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Analysis of Algorithms

## Master Theorem for Divide and Conquer

General Form of Recurrence Relation

$$T(n) = \alpha T\left(\frac{n}{\beta}\right) + f(n)$$

$$\alpha \geq 1 \quad \beta > 1 \quad f(n) = \theta(n^k \log^p n)$$

The Recurrence Equation provides all of these elements including:

- $\log_b^a$
- $k$

Based on these two values of  $\log_b^a$  and  $k$  there are 3 Cases:

Case 1: if  $\log_b^a > k$  then  $\theta(n^{\log_b^a})$

Case 2: if  $\log_b^a = k$  then:

- if  $p > -1 \cong \theta(n^k \log^{p+1} n)$
- if  $p = -1 \cong \theta(n^k \log \log n)$
- if  $p < -1 \cong \theta(n^k)$

Case 3: if  $\log_b^a < k$  then:

- if  $p \geq 0 \cong \theta(n^k \log^p n)$
- if  $p = -1 \cong \theta(n^k \log \log n)$
- if  $p < 0 \cong O(n^k)$