Wednesday: September 2, 2020 L4
Math 360

HW: pg. 68, #'s 110, 111
pg. 72, #'s 129, 134

Review

- $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, $\quad (P(A \mid B) = P(A) \quad \textit{if} \quad A \textit{ and } B \textit{ are independent})$

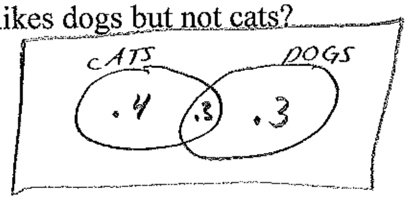
- $P(A \cap B) = P(A) \cdot P(B \mid A)$
  $\qquad\quad = P(B) \cdot P(A \mid B)$ $\quad (P(A \cap B) = P(A) \cdot P(B)$ if A and B are independent)

- $P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (= P(A) + P(B)$ A and B mutually exclusive)

- $P(A) = 1 - P(\overline{A})$

Examples

1. In a survey, 70% likes cats and 60% likes dogs, and everyone in the survey likes at least one of the animals. What is the probability that a randomly chosen person in this survey likes dogs but not cats?

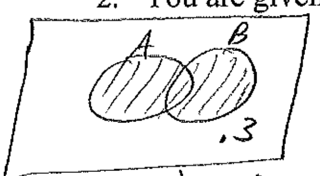


$P(C \cup D) = P(C) + P(D) - P(C \cap D)$

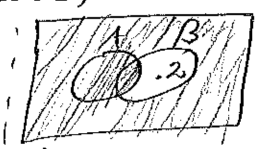
$1 = .7 + .6 - P(C \cap D)$

$P(C \cap D) = .3$ $\boxed{\text{Answer: } P(D \cap \overline{C}) = .3}$

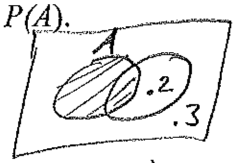
2. You are given $P(A \cup B) = 0.7$ and $(A \cup \overline{B}) = .8$. Determine $P(A)$.



$P(A \cup B) = .7$ so $P(\overline{A \cup B}) = .3$



$(A \cup \overline{B}) = .8$ $\Rightarrow$



$P(A) = .5$

OR

$P((A \cup B) \cap (A \cup \overline{B})) = P(A \cup B) + P(A \cup \overline{B}) - P(A \cup B \cup A \cup \overline{B})$

$P(A) = .7 + .8 - 1$

$P(A) = .5$

3. $P(A) = \frac{1}{6}$, $P(B) = \frac{5}{12}$ and $P(A \mid B) + P(B \mid A) = \frac{7}{10}$. Find $P(A \cap B)$.

$P(A \mid B) + P(B \mid A) = .7$

$\dfrac{P(A \cap B)}{P(B)} + \dfrac{P(B \cap A)}{P(A)} = .7$

$P(A \cap B)\left(\frac{12}{5} + 6\right) = .7 \quad \Rightarrow P(A \cap B) = 1/12$

4. Urn A contains 2 white marbles and 2 black marbles. Urn B contains 2 white marbles and 3 black marbles. An urn is chosen at random and a ball is randomly selected from the urn. Find the probability that a black marble is chosen.

2W 2B urn A

2W 3B URN B

$P(B) = \frac{1}{2}\left(\frac{2}{4}\right) + \frac{1}{2}\left(\frac{3}{5}\right)$

$= \frac{1}{4} + \frac{3}{10} = \frac{5+6}{20} = \frac{11}{20} = .55$

Example:

Of the voters in a city, 40% are Republicans and 60% are Democrats. Of the Republicans, 35% favor the bond issue while 70% of the Democrats favor the bond issue. What percent of the city favor the bond issue?

$$P(F) = \underbrace{P(R)\cdot P(F|R)}_{\text{Republicans}} + P(D)\cdot P(F|D)$$
$$= (.40)(.35) + (.60)(.70)$$
$$= .14 + .42$$
$$= .56$$

Example 2.19:

Two applicants are randomly selected for a job from among five who have applied for a job. Find the probability that exactly one of the two best applicants is selected for the job.

2 are the Best
3 are Not.

$$P(\text{exactly one of the two best}) = \frac{\overset{\text{best}}{\binom{2}{1}}\overset{\text{Rest}}{\binom{3}{1}}}{\binom{5}{2}} = \frac{{}_2C_1 \cdot {}_3C_1}{{}_5C_2} = \frac{2\cdot 3}{10} = .60$$

Example 2.20:

It is known that a patient with a disease will respond to treatment with probability equal to 0.9 . If three (independent) patients with the disease are treated, Find the probability that at least one will respond.

Hard way

P(one will Respond)
+P(2 will Respond)
+ P(3 will Respond)

easy way – Find the opposite

i.e. $P(\text{None will Respond}) = (.1)^3$

$$P(\text{at least one}) = 1 - P(\text{none})$$
$$= 1 - (.1)^3$$
$$= .999$$

Section 2.9 - 2.10 Bays Rule

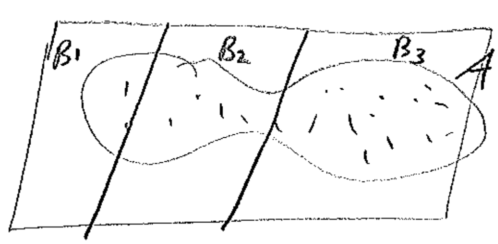
Def. 2.11

A collection of sets $\{B_1, B_2, \ldots, B_k\}$ is said to be *a partition* of *S*, if

i. $S = B_1 \cup B_2 \cup \cdots \cup B_k$

ii. $B_i \cap B_j = \phi$ for $i \neq j$ (ie, B_i's are mutually exclusive)

Then any set *A* can be written as: $A = (A \cap B_1) \cup (A \cap B_2) \cup \cdots \cup (A \cap B_k)$



Theorem 2.8

Assume that $\{B_1, B_2, \ldots, B_k\}$ is a partition of S, such that $P(B_i) > 0$, for $i = 1, 2, \ldots, k$. Then for any event A

$$P(A) = \sum_{i=1}^{k} P(A \mid B_i) \cdot P(B_i)$$

Proof:

$$\begin{aligned} P(A) &= P[(A \cap B_1) \cup (A \cap B_2) \cup \cdots \cup (A \cap B_k)] \\ &= P\left(\cup_{i=1}^{k} (A \cap B_i)\right) \\ &= \sum_{i=i}^{k} P(A \cap B_i) \\ &= \sum_{i=1}^{k} P(A \mid B_i) \cdot P(B_i) \end{aligned}$$

since $P(A/B_i) = \dfrac{P(A \cap B_i)}{P(B_i)}$

Theorem 2.8 **Bayes Rule**

Assume that $\{B_1, B_2, \ldots, B_k\}$ is a partition of S, such that $P(B_i) > 0$, for $i = 1, 2, \ldots, k$. Then

$$P(B_j \mid A) = \frac{P(B_j \cap A)}{P(A)} = \frac{P(A \mid B_j) \cdot P(B_j)}{\sum_{i=1}^{k} P(A \mid B_i) \cdot P(B_i)}$$

$P(B_j/A) = \dfrac{P(B_j \cap A)}{P(A)}$

These two are equal

$P(A/B_j) = \dfrac{P(A \cap B_j)}{P(B_j)}$

$P(B_j/A)P(A) = P(A/B_j) \cdot P(B_j)$

$P(B_j/A) = \dfrac{P(A/B_j) \cdot P(B_j)}{P(A)}$

Examples:

1. Suppose you have three bowls with marbles in them. The first bowl has 2 red and 4 white, the second bowl has 1 red and 2 white, and the third bowl has 5 red and 4 white. In this game, you first select a bowl at random then randomly select a marble from the bowl selected. The probability of selecting bowl is as follow:
   $P(B_1) = \frac{1}{3}$, $P(B_2) = \frac{1}{6}$, *and* $P(B_3) = \frac{1}{2}$

   a. Find the probability of selecting a red marble.

Bowl 1: 2R, 4W, $P(B_1) = 1/3$

B2: 1R, 2W, $P(B_2) = 1/6$

B3: 5R, 4W, $P(B_3) = \frac{1}{2}$

$$P(R) = \frac{1}{3}\left(\frac{2}{6}\right) + \frac{1}{6}\left(\frac{1}{3}\right) + \frac{1}{2}\left(\frac{5}{9}\right)$$
$$= \frac{2}{18} + \frac{1}{18} + \frac{5}{18}$$
$$= \frac{8}{18} = \frac{5}{9}$$

   b. Suppose that Fred picked a red marble, find the probability that this marble came from bowl 1.

$$P(B_1/R) = \frac{P(R/B_1)\cdot P(B_1)}{P(R)} = \frac{\left(\frac{2}{6}\right)\left(\frac{1}{3}\right)}{\frac{4}{9}} = \frac{\cancel{2}}{\cancel{6}2}\cdot\frac{1}{\cancel{3}}\cdot\frac{\cancel{9}}{\cancel{4}2}$$
$$= \frac{1}{4}$$

2. Let D be the event that a person has a disease.
   Let E be the event that the test is positive.

   $P(E \mid D) = .95$

   Suppose that $P(E \mid \overline{D}) = .01$. Find the probability that a person has the disease given

   $P(D) = .005$

   that the test is positive, i.e. find $P(D \mid E)$.

$$P(D/E) = \frac{P(E/D)\cdot P(D)}{P(E)}$$
$$= \frac{(.95)(.005)}{.0147}$$
$$= .32313$$

$$P(E) = P(E/D)\cdot P(D) + P(E/\overline{D})\cdot P(\overline{D})$$
$$= (.95)(.005) + (.01)(.995)$$
$$= .0147$$