multi-legged walking robots and is only on the stage of theoretical research. Periodic gait such as tripod gait can be easily controlled and has an optimal stability margin [6].

A considerable amount of prior work has been devoted to the periodic gait for the special case of straight line constant-speed locomotion over level terrain. As early as 1960, McGhee and Frank [7] proved that a quadruped wave gait has the optimum stability among all quadruped periodic and regular gaits. Bessonov and Umnov [2] extended that result to the hexapod motion in straight lines. Recently, Song and Choi [8] reported the optimally stable ranges of generalized 2n-legged wave gaits. [9] designed one hexapod tripod gaits for straight line motion and crab walking and their stability margin. [10] provided a eight-legged demining robot and corresponding turn gait. [11] proposed a scheme for fault detection and tolerance of the hexapod robot locomotion on even terrain.

However, there are not many researches on turn gaits. On the existing turn gaits, the chassis of robot needs to rotate twice or more times to complete a gait period and the rotate angles of each time are same that affected the rapid and agile locomotion of hexapod robot. In this paper, we propose a new turn gait that the chassis of robot only needs to rotate once to complete a gait period when the rotation angle is smaller than the maximum angle that the robot can rotate. Only when the rotation angle is larger than the maximum rotation angle, the chassis of robot needs to rotate more time to complete a gait period. And the rotation angle of each time can be different and transformable. Accordingly, the rapid and agile locomotion of robot are improved.

The mechanical complexity of walking robots poses a number of control problems due to the large number of degrees of freedom involved in the robot's motion: during the support phase the system performs a closed kinematical chain and an opened kinematical chain in the transfer phase. The inverse kinematics solution is essential for the robot's control, since it allows computing the required joints angles to move the robot's leg in a desired trajectory.

In this paper, we first describe the design and construction

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of this hexapod robot that contains its mechanical configuration and electronic hardware. Then, based on tripod gaits and analysis of reachable area of each leg, the turn gaits which are suitable for hexapod robot are proposed. This gait is proved having static stability since the projection of the center of robot body is inside the stability polygon formed by the support legs and the foothold positions of the support legs are solved. Furthermore, for realizing kinematics control of robot, an analytical inverse kinematics of entire robot is discussed. Finally, a simple obstacle avoidance based on the turn gait is used to verify these algorithms and the results of experiments proved that this turn gaits are feasible and useful.

II. Robot Design

A. Mechanical Configuration

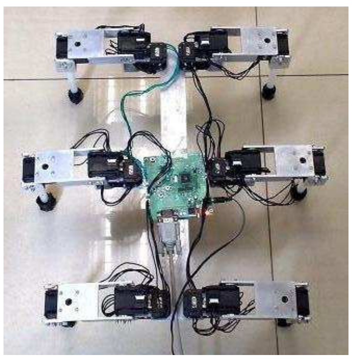

Fig. 1. The hexapod robot

Fig.1 shows the hexapod robot that we have already developed. This hexapod robot has six identical legs aligned symmetrically along the body, being three on each side. Of course, this is only a simple prototype and our ultimate objective is to design a hexapod robot that imitates limb structure and motion control of cockroach. The total length of the robot when the legs are stretched is $668mm$, the height of the body can change from 0 to $190mm$. Each leg has 3DOF, thus the robot has 18 DOF. Fig.2 shows the configuration of single leg and the length of link L_1, L_2 and L_3. The range of joints angles are $\theta_{i1} \in \left[-2\pi/3, \pi/2\right]$, $\theta_{i2} \in \left[-\pi/3, 2\pi/3\right]$, $\theta_{i3} \in \left[-\pi/3, \pi/3\right]$ $(i = 1, 2, ..., 6)$.

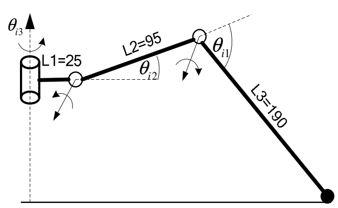

Fig. 2. Configuration of single leg

B. Electronic Hardware

We adopted smart actuator module Dynamixel RX-28 by ROBOTIS as joint actuators. This module is a smart, modular actuator that incorporates a gear reducer, a precision DC motor and a control circuitry with networking functionality, all in a single package. Despite its compact size, it can produce high torque and is made with high quality materials to provide the necessary strength and structural resilience to withstand large external forces. It also has the ability to detect and act upon internal conditions such as changes in internal temperature or supply voltage. Only if the reference motor angle is commanded, the control unit controls the joint with position control. The current position and speed of each joint can be read out.

This work used the control board based on the 16-bit AVR ATmega128.The main controller communicates with the Dynamixel units by sending and receiving data packets. The main controller sends commands to the motors and sensors and read out the status of them. We run the programs for simple tasks on the board computer. The board can also communicate with external computers by RS232. For complicated tasks, the external powerful computer receives the data from board, generates the robot motion and sends the reference joint angles to the board by RS232.

III. Gaits Planning

A. Reachable Area

In this paper, the following assumptions are made for the simplicity of analysis.

1) The contact between a foot and the ground is a point.

2) There is no slipping between the foot and the ground.

3) All the mass of the legs is lumped into the body, and the center of gravity is assumed to be at the centroid of the body.

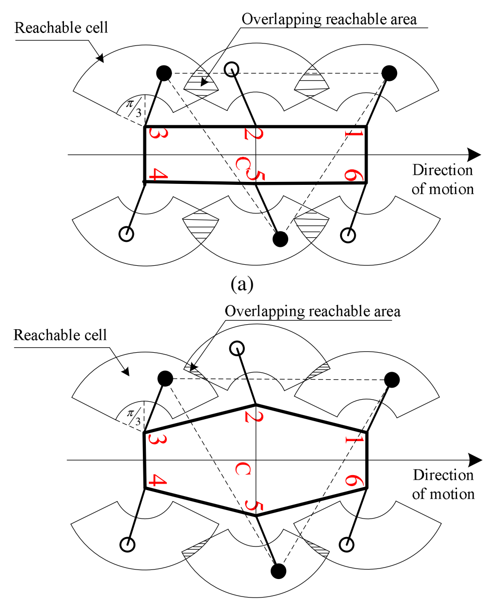

Fig. 3. Schematic top view of robot showing the reachable cell of each leg and overlapping reachable area

Before planning gaits, determining of reachable area of each leg is essential. As Fig.3 shows, each leg has a reachable area in the form of a sector of an annulus. This area is specified by four parameters: minimum angle $\psi_{\min}$, maximum angle $\psi_{\max}$, minimum radius $\gamma_{\min}$, and maximum radius $\gamma_{\max}$ Reachable areas move as the body moves. As we mentioned above, the range of angle $\theta_{i3} (i = 1, 2, \cdots, 6)$ is from $-\pi/3$ to $\pi/3$. So angle $\psi_{\min}$ is $-\pi/3$, and angle $\psi_{\max}$ is $\pi/3$. $\gamma_{\min}$ and $\gamma_{\max}$ are determined by the height of robot body and the length of L_1, L_2 and L_3. For this hexapod robot, $\gamma_{\min}$ is approximate to 0. Limitation of $\gamma_{\max}$ is $L_1 + L2 + L3$ when the height of robot is 0.

As Fig .3(a) shows, feet are always placed in the center of reachable cells. Overlapping reachable areas raise interference problems. That is, a given leg may exclude regions of the reachable area of legs adjacent to it. Calculation of the excluded areas is not easy and is dependent on where the legs are at a given time. According to [12], one way of dealing with this problem is to avoid it altogether by specify two boundary lines perpendicular to the longitudinal axis of the robot. Though these constraints eliminate the interference problem entirely, this is just a passive way to solve the problem and reduces the workspace of each leg greatly.

So, in our work, we solve this problem by modifying mechanism of robot body. As Fig .3 shows, (a) is the most common rectangular structure; (b) is the approximate elliptical structure that we designed. It is obvious that the Overlapping reachable areas of (b) are smaller than (a) and the former lie at the edge of reachable cells which will be rarely used. At the same time, we found that the area of stability polygon (broken lines) of (b) is larger than (a), so (b) will have better static stability than (a) when the robot walks.

B. *Turn Gait Planning*

The alternating tripod gait is widely used because of its static stability and speed on a relatively flat surface. During locomotion, two legs on one side and one leg on the opposite side make ground contacts to move the platform as they deploy into their final joint configurations. The other three legs are raised away from the ground.

Based on the tripod gait, we plan a nominal turn gait that is suitable for our robot. The gait is shown pictorially in Fig.4. Leg1, 3 and 5 are group legs A, leg2, 4 and 6 are group legs B. The first phase (Fig. 4(a)) is the initial state. Leg1, 3 and 5 are raised up. Leg 2, 4 and 6 are on the support phase. The second phase (Fig. 4(b)) is the recovery of the leg2, 4 and 6 to their new positions and leg1, 3 and 5 are raised up. The foothold positions of leg2, 4 and 6 are solved below. The third phase (Fig. 4(c)) is a counterclockwise by γ_1 rotation of the chassis of the robot with leg 2, 4 and 6 on the ground. The next two phases (Fig. 4(d) and (e)) are basically same with forward two phases (Fig. 4(b) and (c)). We only need to exchange the states of group legs A and B.

When the third phase has been completed, the robot recovers to the initial state and can continue to turn or to go forward, so the flexibility of locomotion is improved. We assume that the robot needs to rotate counterclockwise by γ. If γ is less than the maximum $\gamma_{\max}$ that the robot can rotate by one step, then γ_1 are equal to γ and the robot terminates turning when the phase 3 is completed. If γ is larger than $\gamma_{\max}$, then γ_1 are equal to $\gamma_{\max}$ and the robot continue to turn until to complete rotation by γ. $\gamma_{\max}$ is related with initial angles of joints and the body configuration and is discussed in experiment section.

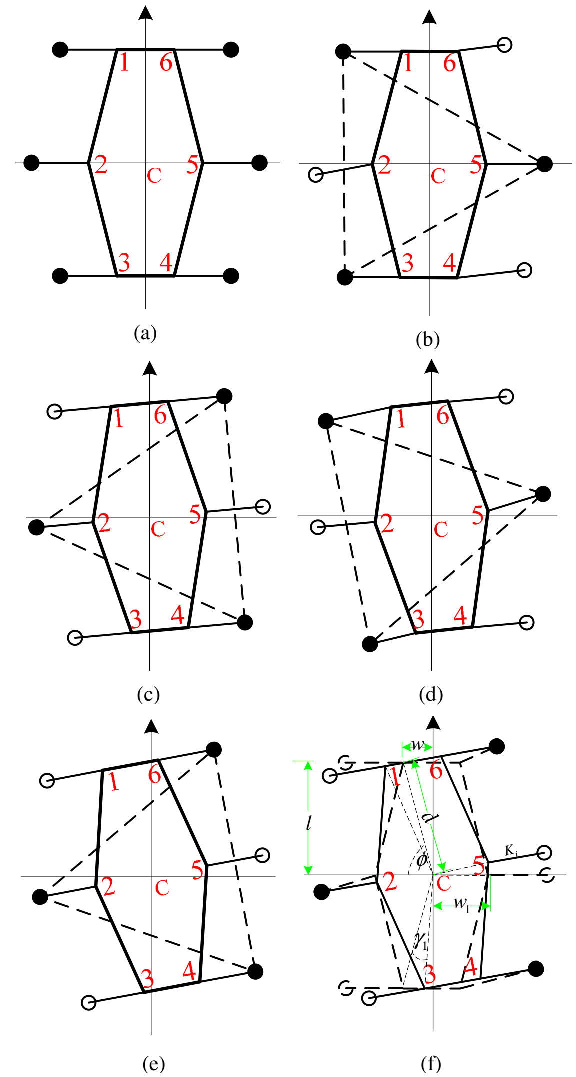

Fig. 4. A plan view of the turn gait

As Fig. 4 shown, the projection of the center of robot is inside the stability polygon (broken lines) formed by the support legs, so this turn gaits have static stability.

C. *Analysis of Footholds*

As Fig .4 (f) shown, broken lines are the second phase and real lines is the third phase. According to the geometrical relationship, we get footholds of the three support legs as following:

$$\begin{aligned}
P_{A2}^{o} &= \left[-(w_1 + K_i)\cos\gamma_1 \quad -(w_1 + K_i)\sin\gamma_1 \quad 0\right] \\
P_{A4}^{o} &= \left[d\cos(\phi-\gamma_1) + K_i\cos\gamma_1 \quad -d\sin(\phi-\gamma_1) + K_i\sin\gamma_1 \quad 0\right] \\
P_{A6}^{o} &= \left[d\cos(\phi+\gamma_1) + K_i\cos\gamma_1 \quad d\sin(\phi+\gamma_1) + K_i\sin\gamma_1 \quad 0\right]
\end{aligned} \tag{1}$$

Where

$$K_i = L_1 + L_2\cos\theta_{i2} + L_3\cos(\theta_{i1} + \theta_{i2}) \tag{2}$$

And

$$d = \sqrt{w^2 + l^2} \tag{3}$$

$$\phi = \arctan\left(\frac{l}{w}\right) \tag{4}$$

$w = 48mm$, $w_1 = 88mm$ and $l = 190mm$ are pre-determined by the configuration of robot. The coordinate systems are established below. It is similar to solve footholds position of the support legs (1, 3 and 5) in the second step, so we shall not talk about it further.

IV. Kinematics Analysis

A. Coordinate Systems

We mainly research on inverse kinematics of the whole body. When using tripod gaits, the three support legs, body of the robot and ground formed a parallel mechanism. The analysis of the whole body's kinematics can be simplified if each leg is considered as an isolated system. Once the kinematics of one leg is computed, it will become simpler to solve the entire robot's kinematics by establishing the legs cooperation in the movement of the robot. As following, we establish coordinate systems in Fig. 5.

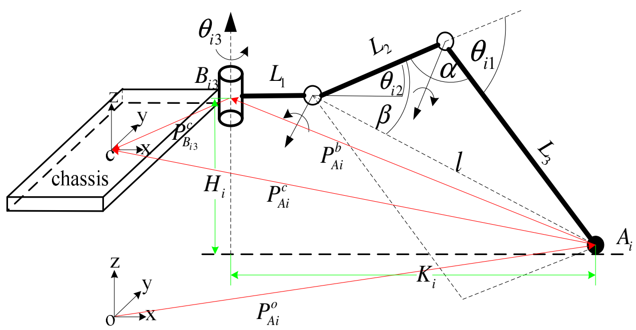

Fig. 5. Configuration of leg mechanism

1)Inertial Frame: the coordinate system fixed on the ground is $O-xyz$, the origin is at the initial center of mass of the body, positive direction of axis y is the same with the forward direction of the body, positive direction of axis z is downward vertically. Axis x is determined by right-hand rule.

2) Locomotive Referenced Coordinate System: the body coordinate system is $C-xyz$. Its origin C is at the center of mass of the body all the time. The plane $x_c y_c$ parallels that of the body. Let the positive directions of the axis be the same with $O-xyz$. The initial position and the pose of $C-xyz$ are respectively the same with $O-xyz$.

3) Coordinate Systems on the Coxa: let $B_{i3}-xyz$ be the coordinate system on leg i. Its origin $B_{i3}\,(i=1,2,\cdots,6)$ is at the connection point with the body. In the inertial frame, B_{i3} and C are at the same height.

Here, every leg can be considered as a serial robot consisting of 3 joints. Fig.5 shows the configuration of one leg. $L_i(i=1,2,3)$ are link lengths. $P_{Ai}^{\,o}(i=1,2,\cdots,6)$ are the vectors from O to points $A_i(i=1,2,\cdots,6)$. $P_{Ai}^{\,b}(i=1,2,\cdots,6)$ are the vectors from $B_{i3}(i=1,2,\cdots,6)$ to points $A_i(i=1,2,\cdots,6)$. $P^c_{B_{i3}}(i=1,2,\cdots,6)$ are the vectors from C to points $B_{i3}(i=1,2,\cdots,6)$.

B. Inverse Kinematics Analysis

Considering the body of cockroach robot, we obtain all the joint driving variables, θ_{i1} , θ_{i2} and θ_{i3} (i=1, 2, 3, 4, 5, 6) according to the body pose T_c^o , footholds $P_{Ai}^o\,(i=1,3,5)$ and the trajectory of leg $P_{Ai}^o\,(i=2,4,6)$.

We assume the coordinate system C be fixed at the center of the robot body. $B_{i3}\,(i=1,2,\cdots,6)$ is the center of the coxa which links *leg i* $(i=1,2,\cdots,6)$ and the body.

Because

$$P_{Ai}^o = p_c^o + R_c \cdot P_{Ai}^c \text{ then } P_{Ai}^c = R_c^{-1}\cdot\left(P_{Ai}^o - p_c^o\right) \quad (13)$$

Also,

$$P_{Ai}^c = P_{B_{i3}}^c + p_{Ai}^b \quad (14)$$

Where $p_{Ai}^b = \left[x_{Ai}^b, y_{Ai}^b, z_{Ai}^b\right]^T$

Considering that R_c is orthogonal matrix and $R_c^{-1} = R_c^{\,T}$, we get

$$P_{Ai}^b = R_c^{\,T}\cdot\left(P_{Ai}^o - p_c^o\right) - P_{B_{i3}}^c \quad (15)$$

In $B_{i3}-xyz$, project the standing leg in plane xy and we get Fig. 6.

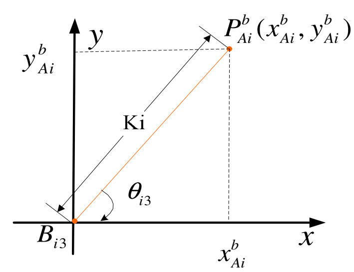

Fig. 6. Projection in plane xy of $B_{i3}-xyz$

According to the geometrical relationship, we get:

$$\theta_{i3} = \arctan(y_{Ai}^b / x_{Ai}^b) \quad (16)$$

And

$$P_{Di}^b = \left[L_1\cdot\cos\left(\theta_{i3}\right), L_1\cdot\sin\left(\theta_{i3}\right), 0\right]^T \quad (17)$$

As shown in Fig. 6, according to the geometrical relationship of the points D_i and A_i , with links L_2 , L_3 and broken lines, we obtain that:

$$\begin{aligned} &\theta_1 = \alpha \pm \pi && \alpha = \arccos\left[\frac{L_2^2 + L_3^2 - L^2}{2L_2L_3}\right] \\ &\theta_2 = \beta \mp \arcsin\left(\frac{P_{Ai}^b - P_{Di}^b}{L}\right) && \beta = \arccos\left[\frac{L^2 + L_2^2 - L_3^2}{2L_2L}\right] \end{aligned} \quad (18)$$

Where

$$L = \sqrt{\left(x_{Ai}^b - x_{Di}^b\right)^2 + \left(y_{Ai}^b - y_{Di}^b\right)^2 + \left(z_{Ai}^b - z_{Di}^b\right)^2} \quad (19)$$

As there may be two groups of solutions for each joint. So, it is important to choose an appropriate group of solution. The trajectory of inverse kinematics is determined by the initial solution, so it is required that each of the inverse solution should be of the same cluster as the initial solution for the stable movements. Our guideline is that we minimize the increment to guarantee the inverse solution each time is in the same cluster with the initial one.

V. EXPERIMENTS

A. Simulation of Maximum Turn Angle

The purpose of simulation is to obtain the approximation of the maximum $\gamma_{\max}$ that the robot can rotate by one step, furthermore to provide some reference for the selection of initial joints angles and rotation angle of turn gaits. As we mentioned before, $\gamma_{\max}$ is related with initial joint angles and the body configuration. Now, the body configuration has been determined, thereby $\gamma_{\max}$ is only related with initial joint angles. However, it is hard to get the analytical expression of them. Our method is as following:

First, we appoint that the height of robot is fixed during turn and the initial angle of joints θ_{i3} is 0 according to the turn gaits above. Then, $\gamma_{\max}$ will be only determined by joints θ_{i1}, θ_{i2}. We select 100 values from the ranges of θ_{i1} θ_{i2} respectively, thus we get 10000 data sets. The process of solving $\gamma_{\max}$ corresponding to each group of initial joints angles is shown as Fig. 7.

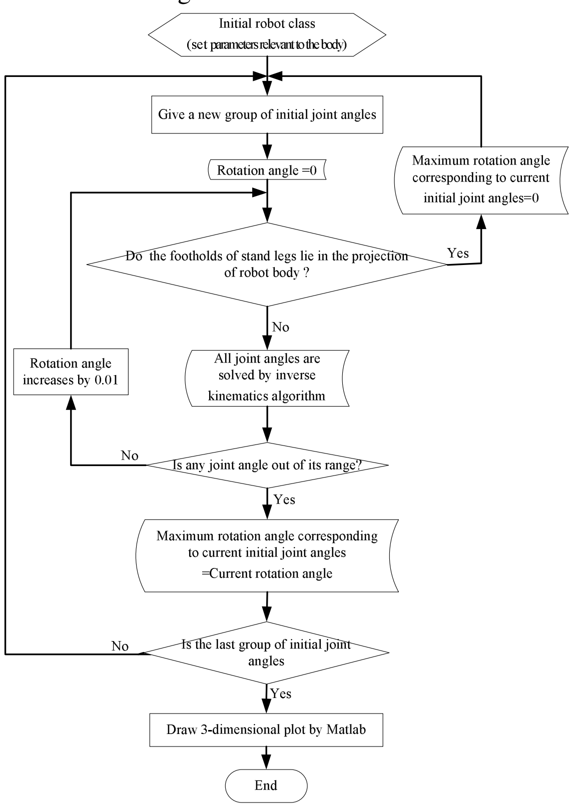

Fig. 7. Flowchart of solving $\gamma_{\max}$

We program under the software platform of VC++6.0 to realize gait planning and kinematics control. Then we draw three-dimensional plot using software platform of MATLAB 7.0(as Fig. 8. shows).

We can see that there have three peak values of $\gamma_{\max}$ as following:

$$\theta_{i1} = 1.53, \theta_{i2} = -0.20, r_{\max} = 0.51$$
$$\theta_{i1} = -1.65, \theta_{i2} = 2.09, r_{\max} = 0.52$$
$$\theta_{i1} = -1.65, \theta_{i2} = -0.34, r_{\max} = 0.52$$

When the first two data sets are given to initial angles, the stand legs of robot can not touch the ground, so they are unusable. We try to select initial angles close to $\theta_{i1} = -1.65, \theta_{i2} = -0.34$ so as to obtain maximum rotation angle.

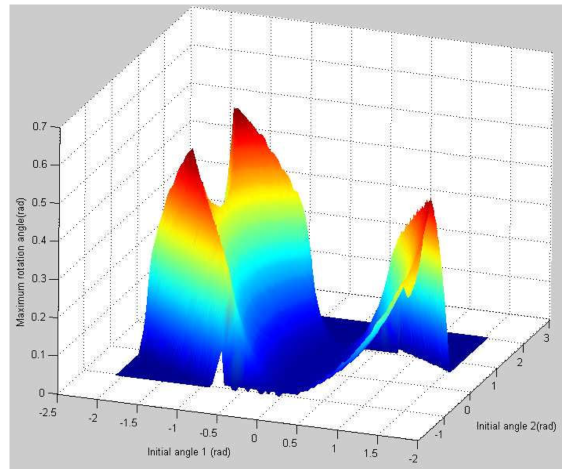

Fig. 8. The three-dimensional plot drawn by Matlab

B. A Simple Obstacle Avoidance Experiment

Fig. 9 shows the human-computer interface with PC. PC sends the reference joint angles to the board by RS232 to control the robot motion and receives the data from board in real time to monitor the state of Dynamixel RX-28.Because we have not assembled sensors, it is impossible to realize real obstacle avoidance. For verifying feasibility of the turn gaits and demonstrating their function, we assume that the sensors data is available.

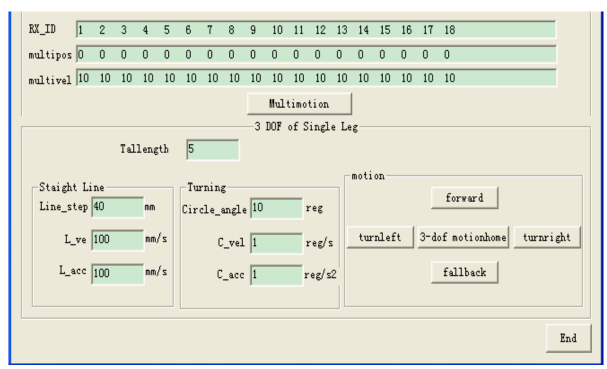

Fig. 9. Human-computer interface with PC

Fig .10 shows the obstacle avoidance experiment process. The hexapod robot goes forward in a straight line with tripod gaits during (a)-(c). Then the sensors data indicate that the distance between robot and obstacle is equal to or smaller than safe distance, so the robot stops to prepare for avoiding obstacle as (d). (e)-(g) shows the robot select to turn left around the obstacle. During turning, PC read the sensors data all the time to judge that the obstacle is avoided. After the robot makes sure that the obstacle is not in the way, it prepare for going forward as (h) shows. At last, the robot continue to

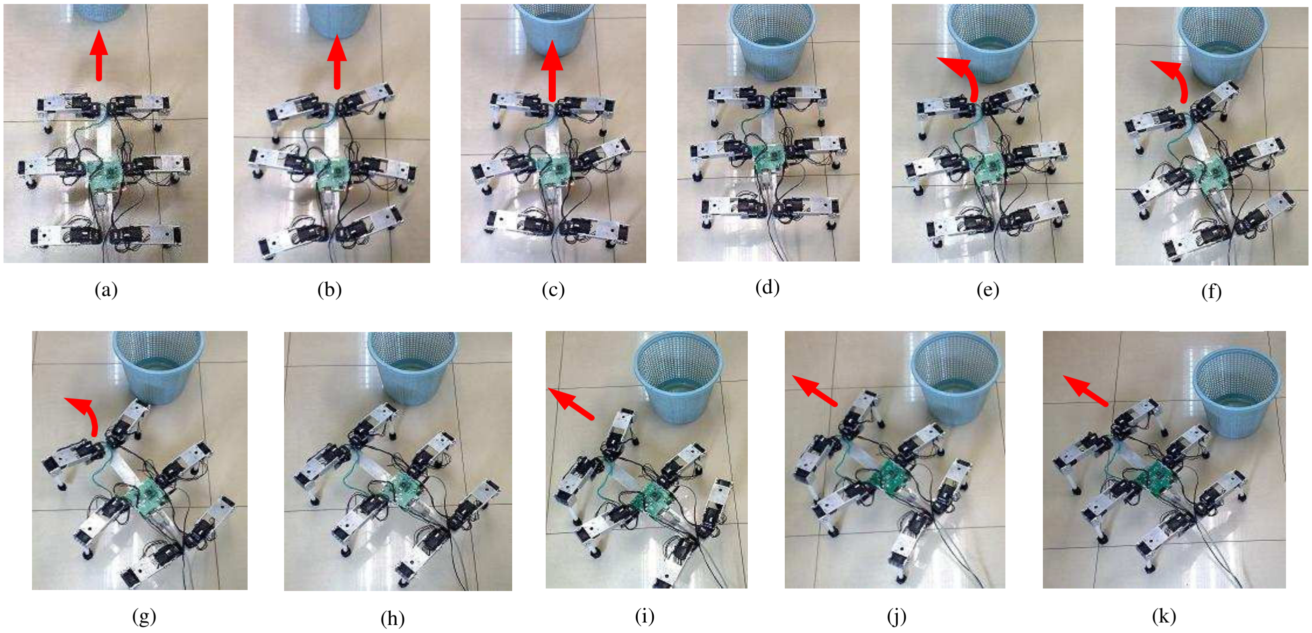

Fig. 10. Snapshots of obstacle avoidance experiment

go forward as (i)-(k) shows.

The experiment results show that the turn gaits are feasible and the robot can walk rapidly and steadily.

VI. CONCLUSION

This paper has discussed the construction, gait planning and kinematics control of a real hexapod robot. The turn gaits are proved having static stability and the footholds of support legs are not out of the reachable cell that we defined. Then, the experiment results show that the turn gaits have static stability and the inverse kinematics analysis of entire hexapod is correct. Future work in this area will involve research on the relationship between maximum rotation angle and the robot configuration, development of free gaits that are easy to realize.

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