L8, Review for Test 1

Sets and Venn diagrams

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\quad$ $(= P(A) + P(B)$ A and B mutually exclusive)

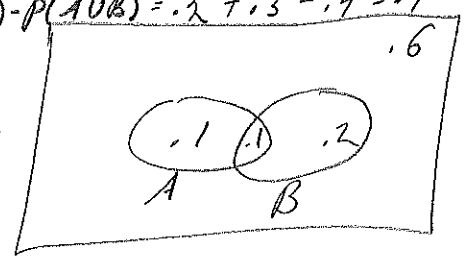
$P(A) = 1 - P(\overline{A})$

Ex. Two events A and B are such that $P(A) = .2$, $P(B) = .3$ and $P(A \cup B) = .4$

i. Find $P(A \cap B)$. $= P(A) + P(B) - P(A \cup B) = .2 + .3 - .4 = .1$

ii. Find $P(A' \cup B) = .9$

iii. Find $P(A' \mid B)$ $= \frac{.2}{.3} = \frac{2}{3}$



Counting Methods and Probability

Ex. Four students are to be randomly selected to fill certain student government positions, from a group of 3 undergraduate and 5 graduate students. Find the probability that exactly 2 undergraduates will be among the four chosen.

$$\frac{\binom{3}{2}\binom{5}{2}}{\binom{8}{4}} = \frac{3 \cdot 10}{70} = \frac{3}{7} \quad (= .429)$$

Ex. From a standard deck of 52 playing cards a hand of 5 cards is dealt. Find the probability of having all five cards being from the same suit (i.e. a flush).

$$\frac{4 \cdot \binom{13}{5}}{\binom{52}{5}} = \frac{4(1287)}{2598960} = .00198$$

Conditional Probability, Independent events and Mutually exclusive events

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)},$$

$$P(A \cap B) = P(B \mid A) \cdot P(A)$$
$$= P(A \mid B) \cdot P(B)$$

Two events are independent if: $P(A \mid B) = P(A)$ $\quad$ or $\quad P(B \mid A) = P(B)$

$P(A \cap B) = P(A) \cdot P(B)$

Two events are mutually exclusive if $(A \cap B) = \phi$

Ex. If A and B are independent events with $P(A) = .5$ and $P(B) = .2$ find $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= .5 + .2 - P(A)P(B)$$
$$= .7 - (.5)(.2)$$
$$= .6$$

Bays Rule

Assume that $\{B_1, B_2, \ldots, B_k\}$ is a partition of *S*, such that $P(B_i) > 0$, for $i = 1, 2, \ldots, k$.

Then

$$P(B_j \mid A) = \frac{P(B_j \cap A)}{P(A)} = \frac{P(A \mid B_j) \cdot P(B_j)}{\sum_{i=1}^{k} P(B_i) \cdot P(A \mid B_i)}$$

Example:

.35 .25 .40

In a certain factory, machines A, B, and C are all producing springs of the same length with defective rates: 2%, 1%, and 3% respectively. Of the total production of springs in the factory, machine A produces 35%, machine B produces 25% and machine C produces 40%.

a) If a spring is chosen at random from a day's production, what is the probability that it is defective? $P(D) = (.02)(.35) + (.01)(.25) + (.03)(.40) = .0215$

b) If a defective spring is selected, what is the probability that it was produced by machine C?

$$P(C/Def) = \frac{P(Def/C) \cdot P(C)}{P(Def)} = \frac{(.03)(.40)}{.0215} = .558$$

Example:

A gambler has in his pocket a fair coin and a two-headed coin. He selects a coin at random and when he flips it, it shows heads. What is the probability that the coin he flipped is the fair coin?

$$P(F/H) = \frac{P(H/F) \cdot P(F)}{P(H)} = \frac{(\frac{1}{2})(\frac{1}{2})}{\frac{3}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

$$P(H) = \underbrace{\tfrac{1}{2} \cdot \tfrac{1}{2}}_{\text{Fair}} + \underbrace{\tfrac{1}{2}(1)}_{\text{2-Headed}} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

Expected Value, mean, Variance

$E[u(y)] = \sum_{y \in S} u(y) \cdot p(y)$, mean $\mu = E(Y)$ Variance $\sigma^2 = E(Y^2) - E(Y)^2$

Example:

Given the pmf : $f(y) = \frac{y}{10}$ *for* $y = 1,2,3,4$. Find the mean and variance.

$$\mu = E(Y) = 1\left(\frac{1}{10}\right) + 2\left(\frac{2}{10}\right) + 3 \cdot \left(\frac{3}{10}\right) + 4 \cdot \frac{4}{10} = \frac{1+4+9+16}{10} = \frac{30}{10} = 3$$

$$E(Y^2) = 1\left(\frac{1}{10}\right) + 2^2\left(\frac{2}{10}\right) + 3^2\left(\frac{3}{10}\right) + 4^2\left(\frac{4}{10}\right) = \frac{1+8+27+64}{10} = \frac{100}{10} = 10$$

$$Var(Y) = \sigma^2 = 10 - 3^2 = 10 - 9 = 1$$

Example:

Let X be a discrete random variable with the distribution shown in the table below: (Since most calculators can calculate the following, work must be show in order to receive credit.)

x	$f(x)$	$x \cdot f(x)$	$x^2 \cdot f(x)$
21	.05	1.05	22.05
22	.20	4.4	96.8
23	.30	6.9	158.7
24	.25	6	144.
25	.15	3.75	93.75
26	.05	1.3	33.8
		23.4	549.1

a) Find E(X). $= 23.4$

b) Find Var(X) $= 549.1 - (23.4)^2 = 1.54$

c) What is the standard deviation of X?

$SD = \sqrt{1.54} = 1.24$

Binomial Probability Distribution

$$p(y) = f(y) = \binom{n}{y} p^y (1-p)^{n-y} \qquad \text{mean} \quad \mu = np \qquad \text{Variance} \quad \sigma^2 = np(1-p)$$

Example:

In a lab experiment involving inorganic syntheses of molecular precursors to organometallic ceramics, the final step of a five-step reaction involves the formation of a metal to metal bond. The probability of such a bond forming is $p = 0.20$. Let X equal the number of successful reactions out of $n = 25$ such experiments.

a. Find the probability that X is at most 4. $P(X \le 4) = .421$ where $n = 25, p = .2$ Table

b. Find the probability that X is at least 5. $P(X \ge 5) = 1 - P(X \le 4) = 1 - .421 = .579$

c. Find the probability that X is equal to 6. Give the mean, variance and standard deviation of X. $P(X=6) = \binom{25}{6}(.2)^6(.8)^{19} = .163$

$\mu = E(X) = np = 25(.2) = 5$

$\sigma^2 = npq = 25(.2)(.8) = 4$

$\sigma = SD = \sqrt{4} = 2$

Geometric Probability Distribution (How long will it take to succeed?)

$p(y) = f(y) = p\,(1-p)^{y-1}$ mean $\mu = \frac{1}{p}$ Variance $\sigma^2 = \frac{1-p}{p^2}$

Example:

The probability of it raining on any day in June is 0.2. Find the expected number of days before it rains in June. $\mu = E(Y) = \frac{1}{p} = \frac{1}{.2} = 5$ days

Example:

(example 3.11 & 3.12) Suppose that the probability of an engine malfunction during any one-hour period is $p = 0.02$,

a) Find the probability that a given engine will survive three hours. (ex. 3.11 ask for two hours.)

$$P(y \geq 4) = 1 - p(y \leq 3)$$
$$= 1 - (.02) - (.98)(.02) - (.98)^2(.02)$$
$$= .9413$$

b) Find the mean and standard deviation.

$$\mu = \frac{1}{p} = \frac{1}{.02} = 50$$
$$\sigma^2 = \frac{1-p}{p^2} = \frac{.98}{(.02)^2} = 2450$$
$$\sigma = \sqrt{2450} = 49.497$$