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Dynamics and Planning for Stable Motion of a Hexapod Robot

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***Abstract*— A dynamics model for a hexapod robot is developed in this paper which avoids extra calculations to yield a compact set of dynamic equations. To this end, foot interaction with the ground is modeled based on a compliant model, and a force distribution method is implemented in finding required friction forces. This leads to minimum slippage possibility and energy consumption. Furthermore, a scheme is proposed to generate an appropriate rhythmic gait and the ZMP criterion is successively used for checking the robot stability during its planned motion. In conclusion, obtained results are presented for robot stable motion based on the designated gait along a straight path.**

I. INTRODUCTION

LEGGED vehicles offer superior mobility over natural terrain against traditional mobile platforms. These robot's structure allows locomotion on inaccessible terrain, which does not need a continuous support surface. However, motion of these robots on natural terrain presents a set of complicated problems (e.g. foot placement, obstacle avoidance, general vehicle stability and the slippage possibility) that must be taken into account both in mechanical structural design and development of control strategies.

Considerable research studies have been done about the dynamic of multi-legged robot. Barreto et al. modeled a hexapod robot by using Denavit-Hartenberg method to drive kinematic equations, and free body diagram based on the dynamic equations of isolated rigid bodies, [1]. Silva et al. have presented a planar hexapod robot modeling procedure, [2]. Explicit dynamics method has been proposed based on Lagrange approach to derive dynamics equation of a robotic system for reducing required calculations, and studies the kinematics and dynamics of the robot, [3].

Multi-legged robots always have some constraint equations that are given by located foot tips on ground. Constraint forces that implemented on foot tips could be considered in various aspects. For instance, these forces can be considered as desirable forces which will act on foots, to introduce the major gait parameters. These forces determine minimum possible values in a trajectory planning for locomotion which avoid foot slippage along the ground, and this method leads to minimum battery usage. This approach in multi-legged robot is same as grasping problem which was studied mainly by Buss, [6], because of under-actuator system and nonlinearity in equality constraints. Chen et al. proposed a method that transforms the friction constraints from the nonlinear inequalities into a combination of linear equalities and linear inequalities, for optimal force distribution of the quadruped robot's legs interaction with the ground, [7]. Also, Santos et al. proposed the method which consists in reducing the maximum foot forces that a legged robot requires in order to support itself by placing the legs strategically around the robot's body via nonlinear optimization methods, [10]-[11]. Another approach considers constraint forces that must be modeled by one of conventional mathematical models to satisfy constraint equations. Again, Silva et al. have modeled a planar hexapod robot and focused on its foot interaction with nonlinear mass-spring model ground. Miller et al. have also analyzed a vertical compliance prosthetic foot via simple mass-spring model, [12].

Gait planning is another major issue in multi-legged robotic systems. Pratihar et al. define "gait" as "a sequence of leg motions coordinated with a sequence of body motions for transporting the body of the legged robot from one place to another", [13]. There are two types of gaits, namely rhythmic gait and non-rhythmic gait; in rhythmic gait the body motion is obtained based on the frequency sequence of legs motion which we name it a cycle, and each cycle consist of some different motions, which we name it a step. Erden et al. developed suitable methods for collision free path generation of a mobile robot in the presence of static as well as moving obstacles separately which concluded about free gait generation and reinforcement learning, [14]. Pratihar used fuzzy-GA method to generate optimal path and gait simultaneously. Porta proposed a reactive controller that generates free gait to follow an arbitrary trajectory, [15].

Dynamic stability margin (DSM) is described when robot's stability satisfies one of dynamic stable criterions. Several researchers have tried to propose a suitable criterion for dynamic tip-over stability evaluation, including the Zero Moment Point (ZMP) [16], Energy-Based measure [17], the force-angle margin [18], the Moment-Height Stability measure (MHS) [19]-[21]. In trajectory-based control, one major criterion in the generation of joint trajectories is the position of the Zero-Moment Point (ZMP). ZMP is defined as the point on the ground where the net moment of the

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inertial forces and the gravity forces has no component along the horizontal axes. For dynamic stable locomotion, the necessary and sufficient condition is to have the ZMP in the support polygon at all stages of the locomotion gait.

The paper is organized as follows. Section II defines a few definitions that are used in the following sections; and section III describes a hexapod robot and derives the dynamic equation of system. Section IV defines the algorithm of path gait generation and studies the modified force distribution method which is mentioned in advance; and section V presents the stability analysis of robot. In section VI obtained simulation results of the robot motion along a given path will be presented. Section VII summarizes the work and presents the conclusions.

II. BASIC DEFINITIONS

- *Leg's workspace* defines a reachable space of a foot's tip relative to the leg's hip joint
- *Leg's reaching area (LRA)* defines an area on the ground that is flat and suitable for foothold selection. Its radius is represented by *RLRA*.
- *Posterior* extreme *point (PEP)* and *anterior extreme point (AEP)* are defined as the rear and front furthest point of LRA along the defined path respectively.
- *Stroke length (SKL)* is defined as a distance that the leg's tip can move along the main body longitude.
- *Stride length (SDL)* is defined as the distance traveled by the body center mass.
- *Extreme Stride length (ESDL)* is defined as the maximum possible length of SDL.
- *Cycle time* is defined as the time for a complete cycle of leg locomotion.

Based on the above concepts, this paper aims to discuss the following points: first, developing a computer routine which can model a system with high degrees of freedom (HDOF) including holonomic or non-holonomic constraints, and determine the contact forces as the minimum slippage and energy consumption possibility. Consequently, System dynamic equation could be derived by either Euler method or Lagrange method. For system with HDOF, clearly, Euler method isn't a conventional method to derive system's dynamic equation; in the other hand. Basically, for the HDOF system isn't suitable directly used Lagrange algorithm because of extensive calculations. Therefore, this paper uses explicit dynamic equation that lead to outcome the system dynamic equation's term as the practical terms directly.

Second, the desired actuators and friction forces are determined such as lead to minimum energy usage and slippage possibility. Finally, proposed a scheme to develop both the path and the gait for locomotion, and checks the robot stability via ZMP criterion.

III. DYNAMICS MODELING

A. KINEMATICS MODEL

The considered hexapod robot has 24 DOF, which includes 6 DOF for the main body, and 6 legs each with 3 DOF as shown in Fig. 1; i.e. each leg has two ankles and a hip joint. Various options exist to install coordinate frames, e.g. D-H parameters maybe used and their instruction to define the coordinate frames. The Euler angels are used with Z-X-Z convention to represent the spatial orientation of the main body frame.

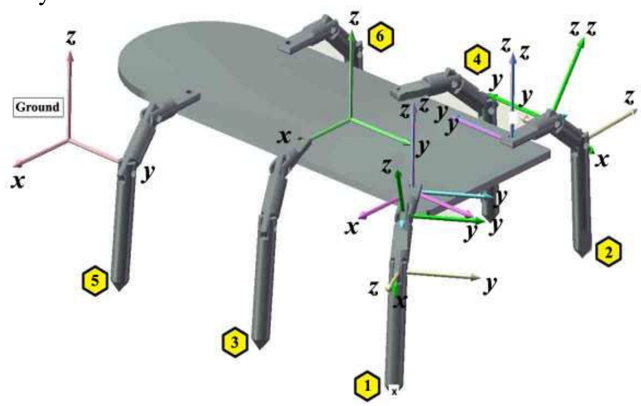

Fig. 1: The hexapod robot with 24 DOF and each leg has two ankles and one hip joint. All installed coordinate frames are shown.

B. DYNAMICS MODEL

Using free body diagram and Newton-Euler method is not suitable for HDOF system; Lagrange method dissembled the internal reaction forces but its sequent derivations are the crucial problem in this method. To overcome this problem explicit dynamic method is proposed that is a customized Lagrange method for robotic system and inherently simplified the dynamic equation terms and implements the minimum derivation so it obviously reduces the calculations. The keys of this method are the analytical Jacobian, linear Jacobian and the definite-positive property of mass matrix that help to subdivide the derivations and avoid calculating large terms. This algorithm derives the robot dynamic equation as

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=J_{\tau}(q,\dot{q})\tau+J_{f}(q,\dot{q})f+d \quad (1)$$

$$M(q)=J_L^T\bar{M}J_L+J_A^TR^T\bar{I}RJ_A \quad (2)$$

$$C_{ij}=\frac{1}{2}\dot{M}_{ij}+C_{C_{ij}}-\frac{1}{2}\sum_{k=1}^{n}\left(\frac{\partial M_{ik}}{\partial \dot{q}_j}-\frac{\partial M_{jk}}{\partial \dot{q}_i}\right)\dot{q}_k \quad (3)$$

where $q\in\Re^{24\times 1}$ vector represents of state variables, $M(q)\in\Re^{24}$ is the inertia matrix, $C(q,\dot{q})\dot{q}\in\Re^{24\times 24}$ represents centripetal and carioles terms, $C_C(q,\dot{q})\dot{q}\in\Re^{24\times 24}$ represents damping terms, $J_L(q,\dot{q})\in\Re^{57\times 24}$ is the Jacobian of mass center velocity, $J_A(q,\dot{q})\in\Re^{57\times 24}$ is the analytical Jacobian.

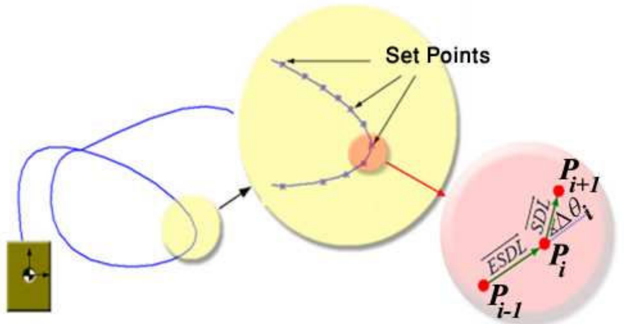

Fig. 2: Locomotion path defined by set of points and body planar orientation is defined by each two points.

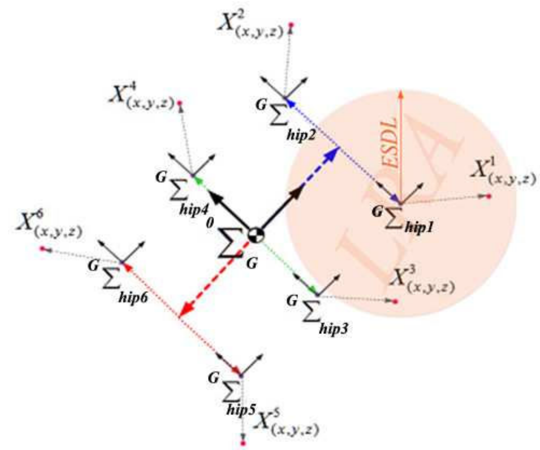

Fig. 3: Foothold locations are defined in the main body coordinate frame.

Compliance method as known method used for modeling the surface stiffness. Simple spring-damper model is used for surface normal reaction forces.

Coulomb friction model is used for the ground friction forces and studies friction forces in two models:

I) *Static friction model,* is used when it is required that friction force remains within the friction cone (i.e. $\sqrt{f_x^2+f_y^2} \le \mu_s f_z$, where μ_s is the static friction coefficient).

II) *Dynamic friction model*, is used when it is required that a friction force keeps the foot static, is be outside of the friction cone (i.e. $\sqrt{f_x^2+f_y^2} = \mu_k f_z$ $\sqrt{f_x^2+f_y^2} \le \mu_s f_z$, where μ_k is the kinetic friction coefficient).

IV. Gait Planning

Desired path is defined by a set of points $P \subseteq \{p_i \mid p_i \in \Re^{(x,y,z)}\}$ which the body CG must trace as shown in Fig. 2. To generate a rhythmic gait a scheme is proposed here as: *Step 1*, determining the number of stance legs; by choosing the number of stance legs all possible combinations of maintaining legs should be determined. *Step 2*, determining the number of steps need for completing operation, we assume that the number of gait steps in a simple locomotion are usually in an order of 2, 3 or 4 steps. *Step 3*, determining the initial conditions for continuous and smooth motion. *Step 4*, swing phase motion, describing the tip motion by a function; like as a piecewise function, a polynomial function, a trigonometric function, and so on, [22].

As shown in Fig. 3 to demonstrate the lactation of a foothold in each step used a simple vector (X40) that measure in the main body coordinate.

By choosing a proper gait and trajectory, dynamic equations of robot yielded as set of linear equations, and it contains unknown actuators' torque and the ground reaction forces as follow

$$F\left(kT_s\right)_{24\times1} = G\left(kT_s\right)_{24\times36}\cdot\{\tau, f\}^T_{36\times1} \qquad (4)$$

where T_s is the time step, $\tau \in \Gamma \subset \Re^n$ is the vector of actuators and $f \in \Lambda \subset \Re^m$ is the vector of contact forces. By using the force distribution method could solve this problem; this method converts friction nonlinear-inequality constraints to a quadratic cost function subjected to linear- inequality ones as

$$\left|f_{x_i}\right| \le \frac{1}{\sqrt{1+\lambda^2}}\cdot\mu_s f_{z_i} \qquad \left|f_{y_i}\right| \le \frac{\lambda}{\sqrt{1+\lambda^2}}\cdot\mu_s f_{z_i} \qquad (5)$$

where λ is a coefficient that noticed as "tangential coefficient" which indicates a portion of f_x and f_y in a tangential force. This method minimizes leg slipping possibility by finding an optimal solution such that minimizes a cost function like

$$J = \frac{1}{2}X^T P X \qquad X = \{\tau, f\}^T \qquad (6)$$

where P is the semi-definite-positive matrix introduced a cost function gain respected to constrains $AX < B$ which contains actuator saturation and friction forces constraints. Finally, a Quadratic programming method used for solving the optimization problem, [23].

V. Dynamic Stable Locomotion

To check dynamic stable of locomotion ZMP criterion is used as Equation 7.

$$x_{ZMP} = \frac{\sum_{i=1}^{n}\left[m_i\left(\left(\ddot{z}_i+g\right)\cdot x_i - \ddot{x}_i\cdot z_i\right) - I_i^{yy}\cdot\alpha_i^y\right]}{\sum_{i=1}^{n}\left[m_i\left(\ddot{z}_i+g\right)\right]}$$

$$y_{ZMP} = \frac{\sum_{i=1}^{n}\left[m_i\left(\left(\ddot{z}_i+g\right)\cdot y_i - \ddot{y}_i\cdot z_i\right) + I_i^{xx}\cdot\alpha_i^x\right]}{\sum_{i=1}^{n}\left[m_i\left(\ddot{z}_i+g\right)\right]} \qquad (7)$$

where x_i, y_i, z_i represent the vector of i^{th} mass center gravity to an original coordinate; $\ddot{x}_i, \ddot{y}_i, \ddot{z}_i$ are the linear acceleration of i^{th} mass; α_i^x, α_i^y represent angular accelerations of i^{th} mass and m_i, I_i^{xx}, I_i^{yy} are the mass and inertia coefficient of i^{th} mass. It's clear that if ZMP located in far from each side of support polygon then locomotion is more stable. Therefore, to measure the robot stability we define a new parameter that measures the neighbourly of ZMP to support polygon. Fig. 4 shows the support polygon, the i^{th} vertex is assigned as P_i and vertexes are countered in clockwise rotation.

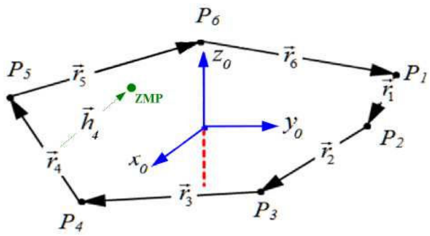

Fig. 4: Support polygon which constructed by stance legs.

Each side is represented by a vector such as

$$\vec{r}_{i-1} = P_i - P_{i-1} \qquad 1 \le i \le n$$
$$\vec{r}_n = P_0 - P_n \tag{8}$$

By definition, the perpendicular vector to the side that crosses the ZMP is given by

$$\vec{h}_i = \left(\overline{ZMP} - \vec{P}_i\right)\cdot\left(1 - \|\vec{r}_i\|\right) \qquad 0 \le i \le n \tag{9}$$

Definition 1. Polygon side stability (PSS) is defined as

$$S_i = \left(\|\vec{g}\| \times \vec{h}_{G_i}\right)\cdot\|\vec{r}_i\| \tag{10}$$

where this equation shows that S_i is the distance of the ZMP to a polygon side so a side named *stable* side if $S_i > 0$; and *unstable side* if $S_i < 0$ And *critical side* if $S_i = 0$

Definition 2. Minimum polygon side stabilities named locomotion stability (LS_{ZMP}):

The robot is stable if LS_{ZMP} becomes positive during locomotion; if it becomes zero then the robot is in critical stability locomotion, accordingly if it becomes negative the robot is unstable.

Proposition. A system is in dynamic instability condition if there exists at least one unstable side in the support polygon.

VI. Simulation Results

The hexapod robot properties are shown in TABEL 1, where the legs tip follow the rectangular path in swing phase and Fig. 5 shows the implemented gait on the robot, as shown in this figure robot stays for 0.1 second, then starts its first step by moving with two swing legs in each steps and after three step will arrive to the final destination point (30cm far away from started point on the straight path).

We planned body states and joints' space used cubic polynomials with issued that the initial velocities and accelerations are zero, Fig. 6 shows the desired calculated states of body.

Fig. 7 shows the desired actuator torques and friction forces of the 1^th leg. Fig. 7 -a shows that solving trajectory via force distribution method can successfully planning desired actuator torques and bounds them into actuators' work spaces, also desired reaction forces are bounded admissible too as shown in Fig. 7 -b. The 1^th leg is in swing phase in the second step during time 0.1sec- 0.6sec, i.e. Fig. 5, so in this period of time all reaction forces must be zero and they are successfully yielded zero in planning process. By looking exactly to Fig. 7 -a we find that an unexpected actuator torque value is occurred in 1.3 sec when for the actuator number 2 it is $\tau_2 = -3.7\ \mathrm{N/m}$ and smaller than the minimum actuator saturation. As results, must notice that the force distribution method that is used here is based on a numerical method in the way of minimizing a cost function so calculated torques and reaction forces are the best fit-able values as them possible. Fig. 8 shows the LS_{ZMP} is positive in during of locomotion and the robot is stable. It shows that t=0.672s is a critical stable locomotion time for robot, i. e. It is predictable because Fig. 6 shows that one of the maximum longitude accelerations of body is occur in t=0.7s also in this time as demonstrated in Fig. 5 legs (1, 3, 4, 6) are stance legs so the front polygon side which is constructed by legs number 2 & 3 are located near body CG.

TABEL 1.
Properties of simulation

Body			
Shape	*Rectangle*		
Mass	*7 kg*		
Dimension	*60×40×30 cm^3*		
Leg			
Shape	*Rectangle*	*Rectangle*	*Rectangle*
Mass	*178.2 gr*	*217.3 gr*	*217.3 gr*
Dimension	*10×10×10 cm^3*	*15×10×10 cm^3*	*15×10×10 cm^3*
Path		Time	
Shape	*Straight*	Step time (*sec*)	$\begin{bmatrix}0.1,0.5,\\0.5,0.5\end{bmatrix}$
Length	*30 cm*	Sample time(*sec*)	*0.032*
SDL	*15 cm*		
Tip height	*2 cm*		
Ground		Force distribution	
μ_s	2.0	λ	$\sqrt{2}/2$
μ_k	0.8	P	$100\times I_{36\times36}$
Actuator saturation			
$\tau^i_{\max}$	3 N/m		
$\tau^i_{\min}$	-3 N/m		

VII. Conclusions

This paper discussed dynamics, and stable gait planning of a hexapod robot. The system was modeled based on Lagrange method. To this end, using cubic polynomials for joint trajectories, a modified force distribution method was used which transformed friction nonlinear constraints to linear constraints in order to use a conventional quadratic programming algorithm; i. e. "quadprog" function had used in MATLAB. Therefore, friction forces and actuator torques guarantee minimum slippage possibility and energy consumption, and compliant method had been used to verify the results. Next, a scheme was proposed to define a locomotion path, and generate a rhythmic stable gait and the suitable foothold selection to follow the given path. Also, the ZMP criterion was used to check the robot stability during its locomotion. Finally, a simple planning procedure was used to move on a given straight path with a stable rhythmic gait via a force distribution method and the obtained results were discussed.

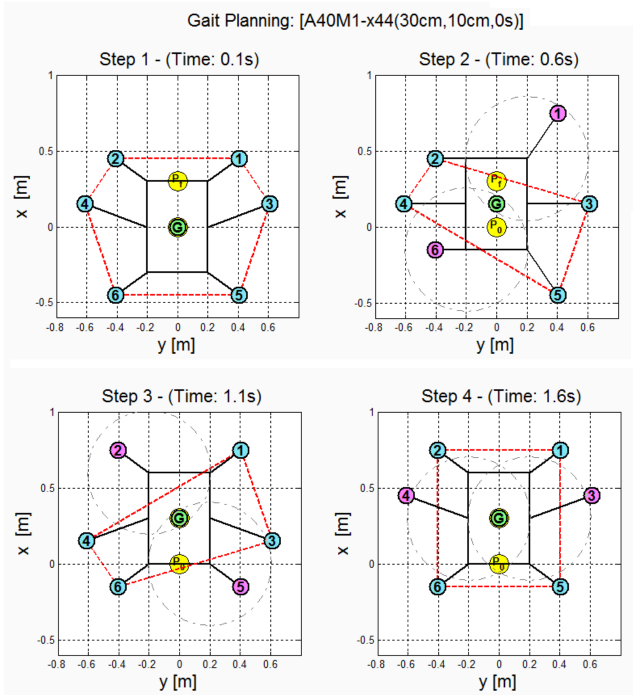

Fig. 5: Planned gait for robot locomotion; swing legs are colored with pink and stance legs are colored with blue, P_0 is started point and P_f are destination point, G is the mass body center gravity of main body.

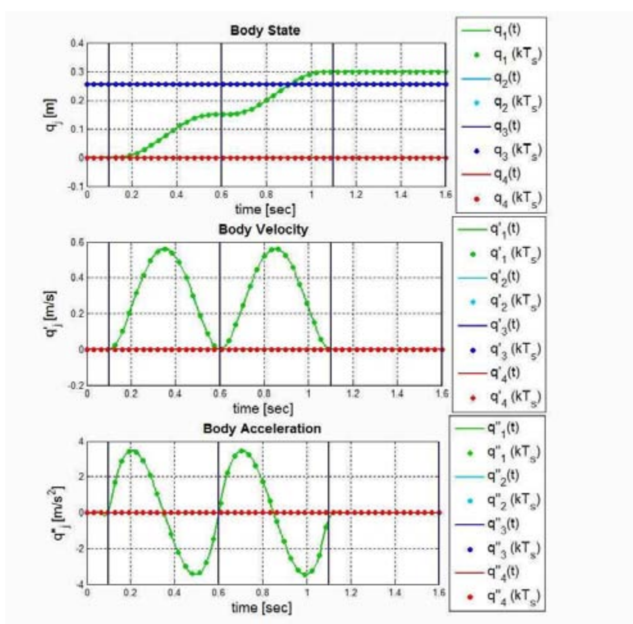

Fig. 6: Body desired states, using cubic polynomials with zeros initial velocities and accelerations.

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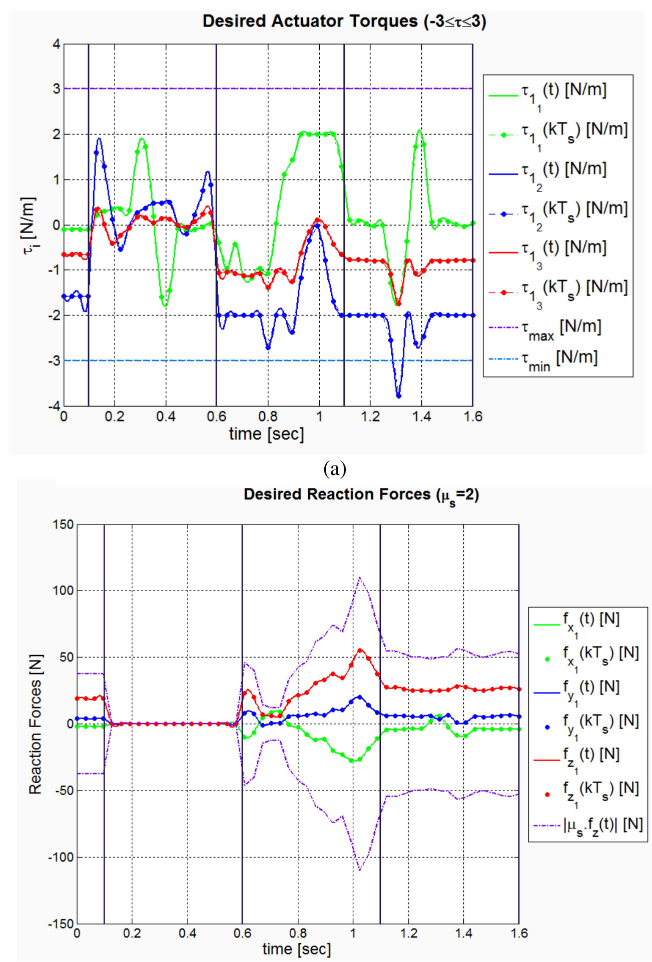

(b)

Fig. 7: (a) Desired actuator torques; (b) Desired reaction forces; sample time is 0.032 second and actuators are bounded in -3 and 3 N/m and static friction coefficient is 2.

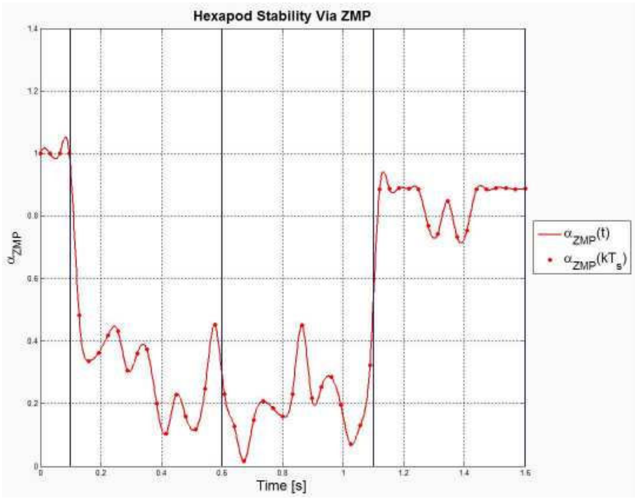

Fig. 8: Stability analysis using ZMP criterions

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